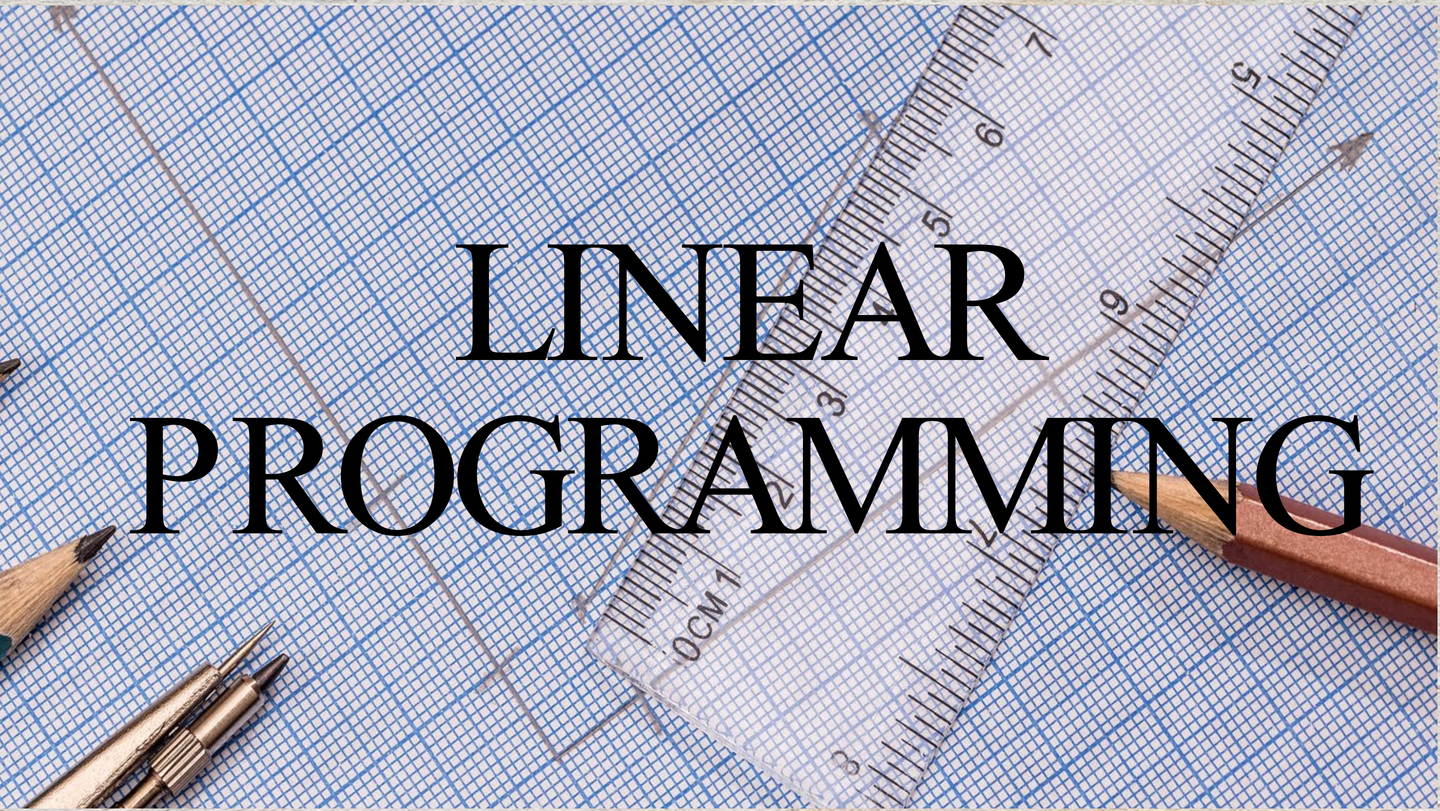
POLITEKNIK MALAYSIA
PORT DICKSON



LINEAR PROGRAMMING

POLYTECHNIC SERIES

NOORAZLINA BINTI ABD.KARIM

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POLYTECHNIC SERIES

POLITEKNIK MALAYSIA
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PREFACE

Assalamualaikum & Peace be Upon You

Allahmdulillah praise be to Alllah S.W.T with His grace and mercy, finally the e-book Linear Programming For Polytechnics has been successfully published. This e-book has been developed based on the latest Engineering Mathematics 3, Polytechnics Course Syllabus which covers the topic of Linear Programming.

This e-book is a useful learning material for students who have never taken additional mathematics in secondary school and as a reference of reference for newly trained lecturers in this field.

I hope that this effort will be an ongoing process to achieve the vision of becoming an excellent academic center. Any comments and suggestions from students or other readers are welcome to ensure that this module can be improved for future editions.

Thank you very much

Azlina

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We would like to convey our utmost gratitude to the Department of Polytechnic and Community College Education particularly the e-learning and instructional Division (BIPD) for funding our e-book project.
We hereby declare that this module is our original work. To the best of our knowledge, it contains no materials previously written or published by another person. However, if there is any due acknowledgement and credit are mentioned according in the e-book.

TABLE OF CONTENTS

INTRODUCTION

INTRODUCTION

What is Linear Programming (LP)

This method use to find the **best solution** to a problem that requires a decision about how best to use a set of limited resources to **achieve a particular goal .**

Linear programming (LP) is applied in real-world scenarios :

Production Planning	Resource Allocation	Transportation and Logistics	Financial Planning	Agriculture

- A simple LP problem may look like this:

Maximize $P = 2x + 4y$ objective function

subject to :

$$5x + 3y \leq 5$$
$$x - y \leq 12$$
$$x, y \geq 0$$

constraints

DON'T FORGET

The problem (constraints) can be represented by a **set of linear inequalities.**

Symbol inequalities	Situation
>	Greater than More than Exceeds In excess of Bigger than Over
≥	Greater than or equal to Greater and equal than Not less than At least Minimum Is not under
<	Less than Fewer than Smaller than Must not be Below Is under Is fewer
≤	Less than or equal to Less and equal than Not more than At most maximum

Table 1: Inequalities to describe situations

INTRODUCTION

Constraint	Inequality
a is more than b a is greater than b a in excess of b	a > b
a is less than b a is fewer than b a smaller than b	a < b
a is not more than b a is at most b a is less than or equal to b	a ≤ b
a is greater than or equal to b a is at least b a is not under b	a ≥ b
a is at least k times of b	a ≥kb
a is at most k times of b	a≤kb

Table 2: Inequalities to interpret situations

Constraints are inequalities that limit the values that variables can take.

CASE 1

Express the inequality for the following constraints :

a. The total sum of p and q is fewer than 75
b. The speed limit (s) of car is does not exceed 110 km/h.
c. x exceeds 2y by 85 or more
d. The ratio of M to N is maximum value of 80.

A

a. The total sum of a and b is fewer than 75

$$\boldsymbol{p + q < 75}$$

b. The speed limit (s) of car is does not exceed 110 km/h.

$$\boldsymbol{s < 110\ km/h}$$

c. x exceeds 2y by 85 or more

$$\boldsymbol{x - 2y \geq 85}$$

d. The ratio of M to N is maximum value of 80.

$$\boldsymbol{\frac{M}{N} \leq 80\ or\ M \leq 80\ N}$$

Let's do this!

Express the inequality for the following constraints :

a. The volume (V) of the revolution is greater than 150m^3.

b. J is not more than three times of K.

c. The value of x is not more than 78.

d. The ratio of the number of scarves purchased (y) to the number of clothes (x) is over than 3:5

e. The minimum score for Engineering Mathematics 3 (Ms) is 40.

f. The value of x must be at least 7.

g. The ratio of y to x is must be more than 2 : 7

h. x is more than y less than 34

SCAN ME!

APPLY PROBLEM FORMULATION

There are three vital stage in the problem formulation, including

- **What are the decision variables?**
- **What are the constraints?**
- **What should be done to maximize/minimize profit/cost?**

IMPORTANT!

Guideline for Linear Programming Model Formulation:

- Recognize the problem completely
- Determine the unknowns/decision variables and give them symbols (ex: x,y,z….or x_1, x_2)
- Identify the constraints and express them in terms of inequalities involving the decision variables .(symbols of constraint is ≥ ,≤ ,=)
- Define the objective function to represents the goal of problem.
- Add the (x,y,z….@ $x_1, x_2, \ldots, x_n \geq 0$) non-negativity requirement.

The term for y is typically put on the left side of an inequality when it is written.

CASE 2

Ayu and Ros run a small business together, making pants and blouse. Each pair of pants takes Ayu and Ros 1 hour each. For each blouse Ayu needs 1 hour and Ros needs half an hour. Ayu has 6 hours every day and Ros has 5 hours every day. The number of pants at least two times blouse. The price of one pair of pants and one blouse is RM35 and RM20 , respectively. Write inequalities and objective functions that satisfy all the above conditions.

A

Step 1: State the unknowns variables

Define

x= number pair of pants

y= number of blouse

Step 2: State the constraints

Ayu time : $x + y \leq 6$

Ros time: $x + 0.5y \leq 5$

$x \geq 2y$

Step 3: State the objective function

$$P = 35x + 20y$$

So, in summary, the LP problem is

Maximize $P = 35x + 20y$

Subject to $x + y \leq 6$

$x + 0.5y \leq 5$

$x \geq 2y$

$x \geq 0; y \geq 0$ non-negativity requirement

CASE 3

A factory produces two components H and R on a given day. The profit from the sale of a piece of component H is RM16 and RM13 for a piece of component R .The income component of the day's component is based on the following constraints:

i. The number of components produced in excess 600

ii The number of component H does not exceed three times the number of component R produced.

iii. The minimum amount of profit for both components is RM5200

Write inequalities and objective functions that satisfy all the above conditions

A

Step 1: State the unknowns/decision variables

Define

x= components H

y= components R

Step 2: State the constraints

i. The number of components produced in excess 600 : $\boldsymbol{x + y > 600}$

ii. The number of components produced H does not exceed three times the number of component R : $\boldsymbol{x < 3y}$

iii. The minimum amount of profit for both components is RM5200 :

$$\boldsymbol{x + y \geq 5200}$$

Step 3: State the objective function

$$P = 16x + 13y$$

So, in summary, the LP problem is

Maximize $P = 16x + 13y$

Subject to $x + y > 600$

$x < 3y$

$x + y \geq 5200$

$x \geq 0; y \geq 0$ non-negativity requirement

Let's do this!

1. The members of "Kelab Kembara" are planning to have a picnic. They have agreed to hire bus x and van y. The rent for a charter bus is RM1200 and for a van RM500. The following restrictions apply to the rental of vehicles for the picnic :

i. The number of bus and vans rented is less than or equal to 7

ii. The number of buses is not less than twice number of van

iii. Maximum quota for rented vehicles is RM6000.

The maximum number of members that can be loaded into their leased vehicles if a bus can be loaded with 48 passengers and a van loaded with 12 passengers. Write inequalities and objective functions that satisfy all of the above conditions

2. To make a muffin, you need100 g of flour and 65 g of sugar to make a bun, you need 150 g of flour and 35 g of sugar. To make muffins and bun, 8.2 kg of flour and 3.5 kg of sugar are needed. The number of muffin made is not more than 2 times the number of buns made. Assume that x is muffin and y is bun. The price of each muffin and bun is RM1.50 and RM2.00. Write inequalities and objective functions that satisfy all the above conditions.

Let's do this!

3. Pak Amin cannot raise more than 500 cows. The number of local cows is more than 250 but the number of export cows is not more than four times the number of local cows raised. The ration between the number of export cows and the number of local cows is at least 2:3. State four inequalities based on the above information.

4. "Syarikat Gemilang" is appointed by "Syarikat Cemerlang" to carry out a survey on the quality of their TV programmers through questionnaires. The constraints fixed by "Syarikat Cemerlang" are:

i. At least 600 people who are to be given the questionnaires are from rural areas.
ii. The number of urban area people who are to be given the questionnaires is not less than the number of rural area people.
iii. The total number of people who are to be given the questionnaires is not more than 1300

Write inequalities and objective functions that satisfy all the above conditions

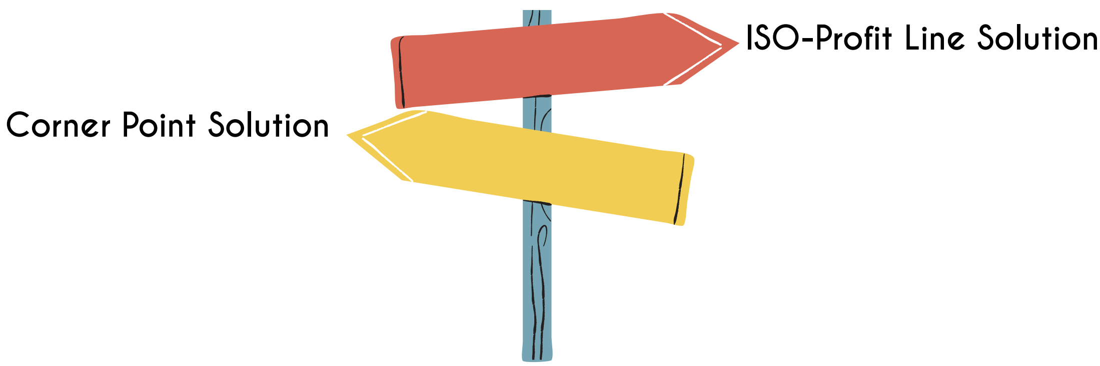
SCAN ME!

