POLITEKNIK MALAYSIA PORT DICKSON

ELECTRICAL CIRCUITS

2024

ALTERNATING CURRENT
SINUSOIDAL STEADY STATE CIRCUIT ANALYSIS
RESONANCE

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We hereby declare that this book is our original work. To the best of our knowledge. It contains no materials previously written or published by another person. However if there is any, due acknowledgment and credit are mentioned accordingly in the e-book.

PREFACE

Electrical circuits is a fundamental to understand the principles of electrical engineering and forms the backbone of many technological advancements in our modern world. This book is designed to provide students with the knowledge related to alternating current of electrical circuits.

The chapters in this book are structured to gradually build the reader's understanding, starting from the basic concepts of alternating current, sinusoidal steady state circuit analysis and resonance. Each chapter includes detailed explanations, illustrative examples, and practical exercises designed to reinforce learning and encourage critical thinking.

In writing this book, we have drawn upon our experiences in teaching electrical engineering. Our goal is to make the subject accessible to all readers, regardless of their prior knowledge, while also challenging them to deepen their understanding and apply what they have learned to real-world problems.

We want to express our gratitude to our colleagues and students who have contributed valuable feedback during the development of this book. Their insights have been instrumental in shaping the content and ensuring that it meets the needs of a diverse readers.

Hopefully this book will serve as a reliable resource for anyone who is interested in the study of electrical circuits and inspire further exploration and innovation in this exciting field.

Ts. AZIHA BINTI MOHD NOR
RAZIMAH BINTI ABDUL
SUHAIDA BINTI ABDUL HALIM

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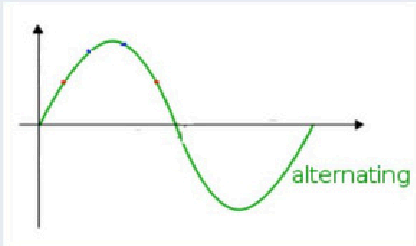
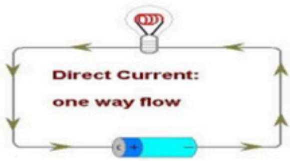
TOPIC 1

ALTERNATING VOLTAGE AND CURRENT

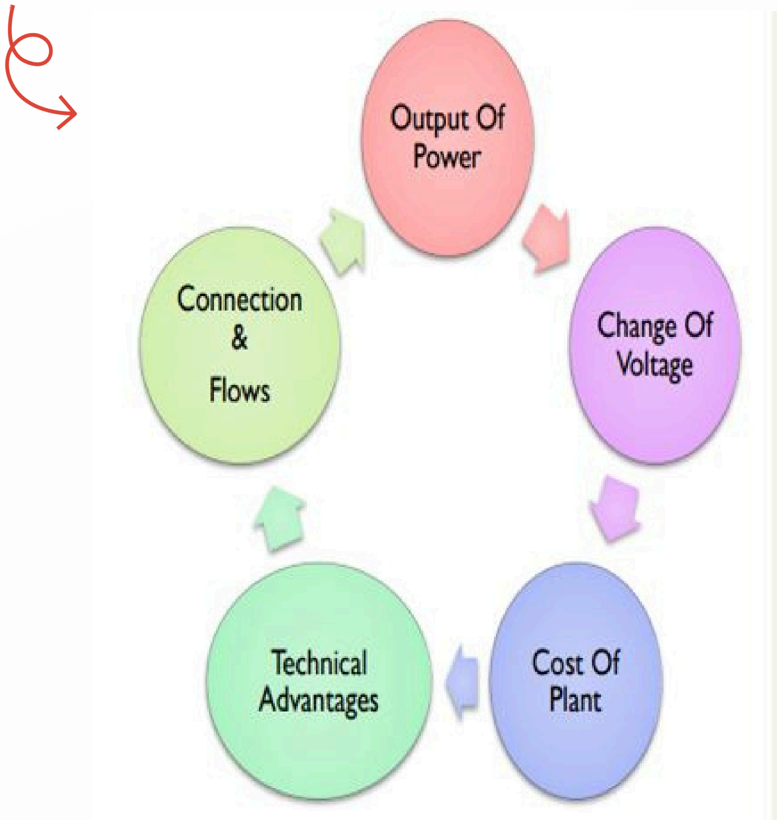
1.1.1 DISTINGUISH BETWEEN DIRECT CURRENT AND ALTERNATING CURRENT

DC	AC
The flow of electrical charge is only in one direction	The movement of electrical charge periodically reverses directions.
	
The output voltage will remain essentially constant over time	AC source of electrical power charges constantly in amplitude & regularly changes polarity

	ALTERNATING CURRENT	DIRECT CURRENT
Amount of energy that can be carried	Safer to transfer over longer city distances and can provide more power	Voltage of DC cannot travel very far until it begins to lose energy
Flow of Electrons:	Electrons keep switching directions- forward and backward	Electrons move steadily in one direction or 'forward'
		

	ALTERNATING CURRENT	DIRECT CURRENT
Cause of the direction of flow of electrons	Rotating magnet along the wire	Steady magnetism along the wire
Frequency	The frequency of alternating current is 50Hz or 60Hz depending upon the country.	The frequency of direct current is zero.
Direction	It reverses its direction while flowing in a circuit	It flows in one direction in the circuit
Current	It is the current of magnitude varying with time	It is the current of constant magnitude

1.1.2 Why AC is used in preference to DC



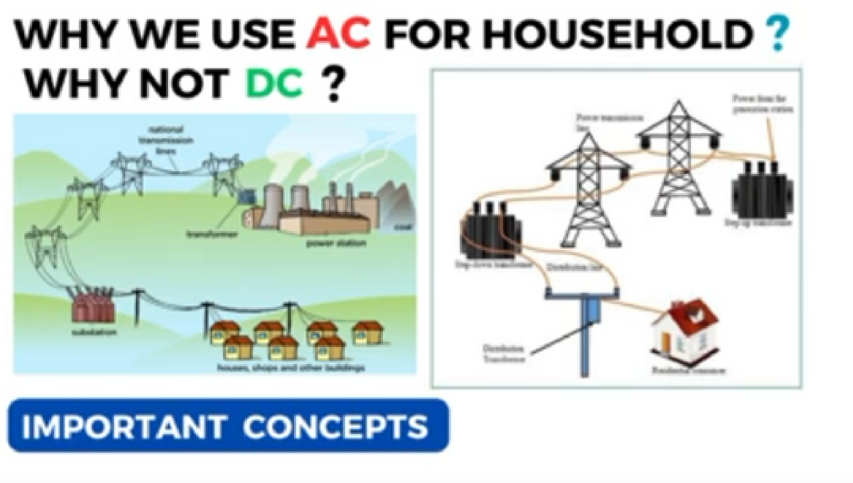
The Output Of Power Stations comes from a rotary turbine, which by its nature Is AC and therefore requires no power electronics to convert to DC

It is much Easier To Change The Voltage of AC electricity for transmission and distribution

The Cost Of Plant associated with AC transmission (Circuit Breakers, Transformers etc) is much lower than the equivalent of DC transmission

AC transmission provides a number of Technical Advantages. When a fault on the network occurs, a large fault current occurs.

It is also easier to meter AC connections and monitor power flows across a network



https://youtu.be/_CytmsGy3xc?feature=shared

1.1.2 The sources of alternating current

- Alternating current generator

- Generating plant

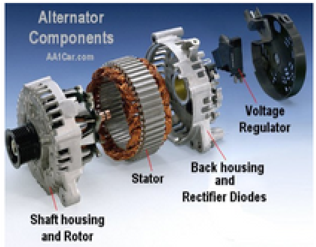
- Wind power station

Any other AC sources?

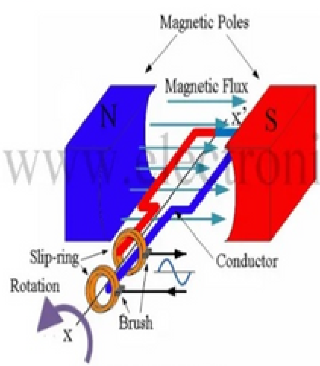
Generation of an alternating current

ALTERNATOR

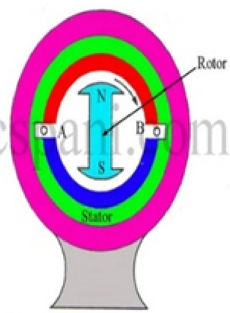
- An alternator is a machine which produces the Alternating Current, by converting mechanical energy into electrical energy.
- It is termed as an AC generator or the synchronous generator.
- Electricity is produced when a magnetic field spins inside the windings of wire.



2 METHODS TO GENERATE ALTERNATING CURRENT



(a) Rotating a coil in Stationary Magntic Field



(a) Rotating of Magnetic Field in Stationary Coil

Alternating voltage may be generated by

* rotating a coil in the magnetic field or
*rotating a magnetic field within a stationary coil.

The value of the voltage generated depends on :
- The number of turns in the coil.
- Strength of the field.
- The speed at which the coil or magnetic field rotates.

1.2.1 Faraday's Law and Lenz's law in generating AC current

Faraday's Law: Any change in the magnetic environment of a coil of wire will cause a voltage (emf) to be induced in the coil.

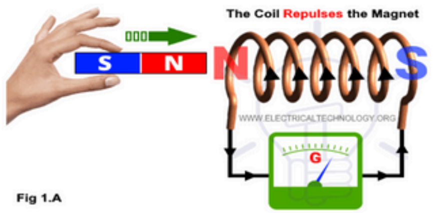
Faraday's law of electromagnetic induction is a basic principle of physics that explains how a changing magnetic field can generate an electric current.

The law was first discovered by English scientist Michael Faraday in 1831.
Faraday's law states that the magnitude of the induced electric current is proportional to the rate of change of the magnetic flux through the area.

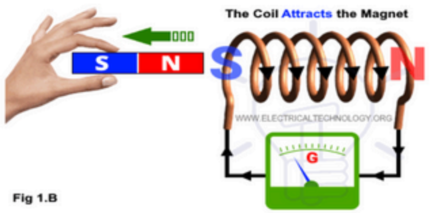
The greater the change in magnetic flux, the greater the induced current. The direction of the current is determined by the direction of the change in flux.

LENZ'S LAW

An induced Current always flows in a direction such that it opposes the change which produced it.



When the "N" Pole of the magnet is moved towards the coil, end of the coil becomes "N" Pole.



When the "N" Poles of the magnet is moved away from the coil, end of the coil becomes "S" Pole.

The Lenz's law states that when an E.M.F is induced in a circuit, the current setup always opposes the motion, or change in current, which produces it. OR

An induced EMF will cause a current to flow in a close circuit in such a direction that its magnetic effect will oppose the change that produced it.

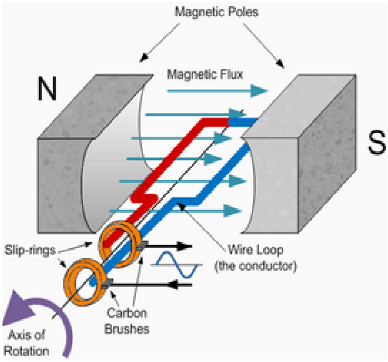
When both the bar magnet and coil is in static position, no current flowing or induced EMF (even the small amount of flux (N pole's of static magnet bar) linked to the coil movement), hence, no deflection in the galvanometer.

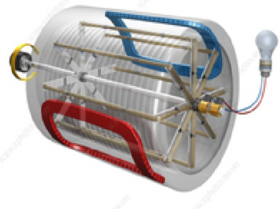
When the magnet bar moves quickly towards the coil, there is rapid deflection in the galvanometer for an instance.The deflection will remain constant until the continuous movement of the magnet bar with respect to the coil (i.e. relative moment between magnet bar and coil). If both the magnet bar and coil achieve the static position, the deflection of the galvanometer will be at Zero position (as shown in fig 1 A).

When the magnet bar is moved away from the coil, there will be again a deflection in the galvanometer until the relative movement between magnet bar and coil becomes at rest or static position. Keep in mind the direction of the galvanometer is in the opposite direction with the respect of fig 1A (as shown in fig 1 B).

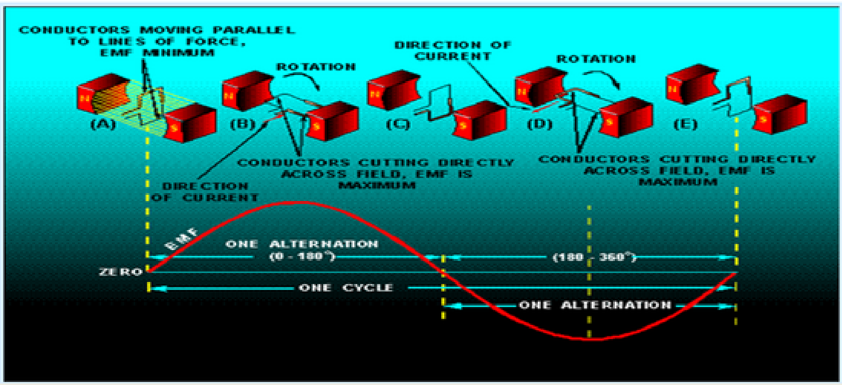
The same happens (Step 2 & 3) if the magnet bar is in a static position while the coil moves towards or moves away from the static magnet bar

BASIC SINGLE COIL AC GENERATOR

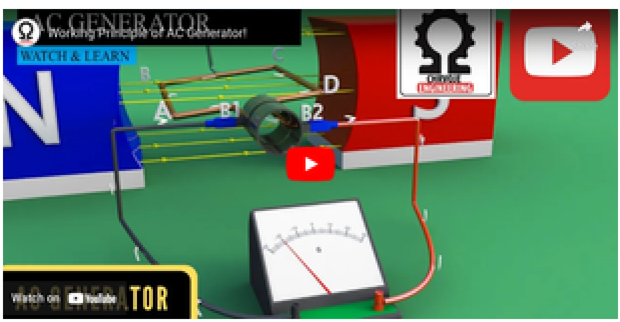




1.2.2 Explain the AC waveforms produced by a simple alternating current generator (one loop in 2-pole magnet)



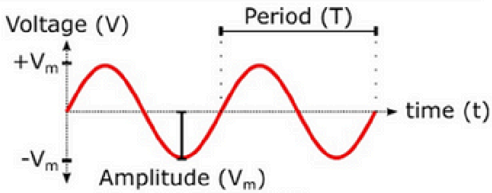
When the coil rotating across the magnetic field in a constant angular velocity, an sinusoidal e.m.f will be produced

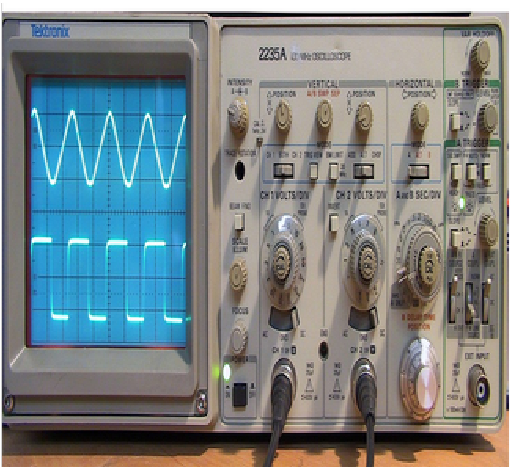


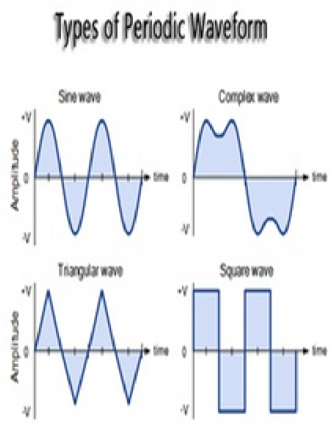
1.2.3 Express equation of a sinusoidal waveform
$e = Em \sin(\omega t + \theta)$

$$e = Em \sin(\omega t \pm \theta)$$

Em = is the peak voltage @ current. (unit: volt @ A) .
ω = is the angular frequency (unit: radians per second; rads)
The angular frequency is related to the physical frequency, (unit = hertz), which represents the number of cycles per second
by the equation .
t = is the time (unit: second).
ϴ = the phase, specifies where in its cycle the oscillation begin at t= o.



