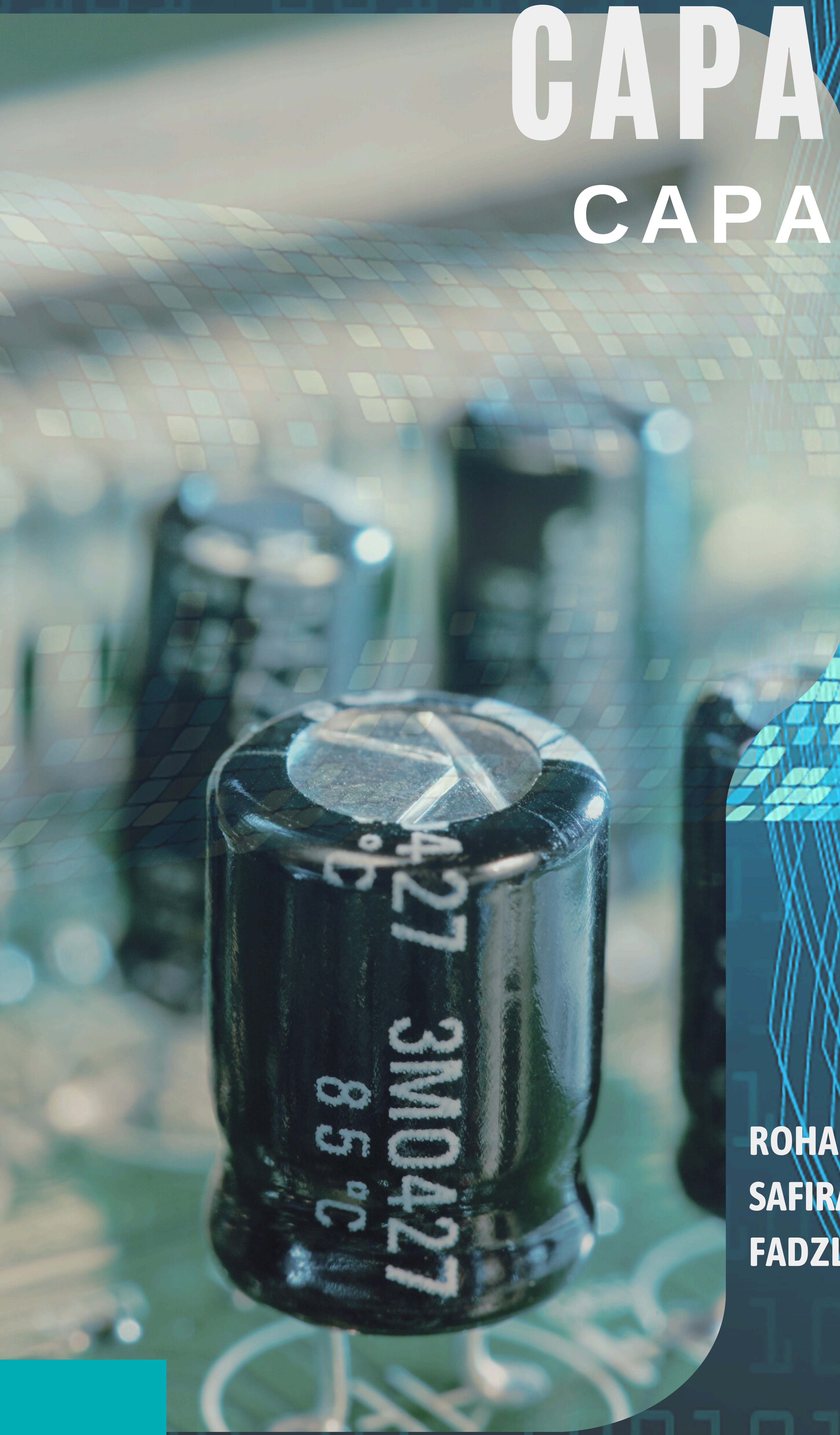
POLITEKNIK MALAYSIA
PORT DICKSON

# ELECTRICAL CIRCUIT 1

# CAPACITORS

## CAPACITANCE

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SAFIRA BINTI DIN
FADZLIDA BINTI SHAMSUDIN

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# QUICK NOTES & EXERCISES

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# PREFACE

This book introduces the basics of capacitors and their role in electric/electronic circuits. It covers the structure, functionality, and behavior of capacitors in various configurations. By examining the principles of capacitance, dielectric properties, and the equations that govern capacitor performance, readers will develop the ability to analyze and apply these components in practical scenarios. The charging and discharging processes are explained with clarity to foster intuitive and technical mastery of time-dependent circuit behavior.

This chapter blends foundational theory with hands-on insight, making it ideal for students and anyone aiming to understand capacitors in a simple way.

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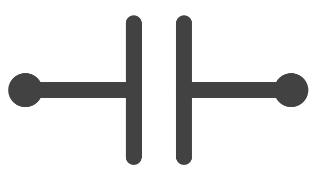
## CAPACITORS AND CAPACITANCE

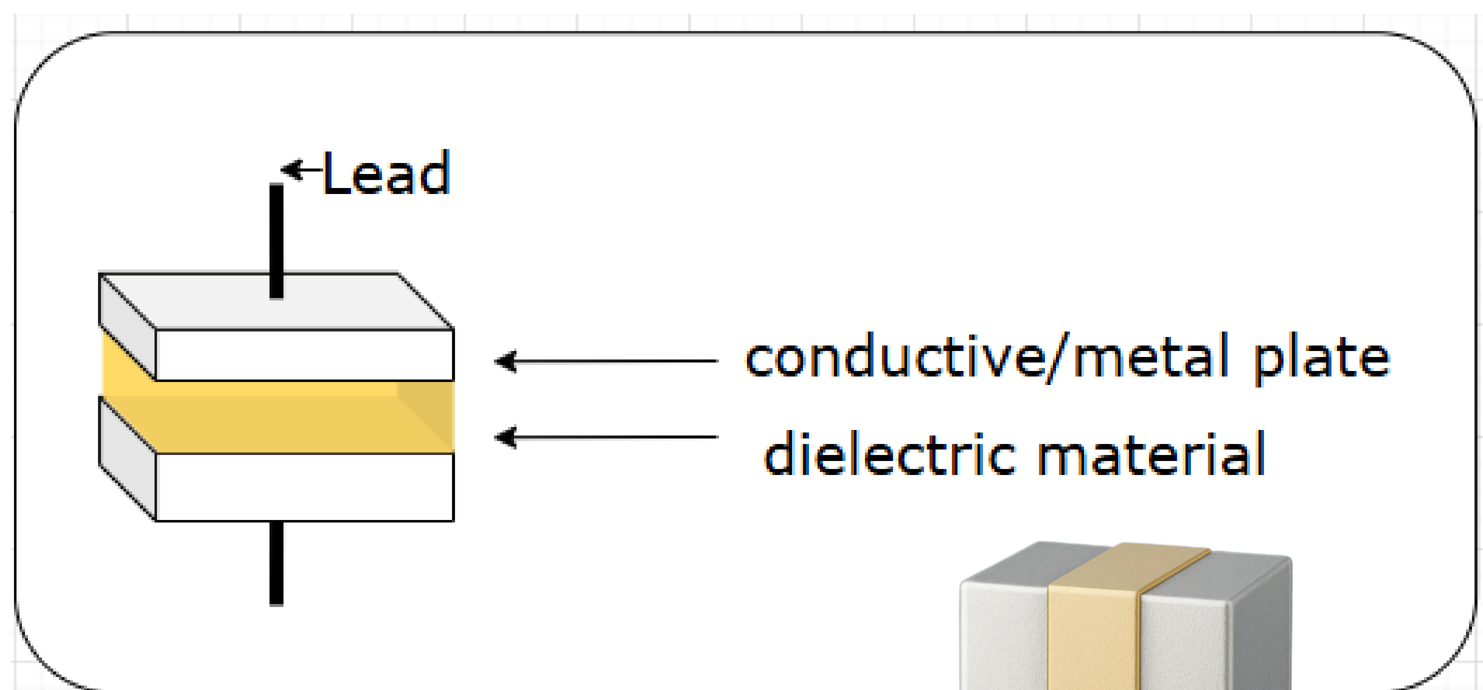
# CAPACITOR

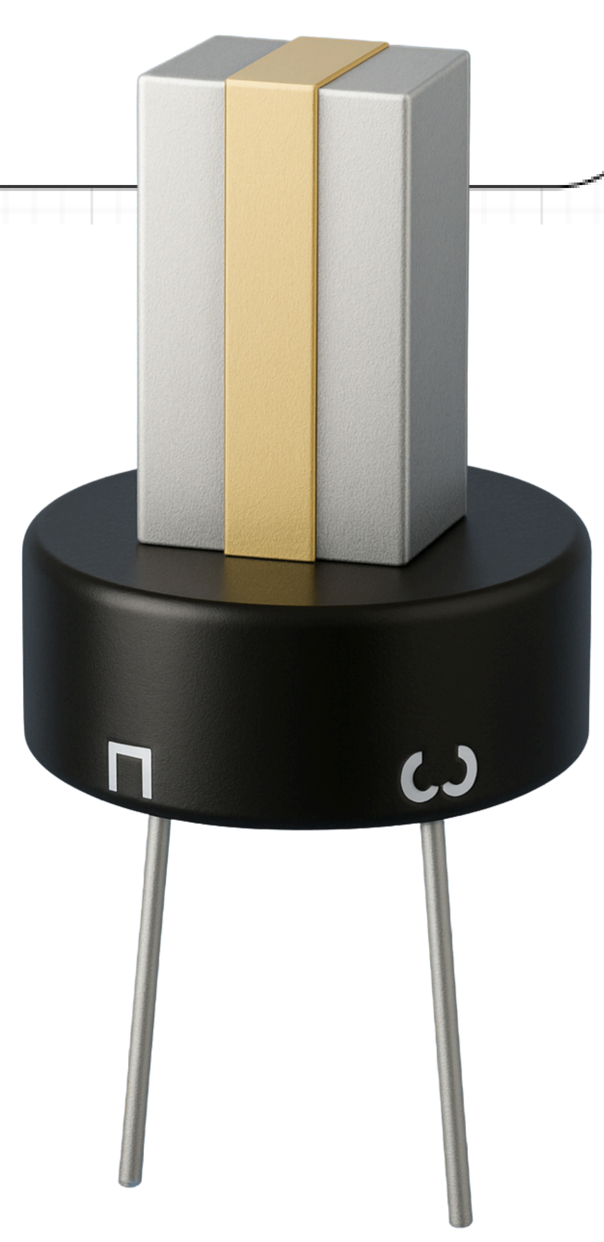


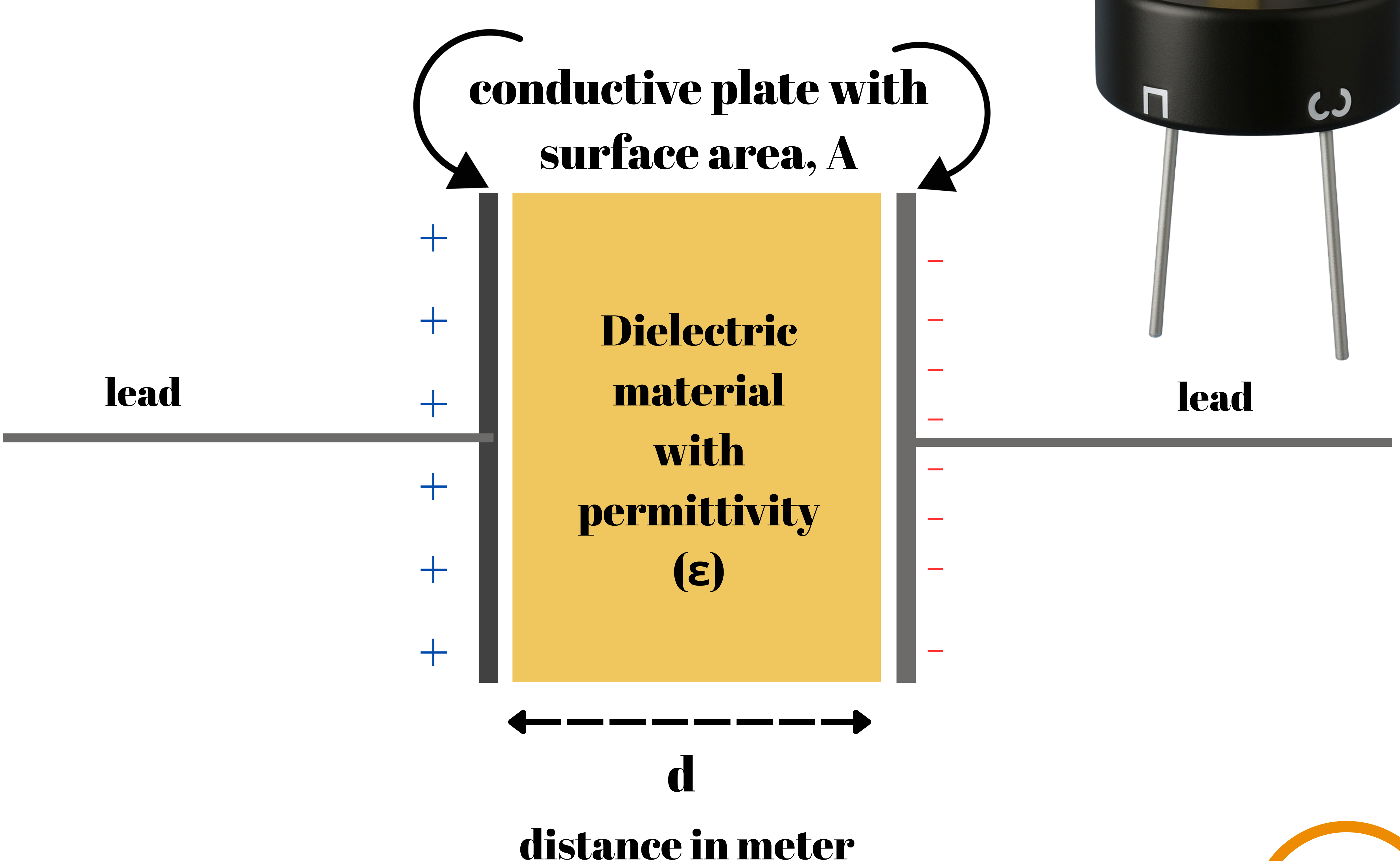
symbol

- Capacitor is an electrical component that stores electrical charge/energy.
- It stores electrical energy in the form of an electric field between its plates
- capacitors can help stabilize voltage and reduce noise.

A Capacitor consists of two conducting plates separated by an insulator (or dielectric).





# CAPACITOR

## HOW IT WORKS

A capacitor is made of two conductive metal plates separated by an insulating material called a dielectric. When voltage is applied across the plates, one plate accumulates positive charge, and the other collects an equal amount of negative charge. The electric field created between these plates stores energy.

**Capacitors are often classified based on the type of dielectric.**

**dielectric (ceramic, paper, vacuum) determines the capacitor's performance.**

**Permittivity (ε) describes how much a dielectric material permits electric field lines to pass through it.**

$$\varepsilon = \varepsilon_r \times \varepsilon_O$$

- Vacuum Permittivity ($\varepsilon_0$):
- The permittivity of free space is $8.85 \times 10^{-12}$ F/m
- Relative Permittivity ($\varepsilon_r$)
- Absolute Permittivity (ε)– The actual permittivity of a material.

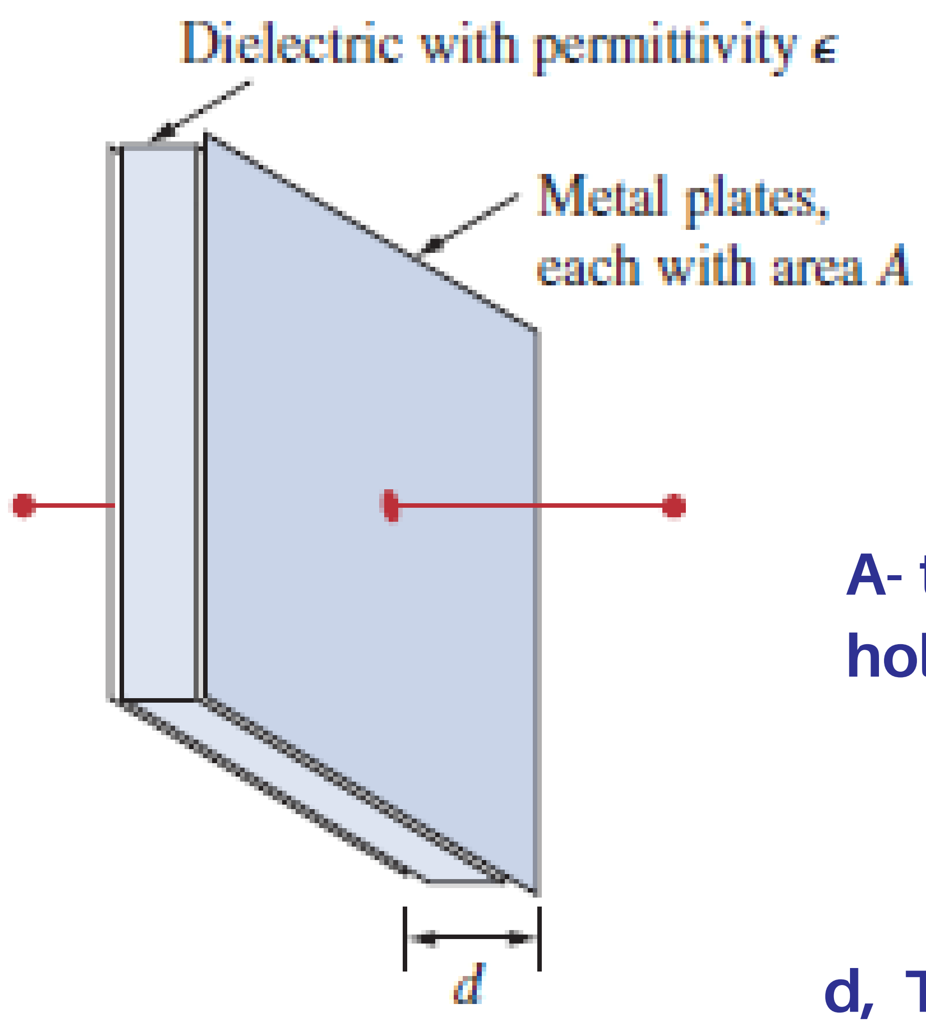



**Figure 6.1**
A typical capacitor.

**the conducting plates : aluminium foil**

**The metal plates store the electric charges.**

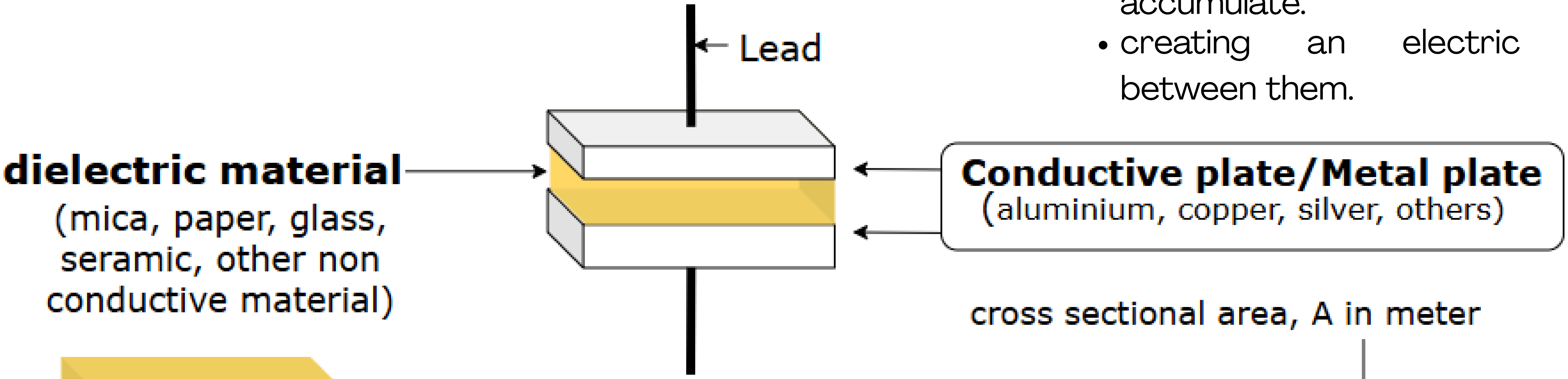
**A- the larger the plate area, the more charge it can hold.**

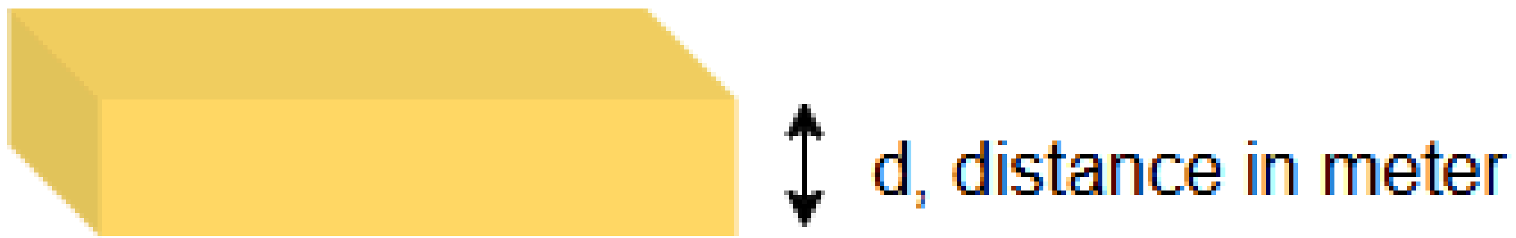
**d, The closer the plates, the stronger the electric field and higher the capacitance.**

The distance between the plates, their surface area, and the type of dielectric between them determine how much charge they can store (the capacitance).

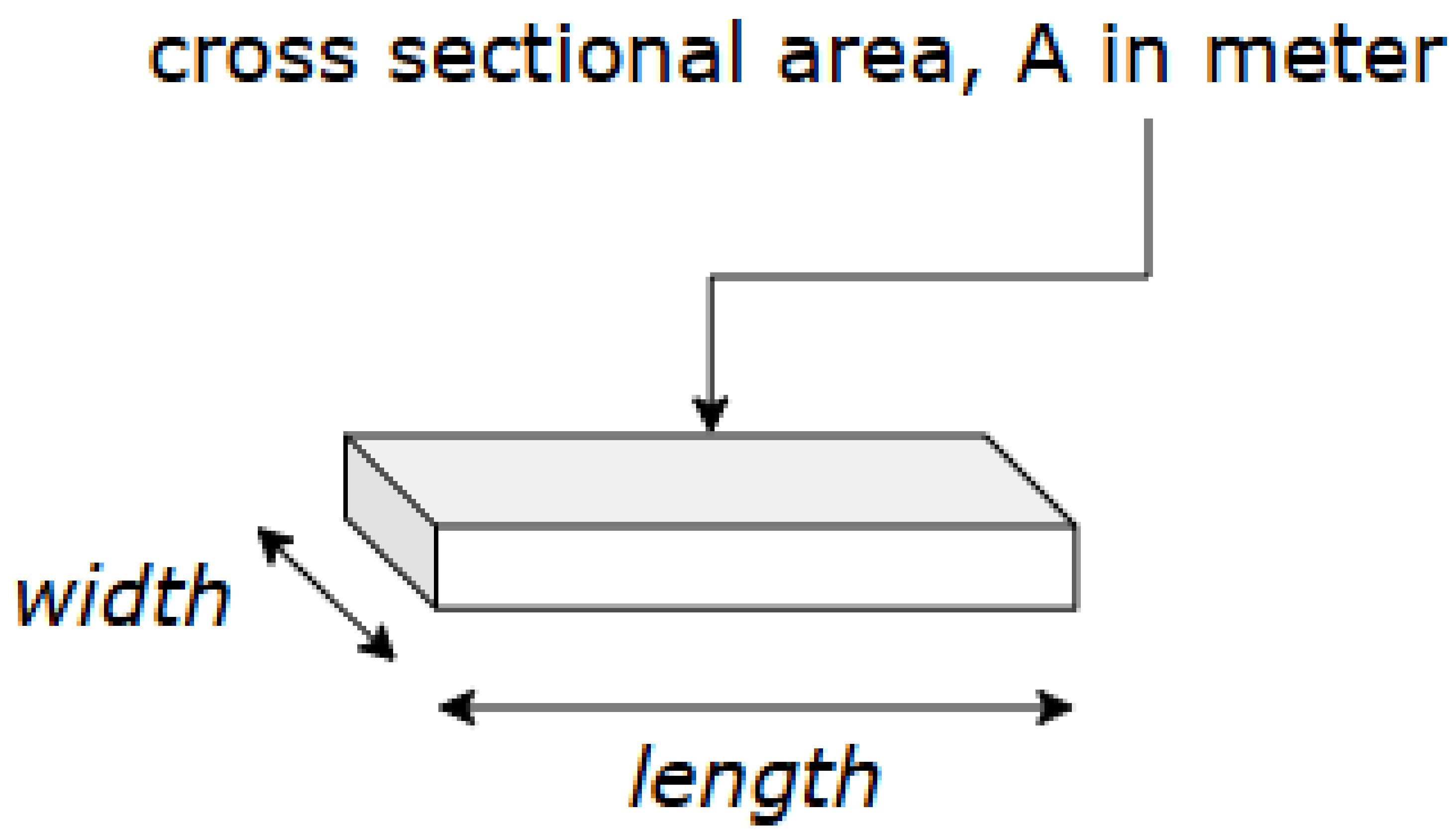
# CAPACITOR CONSTRUCTION

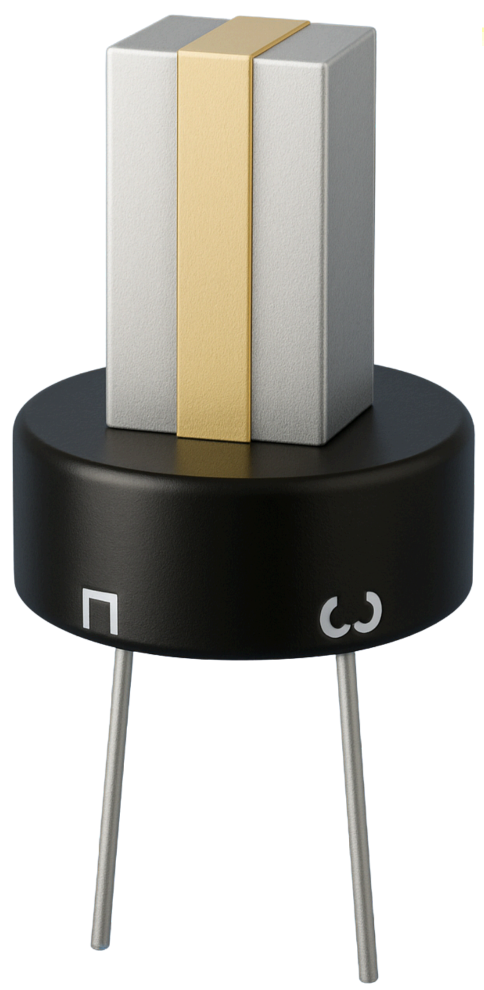
- Metal plates serve as surfaces where electric charge can accumulate.
- creating an electric field between them.




