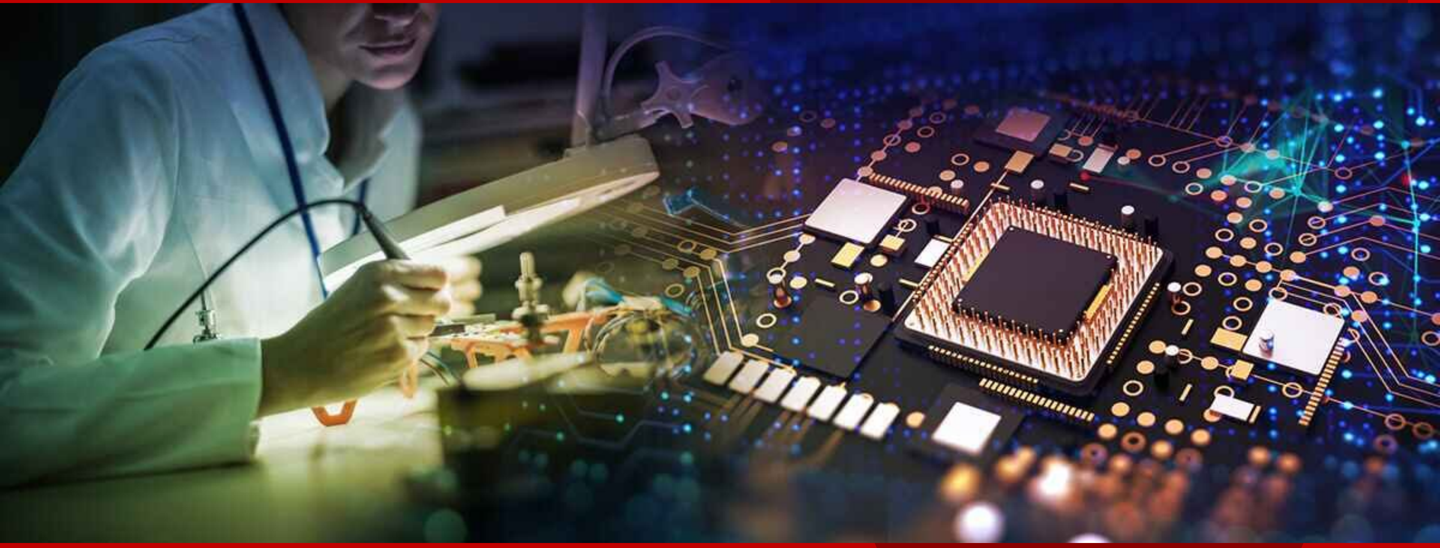
SINUSOIDAL STEADY-STATE CIRCUIT ANALYSIS



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Perpustakaan Negara Malaysia Cataloguing-in-Publication Data

Alizawati Mat Zim, 1982-
SINUSOIDAL STEADY-STATE CIRCUIT ANALYSIS / ALIZAWATI BINTI MAT ZIM,
ROSELINDA BINTI SAMEON, SUHAIDA BINTI ABDUL HALIM.
Mode of access: Internet
eISBN 978-967-2897-15-6
1. Electric circuit analysis.
2. Electric circuits.
3. Government publications--Malaysia.
4. Electronic books.
I. Roselinda Sameon, 1986-. II. Suhaida Abdul Halim, 1975-.
III. Title.
621.3192

PUBLISHED BY:
Politeknik Port Dickson
KM14, Jalan Pantai, 71050 Si Rusa
Port Dickson, Negeri Sembilan

AUGUST 2020

ACKNOWLEDGEMENT

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We would like to convey our utmost gratitude to the Department of Polytechnic and Community College Education particularly the E-learning and Instructional Division (BIPD) for funding our e-book project.

We hereby declare that this module is our original work. To the best of our knowledge it contains no materials previously written or published by another person. However, if there is any, due acknowledgement and credit are mentioned accordingly in the e-book.

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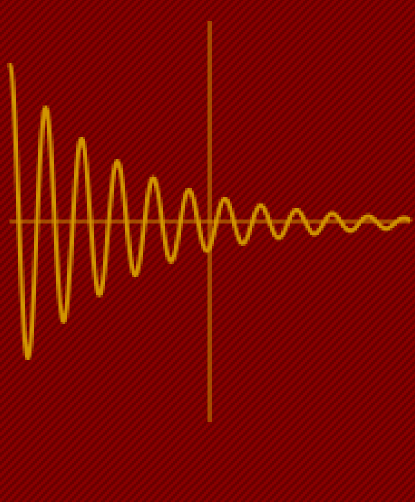
AC BASIC CIRCUIT (SERIES)

Topic 1



PURELY RESISTIVE CIRCUIT AND THE PHASOR DIAGRAM

In AC circuits, a purely resistive circuit consists of a pure resistance of R ohms. Capacitance and inductance do not exist in a purely resistive circuit. The alternating current and voltage move forwards and backwards in both directions in the circuit.

If the current flow through the resistor R is :

$i(t) = I_m \cos(\omega t + \Theta)$

the voltage across it is given by ohm's law as :

$v = iR = R[I_m \cos(\omega t + \Theta)]$

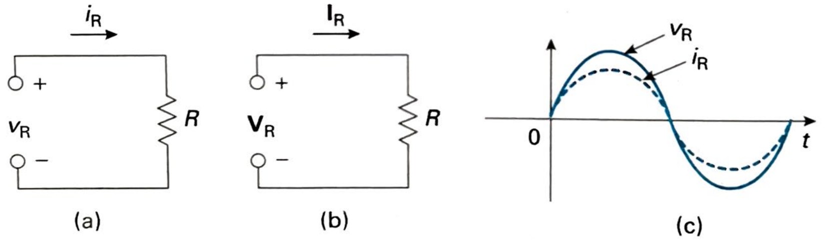
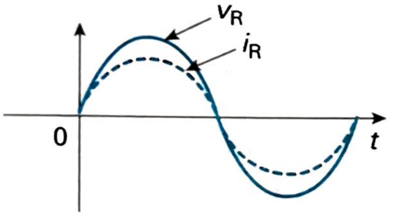


Figure 1.1 Circuit diagrams of a purely resistive circuit.

(a) Time domain circuit

(b) Phasor domain circuit

(c) Voltage and current waveforms.



(c)

As shown in figure (c), the resistor voltage (V_R) and the current (i_R) are all sine waves with the same frequency from the voltage source.

The resistor voltage (V_R) and current (i_R) are in phase with each other.

The phase angle is always 0°.

The resistance R does not change with the frequency.

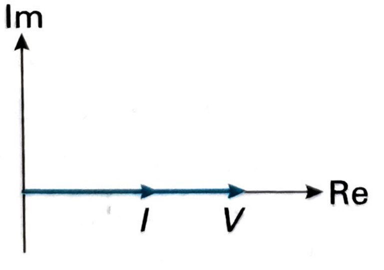
R remains constant even if the frequency is increase or decrease.

- The phasor form of this voltage is V=RI_m $\angle\theta$

- From the expression of the instantaneous applied voltage and the instantaneous current, a purely resistive circuit proves that the applied voltage and current are in phase with each other.

- The impedance Z is Z=R $\angle 0°$

- A phasor diagram shows the magnitude and phase relationship between the voltage across and the current flow through the resistor. The voltage and the current are in phase since they have the same angle as shown in figure :



PURELY INDUCTIVE CIRCUIT AND THE PHASOR DIAGRAM

- When sine-wave variations of current produce an induced voltage, the current lags its induced voltage by exactly 90°, as shown in the figure below.
- The phasors in the figure shows the 90° phase angle between i_L and v_L.
- The 90° phase relationship between i_L and v_L is true in any sine-wave ac circuit, whether L is in series or parallel.

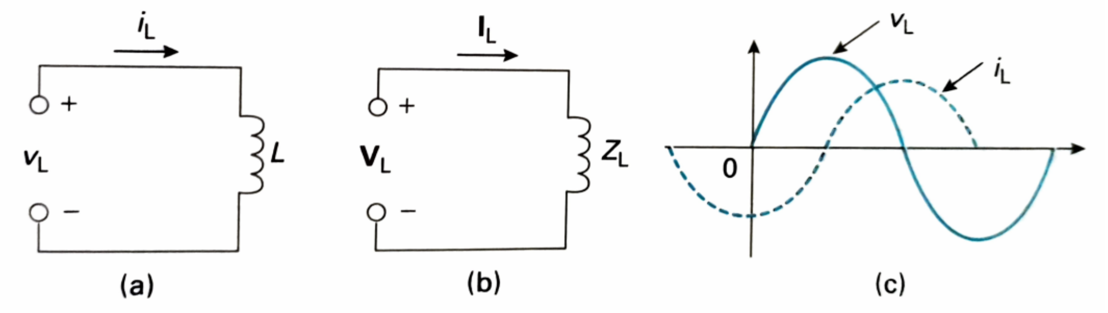


Figure 1.2 Circuit diagrams of a purely inductive circuit.

Circuit diagrams of a purely inductive circuit.

(a) Time domain circuit
(b) Phasor domain circuit
(c) Voltage and current waveforms.

If the current flow through the purely inductive circuit is

$i_L(t) = I_m \cos(\omega t + \Theta)$

The voltage across the inductor is

$V_L(t) = L_\omega I_m \cos(\omega t + \Theta + 90°)$

This equation is transformed to phasor form :

$V_L = j_\omega L I_m \angle\theta$

In phasor presentation, the current is I = $I_m \angle\theta$

Hence $V_L = j_\omega L I$

As defined by Ohm's Law V = IR = IZ the impedance of the purely inductive circuit can be expressed as :

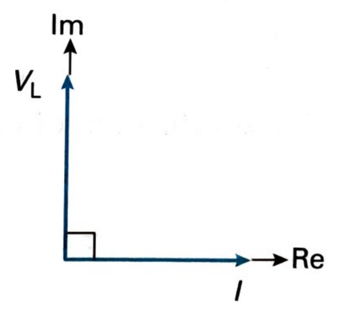
$V_L = \frac{V_L}{I} = j_\omega L = j X_L$

The resistance that opposes the flow of an alternating current by the inductance of the circuit is known as inductive reactance, denoted by X_L, expressed in Ohms as :

$X_L = \omega L = 2\pi fL$

F = frequency (Hz)
L = inductance (H)

The phasor diagram in the figure below shows the phase relationship between the current and voltage of the inductor, where VOLTAGE LEADS CURRENT by 90°.



PURELY CAPACITIVE CIRCUIT AND THE PHASOR DIAGRAM

For any sine wave of applied voltage, the capacitor's charge and discharge current i_c will lead v_c by 90°.

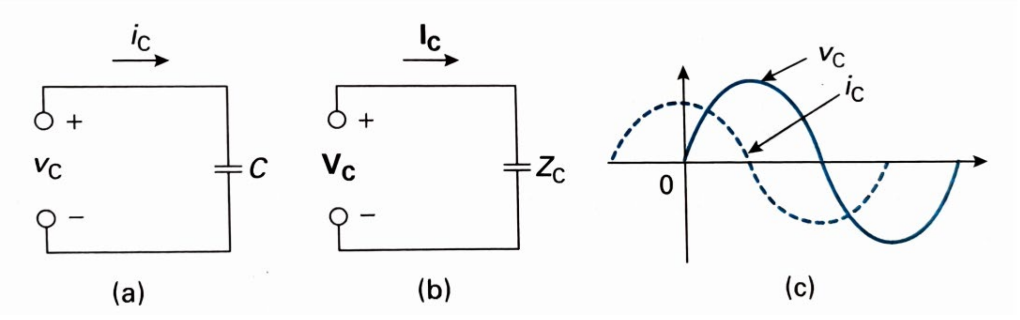


Figure 1.3 : Circuit diagrams of a purely capacitive circuit.