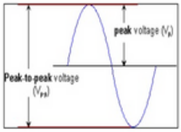




1.3 APPLY THE SINUSOIDAL VOLTAGE AND CURRENT OF A SINE WAVE

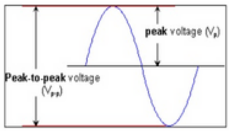
Peak Value / Amplitude

- Peak voltage is the voltage measured from the baseline of an AC waveform to its maximum or peak level.
- Symbol, V_p
- The peak value of a waveform is also called **amplitude**.



$$Amplitude\ or\ Peak\ Value = V_p$$

Peak To Peak Value



$$Peak\ to\ peak\ value = 2V_p @ 2E_m$$

Period

- The time required for completing one full cycle.
- The symbol for period is **T**.
- Period is measured in units of seconds, **s**

$$T = \frac{1}{f} = \frac{1}{\omega/2\pi} = \frac{2\pi}{\omega}$$

Frequency

- The number of cycles that is completed one second
- The symbol for frequency is ***f***
- Frequency is measured in units of cycles per second, or **Hertz, Hz**

$$f = \frac{1}{T}$$

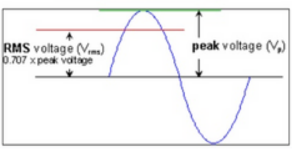
or

$$f = \frac{\omega}{2\pi}$$

rms Value

- RMS voltage is the amount of dc voltage that is required for producing the same amount of power as the ac waveform. For a sine wave:

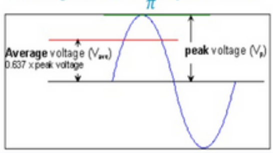
$$rms\ value = \frac{1}{\sqrt{2}} \times peak\ value$$
$$= 0.707\ peak\ value$$



Average value

- The average or mean value of a symmetrical alternating quantity, (such as a sine wave), is the average value measured over a half cycle, (since over a complete cycle the average value is zero).
- For a sine wave:

$$average\ value = \frac{2}{\pi} \times peak\ value$$



Form factor

$$form\ factor = \frac{rms\ value}{average\ value}$$

- For a sine wave,

$$form\ factor = \frac{rms\ value}{average\ value} = \frac{0.707}{0.637} = 1.11$$

Peak factor

$$peak\ factor = \frac{peak\ value}{rms\ value} = \frac{peak\ value}{0.707\ peak\ value}$$

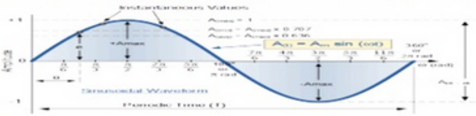
- For a sine wave,

$$peak\ factor = \frac{peak\ value}{rms\ value}$$

$$= \frac{peak\ value}{0.707\ peak\ value} = \frac{1}{0.707} = 1.414$$

Instantaneous value

- Instantaneous values are the values of the alternating quantities at any instant of time. They are represented by small letters, *i, v, e,* etc.



$$a = A_m \sin(\omega t \pm \theta)$$

Where

- a = amplitude (v, i, e, e.m.f)
- A_m = maximum amplitude @ peak amplitude
- ω = angular velocity
- θ = phase angle

SUMMARY

	FORMULA	UNIT
Frequency	$f = \frac{1}{T}$	Hz
Period	$T = \frac{1}{f}$	Sec
Amplitude	Vp @ Ip	Volt @ A
Peak to Peak value	2 x Vp @ 2 x Ip	Volt @ A
RMS value	0.707 x Vp @ 0.707 x Ip	Volt @ A
Average value	0.637 x Vp @ 0.637 x Ip	Volt @ A
Form Factor	$\frac{\text{RMS value}}{\text{Average value}} = 1.11$	-
Peak Factor	$\frac{\text{Peak value}}{\text{RMS value}} = 1.414$	-

EXAMPLE 1

» An alternating voltage is given by v = 282.8 sin 314t v. Find;

a) The r.m.s voltage

b) The frequency

c) The instantaneous value of voltage when t = 4ms

Solution:

a) Vrms = 0.707 x Vp = 0.707 x 282.8V = **200V**

b) ω = 314 rad/s = 2πf

f = 314 / 2π = **50Hz**

a) v = 282.8 sin (314 x 4ms) = 282.8 sin 1.256

= 282.8 sin 71.96° = **268.9V**

EXAMPLE 2

» An alternating voltage is given by v = 310 sin 100πt + 30°. Determine;

a) The amplitude

b) The root mean square voltage

c) The average voltage

d) The instantaneous value of voltage when t = 5ms

e) The time when the voltage first reach maximum value

Solution:

a) Amplitude = Vp = 310V

b) Vrms = 0.707 x Vp = 0.707 x 310V = 219.17 V

c) Vavg = 0.637 x Vp = 0.637 x 310V = 197.47 V

d) V = 310 sin [100 π x 5ms] + 30°

= 310 sin [1.571 + 30°]

= 310 sin [90° + 30°]

= 268.47 V

e) 310 = 310 sin [100πt + 30°]

310 / 310 = sin [100πt + 30°]

1 = sin [100πt + 30°]

$\sin^{-1} 1$ = 100πt + 30°

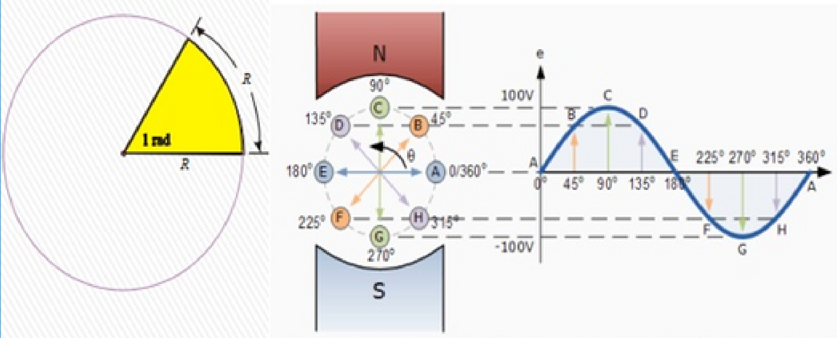
90 - 30° = 100πt

60° = 100πt

1.047 rad = 100πt

so, t = 1.047 rad / 100π = 3.33ms

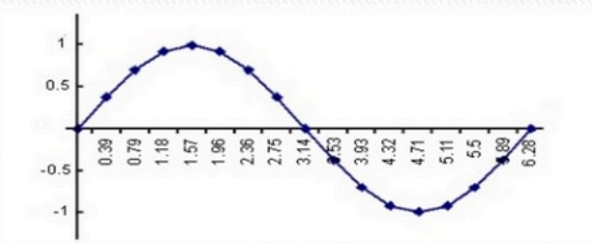
1.4.1 Show how to measure a sine wave in term of angle



- As angle A increases, the values of the trigonometric functions of A undergo a periodic cycle from 0, to a maximum of 1, down to a minimum of -1, and back to 0.

- If we now plot the sine of the angle measured in radians along the Cartesian coordinate system, we see that we again get the characteristic rise and fall.

- If you look closely at this graph you will see that the wave crosses the x-axis at multiples of 3.1416... – the value of pi. One full wave is completed at the value 6.2832..., or 2π, exactly the circumference of the unit circle.



1.4.2 Calculate the phase angle of a sine wave in
a. degree
b. radians

- There are 2 radians in one complete revolution and 360° in a revolution, the conversion between radians and degrees is given by:

$$\text{Radians} = \left(\frac{\pi}{180^0}\right) \times \text{degrees}$$

$$\text{Degrees} = \left(\frac{180^0}{\pi}\right) \times \text{radians}$$

EXERCISE

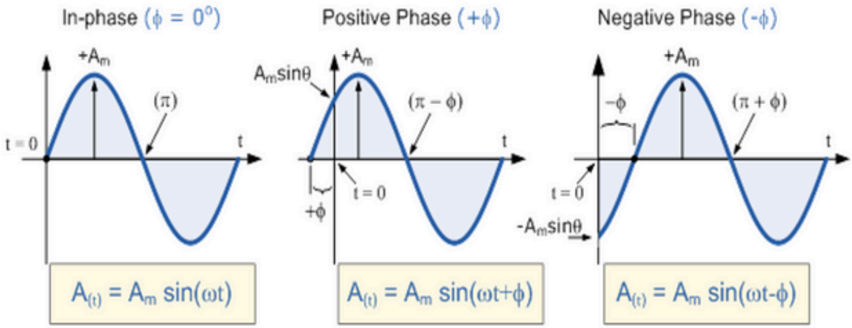
(a) Convert the angle of 5.32 radian to degree unit

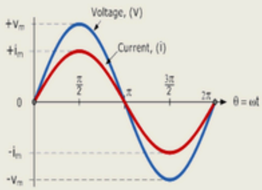
(b) Convert the angle of 314⁰ to radian unit

3 mins Review

Give solution to the above exercises

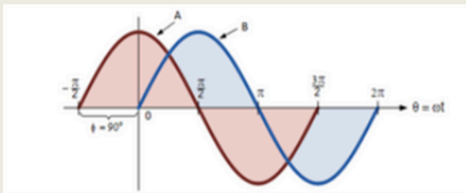
1.5.1 Show how phasors are related to the sine wave formula





Two waveforms are said to be in phase when they have the same frequency

There is no phase difference between them.

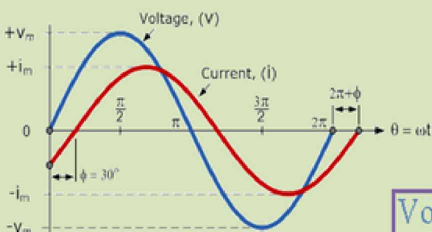


A leading waveform is **one that is ahead of a reference waveform** of the same frequency.

A is ahead B
:- A **"LEADS"** B

A lagging waveform is **one that is behind a reference waveform** of the same frequency

B is behind A
:- B **"LAGS"** A



The current, i is lagging the voltage, v by angle θ and in our example is 30°

$$\text{Voltage}, (v_t) = V_m \sin\omega t$$

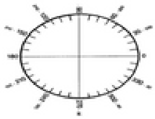
$$\text{Current}, (i_t) = I_m \sin(\omega t - \theta)$$

Q1 : What is the phase relationship between the sinusoidal waveforms of each of the following sets?

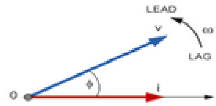
a. $V = 10\sin(\omega t + 30°)$
$I = 5\sin(\omega t + 70°)$

b. $I = 15\sin(\omega t + 60°)$
$v = 10\sin(\omega t - 20°)$

1.5.2 Draw a phasor diagram



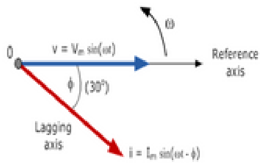
Phasor" is a scaled line whose length represents an AC quantity that has both magnitude ("peak amplitude") and direction ("phase") which is "frozen" at some point in time

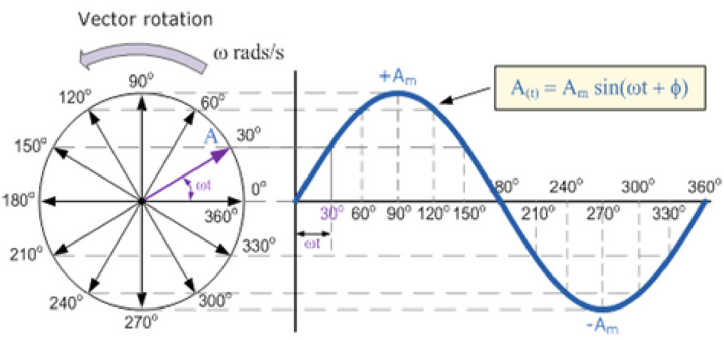


A phasor is a vector that has an arrow head at one end which signifies partly the maximum value of the vector quantity (V or I) and partly the end of the vector that rotates

In the phasor diagram, everything is plotted on a coordinate system.

Phasors are defined relative to the 'reference phasor' which is always chosen to point to the right.





Rotating Phasor

Sinusoidal Waveform in the Time Domain

**The phasor diagram is drawn corresponding to time zero (t = o) on the horizontal axis

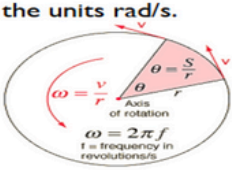
**The lengths of the phasors are proportional to the values of the voltage, (V) and the current, (I) at the instant in time that the phasor diagram is drawn.

**The current phasor lags the voltage phasor by the angle, Φ, as the two phasors rotate in an *anticlockwise* direction as stated earlier, therefore the angle, Φ is also measured in the same anticlockwise direction.

1.5.3 Calculate Angular Velocity

Angular velocity

- Angular velocity can be considered to be a **vector quantity**, with direction along the axis of rotation in the right-hand rule sense.
- For an object rotating about an axis, every point on the object has the same angular velocity.
- The tangential velocity of any point is proportional to its distance from the axis of rotation. Angular velocity has the units rad/s.



$$v = \omega r \;\; or \;\; \omega = \frac{v}{r}$$

EXAMPLE 1

A sinusoidal waveform is defined as $v = 169.8\sin(377t)\ volts$. Calculate the RMS voltage of the waveform, its frequency and the instantaneous value of the voltage after a time of 6ms.

SOLUTION EXAMPLE 1

Generally

$$a = A_m\ sin(\omega t \pm \theta)$$

Then comparing this to our given expression for a sinusoidal waveform above of $v = 169.8\sin(377t)\ volts$;

$$A_m = V_p = 169.8V$$
$$\omega = 377$$

RMS voltage;

$$V_{rms} = 0.707 \times V_p = 0.707\ \times 169.8 = 120\ \text{V}$$

Frequency;

$$Frequency, f = \frac{\omega}{2\pi} = \frac{377}{2\pi} = 60Hz$$

SOLUTION EXAMPLE 1(CONT)

The instantaneous value of the voltage after a time of 6ms

Method 1: **Using angle in radian for calculation.**

Convert calculator mode to rad (radians)

$$v = 169.8\sin(377t)$$
$$v = 169.8\sin(377 \times 6m)$$
$$v = 169.8\sin(2.262)$$
$$v = 130.83\ V$$

The instantaneous value of the voltage after a time of 6ms

Method 2: **Using angle in degree for calculation.**

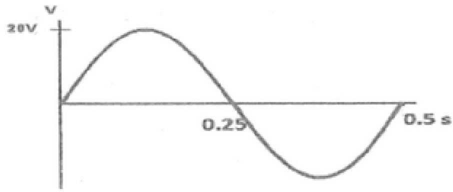
Convert $\omega = 377$ to degree

$$\omega = \frac{180°}{\pi} \times 377 = 21600°$$

$$v = 169.8\sin(21600t)$$
$$v = 169.8\sin(21600 \times 6m)$$
$$v = 169.8\sin(129.61)$$
$$v = 130.83\ V$$

EXAMPLE 2

Based on figure, write the sinusoidal waveform equation.



SOLUTION EXAMPLE 2

The sinusoidal waveform equation

Generally

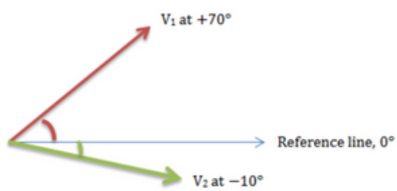
$$a = A_m \, sin(\omega t \pm \theta)$$
$$A_m = peak\ value = V_p = 20$$
$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.5} = 12..57$$
$$\theta = 0°$$
$$\therefore v = 20\, sin(12.57t)$$

EXAMPLE 3

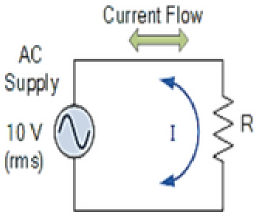
Given $V_1 = 15\sin(\omega t + 70°)$ and $V_2 = 20\sin(\omega t - 10°)$. By using phasor diagram, state which sinusoid is leading and the phase difference.

SOLUTION EXAMPLE 3

- V_1 leads V_2 by 80^0



1.6.1 Apply Ohm's Law to resistive circuits with AC sources



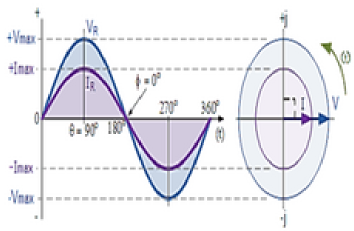
Resistors are "passive" device and do not produce or consume any electrical energy, but convert electrical energy into heat.

In **DC** circuits the linear ratio of voltage to current in a resistor is called its resistance.

However, in **AC** circuits this ratio of voltage to current depends upon the frequency and phase difference or phase angle (Φ) of the supply.

So when using resistors in **AC** circuits the term Impedance, symbol **Z** is the generally used and we can say that **DC** resistance = **AC** impedance, R = Z.

V-I Phase Relationship and Vector Diagram



For resistors in AC circuits the direction of the current flowing through them **has no effect on the behavior of the resistor** so will rise and fall as the voltage rises and falls.

The current and voltage reach maximum, fall through zero and reach minimum at exactly the same time. - "in-phase"

At any point along the horizontal axis that the instantaneous voltage and current are in-phase because the current and the voltage reach their maximum values at the same time, that is their phase angle θ is 0º.