Each material has its **own permittivity,** ε



- dielectric is an insulator, not allow current to flow through it easily

- allows more charge to accumulate on the plates when its molecules is polarized

# CAPACITORS & CAPACITANCE IN A CIRCUIT

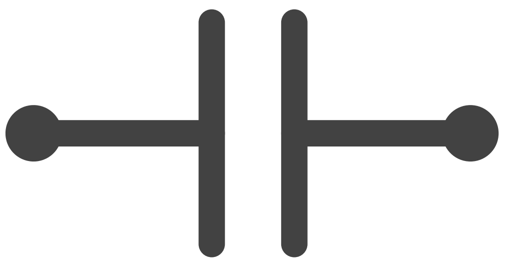
## Capacitor, C

- A capacitor is a passive element designed to store energy in its electric field.
- A capacitor consists of two conducting plates separated by an insulator (or dielectric).
- The amount of charge stored, represented by Q, is directly proportional to the applied voltage and can be expressed mathematically as :

$$Q = Cv$$

- Q is the electric charge stored on the capacitor (in coulombs, C),
- C is the capacitance of the capacitor (in farads, F),
- V is the voltage across the capacitor (in volts, V)

## Capacitance, unit: Farad



- The property of a CAPACITOR to store electricity in a form of electric charge is called its CAPACITANCE, measured in Farad (F).

- Capacitance is the ratio of the charge on one plate of a capacitor to the voltage difference between the two plates, measured in farads (F).

# CAPACITOR APPLICATIONS

**APPLICATIONS**

Capacitive Sensing

Motor Starting & Control

Energy Storage in Smart Agriculture

Wireless Communication & IoT

- Used in RF circuits and antennas
- aquaponic systems- reliable sensor data transmission

Power Supply Smoothing Used in solar-powered systems

- Filters out ripples from DC voltage

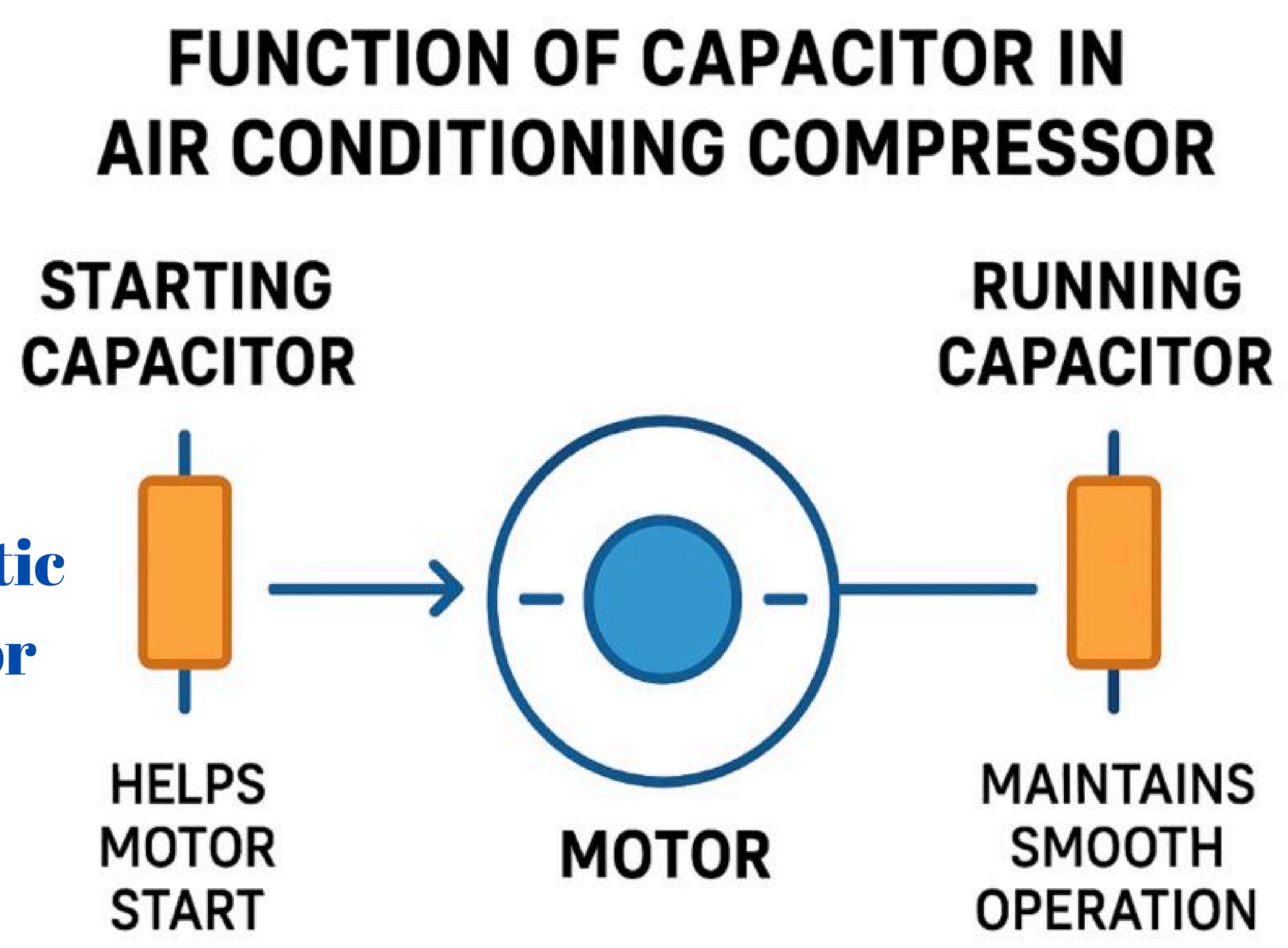
## How Capacitors Store Energy

Capacitors store energy by accumulating electric charge on their plates.

The electric field between the plates holds this energy.

# CAPACITOR APPLICATIONS

electrolytic capacitor




oil-filled/metalized Polypropylene capacitor

**Fluorescent Lamps**

Oil-Filled Capacitor

Metallized Polypropylene Film Capacitor

Electrolytic Capacitor

- Power Factor Correction: Improves efficiency by reducing reactive power
- boosting voltage
- Noise Reduction

# TYPES OF CAPACITOR

- **VARIABLE CAPACITORS**
  - mechanically control
    - trimmer

    - tunning
  - electrically control
- **FIXED CAPACITORS**
  - polar
    - electrolytic

    - super
  - non-polar
    - ceramic

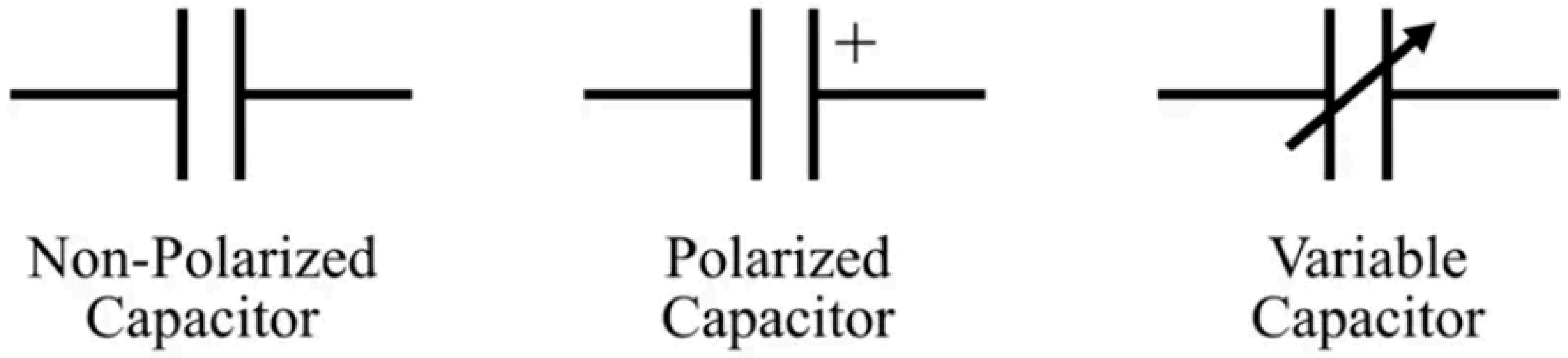
      

    - mica
    - film

## TYPES OF CAPACITOR

| Feature | Fixed Capacitor | Variable (Flexible) Capacitor |
|---|---|---|
| **Capacitance Value** | Constant / Cannot be changed | Can be adjusted / changed manually or automatically |
| **Structure** | Simple, compact | More complex, with movable parts |
| **Polarity** | Can be polarized or non-polarized | Usually, non-polarized |
| **Common Types** | Ceramic, Electrolytic, Film, Tantalum | Tuning Capacitor, Trimmer Capacitor |
| **Application** | Power supply, filtering, decoupling | Radio tuning, oscillators, frequency adjustment |
| **Cost** | Generally cheaper | Usually more expensive |
| **Size** | Small to large | Typically, small |



# CAPACITIVE CIRCUIT

## CAPACITIVE CIRCUIT QUANTITY

### PERMITTIVITY

- Permittivity is the ability of a material to store an electric field in the polarization of the medium.
- The ratio of electric flux density (D) to the electric field strength (E) is called absolute permittivity (ε) of a dielectric and can be expressed as

$$\varepsilon = \frac{D}{E} \text{ (Unit : } \frac{Farad}{metre} \text{)}$$

### ELECTRIC FLUX (Φ)

- Known as the total of electric lines of force that moves out from positive charge in electric field
- electric flux, φ is measured in coulombs
- electric flux is propotional with electric field line

$$\therefore Electric\ Flux, \varphi = \text{Charge, Q}$$

### ELECTRIC FIELD STRENGTH (E)

- the ratio between the potential difference (V) or voltage and the thickness of the dielectric (d), it can be expressed as

$$E = \frac{V\ (unit{:}Volt)}{d\ (unit{:}metre)}$$

DID YOU KNOW

voltage gradient/potential gradient = unit is Volt/meter

A **voltage gradient** is a difference in electrical potential across a distance or space

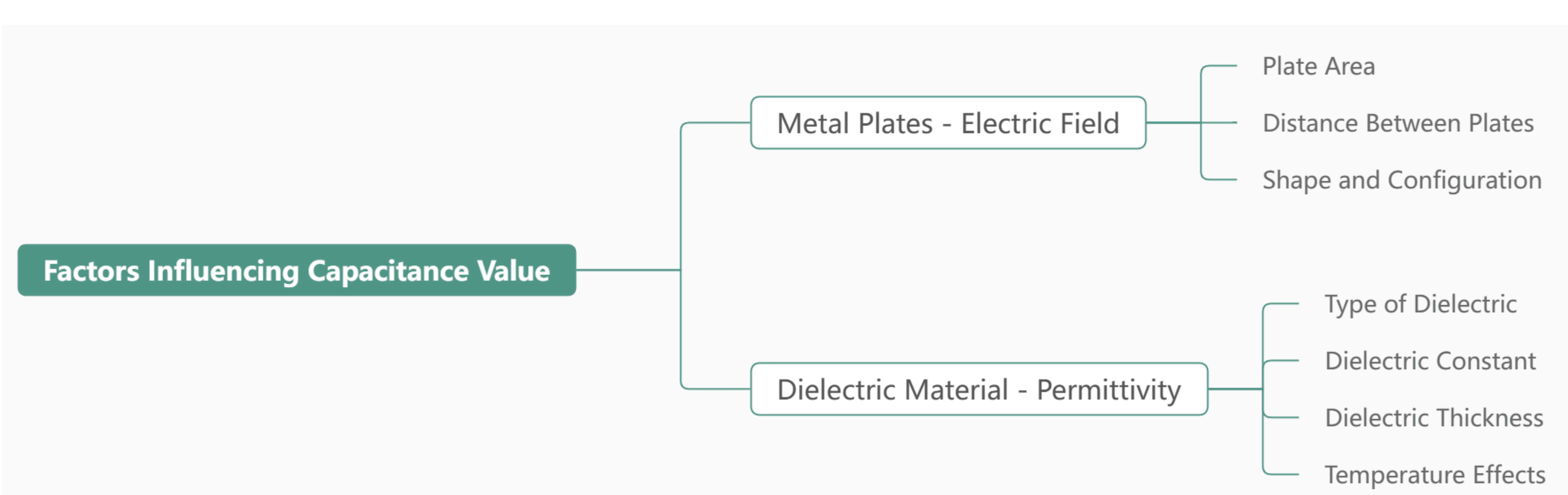
### ELECTRIC FLUX DENSITY (D)

Electric flux density is the ratio between the charge of the capacitor and the surface area of the capacitor plates and can be expressed as

$$D = \frac{Q\ (unit{:}Coulomb)}{A\ (unit{:}metre^2)}$$

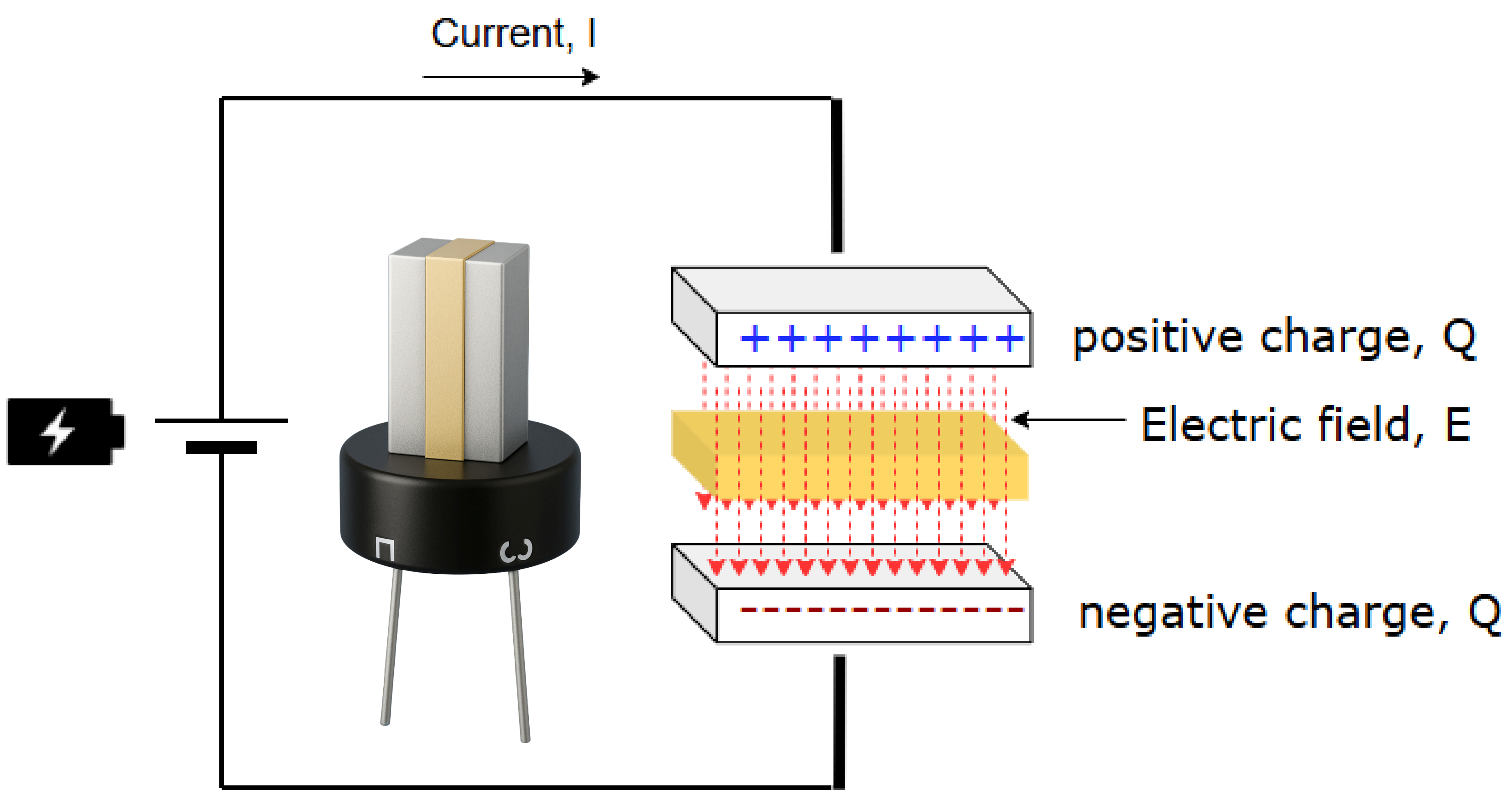
## FACTOR INFLUENCE CAPACITANCE

$$capacitance, C = \frac{Q}{V_c} = \frac{\varepsilon_o\, \varepsilon_r\, A}{d}\ (unit : F)$$

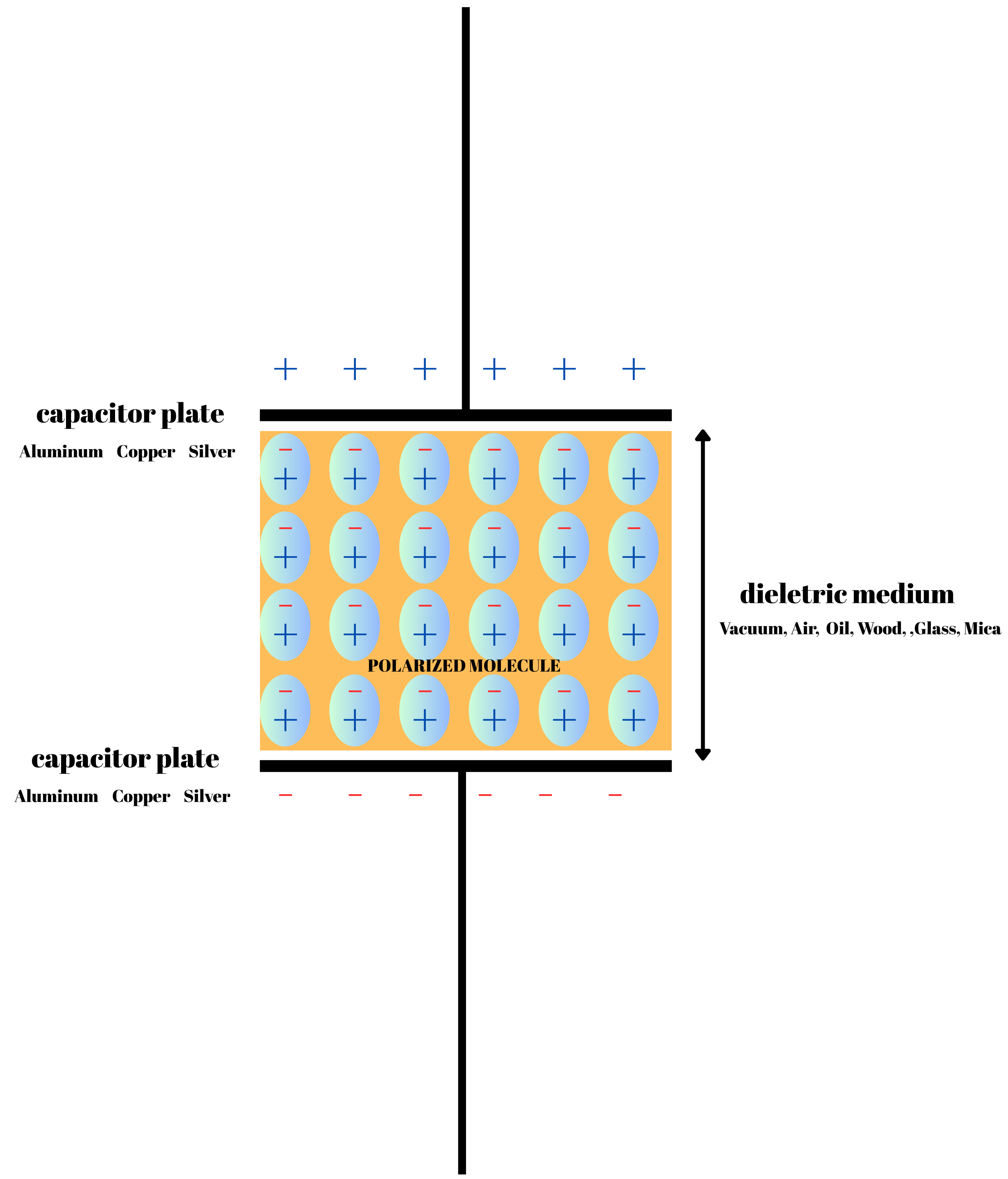


# CAPACITIVE CIRCUIT

## HOW CHARGE CAN BE STORED?

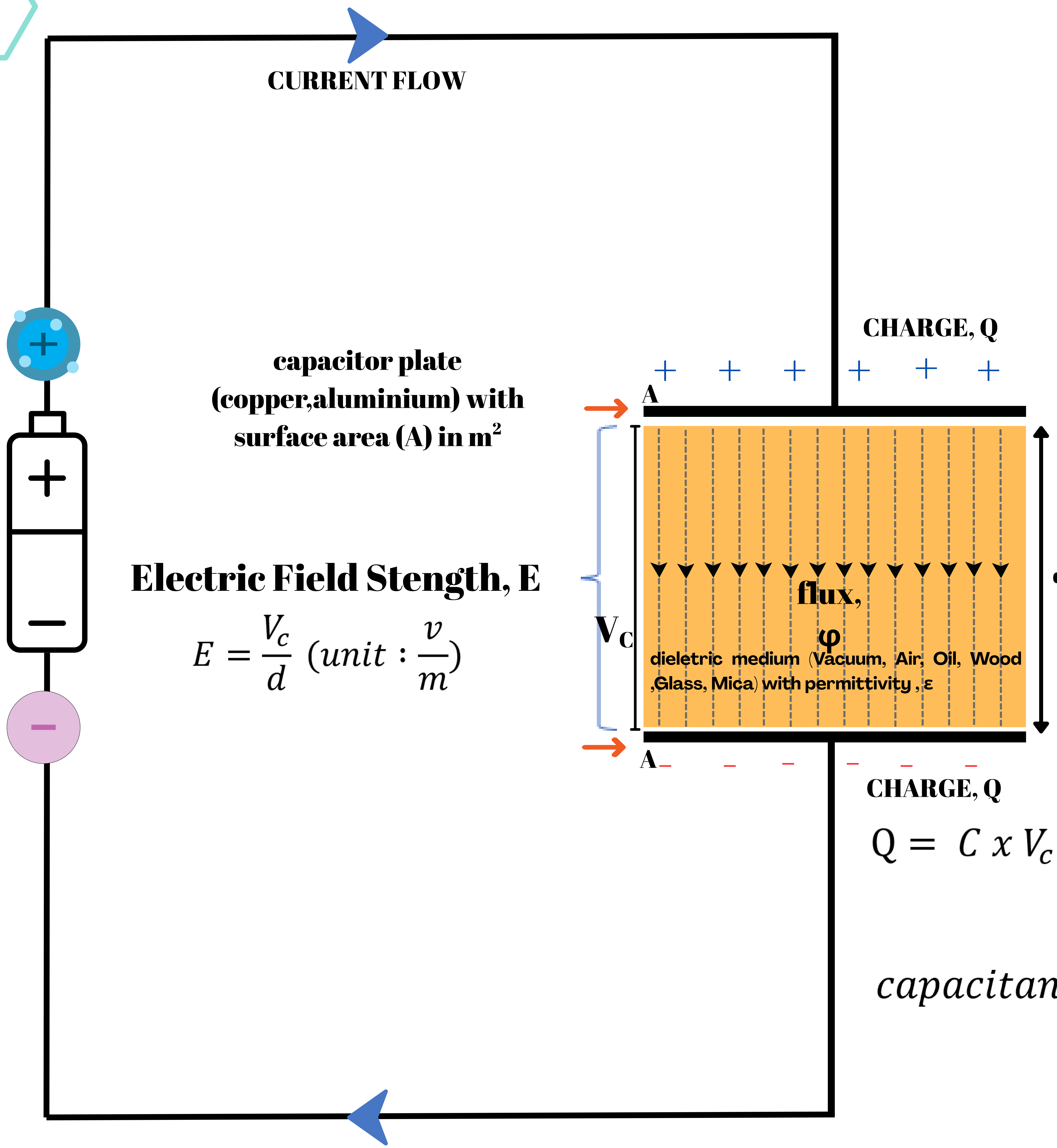



**when a voltage is applied across a capacitor' plates**




**Charge separation** ➡ **Electric field formation** ➡ **Dielectric polarization** ➡ **charge can be stored**

## CAPACITIVE CIRCUIT



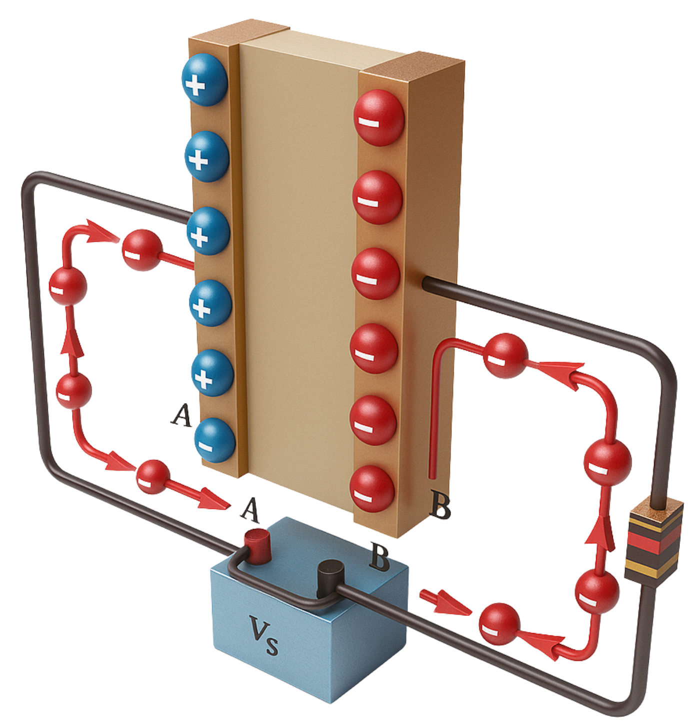

**Electric Field Stength, E**

$$E = \frac{V_c}{d}\ (unit : \frac{v}{m})$$

$$\varepsilon = \varepsilon_o\ x\ \varepsilon_r = \frac{D}{E}\ (unit : \frac{F}{m})$$

$$Q = C\ x\ V_c$$

$$capacitance, C = \frac{Q}{V_c} = \frac{\varepsilon_o\ \varepsilon_r\ A}{d}\ (unit : F)$$




electrons flow from plate A to plate B as the capacitor charges when connected to a battery.

