SIGNAL AND SYSTEMS

TUTORIAL

WITH SOLUTION

VOLUME 1

Suziyana Yaacob
Nurizan Tahir

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TUTORIAL WITH SOLUTIONS

VOLUME 1: LINEAR TIME INVARIENT SYSTEM

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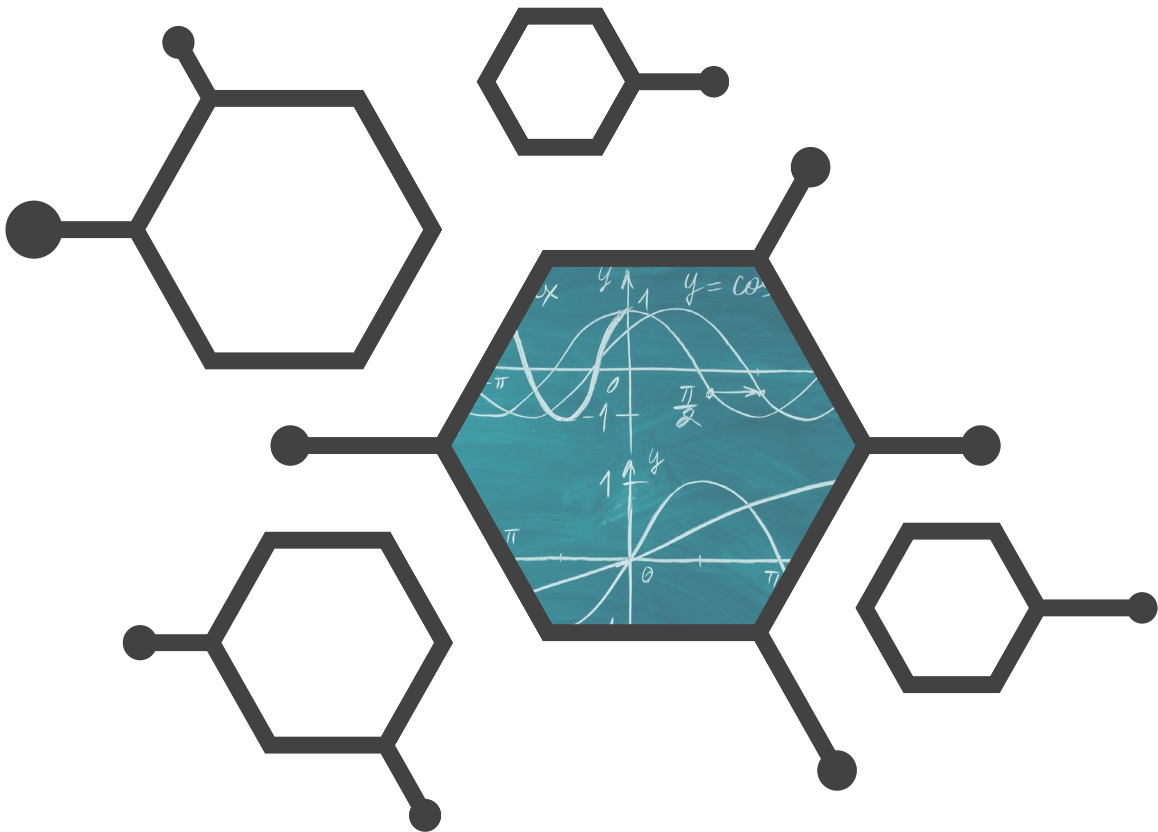
We hereby declare that this module is our original work. To the best of our knowledge, it contains no materials previously written or published by another person. However, if there is any, due acknowledgement and credit are mentioned accordingly in the e-book.

PREFACE

This e-book of Signal and Systems Tutorial with Solution is a collection of selected tutorial from linear time invariant system topic in signal and systems course. This tutorial divided by three subtopics; linear system, convolution sum and convolution integral for clearer and guided step by step solution.

For more informative, this e-book provide interesting notes in chart form to help readers gain more understanding in linear time invariant system topic. Last but not least, few examples been provided to help reader to mastering this topic.



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Short NOTES

LTI SYSTEM

Successful and unsuccessful people do not vary greatly in their abilities. They vary in their desires to reach their potential. – John Maxwell

LTI System

LINEAR TIME INVARIENT (LTI) SYSTEM

LINEAR SYSTEM

Input → SYSTEM → Output

entity act as an input and transform it into output signal

System followed the superposition principle for function F(x)

Homogeneity

$F(ax) = aF(x)$

additivity

$F(x_1 + x_2) = F(x_1) + F(x_2)$

PROPERTIES LTI

- Memory and memoryless
- Causality
- Stability

TIME INVARIENT

behavior & characteristics of the system are fixed over time mathematically, a time shift in the i/p should result in an identical time shift in the o/p signal

CONVOLUTION

refers to a mathematical operation that combines two signal to produce a third signal.

APPLICATION

- Digital Image Processing
- Artificial Intelligent (AI)
- Speech Recognition
- Large - Capacity Data Processing

Convolution

CONVOLUTION INTEGRAL

CONVOLUTION SUM

CONTINOUS TIME SIGNAL

DISCRETE TIME SIGNAL

TYPES OF CONVOLUTION

GOOD START

CONVOLUTION

PROPERTIES

Arithmetic properties of the convolution:

COMMUTATIVE

$h * x = x * h$

ASSOCIATIVE

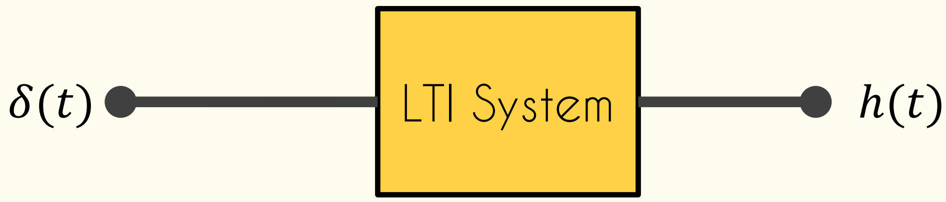
$(h * x) * y = h * (x * y)$

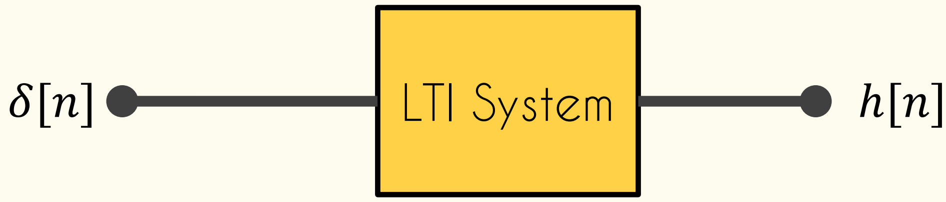
DISTRIBUTIVE

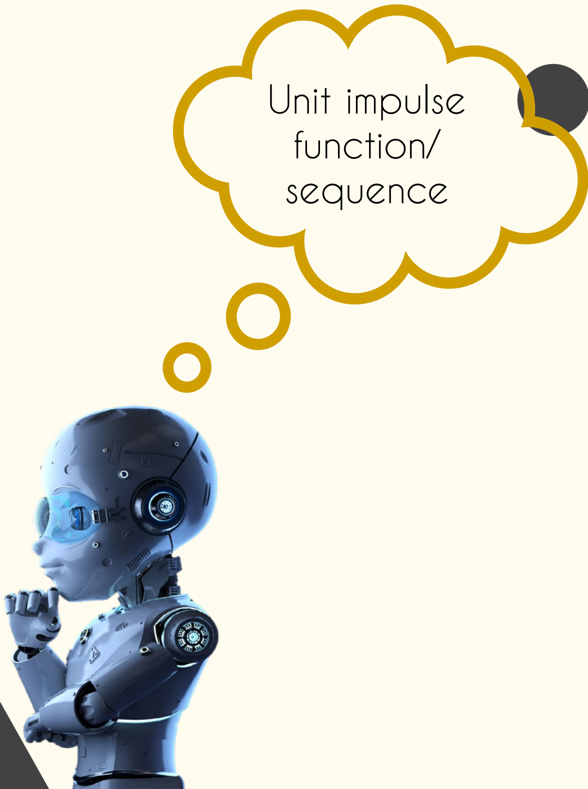
$h * (x + y) = (h * x) + (h * y)$

Impulse Respond

The impulse response (or unit sample response) $\boldsymbol{h(t)}$ of continuous time LLTI system and $\boldsymbol{h[n]}$ of a discrete time LTI system is defined to be the response of the system when the input is $\boldsymbol{\delta(t)}$ ***or*** $\boldsymbol{\delta[n]}$ that is as block diagram below







Convolution Integral

5 Step Operation Process

1
Express each function in terms of dummy variable τ .

2
Reflect one of the function:
$h(t) \rightarrow h(\tau)$

3
Add a time-offset, t:
$h(t-\tau)$
to slide along the τ axis.

4
Find the integral of product whenever the 2-function intersect.

5
The resulting waveform is the convolution of functions f and h.

Convolution Sum: Graphical Method

1 EXPECTATIONS

- ❑ Observe i/p sequence at $n = 0$ for x[n] and h[n]
- ❑ Expectation o/p sequence at, n=n_x+n_h
- ❑ No of sample for o/p sequence = number of sample for x[n]+number of sample h[n] - 1

2 CHANGE INDEX

Change of index from [n] to index [k]

$$x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

3 FOLDING

Fold/ mirror (reverse) the input signal samples. That is replace x[k] into x[−k] or h[k] into h[-k].

4 MULTIPLICATION

Multiplication of the sequence h[k] and the sequence x[n−k]/ x[k] and sequence h[n-k]

5 SUMMATION

Sum all the value of product sequence from ∞ to $-\infty$

02

TUTORIAL

LINEAR SYSTEM

A Person who never made a mistake never tried anything new – Albert Einstein

Question 1

Determine whether the given discrete – time signal is linear or not

$$y[n] = x[n] - x[n-1]$$

superposition principle

$$F(x_1 + x_2) = F(x_1) + F\ (x_2)$$

$$F(ax) = aF(x)$$

Solution:

Let us define $x_1[n]$ and $x_2[n]$ as

$$x_1[n] \rightarrow y_1[n] = x_1[n] - x_1[n-1]$$

$$x_2[n] \rightarrow y_2[n] = x_2[n] - x_2[n-1]$$

Let us define $x_3[n]$ such that

$$x_3[n] = ax_1[n] - bx_2[n]$$

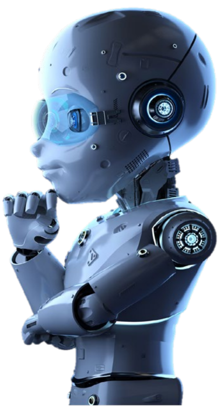
Then, $y_3[n] = x_3[n] - x_3[n-1]$

$$y_3[n] = \{ax_1[n] - bx_2[n]\} - \{ax_1[n-1] - bx_2[n-1]\}$$

$$y_3[n] = a\{x_1[n] - x_1[n-1]\} - b\{x_2[n] - x_2[n-1]\}$$

$$y_3[n] = ay_1[n] - by_2[n]$$

By refering superposition principle, therefore the sistem is linear.



Question 2

Explain the properties of LTI system without memory. Interpret whether the following system are with memory or without memory.

$$y(t) = 5x(t) - 1$$

Solution:

System without memory/memoryless – output of the system depend on present input.

For the system $y(t) = 5x(t) - 1$, Let $t = 2$, thus $y(2) = 5x(2) - 1$

Since the output of the system depend on present input. Therefore, the system is memoryless

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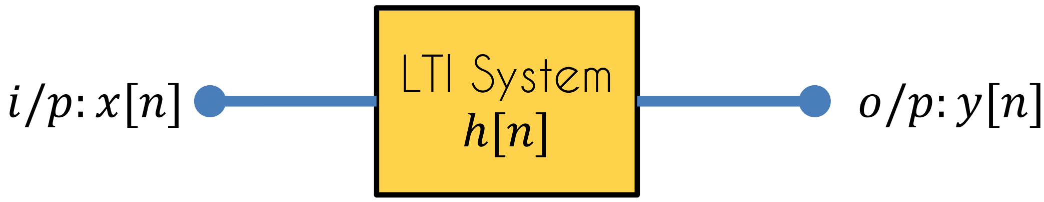
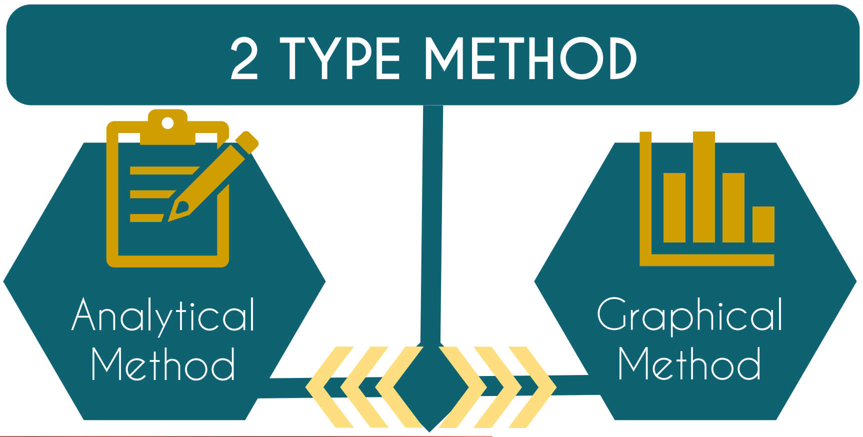
CONVOLUTION SUM

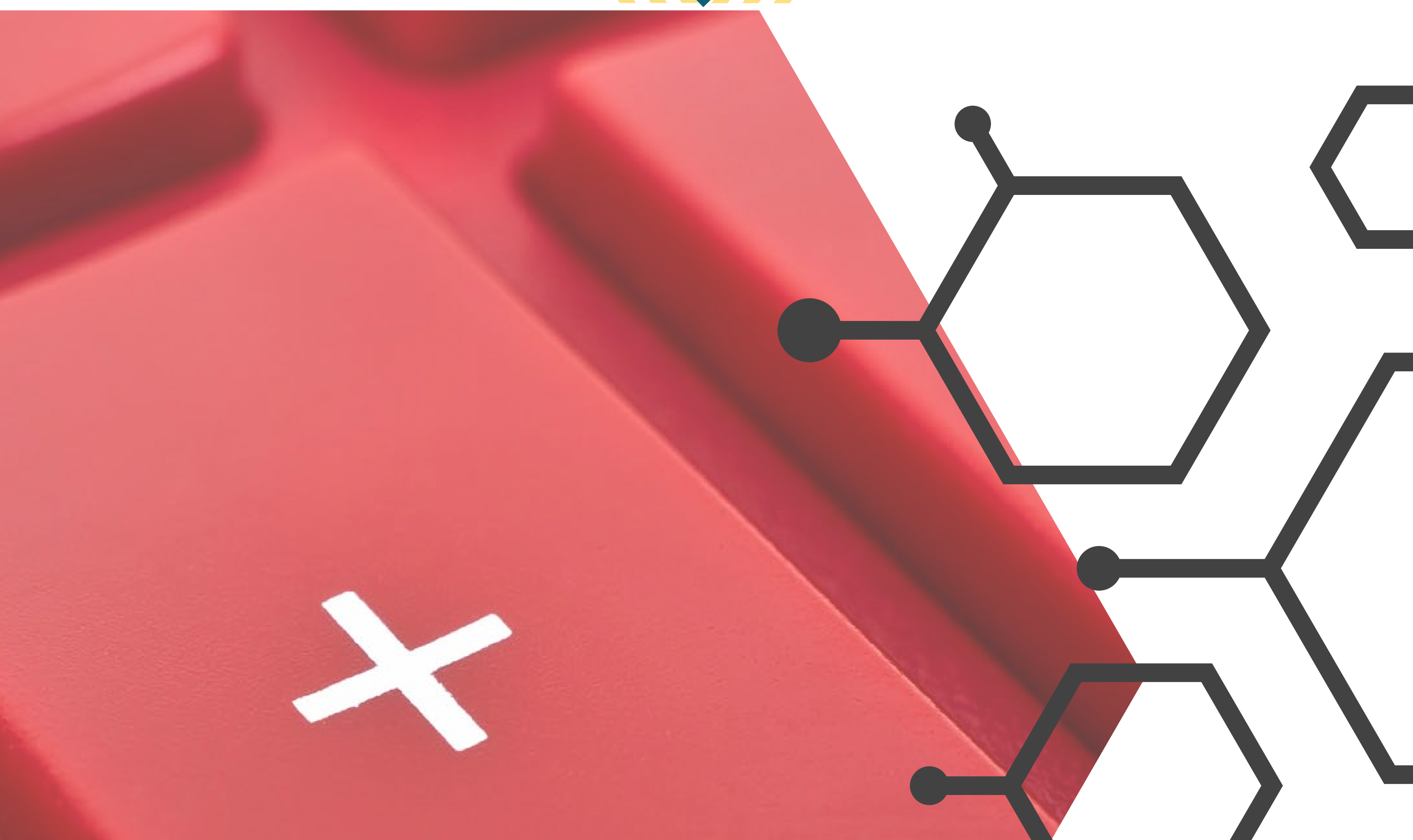
The way to get started is to quit talking and begin doing." – Walt Disney

DT Convolution Sum



$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$





Question 1

Find the convolution of given sequences for x[n] and h[n] using analytical method

$$x[n] = \{\mathbf{1}, 2, 0.5, 1\} \text{ and } h[n] = \{1, \mathbf{2}, 1, -1\}$$

Solution: Analytical Method

Use mathematical properties/ concept to evaluate/ simplify

Convert the given sequence to the equation:

$x[n] = \delta[n] + 2\delta[n-1] + 0.5\delta[n-2] + \delta[n-3]$

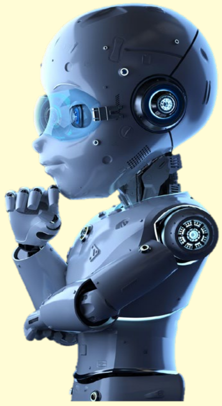
$h[n] = \delta[n+1] + 2\delta[n] + \delta[n-1] - \delta[n-2]$

Then;

$y[n] = x[n] * h[n]$

$y[n] = x[n] * \{\delta[n+1] + 2\delta[n] + \delta[n-1] - \delta[n-2]\}$

$\boldsymbol{y[n] = x[n+1] + 2x[n] + x[n-1] - x[n-2]}$



Substitute;

$\boldsymbol{x[n+1]} = \delta[n+\mathbf{1}] + 2\delta[n-1+\mathbf{1}] + 0.5\delta[n-2+\mathbf{1}] + \delta[n-3+\mathbf{1}]$

$\qquad = \delta[n+1] + 2\delta[n] + 0.5\delta[n-1] + \delta[n-2]$

$\boldsymbol{2x[n]} = \mathbf{2}\{\delta[n] + 2\delta[n-1] + 0.5\delta[n-2] + \delta[n-3]\}$

$\qquad = 2\delta[n] + 4\delta[n-1] + \delta[n-2] + 2\delta[n-3]$

$\boldsymbol{x[n-1]} = \delta[n-\mathbf{1}] + 2\delta[n-1-\mathbf{1}] + 0.5\delta[n-2-\mathbf{1}] + \delta[n-3-\mathbf{1}]$

$\qquad = \delta[n-1] + 2\delta[n-2] + 0.5\delta[n-3] + \delta[n-4]$

$\boldsymbol{x[n-2]} = \delta[n-\mathbf{2}] + 2\delta[n-1-\mathbf{2}] + 0.5\delta[n-2-\mathbf{2}] + \delta[n-3-\mathbf{2}]$

$\qquad = \delta[n-\mathbf{2}] + 2\delta[n-\mathbf{3}] + 0.5\delta[n-\mathbf{4}] + \delta[n-\mathbf{5}]$

Therefore;

$$\boldsymbol{y[n]} = \boldsymbol{x[n+1]} + \boldsymbol{2x[n]} + \boldsymbol{x[n-1]} - \boldsymbol{x[n-2]}$$

$$= \{\delta[n+1] + 2\delta[n] + 0.5\delta[n-1] + \delta[n-2]\}$$

$$+ \{2\delta[n] + 4\delta[n-1] + \delta[n-2] + 2\delta[n-3]\}$$

$$+ \{\delta[n-1] + 2\delta[n-2] + 0.5\delta[n-3] + \delta[n-4]\} - \{\delta[n-2]$$

$$+ 2\delta[n-3] + 0.5\delta[n-4] + \delta[n-5]\}$$

$$= \delta[n+1] + 4\delta[n] + 5.5\delta[n-1] + 3\delta[n-2] + 0.5\delta[n-3] + 0.5\delta n$$

$$- 4] - \delta[n-5]$$

The answer in sequences is;

$$y[n] = \{1, \mathbf{4}, 5.5, 3, 0.5, 0.5, -1\}$$

Question 1

Find the convolution of given sequences for x[n] and h[n] using analytical method

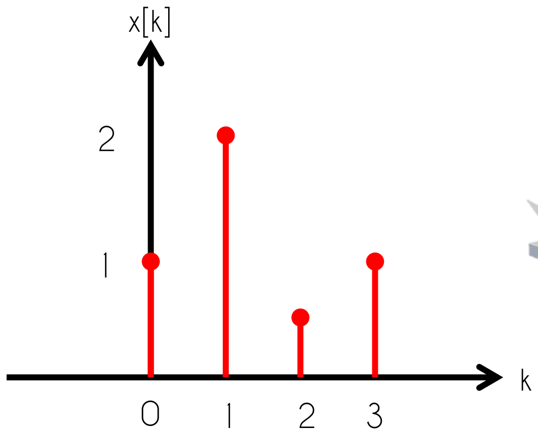
$x[n] = \{\mathbf{1}, 2, 0.5, 1\}$ and $h[n] = \{1, \mathbf{2}, 1, -1\}$

Solution: Graphical Method

Let's solve the same question using the graphical method and remember the steps of that method.