GRAPHICAL SOLUTION

GRAPHICAL SOLUTION

- Method of solving LP problems graphically by plotting the feasible region and finding the optimal solution by visual inspection.
- The feasible set is the set of all feasible combination of the decision variables that satisfy the constraints of the problem
- This method is suitable for solving the linear programming with two decision variables (x and y).

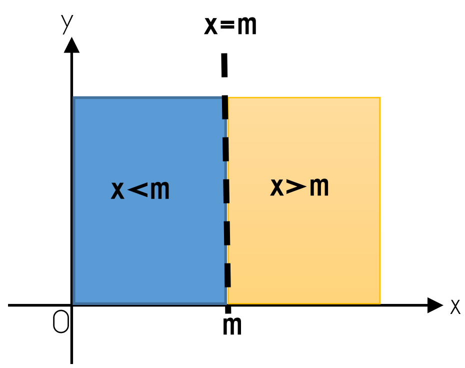
GRAPHICAL METHOD

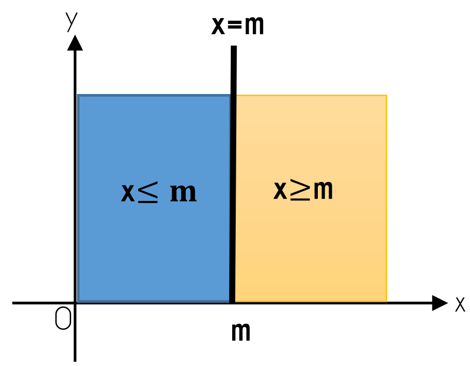


DON'T FORGET

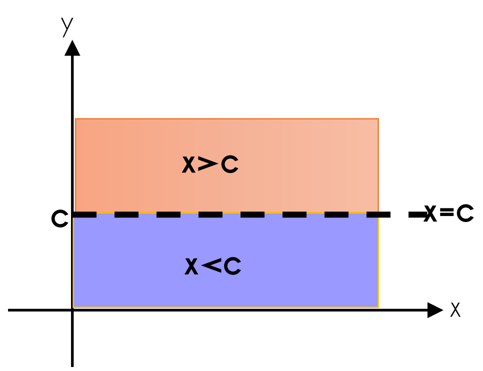
Before we start with linear programming, let us look at the graphical representation of **linear inequalities with two variables.**

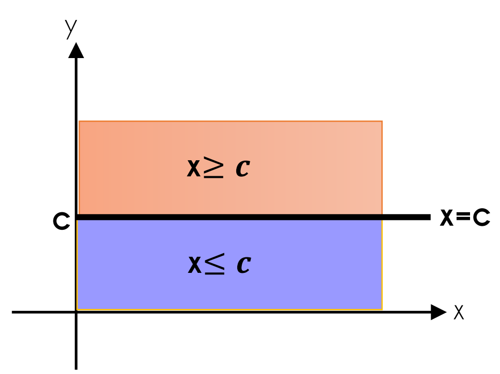
x=m where m is a constant





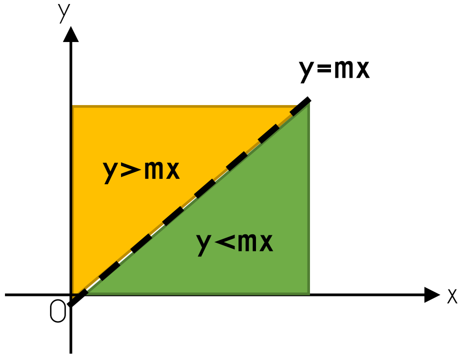
x=c where c is a constant

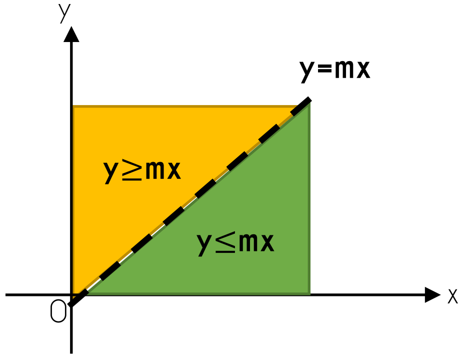




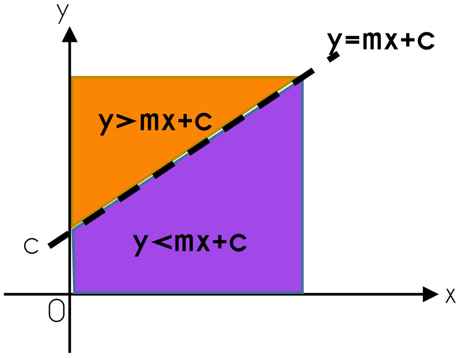
DON'T FORGET

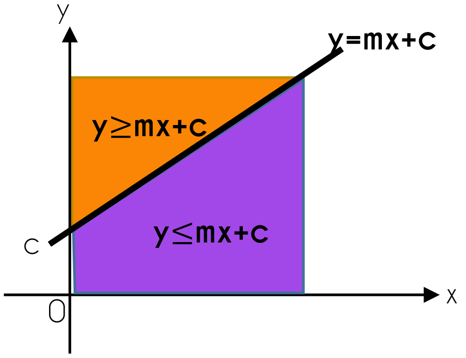
y=mx where $m > 0$





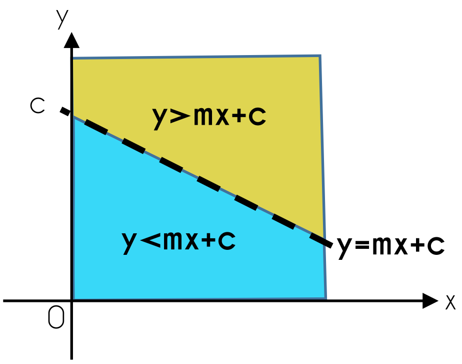
y=mx+c where $m > 0$ *and c is constant*

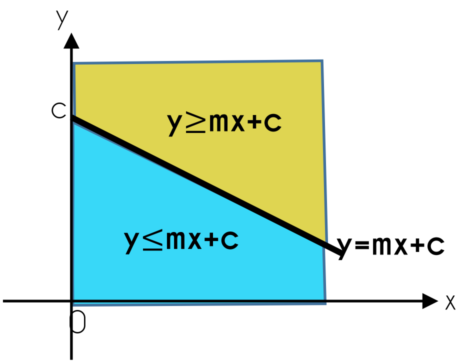




DON'T FORGET

$y=mx+c$ where $m < 0$ and c is constant





A **solid line** is used when the shaded region includes the points lying on the line. **This is indicated by the ≥ or ≤ sign in the inequality**.

A **dashed line** is used when the shaded region includes the points lying on the line**. This is indicated by the > or < sign in the inequality**.

CASE 4

Determine the solution set for the inequality by using graph

A

$$x + 3y \geq 12$$

Step 1:

Replacing the inequality" ≥" with equality "="

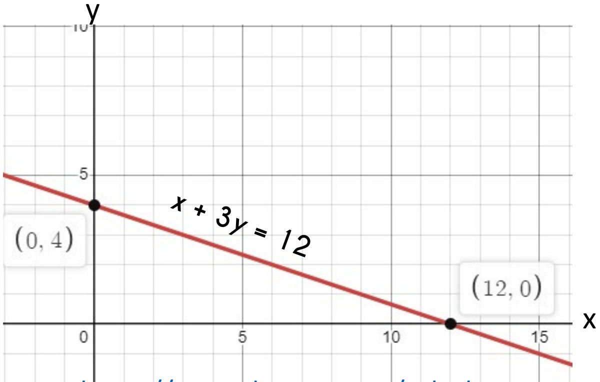
$$x + 3y = 12$$

Step 2:

Sketch the straight **line** $x + 3y = 12$.

x	0	12
y	4	0

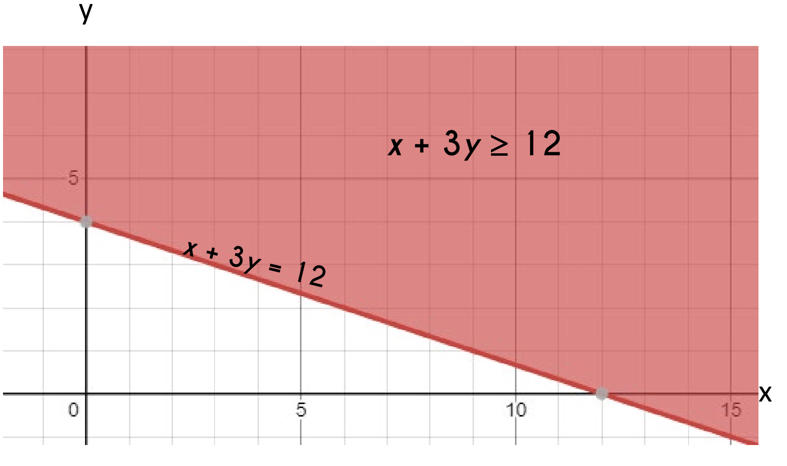
(0,4)
(12,0)



https://www.desmos.com/calculator

Step 3:

Choosing the origin as the test point, we find 1(0) + 3(0) ≥ 12, **or 0 ≥ 12, which is false.** Since the inequality sign is **≥** **a solid line** should be used and the area above the straight line $x + 3y = 12$ must be shaded.



https://www.desmos.com/calculator

CASE 5

Determine the solution set for the inequality by using graph

A

$$x > 8-y$$

Step 1:

Move the terms in y to the left side of the inequality and at the same time, make sure that their **coefficient is positive** .

$x > 8-y$ → $y + x > 8$ **or** $-x<-8+y$

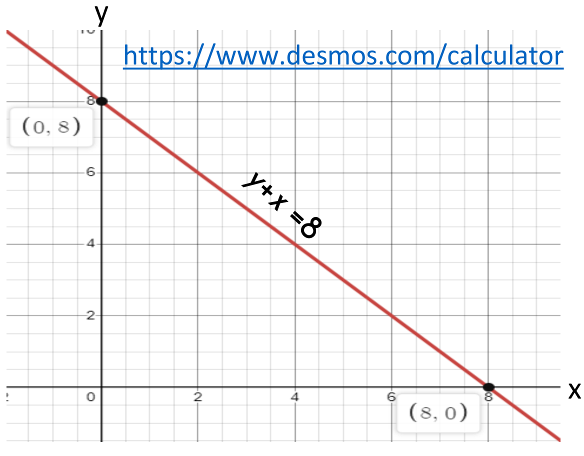
Replacing the inequality > with equality =

y+x=8

Step 2:

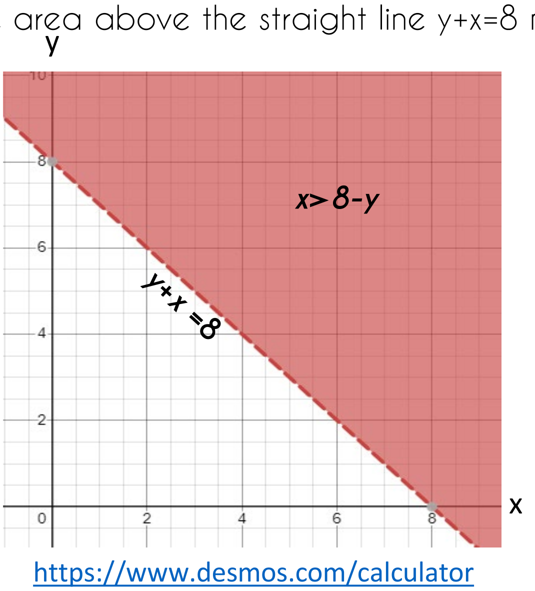
x	0	8
y	8	0

(0,8)
(8,0)



Step 3:

Choosing the origin as the test point, we find 1(0) > 8-(0) , **or 0 >8, which is false.** Since the inequality sign is **> , a dashed line** should be used and the area above the straight line y+x=8 must be shaded.



https://www.desmos.com/calculator

CASE 6

Determine the solution set for the inequality by using graph

$$x \leq 3y$$

A

Step 1:

Move the terms in y to left side of the inequality while making sure that their **coefficient is positive** .