## DIELECTRIC MATERIAL

The purpose of permittivity is to describe how well a material can store electrical energy when exposed to an electric field.

### DIELECTRIC

- A dielectric in a capacitor is a type of insulating material placed between the two conducting plates of the capacitor, or
- insulating material that is sandwiched between the two conducting plates.

-Examples : air, mica, ceramic, paper

### DIELECTRIC ROLE :

- to increase the capacitor’s ability to store electrical energy
- to reduce the electric field between the plates
- preventing charge from directly passing through.

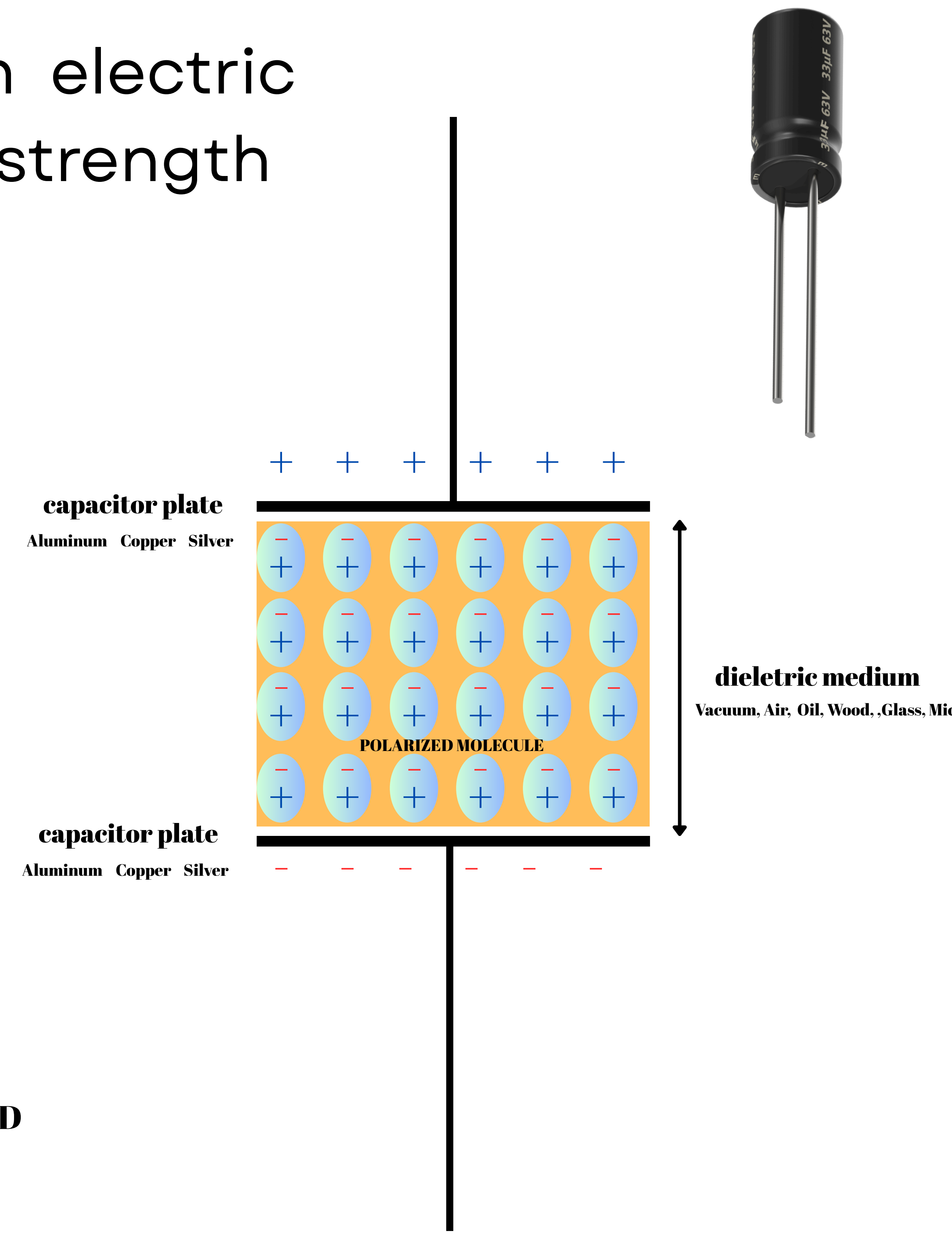
### HOW A CAPACITOR ABLE TO HOLD CHARGE ??

insulating material become polarized (molecular polarization-molecules can shift slightly) when placed in an electric field. The polarization enhances the capacitor's performance by increasing its capacitance. Higher permittivity means greater capacitor.

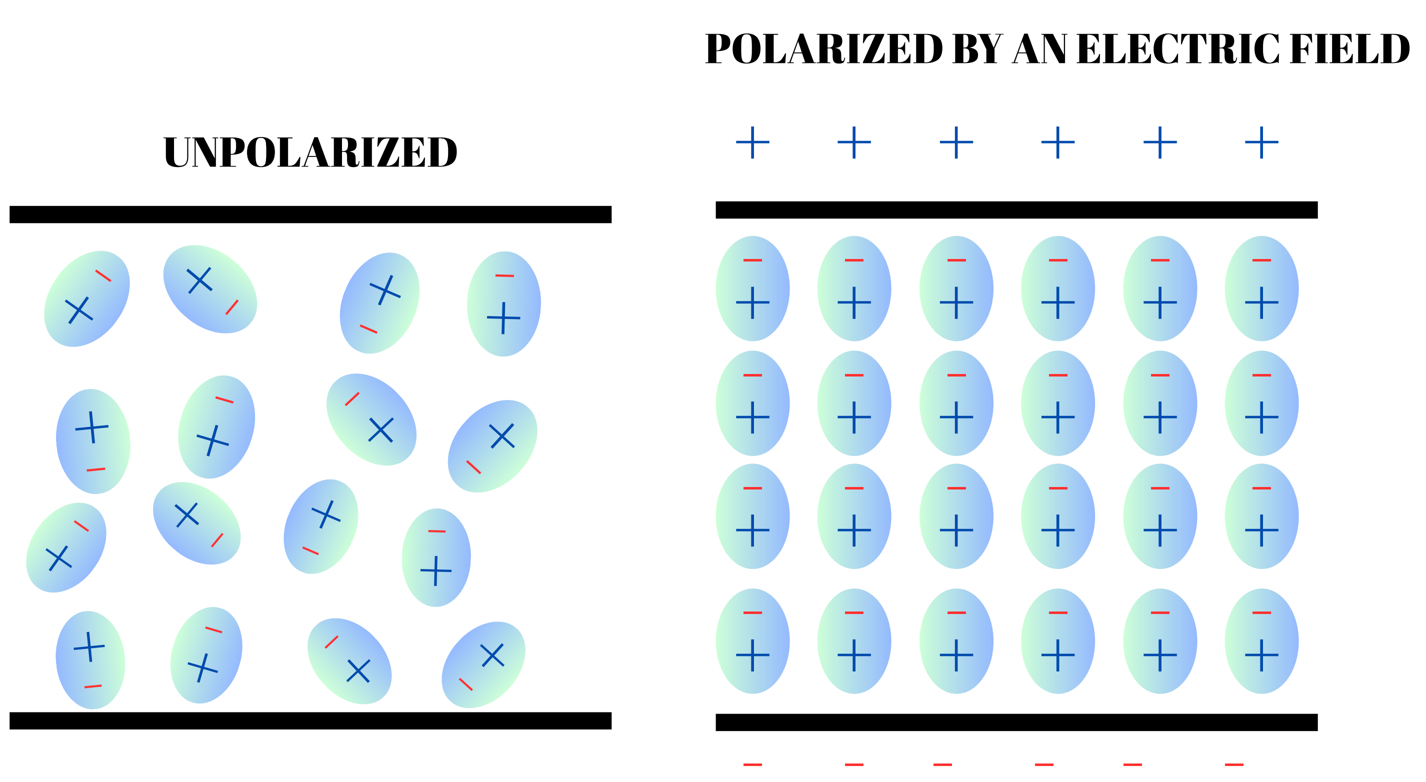
# CAPACITIVE CIRCUIT

## DIELECTRIC MATERIAL

Every dielectric has a maximum electric field it can withstand - dielectric strength

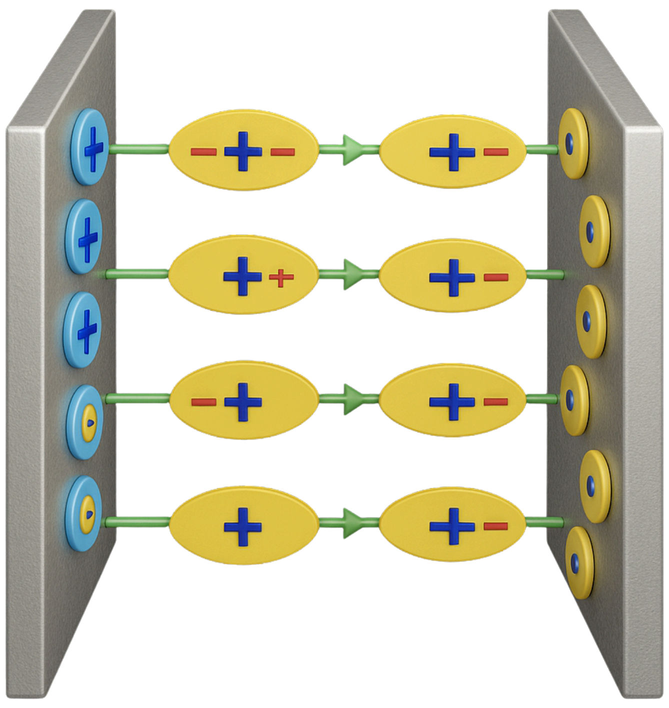



- external electric field is created when voltage aplied across the capacitor plates.
- free charges accumulated on the plates.
- dielectric molecul is polarized




A dielectric material in a capacitor serves to polarize its molecules when placed within an electric field between the metal plates

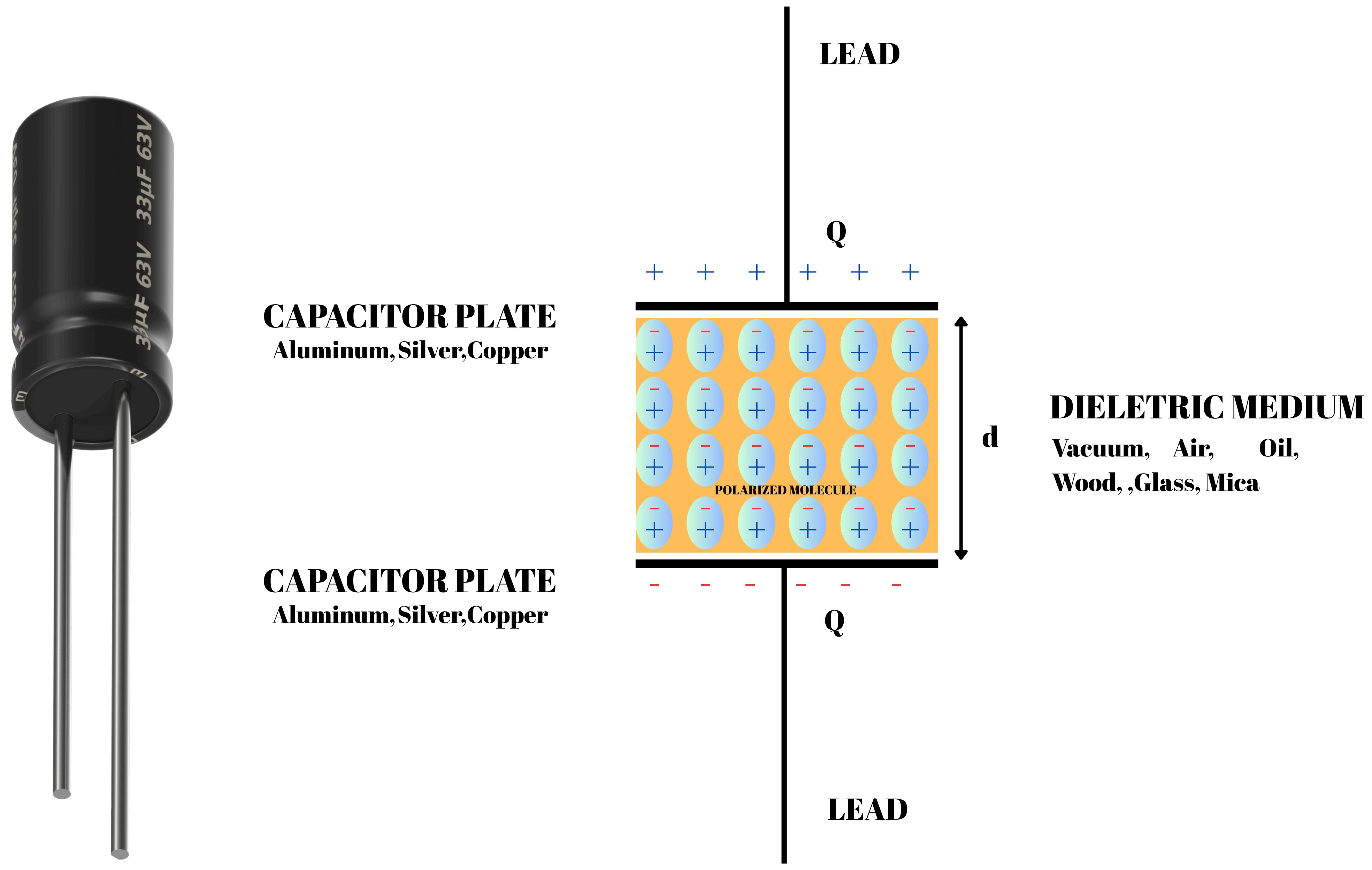
dielectric to weaken the field (external field) when its being polarized.



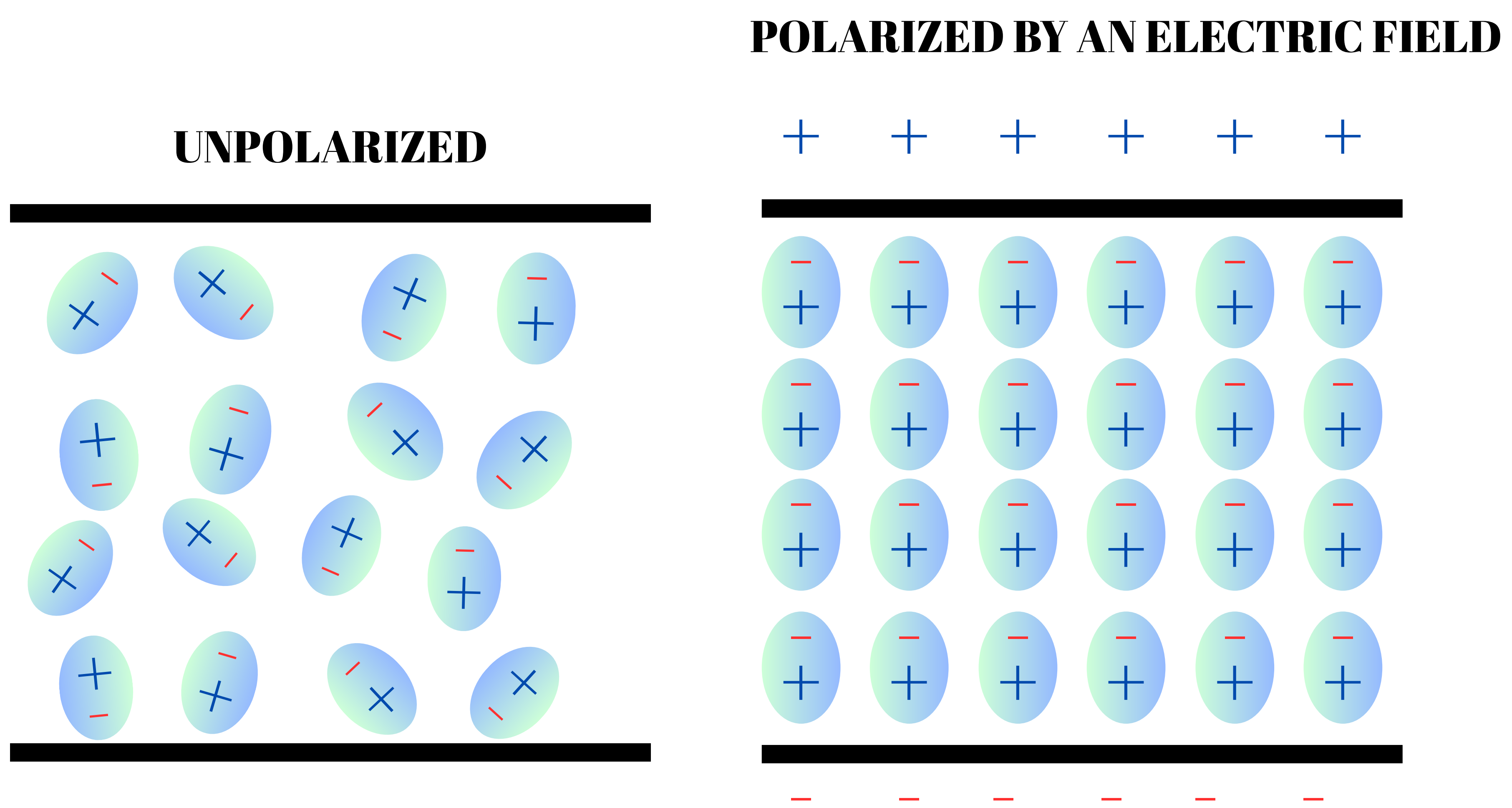
induced field opposes the external electric field.

# CAPACITIVE CIRCUIT

## DIELECTRIC




**Dielectric materials consist of molecules that can become polarized—their positive and negative charges slightly separate—when placed in an electric field.**




- This polarization opposes the external electric field, effectively reducing the net electric field within the capacitor
- A material with higher permittivity can polarize more strongly, which means it provides greater opposition to the electric field, leading to more charge storage for the same voltage.
- higher permittivity - increases the ability of a dielectric to weaken the internal electric field, which results in increased capacitance

$$C = \frac{\varepsilon_o \, \varepsilon_r \, A}{d} \quad (unit : F)$$

**higher permittivity ⇒ lower internal electric field ⇒ higher capacitance.**

# PERMITTIVITY (ε)

- Permittivity is the ability of a material to store an electric field in the polarization (*surface flux density*) of the medium.
- The ratio of electric flux density (D) to the electric field strength (E) is called absolute permittivity (ε) of a dielectric and can be expressed as :

$$\varepsilon = \frac{D}{E} \text{ (Unit : } \frac{Farad}{metre}\text{)}$$

$$D = \frac{Q}{A} \;;$$

$$E = \frac{D}{\varepsilon} = \frac{Q}{A\varepsilon} \;;\; \varepsilon = \varepsilon_0 \, \varepsilon_r$$

Permittivity of the medium (how well the medium can support an electric field)

In a capacitor, if you insert a dielectric with high permittivity, the surface flux density increases — allowing the plates to store more charge without increasing the voltage.

"Surface flux density, often called electric flux density (D) or displacement flux, describes how much electric field (or electric flux) passes through a given area in a material (the "medium")."

# TYPES OF PERMITTIVITY

1. MATERIAL PERMITTIVITY ($\varepsilon$) $= \varepsilon_r \times \varepsilon_0$

2. DIELECTRIC CONSTANT (K) also known RELATIVE PERMITTIVITY ($\varepsilon_r$)

3. ELECTRIC CONSTANT also known vacuum permittivity ($\varepsilon_0$)

DIELECTRIC CONSTANT = RELATIVE PERMITTIVITY

$$K = \varepsilon_r = \frac{\varepsilon}{\varepsilon_o}$$

1 MATERIAL PERMITTIVITY

2 (dielectric constant = relative permittivity)

3 VACCUMM PERMITTIVITY

**8.854 × 10^-12 F/m**

$$\varepsilon = \varepsilon_o \ x \ \varepsilon_r = \frac{D}{E} \ (unit : \frac{F}{m})$$

the dielectric constant (K) is a measure of the ability of a material to store electrical energy and is defined as the ratio of the capacitance (or permittivity) of the dielectric material to the capacitance of a vacuum. Therefore, all the capacitance values are related to the permittivity of vacuum. Each different dielectric material has its own value of permittivity.

$\frac{\varepsilon_m}{\varepsilon_0} = dielectric\ constant(K) = relative\ permittivity(\varepsilon_r)$ (no unit)

$\varepsilon_r = \frac{\varepsilon_m}{\varepsilon_0}$ ; $\varepsilon_m = \varepsilon = \varepsilon_r$ x $\varepsilon_0$ [ unit $\frac{F}{m}$ ]

The permittivity of a dielectric material , $\varepsilon_m$ or $\varepsilon$ ( is measured in Farad per meter) (F/m or $F.m^{-1}$)

The permittivity of vacuum, $\varepsilon_0$, sometimes called the electric constant is $8.85\times10^{-12}$ F/m.