- The instantaneous voltage across the resistor, V_R is equal to the supply voltage, V_t and is given as:

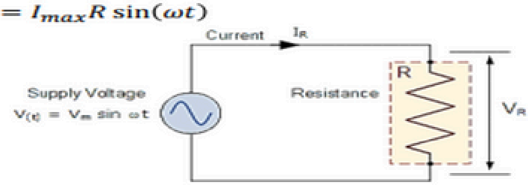
$$V_R = V_{max} \sin(\omega t)$$

- The instantaneous current flowing in the resistor will therefore be:

$$I_R = \frac{V_R}{R} = \left(\frac{V_{max}}{R}\right) \sin(\omega t) = I_{max} \sin(\omega t)$$

- As the voltage across a resistor is given as V_R = I.R, the instantaneous voltage across the resistor above can also be given as:

$$V_R = I_{max} R \sin(\omega t)$$

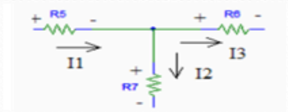


1.6.2 Apply Kirchhoff's Voltage Law and Current Law to resistive circuits with AC sources

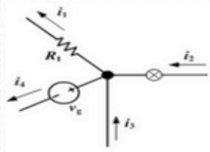
» Kirchhoff's Voltage and Current Laws apply to all AC circuits as well as DC circuits.

Kirchhoff's Current Law:

- The sum of current into a junction equals the sum of current out of the junction.
- $i_2 + i_3 = i_1 + i_4$
- The sum of all currents at a node must equal to zero.



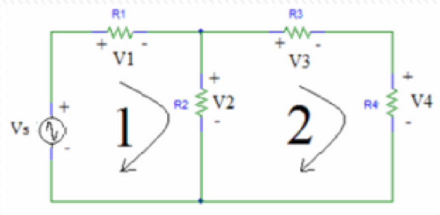
$$I_1 - I_2 - I_3 = 0 .$$



A diagram of Kirchhoff's Current Law.

Kirchhoff's Voltage Law:

- The algebraic sum of the voltage (potential) differences in any loop must equal zero.
- *Example:*



V1 + V2 – Vs = 0

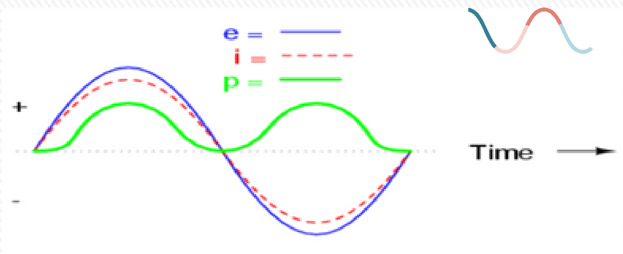
1.6.3 Apply Kirchhoff's Voltage Law and Current Law to resistive circuits with AC sources

- In a direct current circuit the power is equal to the voltage times the current, or P = E X I.
- The **TRUE POWER** depends upon the phase angle between the current and voltage.
- True power of a circuit is the power actually used in the circuit.
- Measured in watts.

P = true power $\qquad P = I^2R \qquad P = \frac{E^2}{R}$

Measured in units of ***Watts***

- Note that the waveform for power is always positive, never negative for this resistive circuit.
- This means that power is always being dissipated by the resistive load, and never returned to the source as it is with reactive loads.



Current is in phase with voltage in a resistive circuit.

END OF CHAPTER

01 List TWO(2) methods of generating alternating current

ANSWER

- Permanent Magnet Moving Coil (PMMC)
- Permanent Coil Moving Magnet(PCMM))

02 Explain THREE (3) reasons why AC is preferred over DC.

ANSWER

- i.Power plants use rotary turbines as their source of output, which are by nature AC and don't need power electronics to convert to DC.
- ii. Changing the voltage of AC electricity for distribution and transmission is far simpler.
- iii. Compared to DC transmission, the cost of plant related to AC transmission (transformers, circuit breakers, etc.) is substantially lower.
- iv. There are a few technical benefits to AC transmission. Large fault currents happen when there is a network fault.
- v. Metering AC connections and keeping an eye on power flows over a network is also simpler.

***Choose any 3 answers*

03

Explain TWO (2) laws related to generating alternating current.
[4 marks]

ANSWER

- Faraday's Law states that, a coil of wire will produce a voltage (emf) in response to any changes in its magnetic surroundings.
- Lenz's Law states that the current configuration in a circuit always opposes the motion, or change in current, that induces an E.M.F.

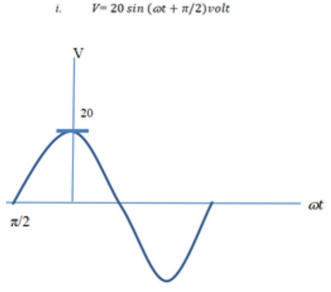
04

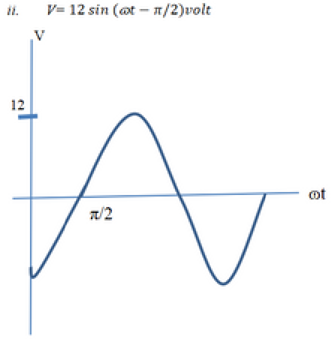
Convert from AC voltage equation to AC voltage waveform for:

i. $V = 20\ sin\ (\omega t + \pi/2) volt$

ii. $V = 12\ sin\ (\omega t - \pi/2) volt$

ANSWER





05 (a) An Alternating Voltage equation is given by V . Calculate the value of amplitude, frequency, phase angle in degree and the the voltage when t=5.5ms

ANSWER

The amplitude, Vp = 215V

The frequency $\omega = 100\pi \, rad/s$

$$2\pi f = 100\,\pi$$

$$f = \frac{100\pi}{2\pi} = 50\text{Hz}$$

phase angle in degree

$$0.25x\frac{360}{2\pi} = 14.32°$$

The value of voltage when t=5.5ms

$$V = 215\, sin\,(100\pi t\ + 0.25)\text{A}$$

$$V = 215\, sin\,(100\pi(5.5m) + 0.25)\text{A}$$

$$= 215\,\sin\,(99° + 14.32°)$$

$$= 215\,\sin\,(113.32°)$$

$$= 197.43v$$

EXERCISE

1

Determine the rms voltage of the 14V peak to peak voltage.

Answer : 4.95V

2

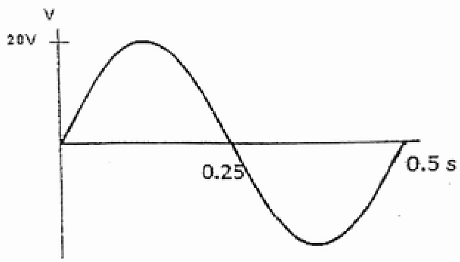


Figure 2a

Referring to figure 2a,

i) Give the defination of time period, T of a sinusoidal waveform

ii) Find the value of time period

iii) State the peak voltage

iv) Write the sinusoidal equation

Answer : T= 0.5s, Vp = 20V, V= 20 sin 12.57t

3

An alternating voltage is given by V(t) = 282.8 sin314t Volt. Find:

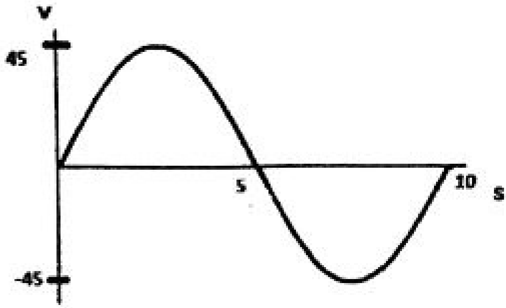
i) Average Voltage

ii) Frequency

iii) The instantaneous voltage value when t= 4ms

Answer : Vavg = 180.14V, f= 49.97 Hz, V(t= 4ms) = 268.9V

4



Based on the above figure calculate peak to peak voltage, root mean square voltage,average voltage and write the sinusoidal equation

Answer : Vpp= 90V, Vrms = 31.815V, Vavg = 28.66V,
V= 45 sin0.628t

5

The current in AC circuit at any time t seconds is given by $i = 20sin(60\pi t + 0.38)A$. Calculate

i. The period time and frequency

ii. The value of current when t= 0

Answer: T= 0.033s, f= 30Hz, i= 7.42A

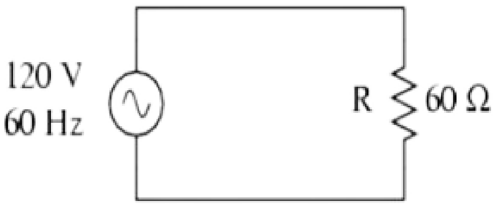
6

An alternating voltage is given by V= 15 sin (314t - 0.52)mA. Determine

i) The amplitude
ii) The peak to peak value
iii) The periodic time
iv) The phase angle in degree

Answer : Vp= 15mV, Vpp= 30mV, T= 20ms, 29.8

7



Based on the figure 7a, find:

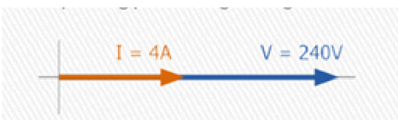
i) The total current in the circuit
ii) Voltage drop across R
iii) True power of the circuit

Answer : I= 2A, V= 120, P= 240W

8

Calculate the current and power consumed in a single phase 240V AC circuit by a heating element which has an impedance of 60 ohm. Also draw the corresponding phasor diagram

Answer: I= 4A, P = 960 W

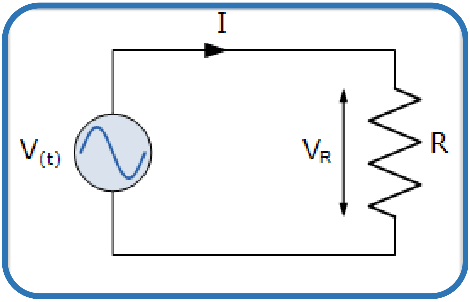


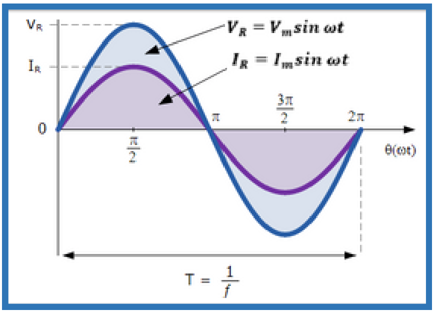
TOPIC 2

SINUSOIDAL STEADY-STATE CIRCUIT ANALYSIS

Purely Resistive Circuit with AC Source

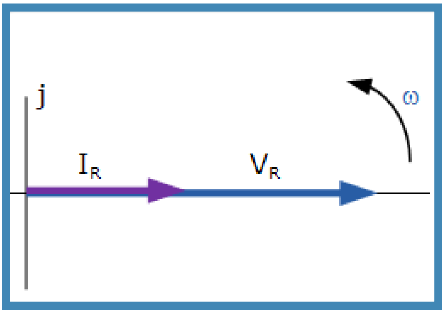
In a purely resistive AC circuit, the voltage and current are in phase, meaning the phase angle difference between them is zero. This can be understood by examining the relationship between voltage and current in a resistor and the behaviour of sinusoidal waveforms, as shown in Figure below.





Sinusoidal waveform showing V_R is in phase with I_R:

$$V_R = V_m sin\ \omega t$$
$$I_R = I_m sin\ \omega t$$



Phasor diagram: V_R is in phase with I_R. (Phase angle = 0º)

$$V_R = V_m \angle 0°$$
$$I_R = I_m \angle 0°$$

- In the resistive circuit, current, voltage, and voltage source are the sine waves.
- The phase angle is 0 degrees.
- The frequency does not affect the Resistance, R.
- Whether the frequency rises or falls, R never changes.
- Ohm's law is used to solve resistors in AC circuits:

$$V_R = I \times R$$

Example 1

A circuit with 50 Ohms resistance is connected across an AC supply with a 230 Vrms voltage. Calculate the total current and the voltage drop across the resistance. Draw the corresponding phasor diagram, showing the voltage and current phase relationship.

Solution

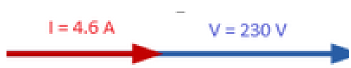
i. Total current:

$$I = \frac{V}{R} = \frac{230}{50} = 4.6\ A$$

ii. Voltage drop across the resistance:

$$V_R = I \times R = 4.6 \times 50 = 230\ V$$

iii. Since a resistive component (θ = 0) has no phase difference, the appropriate phasor diagram is as follows:



Example 2

V(t) = 100sin(ωt + 30) is the equation for a sinusoidal voltage source that is connected to a 50 Ohms resistance. Calculate the peak current and draw the phasor diagram.

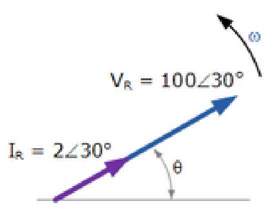
Solution

$$V_{R(t)} = 100sin(\omega t + 30°) \longrightarrow V_R = 100\angle 30°\ V$$

By using Ohm's Law

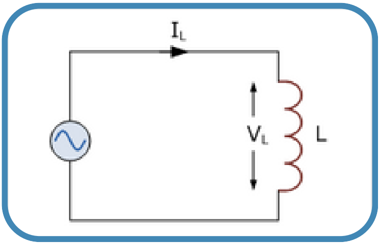
$$I_R = \frac{V_R}{R} = \frac{100\angle 30°}{50} = 2\angle 30°\ A$$

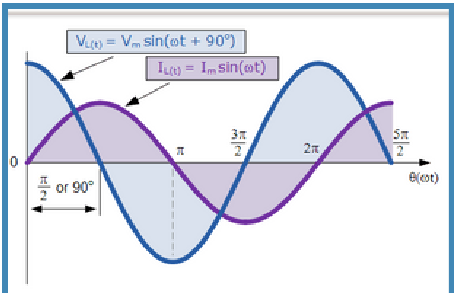
The phasor diagram:



Pure Inductive Circuit with AC Source

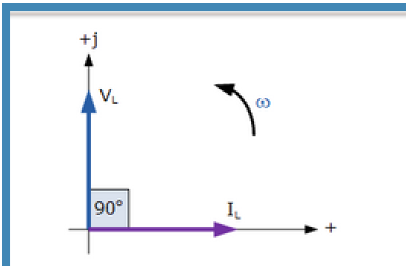
The current fluctuates against an inductor. An AC voltage placed across an inductor causes it to produce a reverse electromotive force, or EMF, which opposes the change in current.





Sinusoidal waveform showing V_L lead I_L by 90º or $\frac{\pi}{2}$:

$$V_L = V_m sin(\omega t + 90°)$$
$$I_L = I_m sin\ \omega t$$



Phasor diagram: Voltage V_L lead current I_L by a phase angle of 90º (Phase angle difference=90º)

$$V_L = V_m\angle 90°$$
$$I_L = I_m\angle 0°$$

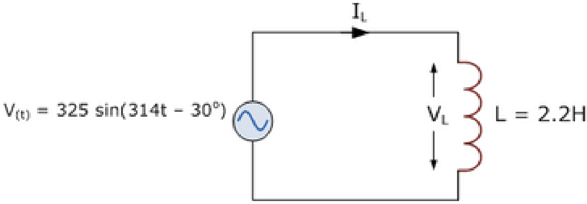
Inductive Reactance

- Inductive reactance, represented by the symbol XL, is the resistance of an inductive circuit with unit ohm.
- The inductive reactance XL rises in proportion to the frequency across it.
- As a result, inductive reactance and frequency are directly proportional, and the formula yields XL. $X_L = 2\pi f L$
- Since ω = 2πf, then inductive reactance will be XL = ωL.
- The relationship between an inductor's voltage and current is expressed using a formula similar to Ohm's law:

$$V_L = I_L \times X_L$$

Example 1

By referring to circuit below, V(t) = 325sin(314t – 30) and L = 2.2H. Draw the phasor diagram and calculate the rms current at the coil.



Solution

The supply voltage will give the coil's rms voltage. If the peak voltage is 325V, and the equivalent rms value is 230V. We get VL=230<-30 V.

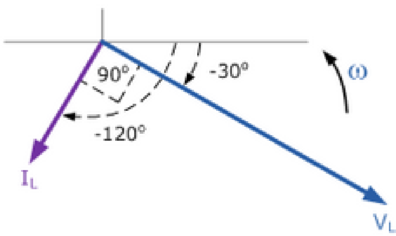
Inductive reactance:

$$X_L = \omega L = 314 \times 2.2 = 690\Omega.$$

The current IL:

$$I_L = \frac{V_L}{jX_L} = \frac{230\angle -30^\circ}{690\angle 90^\circ} = 0.33\ A\angle -120^\circ$$

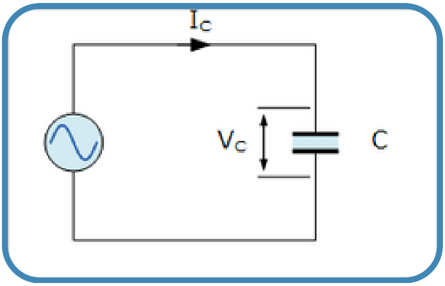
Phasor diagram

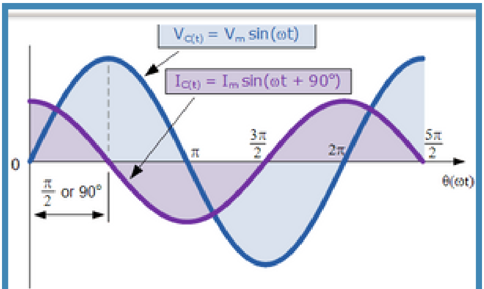


Pure Capacitive Circuit with AC Source

In a purely capacitive AC circuit, the current leads the voltage by 90 degrees (π/2 radians). This phase relationship and the concept of capacitive reactance can be understood by examining the behavior of a capacitor in an AC circuit and the mathematical relationships that describe this behavior.