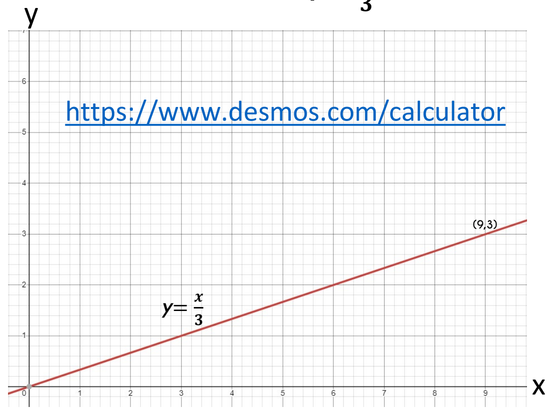
$x \leq 3y$ → $y \geq \frac{x}{3}$

Step 2:

Replacing the inequality " $\leq$" with equality "=" → $y = \frac{x}{3}$

x	0	9
y	0	3

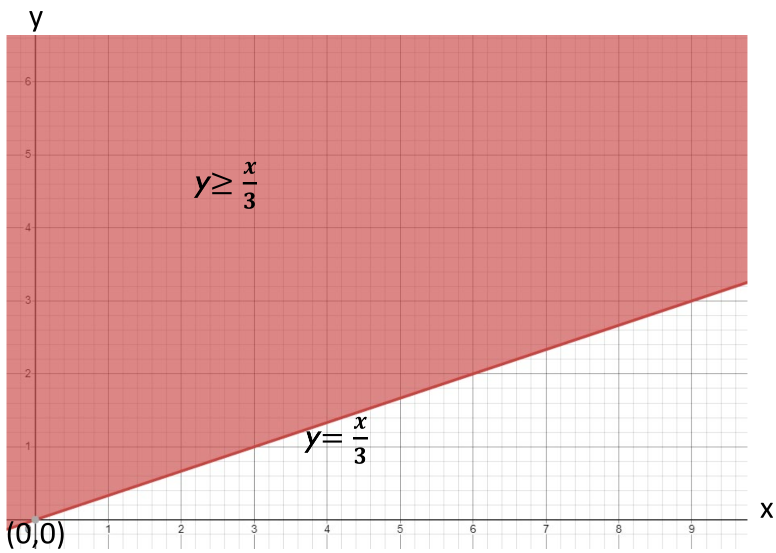
(0,0)
(9,3)



Step 3:

Since the inequality sign is **≥ a solid line** should be used and the area above the straight line $x \leq 3y$ must be shaded..

linear inequality is in the form **y≥, shade the area <u>above</u> the graph**.



https://www.desmos.com/calculator

CASE 7

Construct and shaded the region that satisfies the inequalities $x<7$, $y\leq x-2$ and $3x+6y\geq30$

A

Step 1:

Sketch the line

i. $x= 7$ **(dashed line)**

ii. $y=x-2$ **(solid line))**

x	0	2
y	-2	0

(0,-2) ; (2,0)

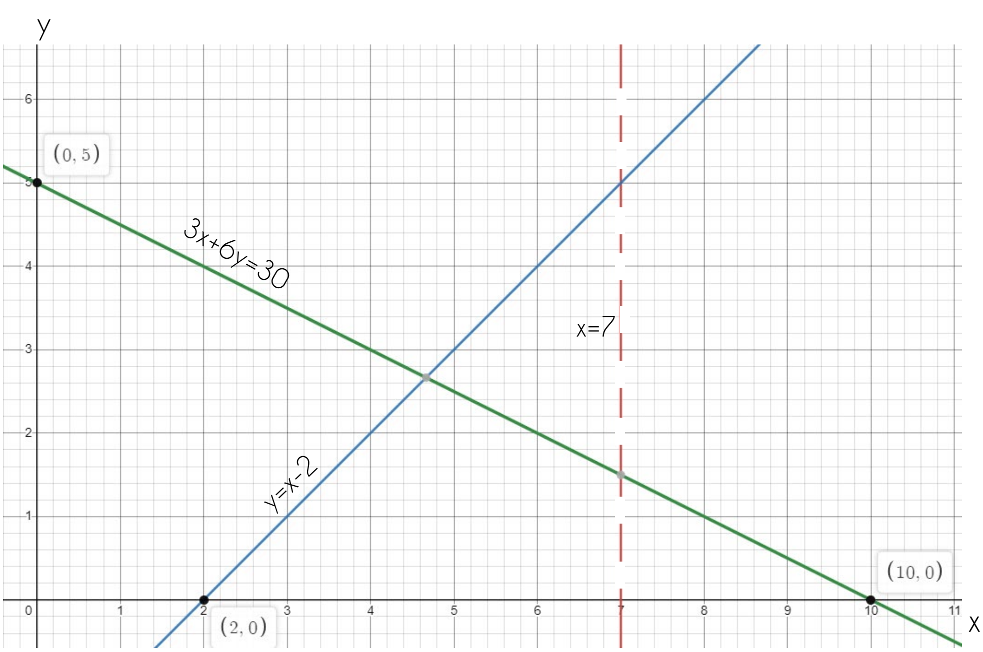
ii. $3x+6y=30$ **(solid line)**

x	0	10
y	5	0

(0,5) ; (10,0)

Step 2:

Plot graph



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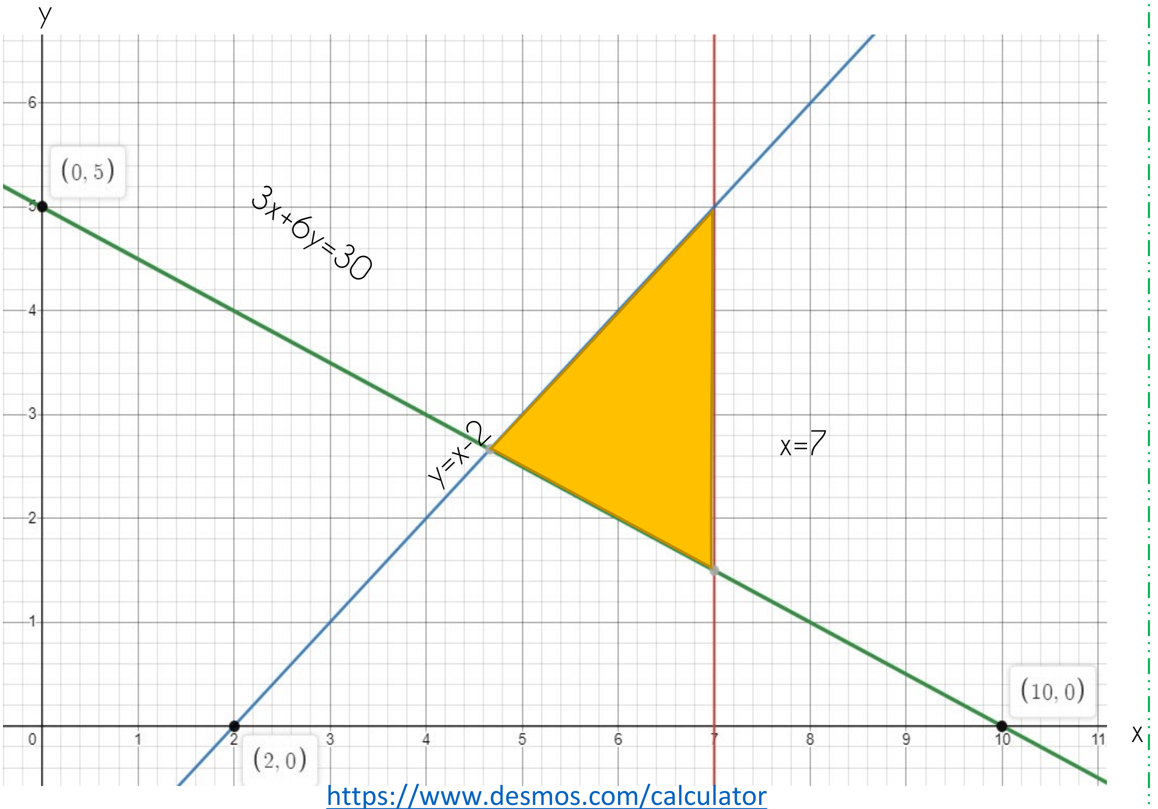
CASE 7

A

Step 3:

Shade the region R.

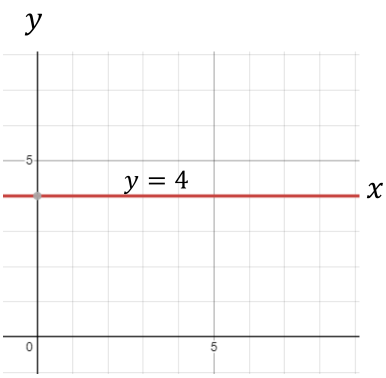


https://www.desmos.com/calculator

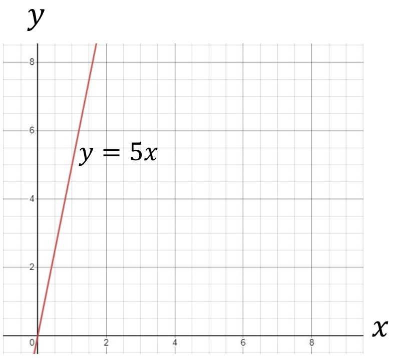
Let's do this!

1. Shade the region on the graph that satisfies the linear inequality.

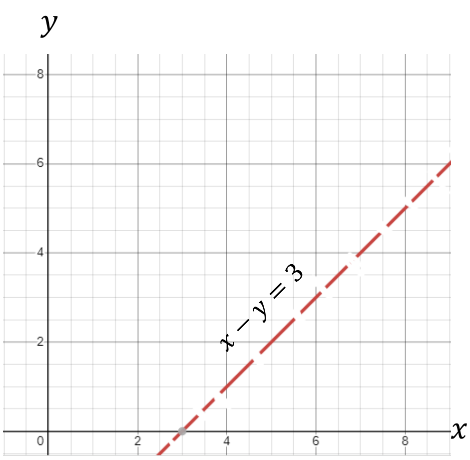
i) $y \geq 4$



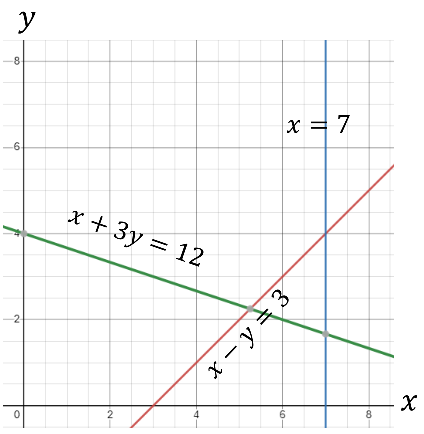
ii) $y \leq 5x$



iii) x-y > 3



iv) x-y $\leq 3; x \geq 7; x + 3y \geq 12$



Let's do this!

2. Plot and shade the region that satisfies each of the following inequalities

i. $5x + 2y \leq 10$

ii. $x - 2y > 4$

iii. $2y \geq 18 - 3x$

iv. $x > 5y$

3. Plot and shade the region that satisfies the following inequalities in one graph.

$$x + y \geq 12$$
$$y \leq 2x$$
$$x < 8$$

SCAN ME!

Method 1: Corner Point Solution

- ❑ A method used in linear programming to find the optimal solution of a problem with multiple constraints and decision variables.
- ❑ It is also called the vertex method.

IMPORTANT!

Step 1	Formulate the Linear Programming
Step 2	Construct a graph and plot the constraints lines
Step 3	❑ Find the valid side of each constraint line ❑ Find the range feasible solution ❑ Find the optimal points
Step 4	❑ Calculate the coordinates of the optimal points ❑ Evaluate the objective function of the optimal points to obtain the required maximum/minimum value of the objective function

CASE 8

A company produces two products D and E . The profit from the sale of products D and E is RM35 and RM45 respectively. The production of products D and E must satisfy the following conditions:

i. The total number of units of products D and E produced in one day is not more than 550 units.

ii. The number of units of product E produced in one day is not more than two times the number of units of product D produced.

iii. The number of units of product E produced in one day is at least 150 units.

Using the graphical method, determine the maximum profit that can be made.