Relative permittivity $\varepsilon_r$ of free space/vacuum is 1

# DIELECTRIC CONSTANT ALSO KNOWN AS RELATIVE PERMITTIVITY

## ($\varepsilon r$)

| No. | Dielectric Material | Relative Permittivity ($\varepsilon r$) |
|---|---|---|
| 1 | Vacuum | 1 |
| 2 | Air | 1.0006 ≈ 1.0 |
| 3 | Paper | 2.0 – 3.0 |
| 4 | Mica | 5.0 – 7.0 |
| 5 | Glass | 4.0 – 10.0 |
| 6 | Ceramic (Class I) | 6.0 – 12.0 |
| 7 | Ceramic (Class II) | 100 – 10,000 |
| 8 | Teflon (PTFE) | 2.1 |
| 9 | Polyester (Mylar) | 3.0 – 3.5 |
| 10 | Polypropylene | 2.2 – 2.3 |
| 11 | Polystyrene | 2.5 – 2.7 |
| 12 | Barium titanate ($BaTiO_3$) | 1,000 – 7,000+ |

where $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m (vacuum permittivity).

$$\varepsilon = \varepsilon_r \times \varepsilon_0$$

# PERMITTIVITY (ε) VERSUS CAPACITANCE (C)

Thus, the capacitance of a capacitor also could be obtained from:

$$\varepsilon = \frac{D}{E} ;\ E = \frac{D}{\varepsilon}$$

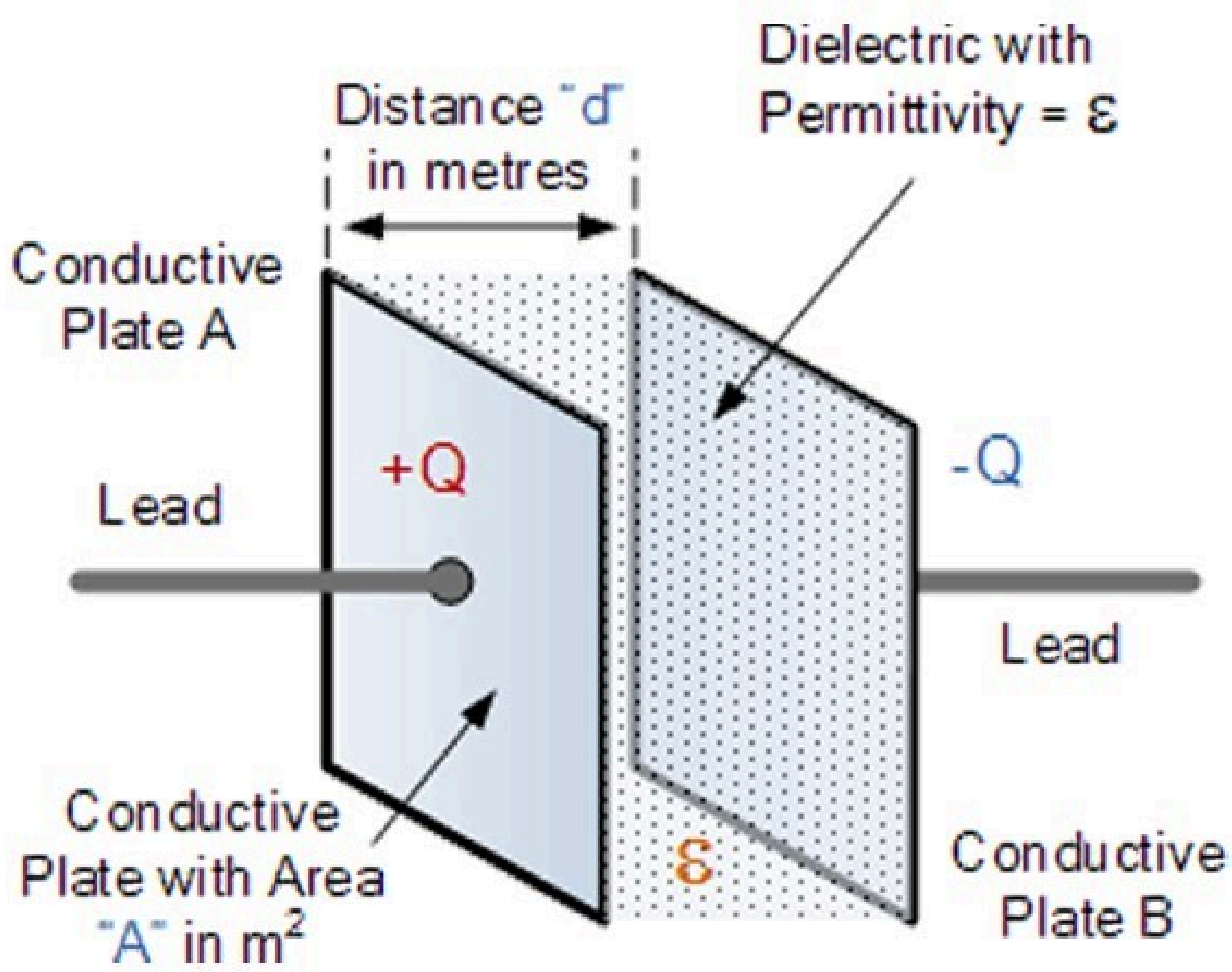
$$D = \frac{Q}{A} ;\ Q = DA \qquad \varepsilon = \varepsilon_0 \ x \ \varepsilon_r$$

$$E = \frac{V}{d} ;\ V = Ed$$

$$C = \frac{Q}{V} = \frac{Q}{Ed} = \frac{Q}{Ed} = \frac{DA}{Ed} = \frac{DA}{\left(\frac{D}{\varepsilon}\right)d} = \frac{\varepsilon A}{d} = \frac{\varepsilon_0 \ x \ \varepsilon_r \ x \ A}{d}$$

$$C = \frac{\varepsilon_r \varepsilon_0 A}{d}$$

$\varepsilon_r$ = relative permittivity
$\varepsilon_0$ = permittivity of free space / electric constant (F/m)
A = surface area of plate ($m^2$)
d = distance between the plate (m)




$$capacitance, C = \frac{Q}{V_c} = \frac{\varepsilon_o \ \varepsilon_r \ A}{d} \ (unit : F)$$

## CAPACITANCE

capacitance, C of a capacitor is the ratio of the charge Q per plate to the applied voltage.

$$C = \frac{Q}{V_C}$$

| | |
|---|---|
| $C$ | = Capacitance |
| VC | = Potential Different of capacit |
| Q | = charge (coulombs) |

$$C = \frac{Q}{V_C} = \frac{\varepsilon A}{d}$$

| | |
|---|---|
| $C$ | = Capacitance |
| d | = spacing between the plates $(m)$ |
| A | = surface area $(m^2)$ |
| ε | = permittivity $(farad/meter)$ |

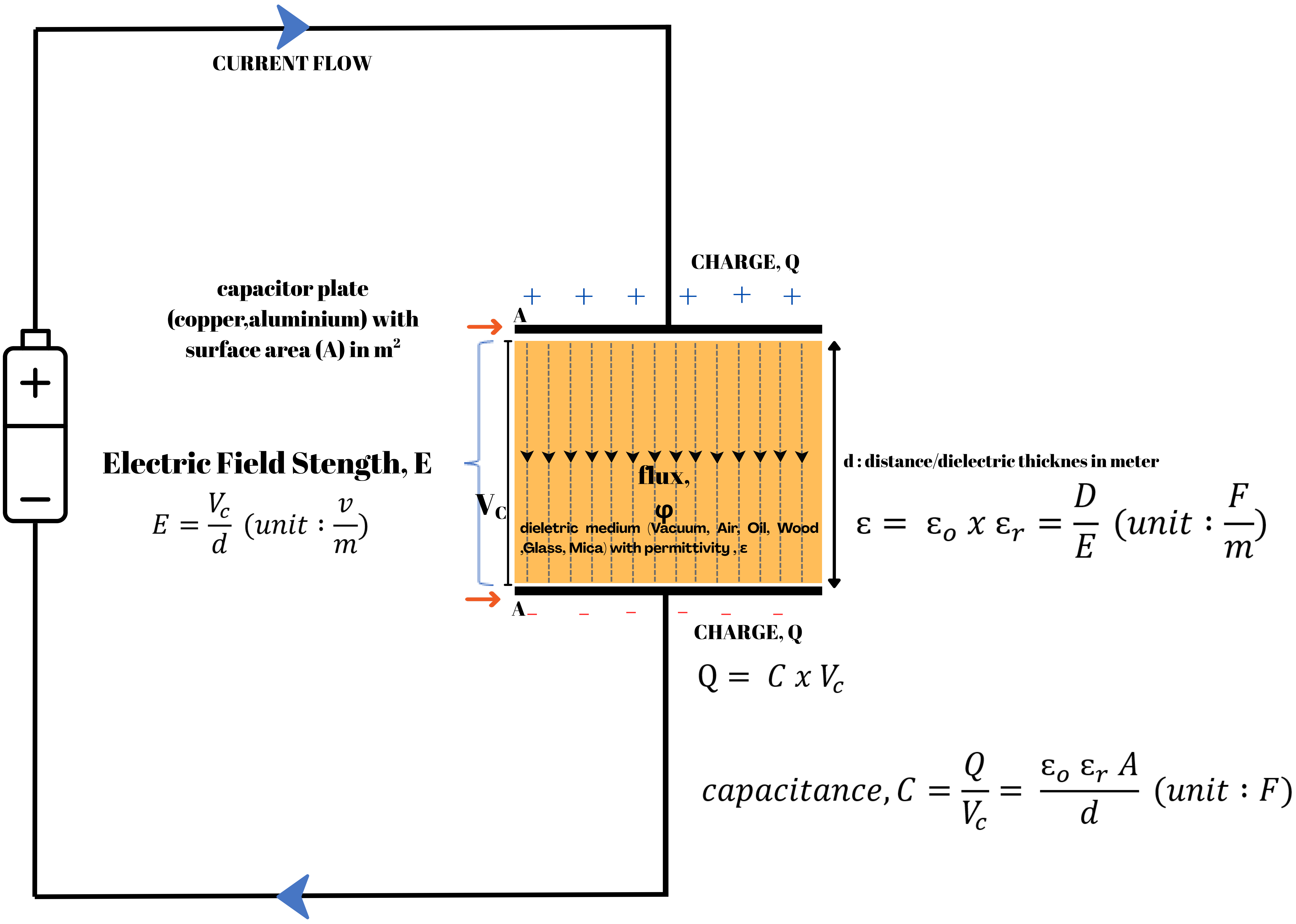
**3 factors determine the value of the capacitance:**

1. The surface area of the plates—the larger the area, the greater the capacitance.

2. The spacing between the plates—the smaller the spacing, the greater the capacitance.

3. The permittivity of the material—the higher the permittivity, the greater the capacitance.

# CAPACITIVE CIRCUIT - EQUATION




permittivity

$$\varepsilon = \frac{D}{E} \;\; (\text{Unit} : \frac{Farad}{metre})$$

permittivity

$$\varepsilon = \varepsilon_r \; x \; \varepsilon_0 \;\; (\text{Unit} : \frac{Farad}{metre})$$

$\varepsilon_0 = 8.854 x \; 10^{-12} F/m$

permittivity

$$\varepsilon_r \; x \; \varepsilon_0 = \frac{D}{E} \;\; (\text{Unit} : \frac{Farad}{metre})$$

ELECTRIC FIELD STRENGTH (E)

$$E = \frac{V \; (unit{:}Volt)}{d \; (unit{:}metre)}$$

Flux density

$$D = \frac{Q \; (unit{:}Coulomb)}{A \; (unit{:}metre^2)}$$

capacitance

$$C = \frac{\varepsilon_r \varepsilon_0 A}{d}$$

$$C = \frac{Q \; (Charge)}{V \; (Potential \; Diff.)}$$

$I = \frac{Q(charge)}{t(time)}$, thus Q = It

Q = charge (coulomb)

t = time (second)

# EXERCISE

1. The flux density between two plates separated by mica of relative permittivity 5 is 2 μC/m2. Find the voltage gradient between the plates.

Ans : 45.2 Kv/m

2. Two parallel plates having a potential difference of 200V between them are spaced 0.8mm apart. What is the electric field strength? Find also the electric flux density when the dielectric between the plates is (a) air, and (b) polythene of relative permittivity 2.3

Ans :
250 Kv/m

2.213 $\mu Cm^{-2}$

5.089 $\mu Cm^{-2}$

**QUICK TIPS**

$E = \frac{v}{d}$ (Unit : $\frac{volt}{meter}$ )

$\varepsilon = \frac{D}{E}$ (Unit : $\frac{Farad}{metre}$ )

*voltage gradient means the electric field strength between the plates*

# EXERCISE

## FINAL SESI 2 : 2022/2023

1. With the aid of a diagram, describe the construction of a capacitor.
2. With aid of circuit diagram, express the formula for the total capacitance inseries and parallel connections.
3. A capacitor is charged to 150V and then discharged through a 60kΩ resistor. If the time constant of the circuit is 0.8s, calculate the value of the capacitor, the time for the capacitor voltage to fall to 20V, the current flowing when the capacitor has been discharging for 0.5s and the voltage drop across the resistor when the capacitor has been discharging for one second.

SESI 2 : 2023/2024

a) List TWO (2) types of polarity capacitor and TWO (2) types of non-polarity capacitor.

b) With the aid of related formula, explain the factors affecting capacitance.

c) Based on Figure A2(c), when switch is closed, calculate the time constant, current and potential difference through the capacitor after 6 seconds and energy stored in capacitor.

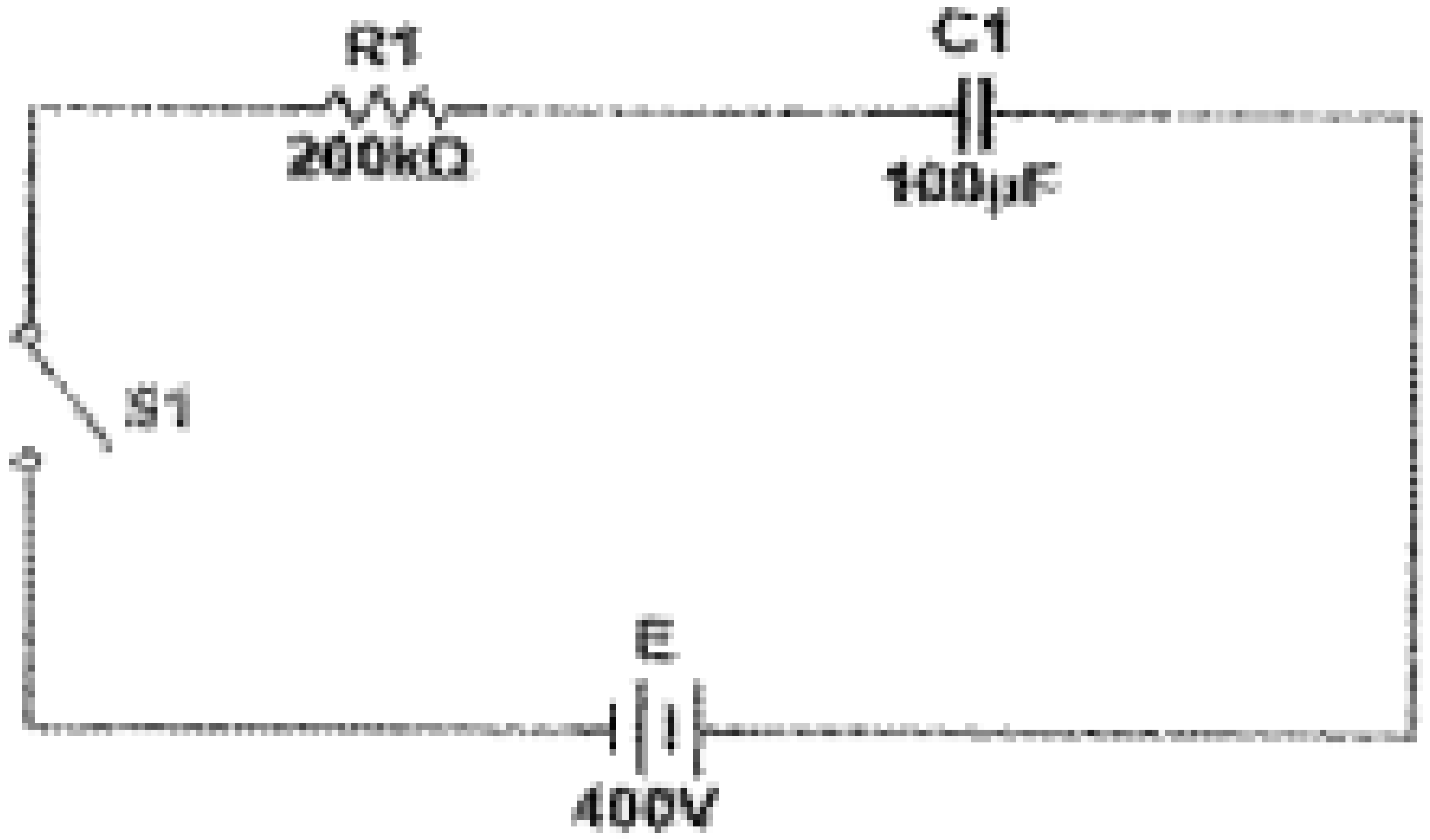
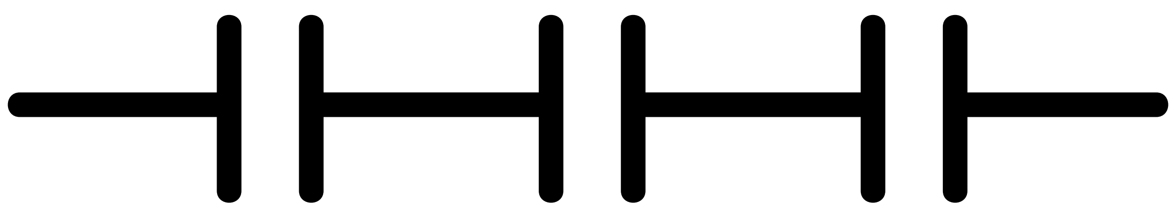


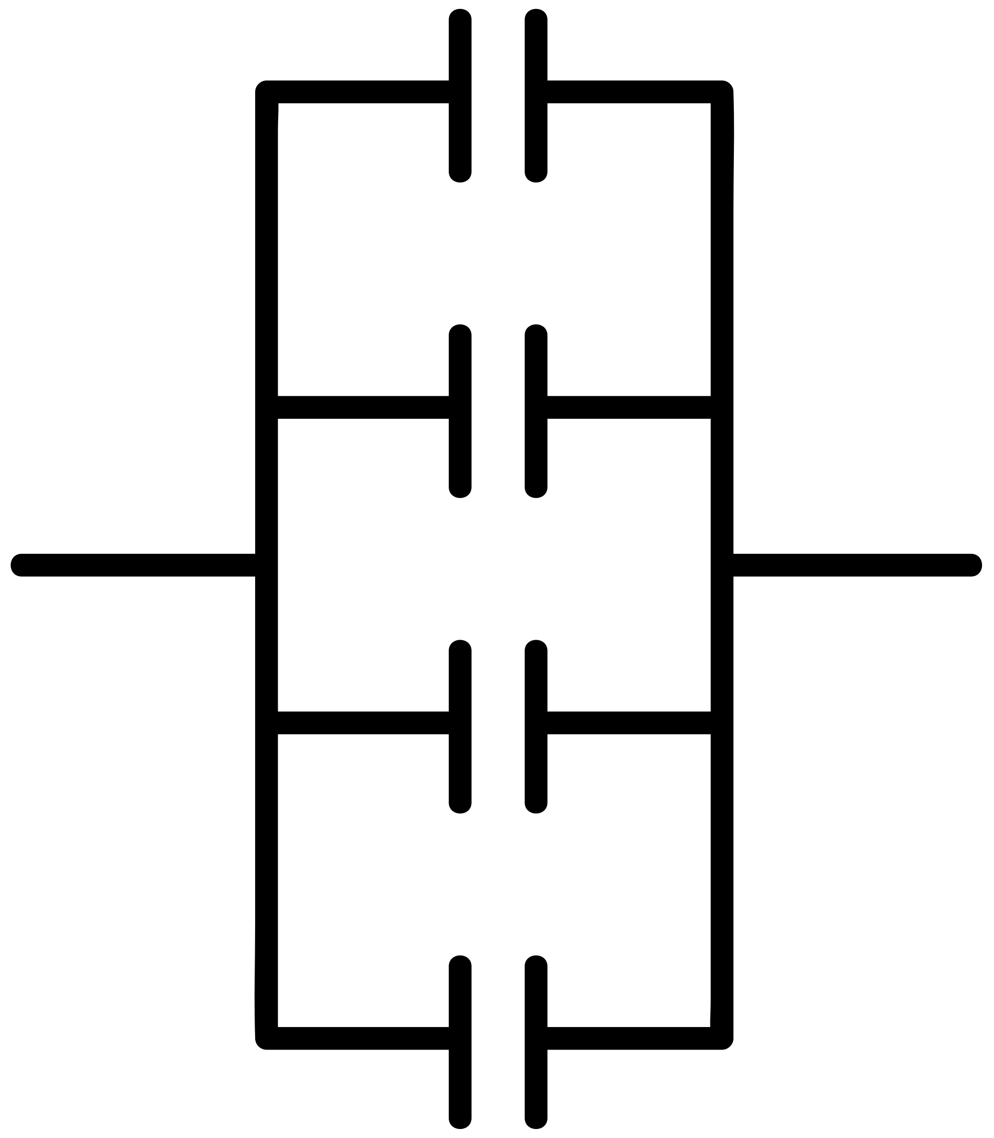
Figure A2(c)/ *Rajah A2(c)*

# CIRCUIT'S CONNECTION

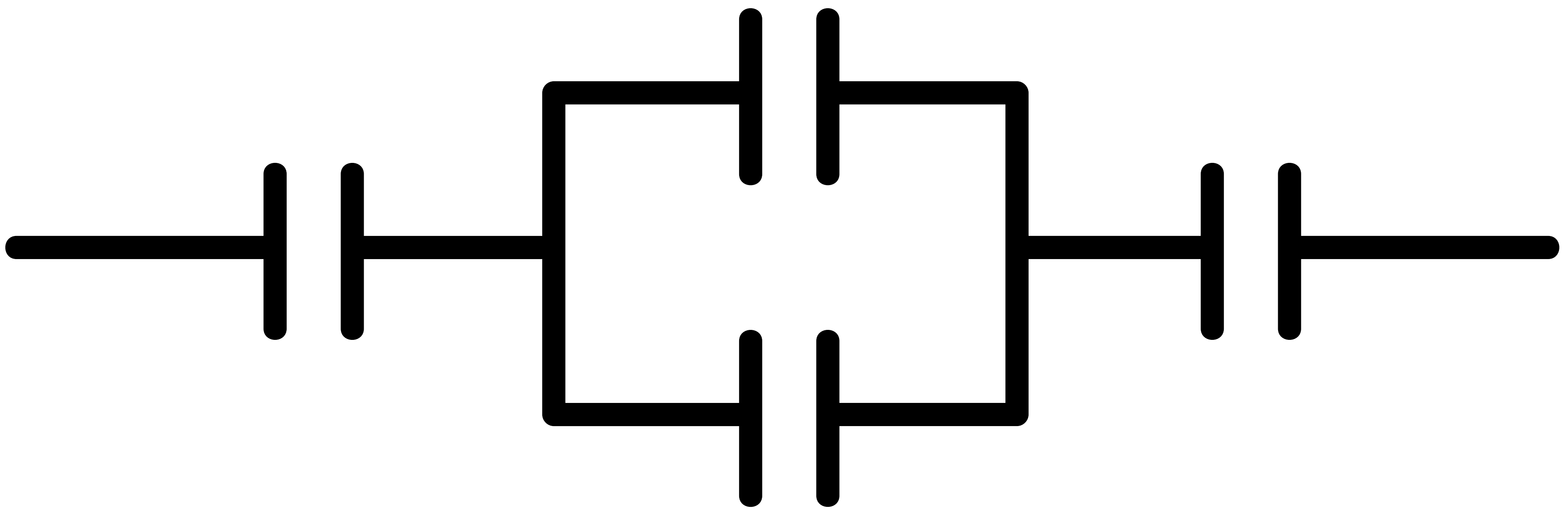
## SERIES



## PARALLEL

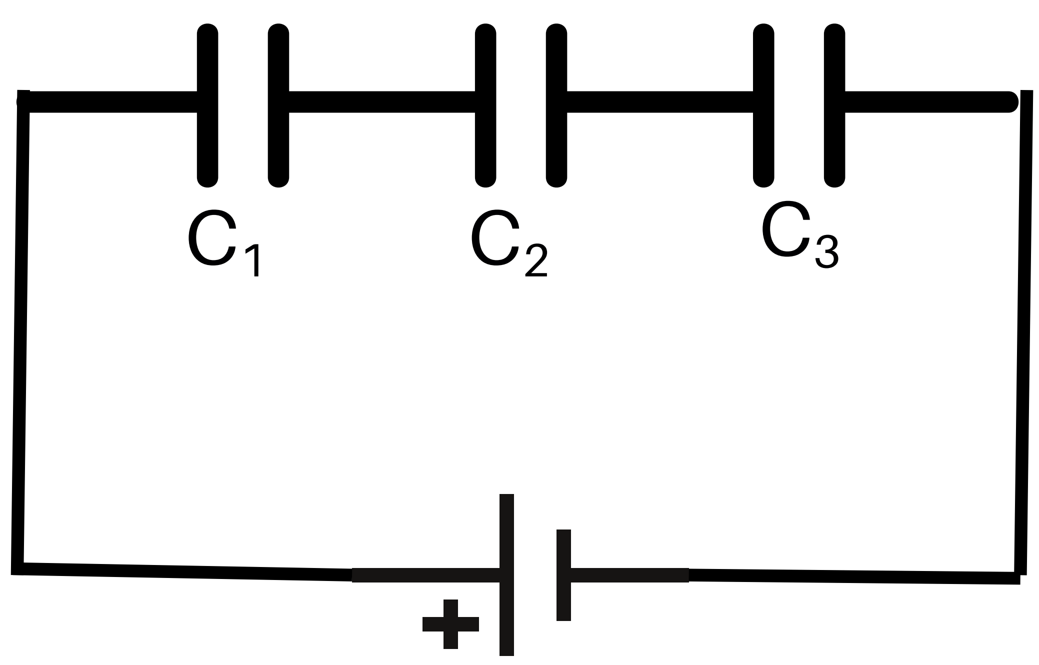


## SERIES - PARALLEL

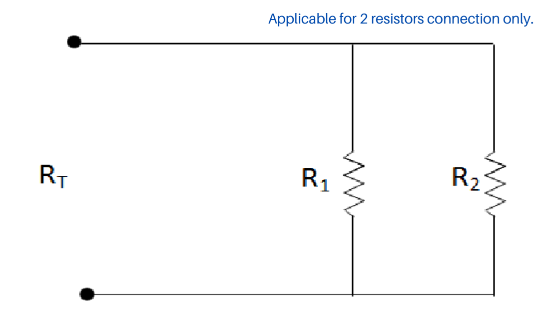


# CAPACITANCE EQUIVALENT CIRCUITS FOR SERIES AND PARALLEL CONNECTIONS

## SERIES




Equivalent resistance in parallel (2 resistors case)




$$R_T = \frac{R_1 \times R_2}{R_1 + R_2}$$

For two capacitors only:

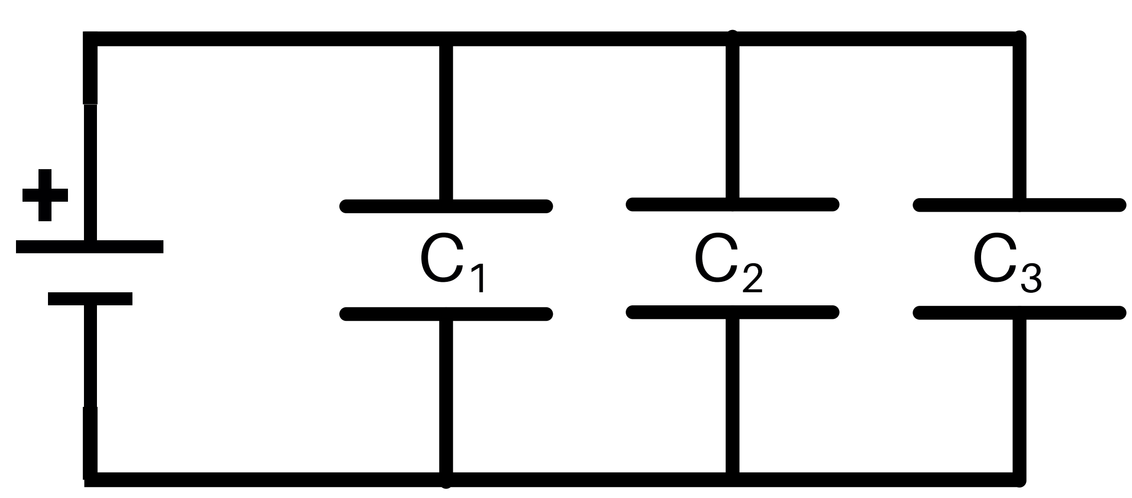
$$C_T = \frac{C_1 \ x \ C_2}{C_1 + C_2} \ (unit : F)$$

TOTAL CAPACITANCE

$$C_T = \frac{1}{\frac{1}{C1}+\frac{1}{C2}+\frac{1}{C3}} = \left(\frac{1}{C1}+\frac{1}{C2}+\frac{1}{C3}\right)^{-1}$$