Circuit diagrams of a purely inductive circuit.

(a)Time domain circuit
(b)Phasor domain circuit
(c)Voltage and current waveforms.

The current $I_c = j\omega CV_c$

Therefore,

$$V_c = \frac{I_c}{j\omega C} = \frac{I_c}{\omega C}\angle 90°$$

As defined by Ohm's Law, V = IR = IZ, the impedence of the purely inductive circuit can be expressed as :

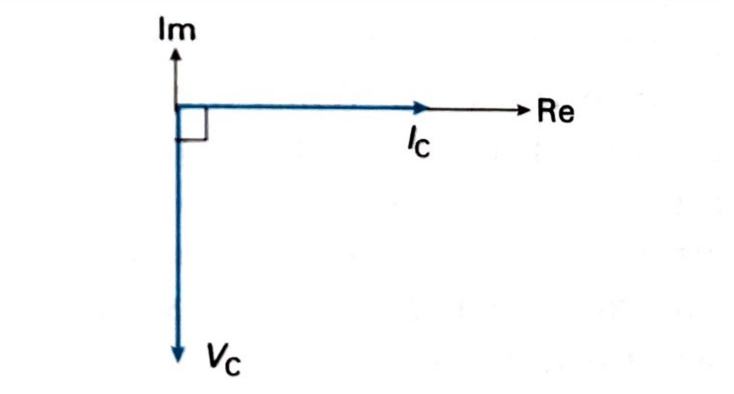
$$Z_c = \frac{1}{j\omega C} = -j\, X_c$$

The opposition to the flow of an alternating current of a capacitor is called capacitive reactance which is expressed in Ohms and derived as :

$$X_c = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

F = frequency (Hz)
L = capacitance (F)

The phasor diagram in the figure below, shows the phase relationship between the current and voltage of the capacitor, where CURRENT LEADS VOLTAGE by 90°.



RL SERIES CIRCUIT AND PHASOR DIAGRAMS

In an RL series circuit, a pure resistance R is connected in series with the inductor L as shown below

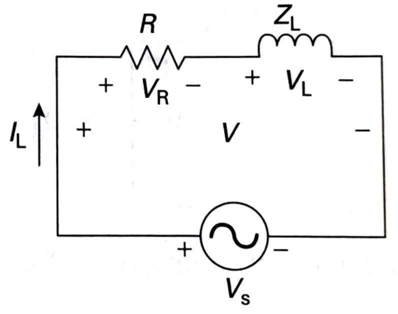


Figure 1.4 : RL series circuit

The total impedance of the circuit is :

Z = Z_R + Z_L= R - jX_L = $\sqrt{R^2 + x_{L^2}}$ + $|Z|\angle - \theta$

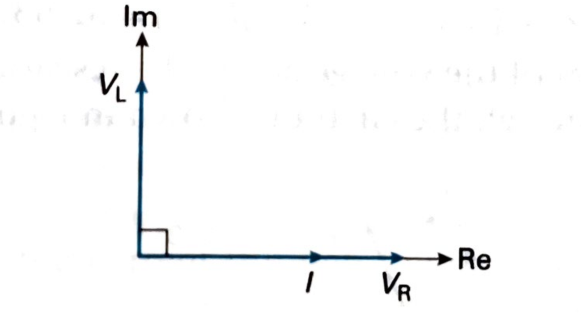
- The current flow through an RL series circuit is I = I_L :

I = $\frac{V}{Z}$ = $\frac{V_S\angle-0°}{|Z|\angle-\theta}$ + $|I|\angle 0°$ - (-θ) = $|I|\angle\theta \; A$

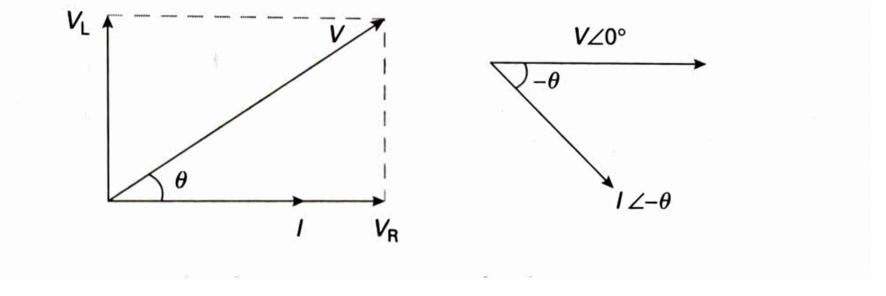
- The voltage across the resistor is :

V_R = I_LR =IR
V_L = $I_L(Z_L)$ =I(- jX_L) = $|V_L|\angle - 90°$ V

The voltage across the V_R is in phase with the current vector while the voltage across the V_L leads the current vector by 90°.

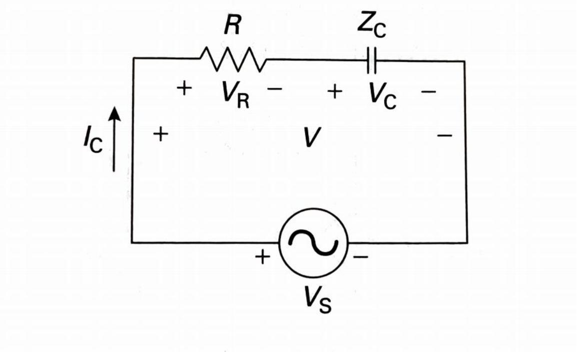


The total voltage is the vector sum of the voltage across the resistor and inductor.



RC SERIES CIRCUIT AND PHASOR DIAGRAMS

In an RC series circuit, a pure resistance R is connected in series with the capacitor C as shown below :



- The total impedance of the circuit is :

Z = Z_R + Z_C = R - jX_C = $\sqrt{R^2 + x_{C^2}}$ + $|Z|\angle - \theta$

- The current flow through an RC series circuit is I = I_c :

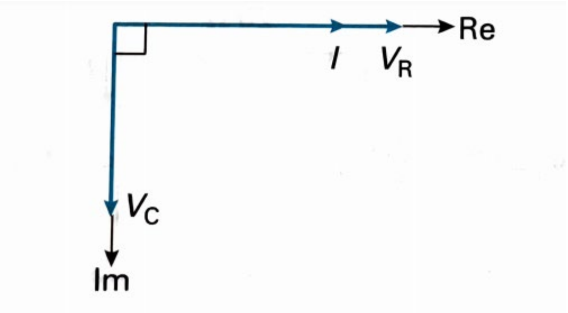
I = $\frac{V}{Z}$ = $\frac{V_S\angle -0°}{|Z|\angle -\theta}$ + $|I|\angle 0°$ - (-θ) = $|I|\angle\theta\ A$

- The voltage across the resistor is :

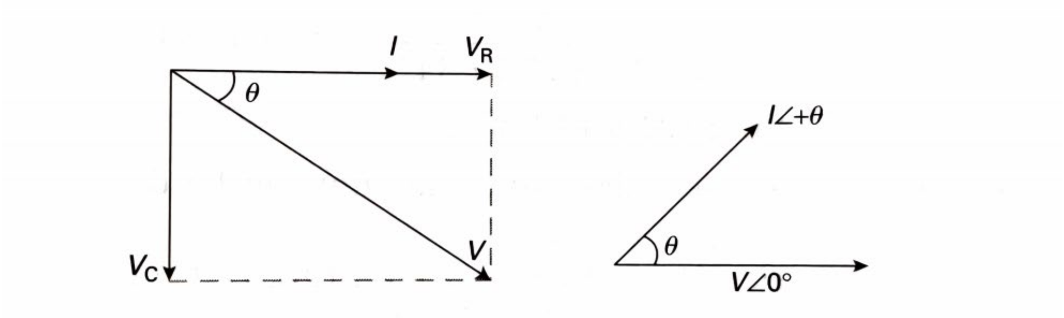
V_R = I_CR =IR
V_C = $I_C(Z_C)$ =I(- jX_C) = $|V_C|\angle - 90°$ V

RLC SERIES CIRCUIT AND PHASOR DIAGRAMS

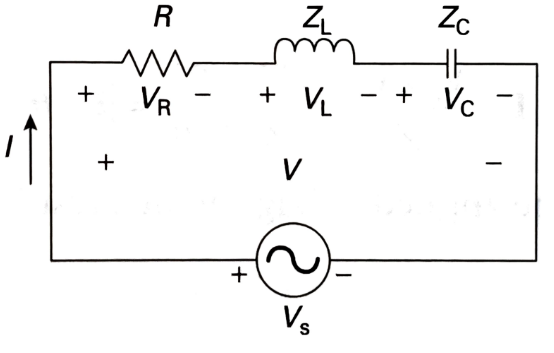
The voltage across the V_R is in phase with the current vector while the voltage across the V_c lags the current vector by 90°.



The total voltage is the is vector sum of the voltage across the resistor and capacitor.



An RLC series circuit consists of a resistor R, inductor L and capacitor C. The applied voltage, V is the phasor sum of V_R,V_L and V_C. The figure shows an RLC series circuit with the total phasor voltage V.



The total impedance of the circuit is :

$$\begin{aligned} Z_T &= Z_R + Z_L + Z_C \\ &= \text{R - j}X_L \text{ - j}X_C \\ &= \text{R} - \text{j}(X_L - X_C) \\ &= \text{R}\sqrt{R^2 + (X_L - X_C)^2} \\ &= |Z|\angle \pm \theta \end{aligned}$$

There are three cases of RLC series circuits to be considered:

CASE 1:

When : $X_L > X_C$

The total impedance of the circuit is : Z = $|Z|\angle + \theta$

The phase angle 0 of the impedance is positive. In this case, the RLC series circuit behaves as an RL series circuit. If the applied voltage is :

V(t) = V_m sin ωt
= $V_m \angle 0^0$

The resulting current is :

I = $\frac{V_m \angle 0^0}{Z}$ = $\frac{V_m \angle 0^0}{|Z|\angle -\theta}$ = $|\text{I}|\angle - \theta$

The current will lag behind the applied voltage, which results in the lagging power factor.

CASE 2:

When,

$X_L < X_C$

The total impedance of the circuit is : Z = $|Z|\angle-\theta$

The phase angle of the impedance is negative. In this case, the RLC circuit behaves as an RC circuit. If the applied voltage is :

V = $V_m \angle 0^0$

The resulting current is : I = $\frac{V_m \angle 0^0}{Z}$ = $\frac{V_m \angle 0^0}{|Z|\angle -\theta}$ = $|\mathrm{I}|\angle\theta$

The current will lag behind the applied voltage, which results in the leading power factor.