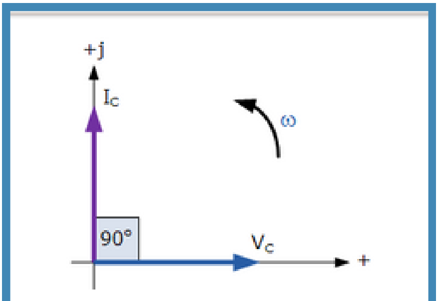
A capacitor stores energy in an electric field created by a voltage across its plates. When an AC voltage is applied, the voltage across the capacitor changes over time, causing a time-varying current to flow as the capacitor charges and discharges.





Sinusoidal waveform showing Ic lead Vc by 90 º or $\frac{\pi}{2}$:

$$I_C = I_m sin(\omega t + 90°)$$
$$V_C = V_m sin\ \omega t$$



Phasor diagram: Ic leads V_C by 90º or $\frac{\pi}{2}$:

(Phase angle = 90º)

$$I_C = I_m \angle 90°$$
$$V_C = V_m \angle 0°$$

Capacitive Reactance

- In circuits that are exclusively capacitive, capacitive reactance inhibits the passage of current.
- Capacitive reactance is denoted by the letter XC. We use the ohm as its unit.
- As the frequency across it rises, the capacitor's XC decreases.
- Thus, the frequency and capacitive reactance have an inverse relationship.
- The formula yields XC as follows:

XC = 1/2πfC Ω

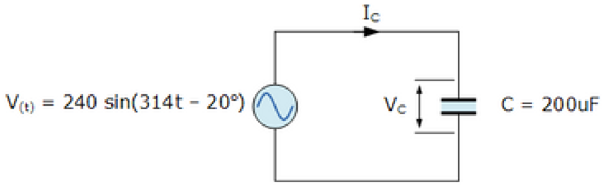
Given that ω (the angular frequency) equals 2πf,

- The equation that relates a capacitor's voltage and current is comparable to Ohm's law:

Vc= IC x XC

Example 1

An ac supply voltage V(t) = 240 sin(314t – 20) is connected in series with a 200uF capacitance. Calculate the current following the capacitor and draw a phasor diagram.



Solution

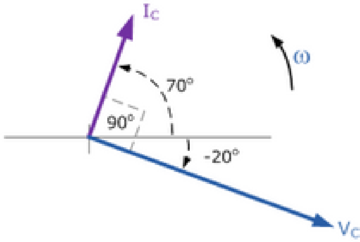
VC = 240 ∠-20 V.

Reactance capacitance:

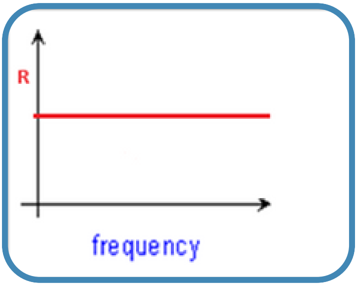
$$X_C = \frac{1}{\omega C} = \frac{1}{314(200\mu)} = 16\angle -90°\Omega$$

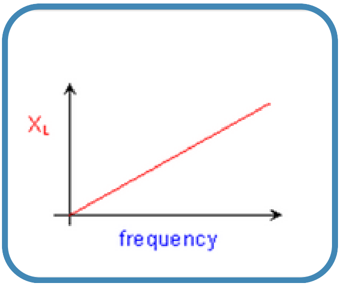
$$I_C = \frac{V_C}{jX_C} = \frac{240\angle -20°}{16\angle -90°} = 15\angle 70°\text{A}$$

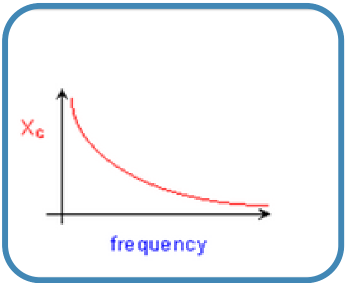
Phasor diagram:



A graph illustrating the frequency-related relationship between R, XL, and XC







List of differences between resistance and impedance

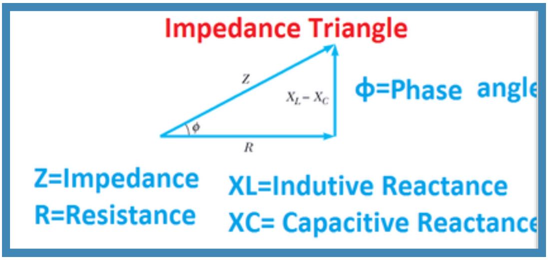
Aspect	Resistance	Impedance
Nature of Opposition	Opposition to direct current (DC).	Opposition to alternating current (AC).
Components	Only resistance (R).	Combination of resistance (R) and reactance (X).
Frequency Dependence	Independent of frequency.	Dependent on frequency.
Reactance	No reactance.	Includes reactance (capacitive X_C and inductive X_L).
Phase Relationship	Voltage and current are in phase.	Voltage and current can be out of phase.
Mathematical Representation	Represented as R.	Represented as a complex number Z=R+jX.
Power Dissipation	$P= I^2R$	$P= I^2Z\cos\theta$ or $P=IV\cos\theta$
Calculation	R=V/I for DC circuits.	Z=V/I for AC circuits (complex quantities).

Impedance Triangle

Z is the impedance in AC circuits.

$$Impedance = \frac{Voltage}{current}$$

$$Z = \frac{V}{I}$$



In general, the impedance triangle can be used to calculate the impedance, Z.

If XL > XC, the impedance become:

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

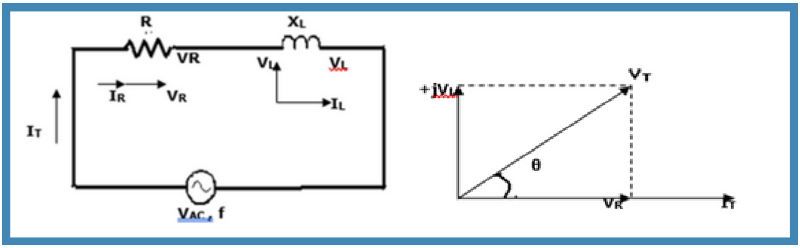
If XC > XL, the impedance become:

$$Z = \sqrt{R^2 + (X_C - X_L)^2}$$

Inductive and capacitive load in a series circuit

In series circuit, current is used as a reference.

R-L series

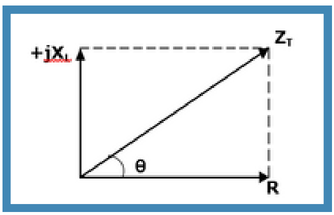


- The current and all of the voltage drops in an RL circuit are sinusoidal waves.
- The source voltage (VT) in an RL circuit always leads to the total current (IT).
- VR and IT are in phase or zero phase angle.
- VL leads IT by 90°.
- The impedance of an RL circuit is the total resistance to the flow of AC current.
- An RL circuit's total impedance (ZT) can be calculated using this equation:

$$Z = \sqrt{X_L^2 + R^2}$$

or in complex form:

$$Z = R + jXL$$



The total voltage:

$$V_T = \sqrt{V_R^2 + V_L^2}$$

or in complex form:

$$V_T = V_R + jV_L$$

Where:
VT = total voltage
VR = voltage across resistor R
VL = voltage across inductor L

The total phase angle:

$$\theta = \tan^{-1}(\frac{X_L}{R})$$ or $$\theta = \tan^{-1}(\frac{V_L}{V_R})$$

RL Series Circuit Analysis:

1) Inductive reactance: $X_L = 2\pi f L$

2) Total impedance: $Z = \sqrt{R^2 + X_L^2}$

or in complex form, $Z = R + jX_L$

3) The total current:

$$I_T = \frac{V_T}{Z_T}$$

4) $I_T = I_L = I_R$ (current is the same in a series circuit)

5) The voltages across R and L:

$V_R = I_T \times R$ and $V_L = I_T \times X_L$

6) The phase angles for R and L.

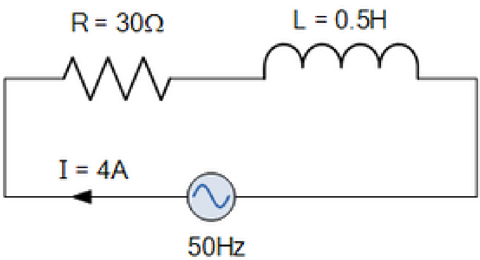
$\theta_R = 0^\circ$ $\theta_L = 90^\circ$

7) The total phase angle for the circuit:

$$\theta_T = \tan^{-1}(\frac{X_L}{R})$$

Example

A coil has a resistance of 30Ω and an inductance of 0.5H. What will be the supply's rms value if the coil is receiving 4 amps of current and the supply voltage has a frequency of 50 Hz?



Solution

The circuit's impedance will be:

$$X_L = 2\pi f L = 2\pi(50)(0.5) = 157$$

$$Z = R + jX_L$$

$$Z = 30 + j157$$

$$Z = 159.84\angle 79.18°\ \Omega$$

Next, each component's voltage drop across is computed as follows:

$$V_R = I(R) = 4(30) = 120\ V$$

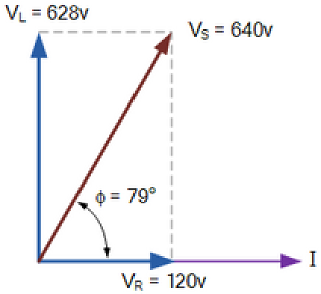
$$V_L = I(X_L) = 4(j157) = j628\ V$$

$$V_T = V_R + jV_L = 120 + j628 = 639.36\angle 79.18°\ V$$

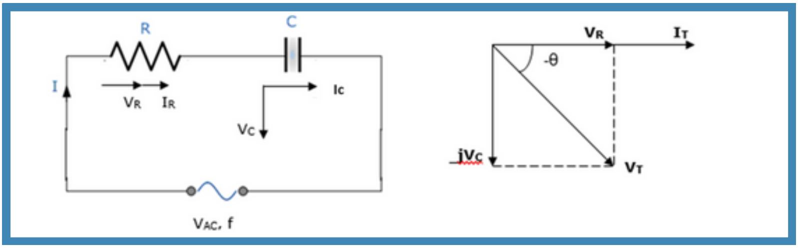
or

$$V_S = I(Z) = 4(159.84\angle 79.18°) = 639.36\angle 79.18°\ \text{V}$$

There will be a phasor diagram. .



R-C Series Circuit



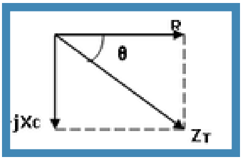
- In an RC circuit, IT leads VT by 90° or VC lags the IT by 90°.
- The VR is in phase with the IT.
- The impedance (Z) of an RC circuit is the total opposition to AC current flow.
- The equation for the total impedance (ZT) of an RC circuit is:

$$Z = \sqrt{R^2 + X_C^2}$$

or

complex form:

$$Z = R - jX_C$$



- The total voltage:

$$V_T = \sqrt{V_R^2 + V_C^2}$$

or

complex form:

$$V_T = V_R - jV_C$$

Where:
VT = total voltage
VR = voltage across resistor R
Vc = voltage across capacitor C

The total phase angle:

$$\theta = -tan^{-1}(\frac{X_C}{R})$$

or

$$\theta = -tan^{-1}\frac{V_C}{V_R}$$

RC Series Circuit Analysis:

1) Capacitive Reactance, Xc: $X_C = \frac{1}{2\pi f C}\Omega$

2) Impedance:

$Z = \sqrt{R^2 + X_C^2}$ or in complex form Z= R – jXc

3) The circuit's current:

$$I_T = \frac{V_T}{Z_T}$$

4) The currents at R and C:

$$I_T = I_C = I_R$$

5) The voltages across R and C:

$V_R = I_T \times R$ and $V_C = I_T \times X_C$

6) The phase angles for R and C.

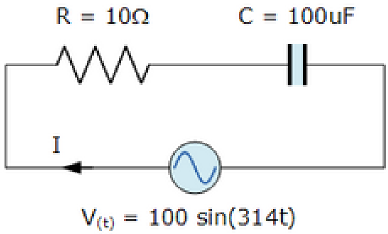
$\theta_R = 0^{\circ}$ and $\theta_C = -90^{\circ}$

7) The total phase angle for the RC circuit:

$\theta_T = -tan^{-1}(\frac{X_C}{R})$ or $\theta = -tan^{-1}\frac{V_C}{V_R}$

Example

V(t) = 100 sin (314t) is the source voltage connected to a 100uF capacitor with 10Ω an internal resistance. Calculate the maximum current flowing into the capacitor at any instantaneous time. Draw a voltage triangle as well, showing each individual voltage drop.



Solution

The circuit impedance and reactance capacitance are computed as follows:

$$I = \frac{V_C}{Z} = \frac{100\angle 0°}{33.38\angle -72.57°} = 3\angle 72.57° A$$

$$X_C = \frac{1}{\omega C} = \frac{1}{314(100\mu)} = 31.85\angle -90° \Omega$$

$$Z = R - jX_C = 10 - j31.85 = 33.38\angle -72.57° \Omega$$

$$Z = \sqrt{R^2 + X_C^2} = \sqrt{10^2 + 31.85^2} = 33.4\Omega$$

Next, the maximum current following the circuit:

$$I = \frac{V_C}{Z} = \frac{100\angle 0°}{33.38\angle -72.57°} = 3\angle 72.57° A$$

$$I = \frac{V_C}{Z} = \frac{100\angle 0°}{33.38\angle -72.57°} = 3\angle 72.57° A$$

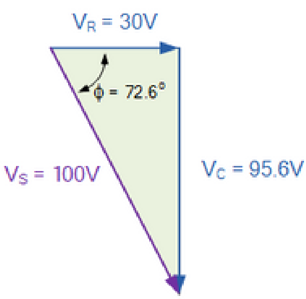
Next, the circuit's individual voltage drops are computed as follows:

$$V_R = I(R) = 3(10) = 30\ V$$

$$V_C = I(X_C) = 3(31.85\angle -90°) = 95.55\angle -90°\ V$$

$$V_T = V_R - jV_C = 30 - j95.55 = 100\angle -72.57°\ V$$

The voltage triangle that results for the computed peak values is then as follows:



RLC Series Circuit

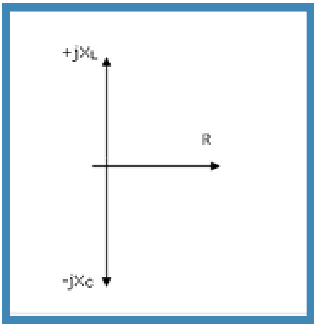
The phase relationships for RLC will change significantly if an AC supply with a sinusoidal waveform is connected to them. The voltage and current are "in-phase" when using a resistor. In an inductance, the voltage "leads" the current by 90°(ELI), while in a capacitance, the voltage "lags" the current by 90° (ICE).