UNIT: FARAD

## PARALLEL




$$C_T = C_1 + C_2 + C_3$$

UNIT : FARAD

# CAPACITANCE EQUIVALENT CIRCUITS FOR SERIES AND PARALLEL CONNECTIONS

## EXERCISE FINAL SESI 1 2023/2024

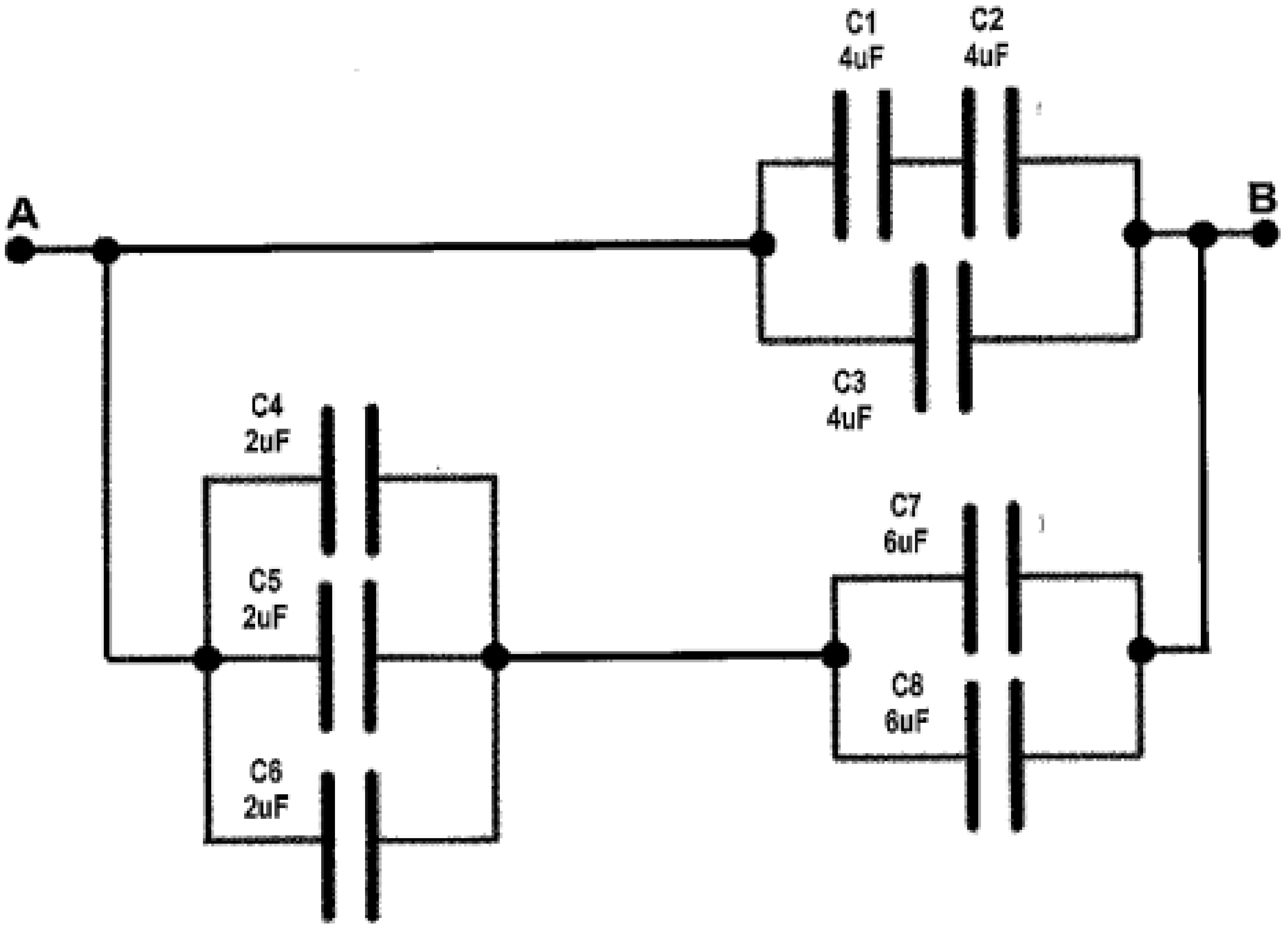



Figure A2(b) / *Rajah A2(b)*

By referring to Figure A2(b), simplify the circuit to obtain the equivalent capacitance value, Cr at terminal A-B.

## EXERCISE FINAL SESI 1 2023/2024

Define a capacitor and label the construction of a capacitor shown in Figure 2(a).

*Takrifkan pemuat dan labelkan pembinaan pemuat seperti yang ditunjukkan dalam Rajah 2(a).*

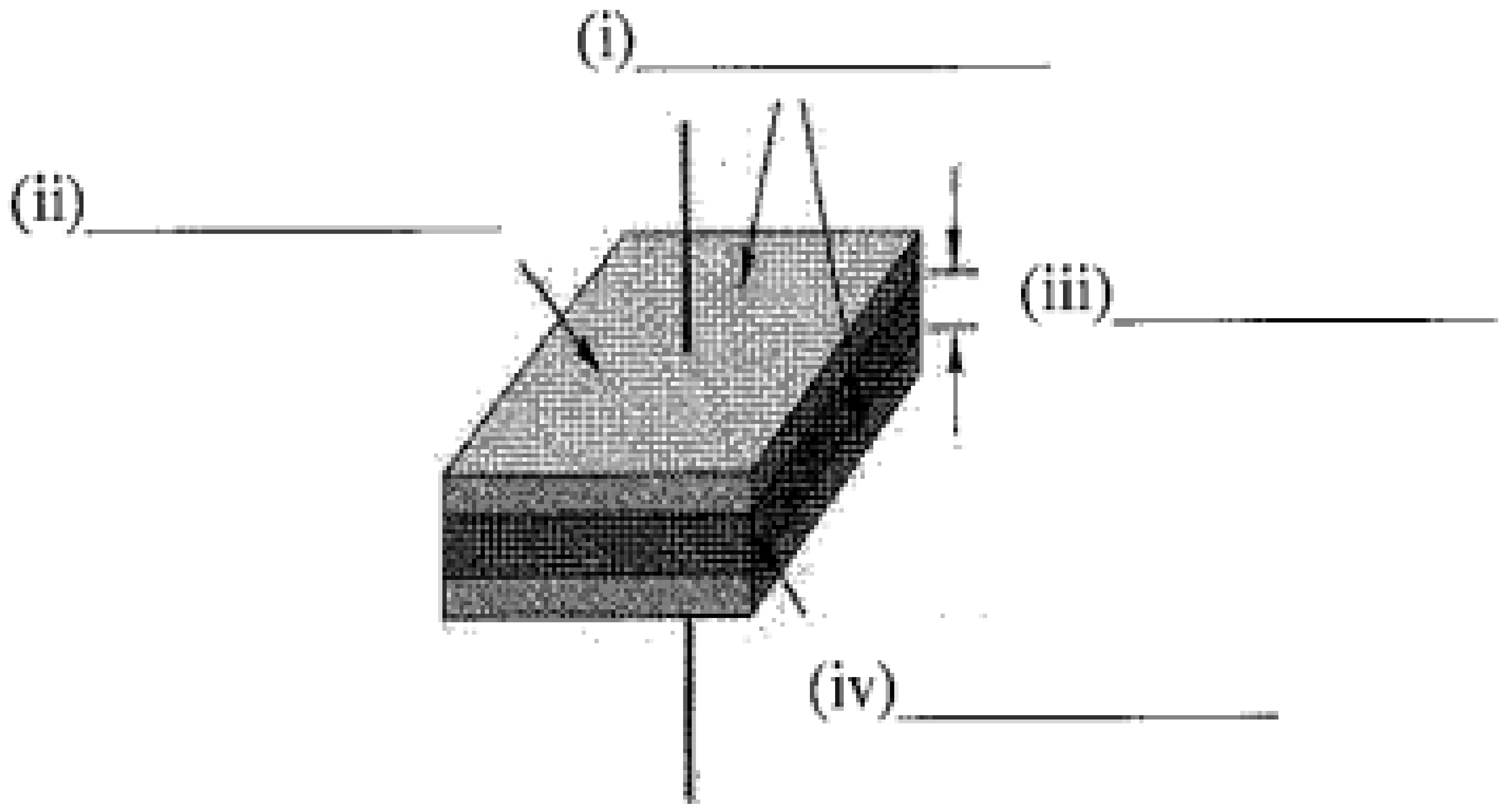



Figure 2(a) / *Rajah 2(a)*

Figure A2(c) shows a schematic diagram that consists of resistive and capacitive loads. If the switch, SW is switched to node a at t = 0 s, calculate the time constant, the instantaneous value of current when t = 2ms and the maximum energy stored by the capacitor.

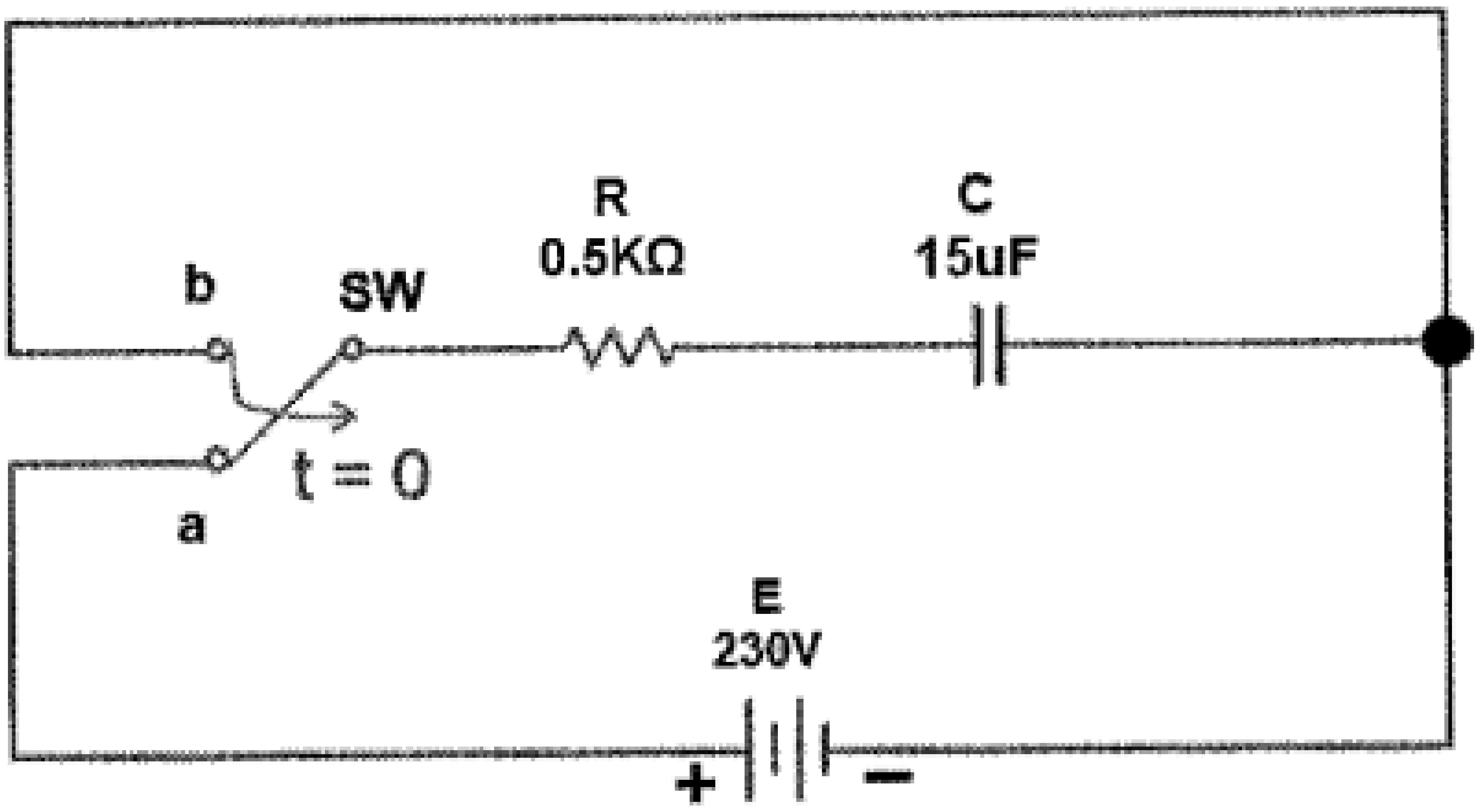



Figure A2(c) / *Rajah A2(c)*

b) Calculate the total capacitance $C_{XY}$ of the network in Figure 2(b).
*Kira jumlah kemuatan $C_{XY}$ bagi rangkaian dalam Rajah 2(b).*

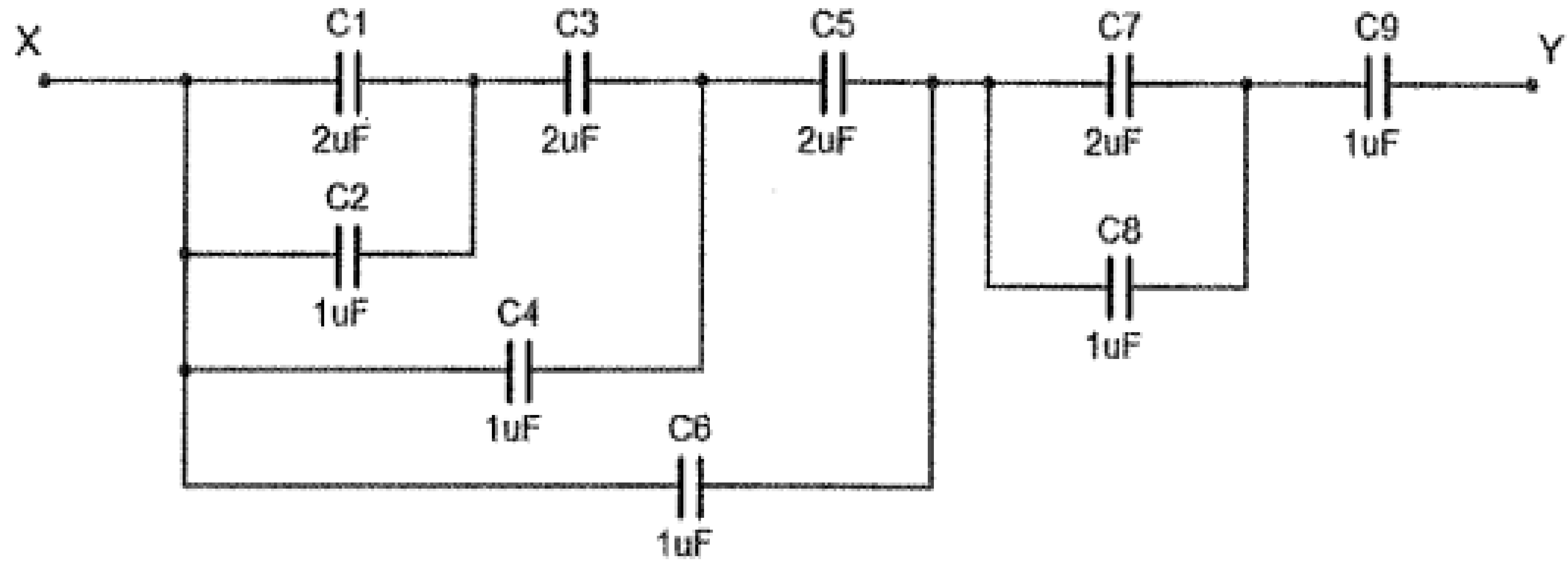



Figure 2(b) / *Rajah 2(b)*

Based on Figure A2 (b), simplify the circuit with capacitors as below.

*Merujuk kepada Rajah A2 (b), permudahkan litar dengan pemuat seperti di bawah.*

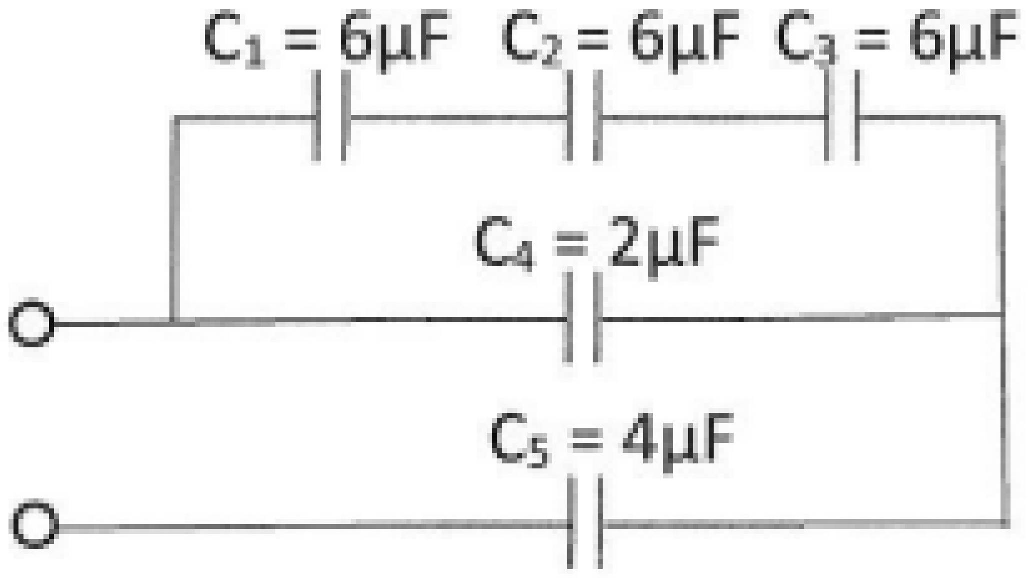
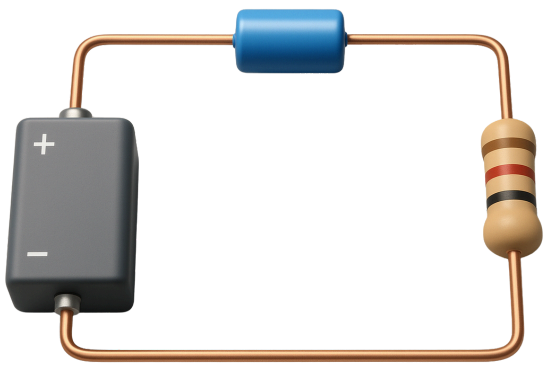


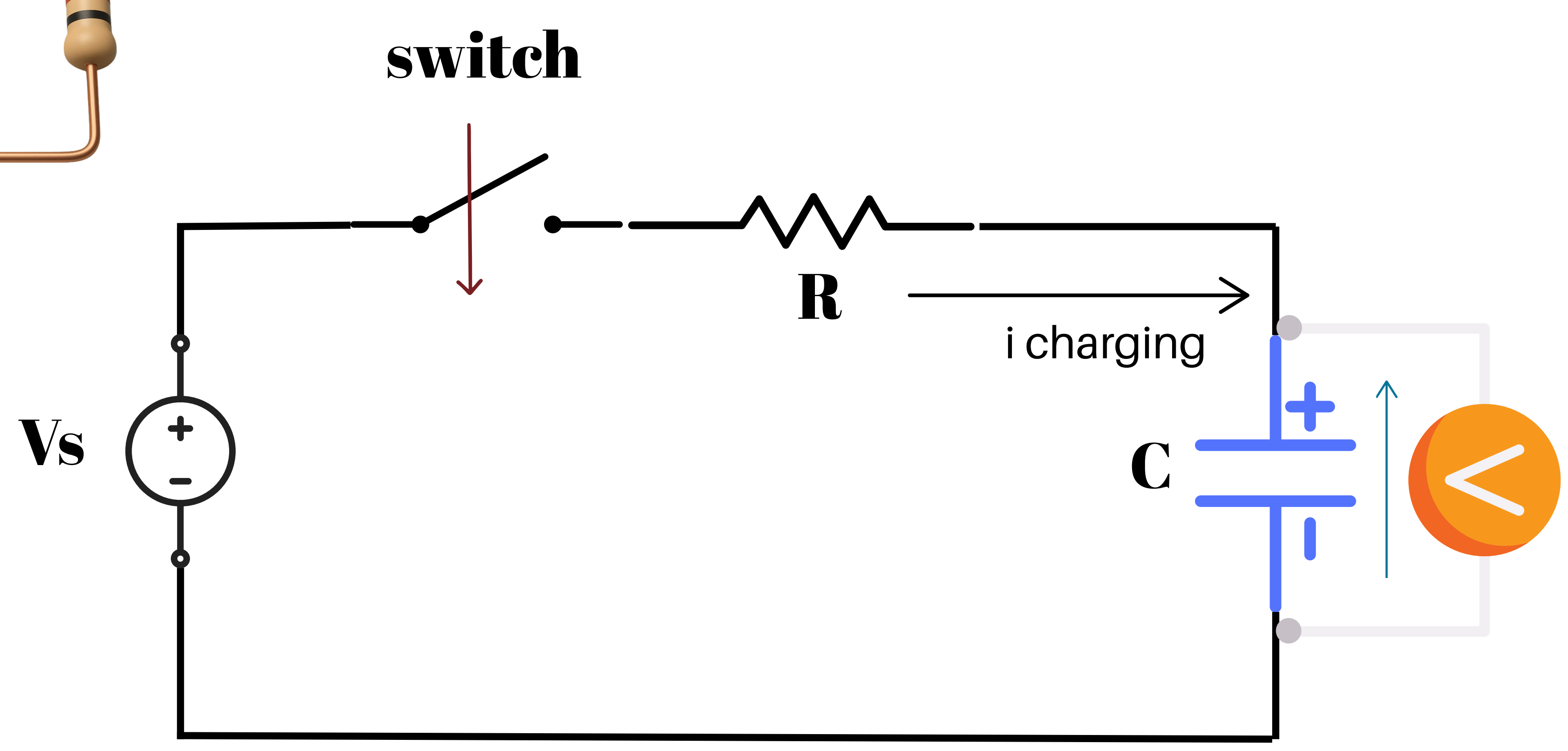

**Figure A2 (b) / *Rajah A2 (b)***

# CAPACITIVE LOAD

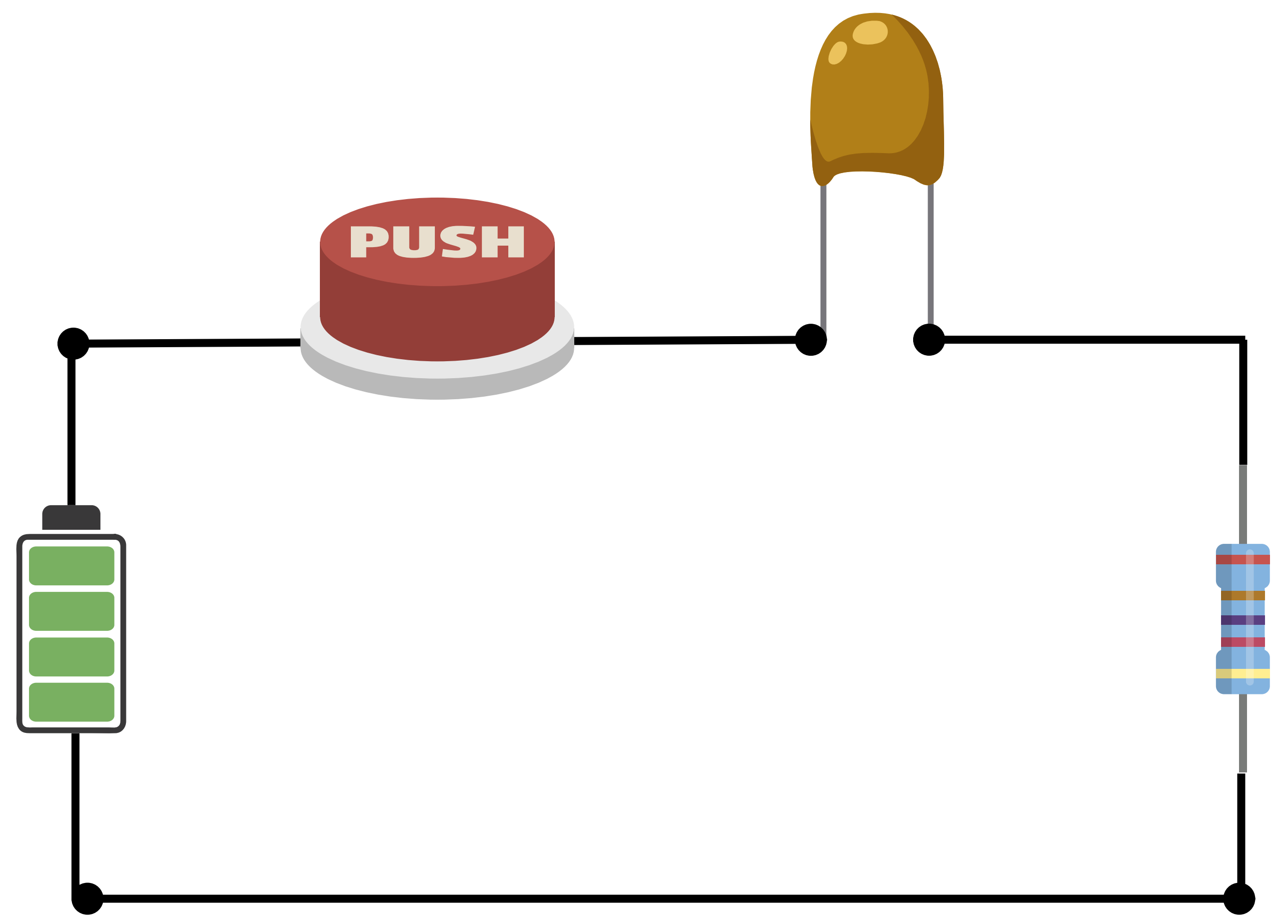
## CHARGING

### WHAT HAPPEN WHEN SWITCH (S) IS PRESSED?






The capacitor is in charging mode when the switch is turned on. When the switch is turned on, the capacitor starts to charge. The circuit is connected to a battery (Vs), so current flows through it. At first (t=0, initial time), the capacitor acts like a wire, so the voltage across it is almost zero ($V_C$ = 0). As it charges, the voltage across the capacitor increases until it reaches the same as the battery, and the current stops flowing



KVL

$$-V_s + V_R + V_C = 0$$

$$V_s = V_R + V_C$$

When a battery connected to circuit, at first (short time) , capacitor like a short circuit, Vc = 0.