CASE 3:

When $X_L > X_C$,

The total impedance of the circuit is : Z = $|Z|\angle + \theta$

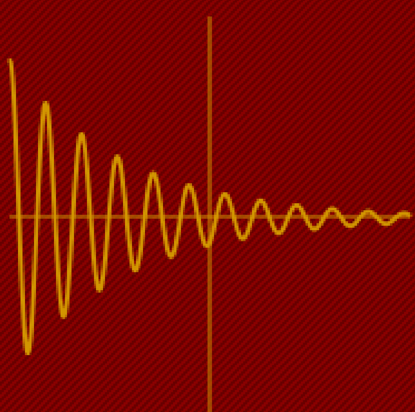
The phase angle is zero. In this case, the RLC series circuit behaves as a purely resistive circuit.

The resulting current will be in phase with the applied voltage and the power factor is unity.

The phasor diagram of the RLC series circuit for each case is based on whether the circuit is inductive or capacitive.

AC BASIC CIRCUIT (PARALLEL)

Topic 2



AC PARALLEL CIRCUIT

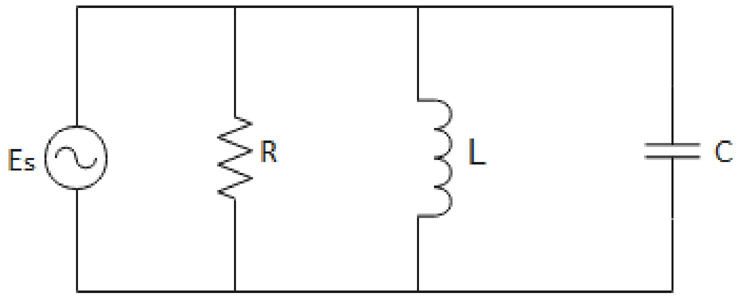


Figure 2.1: R-L-C Parallel Circuit

An AC parallel circuits can be categorized as:

- R-L parallel circuit
- R-C parallel circuit
- R-L-C parallel circuit

Phasor Diagram

If the value of angular frequency 0 is small, the magnitude of I will be greater than 1. Thus, the phasor diagram of the circuit is shown in Figure 2.2.

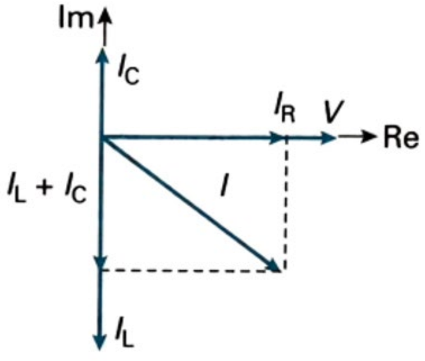


Figure 2.2: Phasor diagram for increased small value of ω

R-L PARALLEL CIRCUIT

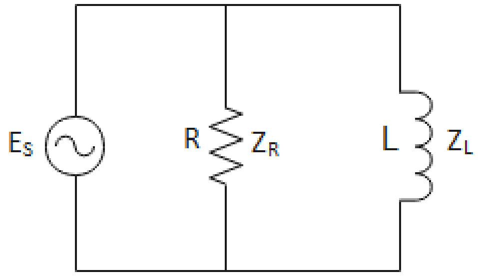


Figure 2.3: R-L Parallel Circuit

1. How to get the total impedance of the circuit (Z_T), refer Figure 2.3:

$$Z_T = Z_R // Z_L$$

$$= \frac{(Z_R)(Z_L)}{(Z_R + Z_L)}$$

OR

$$\frac{1}{Z} = \frac{1}{Z_R} + \frac{1}{Z_L}$$

$$= \frac{1}{R} + \frac{1}{jX_L}$$

$$Z = |Z| < \theta$$

2. How to get the **current**(I):

$$I = \frac{V}{Z_T} = \frac{V < 0^o}{Z_T < \theta}$$, If the applied voltage is $V = V < 0°$

$$= I < -\theta$$

OR

$$I_T = I_R + I_L$$

3. The phasor diagram of the circuit is shown in Figure 2.4.

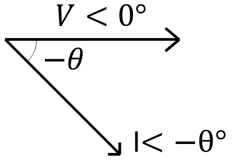


Figure 2.4: Phasor diagram of R-L parallel circuit

EXAMPLE 1: R-L PARALLEL CIRCUIT

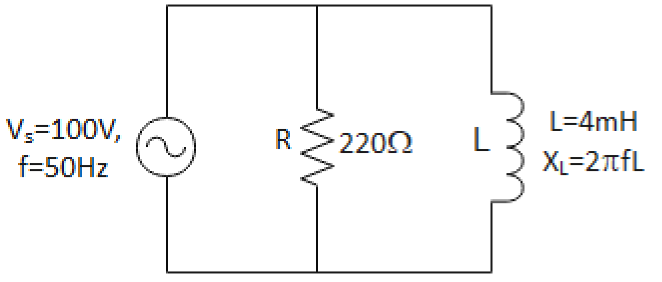


Figure 2.5: Example 1, R-L Parallel Circuit

Refer to the circuit in Figure 2.5 and calculate:

a) Total impedance, Z_T

b) Current I_R, I_L and Total Current I_T

c) Draw the phasor diagram

Answer:

$$X_L = 2\pi f L = 1.26\Omega$$

$$Z_T = 220//j1.26 = 1.26 < 89.67\Omega$$

$$I_R = \frac{100 < 0°}{220 < 0°} = 0.45 < 0°A \ @\ 0.45 + j0A$$

$$I_L = \frac{100 < 0°}{1.26 < 90°} = 79.37 < -90°A \ @\ 0 - j79.37A$$

$$I_T = I_R + I_L = 0.45 + j0 + 0 - j79.37 = 0.45 - j79.37A @\ 79.37 < -89.67°A$$

The phasor diagram of the circuit is shown in Figure 2.6.

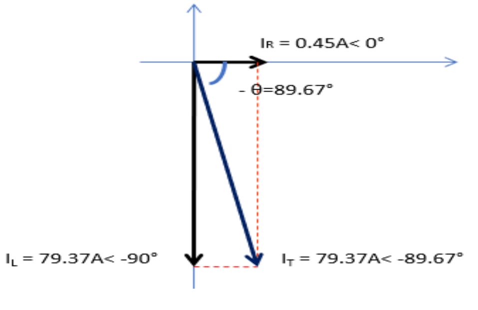


Figure 2.6: Phasor Diagram for example 1

R-C PARALLEL CIRCUIT

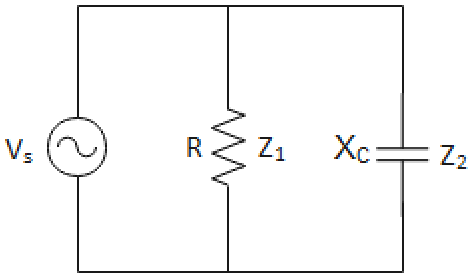


Figure 2.7: R-C Parallel Circuit

1. How to get the total impedance of the circuit (Z_T), refer Figure 2.7:

$$Z_T = Z_1 // Z_2$$

$$= \frac{(Z_1)(Z_2)}{(Z_1 + Z_2)}$$

OR

$$\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2}$$

$$= \frac{1}{R} + \frac{1}{-jX_c}$$

$$Z = |Z| < -\theta$$

2. How to get the **current**(I):

$$I = \frac{V}{Z_T} = \frac{V < 0^o}{Z_T < -\theta}$$, If the applied voltage is $V = V < 0°$

$$= I < \theta$$

OR

$$I_T = I_R + I_c$$

3. The phasor diagram of the circuit is shown in Figure 2.8.

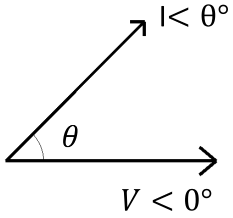


Figure 2.8: Phasor diagram of R-C parallel circuit

EXAMPLE 2: R-C PARALLEL CIRCUIT

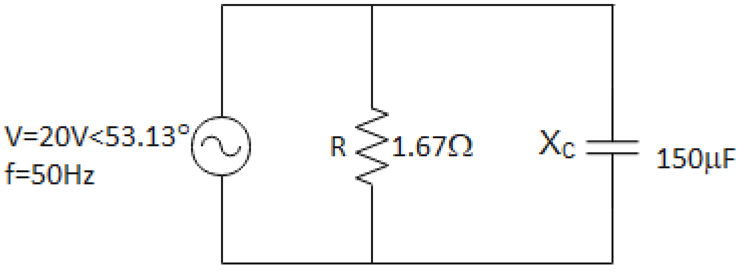


Figure 2.9: Example 2, R-C Parallel Circuit

Refer to the circuit in Figure 2.9 and calculate:

a) Total impedance, Z_T

b) Current I_R, I_C and Total Current I_T

c) Draw the phasor diagram

Answer:

a) $Z_1 = 1.67 + j0 = 1.67\angle 0°\Omega$

$$X_C = \frac{1}{2\pi fC} = 21.22\Omega$$

$$Z_2 = 0 - j21.22 = 21.22\angle -90°\Omega$$

$$Z_T = Z_1//Z_2 = 1.66 < -4.5°\Omega$$

b) $$I_C = \frac{V}{Z_2} = \frac{20\angle 53.13°}{21.22\angle -90°} = 0.943A\angle 143.13°$$

$$I_R = \frac{V}{Z_1} = \frac{20\angle 53.13°}{1.67\angle 0°} = 11.98\angle 53.13°A$$