A

Step 1: Formulate the Linear Programming

State the unknowns/decision variables

Define

x= products D y= products E

State the constraints

$x + y \leq 550$

$y \leq 2x$

$y \geq 150$

$y \geq 0;\ x \geq 0$

State the objective function

$P = 35x + 45y$

CASE 8

A

Step 2: Construct a graph and plot the constraints lines

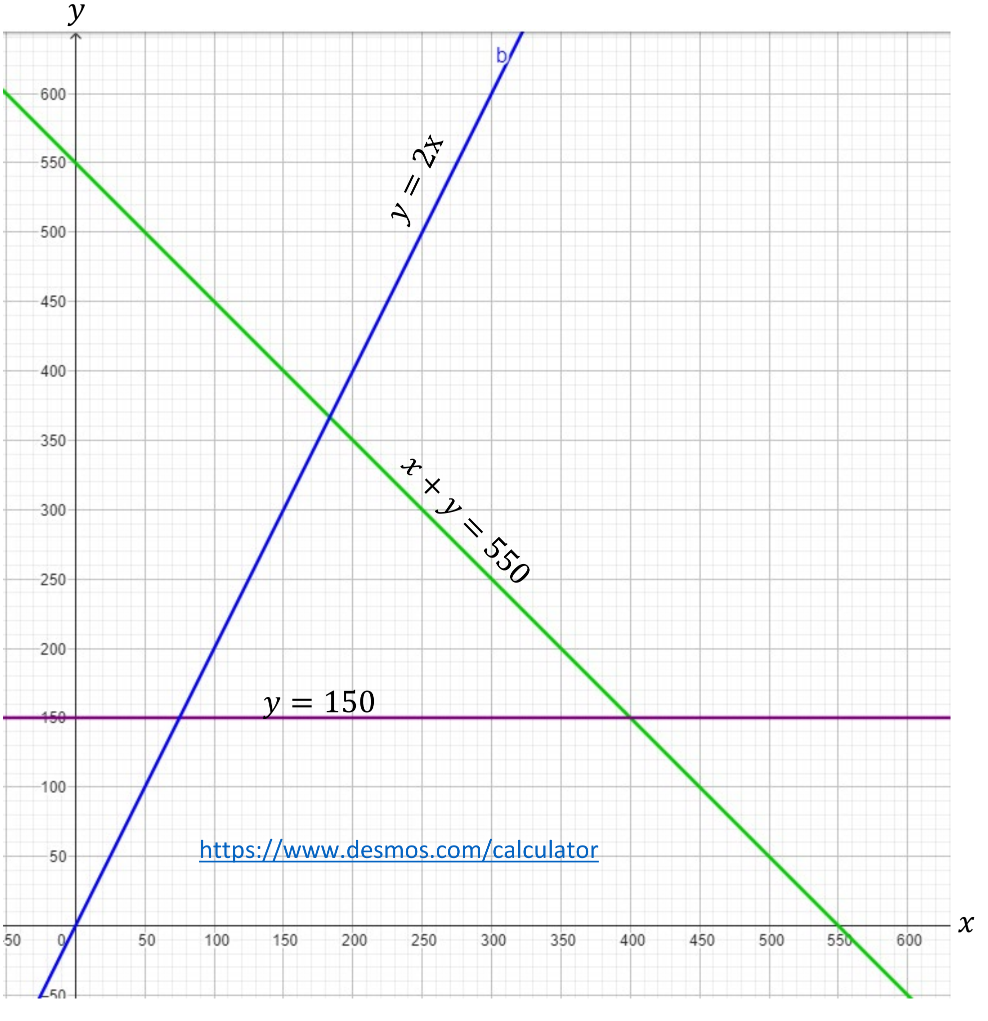
$y = 150$

$x + y = 550$

x	0	550
y	550	0

$y = 2x$

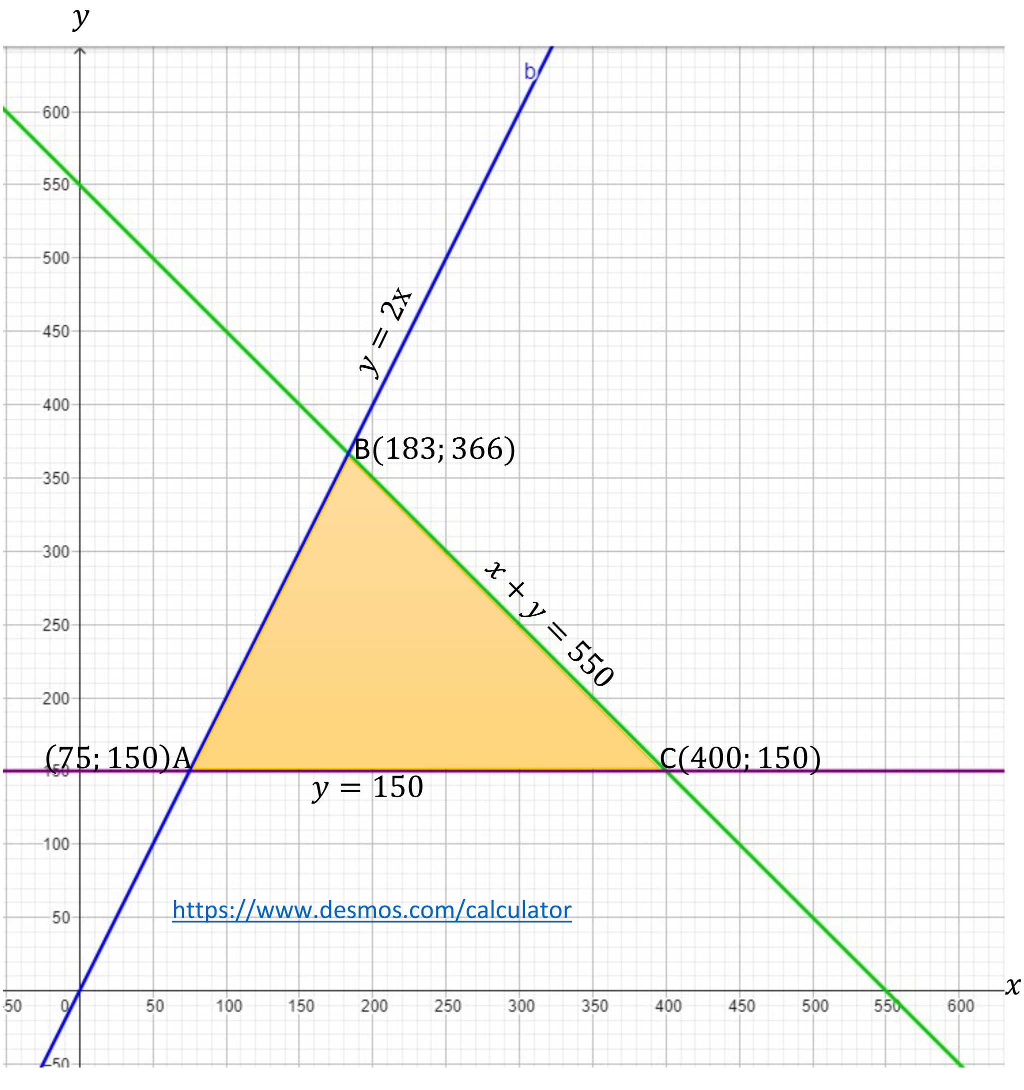
x	0	100
y	0	200



CASE 8

A

Step 3:
- ❑ Find the valid side of each constraint line
- ❑ Find the range feasible solution
- ❑ Find the optimal points



CASE 8

A

Step 4:

- ❑ Calculate the coordinates of the optimal points
- ❑ Evaluate the objective function of the optimal points to obtain the required maximum/minimum value of the objective function,

$$P = 35x + 45y$$

Co-ordinates Point	$P = 35x + 45y$
A (75; 150)	RM 9375
B (183; 366)	RM 22 875
C (400; 150)	RM 20 750

At this point x=183 and y=366, and the corresponding maximum profit is given by

$$P = (183 \times 35) + (366 \times 45) = RM22\,875$$

A factory should sale 183 products D and 366 products E.

CASE 9

A farmer owns 25 acre for growing bananas and pineapples. The farmer must decide how much of each to grow. The cost per acre for bananas is RM30 and for pineapples is RM20. The farmer has budget RM1500. Bananas require 2 man- days per acre and pineapples require 1 man-day per acre. There are 40 man days available. The profit for bananas is RM200 per acre and for pineapples is RM120 per acre. Determine the number of acre of each crop that the farmer should sow to maximize profit.

A

Step 1: Formulate the Linear Programming

State the unknowns/decision variables

Define

x= number of acre of bananas

y= number of acre of pineapples

State the constraints

Land	$x + y \leq 25$
Cost	$30\text{x} + 20y \leq 1500$
Manpower	$2x + y \leq 40$
	$y \geq 0;\ x \geq 0$

State the objective function

$$P = 200x + 120y$$

CASE 9

A

Step 2: Construct a graph and plot the constraints lines

$x + y = 25$

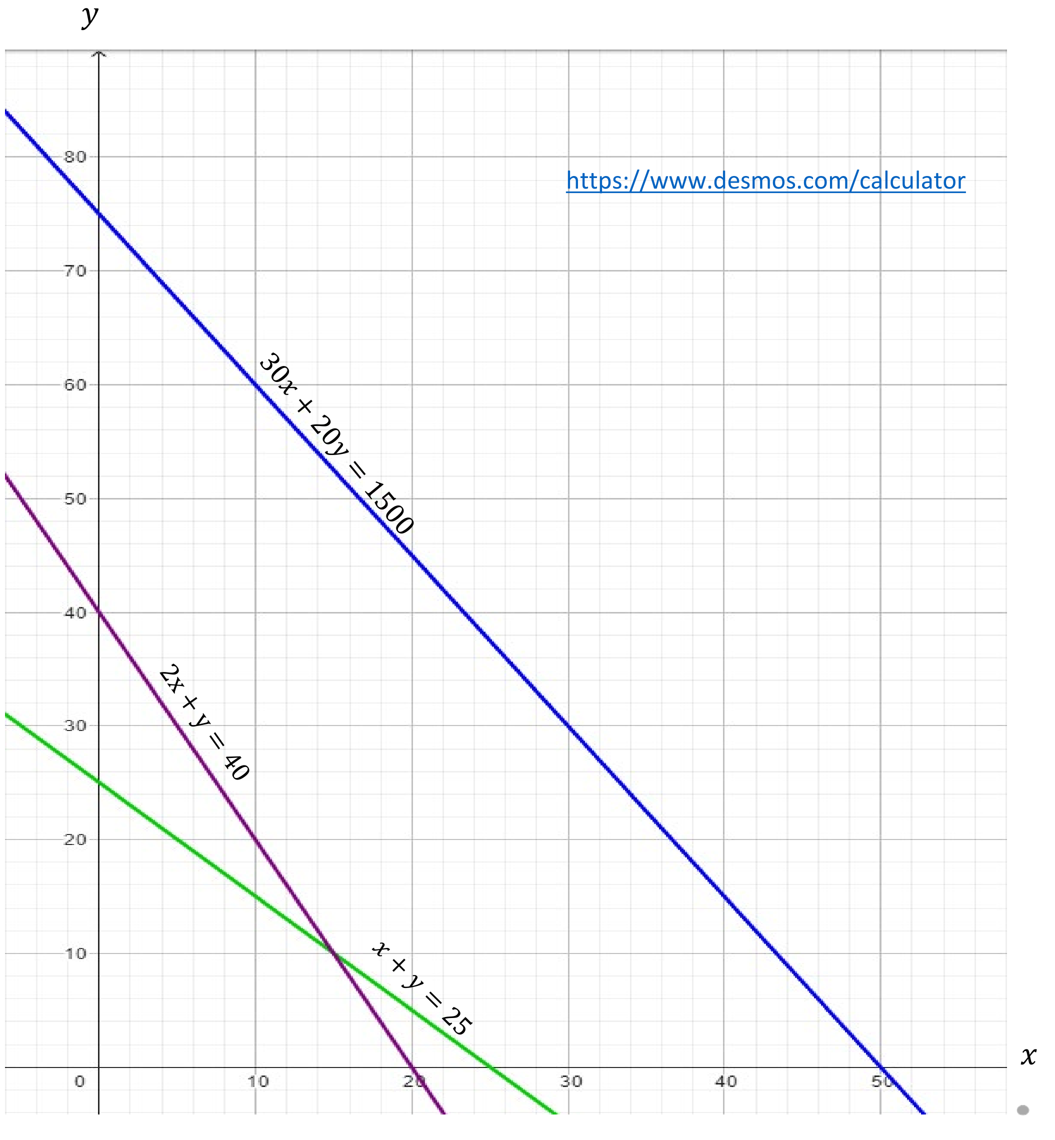
x	0	25
y	25	0

$30x + 20y = 1500$

x	0	50
y	75	0

$2x + y = 40$

x	0	20
y	40	0



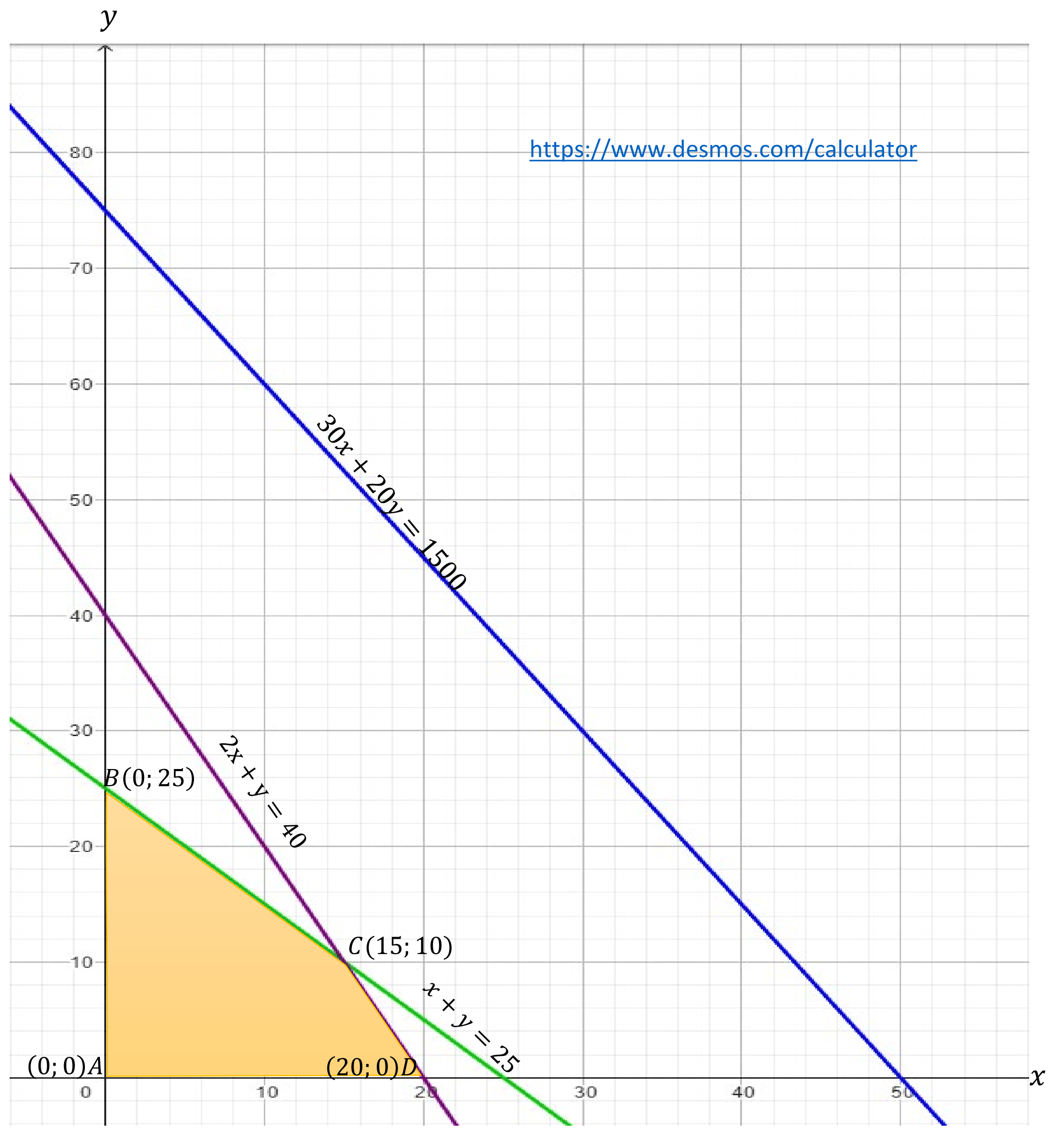
CASE 9

A

Step 3: ❑ Find the valid side of each constraint line

❑ Find the feasible solution region

❑ Find the optimum points



CASE 9

A

Step 4:

- Calculate the co-ordinates of optimum points
- Evaluate the objective function at optimum points to get the required maximum/minimum value of the objective function,

$$\mathrm{P} = 200x + 120y$$

Co-ordinates Point	$P = 200x + 120y$
A (0; 0)	RM 0
B (0; 25)	RM 3 000
C (15; 10)	RM 4 200
D (20; 0)	RM 4 000

At this point x=15 and y=10, and the corresponding maximum profit is given by

$$P = (15 \times 200) + (10 \times 120) = RM4\,200$$

The farmer should sow 15 hectares with banana and 10 hectares with pineapple

Method 2: ISO-profit Line Solution

- ❑ ISO-profit line method is an approach to identifying the optimal point in a graphical linear programming problem.