$$V_s = V_R + 0$$

$$V_s = V_R$$

Initial current, $i_0 = \frac{Vs}{R}$

$i_0 = i_{max}$

$$i = i_{max} \, e^{-\frac{t}{RC}}$$

Capacitor charging,

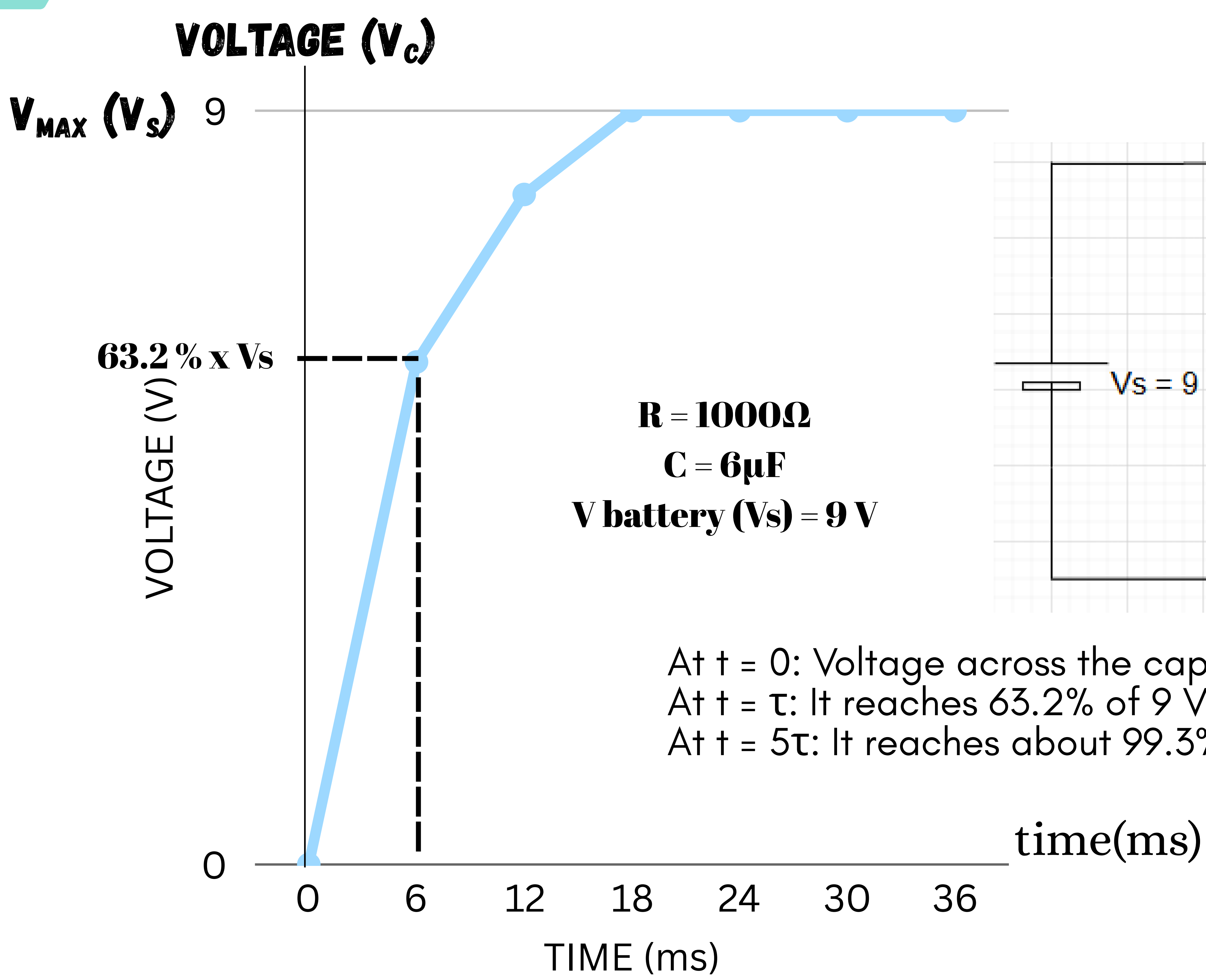
$V_C = V_s \, (1 - e^{-\frac{t}{\tau}})$

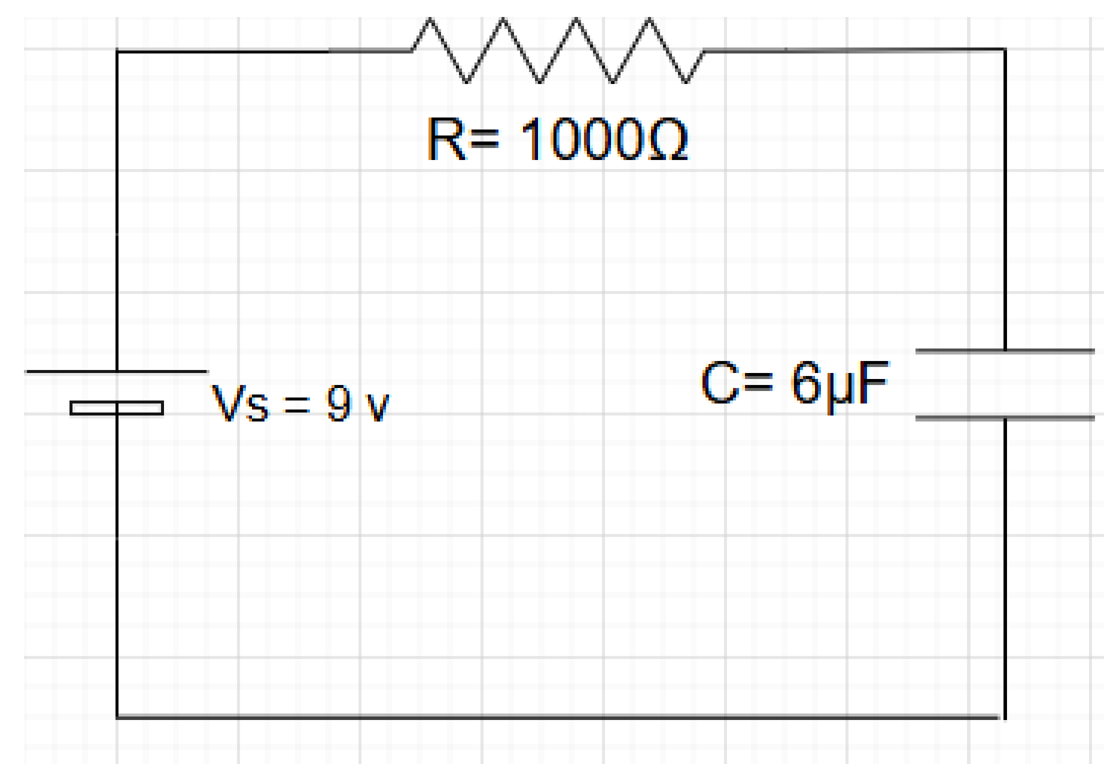
Time constant, $\tau = RC$

At long time, capacitor like an open circuit,
Vc = Vs, i=0 ; t=∞

# CHARGING

## CHARGING CAPACITOR VOLTAGE




At t = 0: Voltage across the capacitor is 0 V.
At t = τ: It reaches 63.2% of 9 V → about 5.69 V.
At t = 5τ: It reaches about 99.3% of 9 V → practically full.

Capacitor charging,

$$V_C = V_s \left(1 - e^{-\frac{t}{\tau}}\right)$$

**Charging** — **Vc versus Time**

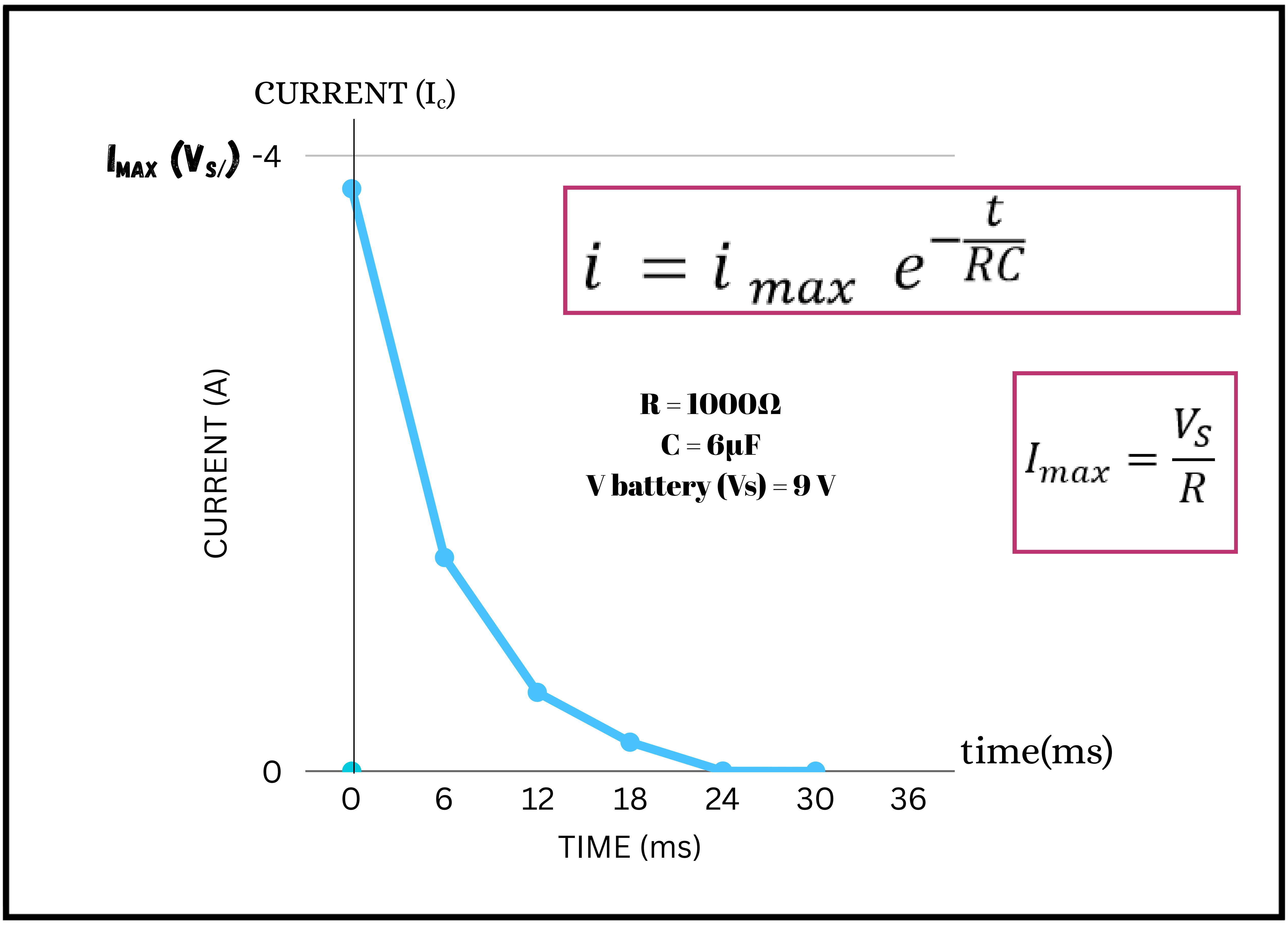
| t | $e^{-\frac{t}{RC}}$ | $\frac{V_C}{V_S} = (1 - e^{-\frac{t}{RC}})$ | $V_C = V_s\ (1 - e^{-\frac{t}{RC}})$ |
|---|---|---|---|
| 0 | 1 | 0 | 0 |
| $\tau_1$ = 6ms | $e^{-\frac{t}{RC}} = e^{-\frac{6ms}{6ms}} = e^{-1} = 0.368$ | 1-0.368 = 0.632 | 5.688 |
| $\tau_2$ = 12ms | $e^{-2}$ = 0.135 | 0.865 | 7.785 |
| $\tau_3$= 18ms | $e^{-3}$ = 0.0498 | 0.95 | 8.55 |
| $\tau_4$ = 24ms | $e^{-4}$ = 0.0183 | 0.982 | 8.835 |
| $\tau_5$ = 30ms | $e^{-5}$ = 0.0067 | 0.9933 | 8.94 |
| ….100% /t=∞ | 0 | 1 ($V_C = V_S$) | 9.00 |

$V_s = 9v$ Time constant, $\tau = RC$ $RC = 1000\ X\ 6\mu F = 6ms$

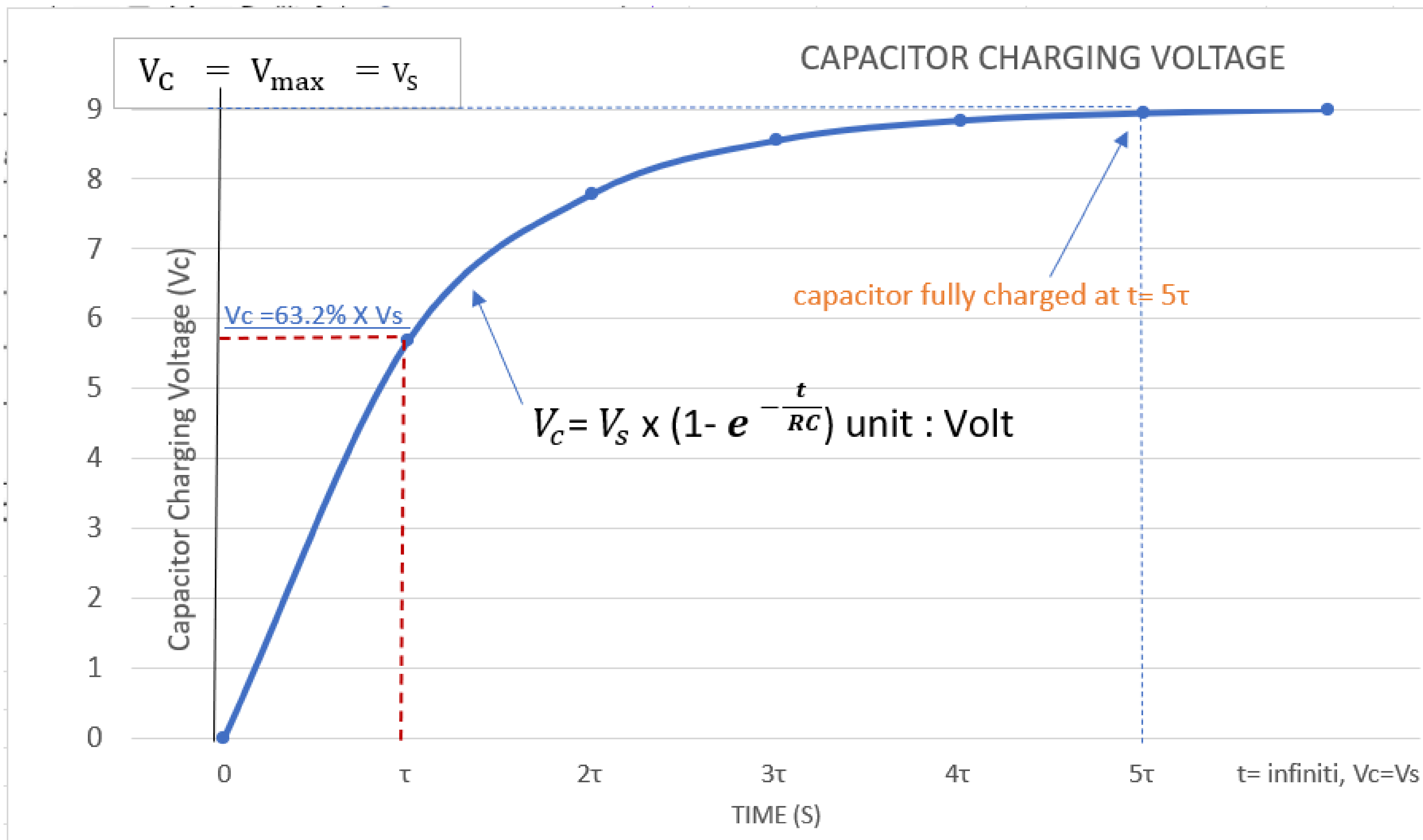
| Voltage | Component |
|---|---|
| $V_s$ | Battery |
| $V_C$ | Capacitor |
| $V_R$ | Resistor |

# CHARGING

## CHARGING CAPACITOR CURRENT

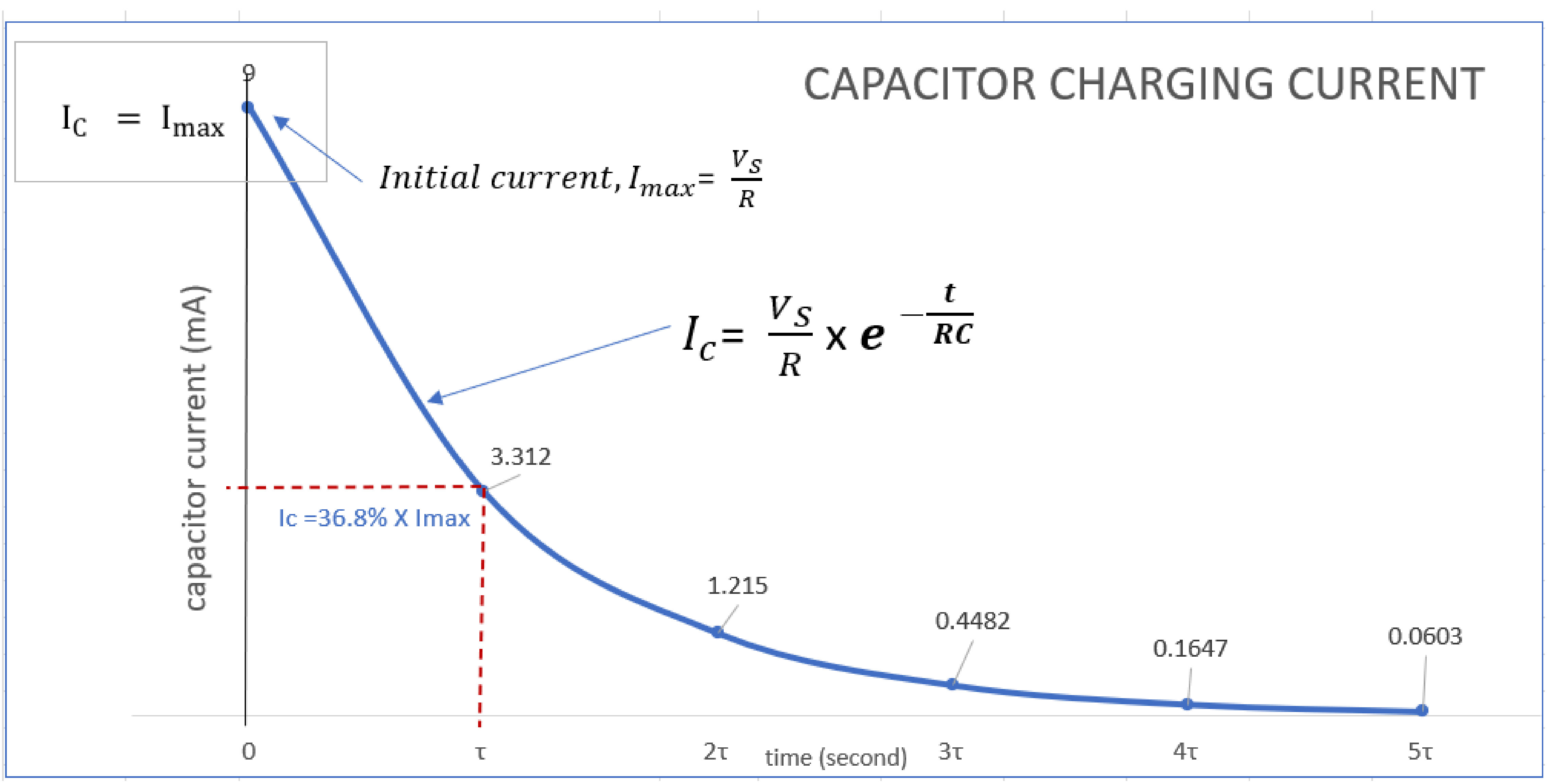



| | | τ = RC = 1000 x 6μ = 6ms | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | 0 | τ | 2τ | 3τ | 4τ | 5τ | T= INFINITI |
| | | 0 | 6ms | 12ms | 18ms | 24ms | 30ms | |
| | | e-0 | e-1 | e-2 | e-3 | e-4 | e-5 | |
| $e^{-\frac{t}{RC}}$ | | 1 | 0.368 | 0.135 | 0.0498 | 0.0183 | 0.0067 | 0 |
| $1 - e^{-\frac{t}{RC}}$ | | 0 | 0.632 | 0.865 | 0.95 | 0.982 | 0.9933 | 1 |
| $I_{max} = \frac{V_S}{R}$ | A | 0.009 | 0.009 | 0.009 | 0.009 | 0.009 | 0.009 | |
| $I_c = \frac{V_S}{R} \times e^{-\frac{t}{RC}}$ | mA | 9 | 3.312 | 1.215 | 0.4482 | 0.1647 | 0.0603 | |
| $V_c = V_s \times (1 - e^{-\frac{t}{RC}})$ | V | 0 | 5.688 | 7.785 | 8.55 | 8.835 | 8.94 | 9 |




At long time, capacitor like an open circuit,

Vc = Vs, i=0 ; t=∞




| [redacted] | | τ = RC = 1000 x 6μ = 6ms | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | 0 | τ | 2τ | 3τ | 4τ | 5τ | T= INFINITI |
| | | 0 | 6ms | 12ms | 18ms | 24ms | 30ms | |
| | | e-0 | e-1 | e-2 | e-3 | e-4 | e-5 | |
| $e^{-\frac{t}{RC}}$ | | 1 | 0.368 | 0.135 | 0.0498 | 0.0183 | 0.0067 | 0 |
| $1 - e^{-\frac{t}{RC}}$ | | 0 | 0.632 | 0.865 | 0.95 | 0.982 | 0.9933 | 1 |
| $I_{max} = \frac{V_S}{R}$ | A | 0.009 | 0.009 | 0.009 | 0.009 | 0.009 | 0.009 | |
| $I_c = \frac{V_S}{R} \times e^{-\frac{t}{RC}}$ | mA | 9 | 3.312 | 1.215 | 0.4482 | 0.1647 | 0.0603 | |
| $V_c = V_s \times (1 - e^{-\frac{t}{RC}})$ | V | 0 | 5.688 | 7.785 | 8.55 | 8.835 | 8.94 | 9 |

At long time, capacitor like an open circuit,
Vc = Vs, i=0 ; t=∞

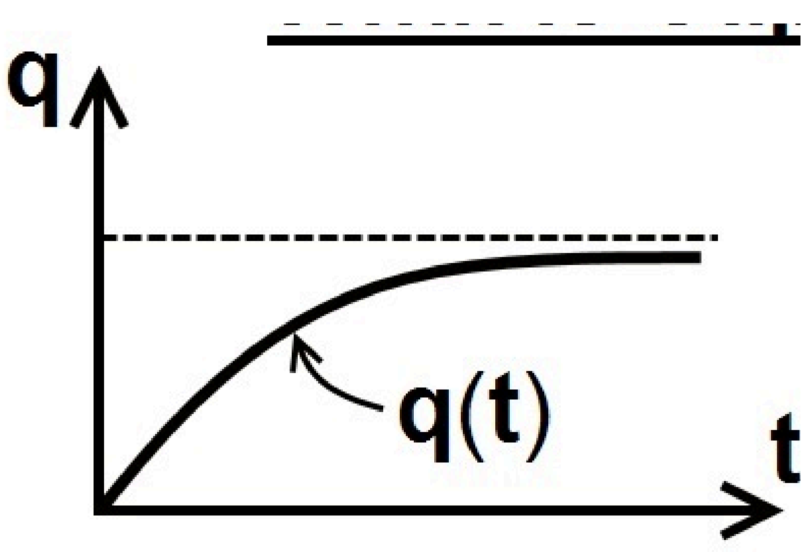
Capacitor charging,

$$V_C = V_s \ (1 - e^{-\frac{t}{\tau}})$$

Time constant, $\tau = RC$

$$i = i_{max} \ e^{-\frac{t}{RC}}$$

$$I_{max} = \frac{V_S}{R}$$




or Q = CV

$$C = \frac{Q}{V}$$

# time constant (τ)

- describes how quickly the circuit responds to a change
- the time constant is defined as $\tau = RC$ for an RC circuit, and $\tau = L/R$ for an RL circuit.
- After one time constant, the voltage or current decays to about 36.8% of its initial value.(discharge or decay)
- After one time constant, the voltage or current has reached about 63.2% of its final value. (charging or rising)
- After about 5τ, the response is considered to have settled, reaching over 99% of its final value.

## Resistor Functions:

- Controls Charging Speed
- as a discharge path
- It prevents a sudden rush of current when the switch is turned on (limit currents).
- Defines Time Constant (RC).

A capacitor stores energy in the form of an electric field.

If there is no path (like a resistor or wire) for current to flow and no leakage, the stored energy stays in the capacitor.

