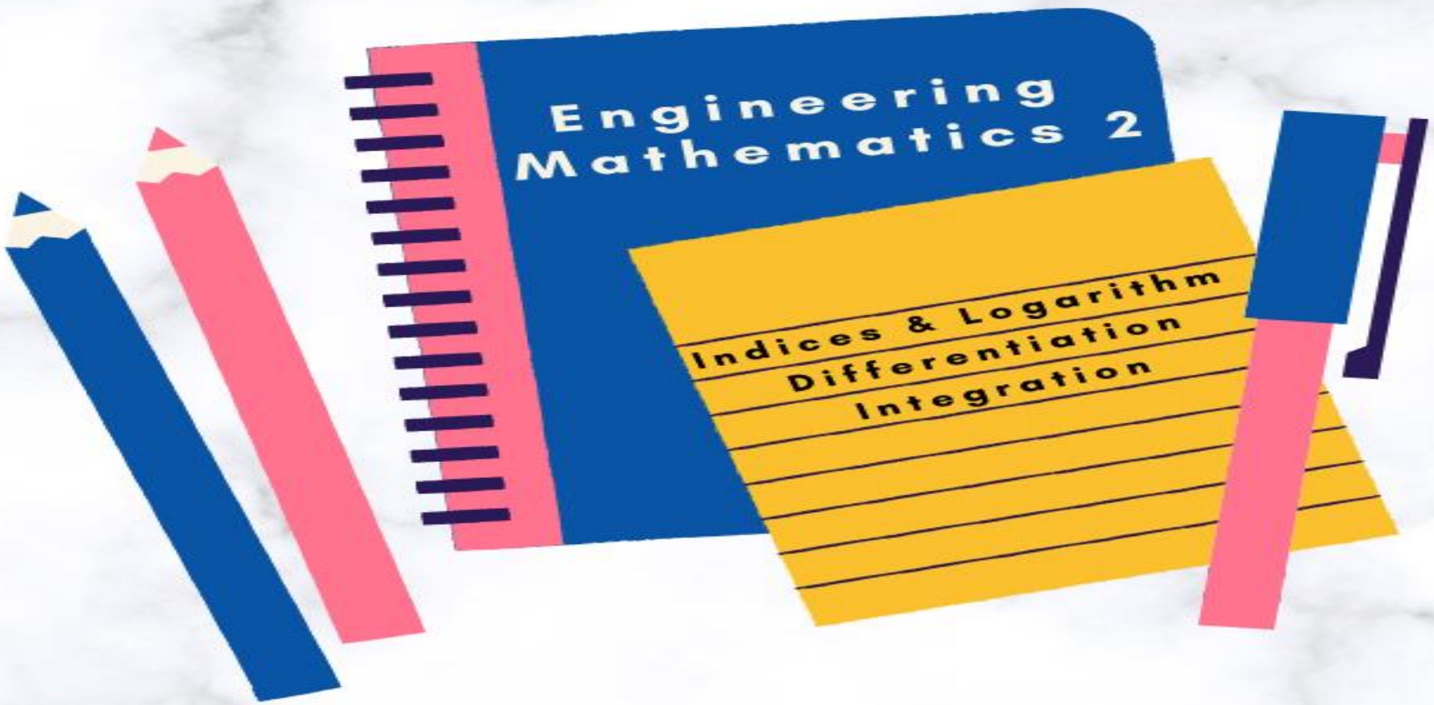

POCKET NOTES eBOOK

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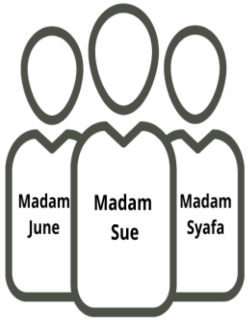
PUBLISHER
POLITEKNIK PORT DICKSON

ENGINEERING MATHEMATICS 2



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Engineering Mathematics 2
-Pocket Notes eBook-

PREFACE

This pocket notes eBook is written by the experienced lecturers from Mathematics, Science and Computer Department, Polytechnic Port Dickson. It is written based on the following topics

Chapter 1 : Indices & Logarithm

Chapter 2 : Differentiation

Chapter 3 : Integration

However, this pocket notes eBook only touches on the basics for the following topics. The purpose is to strengthen students' understanding of the basics of the topics studied before applying them and to make it easier for students to make revision before facing quizzes, test and exam.