- ❑ The line touching a particular point of the feasible region determines the optimal solution. Once the feasible region has been graphed, one can find the optimal solution to the problem.

- ❑ The optimal solution is the point that lies in the feasible region and gives the highest profit.

IMPORTANT!

Step 1	Formulate the Linear Programming
Step 2	Construct a graph and plot the constraints lines
Step 3	❑ Draw the half planes of all constraints ❑ Shade the intersection of all the half planes which is the feasible region
Step 4	❑ Since the objective function is Z=ax+by, draw a dashed line for the equation ax+by=k, where k is an arbitrary constant. To maximize Z, draw a line that is parallel to ax +by=k and farthest from the origin ❑ Find the coordinates of this point by solving the equation of the straight line on which it lies. OR Get the ISO line from the equation k=ax+by, then take the ruler and move it parallel until you get the maximum point

CASE 10

One institution offers two short courses in electrical engineering, E and F . The following conditions apply to the enrollment of the participants :

i. The total number of participants is not more than 70 .

ii. The number of participants for course F must exceed the number of participants for course E by at least 4.

iii. The number of participants for Course F is not more than 3 time the number of participants for Course E.

Using the ISO-profit graph method, determine the maximum total fees that can be collected if the fees per month for courses E and F are RM40 and RM50 respectively.

A

Step 1: Formulate the Linear Programming

State the unknowns/decision variables

Define

x= courses E y= courses F

State the constraints

$$x + y \leq 70$$

$$y - x \geq 4$$

$$y \leq 3x$$

$$y \geq 0; \; x \geq 0$$

State the objective function

$$P = 40x + 50y$$

CASE 10

A

Step 2: Construct a graph and plot the constraints lines

$x + y = 70$

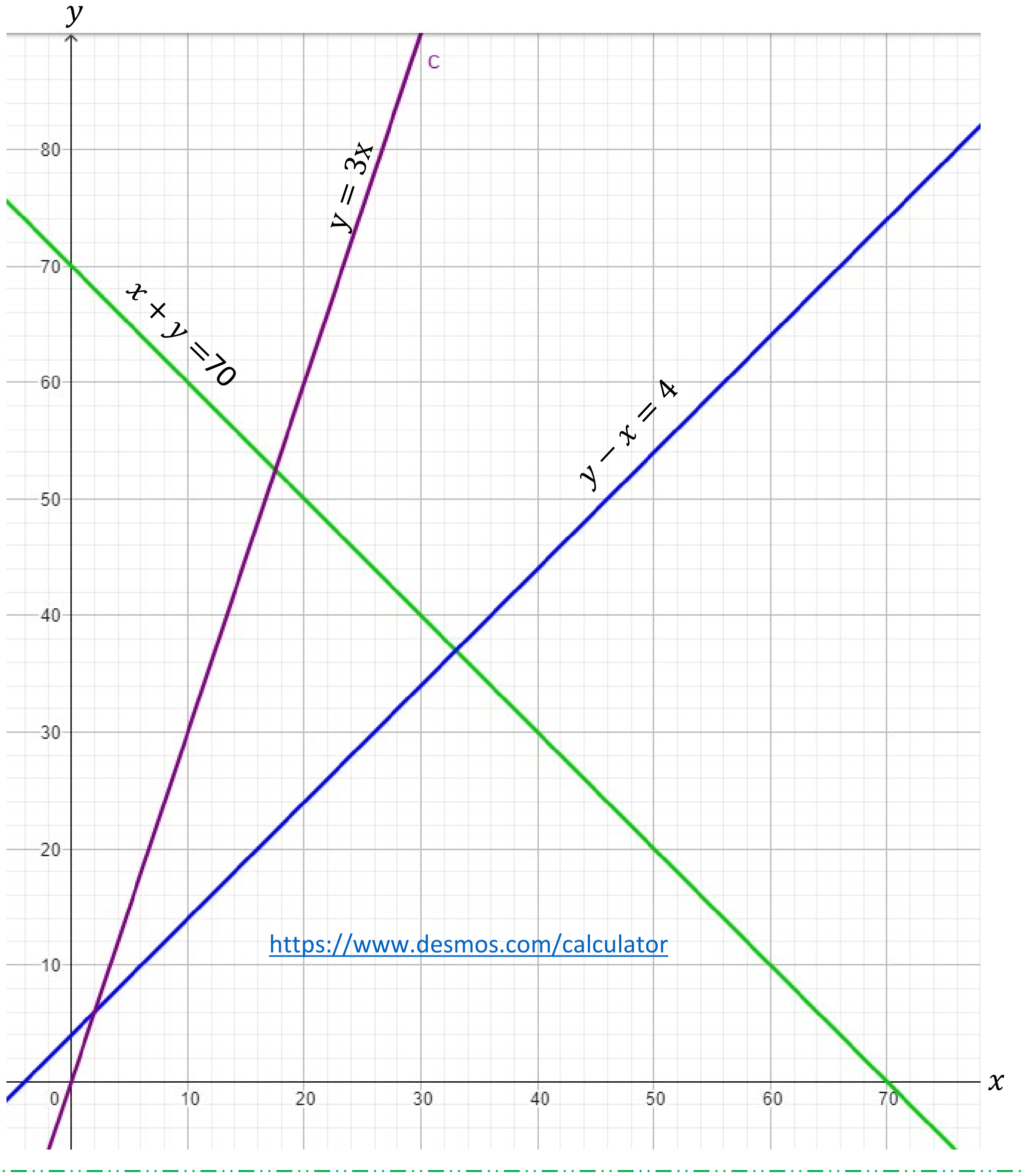
x	0	70
y	70	0

$y - x = 4$

x	0	10
y	4	14

$y = 3x$

x	0	20
y	0	60

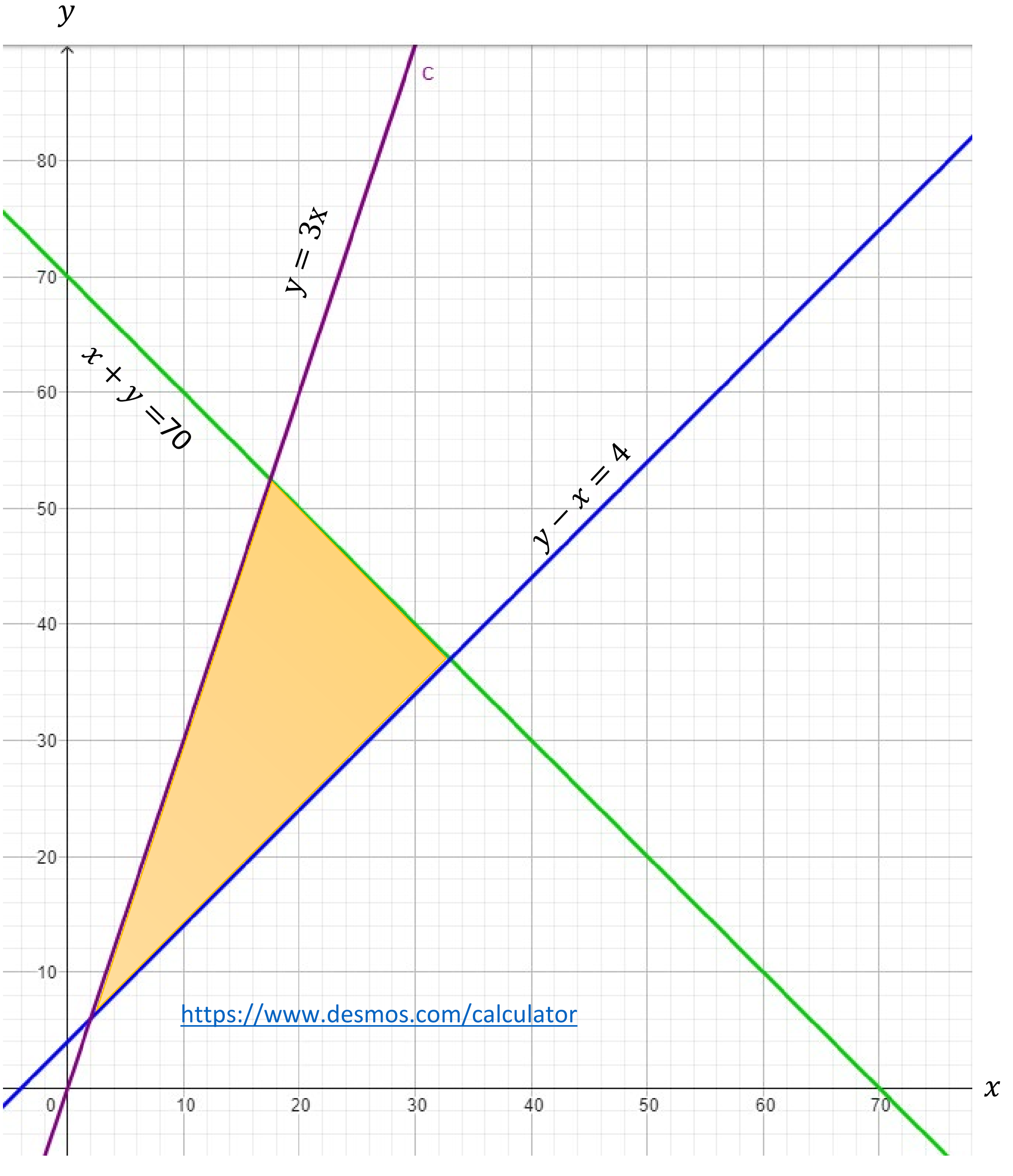


CASE 10

A

Step 3: ❑ Draw the half planes of all constraints

❑ Shade the intersection of all the half planes which is the feasible region



CASE 10

A

Step 4:

- Since the objective function is Z=ax+by, draw a dashed line for the equation ax+by=k, where k is an arbitrary constant. To maximize Z, draw a line that is parallel to **ax +by=k** and farthest from the origin
- Find the coordinates of this point by solving the equation of the straight line on which it lies.

$$given\ \ P = 40x + 50y$$

$$k = 40x + 50y$$

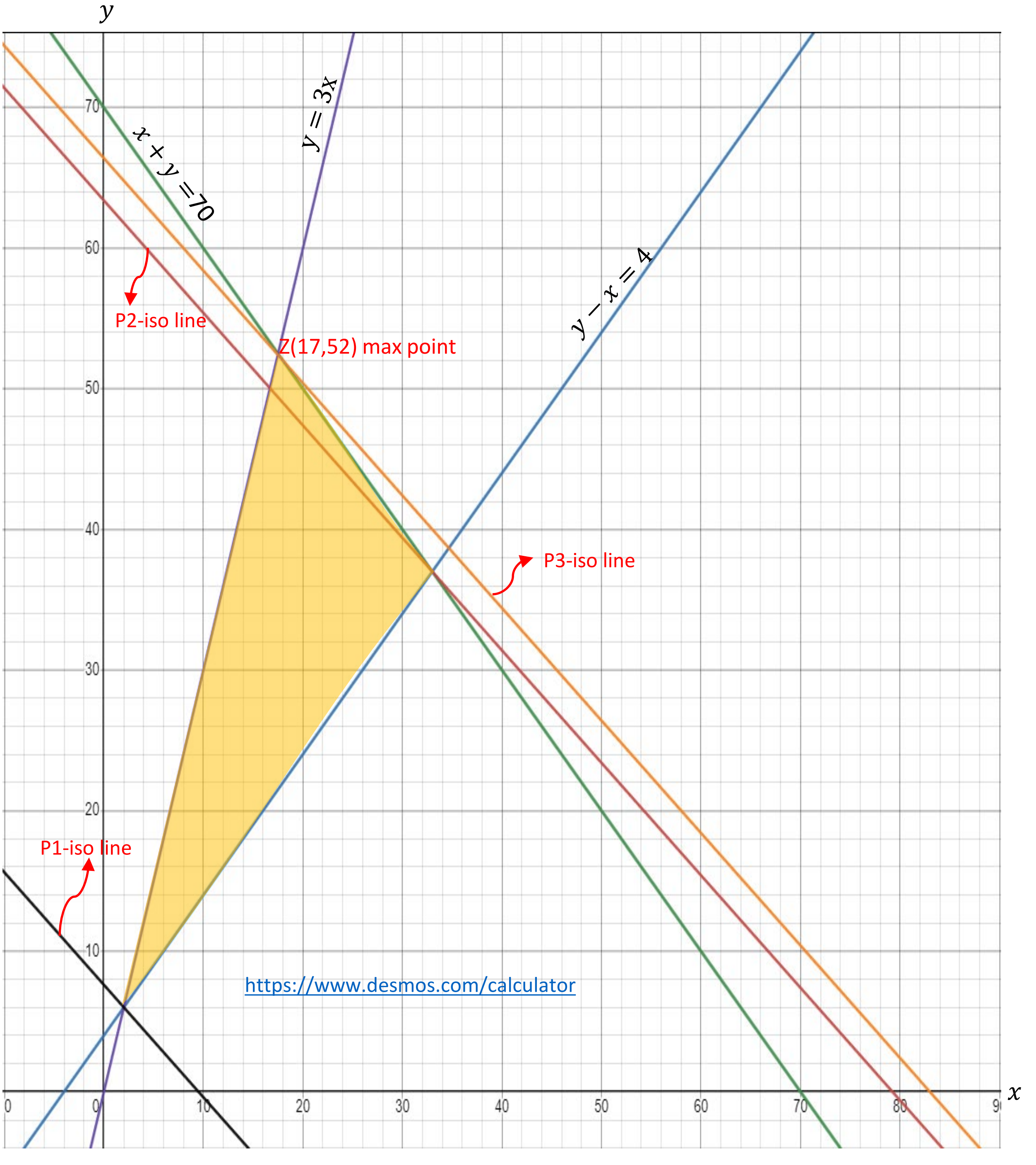
Any point in the feasible region	Value of constant	ISO profit equation	ISO profit line coordinate
P1 (2;6)	k=380	40x+50y=380	(0,7.6) ; (9.5,0)
P2 (33;37)	k=3170	40x+50y=3170	(0,63.4); (79.3,0)
P3 (18;52)	k=3320	40x+50y=3320	(0,66.4); (83,0)