**-retains its charge**


https://www.electrical4u.com/discharging-a-capacitor/

# UNDERSTANDING THE CAPACITOR CHARGING EQUATION

$$V_s = V_R + V_C = i(t)R + \frac{q(t)}{C}$$

$$dQ = \left[\frac{V_s}{R} - \frac{Q}{RC}\right] dt = \left[\frac{CV_s}{RC} - \frac{Q}{RC}\right] dt$$

$$\frac{dq}{dt} + \frac{1}{RC}q = \frac{V_s}{R}$$

$$V_C(t) = \frac{q(t)}{C} = V_s\left(1 - e^{-t/RC}\right)$$

$$q(t) = C \cdot V_s \cdot \left(1 - e^{-\frac{t}{R \cdot C}}\right)$$

rate of change of current is precisely governed by that exponential

$$e^{-\frac{t}{\tau}} = e^{-\frac{t}{RC}}$$

$$C = \frac{Q}{V_C} = \frac{\varepsilon A}{d}$$

increasing current doesn't increase capacitance

$$V_c = \frac{Q}{C}$$

capacitor voltage depends on charge . Charge is depending on current value.

$$I_t = \frac{dQ}{dt}$$

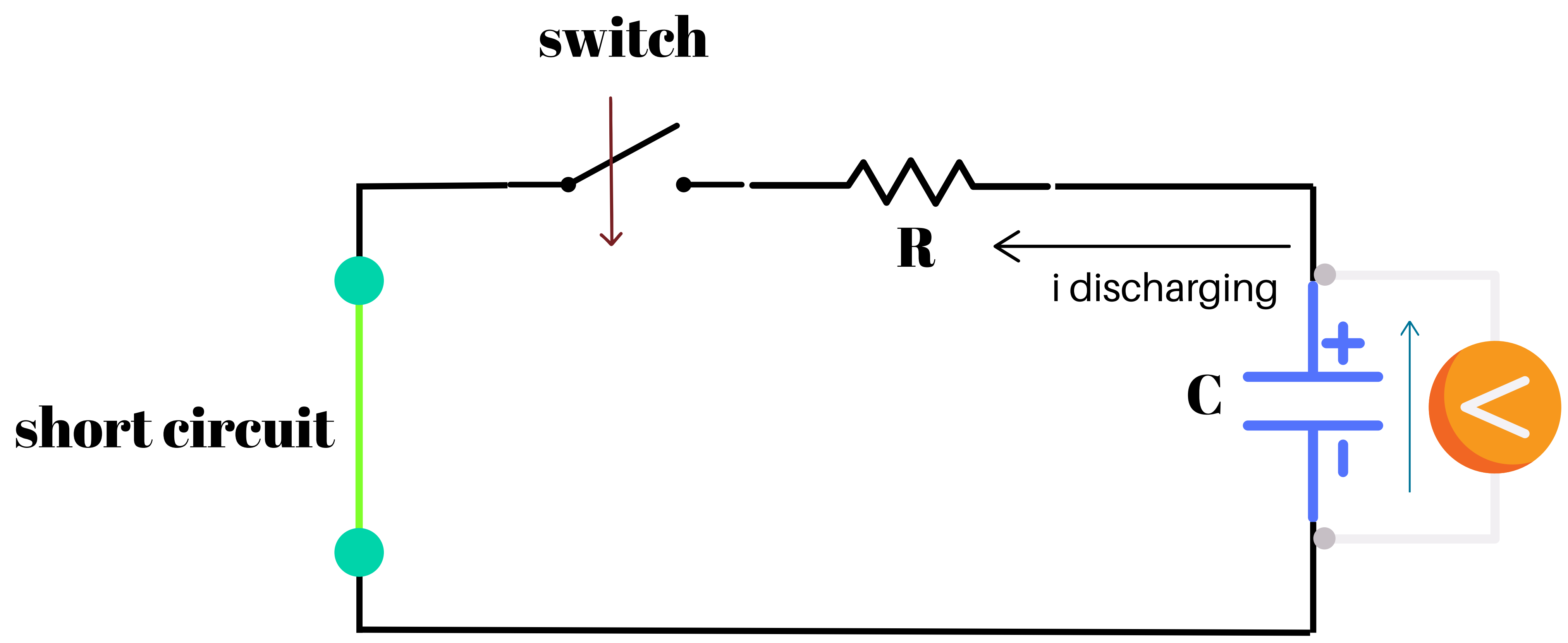
current is the rate of change of charge over time.

If current flows, charge accumulates—and that increases the voltage across the capacitor.

current influences charge, and charge determines voltage.

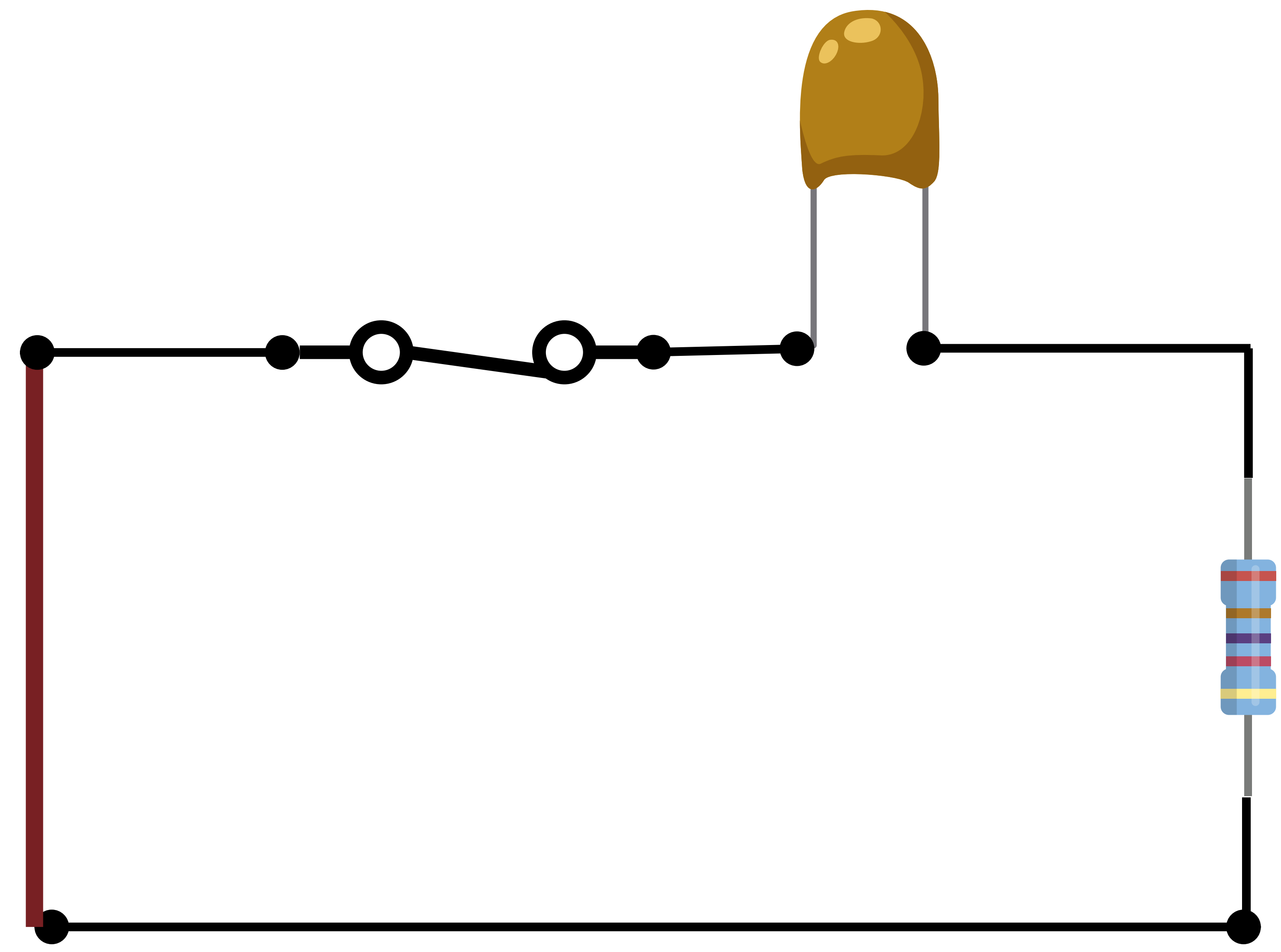
# DISCHARGING

## WHAT HAPPEN WHEN BATTERY IS DISCONNECTED?



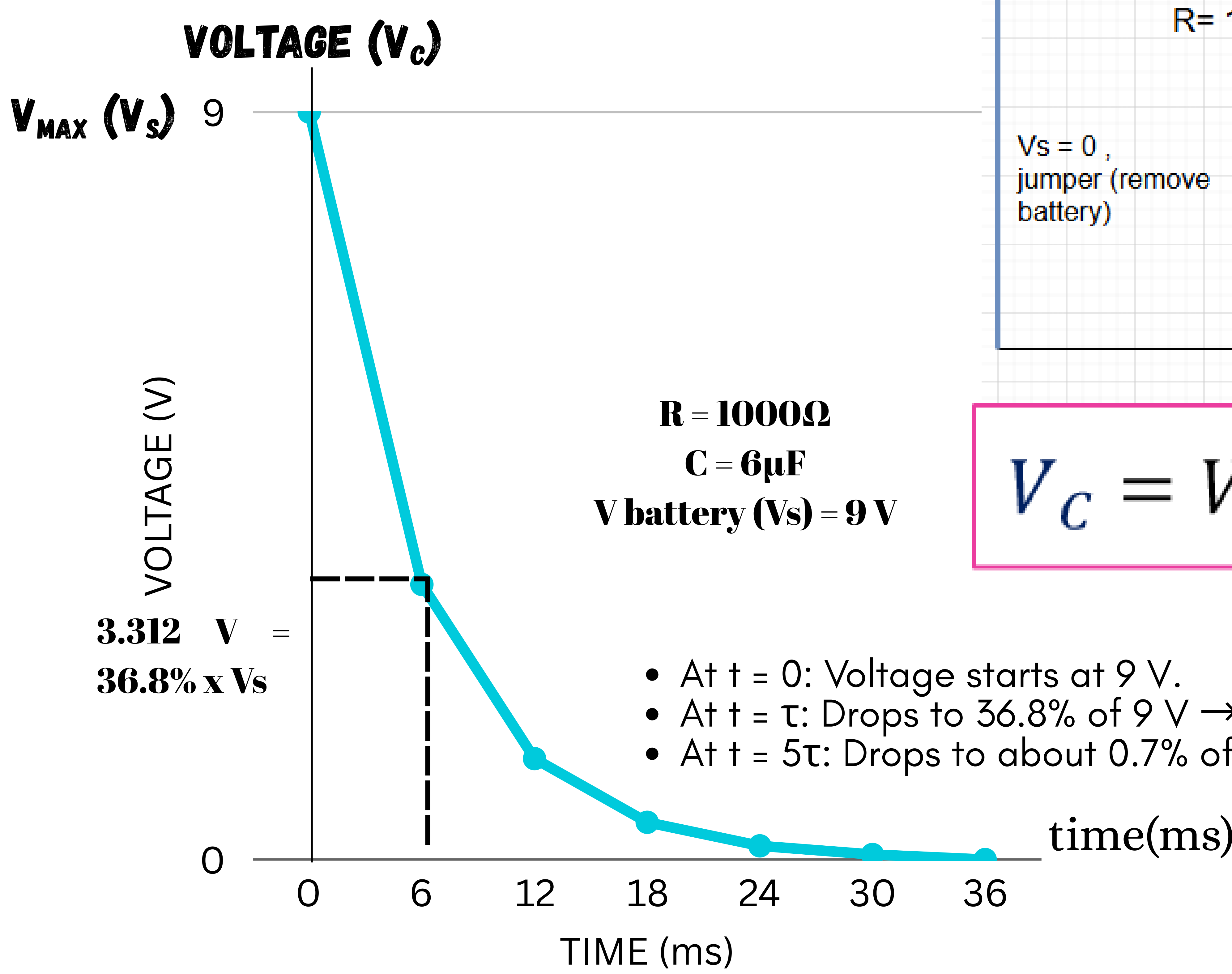

When the switch is pressed, the battery is disconnected (short circuit), and no current flows through the circuit from the battery. The capacitor begins to release its stored energy by discharging through the resistor. The voltage across the capacitor gradually decreases over time. When the capacitor is fully discharged, the voltage across it becomes constants at zero volts (steady state). However, if there is no resistor or discharge path, the capacitor retains its charge for a while.

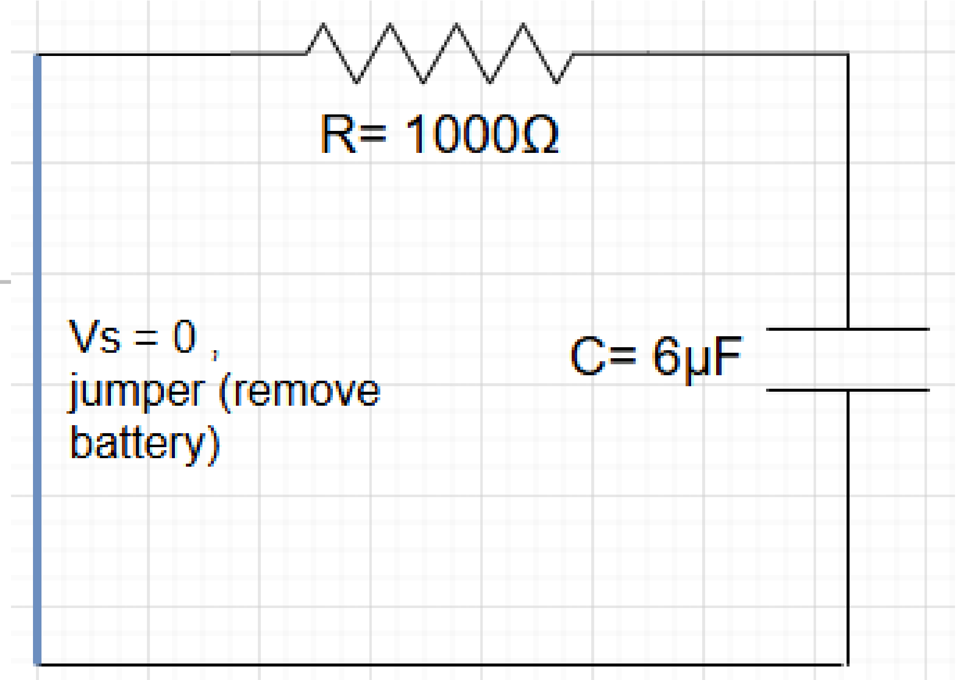
When discharging, both the current and voltage of the capacitor drop gradually and follow an exponential curve until they reach zero



# DISCHARGING

## CAPACITOR VOLTAGE (VC) VERSUS TIME




$$V_C = V_s \ (e^{-\frac{t}{\tau}})$$

- At t = 0: Voltage starts at 9 V.
- At t = τ: Drops to 36.8% of 9 V → about 3.31 V.
- At t = 5τ: Drops to about 0.7% of 9 V → essentially 0 V.

## Discharging Vc versus Time

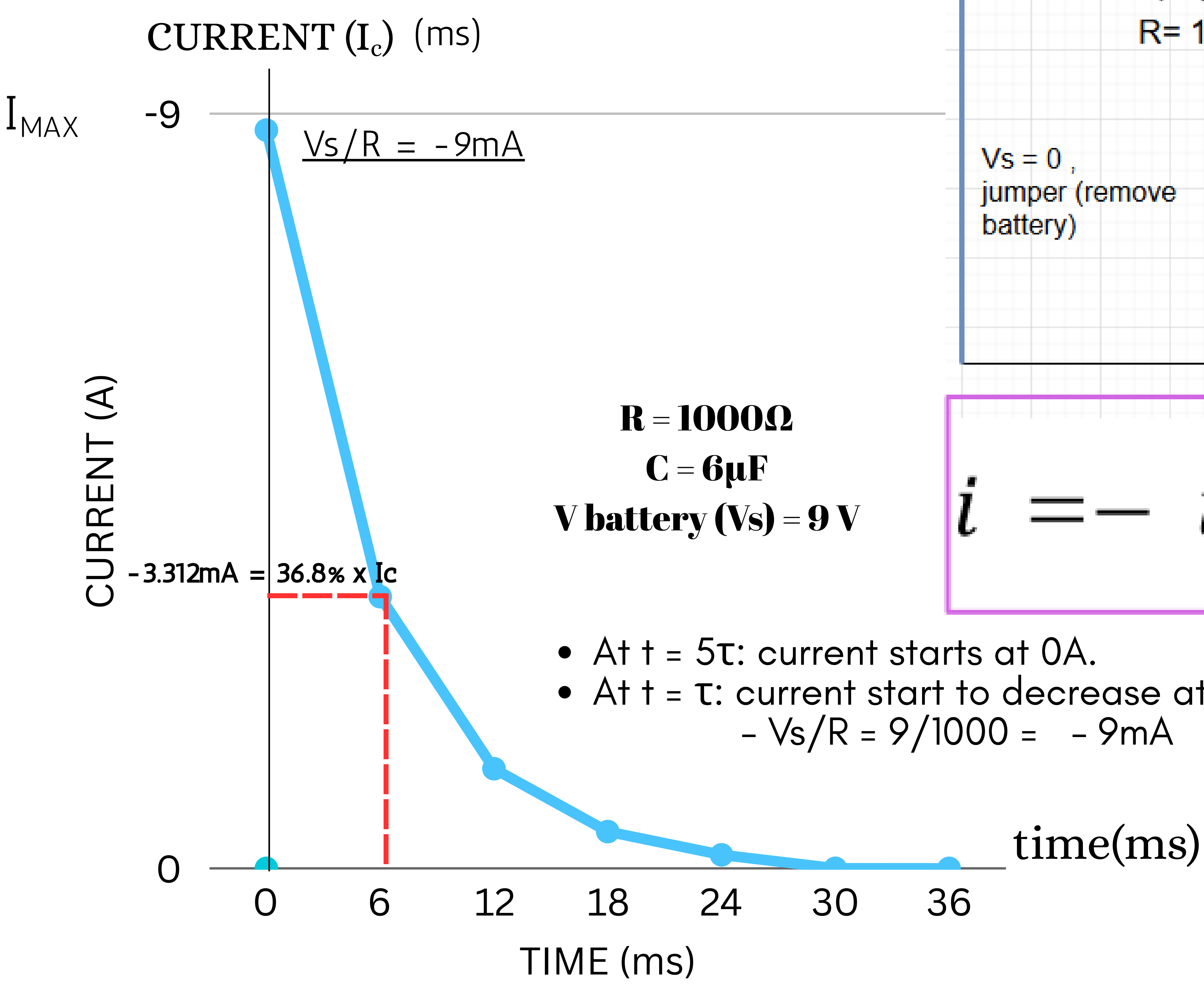
| t | $e^{-\frac{t}{RC}}$ | $\frac{V_C}{V_S} = (e^{-\frac{t}{RC}})$ | $V_C = V_s \ (e^{-\frac{t}{RC}})$ |
|---|---|---|---|
| 0 | 1 | 1 | **9** |
| $\tau_1$ = 6ms | $e^{-\frac{t}{RC}} = e^{-\frac{6ms}{6ms}} = e^{-1} = 0.368$ | $e^{-1}$ = 0.368 | 3.312 |
| $\tau_2$ = 12ms | $e^{-2}$ = 0.135 | $e^{-2}$ = 0.135 | 1.215 |
| $\tau_3$= 18ms | $e^{-3}$ = 0.0498 | $e^{-3}$ = 0.0498 | 0.4485 |
| $\tau_4$ = 24ms | $e^{-4}$ = 0.0183 | $e^{-4}$ = 0.0183 | 0.1647 |
| $\tau_5$ = 30ms | $e^{-5}$ = 0.0067 | $e^{-5}$ = 0.0067 | 0.0603 |
| ....100% /t=∞ | 0 | 0 | 0 |

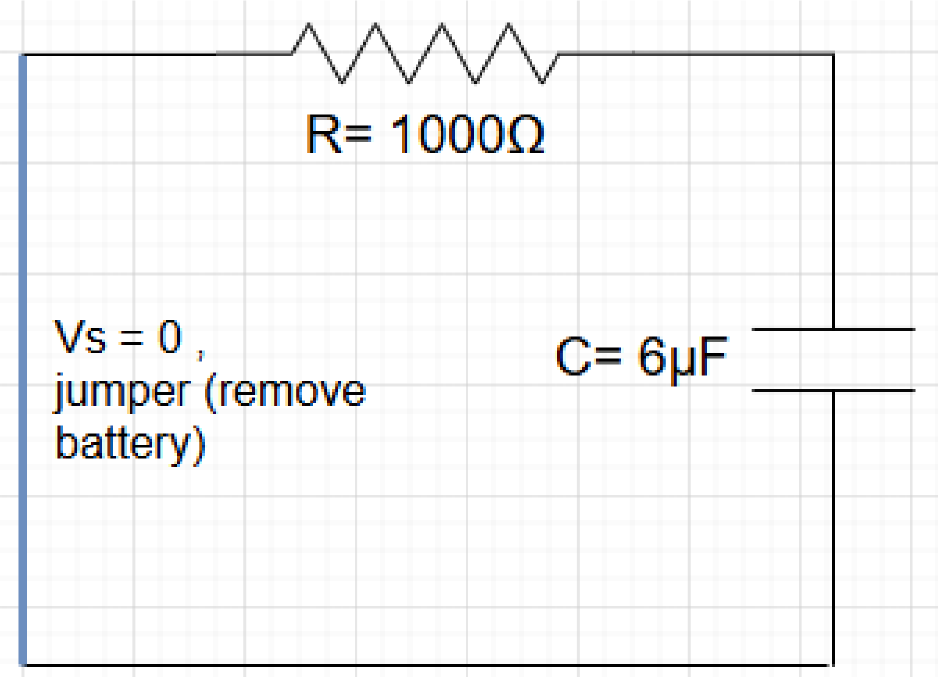
$V_s = 9v$ Time constant, $\tau = RC$ $RC = 1000ohm \ X \ 6µF = 6ms$

| *Voltage* | Component |
|---|---|
| $V_s$ | Battery |
| $V_C$ | Capacitor |
| $V_R$ | Resistor |