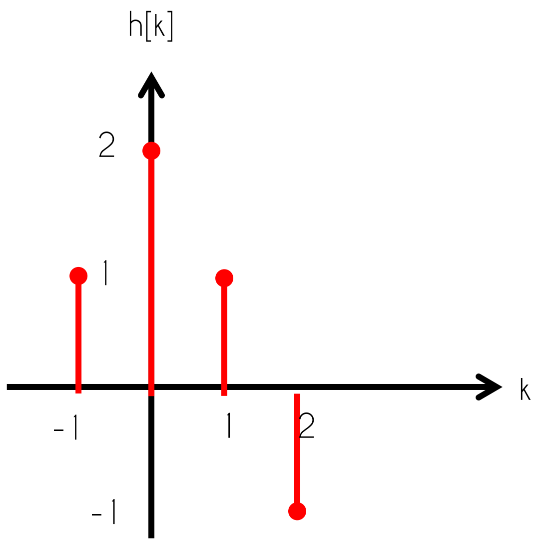
Step 1: EXPECTATION

- Input sequence at $n = 0$ for x[n] is 1 and h[n] is 2 (sequence for h[n] start with $n = -1$
- Expectation output sequence at $n = 0 + (-1) = -1$
- No of sample = $4 + 4 - 1 = 7$ {sequence no. should be start from n, -1, 0, 1, 2, 3, 4 &5.



Step 2: CHANGE INDEX

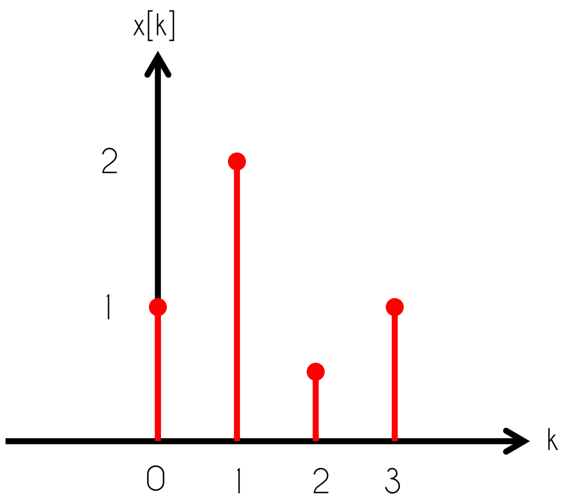
- $x[k] = \{\mathbf{1}, 2, 0.5, 1\}$

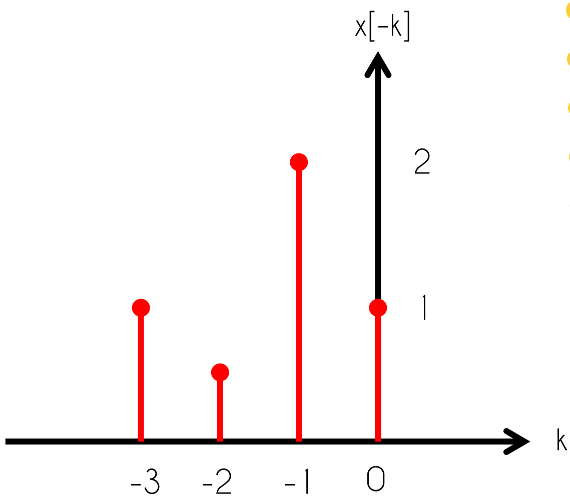


- $h[k] = \{1, \mathbf{2}, 1, -1\}$

Step 3: FOLDING (MIRROR/REVERSE)

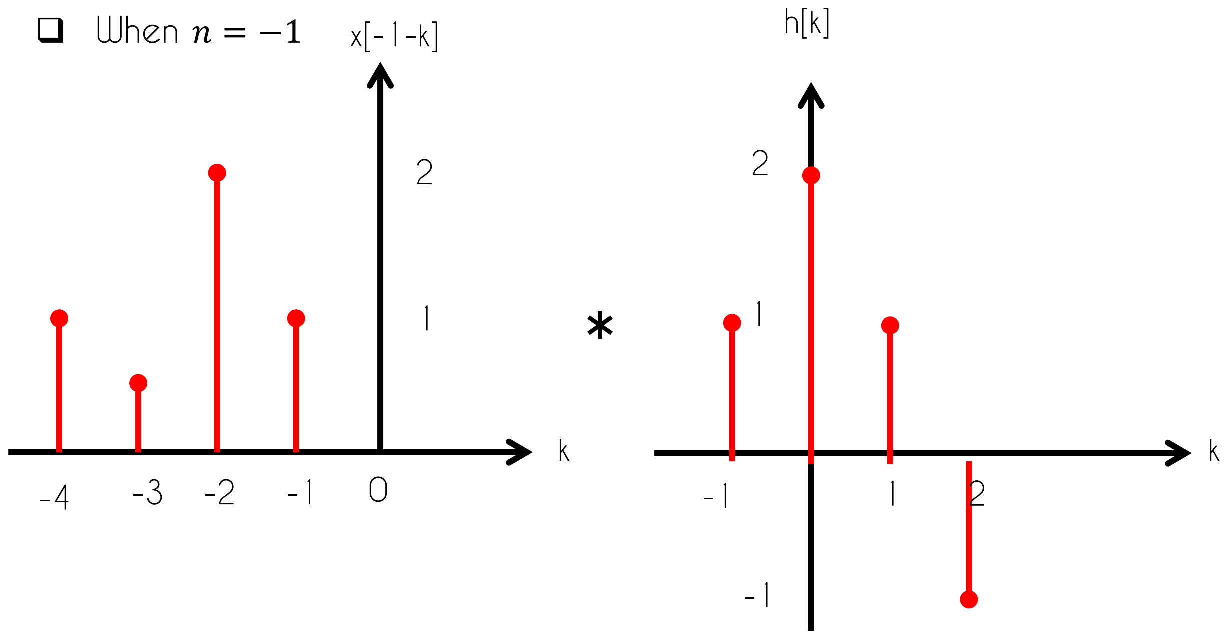
- Can folding either one, x[k] or h[k]
- This solution use to fold for x[k] to x[-k]





Step 3 & 4: MULTIPLICATION & SUMMATION

- Multiplication of the sequence h[k] and the sequence x[n−k] for each amplitude k at each $n = -1, 0, 1, 2, 3, 4, 5$
- When $n = -1$



Multiplication of the sequence h[k] and the sequence x[-1−k] for each k is;

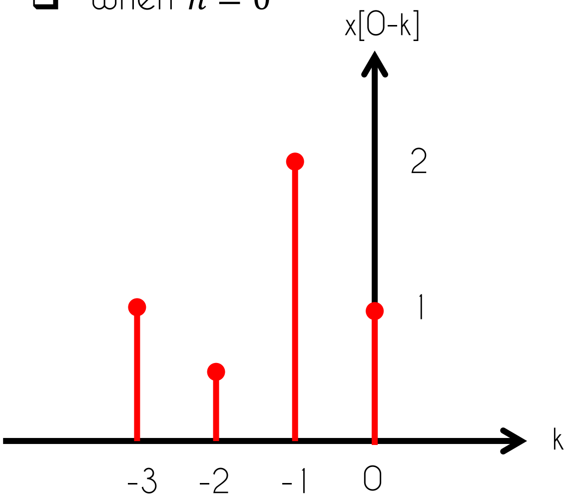
$$y[k] = 0 + 0 + 0 + 1 + 0 + 0 + 0$$

Therefore, summation for y at $n = -1$ is;

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$$y[-1] = 1$$

❑ When $n = 0$



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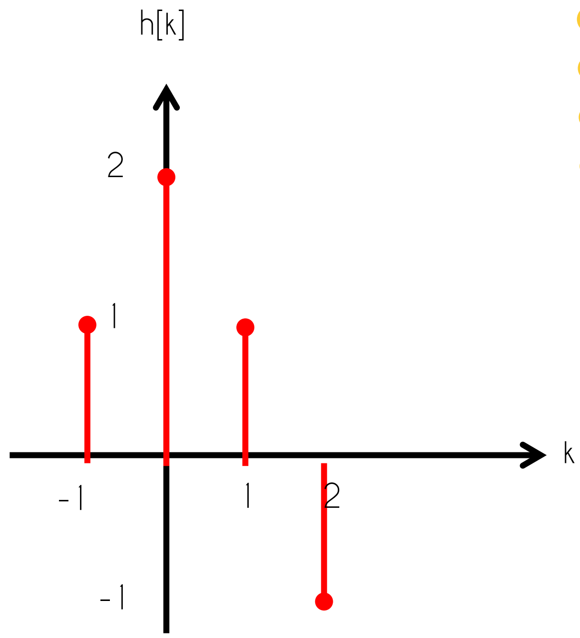


Multiplication of the sequence h[k] and the sequence x[−k] for each k is;

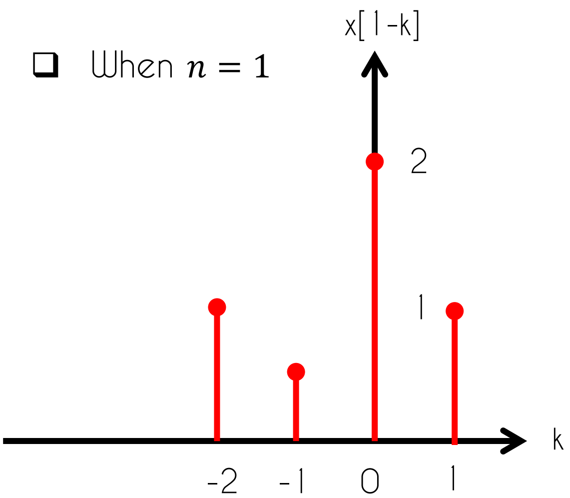
$$y[k] = 0 + 0 + 2 + 2 + 0 + 0$$

Therefore, summation for y at $n = 0$ is;

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$$y[0] = 4$$

❑ When $n = 1$



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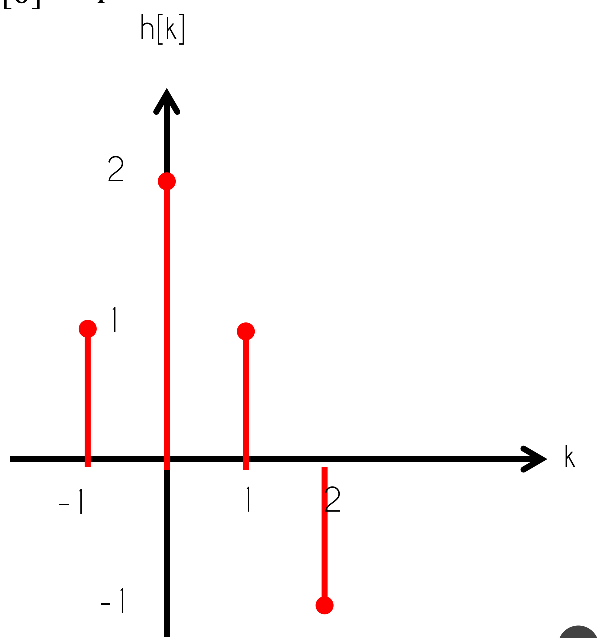


Multiplication of the sequence h[k] and the sequence x[1−k] for each k is;

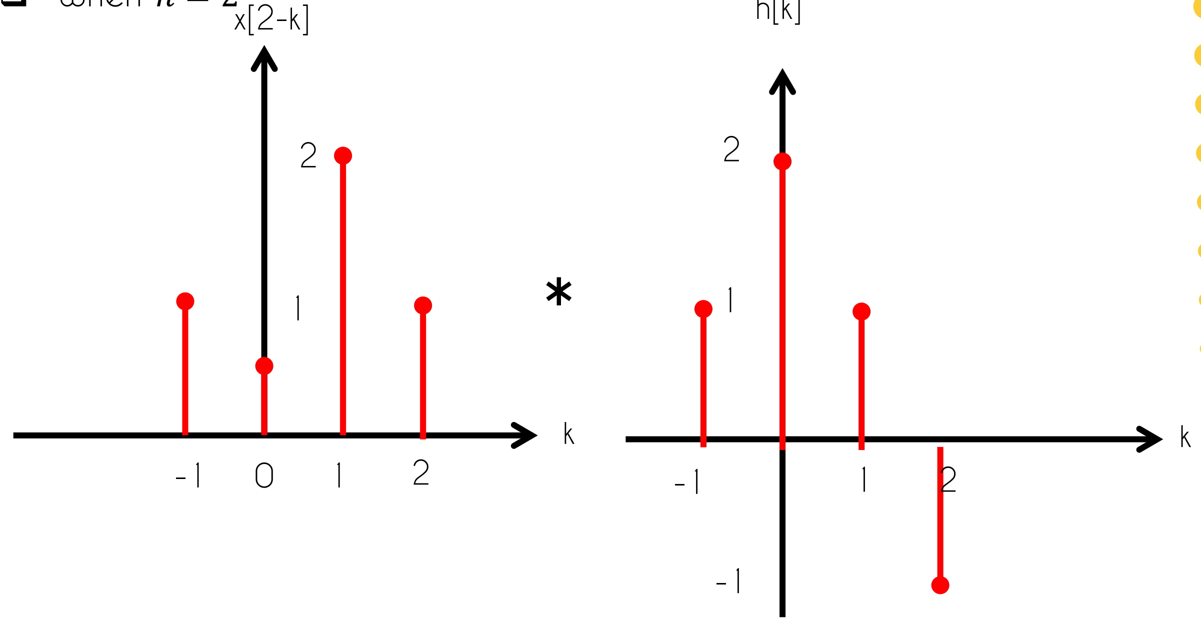
$$y[k] = 0 + 0.5 + 4 + 1 + 0$$

Therefore summation for y at $n = 0$ is;

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$$y[1] = 5.5$$

- When $n = 2$



Multiplication of the sequence h[k] and the sequence x[2−k] for each k is;

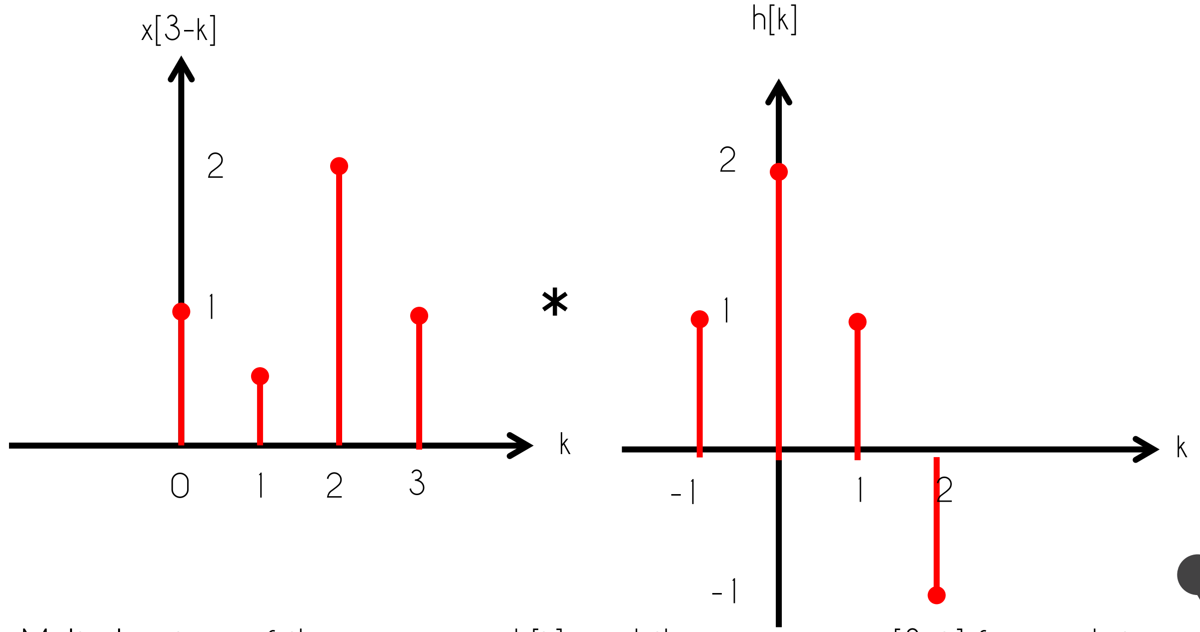
$$y[k] = 1 + 1 + 2 - 1$$

Therefore, summation for y at $n = 0$ is;

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$$y[0] = 3$$

- When $n = 3$



Multiplication of the sequence h[k] and the sequence x[3−k] for each k is;

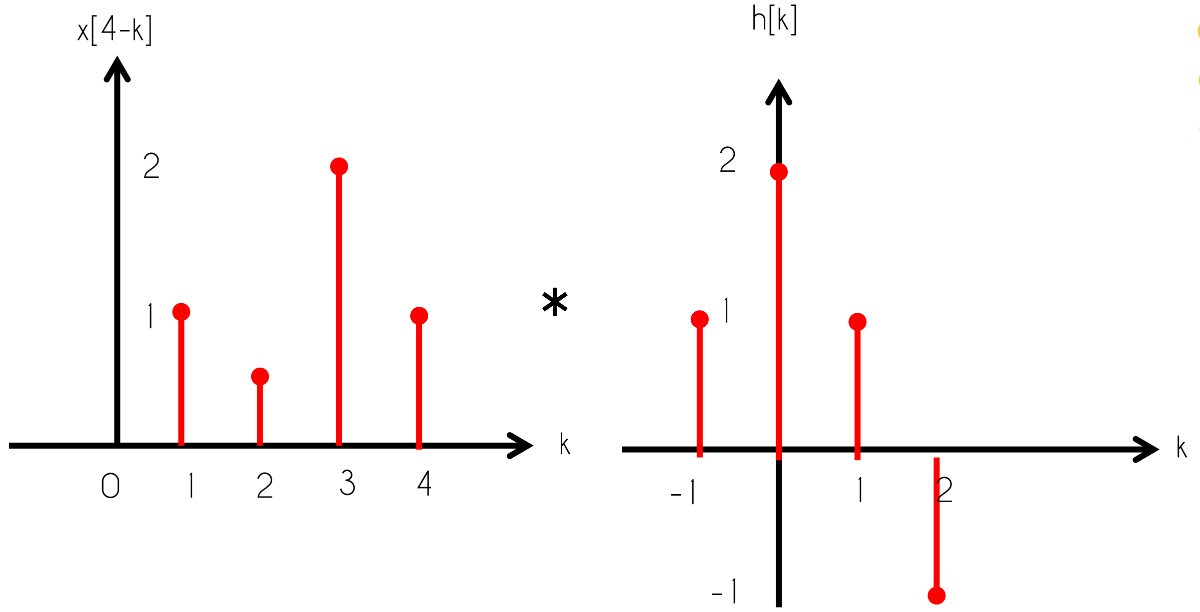
$$y[k] = 0 + 2 + 0.5 - 2 + 0$$

Therefore summation for y at $n = 0$ is;

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$$y[1] = 0.5$$

- When $n = 4$



Multiplication of the sequence h[k] and the sequence x[4−k] for each k is;