$$I_T = I_R + I_C = 6.44 + j10.14\ @\ 12.01 < 57.58°A$$

c) Phasor Diagram

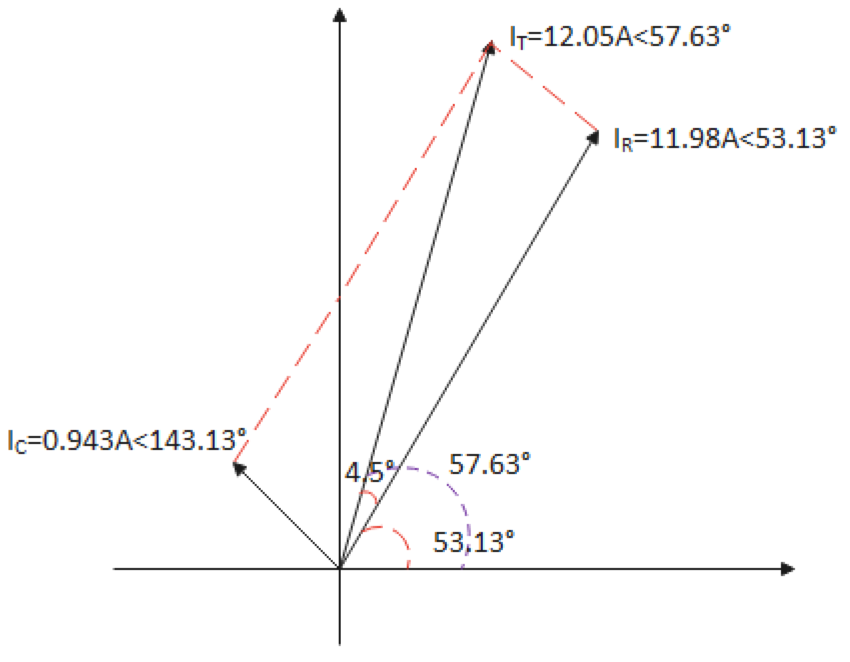


Figure 2.10: Phasor diagram

R-L-C PARALLEL CIRCUIT

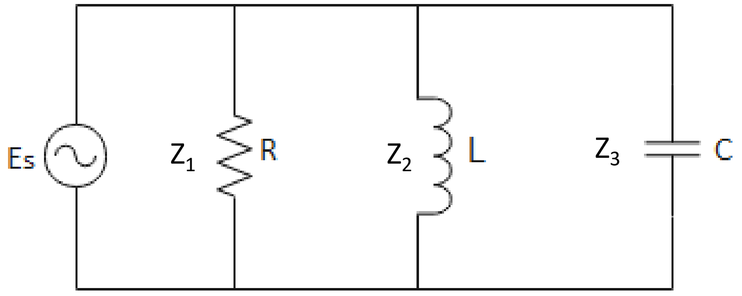


Figure 2.11: R-L-C Parallel Circuit

1. How to get the total impedance of the circuit (Z_T), refer Figure 2.11:

$$\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}$$

$$\frac{1}{Z_T} = \frac{1}{R} + \frac{1}{jX_L} + \frac{1}{-jX_c}$$

2. How to get the **current**(I):

Where $V = V_R = V_L = V_C$, connected in parallel

$$I_R = \frac{V}{Z_1}, I_L = \frac{V}{Z_2}, I_C = \frac{V}{Z_3}$$

$$I_T = I_R + I_L + I_c$$

3. The phasor diagram of the circuit is shown in Figure 2.12.

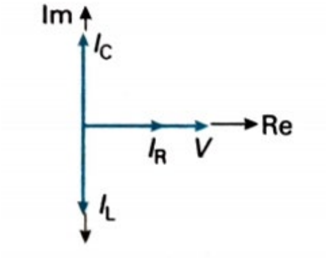


Figure 2.12: Phasor diagram for purely resistive,

purely inductive an purely capacitive circuits

EXAMPLE 3: R-L-C PARALLEL CIRCUIT

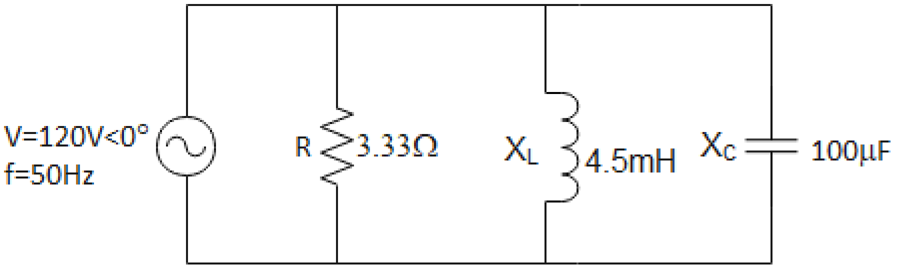


Figure 2.13: Example 3, R-L-C Parallel Circuit

Refer to the Figure 2.13 , calculate;

a) Z_1, Z_2, Z_3, Z_T

b) I_R, I_L, I_C and total current

c) Draw the current vector diagram

$a)$

$$Z_1 = R = 3.33\Omega\angle 0^0$$

$$Z_2 = j2\pi fL = 2\pi(50)(4.5m) = j1.41\Omega\ @1.41\angle 90^0\,\Omega$$

$$Z_3 = -j\frac{1}{2\pi fc} = -j\frac{1}{2\pi(50)100\mu} = -j31.83\Omega\ @31.83 < -90°$$

b)

$$I_R = \frac{V}{Z_1} = \frac{120 < 0°}{3.33 < 0°} = 36.04 < 0°A\ @\ 36.04 + j0A$$

$$I_L = \frac{V}{Z_2} = \frac{120 < 0°}{1.41 < 90°} = 85.11 < -90°A\ @\ 0 - j85.11A$$

$$I_C = \frac{V}{Z_3} = \frac{120 < 0°}{31.83 < -90°} = 3.77 < 90°A\ @\ 0 + j3.77A$$

$$I_T = I_R + I_L + I_C$$

$$I_T = (36.04 + j0) + (0 - j84.87) + (0 + 3.77)$$

$$I_T = 36.04 - j81.11 \text{ @ } 88.76 < -66.05°A$$

$$Z_T = \frac{V}{I_T} = \frac{120 < 0°}{88.76 < -66.05°} = 1.35 < 66.05°\Omega$$

C) Current Phasor diagram

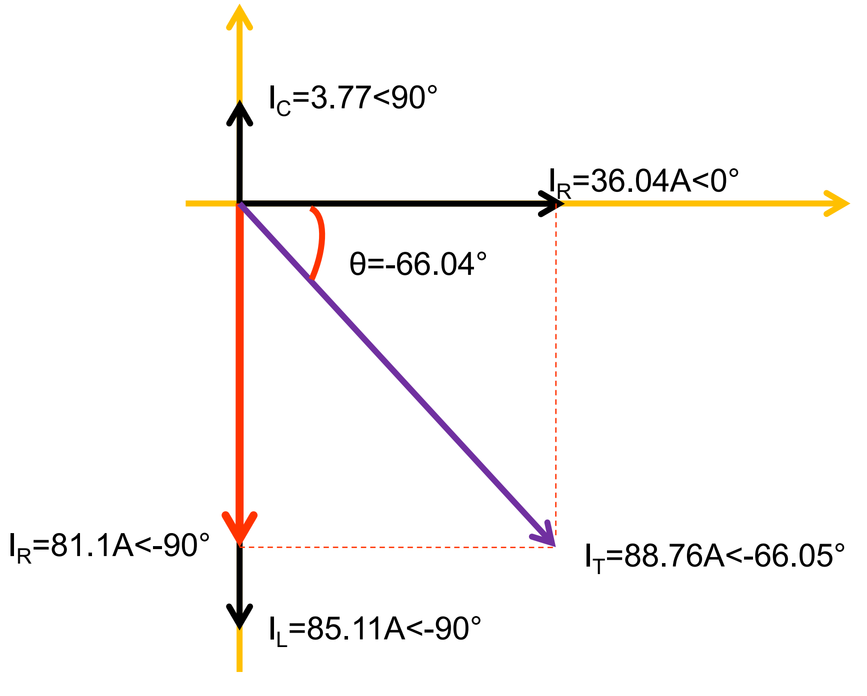


Figure 2.14: Phasor diagram

POWER IN AC CIRCUIT

- The rate at which energy is consumed is referred to as power. The abbreviation for **POWER** is **P.**
- To put it another way, **POWER (P)** is the quantity of energy utilised in a certain length of time (or energy per time). In the form of an equation is,

  P = E / t joules per second or **WATT (W)**

- Electric energy is converted to heat energy when current flows through a resistance. The POWER is the rate of energy conversion.

  $P = I^2 \times R$ $\quad$ $P = I \times V$ $\quad$ $P = V^2 / R$

- R is the resistance's in ohm (Ω), V is the voltage in volts (V) across the resistance, and I is the current in amps (A) through the resistance in each of these equations.

Actual, Reactive and Apparent Power

- Actual, Reactive and Apparent Power are the 3 forms of power used in AC circuits.
- The amount of real power consumed or dissipated in the circuit is known as actual power P_{TRUE}.

$$Actual\ Power, P_{TRUE} = I_{rms} V_{rms} cos\theta$$

- The power obtained from the reactance, measured in VAR, is known as **reactive power Q**.

$$Q = I_{rms} V_{rms}\ Sin\theta$$

- The product of the rms values of the voltage across the load and the current through the load is the apparent power S. Volt-amperes are a unit of measurement for **apparent power** (VA), is given by:

$$S = I_{rms} V_{rms}$$

Power in Purely Resistive Circuit

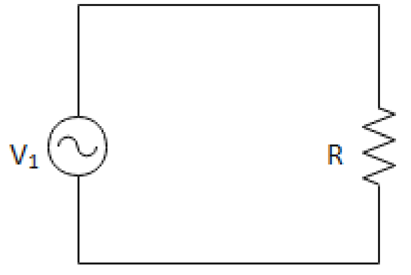


Figure 2.15: Resistive circuit

The symbol for **True, Real, Active or Actual Power** is P_{TRUE} in units of **Watts (W)** is a measure of the rate at which a component or circuit loses energy. The loss of energy is usually caused by heat dissipation (as in a resistor) or conversion to another form of energy (as in a motor that converts electrical energy to motion)

Refer to the Figure 2.16. The current and voltage waveforms in a resistive circuit are in phase.

Average Power $P = I_{RMS} \times V_{RMS}$

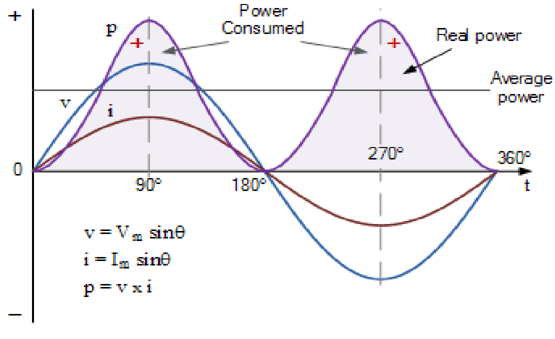


Figure 2.16: The waveforms of I,V and P

Refer to the Figure 2.15, the voltage and current of the resistor in an AC circuit must be in rms values (effective values).

$P_{true} = I_{rms}^2 \times R$ / $P_{true} = V_{rms}^2 / R$ / $P_{true} = V_{rms} \times I_{rms}$

True power or Active power refers to the amount of energy absorbed by the resistor.

Power in Purely Capacitive Circuit

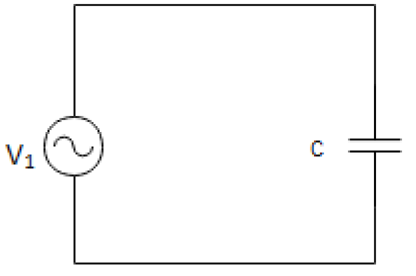


Figure 2.17: Capacitive circuit

- The true power in capacitors is zero, and capacitors do not lose any energy as heat. Refer to the circuit in Figure 2.17.
- The rate at which a component stores energy to the circuit is measured by reactive power.
- The VAR is a unit of reactive power with the symbol Q (Reactive Volt-Ampere)
- These formulas apply:

$$Q = I_{rms} x V_{rms}$$

$$Q = \frac{V^2{}_{rms}}{X_c}$$

$$Q = X_c x I^2{}_{rms}$$

Power in Purely Inductive Circuit

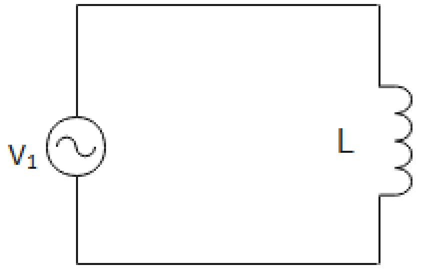


Figure 2.18: Inductive circuit

- Refer to the circuit in Figure 2.18. When there is current flowing through an inductor, it stores energy in its magnetic field.
- An ideal inductor does not dissipate energy but rather retains it (assuming no winding resistance).
- When an AC voltage is supplied to an ideal inductor, the inductor stores energy for a portion of the cycle before returning it to the source for the remainder of the cycle.
- Ideally, during the negative portion of the power cycle, all of the energy stored by an inductor during the positive portion is returned to the source.