This pocket notes eBook is written based on the latest syllabus of the Malaysian Polytechnic for the Engineering Mathematics 2 course.

By the way, we would like to thank all the lecturers involved for their contribution and efforts in producing this great pocket notes eBook. Any views or suggestions for improvement are most welcome. Thank You Very Much.

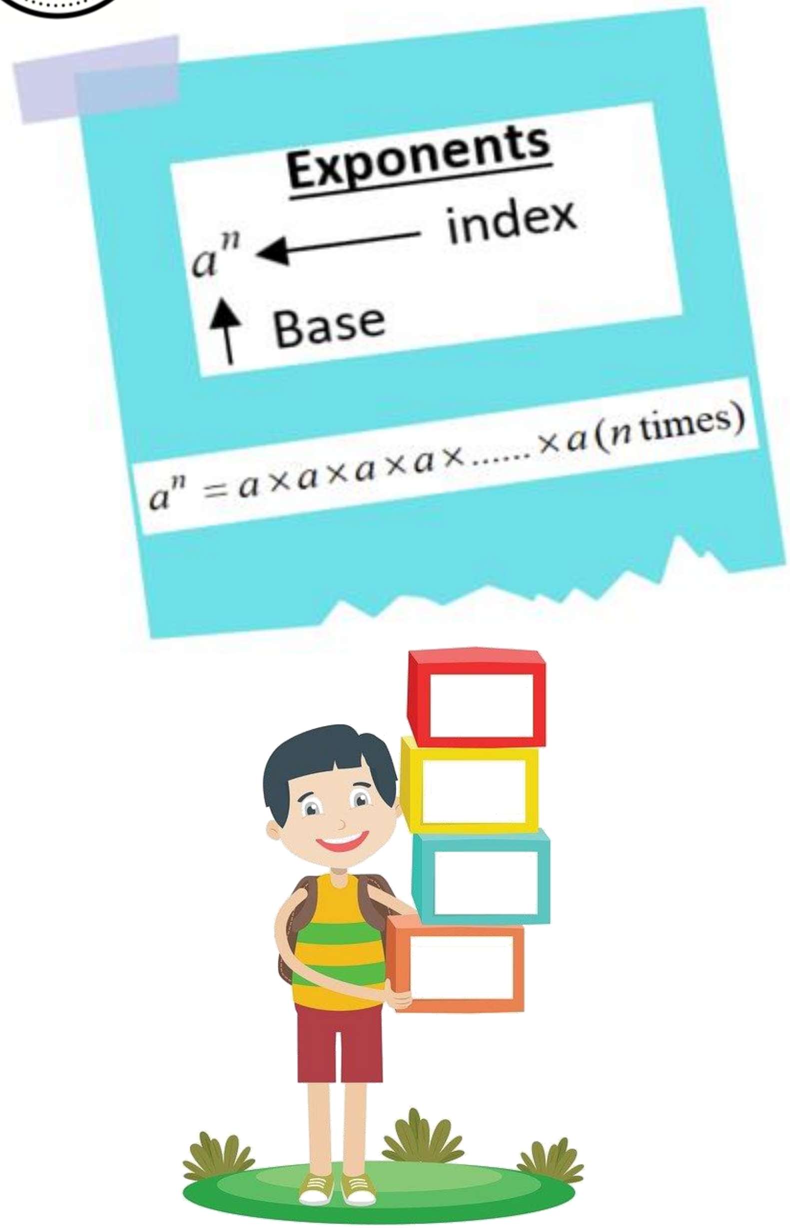
Suhana Binti Ramli

Editor

INDICES & LOGARITHM

- Laws of Indices
- Define Index
- Laws of Logarithm
- Define Logarithm
- Simplify Logarithm Expressions

1



LAW OF INDICES

RULES	EXAMPLES
a) $e^m \times e^n = e^{m+n}$	$e^5 \times e^2 = e^{5+2} = e^7$
b) $e^m \div e^n = e^{m-n}$	$e^5 \div e^2 = e^{5-2} = e^3$
c) $(e^m)^n = e^{mn}$	$(e^2)^3 = e^6$
d) $(mn)^2 = m^2n^2$	$(2x)^2 = 2^2x^2 = 4x^2$
e) $m^{-n} = \frac{1}{m^n}$	$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$
g) $e^0 = 1$	$2^0 = 1$
f) $\sqrt[n]{e^m} = e^{\frac{m}{n}}$	$\sqrt[5]{2^3} = 2^{\frac{3}{5}}$

2

LAW OF INDICES

There are 2 methods for finding the value of an index

METHOD 1 :
Change to the SAME BASE (if can)

METHOD 2 :
Place the base log 10 on the left and right of the equation

3

LAW OF INDICES

Example 1 Find the value of *x* for the equation $8^x = 32$.