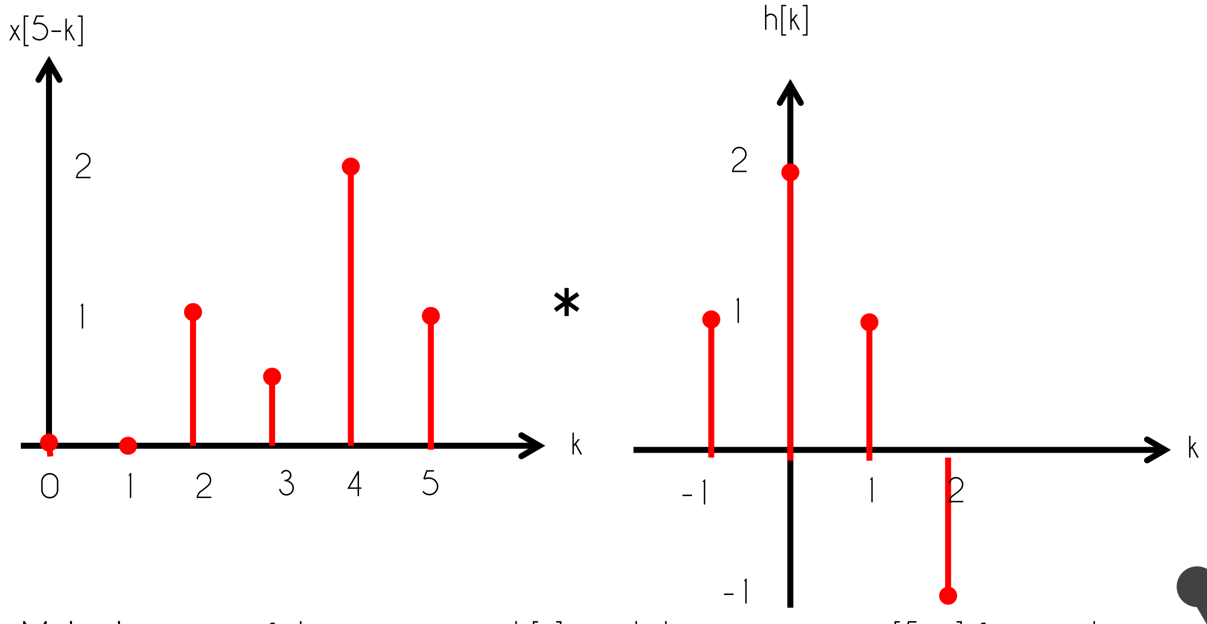
$$y[k] = 0 + 0 + 1 - 0.5 + 0 + 0$$

Therefore summation for y at $n = 0$ is;

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$$y[0] = 0.5$$

- When $n = 5$



Multiplication of the sequence h[k] and the sequence x[5−k] for each k is;

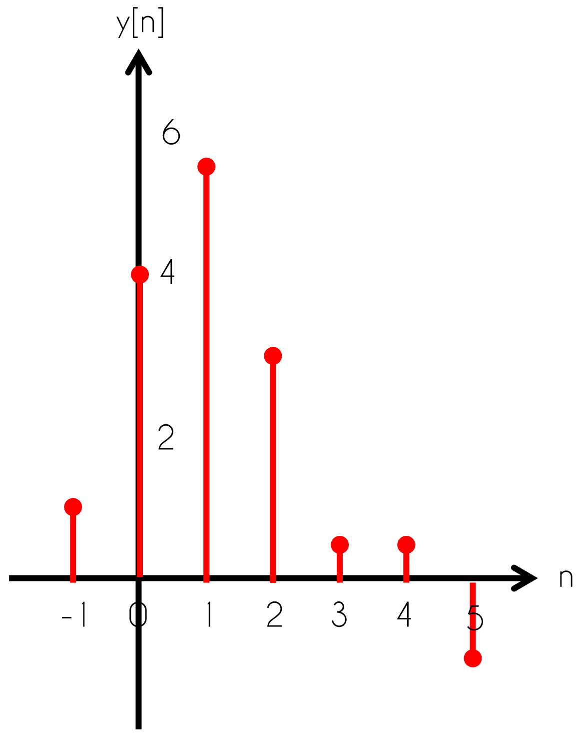
$$y[k] = 0 + 0 + 0 - 1 + 0 + 0 + 0$$

Therefore summation for y at $n = 0$ is;

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$$y[1] = -1$$

- Therefore output sequence $\mathbf{y}[n] = \{1, \mathbf{4}, 5.5, 3, 0.5, 0.5, -1\}$



Question 2

Consider an input $x[n]$ and impulse response $h[n]$ of a discrete – time LTI system given by:

$x[n] = 2\delta[n-2] + 3\delta[n+1]$ and $h[n] = \delta[n] + \delta[n-1] + \delta[n-2]$

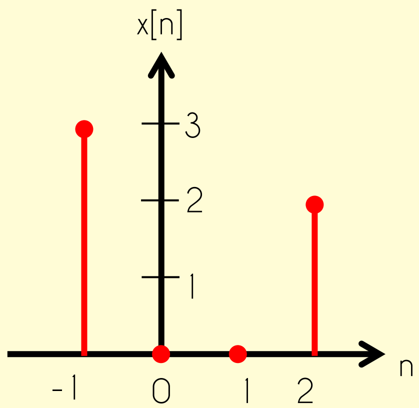
a. Sketch the impulse response $h[n]$ and input $x[n]$.

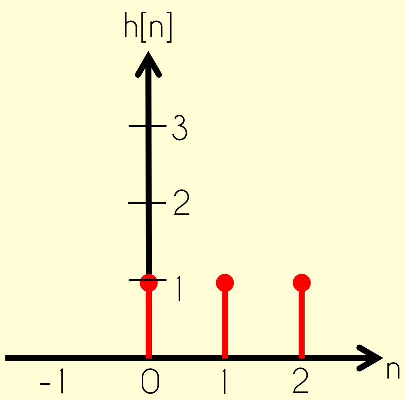
b. Compute the convolution $y[n] = x[n] * h[n]$by using analytical method

c. Sketch the output $y[n]$

Solution: Analytical Method

a. The impulse response $h[n]$ and input $x[n]$ as below:





b. Compute the convolution $y[n] = x[n] * h[n]$ by using analytical method.

$$y[n] = x[n] * h[n]$$

Substitute $h[n]$ with $\delta[n] + \delta[n-1] + \delta[n-2]$;

Therefore,

$y[n] = x[n] * \{\delta[n] + \delta[n-1] + \delta[n-2]\}$

$y[n] = x[n] * \delta[n] + x[n] * \delta[n-1] + x[n] * \delta[n-2]$

$y[n] = x[n] + x[n-1] + x[n-2]$

Substitute;

$x[n] = 3\delta[n+1] + 2\delta[n-2]$

$x[n-1] = 3\delta[n+1-1] + 2\delta[n-2-1]$

$= 3\delta[n] + 2\delta[n-3]$

$x[n-2] = 3\delta[n+1-2] + 2\delta[n-2-2]$

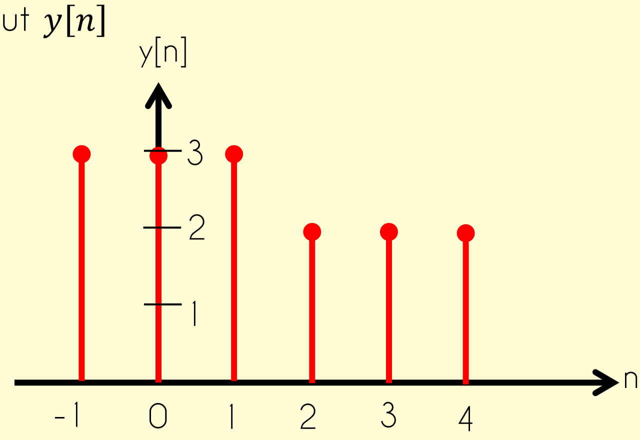
$= 3\delta[n-1] + 2\delta[n-4]$

Therefore,

$y[n] = 3\delta[n+1] + 2\delta[n-2] + 3\delta[n] + 2\delta[n-3] + 3\delta[n-1] + 2\delta[n-4]$

$y[n] = 3\delta[n+1] + 3\delta[n] + 3\delta[n-1] + 2\delta[n-2] + 2\delta[n-3] + +2\delta[n-4]$

c. Sketch the output $y[n]$



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Consider an input $x[n]$ and impulse response $h[n]$ of a discrete – time LTI system given by:

$x[n] = 2\delta[n-2] + 3\delta[n+1]$ and $h[n] = \delta[n] + \delta[n-1] + \delta[n-2]$

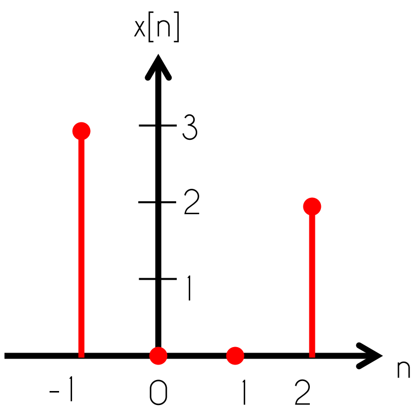
a. Sketch the impulse response $h[n]$ and input $x[n]$.

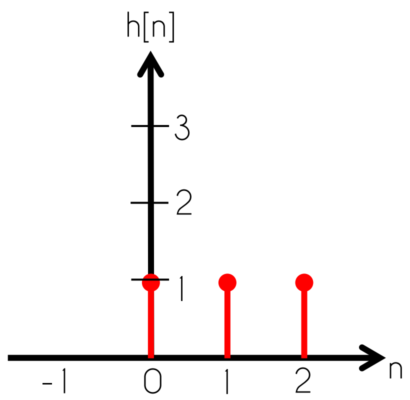
b. Compute the convolution $y[n] = x[n] * h[n]$by using analytical method

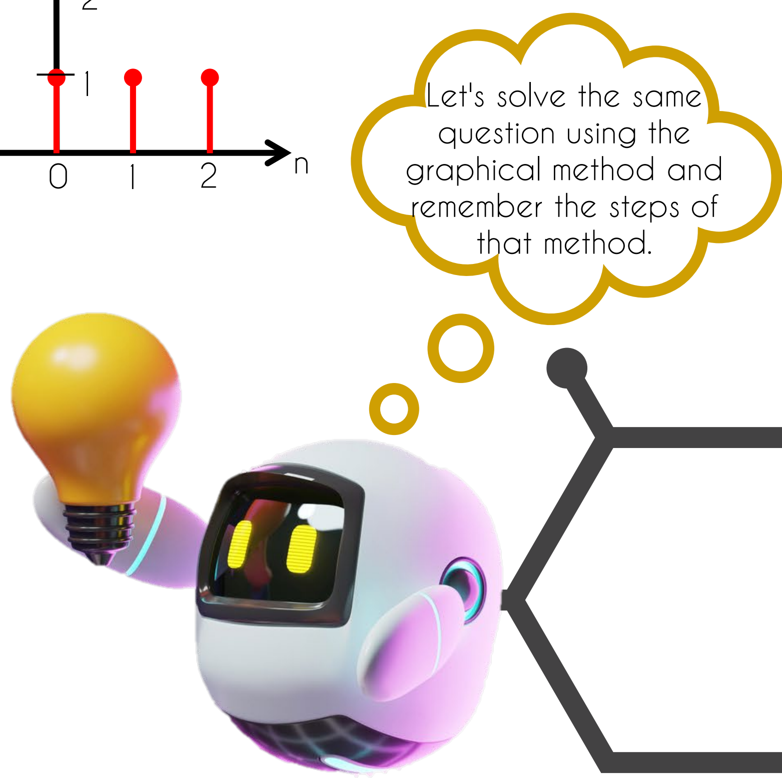
c. Sketch the output $y[n]$

Solution: Graphical Method

a. The impulse response $h[n]$ and input $x[n]$ as below:







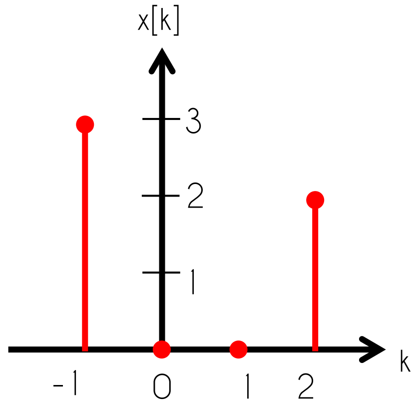
b. Compute the convolution $y[n] = x[n] * h[n]$ by using analytical method.

Step 1: EXPECTATION

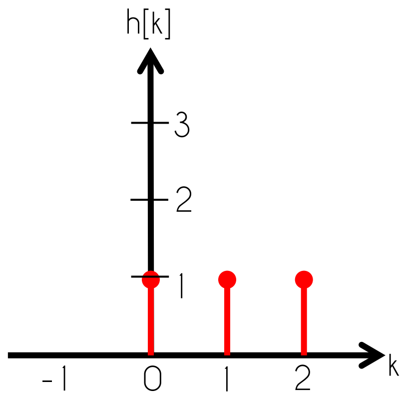
- Input sequence at $n = -1$ for x[n] is 0 and h[n] is 1 (sequence for h[n] start with $n = 0$
- Expectation output sequence at $n = -1 + (0) = -1$
- No of sample = $4 + 3 - 1 = 6$ {sequence no. should be start from n, -1, 0, 1, 2, 3, & 4.

Step 2: CHANGE INDEX

- $x[k] = \{3, \mathbf{0}, 0, 2\}$

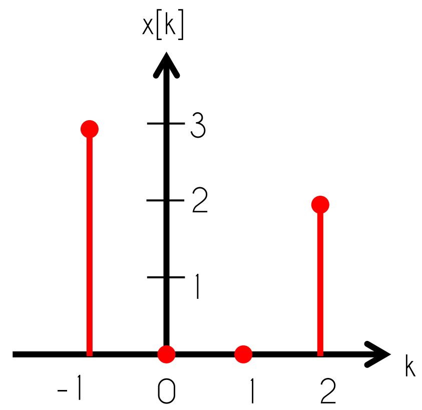


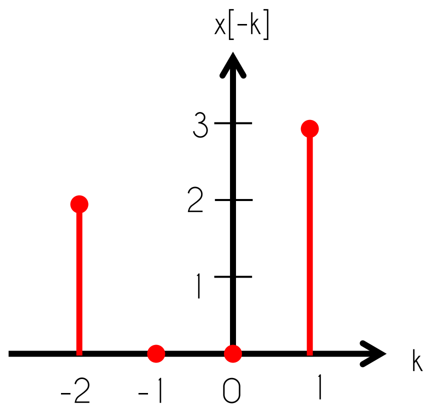
- $h[k] = \{\mathbf{1}, 1, 1\}$



Step 3: FOLDING (MIRROR/REVERSE)

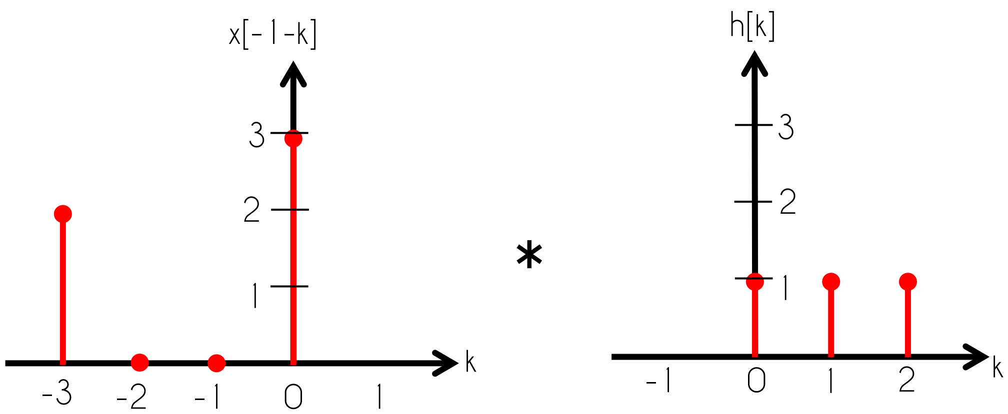
- ❑ Can folding either one, x[k] or h[k]
- ❑ This solution use to fold for x[k] to x[-k]





Step 3 & 4: MULTIPLICATION & SUMMATION

- ❑ Multiplication of the sequence h[k] and the sequence x[n−k] for each amplitude k at each $n = -1, 0, 1, 2, 3, 4$
- ❑ When $n = -1$



Multiplication of the sequence h[k] and the sequence x[-1−k] for each k is;

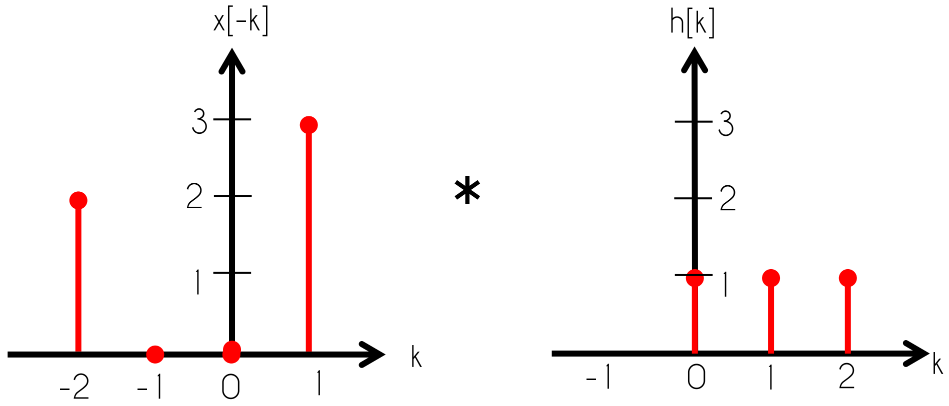
$$y[k] = 0 + 0 + 0 + 3 + 0 + 0$$

Therefore summation for y at $n = -1$ is;

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$$y[-1] = 3$$

- When $n = 0$



Multiplication of the sequence h[k] and the sequence x[0–k] for each k is;

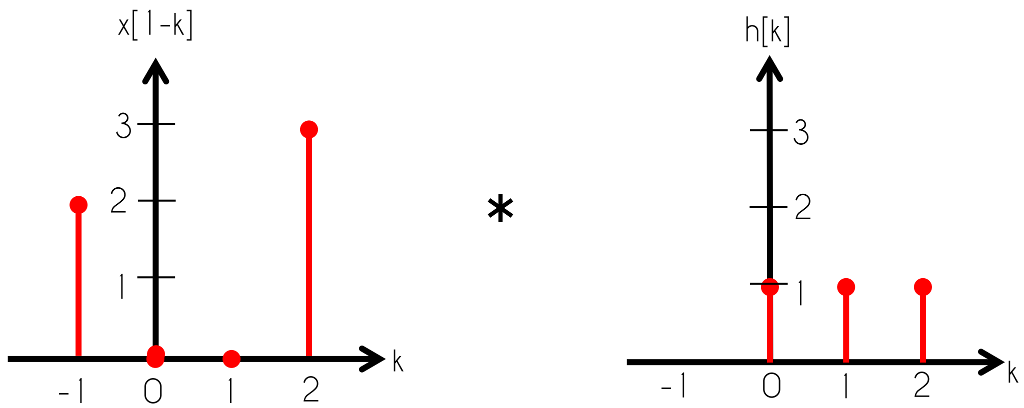
$$y[k] = 0 + 0 + 0 + 3 + 0$$

Therefore, summation for y at $n = 0$ is;

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$$y[0] = 3$$

- When $n = 1$



Multiplication of the sequence h[k] and the sequence x[1–k] for each k is;

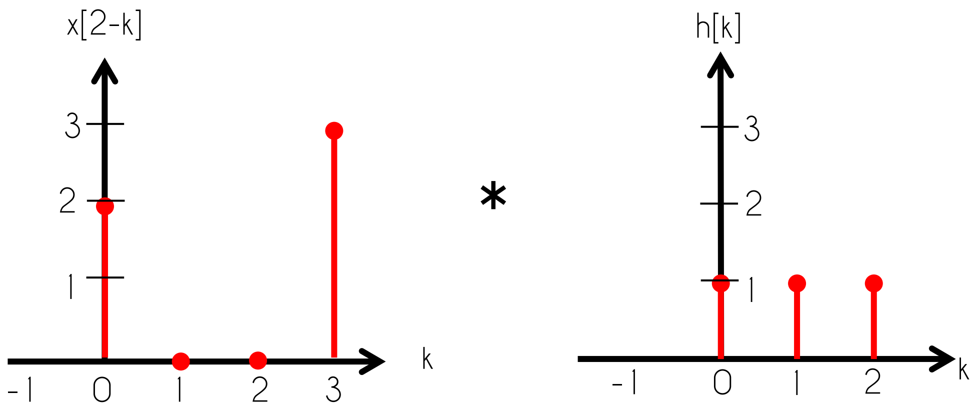
$$y[k] = 0 + 0 + 0 + 3$$

Therefore, summation for y at $n = 1$ is;

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$$y[1] = 3$$

❑ When $n = 2$



Multiplication of the sequence h[k] and the sequence x[2–k] for each k is;