In an ideal inductor, no net energy is wasted owing to heat conversion.
So the ***True power (P_{true}), unit watt is zero.***

Reactive Power (Q), unit VAR (Volt Ampere-Reactive), is the rate at which an inductor stores or recovers energy.

These formulas apply:

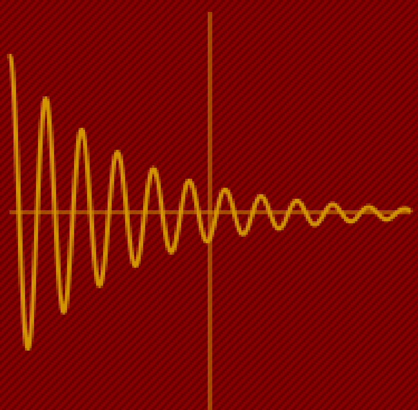
$$Q = I_{rms} x V_{rms}$$

$$Q = \frac{V^2{}_{rms}}{X_L}$$

$$Q = X_L x I^2{}_{rms}$$

RESONANCE

Topic 3



RESONANCE IN A CIRCUIT

In electrical circuit, when the inductive reactance and the capacitive reactance were equal in magnitude, electrical energy will oscillate between the inductor's magnetic field and the capacitor's electric field.

#1: Resonance occurs when the inductor's collapsing magnetic field generates an electric current in its windings, which charges the capacitor, and the discharging capacitor gives an electric current, which builds the magnetic field in the inductor, and so on.

#2: At resonance, the parallel impedance of the two elements is maximum, and the series impedance of the two elements is minimal.

Resonance in electronic systems can only exist when a circuit consists of two storage elements, a capacitor and an inductor. When the capacitive and inductive reactance are identical in magnitude, the impedance becomes entirely resistive. A tuned circuit is another name for a resonant circuit. Resonant circuits are essential to the operation of a wide range of modern electrical and electronic systems. The frequency of the source is equal to the frequency of its own circuit in electrical resonance. If the receiver has a parallel combination of R, C, L, and is a true condition equal to 0, the receiver is in a parallel resonance, also known as the current resonance.

RESONANCE PHENOMENON

The capacitive and inductive reactance in an RLC circuit are identical in magnitude, resulting in a completely resistive impedance. Musical instruments (e.g. guitars, pianos) use resonant systems to generate vibrations of a certain frequency, or to choose specific frequencies from a complex vibration containing numerous frequencies (e.g. filters).

Resonant circuits are those that treat a small band of frequencies considerably different than the rest of the spectrum. A very resonant circuits gain reaches a sharp maximum or minimum at its resonant frequency, with much lower variation outside of a small range of frequencies surrounding the resonant frequency.

Resonant circuits are mostly used in communication systems (e.g. antennas) and power systems (e.g. motors). The configuration of storage elements (capacitor and inductor), whether in series or parallel, may affect the resonant frequency.

Thus, the characteristics of both elements in terms of reactance in the circuit needs to be investigated.

THE CAPACITIVE AND INDUCTIVE REACTANCE

The capacitor and inductor are two reactive components that can store energy. Ideally, both components should have zero resistance and have purely imaginary impedance.

Reactance for a capacitor is given by :

$$X_C = \frac{1}{2\pi f C} \quad = \frac{1}{\omega C}$$

X_C is the capacitive reactance in ohms

ω = $2\pi f$ is the angular velocity in rad/s

F is the frequency in hertz

C is the capacitance in farads.

Capacitive reactance is inversely proportional to the frequency. Reactance for an inductor is given by :

$$X_L = 2\pi f L \;= \omega_L$$

X_L is the inductive reactance in ohms

ω = $2\pi f$ is the angular velocity in rad/s

f is the frequency in hertz

L is the inductance in henrys.

Impedance Z in an RLC circuit is given by :

$$Z = R + j(X_L - X_C)$$

$$= R + j(\omega_L - \frac{1}{\omega C})$$

Resistance (R) does not depend on the value of frequency. Thus, even with the variation in frequency, R will have the same impedance value as the resistance value. However, the inductor can be assumed to be a short circuit when the frequency is zero, while the capacitor is an open circuit. Since the frequency, *f* applied to the capacitor is inversely proportional with impedance Z, a low *f* gives a high X, whereas a high *f* gives a low Xc.

Table 1 : Frequency Changes as a Function of Impedance

Impedance	Frequency			
	0	Low	High	∞
R	R	R	R	R
XL	0 (L = short circuit)	Low	High	∞ (L= open circuit)
XC	∞ (C= open circuit)	High	Low	0 (C = short circuit)

For a high frequency circuit, where the value of frequency is infinity, the inductor in the circuit can be assumed to be an open circuit since the reactance value is so high while the capacitor is considered as a short circuit since the capacitive reactance is approaching zero.

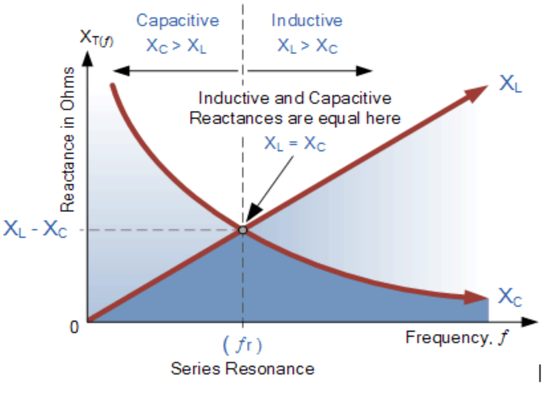


Figure 3.1 : Capacitive reactance with inductive reactance relationship.

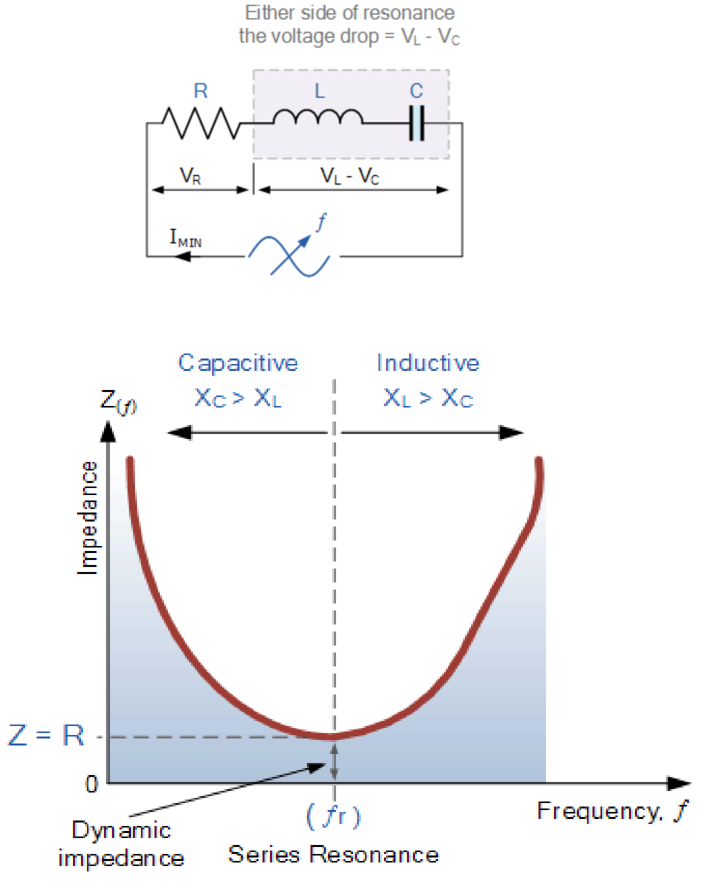
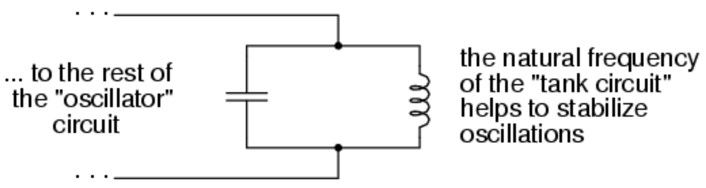


Figure 3.2 : Impedance versus frequency for an RLC series circuit

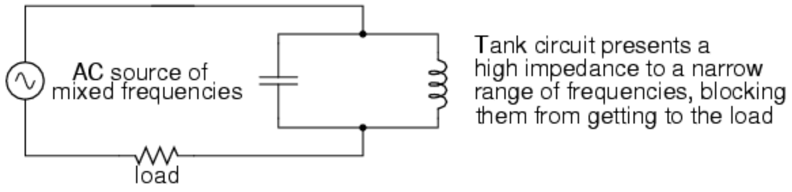
The magnitude of impedance depends on the difference between the inductive and the capacitive reactance. At the resonant frequency, when capacitive and inductive reactance cancels out each other, which results in a short circuit LC, the total impedance is at its lowest point, which is equal to R. With zero imaginary impedance, the circuit becomes purely resistive.

RESONANCE FUNCTIONS

The resonance circuit acts as a frequency source that is stable. The tank circuit's frequency is exclusively determined by the value of L and C.



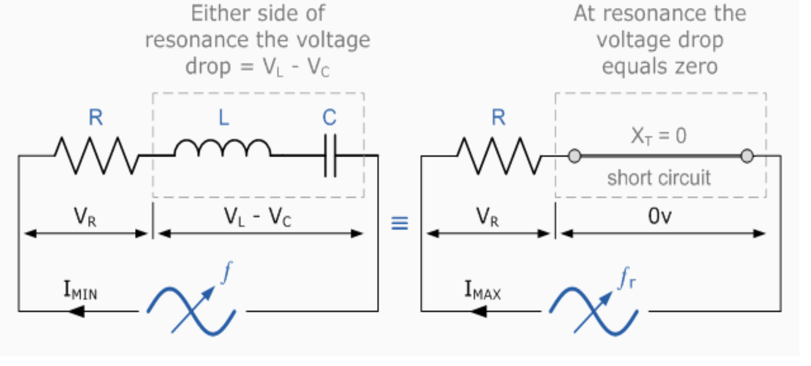
The resonance circuit acts as a filter.



Acting as a frequency "filter" to separate particular frequencies from a mixture of others.

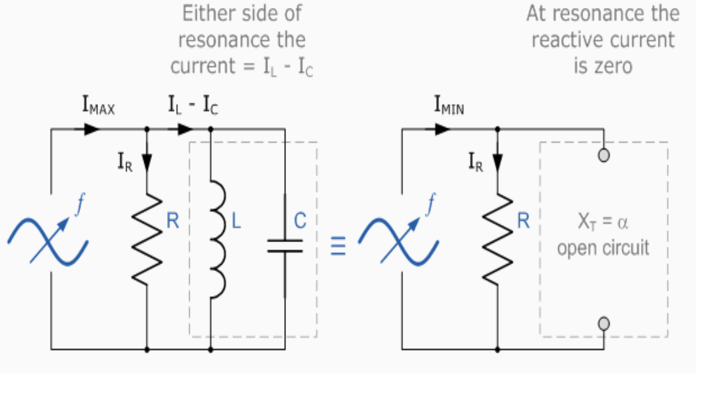
RESONANCE OF A SERIES

The current will be at its maximum and the impedance will be at its lowest.



RESONANCE IN PARALLEL

The current will be as low as possible while the impedance will be as high as possible.



SERIES RLC RESONANCE CIRCUIT

The condition of resonance for serial resonance is straightforward, and it is defined by minimum impedance and zero phase. Parallel resonance, which is more common in electrical applications, necessitates a more detailed definition.