Method 1

Change to the SAME BASE

By comparing base and index of the terms on the left and on the right

Simplify using law of indices
$(e^m)^n = e^{mn}$

$$8^x = 32$$
$$(2^3)^x = 2^5$$
$$2^{3x} = 2^5$$
$$\therefore 3x = 5$$
$$x = \frac{5}{3}$$

When the BASE is the SAME then the INDEX can be equated

4

LAW OF INDICES

Example 1 Find the value of x for the equation $8^x = 32$.

Method 2 Place the base log 10 on the left and right of the equation

$$8^x = 32$$

$$log_{10}8^x = log_{10}2^5$$

$$xlog8 = 5log2$$

$$\therefore x = \frac{5log2}{log8}$$

$$x = \frac{5}{3}$$

10 base logs can be calculated using a calculator

5

LAW OF INDICES

Example 2

Given $8^{2x} \cdot 4 = 16^{x-1}$. Find the value of x.

Method 1

Change to the SAME BASE

By comparing base and index of the terms on the left and on the right

Simplify using law of indices
$e^m \times e^n = e^{m+n}$

$$8^{2x} \cdot 4 = 16^{x-1}$$

$$(2^3)^{2x} \cdot 2^2 = (2^4)^{x-1}$$

$$2^{6x+2} = 2^{4(x-1)}$$

When the BASE is the SAME then the INDEX can be equated

$$\therefore 6x + 2 = 4(x-1)$$

$$6x + 2 = 4x - 4$$

$$6x - 4x = -4 - 2$$

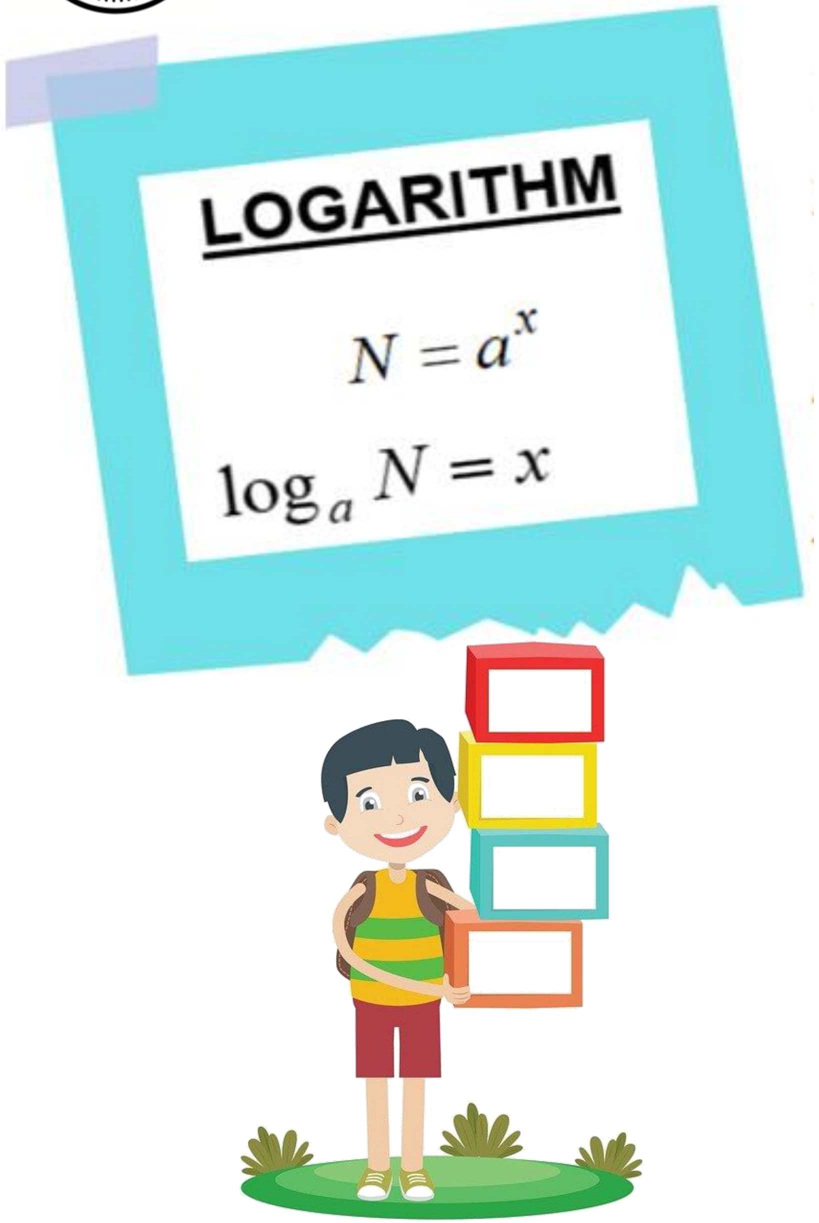
$$2x = -6$$

$$x = -3$$

Remember !
To use the second method, you need to understand the law of logarithms first

6

LAW OF LOGARITHMS



RULES	EXAMPLES
1) $log_a\, a = 1$	$log_3\, 3 = 1$
2) $log_a\, 1 = 0$	$log_3\, 1 = 0$