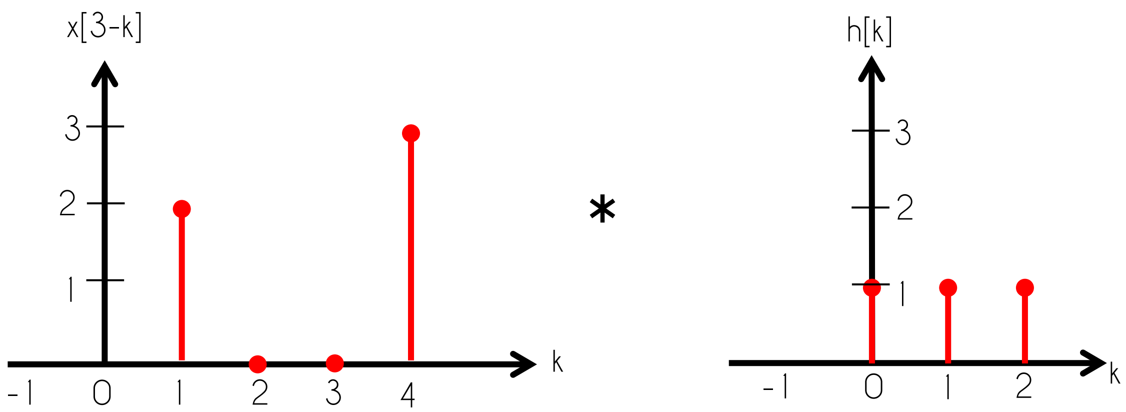
$$y[k] = 2 + 0 + 0 + 0$$

Therefore summation for y at $n = 2$ is;

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$$y[2] = 2$$

❑ When n =3



Multiplication of the sequence h[k] and the sequence x[3–k] for each k is;

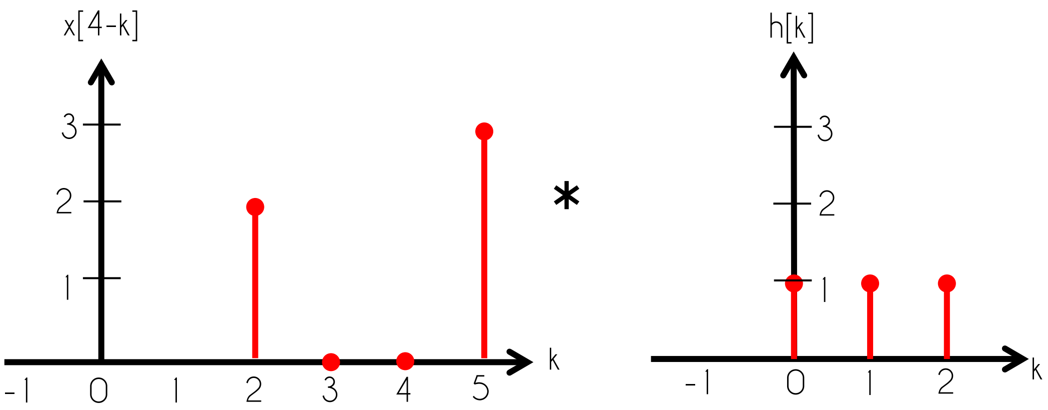
$$y[k] = 0 + 2 + 0 + 0 + 0$$

Therefore summation for y at $n = 3$ is;

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$$y[3] = 2$$

❑ When $n = 4$



Multiplication of the sequence h[k] and the sequence x[4−k] for each k is;

$$y[k] = 0 + 0 + 2 + 0 + 0 + 0$$

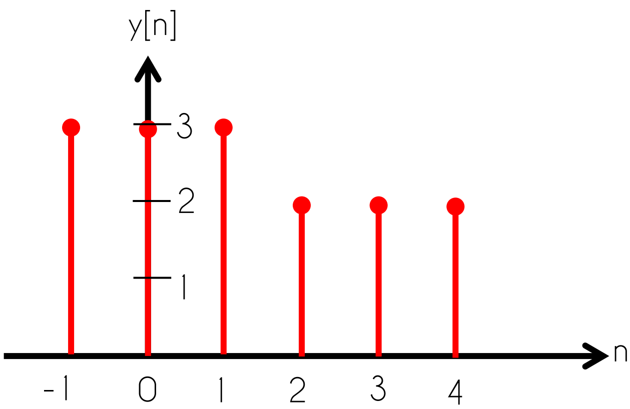
Therefore summation for y at $n = 4$ is;

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$$y[4] = 2$$

c. Sketch the output $y[n]$

❑ Therefore output sequence $\mathbf{y}[n] = \{3, \mathbf{3}, 3, 2, 2, 2\}$



Question 3

Given impulse response of system $h[n] = \delta[n] + \delta[n-1] + \delta[n-2] - \delta[n-3] - \delta[n-4]$ and the input to the signal is $x[n] = \delta[n-2] - \delta[n-4]$

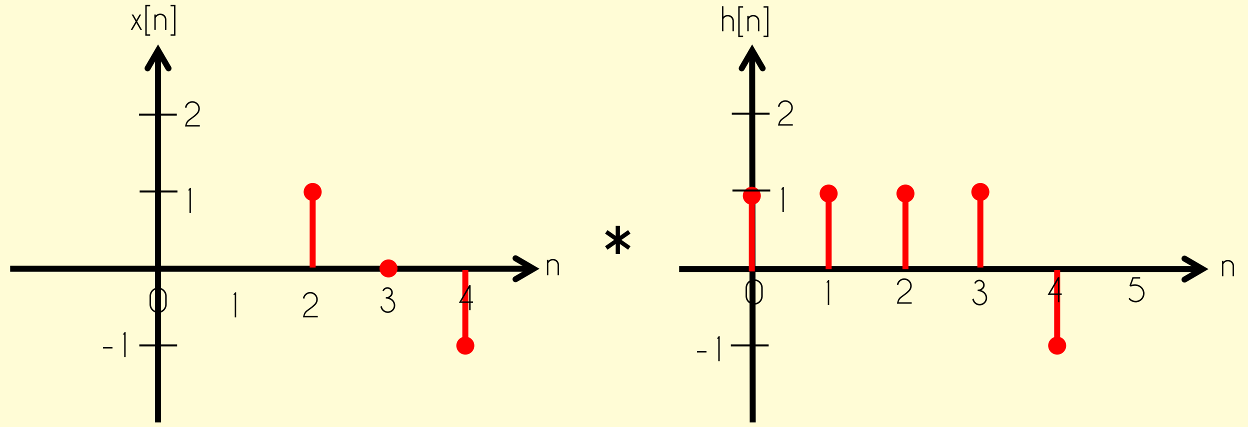
a. Sketch the impulse response $h[n]$ and input $x[n]$.

b. Compute the convolution $y[n] = x[n] * h[n]$ by using analytical method

c. Sketch the output $y[n]$

Solution: Analytical Method

a. The impulse response $h[n]$ and input $x[n]$ as below:



b. Compute the convolution $y[n] = x[n] * h[n]$ by using analytical method.

$$y[n] = x[n] * h[n]$$

Substitute $x[n]$ with $\delta[n-2] - \delta[n-4]$;

Therefore,

$y[n] = h[n] * \{\delta[n-2] - \delta[n-4]\}$

$y[n] = h[n] * \delta[n-2] - h[n] * \delta[n-4]$

$y[n] = h[n-2] - h[n-4]$

Substitute;

$h[n-2] = \delta[n-2] + \delta[n-1-2] + \delta[n-2-2] - \delta[n-3-2] - \delta[n-4-2]$

$= \delta[n-2] + \delta[n-3] + \delta[n-4] - \delta[n-5] - \delta[n-6]$

$h[n-4] = \delta[n-4] + \delta[n-1-4] + \delta[n-2-4] - \delta[n-3-4] - \delta[n-4-4]$

$= \delta[n-4] + \delta[n-5] + \delta[n-6] - \delta[n-7] - \delta[n-8]$

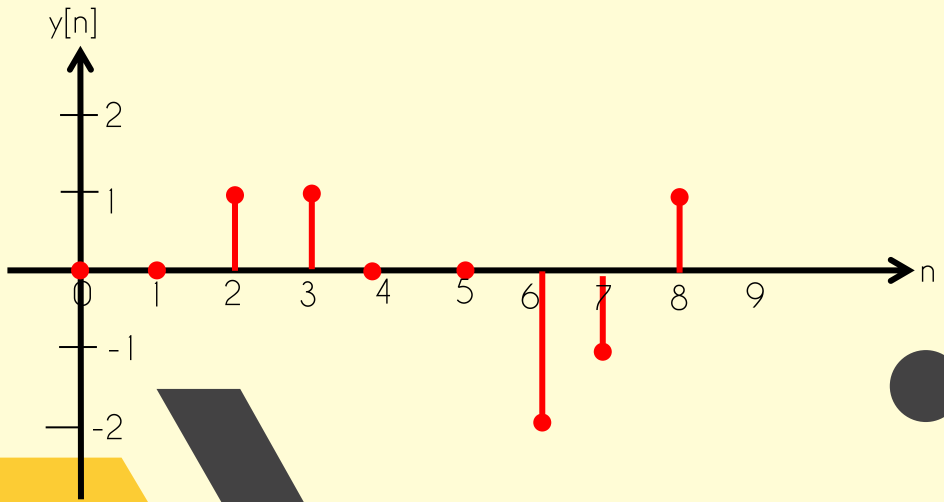
Therefore, $y[n]$ is,

$= \{\delta[n-2] + \delta[n-3] + \delta[n-4] - \delta[n-5] - \delta[n-6]\} - \{\delta[n-4] + \delta[n-5]$
$+ \delta[n-6] - \delta[n-7] - \delta[n-8]\}$

$= \delta[n-2] + \delta[n-3] + \delta[n-4] - \delta[n-5] - \delta[n-6] - \delta[n-4] - \delta[n-5]$
$- \delta[n-6] + \delta[n-7] + \delta[n-8]$

$= \delta[n-2] + \delta[n-3] - 2\delta[n-6] + \delta[n-7] + \delta[n-8]$

c. Sketch the output $y[n]$



Question 3

Given impulse response of system

$h[n] = \delta[n] + \delta[n-1] + \delta[n-2] - \delta[n-3] - \delta[n-4]$ and the input to the signal is $x[n] = \delta[n-2] - \delta[n-4]$

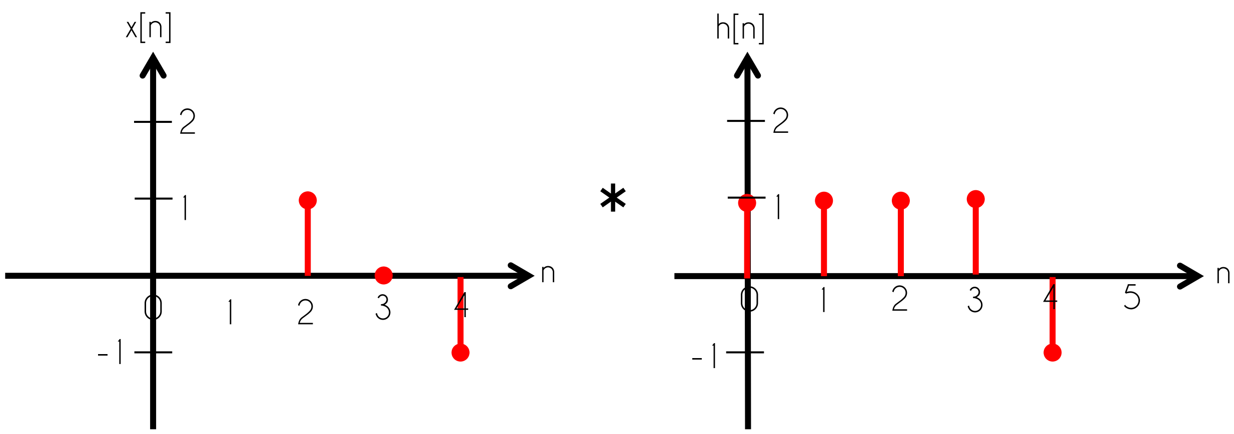
a. Sketch the impulse response $h[n]$ and input $x[n]$.

b. Compute the convolution $y[n] = x[n] * h[n]$ by using analytical method

c. Sketch the output $y[n]$

Solution: Graphical Method

a. The impulse response $h[n]$ and input $x[n]$ as below:



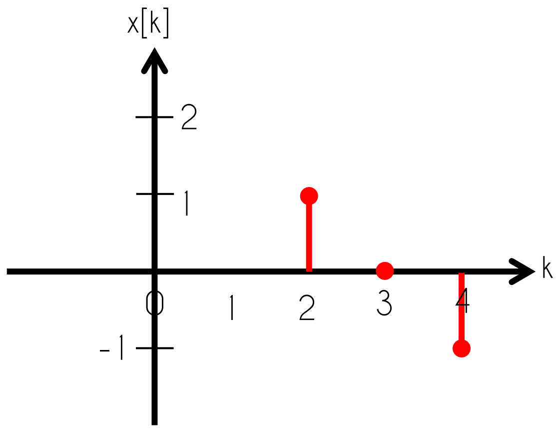
b. Compute the convolution $y[n] = x[n] * h[n]$ by using analytical method.

Step 1: EXPECTATION

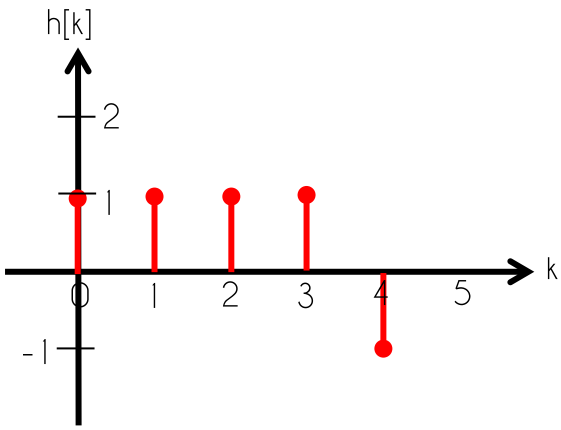
- Input sequence at $n = 0$ for x[n] is 0 and h[n] is 1 (sequence for h[n] start with $n = 0$
- Expectation output sequence at $n = 0 + (0) = 0$
- No of sample = $5 + 5 - 1 = 9$ {sequence no. should be start from n, 0, 1, 2, 3, 4, 5, 6, 7, & 8.

Step 2: CHANGE INDEX

- $x[k] = \{\mathbf{0}, 0, 1, 0, -1\}$

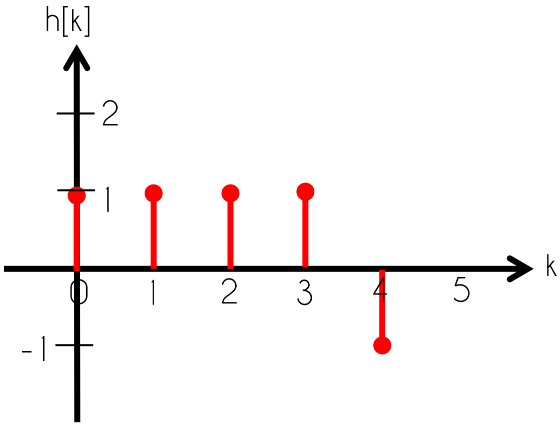


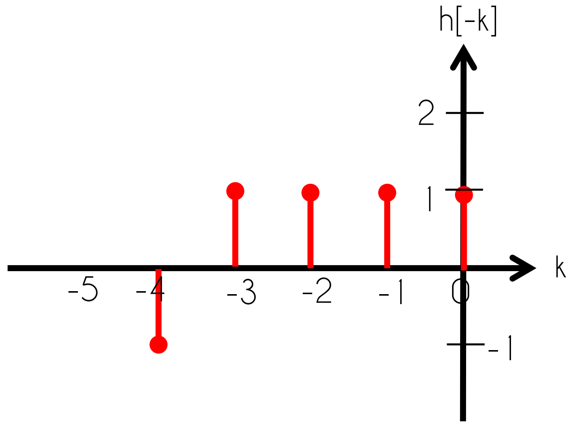
- $h[k] = \{\mathbf{1}, 1, 1,1, -1, -1\}$



Step 3: FOLDING (MIRROR/REVERSE)

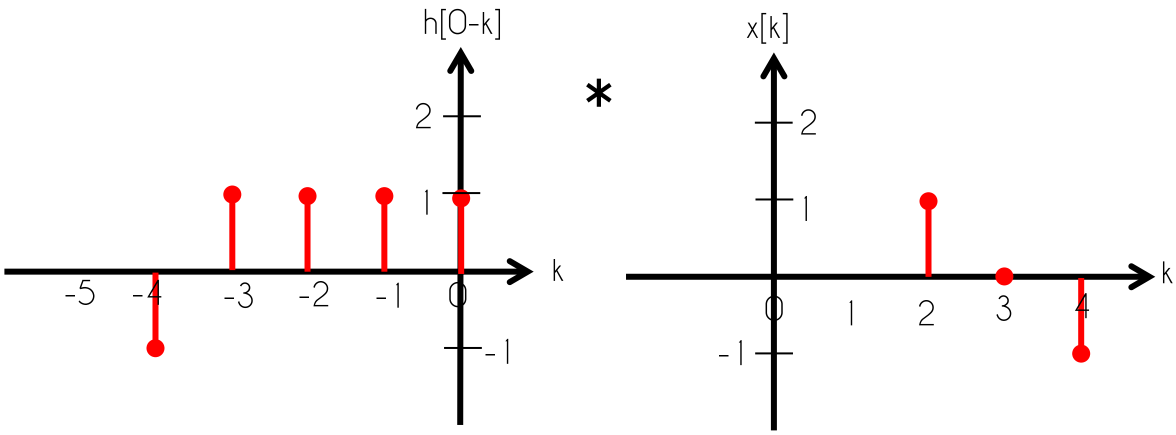
- Can folding either one, x[k] or h[k]
- This solution use to fold for h[k] to h[-k]





Step 3 & 4: MULTIPLICATION & SUMMATION

- Multiplication of the sequence h[k] and the sequence x[n−k] for each amplitude k at each $n = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9$
- When $n = 0$



Multiplication of the sequence h[k] and the sequence x[0−k] for each k is;

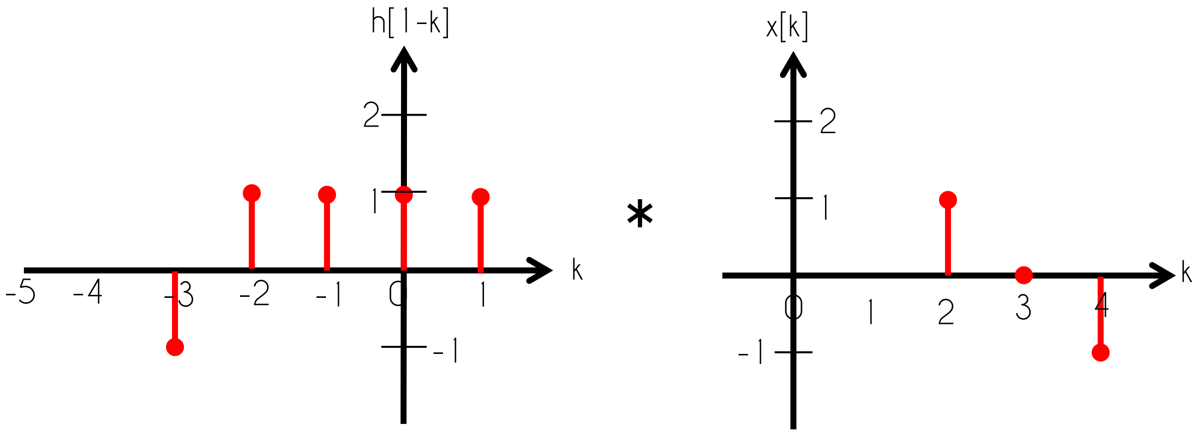
$$y[k] = 0 + 0 + 0 + 0 + 0 + 0$$

Therefore, summation for y at $n = 0$ is;

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$$y[0] = 0$$

- When $n = 1$



Multiplication of the sequence h[k] and the sequence x[1−k] for each k is;

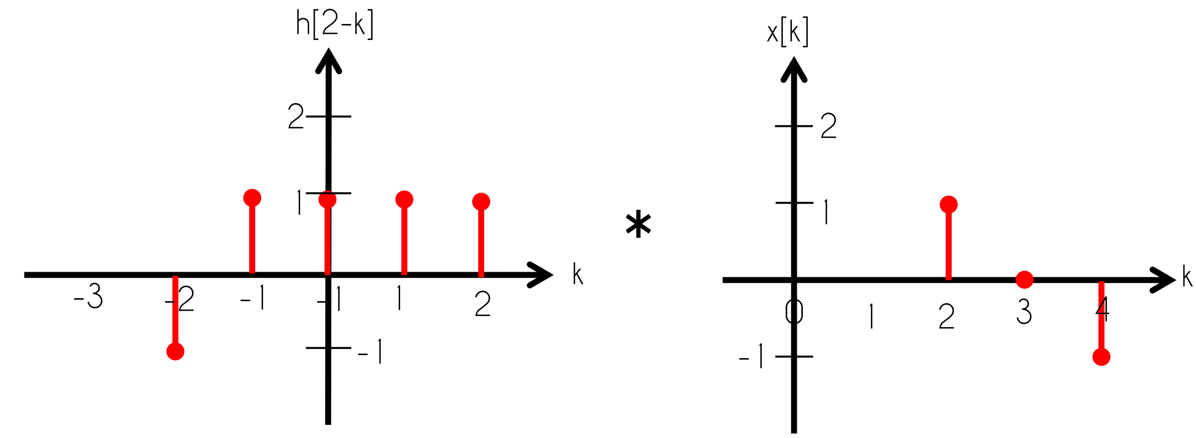
$$y[k] = 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0$$