In an AC circuit, electrical resonance occurs when two reactance that are opposing and equal cancel each other out as XL = XC, and the spot on the graph where this happens is where the two reactance curves intersect. Because XL and Xc have opposing phase angles, they cancel each other out, and the overall reactance of the circuit is less than each individual reactance: XT < XL and XT < Xc

The reactance of a series RLC circuit changes as the frequency of the voltage source changes. Its overall impedance varies as well. Inductive reactance (XL) and capacitive reactance (XC) are both present in a series RLC circuit (Xc).When Xc > XL at low frequencies, the circuit is mainly capacitive. When XL > Xc at high frequencies, the circuit is mainly inductive.

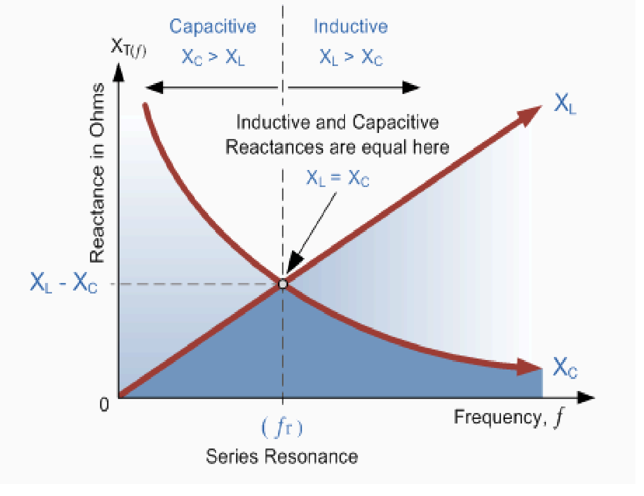
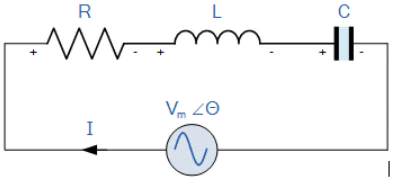


Figure 3.3 : Reactance versus frequency for an RLC parallel circuit

In a series resonant circuit, the resonant frequency, f_r point can be calculated as follows.



The total impedance of the circuit is derived as follow:

$$Z = \left[R + J\omega L + \frac{1}{j\omega C}\right] \quad = \sqrt{R^2 + {X_T}^2}$$

At resonance, $X_L = X_C$ so the total impedance becomes Z = R.

The total current (Z_{Tmin}, I_{mx}) is

$$I = \frac{V}{Z} \qquad = \frac{V_{max}}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

The resonant frequency is

$$X_L = X_C$$
$$2\pi f L = \frac{1}{2\pi f L}$$
$$f = \sqrt{\frac{1}{4\pi^2 LC}}$$
$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

or

$$\omega L = \frac{1}{\omega C}$$
$$\omega^2 = \frac{1}{LC}$$
$$\omega = \frac{1}{\sqrt{LC}}$$

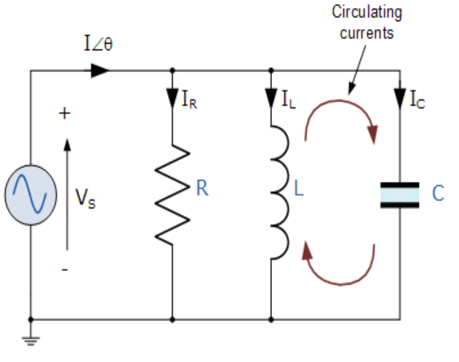
PARALLEL RLC RESONANCE CIRCUIT

Reactance change as you change the voltage source's frequency.

At low frequencies, XL < Xc and the circuit is primarily inductive.

At high frequencies, Xc< XL and the circuit is primarily capacitive.

Its total impedance also changes. The smaller reactance dominates, since a smaller reactance results in a larger branch current.



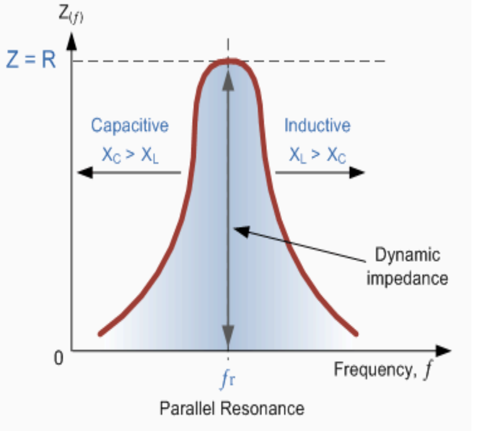


Figure 3.4 : Total impedance vs frequency

The total impedance at resonance is

$$Z = \frac{1}{Y} = R$$

The total impedance of the circuit is derived as follows.

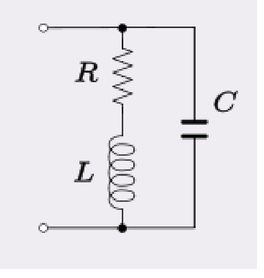
$$Y = Y_R + Y_L + Y_C$$

$$= \frac{1}{R} + \frac{1}{j\omega L} - \frac{1}{\frac{j}{\omega C}}$$

$$= \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)$$

RESONANT FREQUENCY USING IMPURE COMPONENTS

The value of resistance, R = 0.



$$f_r = \frac{1}{2\pi}\sqrt{\left(\frac{1}{LC} - \frac{R^2}{L^2}\right)}$$

Therefore,

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

BANDWIDTH

BANDWIDTH IN RLC CIRCUIT

Band frequencies are those that define the points on the resonance curve that are 0.707 of the peak current or voltage.

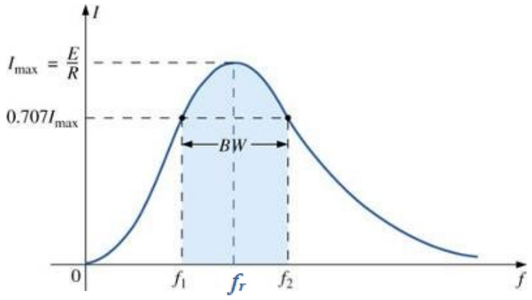


Figure 3.5 : Frequency Bandwidth

Bandwidth (BW) is the range of frequencies between the band, or ½ power frequencies. Z at half-power frequency is obtained from $\sqrt{2}R$.

Thus,

$$Z = \sqrt{2}R = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$\omega_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \quad \text{and} \quad \omega_1 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

Then, the bandwidth is

$$BW = f_2 - f_1$$

The resonance frequency can be written as;

$$f_{1,2} = f_r \pm \frac{BW}{2}$$

QUALITY FACTOR IN RLC CIRCUIT

The quality factor (Q) of a series resonant circuit is defined as the ratio of the reactive power of either the inductor or the capacitor to the average power of the resistor at resonance.

Q can be found several ways:

$$Q = \frac{f_r}{BW} = \frac{X_L}{R} = \frac{\omega_r L}{R} = \frac{2\pi f_r L}{R}$$

This also gives an alternate way to find BW:

$$BW = \frac{f_r}{Q}$$

APPLICATION OF RESONANCE RLC CIRCUITS

Resonant circuits are extensively used in tuning television and radio transmitters and receivers. They are also widely used as tank circuits. An RLC circuit can function as a band-pass, band-stop, low-pass, or high-pass filter. Besides that, RCL circuits can also be used in ignition systems in automobiles and in biomedical applications.

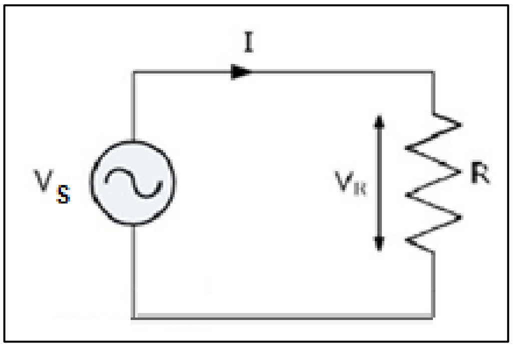
EXERCISE

TOPIC 1 – AC BASIC CIRCUITS (SERIES)

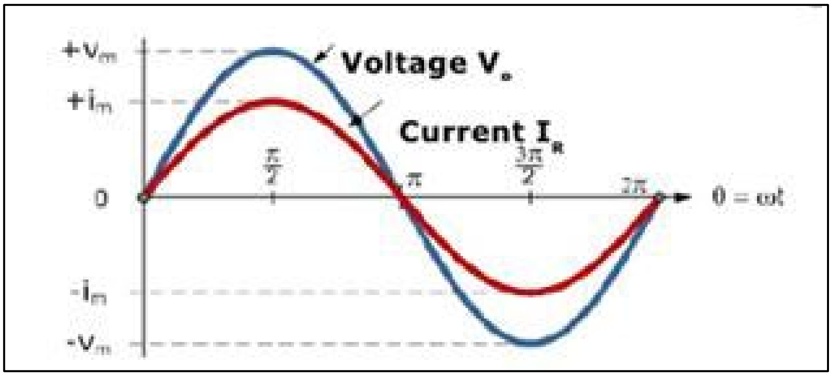
EXAMPLE 1.1

Draw a purely resistive circuit and a sinusoidal waveform of current and voltage for the circuit.

SOLUTION 1.1



Purely resistive circuit.

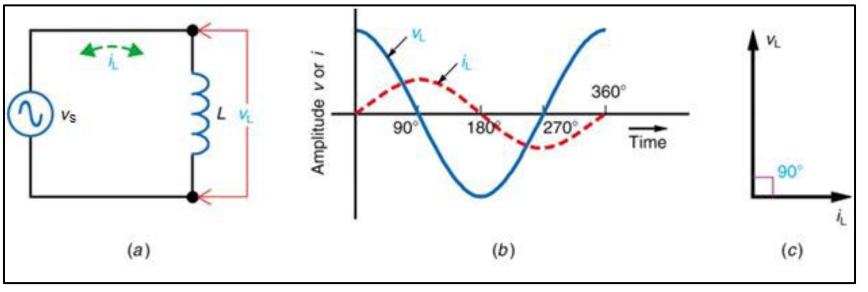


Sinusoidal waveform current, I and voltage,V are in phase

EXAMPLE 1.2

With the aid of a diagram, state the relationship between the voltage and the current for a pure inductive circuit.

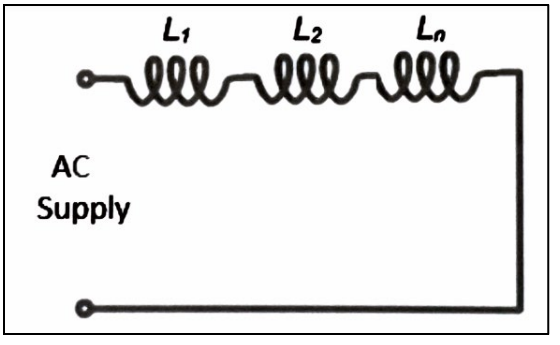
SOLUTION 1.2



- Figure (a) shows a purely inductive circuit
- Figure (b) shows current and voltage waveform
- Figure (c) shows a phasor diagram iL lags VL by 900

EXAMPLE 1.3

By referring to the figure below, calculate the total inductance in the circuit if L_1 = 0.02mH, L_2 = 0.03 and L_n = 0.05mH.