3) $log_N\, M = \frac{log_a\, M}{log_a N}$	$log_3\, 5 = \frac{log_{10}\, 5}{log_{10} 3}$
4) $log_a\, MN = log_a\, M + log_a\, N$	$log_3\, 2 \cdot 5 = log_3\, 2 + log_3\, 5$
5) $log_a\, \frac{M}{N} = log_a\, M - log_a\, N$	$log_3\, \frac{2}{5} = log_3\, 2 - log_3\, 5$
6) $log_a M^b = b\, log_a\, M$	$log_3 2^5 = 5\, log_3\, 2$
7) $M^{log_m N} = N$	$3^{log_3 25} = 25$

7

LAW OF LOGARITHMS

There are 3 methods for finding the value of logarithm

METHOD 1 : Change to the SIMPLEST FORM (if can)

METHOD 2 : Change to INDEX FORM

METHOD 3 : Change to the BASE 10

You can choose one

LAW OF LOGARITHMS

Example 1

Find the value of $log_2 8$

Method 1

Change to the SIMPLEST FORM

$= log_2 8$

change to the form

$= log_2 2^3$

move the power

$= 3log_2 2$

Law: $log_a a = 1$

$= 3(1)$

$= 3$

Method 2

Change to INDEX FORM

$log_2 8 = x$

change from log form to index form

$8 = 2^x$

Change to the same BASE

$2^3 = 2^x$

$\therefore 3 = x$

When the BASE is the SAME then the INDEX can be equated

9

LAW OF LOGARITHMS

Example 1

Find the value of $log_2 8$

Method 3 Rule #

Change to the BASE 10

$$= log_2\ 8$$

change to the base 10

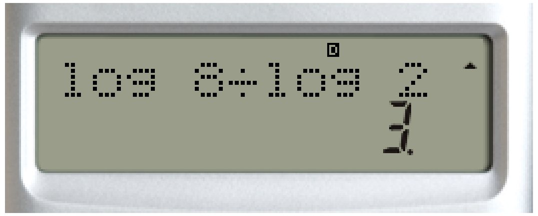
$$= \frac{log_{10}8}{log_{10}2}$$

$$= \frac{0.903}{0.301}$$

Calculate by using calculator

$$= 3$$

HOW TO PRESS THE CALCULATOR



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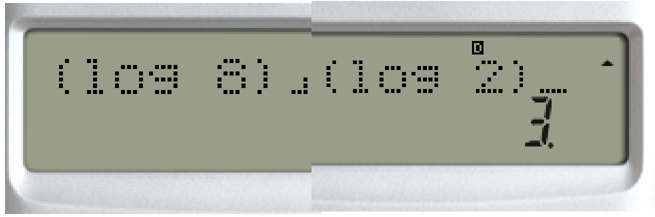