Therefore, summation for y at $n = 1$ is;

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$$y[1] = 0$$

- When $n = 2$



Multiplication of the sequence h[k] and the sequence x[1−k] for each k is;

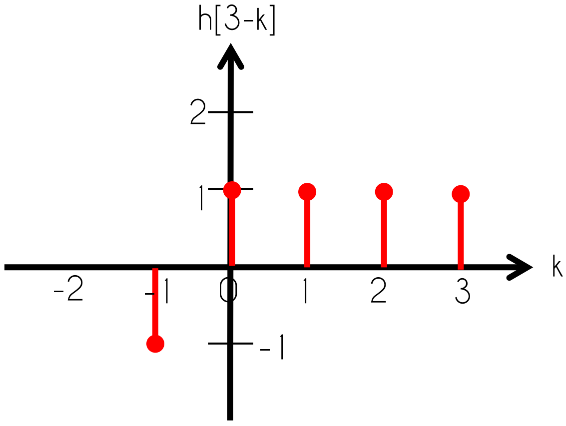
$$y[k] = 0 + 0 + 0 + 0 + 0 + 1 + 0 + 0$$

Therefore, summation for y at $n = 2$ is;

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$$y[2] = 1$$

- When $n = 3$



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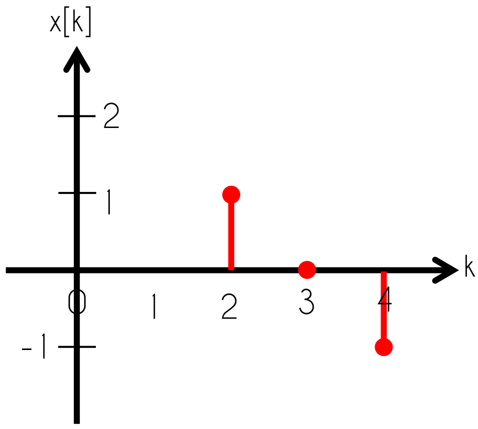


Multiplication of the sequence h[k] and the sequence x[1−k] for each k is;

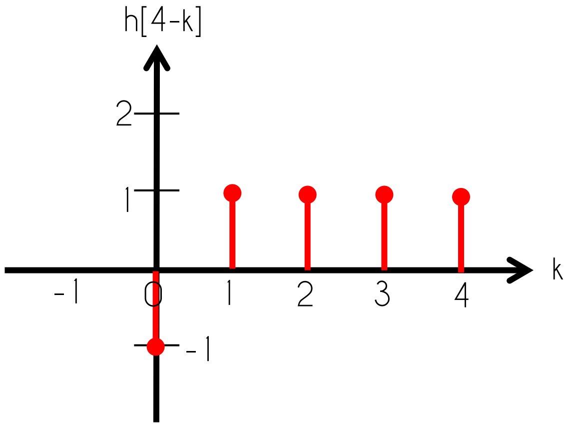
$$y[k] = 0 + 0 + 0 + 0 + 1 + 0 + 0$$

Therefore, summation for y at $n = 3$ is;

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$$y[3] = 1$$

- When n =4



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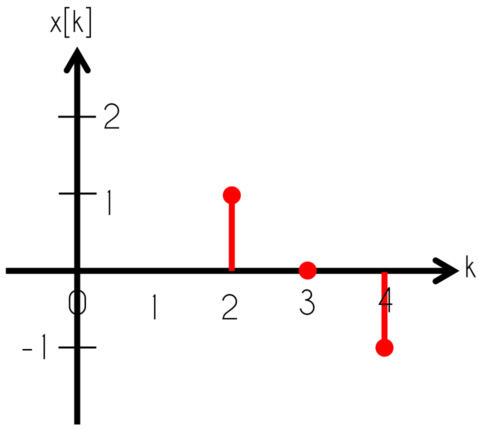


Multiplication of the sequence h[k] and the sequence x[1−k] for each k is;

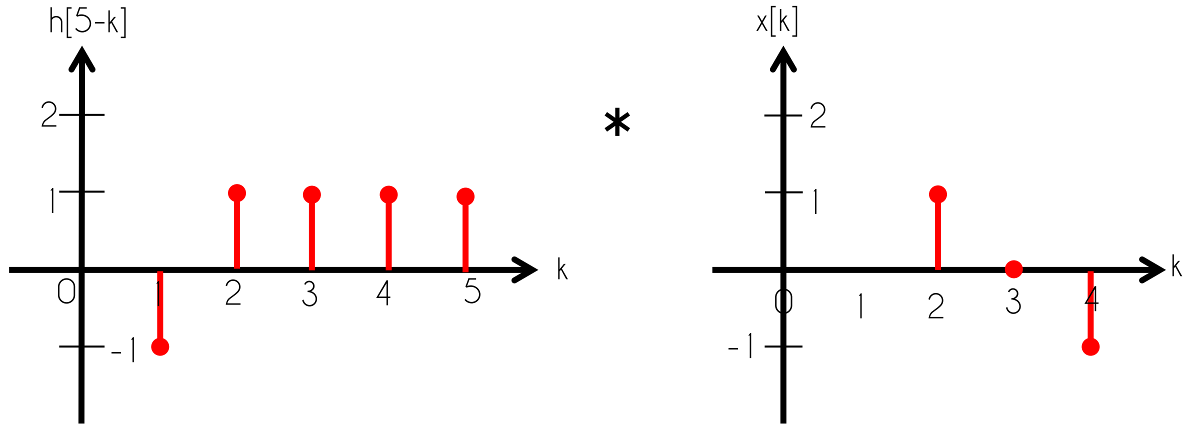
$$y[k] = 0 + 0 + 0 + 1 + 0 - 1$$

Therefore summation for y at $n = 4$ is;

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$$y[4] = 0$$

- When $n = 5$



Multiplication of the sequence h[k] and the sequence x[1−k] for each k is;

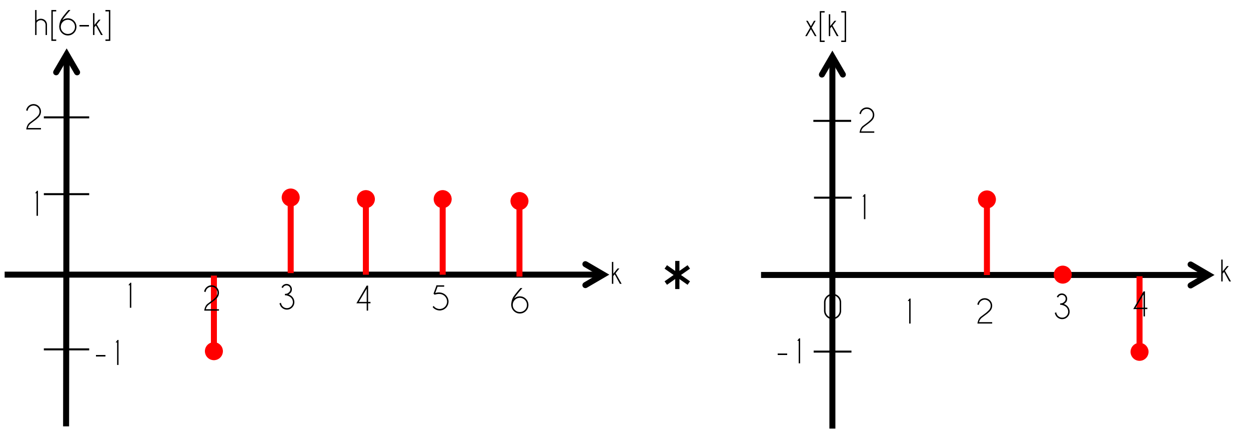
$$y[k] = 0 + 0 + 1 + 0 - 1 + 0$$

Therefore, summation for y at $n = 5$ is;

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$$y[5] = 0$$

- When $n = 6$



Multiplication of the sequence h[k] and the sequence x[1−k] for each k is;

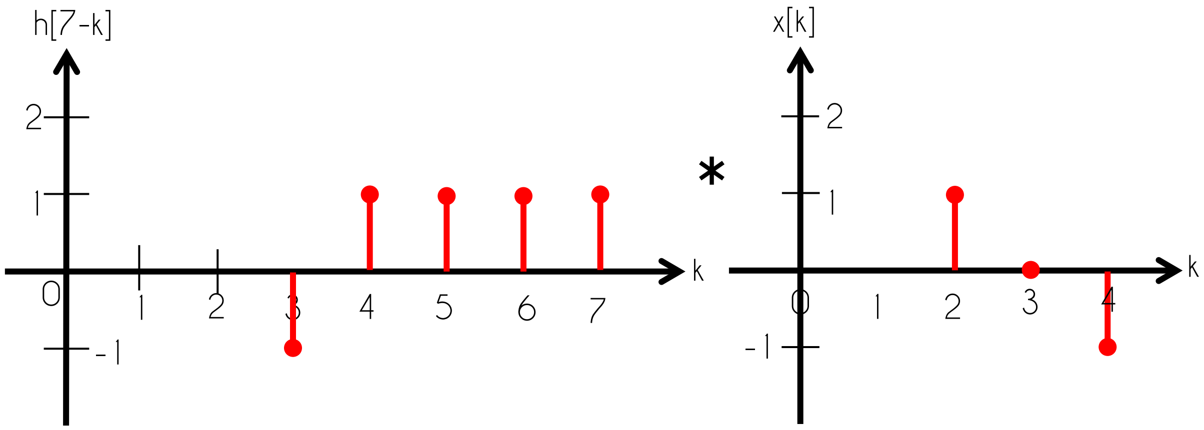
$$y[k] = 0 + 0 - 1 + 0 - 1 + 0 + 0$$

Therefore, summation for y at $n = 6$ is;

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$$y[6] = -2$$

- When $n = 7$



Multiplication of the sequence h[k] and the sequence x[1−k] for each k is;

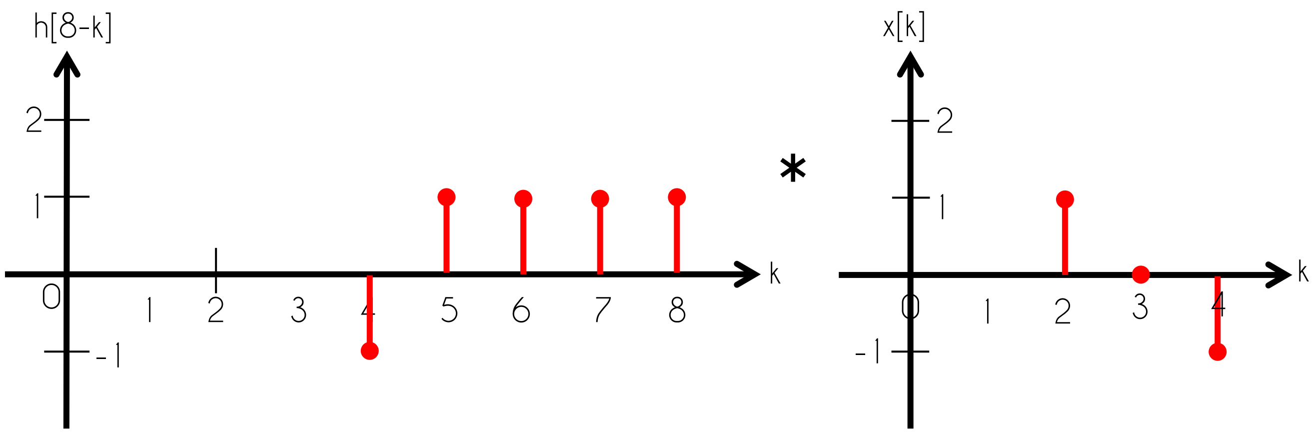
$$y[k] = 0 + 0 + 0 - 1 + 0 + 0 + 0 + 0$$

Therefore, summation for y at $n = 7$ is;

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$$y[7] = -1$$

- When $n = 8$



Multiplication of the sequence h[k] and the sequence x[1−k] for each k is;

$$y[k] = 0 + 0 + 0 + 0 + 1 + 0 + 0 + 0 + 0$$

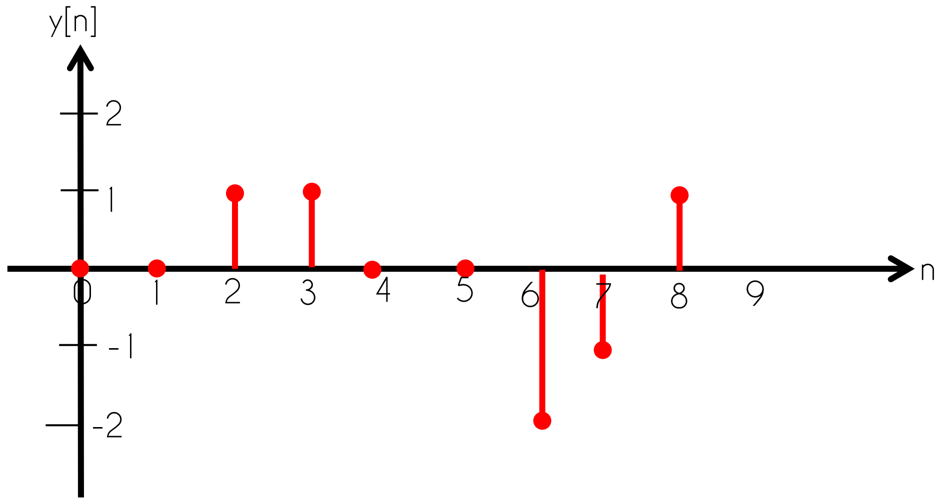
Therefore, summation for y at $n = 8$ is;

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$$y[8] = 1$$

c. Sketch the output $y[n]$

❑ Therefore output sequence $\mathrm{y}[n] = \{0, 0, 1, 1, 0, 0, -2, -1, 1\}$



Question 4

Solve the convolution sum of the given data sequences by using analytical method.

$$x[n] = \{1,2,3\}$$
$$h[n] = \{1,1,1\}$$

Solution: Analytical Method

$$y[n] = x[n] * h[n]$$

Convert the given sequence to the equation:

$x[n] = \delta[n] + 2\delta[n-1] + 3\delta[n-2]$

$h[n] = \delta[n] + \delta[n-1] + \delta[n-2]]$

Substitute $h[n]$ with $\delta[n] + \delta[n-1] + \delta[n-2]$;

Therefore,

$y[n] = x[n] * \{\delta[n] + \delta[n-1] + \delta[n-2]\}$

$y[n] = x[n] * \delta[n] + x[n] * \delta[n-1] + x[n] * \delta[n-2]$

$y[n] = x[n] + x[n-1] + x[n-2]$

Substitute;

$x[n] = \delta[n] + 2\delta[n-1] + 3\delta[n-2]$

$x[n-1] = \delta[n-1] + 2\delta[n-1-1] + 3\delta[n-2-1]$

$\quad = \delta[n-1] + 2\delta[n-2] + 3\delta[n-3]$

$x[n-2] = \delta[n-2] + 2\delta[n-1-2] + 3\delta[n-2-2]$

$\quad = \delta[n-2] + 2\delta[n-3] + 3\delta[n-4]$

Therefore, $y[n]$ is,

$= \{\delta[n] + 2\delta[n-1] + 3\delta[n-2]\} + \{\delta[n-1] + 2\delta[n-2] + 3\delta[n-3]\}$
$+ \{\delta[n-2] + 2\delta[n-3] + 3\delta[n-4]\}$

$= \delta[n] + 2\delta[n-1] + 3\delta[n-2] + \delta[n-1] + 2\delta[n-2] + 3\delta[n-3]$
$+ \delta[n-2] + 2\delta[n-3] + 3\delta[n-4]$

$= \delta[n] + 3\delta[n-1] + 6\delta[n-2] + 5\delta[n-3] + 3\delta[n-4]$

The answer in sequence;

$$y[n] = \{\mathbf{1}, 3, 6, 5, 3\}$$

Question 4

Solve the convolution sum of the given data sequences by using analytical method.

$$x[n] = \{1,2,3\}$$

$$h[n] = \{1,1,1\}$$

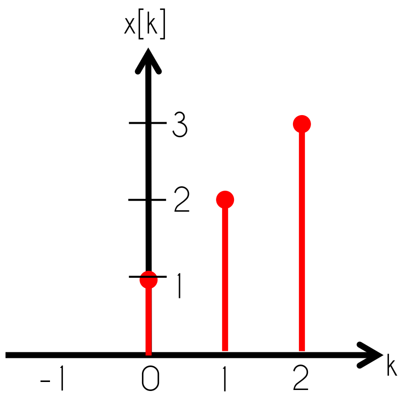
Solution: Graphical Method

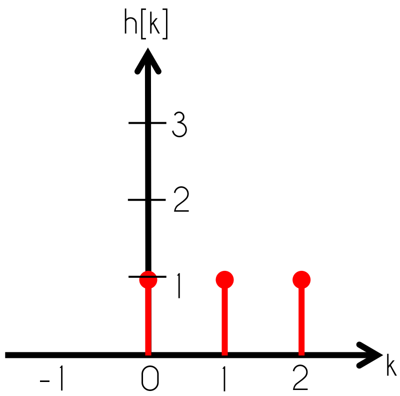
Step 1: EXPECTATION

- Input sequence at $n = 0$ for x[n] is 1 and h[n] is 1 (sequence for h[n] start with $n = 0$
- Expectation output sequence at $n = 0 + (0) = 0$
- No of sample = $3 + 3 - 1 = 5$ {sequence no. should be start from n, 0, 1, 2, 3, & 4.