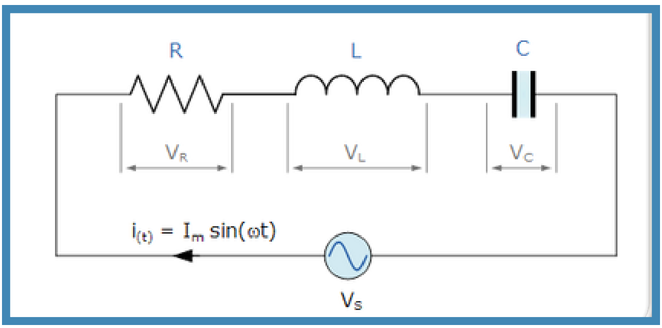
The phase difference depends on how reactive the components are. It is well known that reactance (X) is negative in capacitive elements, positive in inductive components, and zero in resistive elements. The following impedance values can be obtained from these values:

$$Z_R = R = R\angle 0^{\circ}$$

$$Z_L = j\omega L = \omega L\angle 90^{\circ}$$

$$Z_C = \frac{1}{j\omega C} = \frac{1}{\omega C}\angle -90^{\circ}$$





RLC Series Circuit Analysis:

1) Find XC and XL :

$$XC = 1 / (2\pi fC) \text{ and } XL = 2\pi fL$$

2) Impedance:

$$ZT = R + jXL - jXC$$

3) Total current:

$$IT = VT / ZT$$

4) The currents through R, L and C. Since this is a series circuit:

$$IT = IC = IL = IR$$

5) The voltages across R, L and C:

$$VR = IT \times R, \quad VC = IT \times XC \text{ and } VL = IT \times XL$$

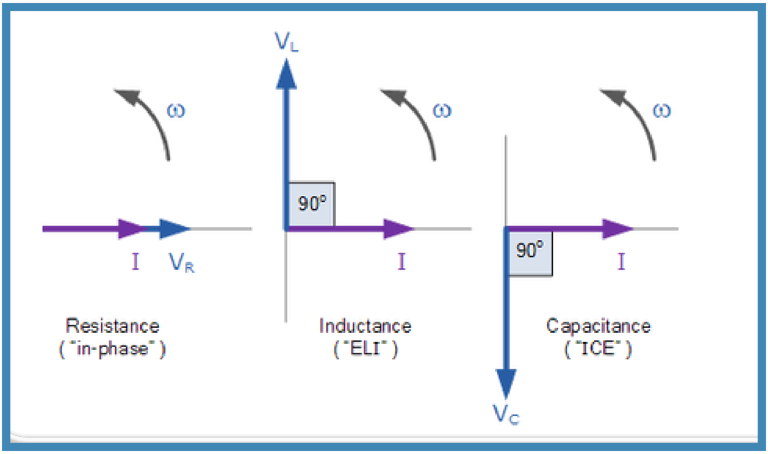
6) Show that the total voltage in a series circuit is equal to the sum of the vectors of voltages across R, L, and C's components.

$$VT = VR + jVL - jVC$$

7) The total phase angle for the R-L-C series circuit:

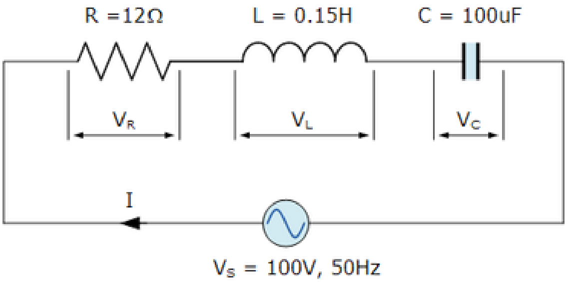
$$\theta T = -\tan^{-1}(XC - XL / R) \quad \text{if } XC > XL$$
$$\theta T = \tan^{-1}(XL - XC / R) \quad \text{if } XL > XC$$

The R-L-C series phasor



Example

An RLC series circuit with a 12 Ω resistance, 0.15H inductance, and a 100uF capacitor is connected in series across a 100V, 50Hz supply. Calculate the voltage phasor diagram, total impedance, and current in the circuit.



Solution

Calculate XL.

$$X_L = 2\pi f L = 2\pi \times 50 \times 0.15 = 47.13\Omega$$

Calculate XC.

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 100 \times 10^{-6}} = 31.83\Omega$$

Calculate, Z.

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$Z = \sqrt{12^2 + (47.13 - 31.83)^2}$$

$$Z = \sqrt{144 + 234} = 19.4\Omega$$

Total current, I.

$$I = \frac{V_S}{Z} = \frac{100}{19.4} = 5.14\text{Amps}$$

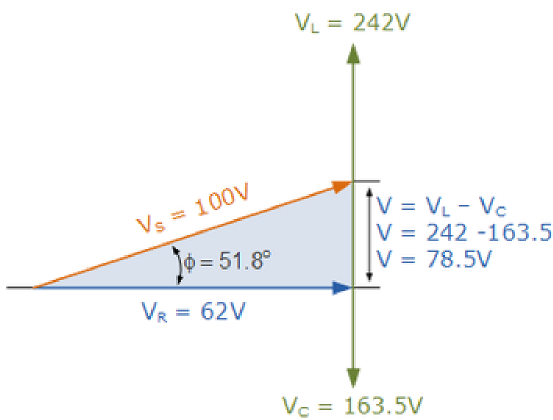
The voltages of VR, VL, VC.

$$V_R = I \times R = 5.14 \times 12 = 61.7\text{ volts}$$

$$V_L = I \times X_L = 5.14 \times 47.13 = 242.2\text{ volts}$$

$$V_C = I \times X_C = 5.14 \times 31.8 = 163.5\text{ volts}$$

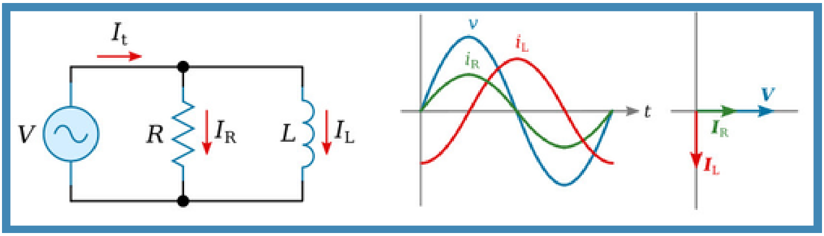
The Phasor Diagram.



RL Parallel Circuit

In parallel circuit, voltage is used as a reference.

R-L parallel Circuit



Parallel RL Circuit Analysis:

1) Inductive Reactance, XL:

XL = 2πfL or jωL

2) The voltages for R and L

VT = VL = VR

3) The current for R and L:

IR = VT / R and IL = VT / XL

4) The total current:

$I_T = \sqrt{I_R^2 + I_L^2}$ or in complex form: $I_T = I_R - jI_L$

5) Impedance:

ZT = VT / IT

6) Phase angles for R and L.

θR = 0º and θL = -90º

7) Total phase angle:

θT = -tan-1(IL/ IR)

8) Impedance:

$$Z_T = \frac{R \times X_L}{R + X_L}$$ or $$\frac{1}{Z_T} = \frac{1}{R} + \frac{1}{X_L}$$

Example

Calculate the phase angle, Zt, It, IR, and IL in a parallel RL circuit with a 10 V source operating at 4 kHz, a 1.4 mH coil, and a 25 Ω resistor.

Solution

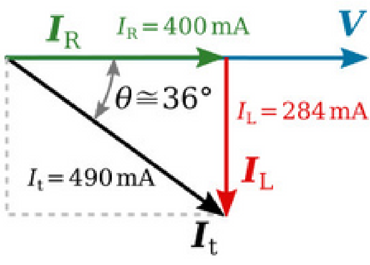
$$X_L = 2\pi f L = 2\pi(4k)(1.4m) = 35.17\Omega$$

$$I_R = \frac{V}{R} = \frac{10}{25} = 0.4A$$

$$I_L = \frac{V}{X_L} = \frac{10}{j35.17} = -j0.284A$$

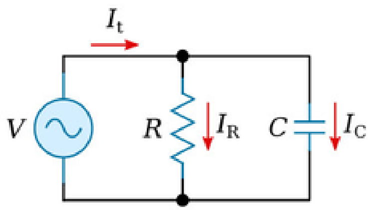
$$I_T = I_R + I_L = 0.4 - j0.284 = 0.491\angle -35.4^\circ A$$

$$Z_T = \frac{V}{I_T} = \frac{10\angle 0^\circ}{0.491\angle -35.4^\circ} = 20.37\angle 35.4^\circ\Omega$$

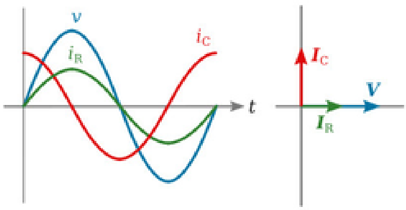


RC Parallel Circuit

Parallel RC circuits are similar to parallel RL circuits in that they can be solved similarly. Below is an example of a parallel resistor circuit.
Here is an illustration of the circuit conditions showing a diagram. The total current needs to be determined using phasor addition because the phasors IR and IC are currently out of phase. When addressing an RC circuit, the same steps are taken as when addressing an LR circuit.



Parallel RC Circuit.



Waveforms and phasors.

RC Parallel Circuit Analysis:

1) Capacitive Reactance, XC:

XC = 1/(2πfC)

2) The voltages for R and L:

VT = VC = VR

3) The currents for R and L:

IR = VT / R and IC = VT / XC

4) Total current:

$$I_T = \sqrt{I_R^2 + I_C^2}$$ or in complex form: **IT = IR + jIC**

5) Impedance:

ZT = VT / IT

6) Phase angles for R and L:

θR = 0º and θC = 90º

7) Total phase angle:

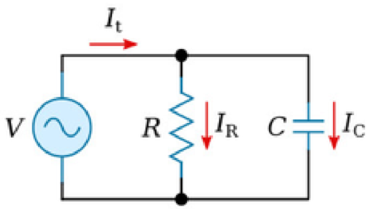
θT= tan-1(IC/IR)

8) Impedance:

$$Z_T = \frac{R \times X_C}{R + X_C}\ \Omega$$ or $$\frac{1}{Z_T} = \frac{1}{R} + \frac{1}{X_C}$$

Example

A 36-ohm resistor and a 6 microfarad capacitor are connected in parallel as shown in the circuit below. They are both connected across a 1.46 kHz, 25-volt source. Calculate the following: XC, IR, IC, It, and Zt.



Solution

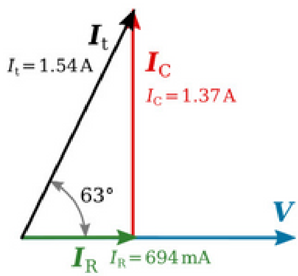
$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(1.46k)(6\mu)} = 18.2\Omega$$

$$I_R = \frac{V}{R} = \frac{25}{36} = 694mA$$

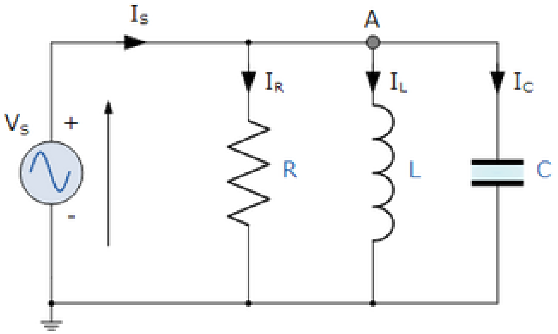
$$I_C = \frac{V}{-jX_C} = \frac{25}{-j18.2} = j1.37A$$

$$I_T = I_R + I_C = 694m + j1.37 = 1.54\angle 63.1^\circ A$$

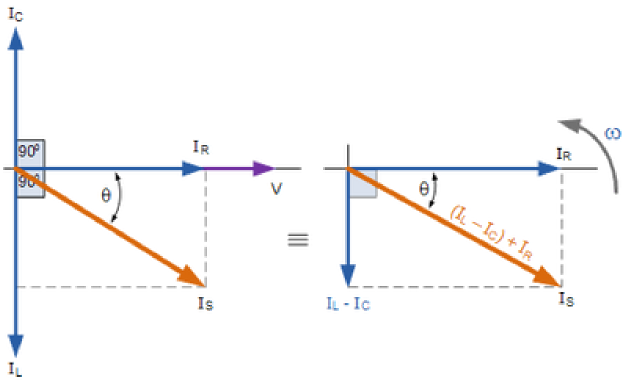
$$Z_T = \frac{V}{I_T} = \frac{25\angle 0^\circ}{1.54\angle 63.1^\circ} = 16.2\angle -63.1^\circ\Omega$$



R-L-C parallel Circuit



The Phasor diagram:



RLC Parallel Circuit Analysis

1) Calculate X_L and X_C

- $X_L = 2\pi f L$
- $X_C = 1/2\pi f L$

2) Calculate the voltages for R, L and C. Since this is a parallel circuit:

- $V_T = V_L = V_C = V_R$

3) By using Ohm's Law, calculate the currents for R and L:

- $I_R = V_T/R$
- $I_L = V_T/X_L$
- $I_C = V_T/X_C$

4) Calculate the total current:

- $I_T = \sqrt{I^2 R + (I_L - I_C)^2}$ or in complex form
- $I_T = I_R + jI_C + (-jI_L)$

5) By using Ohm's Law, calculate the total impedance:

- $Z_T = V_T / I_T$

6) Calculate the phase angles for R and L. Phase angles for these components in a parallel circuit are always:

- $\theta R = 0^{\circ}$
- $\theta C = 90^{\circ}$
- $\theta L = -90^{\circ}$

7) Calculate the total phase angle for the circuit:

- $\theta T = tan^{-1}\ (I_C - I_L)/I_R$

A Parallel RLC Circuit's Impedance

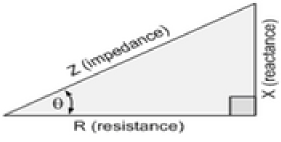
$$R = \frac{V}{I_R} \quad X_L = \frac{V}{I_L} \quad X_C = \frac{V}{I_C}$$

$$Z = \frac{1}{\sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}}$$

$$\therefore \frac{1}{Z} = \sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}$$

Impedance Triangle

In AC circuit analysis, the relationship between the three elements of impedance—capacitive reactance (Xc), inductive reactance (XL), and resistance (R)—is visually depicted using an impedance triangle. This triangle helps explain how various components work together to produce the overall impedance (Z) of a circuit.



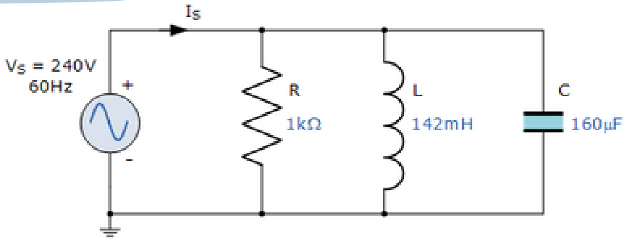
Impedance Triangle

When examining AC circuits, this triangle is a helpful tool, especially when it comes to power calculations and comprehending how the circuit behaves when alternating current is applied.

Example 1

A 240V, 60Hz supply is used to connect a 160uF capacitor, a 142mH coil, and a 1kΩ resistor in parallel. Calculate the supply current as well as the parallel RLC circuit's impedance.

Solution:



Find XL :

$$X_L = \omega L = 2\pi f L = 2\pi.60.142\times10^{-3} = 53.54\Omega$$

Find XC:

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{2\pi.60.160\times10^{-6}} = 16.58\Omega$$

Calculate an Impedance, (Z):

$$Z = \frac{1}{\sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}} = \frac{1}{\sqrt{\left(\frac{1}{1000}\right)^2 + \left(\frac{1}{53.54} - \frac{1}{16.58}\right)^2}}$$

$$Z = \frac{1}{\sqrt{1.0\times10^{-6} + 1.734\times10^{-3}}} = \frac{1}{0.0417} = 24.0\Omega$$

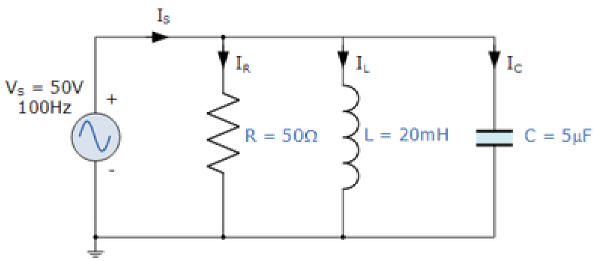
Calculate the Supply Current, (Is):

$$I_S = \frac{V_S}{Z} = \frac{240}{24} = 10 \text{ Amperes}$$

Example 2

Three components are connected in parallel over a 50V, 100Hz supply: a 5uF capacitor, a 20mH coil, and a 50Ω resistor. Calculate the total current drawn from the source, the current flowing through each branch, the circuit's total impedance, and the phase angle. Draw the triangles that currently represent the circuit as well.