SOLUTION 1.3

Given L_1 =0.02mH, L_2 =0.03mH and L_n=0.05mH

Formula for series inductance is:

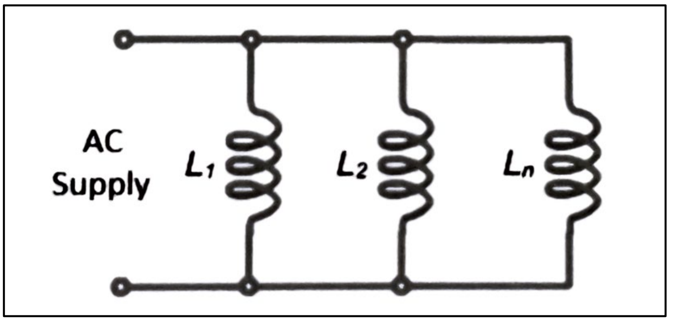
$$L_{Total} = L_1 + L_2 + L_n$$

= 0.02mH+0.03mH+0.05mH

= 0.10mH

EXAMPLE 1.4

By referring to the figure below, calculate the total inductance in the circuit if L1 = 0.02mH,L2 = 0.03 and Ln = 0.05mH.



SOLUTION 1.4

Given L_1 =0.02mH, L_2 =0.03mH and L_n=0.05mH

Formula for series inductance is:

$$L_{Total} = L_1 + L_2 + L_n$$

= 0.02mH+0.03mH+0.05mH

= 0.10mH

EXAMPLE 1.5

Express Z = $7\angle 30°$ in rectangular form

SOLUTION 1.5

a = 7 cos 300 = 6.06

b = 7 sin 300 = 3.5

So, Z = 6.06 + j3.5

EXAMPLE 1.6

Express Z = $7\angle 30°$ in rectangular form

SOLUTION 1.6

a = 7 cos 300 = 6.06

b = 7 sin 300 = 3.5

So, Z = 6.06 + j3.5

EXAMPLE 1.7

Find the total impedance of the series and parallel circuit in Figure 1.1a below if the voltage supply of the circuit is 10V, 60Hz.

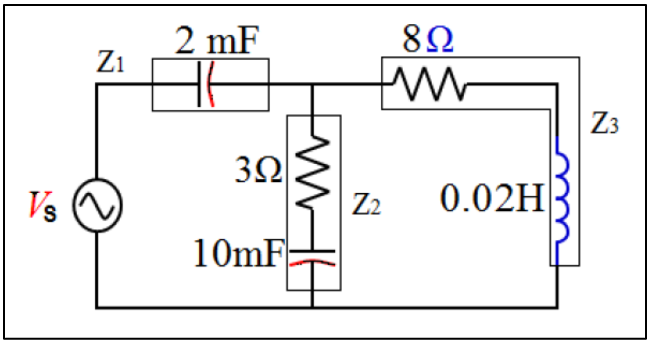


Figure 1.1a

SOLUTION 1.7

- Z1 Impedance of the 2mF
- Z2 Impedance of the 3Ω resistor series with 10mF capacitor
- Z3 Impedance of the 8Ωresistor series with 0.2H inductor

Z_1 = -jX_C = -j $\frac{1}{2\pi fc}$ = $\frac{1}{2\pi \text{ x } 60 \text{ x } 2\text{m}}$ = -j1.33Ω

Z_2 = 3 -jX_C = 3 -j $\frac{1}{2\pi fC}$= 3 -j $\frac{1}{2\pi \text{ x } 60 \text{ x } 10\text{m}}$ = 3 -j0.27Ω

Z_3 = 8 + jX_L = 8 +j$2\pi fL$= 8 + j $(2 \text{ x } \pi \; x \; 60 \; x \; 0.02H)$= 8 + j7.54Ω

$$Z_{Total} = Z_1 + (Z_2 \parallel Z_3) = Z_1 + \frac{Z_2\, Z_3}{Z_2 + Z_3}$$

$$= -j1.33 + \frac{(3 - j0.27\Omega)\,(8 + j7.54\Omega)}{(3 - j0.27\Omega) + (8 + j7.54\Omega)}$$

$$= -j1.33 + \frac{(3.01\angle -5.14^\circ)\,(10.99\angle 43.30^\circ)}{11 + j7.27}$$

$$= -j1.33 + \frac{(33.08\angle 38.16^\circ)}{(13.19\angle 33.46^\circ)} = -j1.33 + (2.51\angle 4.70^\circ)$$

$$= -j1.33 + 2.50 + j0.21 = 2.5 - j1.12\Omega$$

EXAMPLE 1.8

A 15Ω resistor, an inductor with 8Ω inductive reactance, and a capacitor with 12Ω capacitive reactance are in parallel across an AC voltage source. The circuit impedance is...

SOLUTION 1.8

Given

R = 15∠0° , X_L = 8∠90° and X_C = 12∠ − 90°

Impedance's formula for parallel circuit is:

$$\frac{1}{Z_T} = \frac{1}{X_1} + \frac{1}{X_2} + \frac{1}{X_3} + \ldots.$$

$$\frac{1}{Z_T} = \frac{1}{15\angle 0°} + \frac{1}{8\angle 90°} + \frac{1}{12\angle -90°}$$

$$= \frac{1}{0.0782\angle -36.079°}$$

$$Z_T = 12.787\angle 36.079°$$

EXERCISE

TOPIC 2 – AC BASIC CIRCUITS (PARALLEL)

EXAMPLE 2.1

Referring to figure 2.1a, a 7Ω resistor, a 31.4mH inductor and a 100µF capacitor are connected parallel. An AC sinusoidal waveform 100V, 50Hz is used as a supply to this circuit. Calculate the total current, I_T flowing in the circuit.

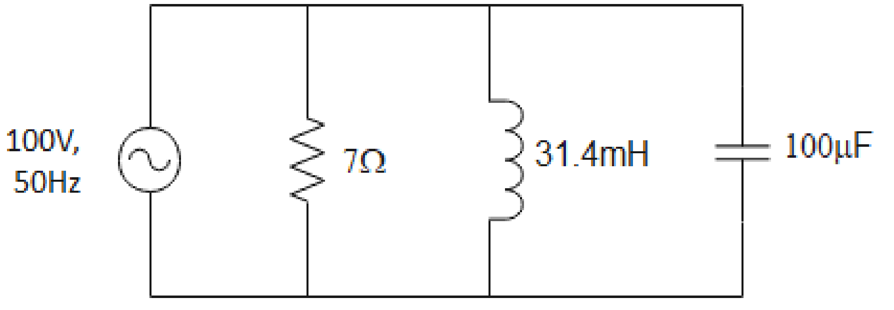


Figure 2.1a: Circuit for example 2.1

SOLUTION 2.1

$$X_C = \frac{1}{2\pi fc} = \frac{1}{2\pi(50)(100\mu)} = 31.83k\Omega$$

$$X_L = 2\pi fL = 2\pi(50)(31.4m) = 9.86\Omega$$

$$I_R = \frac{V}{Z_R} = \frac{100 < 0°}{7 < 0°} = 14.3 < 0°A@14.3 + j0$$

$$I_L = \frac{V}{Z_L} = \frac{100 < 0°}{9.86 < 90°} = 10.14 < -90°A@ - j10.14$$

$$I_C = \frac{V}{Z_C} = \frac{100 < 0°}{31.83 < -90°} = 3.14 < 90°A@j3.14$$

$$I_T = I_R + I_L + I_C$$

$$I_T = 14.3 + j0 - j10.14 + j3.14$$

$$I_T = 14.3 - j7A@15.92 < -26.1°A$$

EXAMPLE 2.2

Referring to the figure 2.2a, calculate the line current I_1, I_2, I_{TOTAL} and draw the phasor diagram of current. Find the power factor, true power and the apparent power as well.

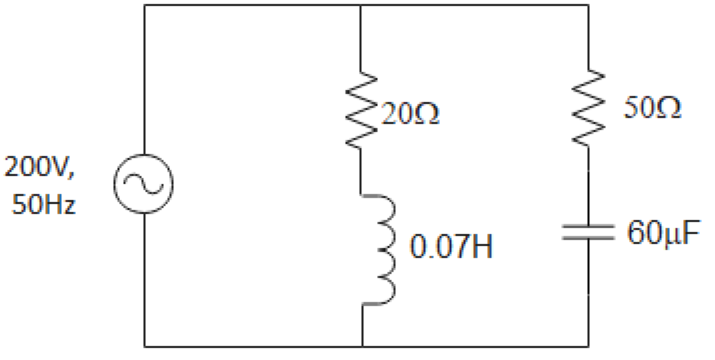


Figure 2.2a: Circuit for example 2.2

SOLUTION 2.2

$$X_C = \frac{1}{2\pi fc} = \frac{1}{2\pi(50)(60\mu)} = 53.1\Omega$$

$$X_L = 2\pi fL = 2\pi(50)(0.07) = 22\Omega$$

$$Z_1 = 20 + j22\Omega @ 29.73 < 47.7°\Omega$$

$$Z_2 = 50 - j53.1\Omega @ 73 < -46.7°\Omega$$

$$I_1 = \frac{V}{Z_1} = \frac{200 < 0}{29.73 < 47.7°} = 6.73 < -47.7°A @ 4.53 - j5$$

$$I_2 = \frac{V}{Z_2} = \frac{200 < 0}{73 < -46.7°} = 2.74 < 46.7°A @ 1.88 + j2$$

$$I_T = I_1 + I_2$$

$$I_T = 4.53 - j5 + 1.88 + j2$$

$$I_T = 6.41 - j3 @ 7.08 < -25.1°A$$

EXAMPLE 2.3

Referring to the figure 2.3a, a coil of inductance 0.12H and resistance 3kΩ is connected in parallel with a 0.02µF capacitor across a 240V, 50Hz supply. Find the value of the total current I, flows in the circuit.

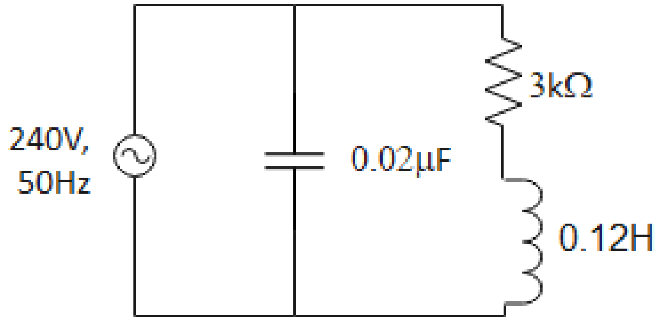


Figure 2.3a: Circuit for example 2.3

SOLUTION 2.3

$$X_C = \frac{1}{2\pi fc} = \frac{1}{2\pi(50)(0.02\mu)} = 159.15k\Omega$$

$$X_L = 2\pi fL = 2\pi(50)(0.12) = 37.7\Omega$$

$$Z_1 = 159.15k < -90°\Omega @ 0 - j159.15k\Omega$$

$$Z_2 = 3k < 0.72°\Omega @ 3k + j37.7\Omega$$

$$Z_T = Z_1 \,//\, Z_2$$

$$Z_T = \frac{159.15k < -90°(3k < 0.72°)}{0 - j159.15k + 3k + j37.7}$$

$$Z_T = \frac{477.45M < -89.28°}{3k + j159.11k} @ \frac{477.45M < -89.28°}{159.14k < 88.92°}$$

$$Z_T = 3k < -178.2°\Omega$$

$$I_T = \frac{V_T}{Z_T} = \frac{240 < 0}{3k < -178.2°} = 0.08 < 178.2°A$$

EXAMPLE 2.4

With reference to the figure 2.4a. Find the value of the total current, I_T flowing in the circuit.