Step 2: CHANGE INDEX

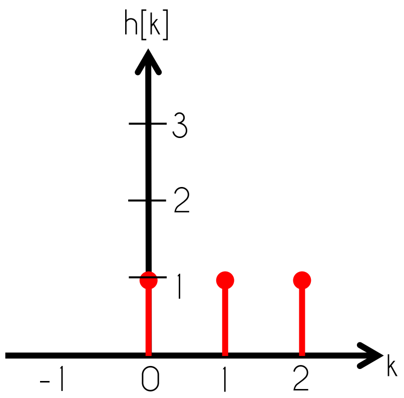
- $x[k] = \{1, 2, 3\}$ $h[k] = \{1, 1, 1\}$

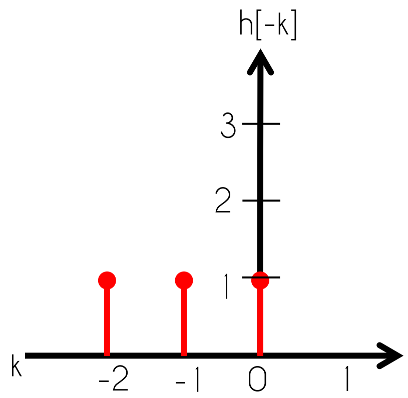




Step 3: FOLDING (MIRROR/REVERSE)

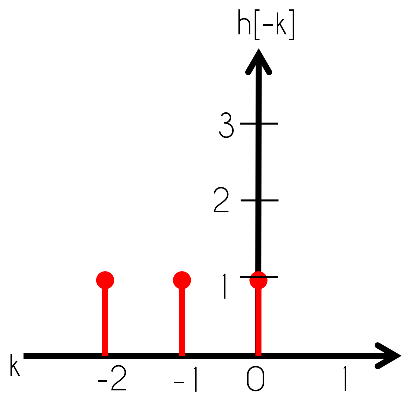
- ❑ Can folding either one, x[k] or h[k]
- ❑ This solution use to fold for h[k] to h[-k]





Step 3 & 4: MULTIPLICATION & SUMMATION

- ❑ Multiplication of the sequence h[k] and the sequence x[n−k] for each amplitude k at each $n = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9$
- ❑ When $n = 0$



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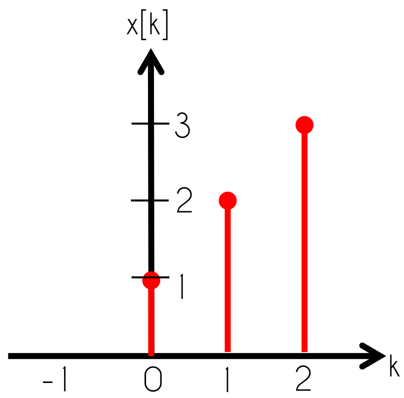


Multiplication of the sequence h[k] and the sequence x[0−k] for each k is;

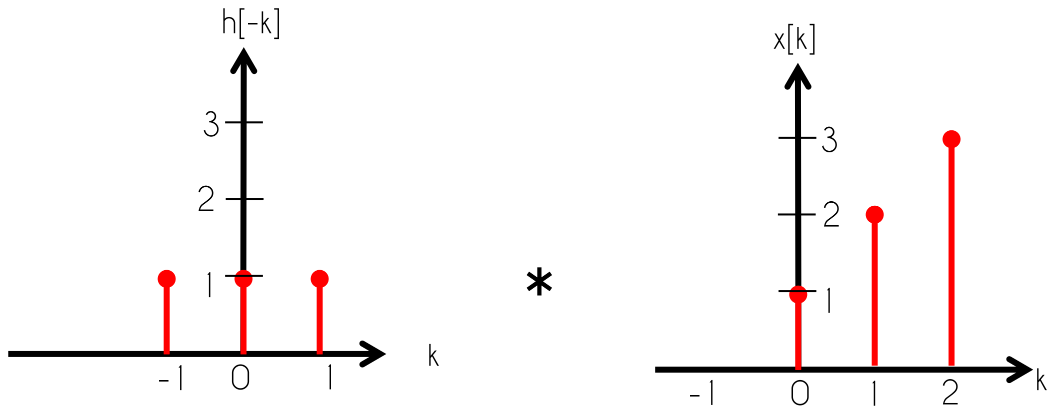
$$y[k] = 0 + 0 + 1 + 0 + 0$$

Therefore, summation for y at $n = 0$ is;

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$$y[0] = 1$$

- When $n = 1$



Multiplication of the sequence h[k] and the sequence x[0–k] for each k is;

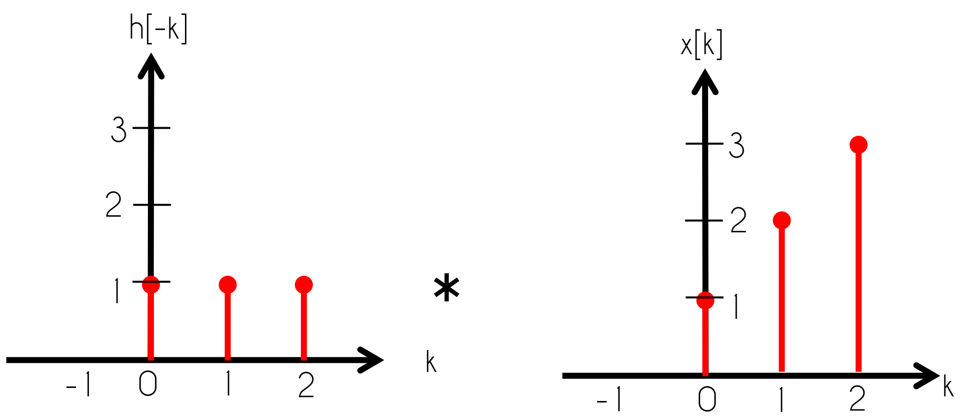
$$y[k] = 0 + 1 + 2 + 0$$

Therefore, summation for y at $n = 1$ is;

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$$y[1] = 3$$

- When $n = 2$



Multiplication of the sequence h[k] and the sequence x[0–k] for each k is;

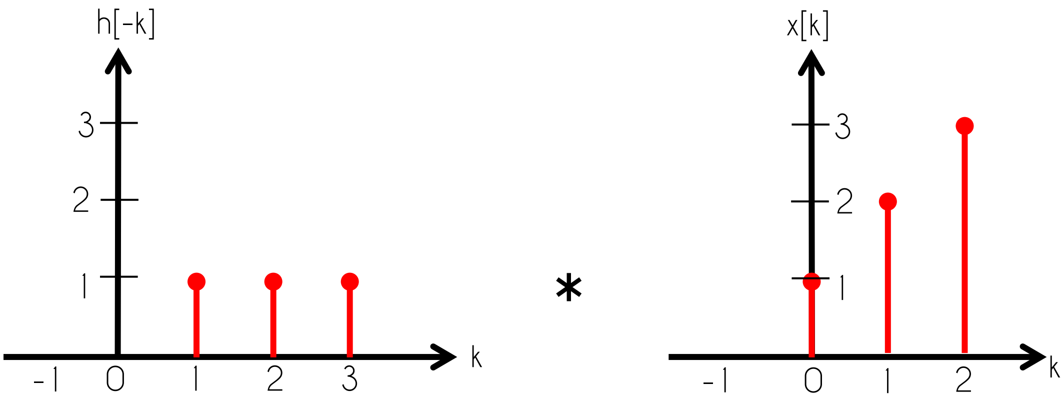
$$y[k] = 1 + 2 + 3$$

Therefore, summation for y at $n = 2$ is;

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$$y[2] = 6$$

- When $n = 3$



Multiplication of the sequence h[k] and the sequence x[0–k] for each k is;

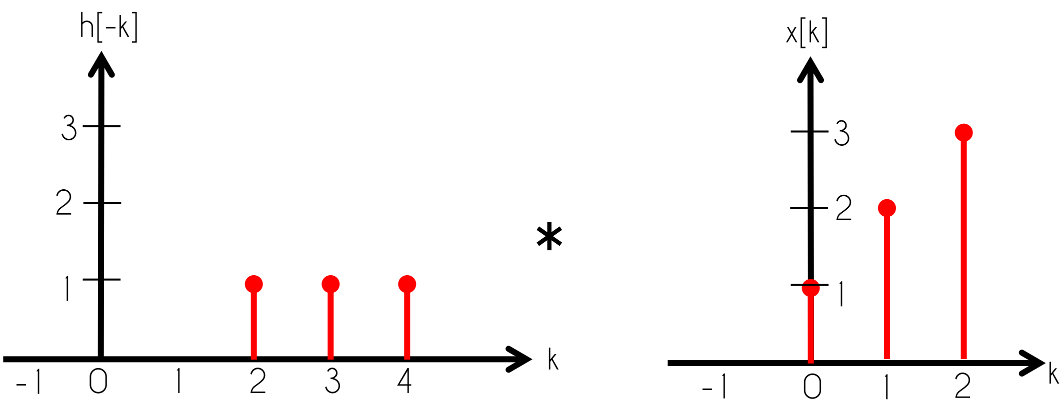
$$y[k] = 0 + 2 + 3 + 0$$

Therefore, summation for y at $n = 3$ is;

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$$y[3] = 5$$

- When $n = 4$



Multiplication of the sequence h[k] and the sequence x[0–k] for each k is;

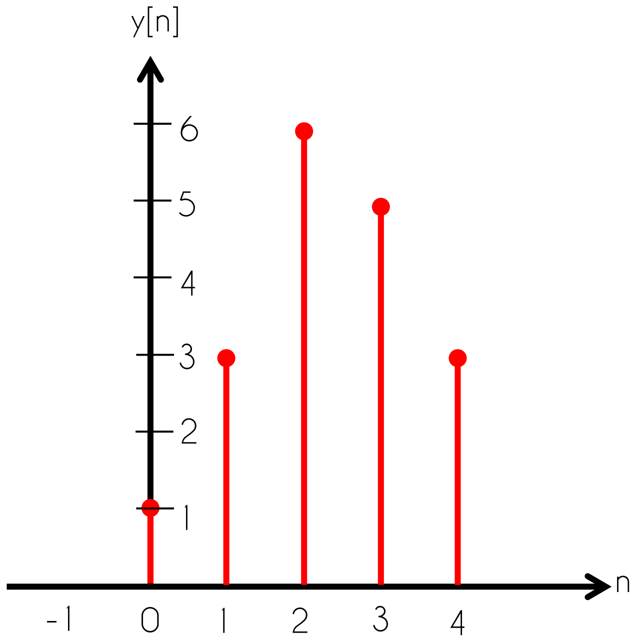
$$y[k] = 0 + 0 + 3 + 0 + 0$$

Therefore, summation for y at $n = 4$ is;

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$$y[4] = 3$$

- Therefore output sequence $\mathbf{y}[n] = \{1, 3, 6, 5, 3\}$



Question 5

Sketch the output $y[n] = x[n] * h[n]$ where $x[n] = [\mathbf{2}, 2, 2]$and $h[n] = [2, \mathbf{3}, 2, -1]$. Use the analytical method to find $y[n]$.

Solution: Analytical Method

$$y[n] = x[n] * h[n]$$

Convert the given sequence to the equation:

$x[n] = 2\delta[n] + 2\delta[n-1] + 2\delta[n-2]$

$h[n] = 2\delta[n+1] + 3\delta[n] + 2\delta[n-1] - \delta[n-2]$

Substitute $h[n]$ with $2\delta[n+1] + 3\delta[n] + 2\delta[n-1] - \delta[n-2]$;

Therefore,

$y[n] = x[n] * \{2\delta[n+1] + 3\delta[n] + 2\delta[n-1] - \delta[n-2]\}$

$y[n] = x[n] * 2\delta[n+1] + x[n] * 3\delta[n] + x[n] * 2\delta[n-1] - x[n] * \delta[n-2]$

$y[n] = 2x[n+1] + 3x[n] + 2x[n-1] - x[n-2]$

Substitute;

$$2x[n+1] = 2\{2\delta[n+1] + 2\delta[n-1+1] + 2\delta[n-2+1]\}$$
$$= 4\delta[n+1] + 4\delta[n] + 4\delta[n-1]$$

$$3x[n] = 3\{2\delta[n] + 2\delta[n-1] + 2\delta[n-2]\}$$
$$= 6\delta[n] + 6\delta[n-1] + 6\delta[n-2]$$

$$2x[n-1] = 2\{2\delta[n-1] + 2\delta[n-1-1] + 2\delta[n-2-1]\}$$
$$= 4\delta[n-1] + 4\delta[n-2] + 4\delta[n-3]$$

$$x[n-2] = 2\delta[n-2] + 2\delta[n-1-2] + 2\delta[n-2-2]$$
$$= 2\delta[n-2] + 2\delta[n-3] + 2\delta[n-4]$$

Therefore, $y[n]$ is,

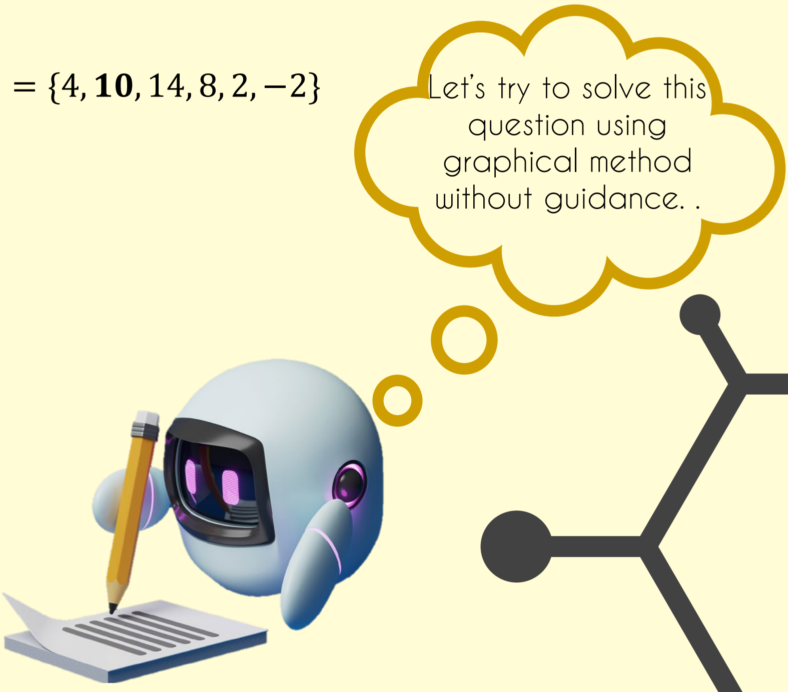
$$= \{4\delta[n+1] + 4\delta[n] + 4\delta[n-1]\} + \{6\delta[n] + 6\delta[n-1] + 6\delta[n-2]\}$$
$$+ \{4\delta[n-1] + 4\delta[n-2] + 4\delta[n-3]\} - \{2\delta[n-2] + 2\delta[n-3] + 2\delta[n-4]\}$$

$$= 4\delta[n+1] + 4\delta[n] + 4\delta[n-1] + 6\delta[n] + 6\delta[n-1] + 6\delta[n-2] + 4\delta[n-1]$$
$$+ 4\delta[n-2] + 4\delta[n-3] - 2\delta[n-2] - 2\delta[n-3] - 2\delta[n-4]$$

$$= 4\delta[n+1] + 10\delta[n] + 14\delta[n-1] + 8\delta[n-2] + 2\delta[n-3] - 2\delta[n-4]$$

The answer in sequence;

$$y[n] = \{4, \mathbf{10}, 14, 8, 2, -2\}$$



04

TUTORIAL

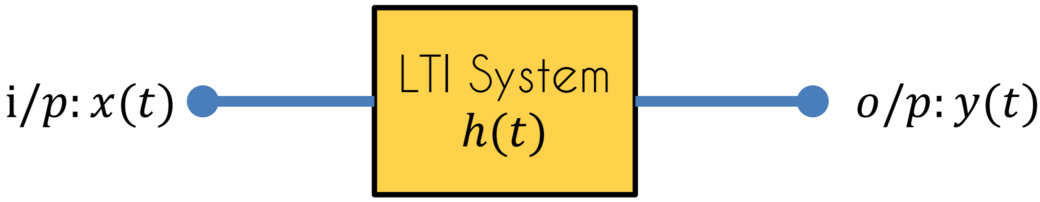
CONVOLUTION INTEGRAL

Call upon me, I will respond. (Quran 40:60)

CT Convolution Integral

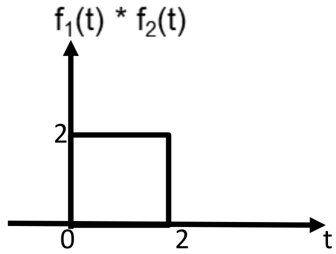
$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

Equation above is called as convolution integral. Fundamental result are the output of any continuous – time LTI system is the convolution of the input $x(t)$ with the impulse response $h(t)$ of the system. The block diagram below describe relationship between input $x(t)$, impulse response $h(t)$ and output, $y(t)$ as equation above.



Question 1

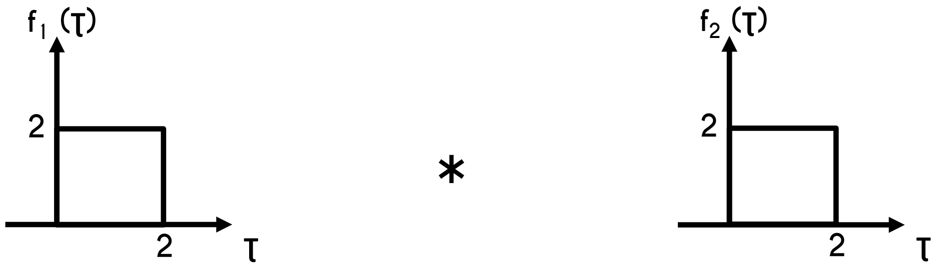
Sketch the output y(t) = f1(t) * f2(t) of the function in Figure below:



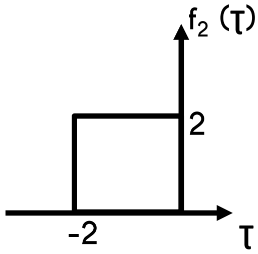
Solution:

$$CT\ Convolution\ Integral\ \rightarrow y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

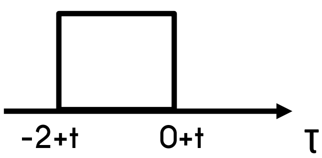
Step 1: EXPRESS FUNCTION IN VARIABLE, τ



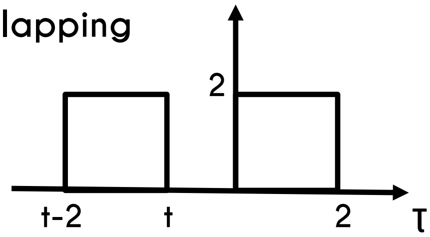
Step 2: REFLECT ONE OF THE FUNCTION: $h(t) \rightarrow h(\tau)$



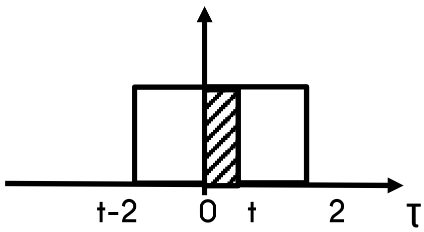
Step 3 & 4: TIME-OFFSET, T: $h(t-\tau)$**, SLIDE ALONG** $\tau - AXIS$ **& FIND THE INTEGRAL OF PRODUCT WHENEVER THE 2-FUNCTION INTERSECT**



Case 1: $t \leq 0$ →No overlapping



Case 2: Overlapping



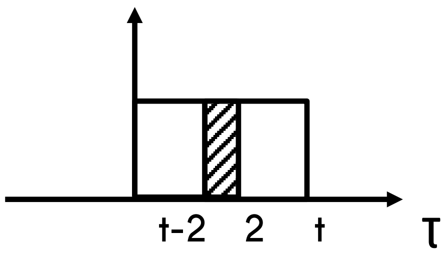
$$t - 2 \leq 0$$

$$\left.\begin{matrix} t \geq 0 \\ t \leq 2 \end{matrix}\right\} \rightarrow 0 \leq t \leq 2$$

$$f_2(\tau) = \int_0^t x(\tau)h(t-\tau)d\tau = \int_0^t (2)(2)d\tau$$

$$f_2(\tau) = 4\big|_0^t = 4t$$

Case 3: Overlapping



$$t - 2 \leq 2$$

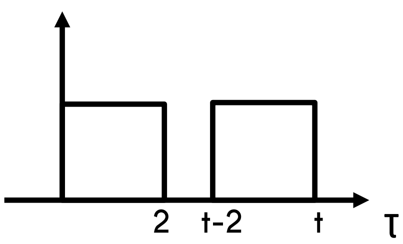
$$\left.\begin{matrix} t \geq 2 \\ t \leq 4 \end{matrix}\right\} \rightarrow 2 \leq t \leq 4$$

$$f_2(\tau) = \int_0^t x(\tau)h(t-\tau)d\tau = \int_0^t (2)(2)d\tau$$

$$f_2(\tau) = 4t\big|_{t-2}^{2} = 4(2) - 4(t-2)$$

$$f_2(\tau) = -4t + 16$$

Case 4: No overlapping

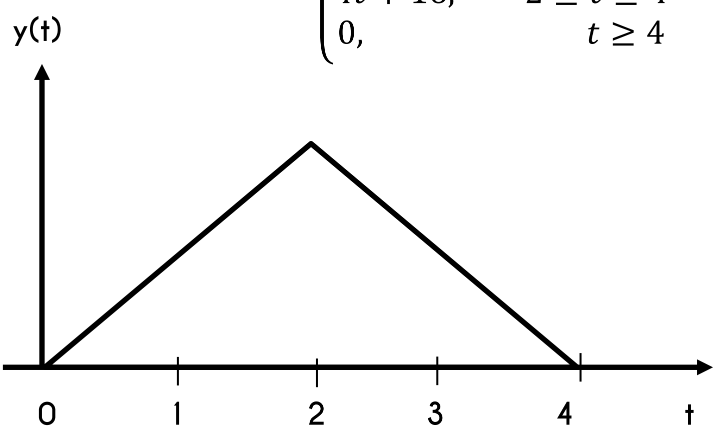


$$t - 2 \geq 0$$

$$t \geq 4$$

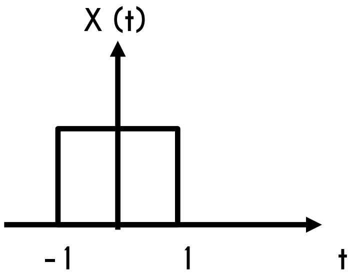
Step 5: THE RESULTING WAVEFORM IS THE CONVOLUTION OF FUNCTIONS f & h.

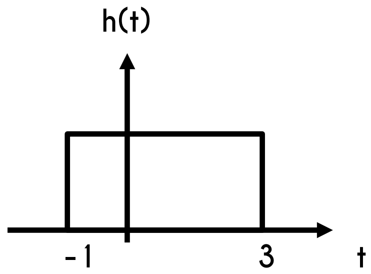
$$y(t) = \begin{cases} 0, & t - 2 \leq 0 \\ 4t, & 0 \leq t \leq 2 \\ 4t + 16, & 2 \leq t \leq 4 \\ 0, & t \geq 4 \end{cases}$$



Question 2

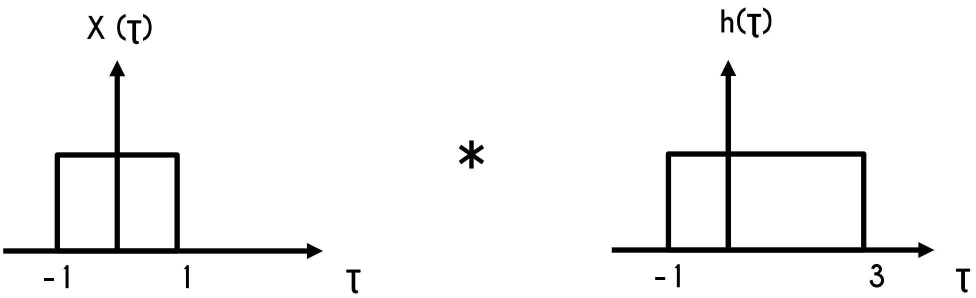
Evaluate the function $y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$ for the signal given below.