Solution:



1) Find XL:

$$X_L = \omega L = 2\pi f L = 2\pi.100.0.02 = 12.6\Omega$$

2) Find XC

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{2\pi.100.5\times10^{-6}} = 318.3\Omega$$

3) Calculate an Impedance, (Z):

$$Z = \frac{1}{\sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_C} - \frac{1}{X_L}\right)^2}} = \frac{1}{\sqrt{\left(\frac{1}{50}\right)^2 + \left(\frac{1}{318.3} - \frac{1}{12.6}\right)^2}}$$

$$Z = \frac{1}{\sqrt{0.0004 + 0.0058}} = \frac{1}{0.0788} = 12.7\Omega$$

4) Calculate the current through resistance, (I_R):

$$I_R = \frac{V}{R} = \frac{50}{50} = 1.0\,(A)$$

5) Calculate the current through inductor, (I_L)

$$I_L = \frac{V}{X_L} = \frac{50}{12.6} = 3.9\,(A)$$

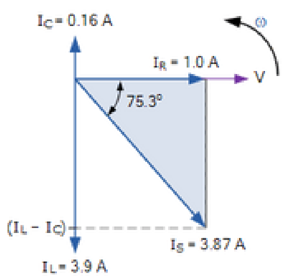
6) Calculate the current through capacitor, (I_C)

$$I_C = \frac{V}{X_C} = \frac{50}{318.3} = 0.16\,(A)$$

7) Calculate the total supply current, (I_S)

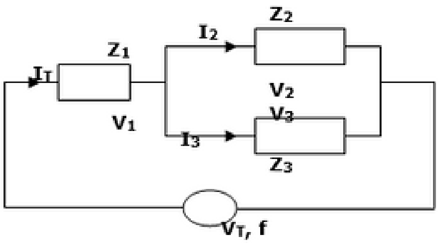
$$I_S = \sqrt{I_R^2 + (I_L - I_C)^2} = \sqrt{1^2 + (3.9 - 0.16)^2} = 3.87\,(A)$$

Draw the current Triangle



Current Triangle

R-L-C circuit combination using series and parallel



- Every component in a series connection needs to have the same current.
- The voltage of components connected in parallel must be the same.
- These two guidelines cover both individual parts and sections of circuits.

i) A series-parallel circuit's total impedance .

$$Z_T = Z_1 + Z_2 \| Z_3$$
$$Z_T = Z_1 + \frac{Z_2 \times Z_3}{Z_2 + Z_3}$$

ii) Total current:
The supply current, IT, is equal to the vector sum of the branch currents, I2 and I3.

$$I_T = \frac{V_T}{Z_T} \qquad I_2 = \frac{V_2}{Z_2} \qquad I_3 = \frac{V_3}{Z_3}$$

iii) The voltage drops across each impedance in a series-parallel circuit.

iv) Since Z_2 and Z_3 are parallel, the voltage loss is equivalent, $V_2 = V_3$. The supply voltage, often known as V_T, is equal to the vector sum of V_1 and V_2 or V_1 and V_3.

$$V_1 = I_T \times Z_1$$
$$V_2 = V_3 = I_T \times \frac{Z_2 \times Z_3}{Z_2 + Z_3}$$

Energy and Power

- Physics defines work as the application of energy to overcome an obstacle or change the physical state of a body.
- The work capacity is called energy.
- Energy can take many different forms. Some of these include:
  - heat energy
  - light energy
  - sound energy
  - mechanical energy
  - chemical energy
  - electrical energy.
- Numerous commonplace devices transform energy into different forms. For instance:
  - The conversion of electrical energy to sound energy occurs in a loudspeaker.
  - Thermal energy is produced by a toaster from electrical energy.
  - An LED or light bulb produces light by converting electrical energy.
- The unit of measurement for energy (E) is called a joule, and its symbol is J

Power

- The rate of energy consumption is called power, or simply P.
- Said another way, power is the quantity of energy utilized each time, or energy per time, in a particular amount of time. That's an equation.

$$P = \frac{E}{t}$$, joules per second or WATT (W)

Power Equations

- When current flows through resistance, electric energy is converted to heat energy.
- The rate of this energy conversion is the power.

$$P = I \times V \qquad P = I^2 \times R \qquad P = \frac{V^2}{R}$$

- In each of these equations, R is the size (in ohms) of the resistance, V is the voltage (in volts) across the resistance, and I is the current (in amps) through the resistance.

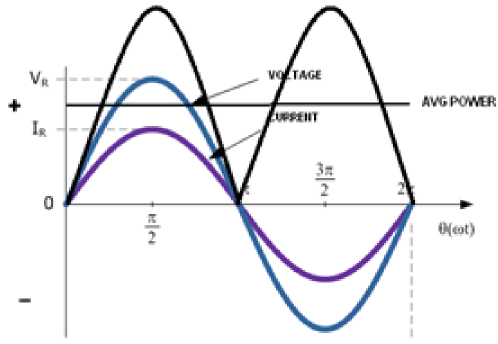
AC circuit power consumption

Pure Resistive Power

- An electrical component or circuit's true or actual power determines how quickly energy is lost from it. Generally, this energy loss is caused by heat dissipation in resistors or by converting one kind of energy into another, as in the case of motors that convert electrical energy into motion.
- In a resistive circuit, current and voltage are in phase,

  Average power:

$$P = I_{rms} \times E_{rms}$$



- The voltage and current of the resistor in AC circuits must be in rms values or effective values:

$$P_{true} = V_{rms} \times I_{rms}$$

$$P_{true} = I_{rms}^2 \times R$$

$$P_{true} = \frac{V_{rms}^2}{R}$$

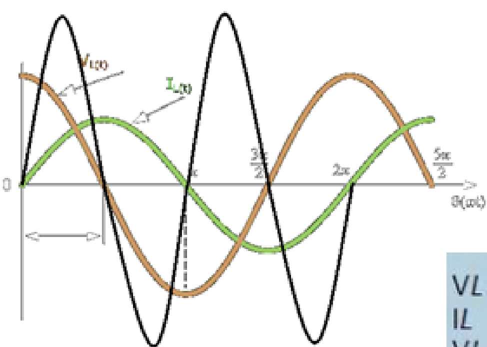
- When a resistor dissipates power, it indicates that a portion of the electrical energy in the circuit is being converted to thermal energy. Due to the heating of the air surrounding the resistor, this energy is lost from the circuit. This is called True Power.

Inductance and capacitance

a) Electrical Power in an Inductor

- The power in an inductor is associated with the energy stored in its magnetic field.
- The voltage across an inductor is related to the rate of change of current through it.
- When an AC voltage is applied to an ideal inductor, energy is periodically stored and then returned to the source over each cycle.
- During one part of the cycle, energy is stored in the inductor's magnetic field, and during another part, this energy is returned to the source.
- In an ideal inductor:
  - True Power (Ptrue): The net power over a full cycle is zero because all the energy stored during the positive part of the cycle is returned during the negative part. As a result, the true power, measured in watts (W), is zero.
  - Reactive Power (Q): The power associated with the energy that is alternately stored and returned by the inductor is called reactive power. It is measured in volt-amperes reactive (VAR).
- TReactive power represents the rate at which energy is stored and returned by the inductor, with no net loss of energy as heat in an ideal scenario.

$$Q = V_{rms} \times I_{rms}$$
$$Q = I_{2rms} \times X_L$$
$$Q = \frac{V_{2rms}}{X_L}$$



VL = Vmax sin (ωt+ 90º)
IL = Imax sin ωt
VL lead IL by 90º

P = I V = 0 watt (because +cycle =-cycle so average power =0)

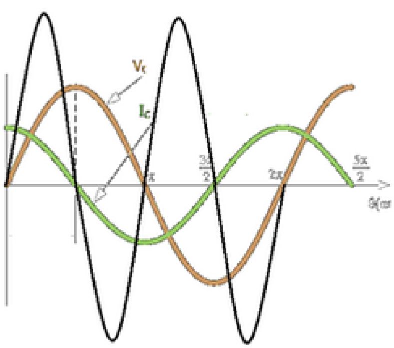
0 - 90º	90º –180º	180º -270º	270º - 360º
VL= +	VL= -	VL= -	VL= +
IL= +	IL= +	IL= -	IL= -

b) Power in A Capacitor

- The power in a capacitor, like in an inductor, relates to the energy stored in its electric field.
- The current through a capacitor is related to the rate of change of the voltage across it
- The power represents the rate at which energy is stored in or released from the capacitor's electric field.
- Like in an inductor, in an ideal capacitor, there is no net energy loss over time because the energy stored during one part of the cycle is returned during another part.

- As demonstrated in the example below, for sinusoidal waves, the current flowing through a capacitor leads the voltage across it by 90 degrees.

$$Q = V_{rms} \times I_{rms}$$
$$Q = I_{2rms} \times X_C$$
$$Q = \frac{V_{2rms}}{X_C}$$



Vc = Vmax sin ωt
Ic = Imax sin (ωt + 90º)
Ic lead Vc by 90º

P = I V = 0 watt (because +cycle =-cycle so, average power (True Power) =0)

0 - 90º	90º –180º	180º -270º	270º - 360º
Vc= +	Vc= +	Vc= -	Ic = +
Ic= +	Ic= -	Ic = -	Vc= -

Reactive Power

- Reactive power (Q) is the power consumed in an AC circuit because of the expansion and collapse of magnetic (inductive) and electrostatic (capacitive) fields.
- Reactive power is expressed in volt-amperes-reactive (VAR).
- Unlike true power, reactive power is not useful power because it is stored in the circuit itself.

Apparent Power

- Apparent Power is the total power in an AC (alternating current) circuit, combining both the real power (which performs actual work) and the reactive power (which oscillates between the source and the reactive components).
- Apparent power represents the product of the circuit's voltage and current without considering the phase angle between them.
- Apparent power is the total power in an AC circuit, representing both the real power that does useful work and the reactive power that cycles back and forth between the source and reactive components.
- It is measured in volt-amperes (VA) and can be visualized using the power triangle, where it is the vector sum of real and reactive power.

Power Triangle

- Power factor (pf), is calculated by dividing true power by its Apparent Power.
- Reactive power is the power that an AC circuit stores
- Real power is the power that the circuit uses.
- Cos Φ is referred to as the power factor (pf) for an AC circuit. where the phase angle between the applied voltage and the current sine waves is represented by the symbol Φ on a power triangle (see power triangle reference).
- How much of the apparent power is true power is indicated by the power factor (pf=cosΦ).
- The circuit is entirely reactive if the power factor is zero, and solely resistive if it is one.

Power Factor (pf) = Cos Φ= P/S =Watt/VA.

where,

cos Φ = power factor (pf)

P = true power (watts)

S = apparent power (VA)

- When the current in a circuit with an inductive load falls behind the voltage, it is referred to as having a lagging power factor.
- A circuit that uses capacitors and has a higher current than voltage is said to have a leading power factor.

Power Factor Correction

- Power Factor Correction is the process of improving the power factor of an electrical system, typically by reducing the amount of reactive power in the system.
- This helps to make the system more efficient by ensuring that more of the power is being used for actual work (real power) and less is being wasted in the form of reactive power.
- Power factor correction is a technique used to improve the power factor of an electrical system, making it more efficient.
- By reducing reactive power, power factor correction lowers energy costs, increases system capacity, reduces losses, and improves voltage stability.
- This is typically achieved through the use of capacitors, synchronous condensers, or other specialized equipment.

Example 2

A 240V, 1kHz voltage source is connected to a parallel 20Ω resistor with a 20mH inductance. Calculate the overall current, the current in each branch, the apparent power, the true power, and the reactive power.

Solution:

$$X_L = 2\pi fL = 2\pi(1k)(20m) = 125.66\Omega$$

$$I_R = \frac{V}{R} = \frac{240}{20} = 12A$$

$$I_L = \frac{V}{X_L} = \frac{240}{125.66} = 1.91A$$

$$I_T = I_R - jI_L = 12 - j1.91 = 12.15\angle -9.04^\circ A$$

True Power, $P = IV\cos\theta = (12.15)(240)\cos 9.04^\circ = 2.88kW$

Reactive Power, $Q = IV\sin\theta = (12.15)(240)\sin 9.04^\circ = 458.17VAR$

Apparent Power, $S = IV = (240)(12.15) = 2.92kVA$

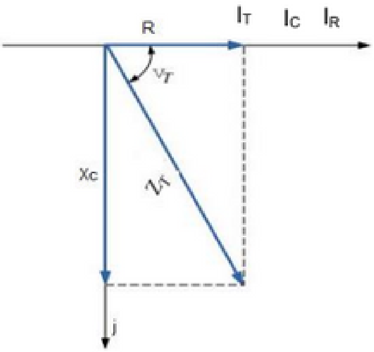
END OF CHAPTER

QUESTION 1:

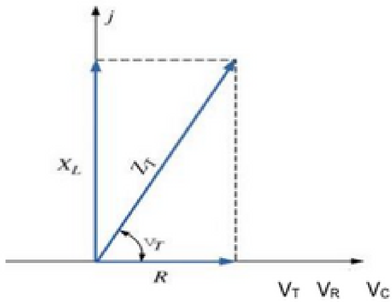
For the below circuit, express the phasor diagram that demonstrates the relationship between VR, VC, VT, IT, IR, and Ic.

ANSWER:

i. Series circuit



ii. Parallel RC circuit



QUESTION 2:

The values of a series RLC circuit are as follows: L = 25 mH, C = 220µF, and R = 8Ω. Suppose the instantaneous voltage in the circuit is Vs = 17 sin (377t) V. Calculate the instantaneous current at t = 5 ms and create a phasor diagram for it.

ANSWER:

$$F = \frac{\omega}{2\pi} = \frac{377}{2\pi} = 60Hz$$

$$X_L = 2\pi f L = 2\pi(60)(25m) = 9.34\Omega$$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(60)(220\mu)} = 12.06\Omega$$

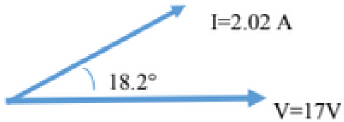
$$Z_T = R + jX_L - jX_C$$

$$Z_T = 8 + j9.34 - j12.06$$

$$Z_T = 8 - j2.63 \quad \text{or} \quad 8.42\angle - 18.19^\circ\Omega$$

$$I_T = \frac{V_S}{Z_T} = \frac{17\angle 0^\circ}{8.42\angle - 18.19^\circ} = 2.02A\angle 18.19^\circ$$

Current leads the voltage by 18.19°
Voltage is given VS = 17 sin(377t) V ,which makes voltage the reference at 0° and I = 2.02sin(377t + 18.2°)



Value of current at 5.0ms,
I = 2.02 sin(377t + 18.2°)A
I = 2.02sin(377(5m) + 18.2°)
I = 1.63 A

QUESTION 3:

A capacitive circuit is fed by a 240V, 50Hz supply. There is a 2A current flow in the circuit and 150 watts of power loss. Calculate the resistance and capacitance values.

ANSWER:

$$PowerDissipated, P = IV\cos\theta$$

$$\cos\theta = \frac{150}{(240)(2)} = 0.3125$$

$$Z_T = \frac{V_S}{I_T} = \frac{240}{2} = 120\Omega$$

$$PowerFactor, \cos\theta = \frac{R}{Z_T}$$

$$R = \cos\theta(Z_T) = (0.3125)(120) = 37.5\Omega$$

$$Z_T = \sqrt{R^2 + X_C^2}$$

$$X_C = \sqrt{Z_T^2 - R^2} = \sqrt{120^2 - 37.5^2} = 114\ \Omega$$

$$X_C = \frac{1}{2\pi f C}$$

$$C = \frac{1}{2\pi(50)(114)} = 27.92\mu F$$

QUESTION 4:

A 240V, 1KHz voltage source is connected in parallel to a 75Ω resistor with a 40mH inductance. Calculate the total current, the true power, apparent power, reactive power, power factor, and the current in each branch.

ANSWER:

$$X_L = 2\pi f L = 2\pi(1k)(40m) = 251.33\Omega$$

$$I_R = \frac{V_R}{R} = \frac{240}{75} = 3.2A$$

$$I_L = \frac{V_L}{X_L} = \frac{240}{251.33} = 0.95A$$

$$I_T = I_R - jI_L = 3.2 - j0.95 = 3.34A\angle - 16.53^\circ$$

True power, P = IV cos θ
True power, P = (3.34) (240) cos16.53° = 768.47 W

Reactive power, Q= IV sin θ
Reactive power, Q= (3.34) (240) sin16.53° = 228.07 VAR

Apparent power, S = IV
Apparent power, S = (3.34) (240) = 801.6 VA

QUESTION 5:

A 120V, 50 Hz voltage supply is used in an RLC series circuit, which also has R = 22Ω, C = 220µF, and L = 110mH.

a. Calculate:-

i. Total Impedance
ii. Total Current
iii. Power factor
iv. True power
v. Reactive power
vi. Apparent

b. Draw the vector diagram.

ANSWER:

$$X_L = 2\pi f L = 2\pi(50)(110m) = 34.56\Omega$$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(50)(220\mu)} = 14.47\Omega$$

$$Z_T = 22 + j34.56 - j14.47 = 22 + j20.1 = 29.8\angle 42.4^\circ\Omega$$

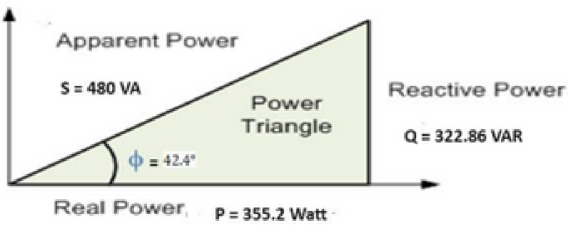
$$I_T = \frac{V_S}{Z_T} = \frac{120\angle 0^\circ}{29.8\angle 42.4^\circ} = 4A\angle - 42.4^\circ$$

$$Power Factor, pf = \cos 42.4^\circ = 0.74 lagging$$

$$True Power, P = IV \cos 42.4^\circ = (4)(120)(0.74) = 355.2W$$

$$Reactive Power, Q = IV \sin 42.4^\circ = (4)(120) \sin 42.4^\circ = 322.86VAR$$

$$Apparent Power, S = IV = (4)(120) = 480VA$$



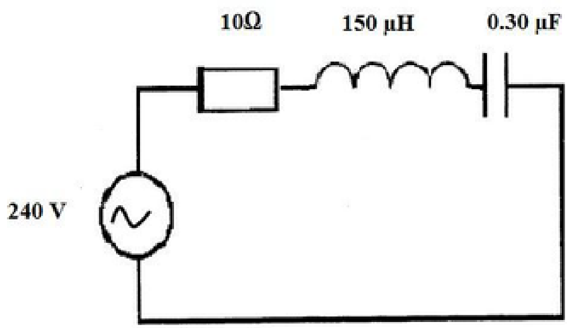
EXERCISE

1 For purely resistive, purely inductive, and purely capacitive AC circuits, draw a phasor diagram to show the relationship between current and voltage.