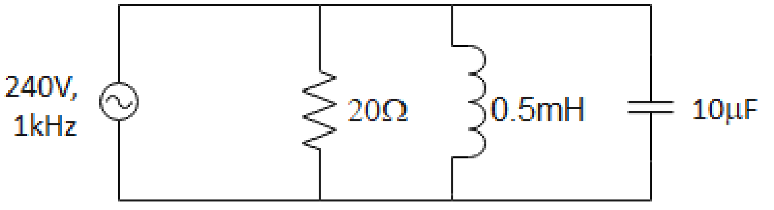


Figure 2.4a: Circuit for Example 2.4

SOLUTION 2.4

$$X_C = \frac{1}{2\pi fc} = \frac{1}{2\pi(1k)(10\mu)} = 15.92\Omega$$

$$X_L = 2\pi fL = 2\pi(1k)(0.5m) = 3.14\Omega$$

$$I_R = \frac{V}{Z_R} = \frac{240 < 0°}{20 < 0°} = 12 < 0°A@12 + j0$$

$$I_L = \frac{V}{Z_L} = \frac{240 < 0°}{3.14 < 90°} = 76.4 < -90°A@ - j76.4$$

$$I_C = \frac{V}{Z_C} = \frac{240 < 0°}{15.92 < -90°} = 15.1 < 90°A@j15.1$$

$$I_T = I_R + I_L + I_C$$

$$I_T = 12 + j0 - j76.4 + j15.1$$

$$I_T = 12 - j61.3A@62.5 < -78.9°A$$

EXAMPLE 2.5

Referring to figure 2.5a, A 15Ω resistor is connected in parallel with an inductance of 40mH across a 240V, 500Hz voltage supply. Calculate the current in each branch, the total current, the true power, reactive power and apparent power .

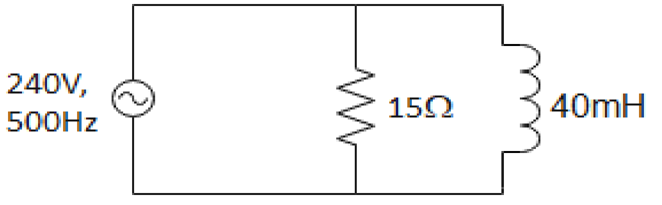


Figure 2.5a: Circuit for Example 2.5

SOLUTION 2.5

$$X_L = 2\pi fL = 2\pi(500)(40m) = 125.66\Omega$$

$$I_R = \frac{240}{15} = 16A$$

$$I_L = \frac{V_L}{X_L} = \frac{240}{125.66} = 1.91A$$

$$I_T = I_R - jI_L = 16 - j1.91 = 16.11 < -6.81°$$

$$True\ Power, P = IVcos\theta = (16.11)(240)cos6.81$$

$$True\ Power, P = 3.84kW$$

$$Reactive\ Power, Q = IVsin\theta = (16.11)(240)sin6.81$$

$$Reactive\ Power, Q = 458.47VAR$$

$$Apparent\ Power, S = IV = (16.11)(240)$$

$$Apparent\ Power, S = 3.87kVA$$

EXAMPLE 2.6

A 100V, 50Hz supply is applied to a capacitive circuit. The current flowing is 2A and power dissipated is 100W. Determine the value of the resistance and capacitance.

SOLUTION 2.6

$$Power\ dissipated, P = IV\ cos\theta$$

$$Cos\theta = \frac{P}{IV}, \qquad cos\theta = \frac{100}{2x100} = 0.5$$

$$I_T = \frac{V_T}{Z_T}$$

$$\therefore Z_T = \frac{V_T}{I_T} = \frac{100}{2} = 50\Omega$$

$$Power\ factor, cos\theta = \frac{R}{Z}$$

$$R = Z\ cos\theta$$

$$R = 50\ (0.5) = 25\Omega$$

$$X_C = \sqrt{50^2 - 25^2} = 43.30\Omega$$

$$X_C = \frac{1}{2\pi fc}\ , \qquad \therefore\ C = \frac{1}{2\pi f X_c}$$

$$C = \frac{1}{2\pi(50)43.30}\ , \qquad \therefore\ C = 73.5\mu F$$

EXAMPLE 2.7

Find the real power absorbed by the resistor in the resistive circuit shown in figure 2.7a.

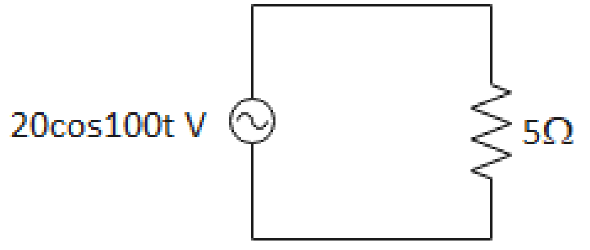


Figure 2.7a: Circuit for example 2.7

SOLUTION 2.7

$$V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{20}{\sqrt{2}} = 14.14V$$

$$I_{rms} = \frac{14.14}{5}, = 2.82A$$

$$\therefore P = I_{rms}\ x\ V_{rms} = 2.82x14.14 = 40W$$

EXAMPLE 2.8

For the inductive circuit shown in figure 2.8a, find the reactive power of the inductor.

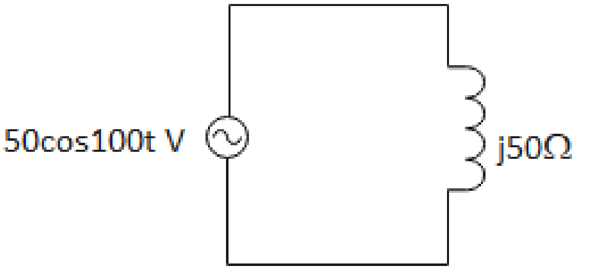


Figure 2.8a: Circuit for example 2.8

SOLUTION 2.8

$$V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{50}{\sqrt{2}} = 35.36V$$

$$I_{rms} = \frac{35.36}{j50} \text{ , } j50 = 50 < 90°$$

$$\therefore I_{rms} = \frac{35.36}{50 < 90°} = 0.707 < -90°$$

$$Q = I_{rms} x V_{rms} sin\theta$$

$$Q = 0.707x35.36sin90°$$

$$Q = 25VAR$$

EXAMPLE 2.9

For the capacitive circuit shown in Figure 2.9a, find the reactive power of the capacitor.

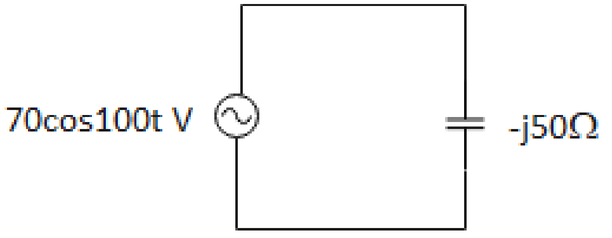


Figure 2.9a: Circuit for example 2.9

SOLUTION 2.9

$$V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{70}{\sqrt{2}} = 49.5V$$

$$I_{rms} = \frac{49.5}{-j50}, -j50 = 50 < -90°$$

$$\therefore I_{rms} = \frac{49.5}{50 < -90°} = 0.99 < 90°$$

$$Q = I_{rms} x V_{rms} sin\theta$$

$$Q = 0.99x59.5sin - 90°$$

$$Q = -49VAR$$

EXERCISE

TOPIC 3 – RESONANCE

EXAMPLE 3.1

Given R = 2Ω, L = 1mH and C = 0.4μF. AC voltage supply is 9V. Calculate the total impedance, maximum current and resonant frequency of the RLC series circuit when the circuit is at resonance. Determine the voltage across the capacitor.

SOLUTION 3.1

Total impedance at resonance

Z = R

= 2Ω

Maximum current

$\mathrm{I} = \frac{V}{Z} = \frac{9}{2} = 4.5A$

Resonant frequency

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$= \frac{1}{\sqrt{(1X10^{-3})(0.4X10^{-6})}}$$

$$= 50K\ rad/s$$

Voltage across capacitor

$$V_C = I\ X_C$$

$$= 2\ X\ \frac{1}{(50X10^{3})(0.4X10^{-6})}$$

$$= 100V$$

EXAMPLE 3.2

In an RLC parallel circuit, given R = 8 kΩ, L = 0.2mH and C = 8μF. The voltage source is 10 sin ωt. Calculate the resonant frequency, total impedance and minimum current.

Solution 3.2

The resonant frequency

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$= \frac{1}{\sqrt{(0.2X10^{-3})(8X10^{-6})}}$$

$$= 25\ Krads/s$$

Total impedance, Z = R = 8KΩ

Minimum current, $I_{min} = \frac{V}{Z_{max}}$

$$= \frac{10\ \angle -90^0}{8000}$$

$$= 1.25\ \ \angle - 90^0$$

EXAMPLE 3.3

In the RLC series circuit shown in Figure 3.3a, voltage supply is 9 V. Calculate the bandwidth and the circuit quality factor.

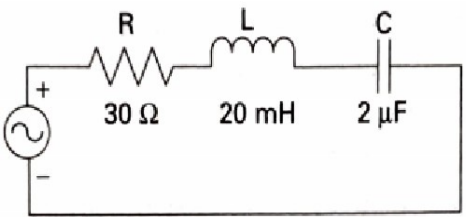


Figure 3.3a Circuit diagram for Example 3.3

Resonance frequency

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$= \frac{1}{\sqrt{(20X10^{-3})(2X10^{-6})}}$$

$$= 5\ Krads/s$$

Bandwidth,

$$B = \frac{R}{L} = \frac{30}{20\ x\ 10^{-3}}$$

$$= \ 1500\ rad/s$$

Or

$$B = \frac{1500}{2\pi}$$

$$= 238.7\ Hz$$

Quality factor.

$$Q = \frac{\omega_0}{B}$$

$$= \frac{5000}{1500}$$

$$= 3.33$$

EXAMPLE 3.4

For a tank circuit, R is 20Ω, the resonant frequency is 120 rad /s and the bandwidth is $0.02f_o$ Hz. Calculate the value of L and C.

Solution 3.4

Resonant frequency

$$f_0 = \frac{\omega_0}{2\pi}$$

$$= \frac{120}{2\pi}$$

$$= 19.09\ Hz$$

Bandwidth, B = $0.02f_o$ = 0.38 Hz

B =0.38 X 2π = 2.388 rad/s

$$B = \frac{1}{RC} = \frac{1}{20C} = 2.388$$

Therefore

$$C \frac{1}{20\ X\ 2.388} = 20.94\ mF$$

To find value of L

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$120\ rad/s = \frac{1}{\sqrt{(L)(20.94X10^{-3})}}$$

$$L = \frac{1}{{\omega_0}^2 C}$$

$$= \frac{1}{(120^2\ x\ 20.94\ x10^{-3})}$$

$$= 3.32mH$$

=

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e ISBN 978-967-2897-15-6
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