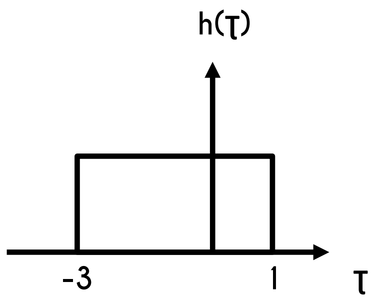


Solution:

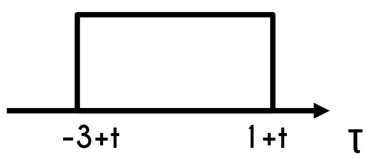
Step 1: EXPRESS FUNCTION IN VARIABLE, τ



Step 2: REFLECT ONE OF THE FUNCTION: $h(t) \rightarrow h(\tau)$



Step 3 & 4: TIME-OFFSET, T: $h(t-\tau)$, SLIDE ALONG $\tau - AXIS$ & FIND THE INTEGRAL OF PRODUCT WHENEVER THE 2-FUNCTION INTERSECT

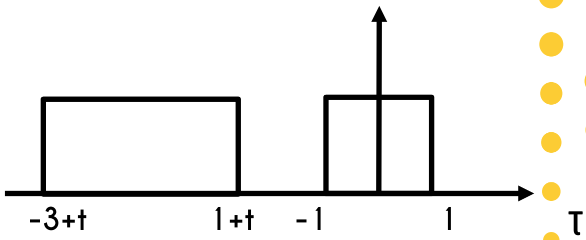


Case 1: $1 + t < -1$ →No overlapping

Integrate $x(\tau)h(t-\tau)$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau = \int_{-\infty}^{\infty} 0\ d\tau = 0$$

Simplify bounds: $1 + t < -1 \rightarrow t < -2$



Case 2: Overlapping

$$1 + t \geq -1$$
$$1 + t < +1$$

Integrate $x(\tau)h(t-\tau)$

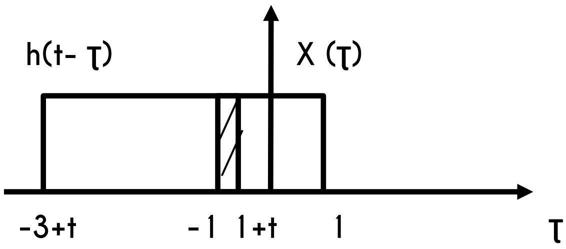
$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau = \int_{-1}^{1+t} (1)(1)\ d\tau$$

$$y(t) = t|_{-1}^{1+t} = (1+t) - (-1) \ = 2 + t$$

Simplify bounds:

$$1 + t \geq -1 \rightarrow t \geq -2$$
$$1 + t < +1 \rightarrow t < 0$$

Therefore, $\mathrm{y(t)} = 2 + t \qquad -2 \leq t < 0$



Case 3: Overlapping

$$1 + t \geq 1$$
$$-3 + t < -1$$

Integrate $x(\tau)h(t-\tau)$

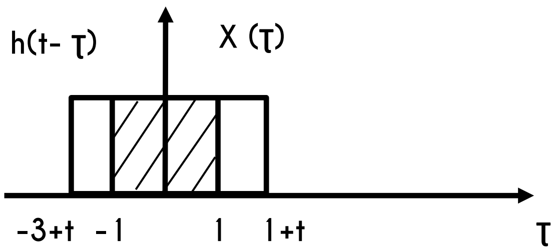
$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau = \int_{-1}^{1} (1)(1)\ d\tau$$

$$y(t) = t|_{-1}^{1} = (1) - (-1) \ = 2$$

Simplify bounds:

$$1 + t \geq 1 \rightarrow t \geq 0$$
$$-3 + t < -1 \rightarrow t < 2$$

Therefore, $\mathrm{y(t)} = 2 \qquad 0 \leq t < 2$



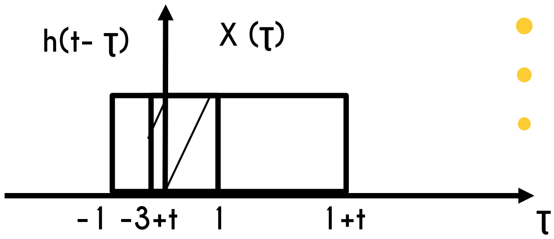
Case 4: Overlapping

$$-3+t \geq -1$$
$$-3+t < 1$$

Integrate $x(\tau)h(t-\tau)$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau = \int_{-3+t}^{1} (1)(1)\,d\tau$$

$$y(t) = t|_{-3+t}^{1} = (1) - (-3+t) = 4 - t$$



Simplify bounds:

$$-3+t \geq -1 \rightarrow t \geq 2$$
$$-3+t < 1 \rightarrow t < 4$$

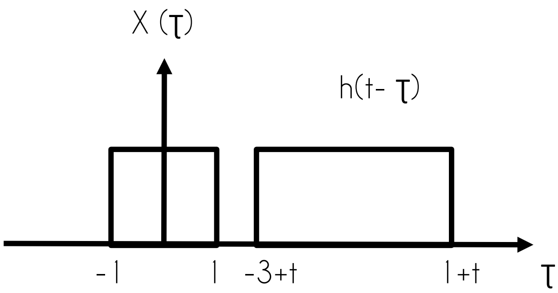
Therefore, $y(t) = 4 - t \qquad 2 \leq t < 4$

Case 5: Overlapping

$$-3+t \geq 1$$

Integrate $x(\tau)h(t-\tau)$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau = \int_{-\infty}^{\infty} 0\,d\tau = 0$$

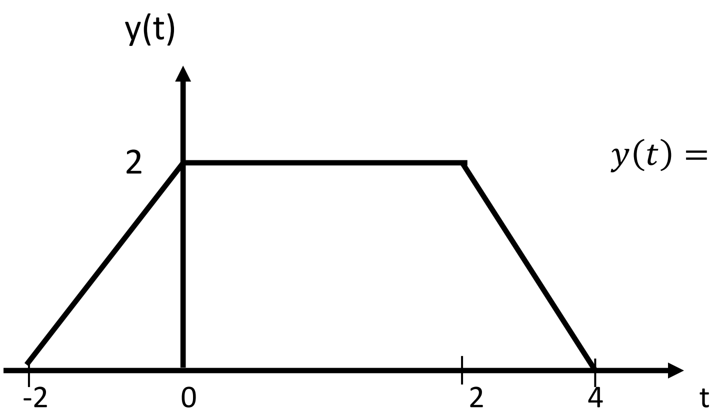


Simplify bounds:

$$-3+t \geq 1 \rightarrow t \geq 4$$

Therefore, $y(t) = 0 \qquad t > 4$

Step 5: THE RESULTING WAVEFORM IS THE CONVOLUTION OF FUNCTIONS f & h.

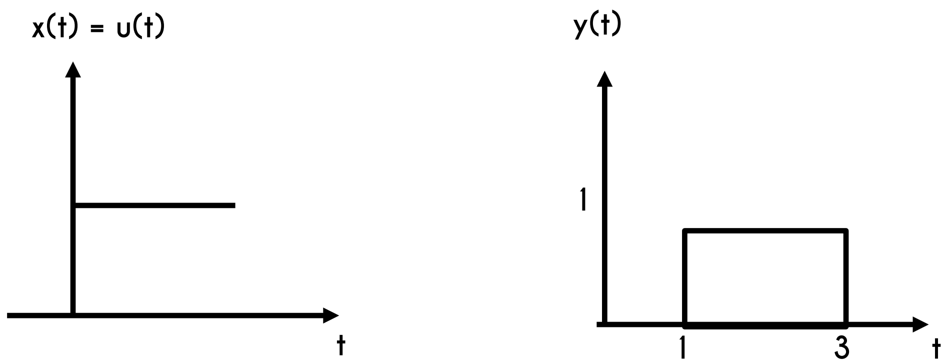


$$y(t) = \begin{cases} 0, & t < 2 \\ 2+t, & -2 \leq t < 0 \\ 2, & 0 \leq t < 2 \\ 4-t, & 2 \leq t < 4 \\ 0 & t > 4 \end{cases}$$

Question 3

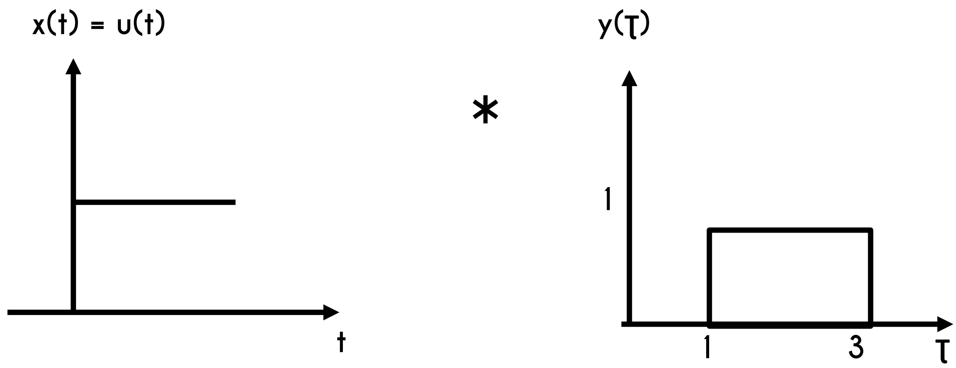
Consider the Continuous – Time signal as below.

Compute $z(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$

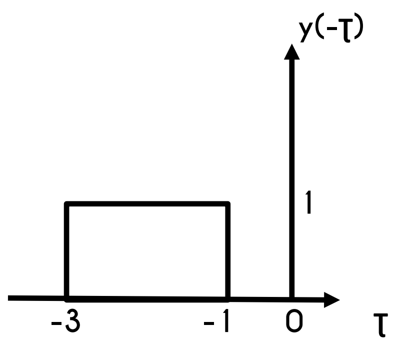


Solution:

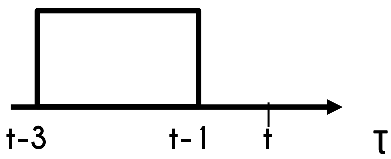
Step 1: EXPRESS FUNCTION IN VARIABLE, τ



Step 2: REFLECT ONE OF THE FUNCTION: $h(t) \rightarrow h(\tau)$



Step 3 & 4: TIME-OFFSET, T: $h(t-\tau)$, SLIDE ALONG $\tau - AXIS$ & FIND THE INTEGRAL OF PRODUCT WHENEVER THE 2-FUNCTION INTERSECT



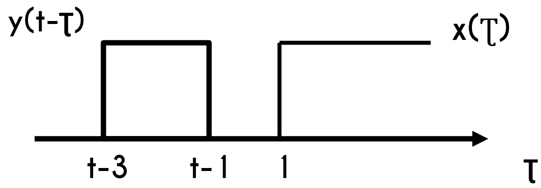
Case 1: No overlapping

$$t-1 \leq 0$$
$$t < 1$$

Integrate $x(\tau)h(t-\tau)$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau = \int_{-\infty}^{\infty} 0\ d\tau = 0$$

Simplify bounds: $1 + t < -1 \rightarrow t < -2$



Case 2: Overlapping

$$t-1 > 0$$
$$t-3 \leq 0$$

Integrate $x(\tau)h(t-\tau)$

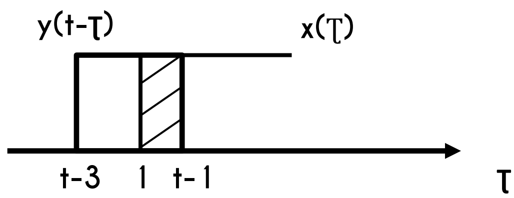
$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau = \int_{0}^{t-1} (1)(1)\ d\tau$$

$$y(t) = t\Big|_{0}^{t-1} = (t-1) - (0)\ = t-1$$

Simplify bounds:

$$t-1 > 0 \rightarrow t > 1$$
$$t-3 \leq 01 \rightarrow t \leq 3$$

Therefore, $\text{y(t)} = t-1 \qquad 1 < t \leq 3$



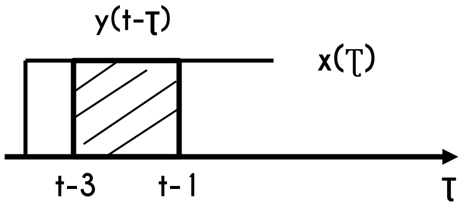
Case 3: Overlapping

$$t-3>0$$

Integrate $x(\tau)h(t-\tau)$

$$y(t)=\int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau=\int_{t-3}^{t-1}(1)(1)\,d\tau$$

$$y(t)=t|_{t-3}^{t-1}=(t-1)-(t-3)\ =2$$

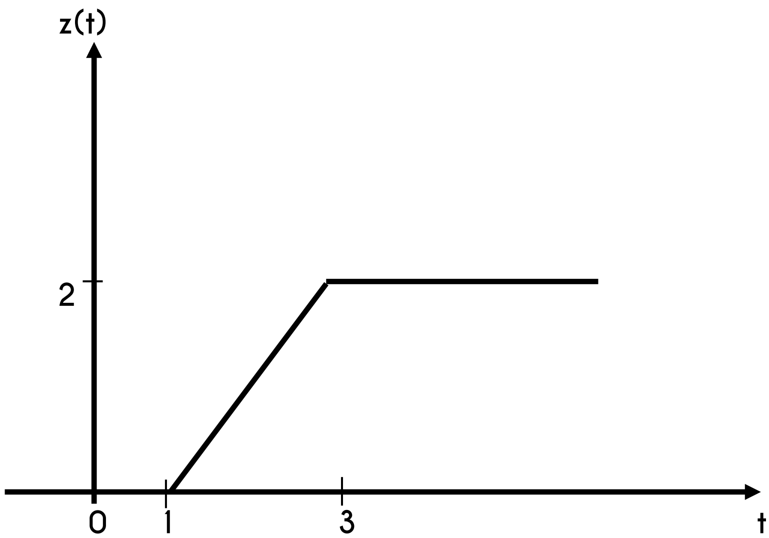


Simplify bounds:

$$t-3>0\rightarrow t>3$$

Therefore, $y(t)=2 \qquad t>3$

Step 5: THE RESULTING WAVEFORM IS THE CONVOLUTION OF FUNCTIONS f & h.

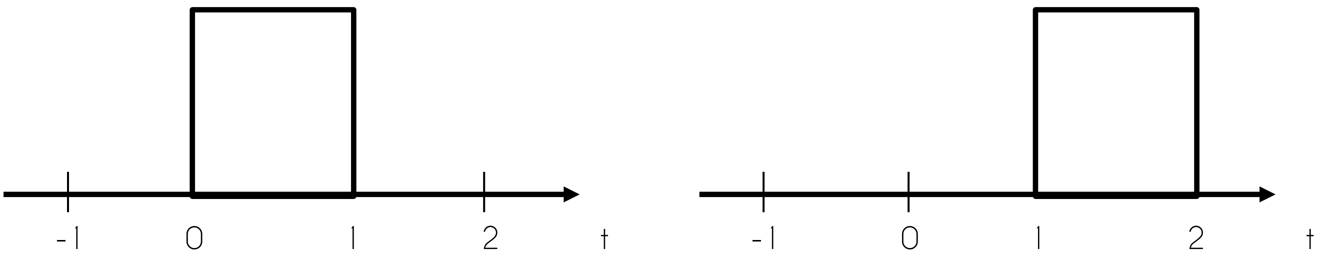


$$y(t)=\begin{cases}0, & t\leq 2\\ t-1, & 1<t\leq 3\\ 2, & t>3\end{cases}$$

Question 4

The input, shown on the left, is a pulse of length one. The impulse response as seen on the right is a pulse of length one delayed by one.

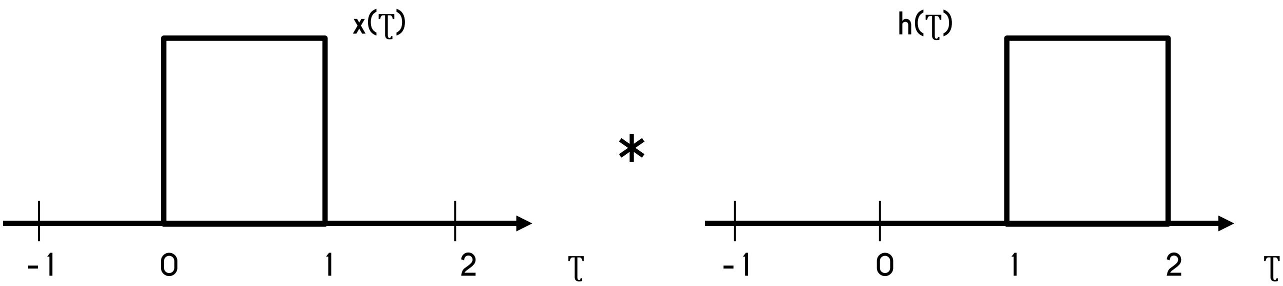
Taking these two functions and converting them into the forms used in the convolution integral



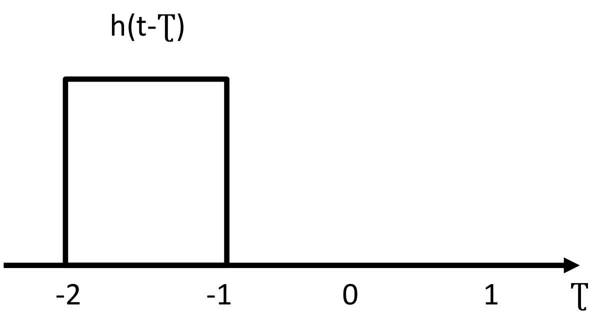
$x(t) = u(t) - u(t-1)$ $\quad$ $h(t) = u(t-1) - u(t-2)$

Solution:

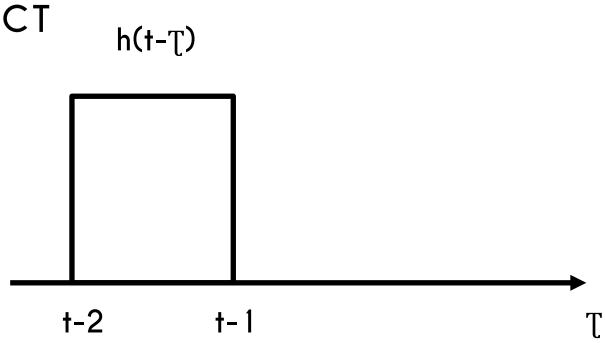
Step 1: EXPRESS FUNCTION IN VARIABLE, τ



Step 2: REFLECT ONE OF THE FUNCTION:$h(t) \rightarrow h(\tau)$



Step 3 & 4: TIME-OFFSET, T: $h(t-\tau)$, SLIDE ALONG $\boldsymbol{\tau - AXIS}$ & FIND THE INTEGRAL OF PRODUCT WHENEVER THE 2-FUNCTION INTERSECT



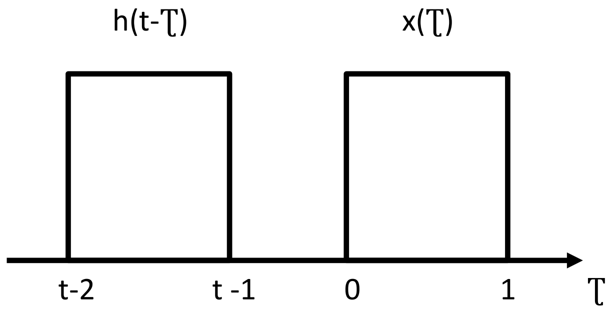
Case 1: No overlapping

$$t - 1 < 0$$

Integrate $x(\tau)h(t-\tau)$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau = \int_{-\infty}^{\infty} 0\, d\tau = 0$$