Use ISO profit equation to get ISO coordinate
e.g: P3 40x+50y=3320

x	y
0	83
66.4	0

CASE 10

Step 5:



CASE 10

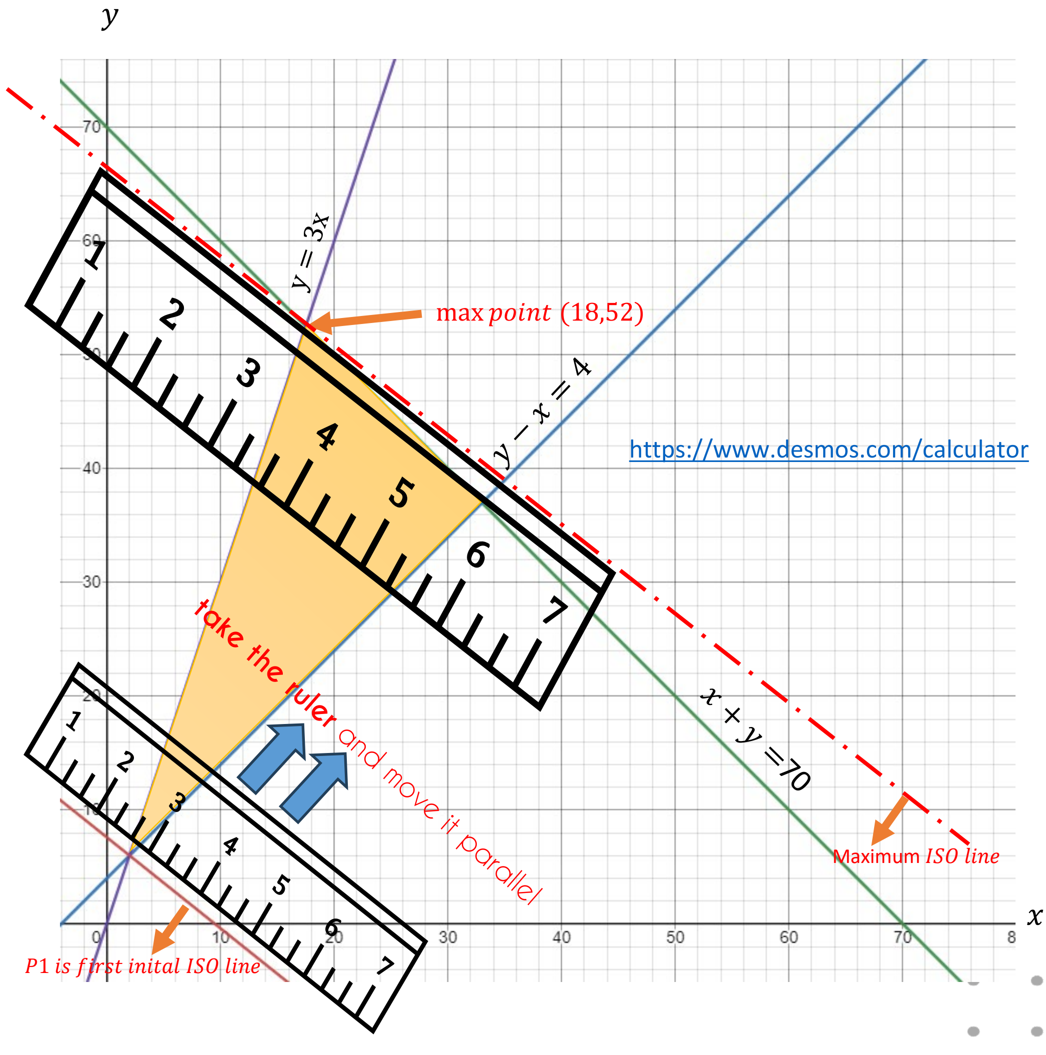
OR Get the ISO line from the equation k=ax+by, then **take the ruler** and move it parallel until you get the maximum point

- Find the coordinates of this point by solving the equation of the straight line on which it lies.

$$given\ P = 40x + 50y$$

$$k = 40x + 50y$$

Any point in the feasible region	Value of constant	ISO profit equation	ISO profit line coordinate
P1 (2;6)	k=380	40x+50y=380	(0,7.6) ; (9.5,0)



A

- if (x_1,y_1) is the point found in step 4, then $x=x_1$, $y=y_1$, is optimal solution of the LPP and ;

$$Z = ax_1 + by_1$$

Coordinate point at P3 (17,52)

P=40x+50y

P=(40x17)+(50x52)

P=3280

the maximum total fees that can be collected is RM3280

CASE 11

Give that maximum, $Z = x + 2y$ is subject to;

$$x + 2y \geq 60$$
$$y \geq 2x$$
$$2x + y \leq 60$$

Calculate the maximum value for the above case by using ISO-profit line method.

A

Step 1: Construct a graph and plot the constraints lines

$x + 2y = 60$

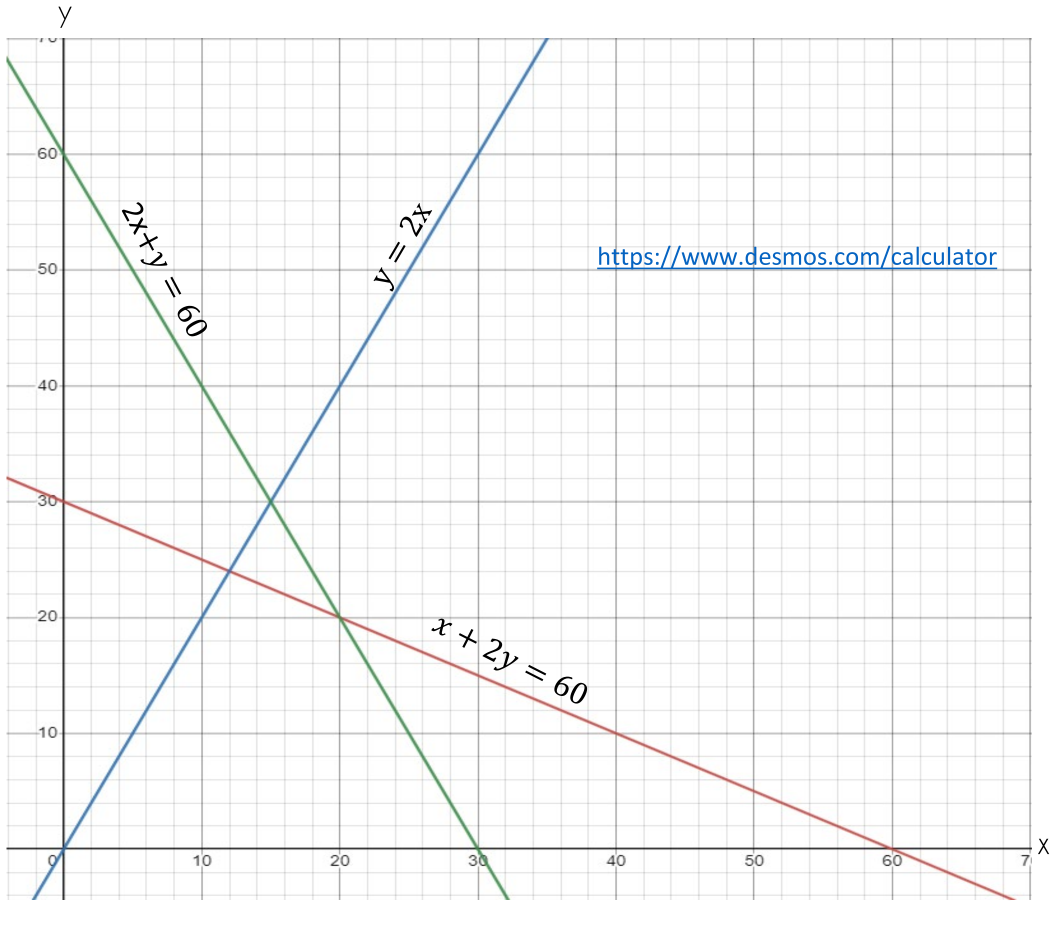
x	0	60
y	30	0

$2x + y = 60$

x	0	30
y	60	0

$y = 2x$

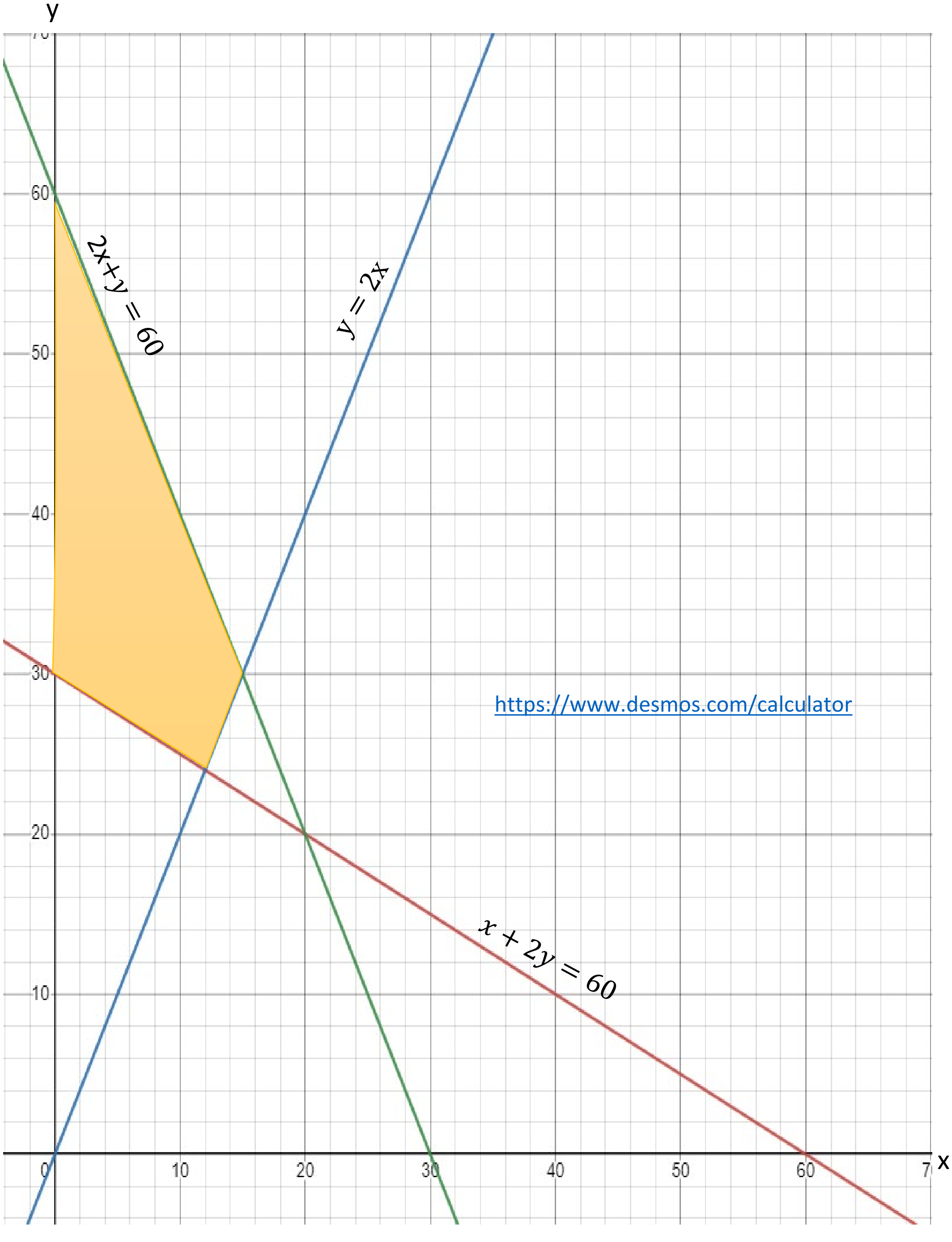
x	0	20
y	0	40



CASE 11

Step 2: ❑ Draw the half planes of all constraints

❑ Shade the intersection of all the half planes which is the feasible region



CASE 11

Step 4:

- Since the objective function is Z=ax+by, draw a dashed line for the equation ax+by=k, where k is an arbitrary constant. To maximize Z, draw a line that is parallel to **ax +by=k** and farthest from the origin

$$given\ Z = x + 2y$$

$$k = x + 2y$$

Any point in the feasible region	Value of constant	ISO profit equation	ISO profit line coordinate
P1 (0;50)	k=100	x+2y=100	(0,100) ; (50,0)
P2 (0;55)	k=110	x+2y=110	(0,110); (55,0)
P3 (0;60)	k=120	x+2y=120	(0,120); (60,0)

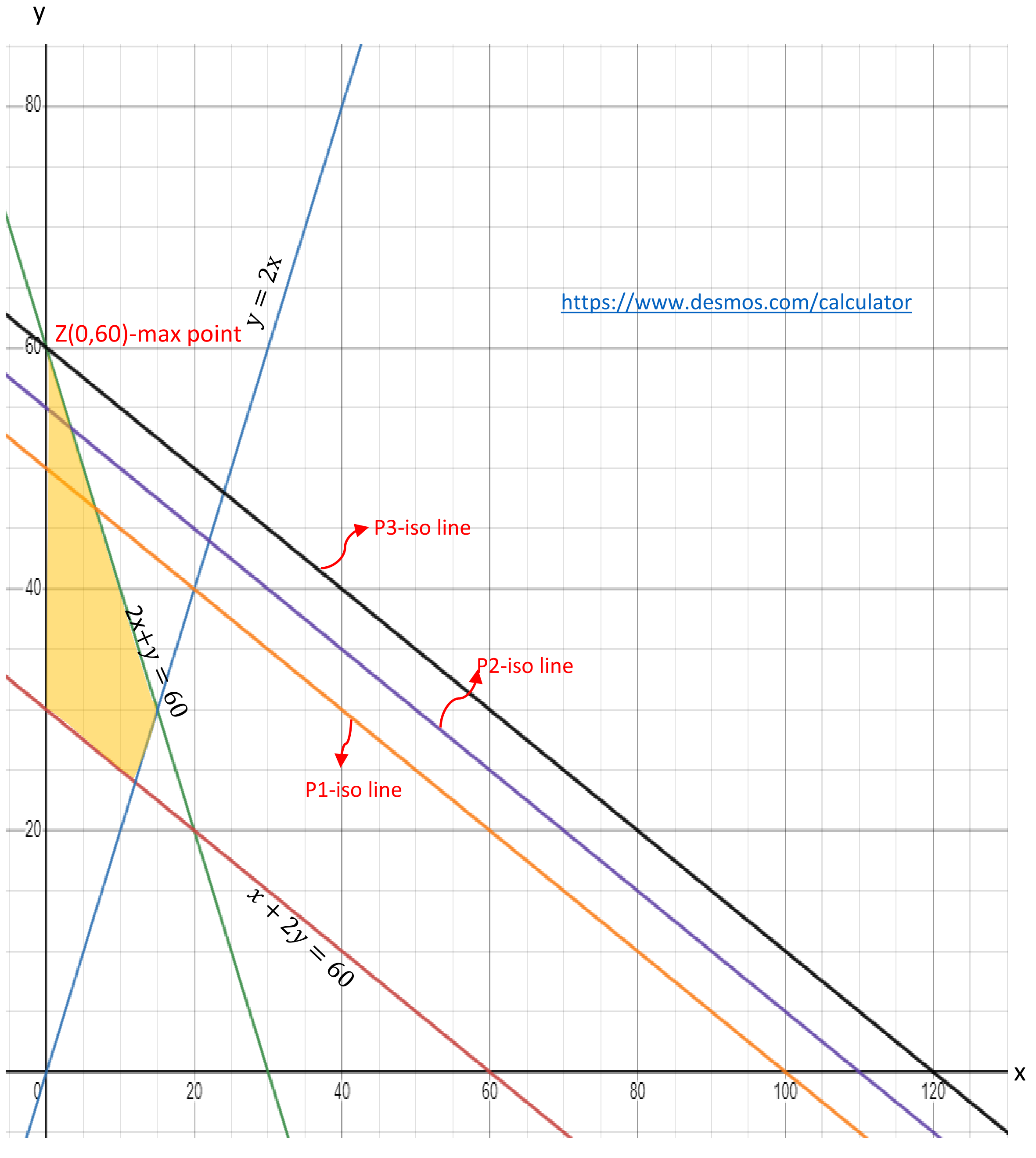
Use ISO profit equation to get ISO coordinate

e.g: P3 x+2y=120

x	y
0	60
120	0

CASE 11

Step 5:



CASE 11

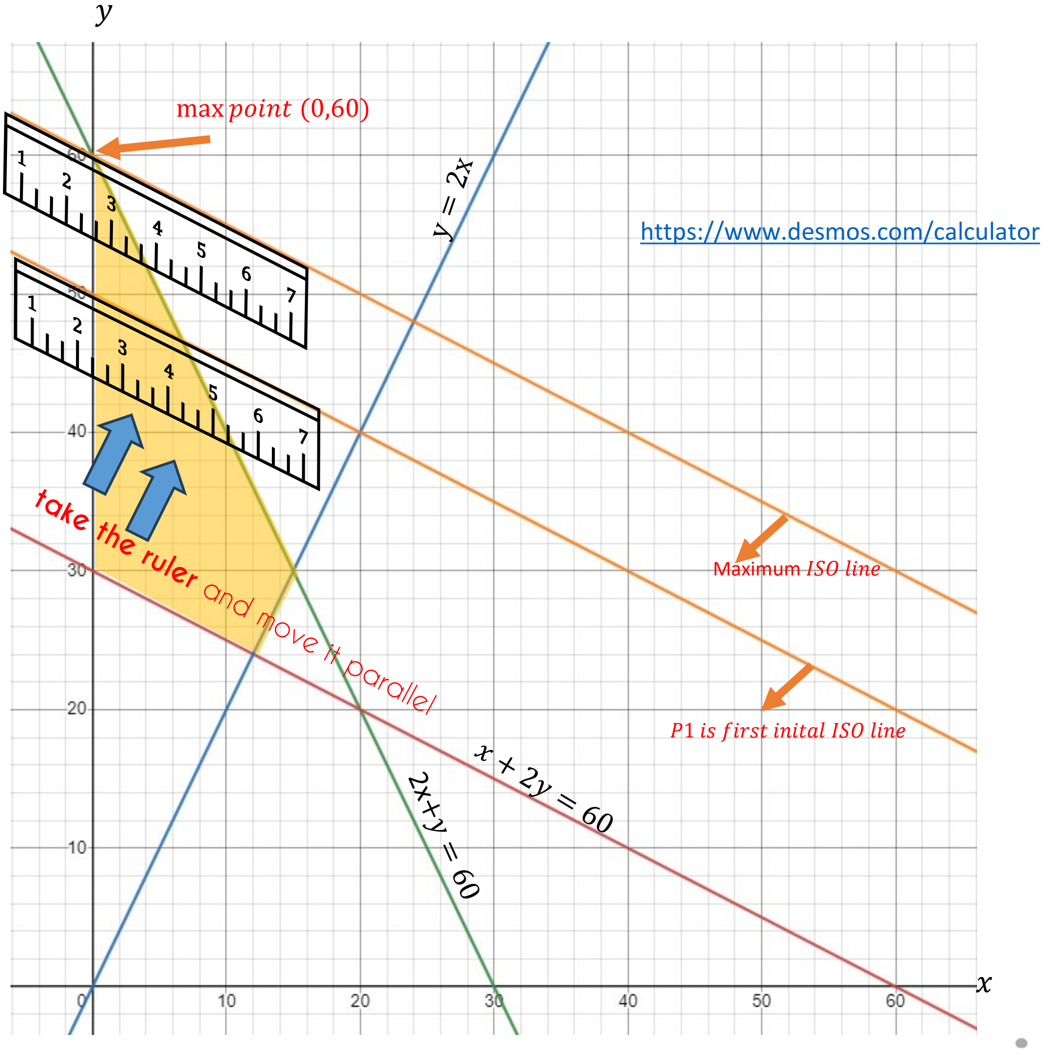
OR Get the ISO line from the equation k=ax+by, then **take the ruler** and move it parallel until you get the maximum point

- ❑ Find the coordinates of this point by solving the equation of the straight line on which it lies.

$$given\ \ P = 40x + 50y$$

$$k = 40x + 50y$$

Any point in the feasible region	Value of constant	ISO profit equation	ISO profit line coordinate
P1 (0,50)	k=100	x+2y=100	(0,100) ; (50,0)



CASE 11

A

- if (x_1, y_1) is the point found in step 4, then $x=x_1$, $y=y_1$, is optimal solution of the LPP and ;

$$Z = ax_1 + by_1$$

Coordinate point at P3 (0,60)

Z=x+2y

Z=(0)+(2x60)

Z=120

Let's do this!

1. A manufacturing plant produces two types of mystery boxes, each requiring the same amount of material. They both pass through a stapling machine and a folding machine. Type A boxes require 3 minutes on the stapling machine and 4 minutes on the folding machine . Type B boxes require 6 minutes on the stapling machine and 3 minutes on the folding machine. Each machine is available for 2 hour. There is a profit of RM 50 for boxes of type A and RM 30 for boxes of type B . How many mystery boxes should be produced of each type to maximize the profit when using corner point method ?

2. Give that maximum, $Z = 3x + 2y$ is subject to;

$$x + y \leq 80$$

$$\mathrm{x} < 40$$

$$2x + y \geq 100$$

$$x \geq 0; y \leq 0$$

Calculate the maximum value for the above case by using graphical method.

Let's do this!

3. Give that maximum, $Z = x + y$ is subject to;

$$x + 4y \geq 8$$

$$2x + 3y \leq 12$$

$$3x + y \leq 9$$

$$x \geq 0; y \leq 0$$

Calculate the maximum value for the above case by using graphical method.

SCAN ME!

SIMPLEX METHOD

- The method is also known as the simplex algorithm.
- An approach to solving linear programming models with more than two variables or problems with a large number of constraints, where the optimal solution to an optimization problem is found using slack variables, tableaus, and pivot variables .
- The simplex method applies only to linear programming problems in standard form where the objective function is to be *maximized.*

IMPORTANT!

To solve a linear programming model using the simplex method the following steps are required:

Step 1.	**Verify that the LP problem is a standard maximization problem in standard form**
Step 2.	Add slack variables to change the inequalities into equations Example: $\mathbf{x}+\mathbf{y}\leq 3$ becomes $\mathbf{x}+\mathbf{y}+\mathbf{s} = 3$

Slack variables - additional variables to convert from inequality constraints to equal constrains.

Step 3	Write the objective function as an equation in the form "left side"= 0 Example: $Z = 3x + 4y$ becomes $-3x - 4y + Z = 0$.
Step 4.	Place the system of equations into the initial tableau. Place the objective equation in the bottom row.
Step 5	Select the pivot column by selecting the smallest negative indicator value in the bottom row.
Step 6	Select pivot row. (Divide the last column by the pivot column for each corresponding entry except the lowest entry and the negative entries. Select the smallest positive result.
Step 7	Find pivot point or pivot element : Circle the pivot entry at the intersection of the pivot column and the pivot row and determine the input and output variable at the mean time. Divide the pivot value in this row by itself to get one. Perform pivoting to make all other entries in this column to zero. This is done in the same way as the Gauss-Jordan method.
Step 8	Do we get all non-negative indicators? If yes, we can stop. Otherwise repeat step 5 to step 7.
	Read the results.

CASE 12

Solve the Linear Programming by using Simplex Method:

Maximize $P = x - y + 2z$

Constraints

$$2x + 2y \leq 8$$

$$z \leq 5$$

$$x \geq 0; y \geq 0; z \geq 0$$

A

Step 1- Check the question is the standard maximization problem (✓)

Step 2- convert to standard form and **add slack variables** into the equation

$$2x + 2y + s = 8$$

$$z + t = 5$$

$$x; y; z; s; t \geq 0$$

Slack variables

Step 3- rewrite the objective function

$$-x + y - 2z + p = 0$$

Step 4- Place the system ***(form the initial tableau).***

1st tableau

x	y	z	s	t	p	b
2	2	0	1	0	0	8
0	0	1	0	1	0	5
-1	1	-2	0	0	1	0

Place the revised **objective function** in the bottom row.