LAW OF LOGARITHMS

Example 2

Simplify each of the following

a) $log_2x + log_2(2x+1)$

$= log_2x(2x+1)$ Rule #4

$= log_2(2x^2 + x)$

b) $log_5 25x - log_5 5x$

$= log_5 \dfrac{25x}{5x}$ Rule #5

$= log_5 5$ Rule #1

$= 1$

c) $log_3 9 - log_3 5 + log_3 15$

$= log_3 \dfrac{9 \cdot 15}{5}$ Rule #4 Rule #5

$= log_3 27$

$= log_3 3^3$ Rule #6

$= 3log_3 3$ Rule #1

$= 3(1)$

$= 3$

BASIC RULES OF DIFFERENTIATION

- Constant Rule
- Constant Multiple Rule
- Power Function Rule
- Sum & Difference Rule
- Composite Function
- Chain Rule
- Product Rule
- Quotient Rule

11

Basic Rules Of Differentiation

Constant Rule

$$y = c$$
$$\frac{dy}{dx} = 0$$

Constant Multiple Rule

$$y = ax$$
$$\frac{dy}{dx} = a$$

Power Function Rule

$$y = x^n$$
$$\frac{dy}{dx} = nx^{n-1}$$

$$y = ax^n$$
$$\frac{dy}{dx} = nax^{n-1}$$

Sum & Difference Rule

$$y = f(x) \pm g(x)$$
$$\frac{dy}{dx} = f'(x) \pm g'(x)$$

12

Basic Rules Of Differentiation

How to differentiate ?

STEPS:

1. **Bring the POWER up front**
2. **POWER is subtracted by 1**

Example 1

Differentiate

$$y = x^5$$

$$\frac{dy}{dx} = \overset{1}{5}x^{\overset{2}{5-1}}$$

$$= 5x^4$$

Example 2

Differentiate

$$y = 3x^4$$

$$\frac{dy}{dx} = \overset{1}{4} \cdot 3x^{\overset{2}{4-1}}$$

$$= 12x^3$$

13

Basic Rules Of Differentiation

CHAIN RULE

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

STEPS:

1. Find u & y
   - u in the bracket & y the remain
2. Differentiate u & y
3. Substitute into the formula
4. Solve/simplify
5. Replace u

PRODUCT RULE

$$\frac{dy}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$$

STEPS:

1. Find u & v
2. Differentiate u & v
3. Substitute into the formula
4. Factorize
5. Solve/Simplify

QUOTIENT RULE

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

STEPS:

1. Find u & v
2. Differentiate u & v
3. Substitute into the formula
4. Factorize
5. Solve/Simplify

14

IMPORTANT !!

a) Must have POWER $y = \sqrt[5]{x^3}$ → $y = x^{\frac{3}{5}}$

b) **POWER** must be **numerator** $y = \dfrac{2}{x^4}$ → $y = 2x^{-4}$

c) $x^0 = 1$

d) Expand $y = (x+2)(1-x)$ → $y = 2 - x - x^2$

$y = (x+2)^2$ → $y = x^2 + 4x + 4$

d) Separate $y = \dfrac{2x^5 - x^2 + 5}{x^2}$ → $y = \dfrac{2x^5}{x^2} - \dfrac{x^2}{x^2} + \dfrac{5}{x^2}$

$y = 2x^3 - 1 + 5x^{-2}$

Example 1

Differentiate $y = 7x^3 - \frac{x^2}{3} + 5x + 1$

ATTENTION!
$\frac{x^2}{3} = \frac{1}{3}x^2$

Let's differentiate

$$\frac{dy}{dx} = (3)7x^{3-1} - 2\frac{x^{2-1}}{3} + (1)5x^{1-1} + 0$$

$$= 21x^2 - \frac{2}{3}x + 5$$

REMEMBER!
$x^0 = 1$

$$s = (3t + 2)^2$$

$$\therefore s = 9t^2 + 12t + 4$$

ATTENTION!
$(3t + 2)^2$
$= (3t + 2)(3t + 2)$
$= 9t^2 + 6t + 6t + 4$
$= 9t^2 + 12t + 4$

Now, let us differentiate

$$\frac{ds}{dt} = (2)9x^{2-1} + (1)12x^{1-1} + 0$$

$$= 18x + 12$$

16

Example 2

Differentiate $y = \dfrac{6x^4 + 3x^3 - \sqrt{5x} + 1}{x}$

ATTENTION!