Simplify bounds: $t - 1 < 0 \rightarrow t < 1$



Case 2: Overlapping

$$t - 1 < 1$$
$$t - 1 > 0$$

Integrate $x(\tau)h(t-\tau)$

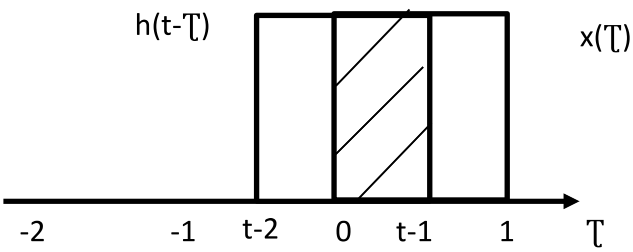
$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau = \int_{0}^{t-1} (1)(1)\, d\tau$$

$$y(t) = t|_{0}^{t-1} = (t-1) - (0) \; = t - 1$$

Simplify bounds:

$$t - 1 < 1 \rightarrow t < 2$$
$$t - 1 > 0 \rightarrow t > 1$$

Therefore, $\mathrm{y(t)} = t - 1 \qquad 1 < t < 2$



Case 3: Overlapping

$$t - 2 < 1$$
$$t - 2 > 0$$

Integrate $x(\tau)h(t-\tau)$

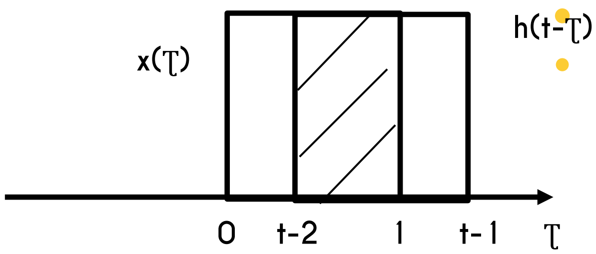
$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau = \int_{t-2}^{1} (1)(1)\, d\tau$$

$$y(t) = t|_{t-2}^{1} = (1) - (t-2) = 3 - t$$

Simplify bounds:

$$t - 2 < 1 \rightarrow t < 3$$
$$t - 2 > 0 \rightarrow t > 2$$

Therefore, $\mathrm{y(t)} = 3 - t \qquad 2 < t < 3$



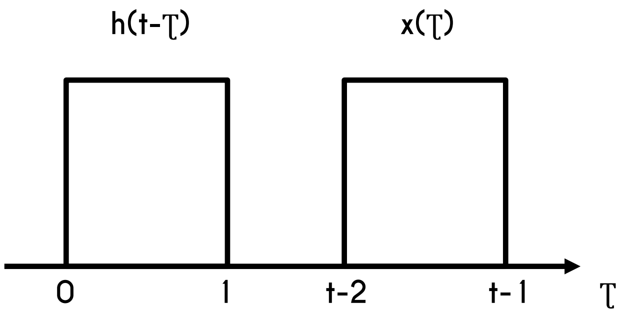
Case 4: No overlapping

$$\mathrm{t} - 2 > 1$$

Integrate $x(\tau)h(t-\tau)$

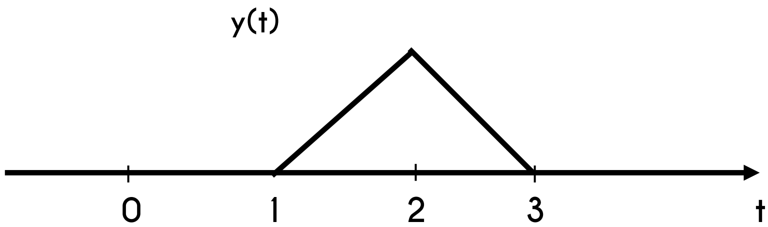
$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau = \int_{-\infty}^{\infty} 0\, d\tau = 0$$

Simplify bounds: $t - 2 > 1 \rightarrow t > 3$



Step 5: THE RESULTING WAVEFORM IS THE CONVOLUTION OF FUNCTIONS f & h.

$$y(t) = \begin{cases} t - 1 & 1 < t < 2 \\ 3 - t, & 2 < t \leq 3 \\ 0, & otherwise \end{cases}$$



Properties of the

CONVOLUTION

Arithmetic properties of the convolution.

Question 1

The Figure 1 below is a basic channel equalization system. Explain the input – output relationship for the block diagram of LTI system with impulse response h_1 and h_2.

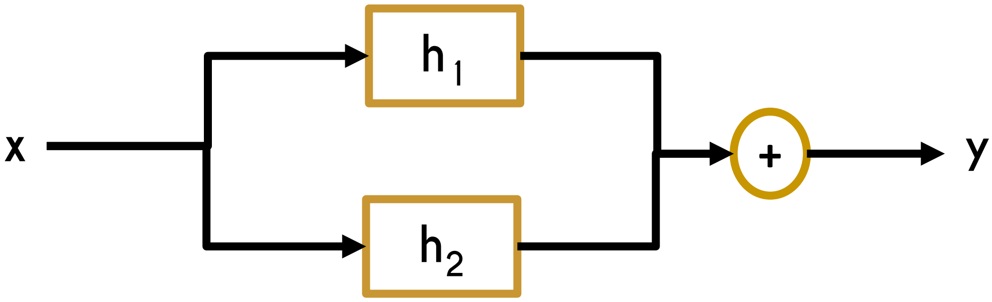
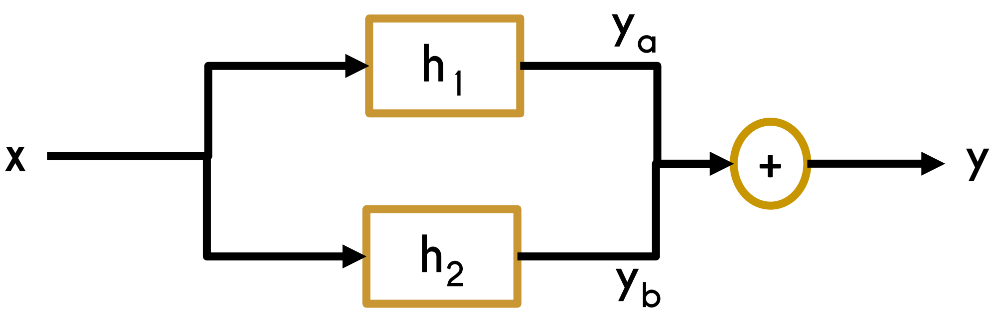


Figure 1

Solution:



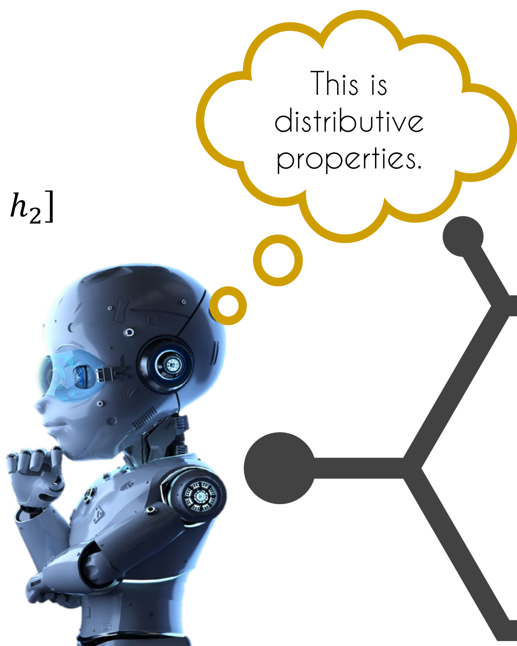
$$y_a = x * h_1$$
$$y_b = x * h_2$$

Therefor;

$$y = y_a + y_b = [x * h_1] + [x * h_2]$$

or can be write;

$$y = x[h_1 + h_2]$$



Question 2

The Figure 2 below is a block diagram of LTI system. Explain the input – output relationship impulse response for h_1 and h_2.

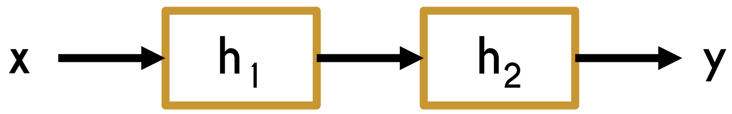
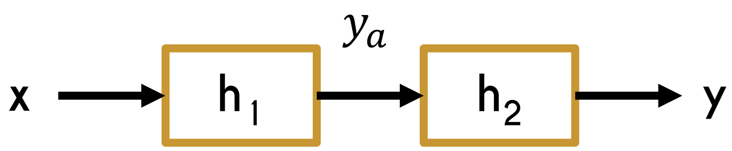


Figure 2

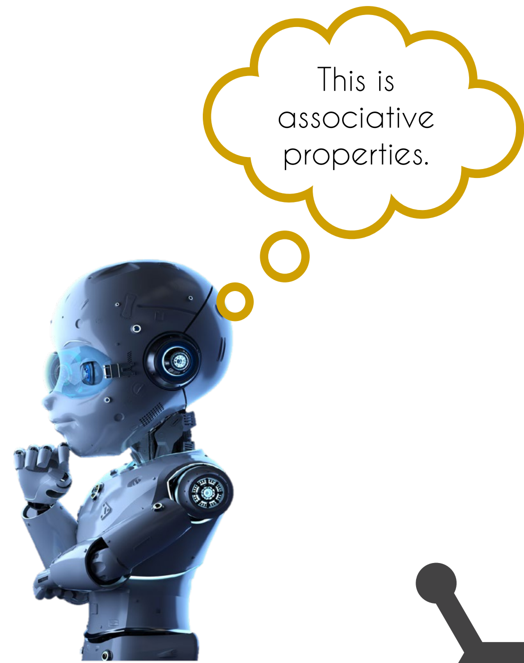
Solution:



$$y_a = x * h_1$$
$$y = y_a * h_2$$

Therefor;

$$y = y_a * h_2 = [x * h_1] * h_2$$



Question 3

Referring to Figure 3 below is, state the input – output relationship for the block diagram of LTI system.

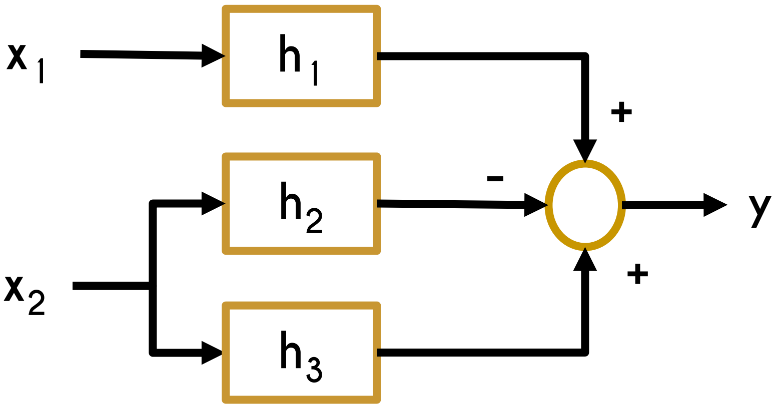
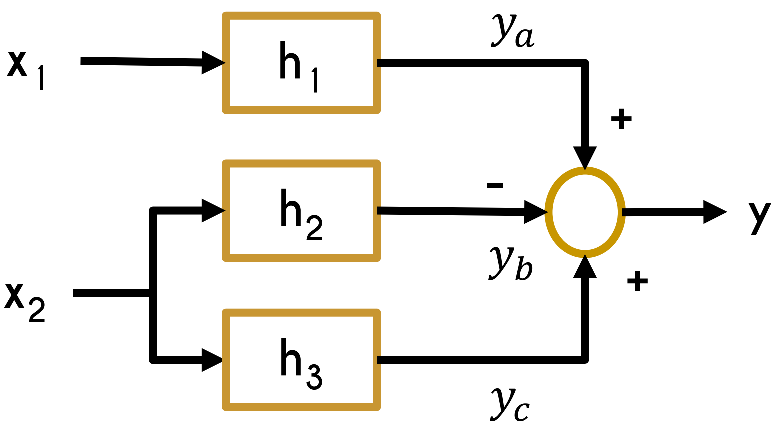


Figure 3

Solution:



$$y_a = x_1 * h_1$$
$$y_b = x_2 * h_2$$
$$y_c = x_2 * h_3$$

Therefor;

$$y = y_a - y_b + y_c = [x_1 * h_1] - [x_2 * h_2] + [x_2 * h_3]$$
$$y = [x_1 * h_1] + x_2[h_3 - h_2]$$

Question 4

Referring to Figure 4 below is, state the input – output relationship for the block diagram of LTI system.

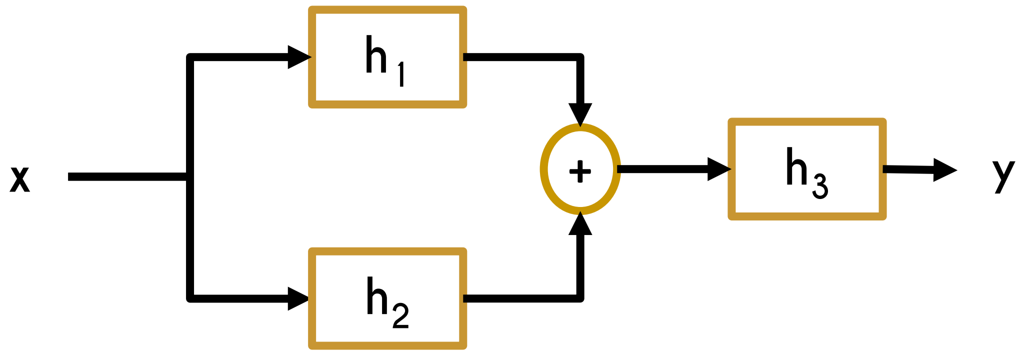
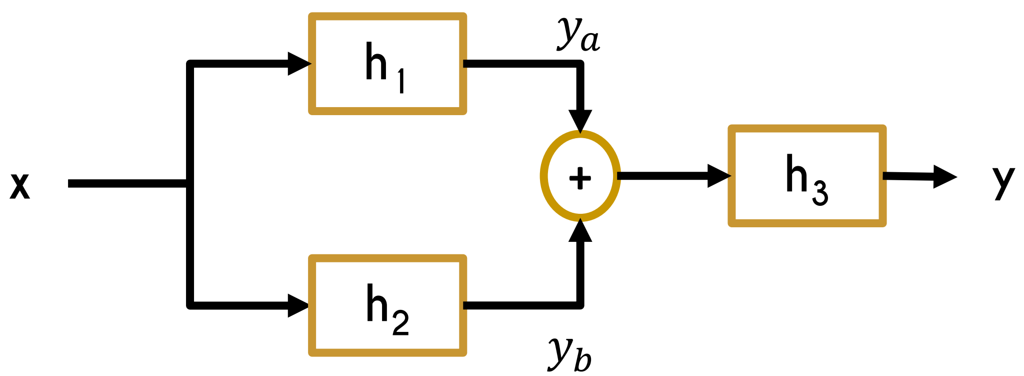


Figure 4

Solution:



$$y_a = x * h_1$$
$$y_b = x * h_2$$
$$y = [y_a + y_b] * h_3$$

Therefor;

$$y = [y_a + y_b] * h_3 = \{[x * h_1] + [x * h_2]\}h_3$$
$$y = x * [h_1 + h_2] * h_3$$

Question 5

Express the input output relationship for the block diagram of LTI systems in Figure 5.

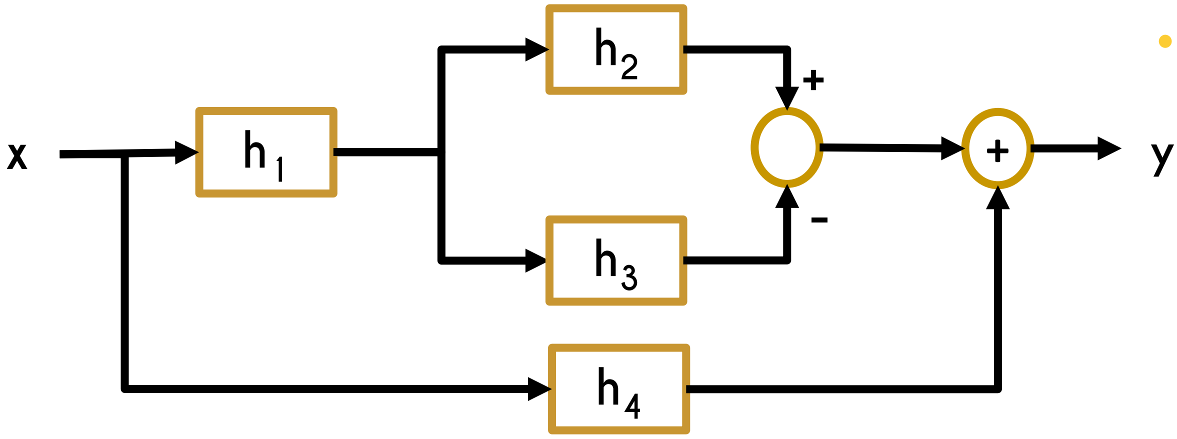
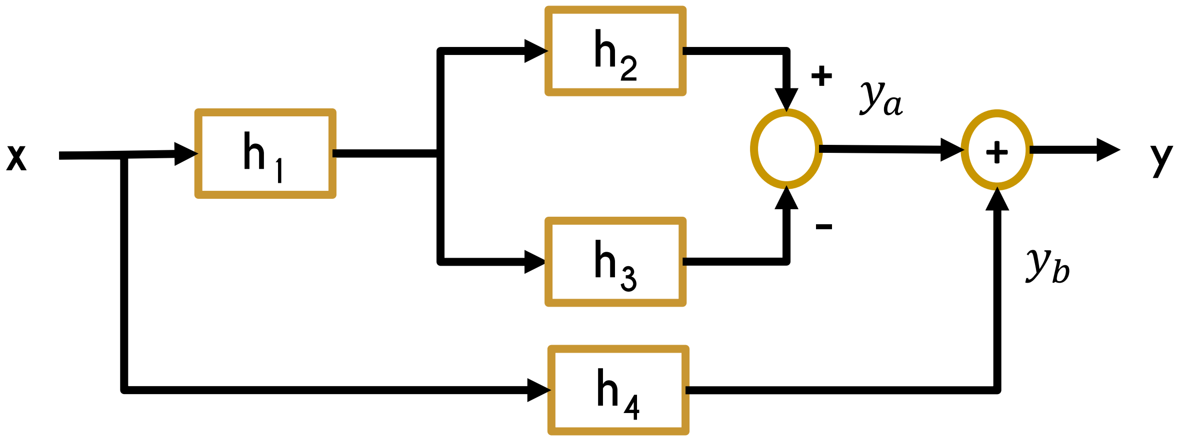


Figure 5

Solution:



$$y_a = x * h_1[h_2 - h_3]$$
$$y_b = x * h_4$$
$$y = y_a + y_b$$

Therefor;

$$y = y_a + y_b = x * h_1[h_2 - h_3] + x * h_4$$
$$y = x[h_1h_2 - h_1h_3 + h_4]$$

05

EXERCISE

COLLECTION

FINAL YEAR QUESTION

"You don't have to be great to start, but you have to start to be great." – Zig Ziglar

Exercises

Question 1

Write the Expression for the impulse response related to input x(t) with output y(t) for the system in Figure 1. [Final Question DEE40113: Signal & System, Session 2:2021/2022]

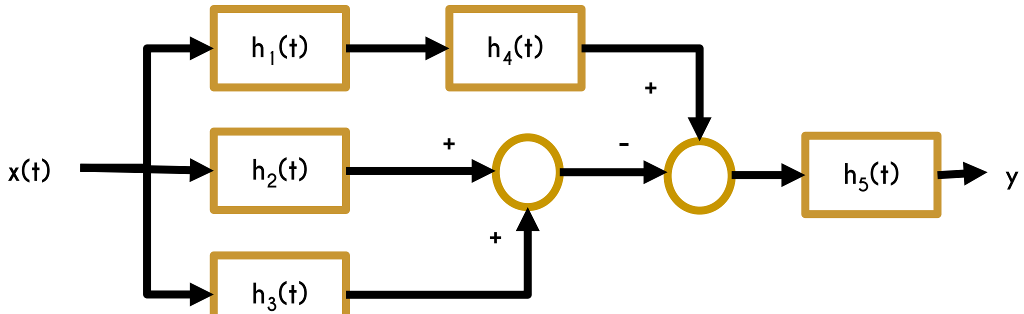


Figure 1.

Question 2

Explain four (4) steps of convolution sum operation in LTI system. [Final Question DEE40113: Signal & System, Session 1:2022/2023]

Question 3

Calculate $y[n] = x[n] * h[n]$ by using analytical technique.

$x[n] = \delta[n] + \delta[n-1] + \delta[n-2]$

$h[n] = 3\delta[n-2] + 3\delta[n-3] + 2\delta[n-4] + 2\delta[n-5] + \delta[n-6] + \delta[n-7]$

[Final Question DEE40113: Signal & System, Session 1:2022/2023]

Question 4

Express the convolution in the time range from 0 to t if $h(t) = e^{-\alpha t}u(t)$ and $x(t) = u(t)$.

[Final Question DEE40113: Signal & System, Session Jun 2019]

Question 5

Compute the convolution from 0 to t if $h(t) = u(t)$ and $x(t) = t^2u(t)$.

[Final Question DEE40113: Signal & System, Session Jun 2018]

Rujukan

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