Answer

Refer to topic 2

2 Calculate the ZT, or total impedance, for the series circuit at a frequency of 30 kHz by referring to the figure below.

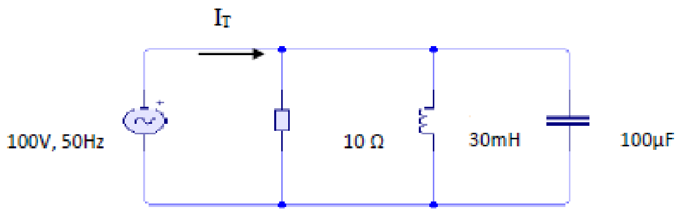


Answer

$$Z_T = 10 + j10.59 = 14.56\angle 46.64^\circ \Omega$$

3

Calculate the total current (IT) following through the circuit by referring to the figure below.



Answer

$$I_T = 10 - j7.84 = 12.49\angle -36.8^\circ A$$

4

Calculate the total impedance and circuit current of an RLC series circuit that is connected in series across a 100V, 50Hz supply with a 12Ω resistance, a 0.15H inductance, and a 100uF capacitor.

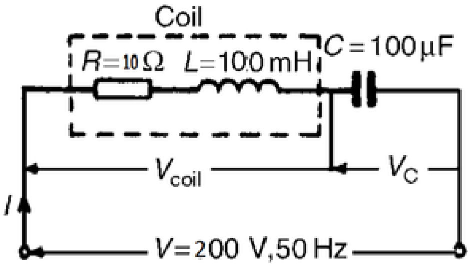
Answer

Z=19.4 Ω

I = 5.15 A

5 As shown in the figure below, a 50Hz power source is connected in series with a 100uF capacitor and a coil of inductance 100mH and resistance 10Ω. Calculate :

i) Circuit impedance, Z
ii) Circuit current, I
iii) Phase angle
iv) Voltage across inductor
v) Voltage across capacitor



Answer

(i) Z = 10 Ω

(ii) I = 20 A

(iii) ϕ = 2.35°

(iv) VL=628.4 V

(v) Vc = 636.6 V

6 An 80µF capacitor, a 0.1H inductor, and a 15Ω resistor are connected in series to power supply 100V at 50Hz. Calculate:

i) Total impedance
ii) Total circuit current
iii) Voltage across each components

Answer

Z=15-j8.37=17.18∠-29.16°Ω

I=5.82∠29.16°A

VR = 87.3 V

VL = 182.86 V

Vc= 234.36 V

7

A 100V, 50Hz supply is connected in series with a 90Ω resistor, 0.3H inductor, 10µF capacitor, and 1H inductor. Calculate the actual power dissipation of the circuit as well as the voltage across each component.

Answer

VR = 71.1 V

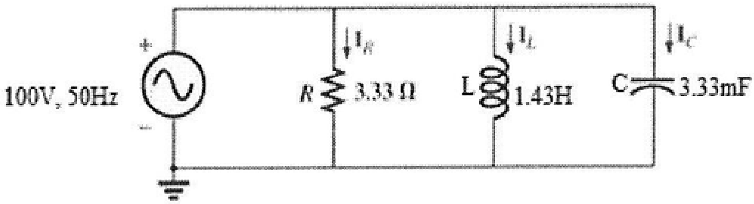
VL = 322.64 V

Vc = 251.5 V

P = 55.83 Watt

8

Calculate the total current IT and total impedance ZT by referring to the figure below.



Answer

$$Z_T = 0.254 + j0.885 = 0.921\angle -73.95\Omega$$

$$I_T = 30.02 + j104.35 = 108.58\angle 73.95A$$

9

A coil has a 40 mH inductance and negligible resistance. Calculate the current that results from its inductive reactance if it is connected to:
i. A 240V, 50 Hz supply
ii. A 100V, 1KHz supply

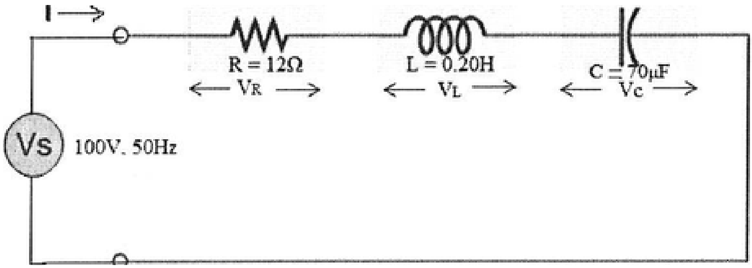
Answer

i. $X_L = 12.57\Omega, I = 19.09A$

ii. $X_L = 251.3\Omega, I = 0.398A$

10

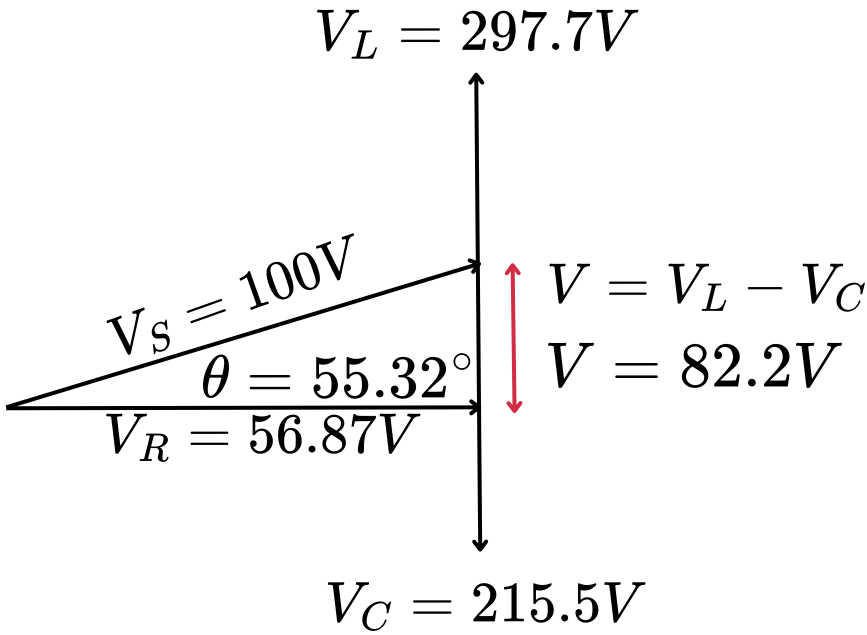
Calculate the circuit's total current flow (I) and draw the voltage phase diagram by referring to the figure below.



Answer

I = 4.739 A

Answer

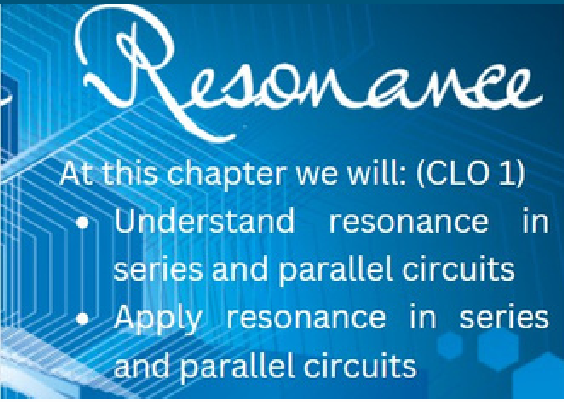


$V_L = 297.7V$
$V_S = 100V$
$\theta = 55.32^\circ$
$V_R = 56.87V$
$V = V_L - V_C$
$V = 82.2V$
$V_C = 215.5V$

TOPIC 3

RESONANCE

INTRODUCTION



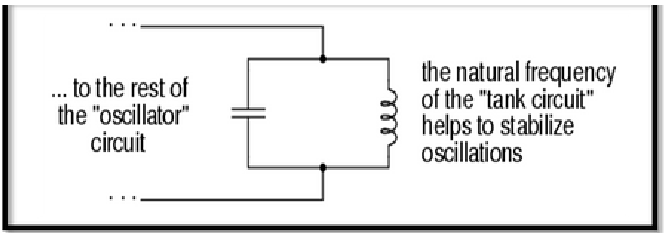
Resonance is a system's tendency to oscillate with greater amplitude at certain frequencies than at others

Resonance occurs when a system can easily store and transfer energy between two or more different storage modes

Resonance is used, among other things, to establish a stable frequency state in circuits intended to generate AC signals

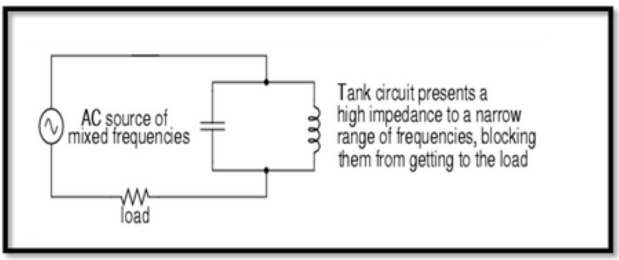
FUNCTIONS OF RESONANCE:

1 The resonance circuit acts as stable frequency source



The frequency set by the tank circuit is safety depending on the value of L & C

2 Resonance circuit serves as filter

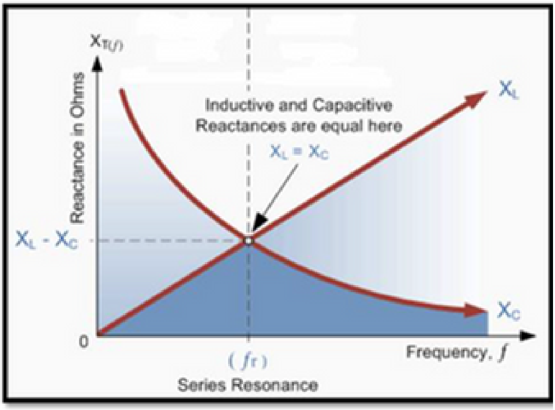


Acting as a short of frequency "filter" to prevent some frequencies from mixing with others.

RESONANCE PHENOMENON

In an RLC circuit, resonance occurs when the capacitive and inductive reactance are the same magnitude, resulting in a completely resistive impedance

$$Z = R + j\left(\omega L - \frac{1}{\omega C}\right) \quad ; \text{note} : \omega L - \frac{1}{\omega C} = 0$$



$$X_L = X_C$$

$$X_L = 2\pi f L \qquad X_C = \frac{1}{2\pi f C}$$

. . . setting the two equal to each other, representing a condition of equal reactance (resonance) . . .

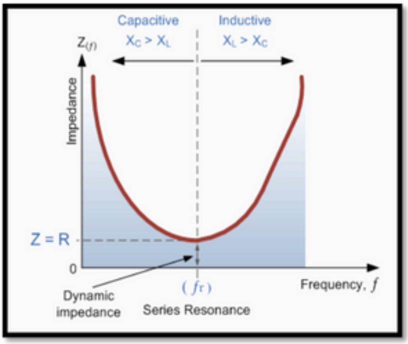
$$2\pi f L = \frac{1}{2\pi f C}$$

RLC Series & Parallel Circuit

RLC Series Circuit

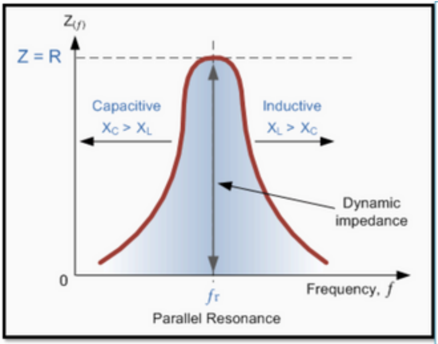
Explain the effect of changing the frequency

- ✓ A series RLC circuit contains both inductive reactance (X_L) and capacitive reactance (X_C)
- ✓ A series RLC circuit's reactance changes as you change the voltage source's frequency.
- ✓ Its total impedance also changes.
- ✓ At **low frequencies**, $X_C > X_L$ **and the circuit is primarily capacitive**.
- ✓ At **high frequencies, $X_L > X_C$ and the circuit is primarily inductive.**



RLC Parallel Circuit

Explain the effect of changing the frequency

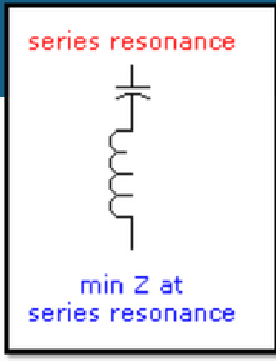


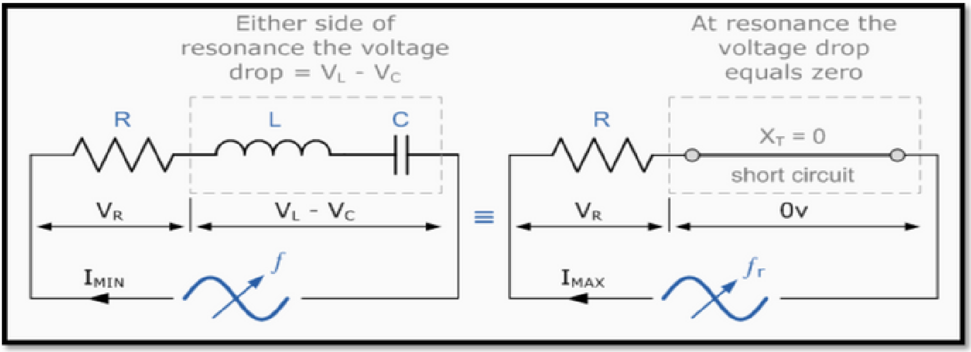
- ✓ Reactance change as you change the voltage source's frequency.
- ✓ **At low frequencies, $X_L < X_C$ and the circuit is primarily inductive.**
- ✓ **At high frequencies, $X_C < X_L$ and the circuit is primarily capacitive.**

Series Resonance

Current will be maximum & offering minimum impedance

- ✓ Consider a RLC circuit in which resistor, inductor and capacitor are connected in series across a voltage supply.
- ✓ This series RLC circuit has distinguishing property of resonating at a specific frequency called resonant frequency
- ✓ In this circuit containing inductor and capacitor the energy is stored in two different ways
- ✓ When a current flows in an inductor, energy is stored in magnetic field
- ✓ When a capacitor is charged, energy is stored in electric field.





$(i)\ V_L = V_C$

$(ii)\ Z = R$ (*i.e. the minimum circuit impedance possible in an RLC circuit*)

$(iii)\ I = \frac{V}{R}$ (*i.e. the maximum current possible in an RLC circuit*).

(iv) Since $X_L = X_C$, then $2\pi f L = \frac{1}{2\pi f C}$

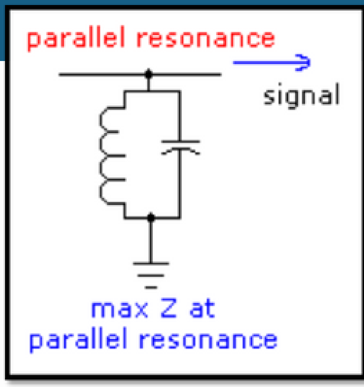
$$f^2 = \frac{1}{2\pi L x 2\pi C} = \frac{1}{4\pi^2 LC} = \sqrt{\frac{1}{4\pi^2 LC}}$$

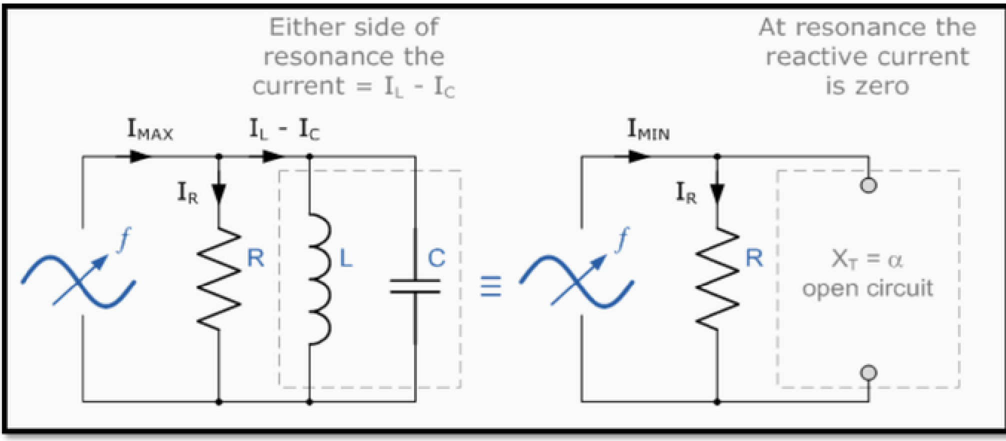
$$f = \frac{1}{2\pi\sqrt{LC}}\ (Hz)$$

Parallel Resonance

Current will be minimum & offering maximum impedance

- Resonance occurs in the two branch network containing capacitance (C) in parallel with inductance (L) and resistance (R) in series when the quadrature (i.e vertical) component of current ILR is equal Ic
- At this condition the supply current I is in-phase with the supply voltage V.