$\sqrt{5x} = \sqrt{5}\sqrt{x}$

$= \sqrt{5}x^{\frac{1}{2}}$

$$y = \frac{6x^4}{x} + \frac{3x^3}{x} - \frac{\sqrt{5x}}{x} + \frac{1}{x}$$

Simplify using law of indices

$\frac{e^m}{e^n} = e^{m-n}$ & $\frac{1}{m^n} = m^{-n}$

$$= 6x^{4-1} - 3x^{3-1} - \sqrt{5}x^{\frac{1}{2}-1} + x^{-1}$$

$$= 6x^3 - 3x^2 - \sqrt{5}x^{-\frac{1}{2}} + x^{-1}$$

Then differentiate

$$y = 6x^3 - 3x^2 - \sqrt{5}x^{-\frac{1}{2}} + x^{-1}$$

$$\frac{dy}{dx} = (3)6x^{3-1} - (2)3x^{2-1} - \left(-\frac{1}{2}\right)\sqrt{5}x^{-\frac{1}{2}-1} + (-1)x^{-1-1}$$

$$= 18x^2 - 6x - \frac{\sqrt{5}}{2}x^{-\frac{3}{2}} - x^{-2}$$

$$= 18x^2 - 6x - \frac{1}{2}\sqrt{\frac{5}{x^3}} - \frac{1}{x^2}$$

17

Basic Rules Of Differentiation

Composite Function

$$y = (ax^n + b)^k$$

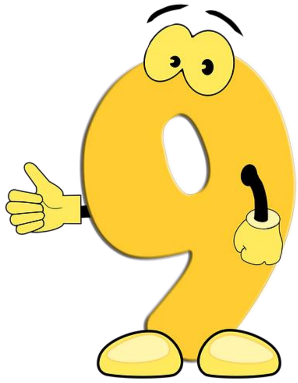
$$\frac{dy}{dx} = k(ax^n + b)^{k-1} \cdot n \cdot ax^{n-1}$$

Example

Differentiate $y = (7 - 6x^2)^8$

$$\frac{dy}{dx} = \overset{1}{\boxed{8(7-6x^2)^{8-1}}} \cdot \overset{2}{\boxed{(-12x)}}$$

$$= -96x(7-6x^2)^7$$



Composite Function

1) Differentiate **OUTSIDE** (power value)
2) Differentiate **INSIDE** (value in bracket)

18

Rules Of Differentiation

Chain Rule

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

STEPS:

1. Find u & y
   - u in the bracket & y the remain
2. Differentiate u & y
3. Substitute into the formula
4. Solve/simplify
5. Replace u

Example

Differentiate $y = (7 - 6x^2)^8$

$u = 7 - 6x^2 \qquad y = u^8$ STEP 1

$\frac{du}{dx} = -12x \qquad \frac{dy}{du} = 8u^7$ STEP 2

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$\frac{dy}{dx} = 8u^7 \cdot (-12x)$ STEP 3

$= -96x(u)^7$ STEP 4

$= -96x(7 - 6x^2)^7$ STEP 5

19

Example

Differentiate $y = \dfrac{4}{\sqrt[3]{5 - x^2}}$

change to the form

$$y = 4(5 - x^2)^{-\frac{1}{3}}$$

Composite Function

$$\frac{dy}{dx} = \left(-\frac{1}{3}\right)4(5 - x^2)^{-\frac{1}{3}-1} \cdot (2)(-x^{2-1})$$

$$= -\frac{4}{3}(5 - x^2)^{-\frac{4}{3}} \cdot -2x$$

$$= \frac{8}{3}x(5 - x^2)^{-\frac{4}{3}}$$

$$= \frac{8x}{3\sqrt[3]{(5 - x^2)^4}}$$

Chain Rule

$$y = 4(5 - x^2)^{-\frac{1}{3}}$$

$$u = 5 - x^2 \qquad y = 4u^{-\frac{1}{3}}$$

$$\frac{du}{dx} = -2x \qquad \frac{dy}{du} = -\frac{4}{3}u^{-\frac{4}{3}}$$

$$\frac{dy}{dx} = -\frac{4}{3}u^{-\frac{4}{3}} \cdot (-2x)$$

$$= \frac{8}{3}x\, u^{-\frac{4}{3}}$$

$$= \frac{8}{3}x\,(5 - x^2)^{-\frac{4}{3}}$$

$$= \frac{8x}{3\sqrt[3]{(5 - x^2)^4}}$$

20

Rules Of Differentiation

Product Rule

$$\frac{dy}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$$

STEPS:

1. Find u & v
2. Differentiate u & v
3. Substitute into the formula
4. Factorize
5. Solve/Simplify

Example 1

Differentiate $y = x^5(2x+8)^6$

$u = x^5$ $\quad v = (2x+8)^6$ STEP 1

$\frac{du}{dx} = 5x^4$ $\quad \frac{dv}{dx} = 6(2x+8)^5(2)$

$= 12(2x+8)^5$ STEP 2

$$\frac{dy}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$$

$\frac{dy}{dx} = (2x+8)^6 \cdot 5x^4 + x^5 \cdot [12(2x+8)^5]$ STEP 3

$= 5x^4(2x+8)^6 + 12x^5(2x+8)^5$

$= x^4(2x+8)^5[5(2x+8)+12x]$ STEP 4

$= x^4(2x+8)^5[10x+40+12x]$

$= x^4(2x+8)^5(22x+40)$

STEP 5

21

Example 2

Differentiate $y = \sqrt{2x+3}(2-3x^2)^5$

$$u = (2x+3)^{\frac{1}{2}}$$

$$\frac{du}{dx} = \frac{1}{2}(2x+3)^{-\frac{1}{2}}(2)$$

$$= (2x+3)^{-\frac{1}{2}}$$

$$v = (2-3x^2)^5$$

STEP 1

$$\frac{dv}{dx} = 5(2-3x^2)^4(-6x)$$

STEP 2

$$= -30x(2-3x^2)^4$$

$$\frac{dy}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$$

$$\frac{dy}{dx} = (2-3x^2)^5 \cdot (2x+3)^{-\frac{1}{2}} + (2x+3)^{\frac{1}{2}}[-30x(2-3x^2)^4]$$

STEP 3

$$= (2-3x^2)^5 \cdot (2x+3)^{-\frac{1}{2}} - 30x(2x+3)^{\frac{1}{2}}(2-3x^2)^4$$

$$= (2-3x^2)^4(2x+3)^{-\frac{1}{2}}[(2-3x^2) - 30x(2x+3)]$$

STEP 4

$$\left.\begin{aligned} &= (2-3x^2)^4(2x+3)^{-\frac{1}{2}}[2-3x^2-60x^2-90x] \\ &= (2-3x^2)^4(2x+3)^{-\frac{1}{2}}[2-63x^2-90x] \end{aligned}\right\}$$

STEP 5

Rules Of Differentiation

Quotient Rule

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

STEPS:

1. Find u & v
2. Differentiate u & v
3. Substitute into the formula
4. Factorize
5. Solve/Simplify

Example 1

Differentiate $y = \frac{x^5}{(2x+8)^6}$

STEP 1

$u = x^5 \qquad v = (2x+8)^6$

STEP 2

$\frac{du}{dx} = 5x^4 \qquad \frac{dv}{dx} = 6(2x+8)^5(2)$

$= 12(2x+8)^5$

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

STEP 3

$$\frac{dy}{dx} = \frac{(2x+8)^6 \cdot 5x^4 - x^5 \cdot [12(2x+8)^5]}{((2x+8)^6)^2}$$

STEP 4

$$= \frac{x^4(2x+8)^5[5(2x+8) - 12x]}{(2x+8)^{12}}$$

STEP 5

$$= \frac{x^4[10x+40-12x]}{(2x+8)^7} = \frac{x^4(40-2x)}{(2x+8)^7}$$

Example 2

Differentiate $s = \frac{(7t+2)^2}{4(t^3+1)^3}$

$$u = (7t+2)^2$$
$$\frac{du}{dt} = 2(7t+2)(7)$$
$$= 14(7t+2)$$

$$v = 4(t^3+1)^3$$
$$\frac{dv}{dt} = 12(t^3+1)^3(3t^2)$$
$$= 36t^2(t^3+1)^3$$

$$\frac{ds}{dt} = \frac{v\frac{du}{dt} - u\frac{dv}{dt}}{v^2}$$

$$\frac{ds}{dt} = \frac{4(t^3+1)^3 \cdot 14(7t+2) - (7t+2)^2 \cdot [36t^2(t^3+1)^3]}{(4(t^3+1)^3)^2}$$