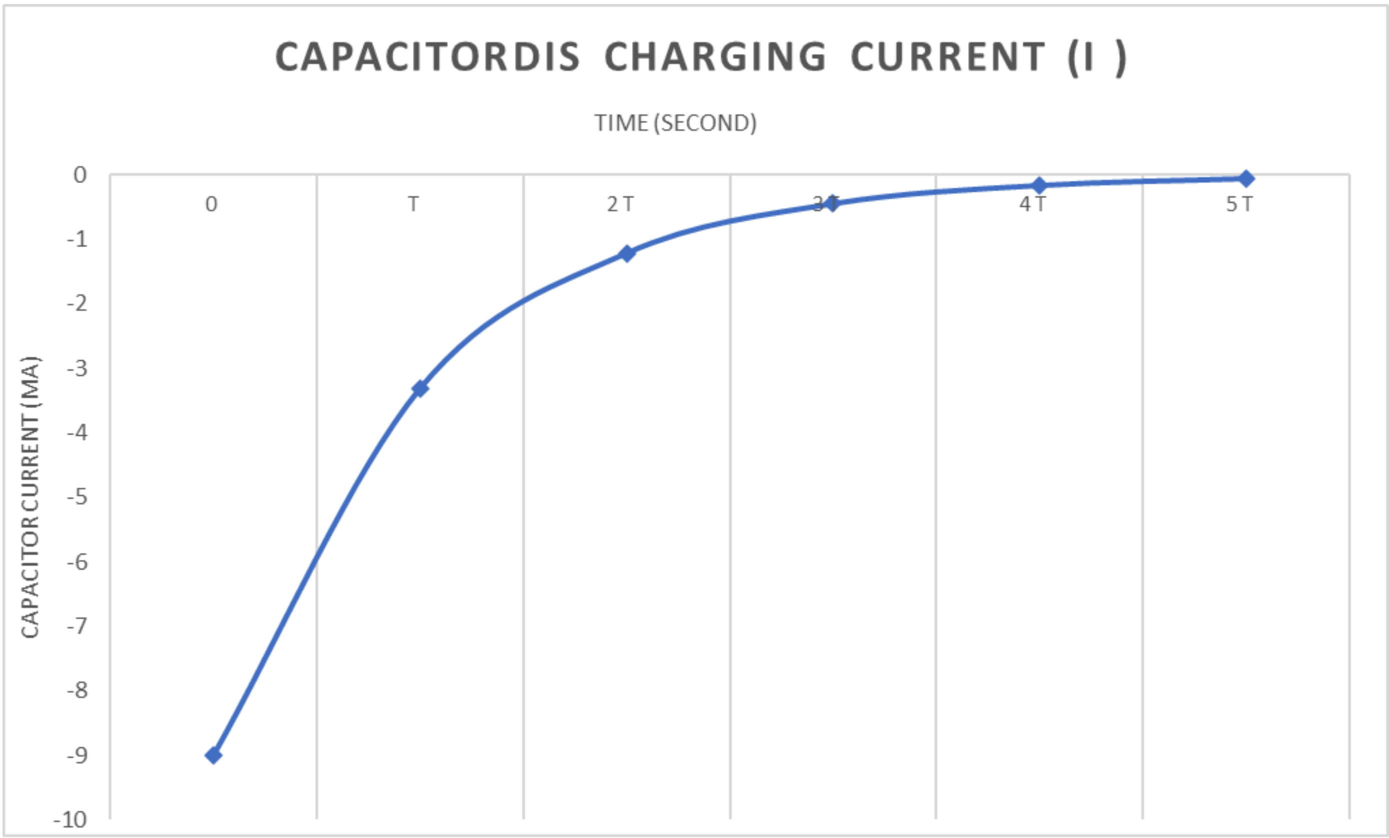
# DISCHARGING

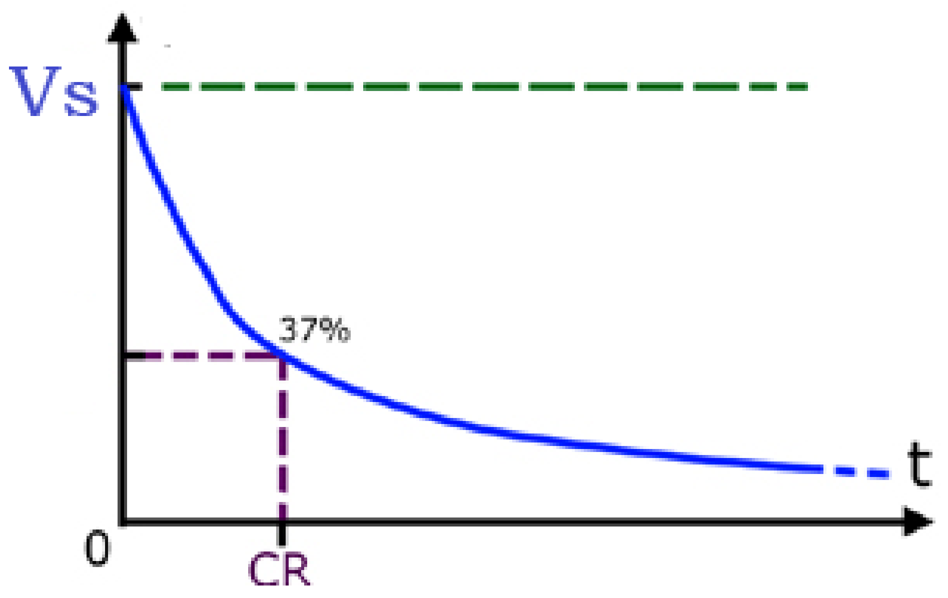
## CURRENT VERSUS TIME



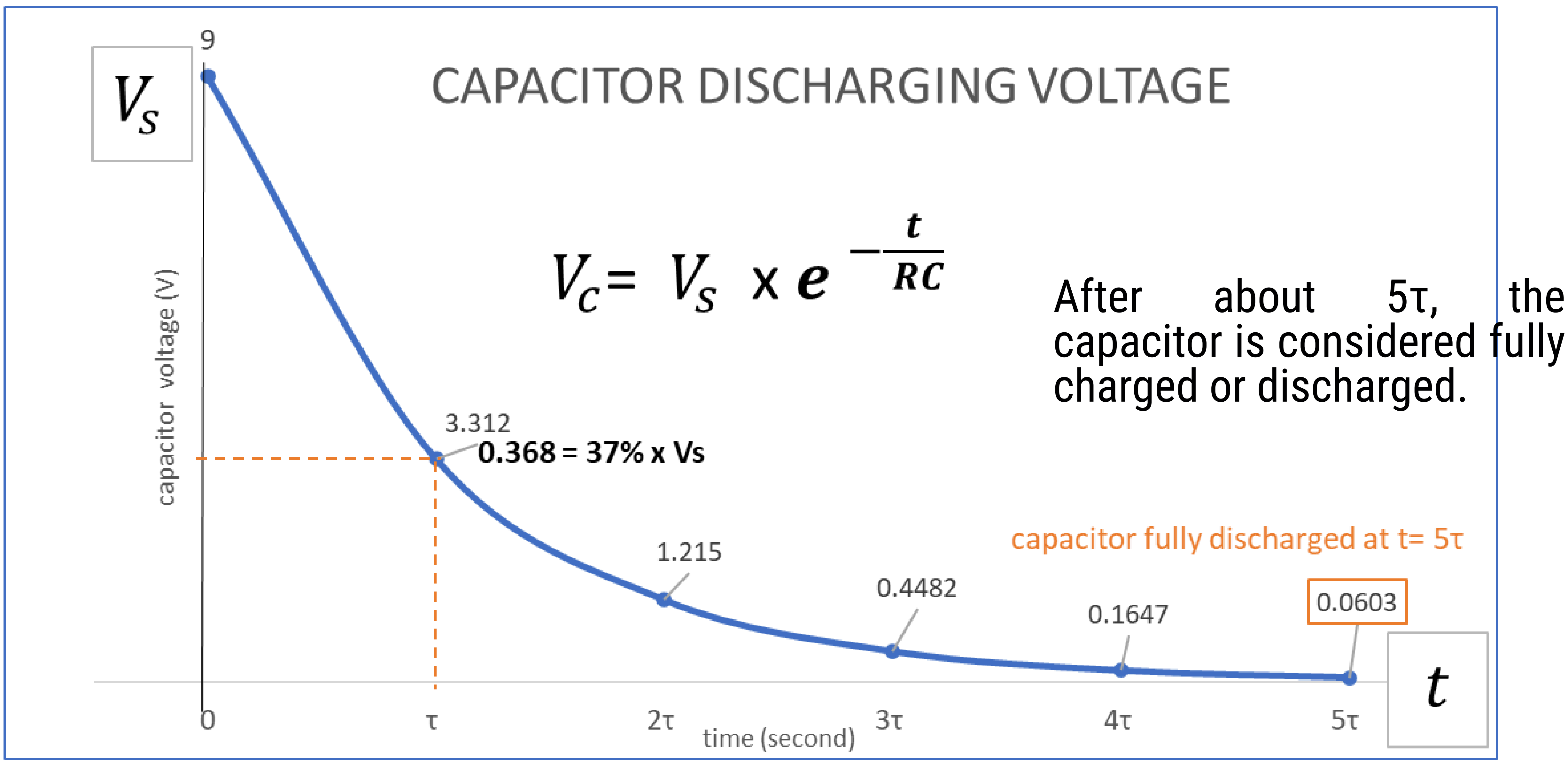

$$i = -\, i_{max}\, e^{-\frac{t}{RC}}$$

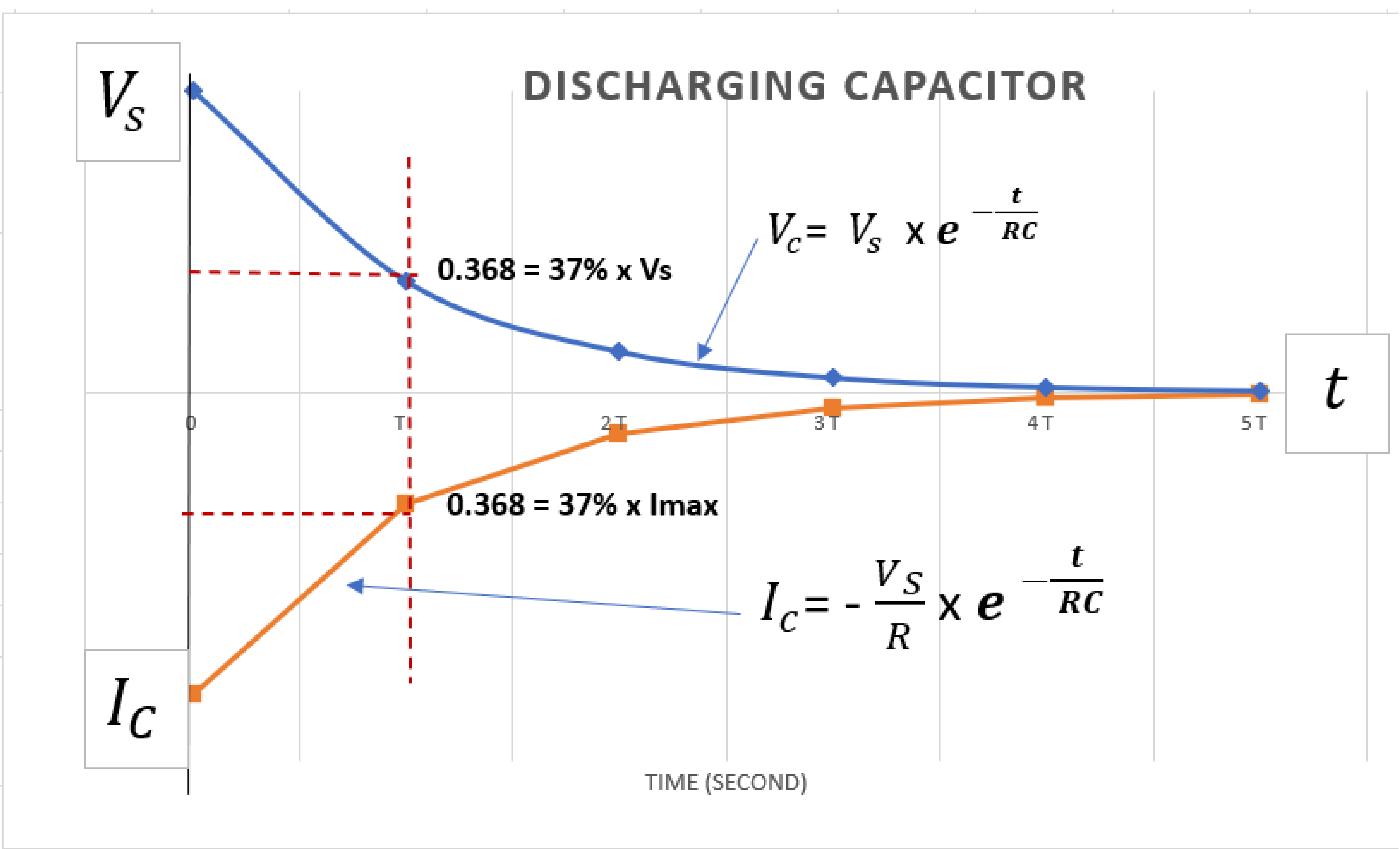


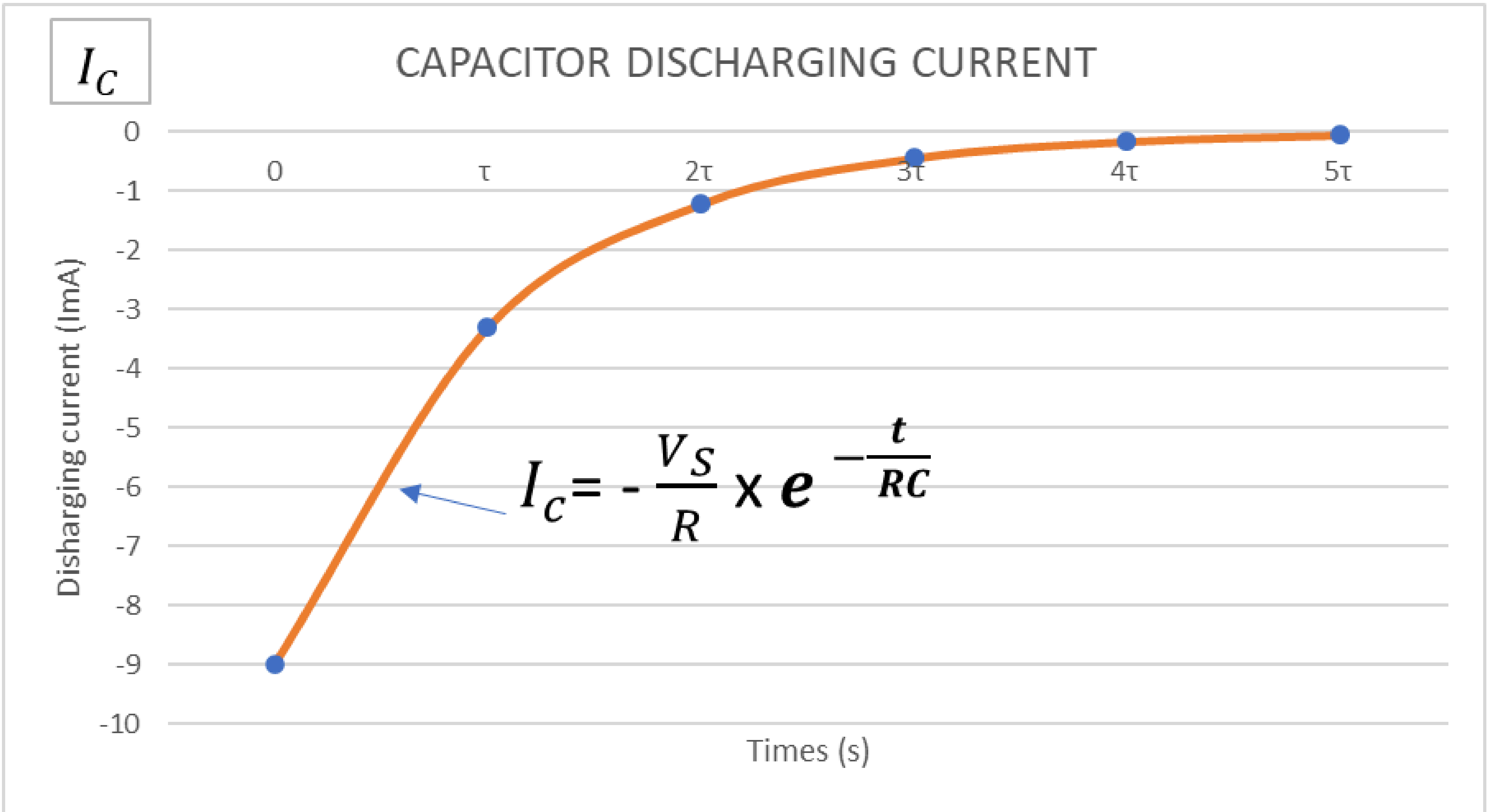




| [redacted] | | τ = RC = 1000 x 6μ = 6ms | | | | | |
|---|---|---|---|---|---|---|---|
| | | 0 | τ | 2τ | 3τ | 4τ | 5τ |
| | | 0 | 6ms | 12ms | 18ms | 24ms | 30ms |
| | | e-0 | e-1 | e-2 | e-3 | e-4 | e-5 |
| $e^{-\frac{t}{RC}}$ | | 1 | 0.368 | 0.135 | 0.0498 | 0.0183 | 0.0067 |
| $I_{max} = -\frac{V_S}{R}$ | A | 0.009 | 0.009 | 0.009 | 0.009 | 0.009 | 0.009 |
| $I_C = -\frac{V_S}{R} \times e^{-\frac{t}{RC}}$ | mA | -9 | -3.312 | -1.215 | -0.448 | -0.165 | -0.0603 |

# DISCHARGING GRAPH



# DISCHARGING

- After about 5τ, the capacitor is considered fully charged or discharged.
- After one time constant (τ), the voltage across the capacitor drops to about 36.8% of its initial value.
- After five time constants, the capacitor is considered fully discharged (less than 1% of initial voltage).

Time constant, $\tau = RC$

**capacitor discharging voltage**

$$V_C = V_s\ (e^{-\frac{t}{\tau}})$$

**capacitor discharging current**

$$i = -\ i_{max}\ e^{-\frac{t}{RC}}$$

When discharging, both the current and voltage of the capacitor drop gradually and follow an exponential curve until they reach zero

# SUMMARY

## CHARGING AND DISCHARGING

| Charging | Discharging |
|---|---|
| $V_C = V_s \ (1 - e^{-\frac{t}{\tau}})$ | $V_C = V_s \ (e^{-\frac{t}{\tau}})$ |
| $i = i_{max} \ e^{-\frac{t}{RC}}$ | $i = - i_{max} \ e^{-\frac{t}{RC}}$ |

Time constant, $\tau = RC$

Time of fully charged, $5\tau = 5RC$

Initial current, $i_0 = i_{max} = \frac{Vs}{R}$

Energy stored, $E = \frac{1}{2} CV^2 \ joule$

| t=0 (initial time) | t=∞ |
|---|---|
| $V_C = 0$ ; | $V_C = V_S = V_{max}$ |
| Q=0 | Q = C X $V_S$ |
| $V_R = V_S$ | $V_R = 0$ |
| $i = \frac{Vs}{R}$ | i = 0 ; |

$V_s$ = Source Voltage/Battery Voltage

$V_c$ = voltage across capacitor

$i_{max} = i_0$ = initial current at t = 0

## EXAMPLE 1

A capacitor with a capacitance of 20 μF which is connected in series to a 200 kΩ resistor is being placed a 250 VDC voltage supply. Calculate the initial current, initial potential different across capacitor, the time constant during charging, the time taken to be fully charge and the energy stored in the capacitor.

## SOLUTION

a) Intial current (t=0) ; I = V/R = 250/200k = 1.25mA
b) intial potential different; Vc=0
c) Time constant , t = RC = 200k x 20μ = 4s
d) time taken to be fully carge, t= 5RC = 5X4s= 20s
e) Energy (E)= $\frac{1}{2}$ X C $V^2$ = $\frac{1}{2}$ X 20μ $250^2$ = 0.625J

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# EXAMPLE 2

Figure C2 shows a 4700μF capacitor is connected in series with a 5.6 kΩ resistor and a 6Vdc supply. When the switch is ON, calculate the time constant, initial charge current, time taken for capacitor voltage to increase to 3V, the voltage across the capacitor at 20s and energy stored in the capacitor.

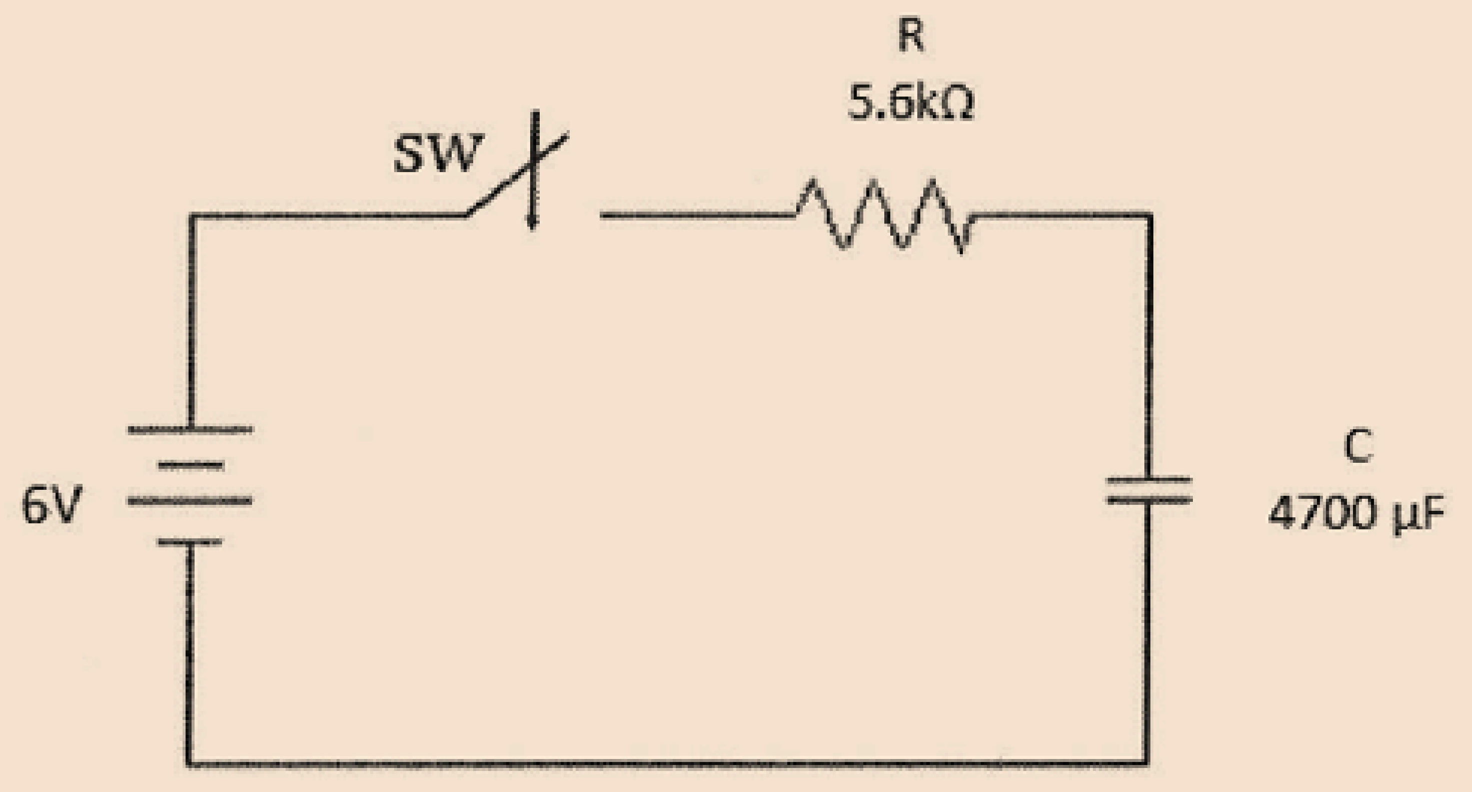



Figure C2

**1** time constant, t $= RX\,C = 5600X$ 4700μ $= 26.32S$

**2** $Initial\ charge\ current, i = \frac{v}{R} = \frac{6}{5600} = 1.0714ms$

**3** the voltage across the capacitor at 20s

$$V_C = V_s\ (1 - e^{-\frac{t}{\tau}})$$

$$V_C = 6(1 - e^{-\frac{20}{26.32}})$$

$V_C = 6(1 - 0.4677)$ = 6x0.532277 = 3.194 v

**4** time taken for capacitor voltage to increase to 3V

$$V_C = V_s\ (1 - e^{-\frac{t}{\tau}})$$

$$3 = 6(1 - e^{-\frac{t}{26.32}})$$

$$\frac{3}{6} = (1 - e^{-\frac{t}{26.32}})$$

$$0.5 = 1 - e^{-\frac{t}{26.32}}$$

$$-0.5 = -e^{-\frac{t}{26.32}}$$

$$0.5 = e^{-\frac{t}{26.32}}$$

$\ln 0.5 = \ln e^{-\frac{t}{26.32}}$ ; In e= 1

$$-0.6931 = -\frac{t}{26.32}$$

t= $26.32\ x\ 0.6931 = \mathbf{18.242s}$

**5** Energy (E)= $\frac{1}{2}$ X C $V^2$ = $\frac{1}{2}$ X 4700μ x $6^2$ = 0.0846J

EXAMPLE 3

A capacitor consists of two circular metal plates, each with a radius of 5 cm. The plates are parallel to each other and separated by a distance of 1 mm. The capacitor also connected to 9V battery across the plates in vacuum. Calculate the capacitance of the capacitor and the charge on each plate.

SOLUTION

Given:
Radius of each plate, r = 5 cm = 0.05 m
Separation, d = 1 mm = 0.001 m
Vacuum between plates → $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m
Voltage across plates, V = 9 V

$$C = \frac{\varepsilon \, x \, A}{d} \; ; \; \varepsilon = \varepsilon_0 \, x \, \varepsilon_r$$

$$C = \frac{\varepsilon \, x \, A}{d} = \frac{\varepsilon_0 \, \varepsilon_r \, x \, A}{d} \; ; \; \varepsilon = \varepsilon_0 \, x \, \varepsilon_r$$

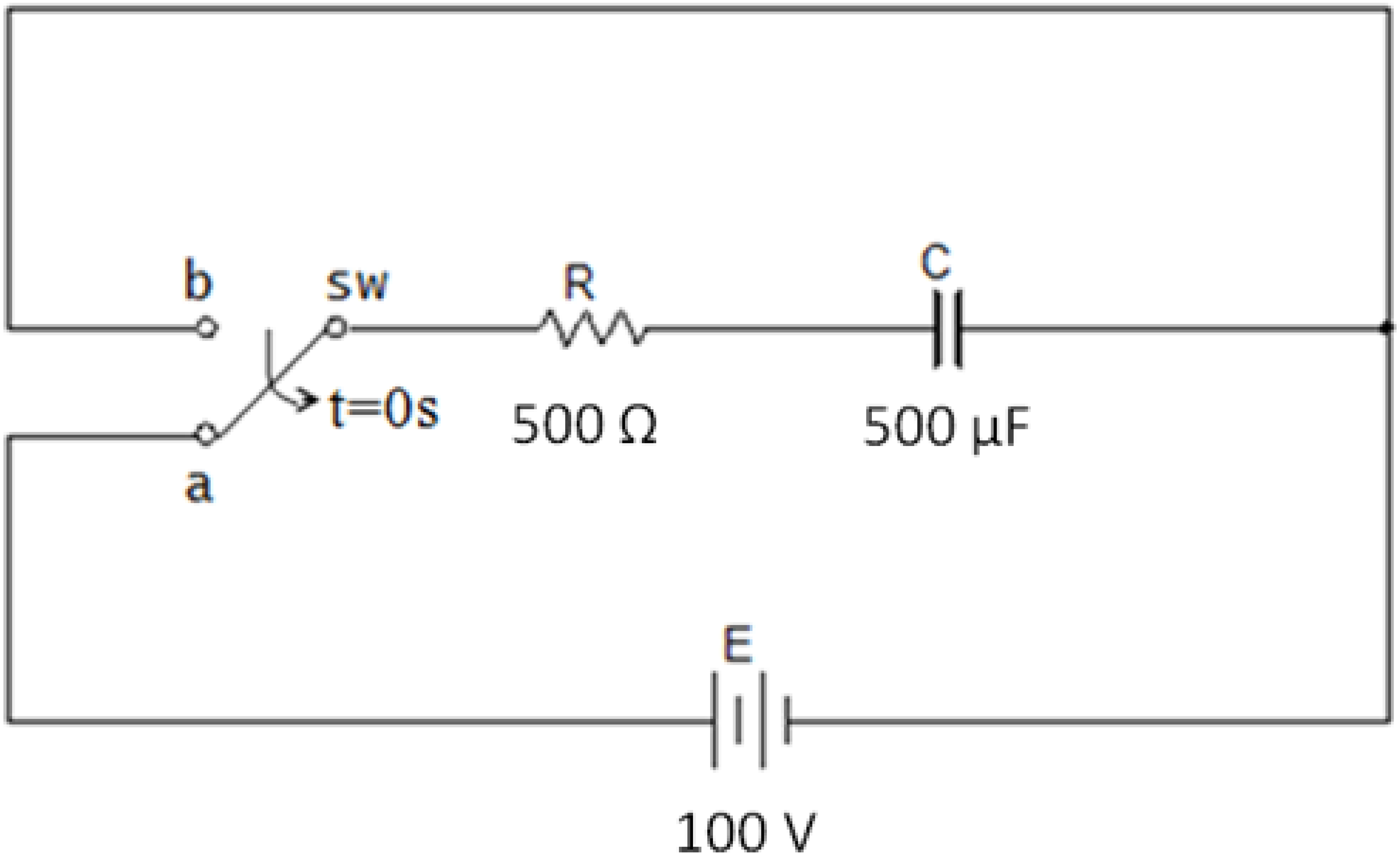
$$A = \pi r^2 = \pi(0.05)^2 = (7.854 \text{ x } 10^{-3}) \, m^2$$

$$C = \frac{(8.85 \text{ x } 10^{-12})(1)x(7.854 \text{ x } 10^{-3})}{0.001} = 6.95 \times 10^{-11} F = \mathbf{69.5 \, pF}$$

$$Q = C \text{ x } V = 69.5 \, pF \text{ x } 9 = \mathbf{625.5 pF}$$

# SELF-EXERCISE

Figure below shows a schematic diagram that consists of resistive and capacitive load. If the switch SW is switched to node a at t= 0s.




a) COMPUTE the time constant, τ

b) PREDICT the maximum voltage of the capacitor.

c) CALCULATE the instantaneous value of current when t = 0.25ms.

d) CALCULATE the time taken to make the instantaneous value of charging voltage equals to 15V.

e) CALCULATE the maximum energy stored by the capacitor

Answer :

a) $\tau$ = 250ms, b) $v_c$ = 100V, c) t = 155.76 mA, d) t = 0.1625 ms

e) $E_c$= 2.5 J

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