CASE 12

A

Step 5- The **pivot column** can be determined using the bottom row of the tableau and the indicator. Select the **pivot column** by finding **the smallest negative indicator value.**

2nd tableau

pivot column

x	y	z	s	t	p	b
2	2	0	1	0	0	8
0	0	1	0	1	0	5
-1	1	-2	0	0	1	0

The smallest negative indicator (Pivot column)

Step 6 - To find the **pivot row** divide the beta values (b) of the linear constraints by the corresponding values of the column containing the possible pivot variable. **The intersection of the row with the smallest non-negative indicator and the smallest negative value in the bottom row becomes the pivot row.**

x	y	z	s	t	p	b	Indicator
2	2	0	1	0	0	8	
0	0	1	0	1	0	5	$R_2 = \frac{5}{1} = 5$
-1	1	-2	0	0	1	0	

pivot column

the smallest positive
(Pivot row)

CASE 12

A

Step 7- Find the **pivot element**: circle the pivot entry at the intersection of the pivot column and the pivot row, and identify the input and output variable at the mean time. **Divide the pivot in this row by itself to get 1.**

pivot element

x	y	z	s	t	p	b
2	2	0	1	0	0	8
0	0	1	0	1	0	5
-1	1	-2	0	0	1	0

*A solution is considered optimal if all values in the bottom row are greater than or equal to zero.

x	y	z	s	t	p	b	
2	2	0	1	0	0	8	
0	0	1	0	1	0	5	
-1	1	-2	0	0	1	0	$R_3 + 2R_2 \rightarrow R_3'$

2nd tableau

x	y	z	s	t	p	b
2	2	0	1	0	0	8
0	0	1	0	1	0	5
-1	1	0	0	2	1	10

*If negative values exist, repeat steps 5 to step 7.

CASE 12

Repeat Step 5 & step 6

x	y	z	s	t	p	b
2	2	0	1	0	0	8
0	0	1	0	1	0	5
-1	1	0	0	2	1	10

Indicator

$$R_1 = \frac{8}{2} = 4$$

Step 7

3rd tableau

pivot element

	x	y	z	s	t	p	b
$\frac{1}{2}R_1 \rightarrow R'_1$	1	1	0	$\frac{1}{2}$	0	0	4
	0	0	1	0	1	0	5
$R_3 - R_1 \rightarrow R'_3$	-1	1	0	0	2	1	10

4th tableau

x	y	z	s	t	p	b
1	1	0	$\frac{1}{2}$	0	0	4
0	0	1	0	1	0	5
0	2	0	$\frac{1}{2}$	2	1	14

Step 8- Do we get all **nonnegative** indicators? If yes, we may stop.

Locate the basic variables in the final tableau. In this case, the basic variables are *x*, *y*, z and *P*.

The optimal value for x is 4.

The optimal value for y is 0.

The optimal value for z is 5.

The optimal value for P is 14

CASE 13

Solve the Linear Programming by using Simplex Method:

Maximize $P = 10x + 12y + 8z$

Constraints

$$2x + 2y \leq 5$$

$$5x + 3y + 4z \leq 15$$

$$x \geq 0; y \geq 0; z \geq 0$$

A

Step 1- Check the question is the standard maximization problem (√)

Step 2- convert to standard form and **add slack variables** into the equation

$$2x + 2y + s = 5$$

$$5x + 3y + 4z + t = 15$$

$$x; y; z; s; t \geq 0$$

Slack variables

Step 3- rewrite the objective function

$$-10x - 12y - 8z + p = 0$$

Step 4- Place the system of equations with slack variables, into a matrix **(*form the initial tableau*).**

1st tableau

x	y	z	s	t	p	b
2	2	0	1	0	0	5
5	3	4	0	1	0	15
-10	-12	-8	0	0	1	0

Place the revised **objective function** in the bottom row.

CASE 13

A

Step 5- The **pivot column** can be determined using the bottom row of the tableau and the indicator. Select the **pivot column** by finding **the smallest negative indicator value.**

x	y	z	s	t	p	b
2	2	0	1	0	0	5
5	3	4	0	1	0	15
-10	-12	-8	0	0	1	0

The smallest negative indicator (Pivot column)

Step 6 - To find the **pivot row** divide the beta values (b) of the linear constraints by the corresponding values of the column containing the possible pivot variable. The intersection of the row with the smallest non-negative indicator and the smallest negative value in the bottom row becomes the pivot row.

x	y	z	s	t	p	b
2	2	0	1	0	0	5
5	3	4	0	1	0	15
-10	-12	-8	0	0	1	0

pivot column

Indicator

$$R_1 = \frac{5}{2} = 2.5$$

$$R_2 = \frac{15}{3} = 5$$

(Pivot row)

the smallest positive

CASE 13

A

Step 7- Find the **pivot element**: circle the pivot entry at the intersection of the pivot column and the pivot row, and identify the input and output variable at the mean time. **Divide the pivot in this row by itself to get 1.**

x	y	z	s	t	p	b
2	2	0	1	0	0	5
5	3	4	0	1	0	15
-10	-12	-8	0	0	1	0

$\frac{1}{2}R_2 \rightarrow R_2'$

2nd tableau

pivot element

x	y	z	s	t	p	b
1	1	0	$\frac{1}{2}$	0	0	$\frac{5}{2}$
5	3	4	0	1	0	15
-10	-12	-8	0	0	1	0

$R_2 - 3R_1 \rightarrow R_2'$

$R_3 + 3R_1 \rightarrow R_2'$

3rd tableau

x	y	z	s	t	p	b
1	1	0	$\frac{1}{2}$	0	0	$\frac{5}{2}$
2	0	4	$\frac{-3}{2}$	1	0	$\frac{15}{2}$
2	0	-8	6	0	1	30

*If negative values exist, repeat steps 5 to step 7.

CASE 13

A

Repeat Step 5 & step 6

x	y	z	s	t	p	b
1	1	0	$\frac{1}{2}$	0	0	$\frac{5}{2}$
2	0	4	$\frac{-3}{2}$	1	0	$\frac{15}{2}$
2	0	-8	6	0	1	30

Indicator

$R_2 = \frac{15}{2} \div 4 = \frac{15}{8}$

Step 7 4th tableau

pivot element

	x	y	z	s	t	p	b
	1	1	0	$\frac{1}{2}$	0	0	$\frac{5}{2}$
$\frac{1}{4}R_2 \rightarrow R_2$	$\frac{1}{2}$	0	1	$\frac{-3}{8}$	$\frac{1}{4}$	0	$\frac{15}{8}$
$R_3 + 8R_2 \rightarrow R'_3$	2	0	-8	6	0	1	30

5 tableau

x	y	z	s	t	p	b
1	1	0	$\frac{1}{2}$	0	0	$\frac{5}{2}$
$\frac{1}{2}$	0	1	$\frac{-3}{8}$	$\frac{1}{4}$	0	$\frac{15}{8}$
6	0	0	3	2	1	45

Step 8- Do we get all **nonnegative** indicators? If yes, we may stop.

Locate the basic variables in the final tableau. In this case, the basic variables are *x*, *y*, z and *P*.

The optimal value for x is 0.

The optimal value for y is $\frac{5}{2}$.

The optimal value for z is $\frac{15}{8}$.

The optimal value for P is 45

Let's do this!

a. Given Linear Programming problem with , Objective function

$$p = x + 2y$$

Constraints

$$x + y + 3z \leq 5$$
$$2x - 3z \leq 1$$
$$x, y, z \geq 0$$

i. Write the problem in Standard Simplex Form.
ii. Convert the following Standard Form into 1st Tableau and find pivot row, pivot column and pivot element.

b. Given Linear Programming problem with , Objective function

$$p = 3x + 2y + z$$

Constraints

$$4x + y + z \leq 30$$
$$2x + 3y + z \leq 60$$
$$x + 2y + 3z \leq 40$$
$$x, y, z \geq 0$$

i. Write the problem in Standard Simplex Form.
ii. Convert the following Standard Form into second Initial Tableau.

Let's do this!

c. Given Linear Programming problem with , Objective function

$$p = 3x + 6y + 2z$$

Constraints

$$3x + 4y + 2z \leq 2$$
$$x + 3y + 2z \leq 1$$
$$x, y, z \geq 0$$

Solve by using Simplex Method:

d. Given Linear Programming problem with , Objective function

$$p = 3x + 8y - 5z$$

Constraints

$$2x - y + z \leq 3$$
$$2x + 5y + 6z \leq 5$$
$$x, y, z \geq 0$$

Solve by using Simplex Method.

SCAN ME!

COMPILATION OF PAST FINAL EXAMINATION QUESTIONS

LINER PROGRAMMING [QUESTION 4]

SCAN ME!

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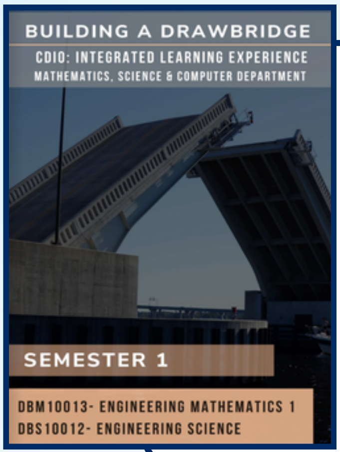

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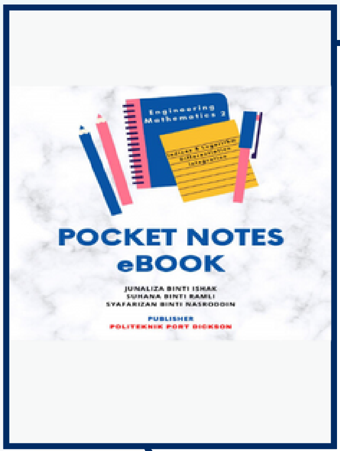

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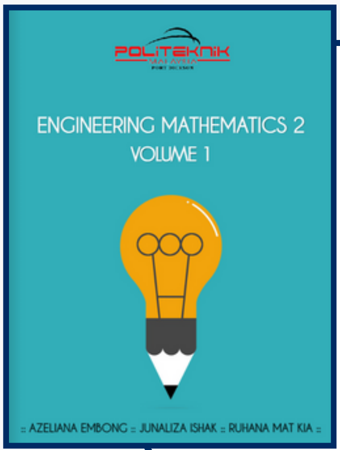

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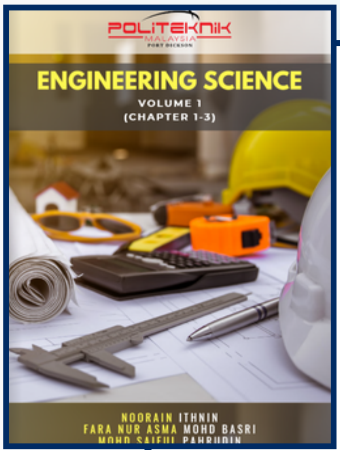

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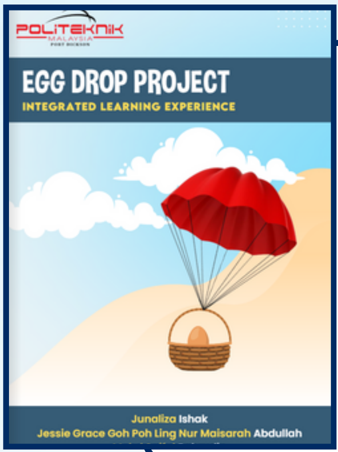

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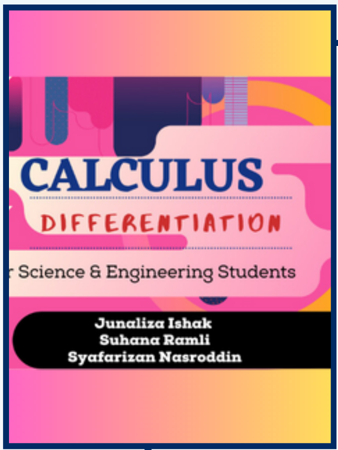

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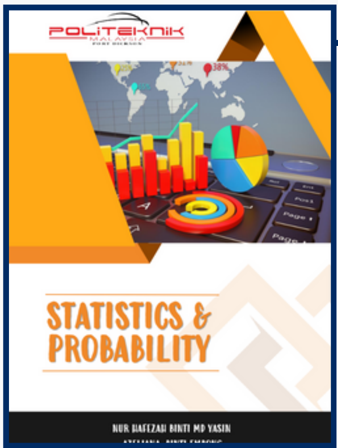

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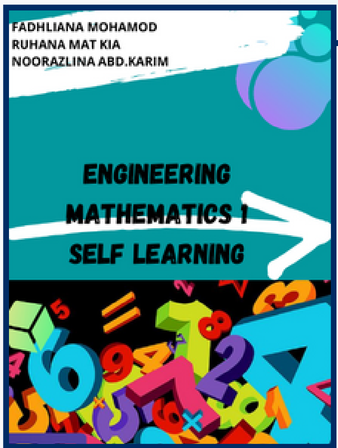

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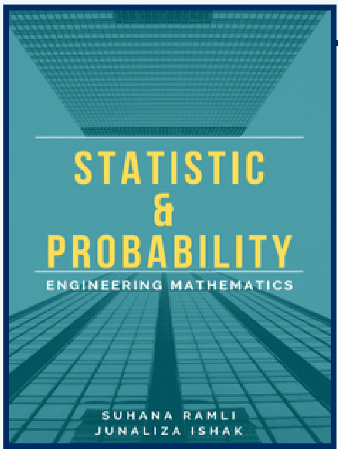

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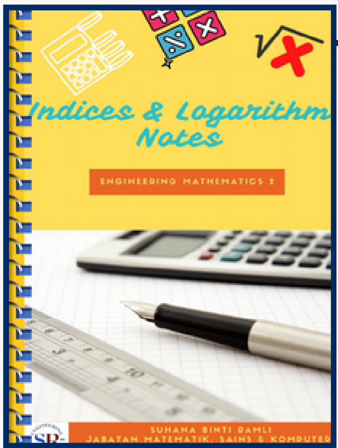

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