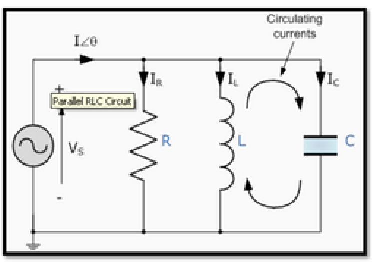
Resonance occurs when $X_L = X_C$ and the imaginary parts of Y become zero.

$$X_L = X_C \Rightarrow 2\pi f L = \frac{1}{2\pi f C}$$

Then:

$$f^2 = \frac{1}{2\pi L \ x \ 2\pi C} = \frac{1}{4\pi^2 LC} = \sqrt{\frac{1}{4\pi^2 LC}}$$

$$\therefore f_r = \frac{1}{2\pi\sqrt{LC}}(Hz) \quad or \quad \omega_r = \frac{1}{\sqrt{LC}}(rads)$$



Think About It!

EXAMPLE 3.1a:

A coil with a resistance of 14Ω and inductance of 120mH is connected in series with a 60µF capacitor across a 140V supply. What frequency produces resonance? Determine the current flowing at the resonance frequency.

ANSWER 3.1a:

Resonant frequency, $f_r = \frac{1}{2\pi\sqrt{LC}}$

$$= \frac{1}{2\pi\sqrt{(120m)(60\mu)}}$$

$$= 59.31\text{Hz}$$

$$At\ resonance, X_L = X_C\ and\ impedance\ Z = R$$

$$Hence\ current, \quad I = \frac{V}{R} = \frac{140}{14} = 10A$$

EXAMPLE 3.1b:

The current at resonance is a series RLC circuit is 100µA. Determine the circuit resistance and capacitance when the applied voltage is 2mV at a frequency of 200kHz and the circuit inductance is 50µH.

ANSWER 3.1b:

$$At\ resonance, impedance\ Z = R$$

$$Hence, R = \frac{V}{I} = \frac{2m}{200\mu} = 10\Omega$$

$$At\ resonance, X_L = X_C$$

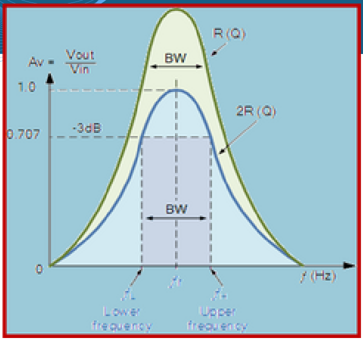
$$Hence, 2\pi f L = \frac{1}{2\pi f C}$$

$$30\mu H = \frac{1}{2\pi f C}$$

$$C = \frac{1}{2\pi(150kHz)x(30\mu H)}$$

$$C = 35.37mF$$

FREQUENCY BANDWIDTH



- Figure show how current I varies with frequency in an RLC series circuit.
- At the resonant frequency , current is a maximum value, shown as Ir.
- Also shown are the points A and B where the current is 0.707 of the maximum value at frequencies and
- The power delivered to the circuit is

- At I =0.707 Ir , the power is $(0.707\ Ir)^2R = 0.5\ Ir^2\ R$, i.e. half the power that occurs at frequency fr .

- The distance between these points, i.e. (f2 –f1), is called the bandwidth, BW.

$$Q = \frac{f_r}{B} = \frac{f_r}{(f_2 - f_1)}$$

$$or \quad Bandwidth, B,$$

$$f_2 - f_1 = \frac{f_r}{Q}$$

The points corresponding to f_L and f_U are called the half-power points.

$$Lower\ frequency, f_L = f_r - \frac{BW}{2}$$

$$Upper\ frequency, f_U = f_r + \frac{BW}{2}$$

QUALITY FACTOR, Q

A) SERIES CIRCUIT

- Q is the ratio of power stored to power dissipated in the circuit reactance and resistance
- Q is the ratio of its resonant frequency to its bandwidth

$$Q = \frac{X_L}{R} = \frac{2\pi f L}{R}$$

$$if;\ f_r = \frac{1}{2\pi\sqrt{LC}}$$

$$Q = \frac{2\pi\left(\frac{1}{2\pi\sqrt{LC}}\right)}{R}$$

$$Q = \frac{L}{R\sqrt{LC}} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

B) PARALLEL CIRCUIT

- Quality factor: the ratio of the circulating branch currents to the supply current.

$$Q = \frac{R}{2\pi f L} = 2\pi f C R = R\sqrt{\frac{C}{L}}$$

Think About It!

EXAMPLE 3.2:

A 100mH coil of inductance is connected in series with a 2μF capacitor and 10Ω resistance across a 240V variable frequency supply. Calculate the resonant frequency, the current at resonance, the voltage across inductor and capacitor at resonance and Quality factor of the circuit.

ANSWER 3.2:

a) *Resonant frequency,* f_r

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

$$f_r = \frac{1}{2\pi\sqrt{100m \; x \; 2\mu}} = 355.88Hz$$

b) *Current at tesonance,* $I = \frac{V}{R} = \frac{240}{10} = 24A$

c) *Voltage across inductor and capacitor at resonance,*

$$V_L = V_C$$

$$V_L = IX_L = (I)(2\pi f L)$$

$$V_L = (8)[2\pi(355.88)(100m)]$$

$$V_L = 5366.54V$$

$$V_L = V_C = 5366.54V$$

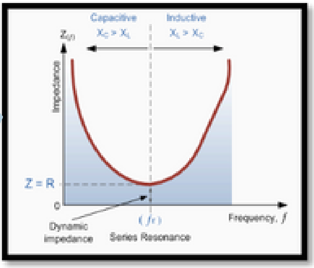
b) $Q - factor, Q = \frac{X_L}{R} = \frac{[2\pi(355.88)(100m)]}{10} = 22.36A$

END OF CHAPTER

01

Explain the effect of changing the frequency on an RLC series circuit using an appropriate diagram

ANSWER 01:



- If XL < XC and the circuit is primarily capacitive (Low frequencies)
- If XC < XL and the circuit is primarily inductive (High frequencies)

02

An RLC series circuit has a resonance frequency of 2kHz and Q-factor of 40. Calculate the inductance, capacitance, bandwidth, lower and upper cut-off frequencies for a circuit with a 30Ω impedance at resonance.

ANSWER 02:

$$At\ Resonant\ frequency, the\ circuit\ impedance, Z = R = 30\Omega$$

$$Q - factor, Q = \frac{X_L}{R} = \frac{2\pi f L}{R}$$

$$L = \frac{QR}{2\pi f} = \frac{(40)(30)}{2\pi(2k)} = 95.5mH$$

$$At\ Resonant, X_L = X_C$$

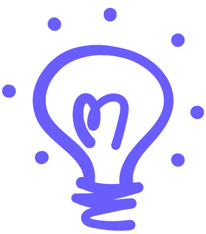
$$2\pi f L = \frac{1}{2\pi f C}$$

$$C = \frac{1}{(2\pi f)^2 L} = \frac{1}{(2\pi 2k)^2 x 95.5m} = 0.0663\mu F$$

$$Bandwidth, BW = \frac{f_r}{Q} = \frac{2k}{40} = 50Hz$$

$$f_L = f_r - \frac{BW}{2} = 2k - \frac{50}{2} = 1975Hz$$

$$f_H = f_r + \frac{BW}{2} = 2k + \frac{50}{2} = 2025Hz$$



03

A series resonance circuit with a 47µF capacitor, 100Ω resistor and 70mH inductor is connected to a sinusoidal source voltage produces a consistent output of 5V at all frequencies.
Calculate, Resonance frequency, Current resonance, Voltage across the inductor and capacitor at resonance and Quality factor, Q

ANSWER 03:

$Resonance\ Frequency, fr$

$$fr = \frac{1}{2\pi\sqrt{LC}}$$

$$fr = \frac{1}{2\pi\sqrt{(70m)(47\mu)}} = 87.74$$

$Current\ at\ Resonance, I$

$$ZR = R = 100\Omega$$

$$Imax\ = \frac{VT}{ZR}$$

$$I = \frac{5}{100} = 0.05A@50mA$$

$Voltage\ across\ inductor, V_L$

$$V_L = I\ x\ X_L$$

$$== 0.05\ x\ 38.6 = 1.93V$$

$Voltage\ across\ capacitor, V_C = V_L\ = 1.93V$

$Quality\ Factor, Q$

$$Q = \frac{XL}{R} = \frac{2\pi fL}{R} = \frac{2\pi(87.74)(70m)}{100}$$

Q = 0.39

04 A 100mH coil inductance is connected in series with a 10μF capacitor and a 100Ω resistance across a 240V, variable frequency supply. Calculate the resonant frequency, the current at resonance, voltage across inductor and capacitor at resonance and Q-factor of the circuit.

ANSWER 04:

Resonance Frequency, fr

$$fr = \frac{1}{2\pi\sqrt{LC}}$$

$$fr = \frac{1}{2\pi\sqrt{(100m)(10\mu)}} = 159Hz$$

Quality Factor, Q

$$Q = \frac{XL}{R} = \frac{2\pi fL}{R} = \frac{2\pi(159)(100m)}{100}$$

$$Q = 1$$

Current at Resonance, I

$$ZR = R = 100\Omega$$

$$Imax = \frac{VT}{ZR}$$

$$I = \frac{240}{100} = 2.4A$$

Voltage across inductor, V_L

$$V_L = I \: x \: X_L$$

$$== 2.4 \: x \: 100 = 240V$$

Voltage across capacitor, $V_C = V_L = 240V$

05

A circuit with of a 900µF capacitor, 90mH inductor and 9Ω resistor is connected in series to a 150V AC source. Calculate the upper and lower cut-off frequencies. Then using the acquired value, draw a resonance graph of Current versus Frequency.

ANSWER 05:

Resonance Frequency, fr

$$fr = \frac{1}{2\pi\sqrt{LC}}$$

$$fr = \frac{1}{2\pi\sqrt{(90m)(900\mu)}} = 17.68Hz$$

Quality Factor, Q

$$Q = \frac{XL}{R} = \frac{2\pi fL}{R} = \frac{2\pi(17.68)(90m)}{9}$$

$$Q = 1.11$$

Bandwidth, BW

$$BW = \frac{fr}{Q} = \frac{17.68}{1.11}$$

$$= 15.91Hz$$

Lower cut – off frequency, fL

$$fL = fr - \frac{BW}{2}$$

$$= 17.68 - \frac{15.91}{2}$$

$$= 9.72Hz$$

Upper cut – off frequency, fH

$$fH = fr + \frac{BW}{2}$$

$$= 17.68 + \frac{15.91}{2}$$

$$= 25.64Hz$$

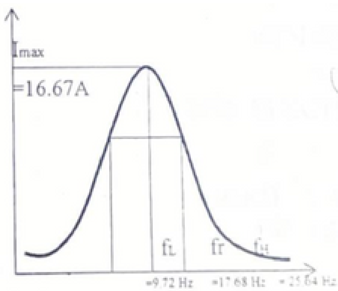
Current at Resonance, I

$$ZR = R = 9\Omega$$

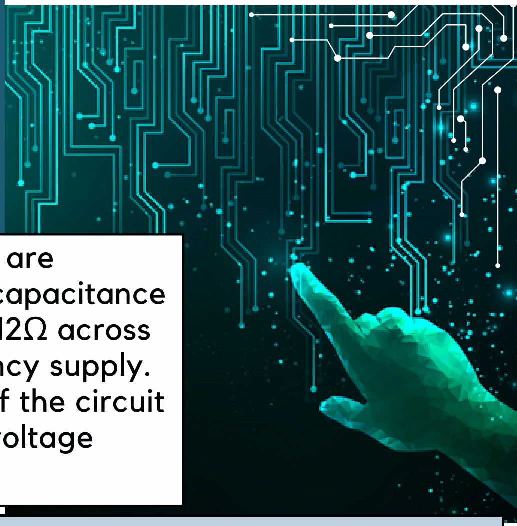
$$Imax = \frac{VT}{ZR}$$

$$I = \frac{150}{9} = 16.67A$$

Resonance Graph (Current vs Frequency)



EXERCISE



1. A coil of inductance 120mH are connected in series with a capacitance of 2μF and a resistance of 12Ω across a 50V and variable frequency supply. Determine the bandwidth of the circuit during the resonance and voltage across each component.

ANSWER 01:

Frequency resonance =324.87Hz
Quality factor, Q = 20.4
Bandwidth, Bw = 15.92Hz
Frequency Low, FL =316.91Hz
Frequency High, FH =332.83Hz

2. A series resonance circuit consists of 20.3μF capacitor, a coil 0.5H and a resistor 40Ω. When the circuit is connected to a 240V AC supply, calculate the frequency resonance, current of the circuit, voltage across each component, the bandwidth and the Q factor.

ANSWER 02:

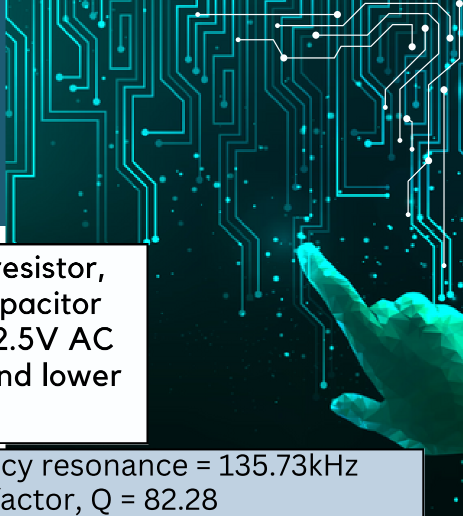
Frequency resonance =49.96Hz
Current, I = 6A
VR = 240V
VL=VC = 941.7V
Bw = 12.74Hz
Quality factor, Q = 3.92

3. A coil with 60mH inductance and 100Ω resistance are connected in series with a capacitance of 0.6μF across a 240V variable frequency supply. Calculate the upper and lower cut-off frequencies during resonance.

ANSWER 03:

Frequency resonance = 838.82Hz
Quality factor, Q = 3.16
Bw = 265.45Hz
Frequency Low, FL = 706.1Hz
Frequency High, FH = 971.5Hz

EXERCISE



4 A circuit which consists 25Ω resistor, 2.5mH inductor and 550pF capacitor is connected in series across 2.5V AC supply. Calculate the upper and lower cut-off frequency.

ANSWER 04:

Frequency resonance = 135.73kHz
Quality factor, Q = 82.28
Bandwidth, Bw = 1.59kHz
Frequency Low, FL = 134.94kHz
Frequency High, FH = 136.53kHz

5 A circuit with a 45mH inductor, 100μF capacitor and 12Ω resistor is connected in series across 220V AC supply. Calculate frequency resonance, quality factor, bandwidth, the upper and lower cut-off frequencies.

ANSWER 05:

Frequency resonance = 75.03Hz
Quality factor, Q = 1.77
Bandwidth, Bw = 42.39Hz
Frequency Low, FL = 53.84Hz
Frequency High, FH = 96.23Hz

6 A series resonance circuit consists of a 8μF capacitor, 40mH inductor and 50Ω resistor. When the circuit is connected to a 10V AC supply, calculate the current, frequency resonance, quality factor Bandwidth and the lower and upper cut-off frequencies during resonance.

ANSWER 06:

Current, I = 0.2A
Frequency resonance = 281.3Hz
Quality factor, Q = 1.414
Bandwidth, Bw = 198.9Hz
Frequency Low, FL = 181.8Hz
Frequency High, FH = 380.8Hz

EXERCISE

7

An RLC series circuit has a resonant frequency of 2kHz and Q-factor at resonance of 40. If the impedance of the circuit at resonance is 30Ω, determine the values of inductance, capacitance, bandwidth, lower and upper cut-off frequencies.

ANSWER 07:

Inductance, L = 9.55mH
Capacitance, C = 0.0663μF
Bandwidth, Bw = 50Hz
Frequency Low, FL = 1975Hz
Frequency High, FH = 2025Hz

8

A series of resonance circuit consists 80mH inductor, a capacitor of 16μF and a resistor of 100Ω. This circuit is connected to a 240V ac supply. Calculate resonance frequency, current during resonance, Quality Factor, bandwidth, lower and upper cut-off frequency.

ANSWER 08:

Frequency resonance = 140.67Hz
Current, I = 2.4A Quality factor, Q = 0.7
Bandwidth, Bw = 198.13Hz
Frequency Low, FL = 41.61Hz
Frequency High, FH = 239.74Hz

9

A 2μF capacitor is connected in series with a 100mH inductor and a 10Ω resistor across a 240V variable frequency power supply. Determine the resonant frequency, the current at resonance, Q-factor, voltages across inductor and capacitor at resonance of the circuit.

ANSWER 09:

Frequency resonance = 355.88Hz
Current, I = 24A
Quality factor, Q = 22.36
VL=VC = 5366.51V

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