$$= \frac{56(t^3+1)^3 \cdot (7t+2) - 36t^2(t^3+1)^3(7t+2)^2}{16(t^3+1)^6}$$

$$= \frac{4(t^3+1)^2(7t+2)[14(t^3+1) - 9t^2(7t+2)]}{16(t^3+1)^6}$$

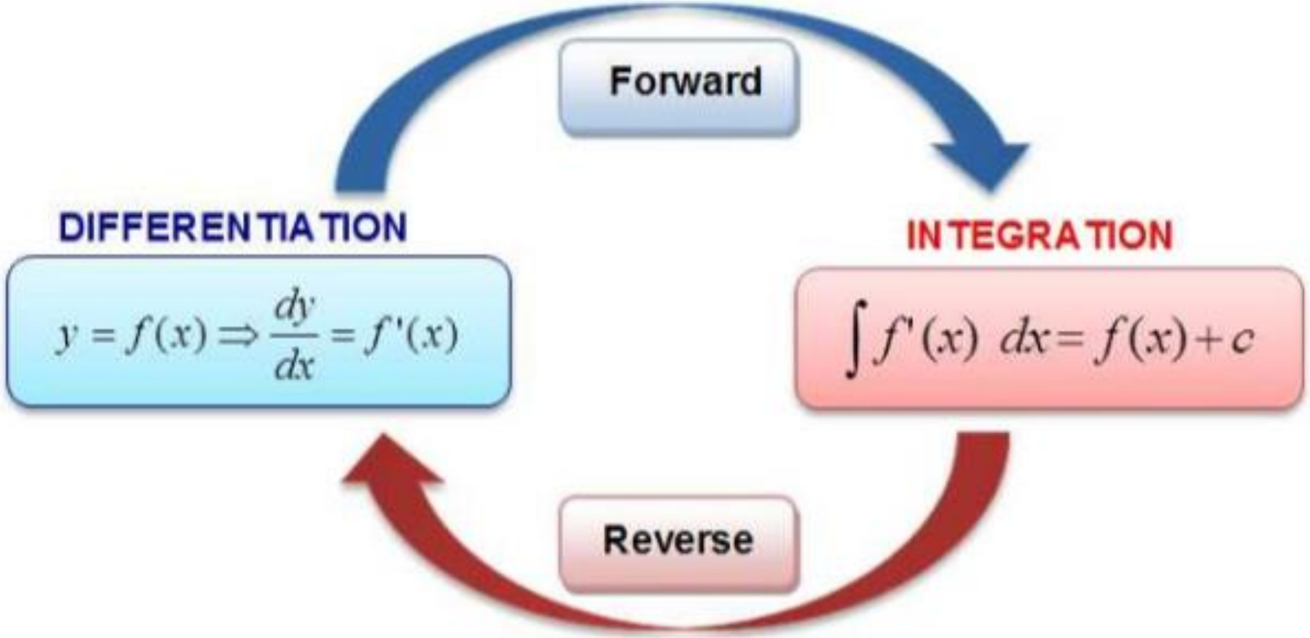
$$= \frac{4(t^3+1)^2(7t+2)[14t^3+14-63t^3-18t^2]}{16(t^3+1)^6} = \frac{(7t+2)(14-18t^2-49t^3)}{4(t^3+1)^4}$$

BASIC OF INTEGRATION FUNCTION

Constant
Basic Algebraic Function
Addition & Subtraction
Composite Function
Substitution Method

Basic of Integration Function

- Integration is the inverse or reverse process of differentiation
- The symbol for integration is $\int \ldots dx$
- There are no limits of integration in indefinite integral
- In indefinite integral, the constant c must be stated



25

Basic of Integration Function

CONSTANT FUNCTION

$$\frac{dy}{dx} = \int f'(x)\,dx \pm \int g'(x)\,dx$$

ADDITION & SUBTRACTION

$$f'(x) = \int f(x) \pm g(x)\,dx$$

ALGEBRAIC FUNCTION

$$\frac{dy}{dx} = \int ax\,dx \quad, n \neq -1$$
$$= \frac{ax^{1+1}}{1+1} + c$$

$$\frac{dy}{dx} = \int ax^{n}\,dx, n \neq -1$$
$$= \frac{ax^{n+1}}{n+1} + c$$

$$\frac{dy}{dx} = \int x^{n}\,dx, n \neq -1$$
$$= \frac{x^{n+1}}{n+1} + c$$

Basic of Integration Function

HOW TO INTEGRATE ?

STEPS:

1. POWER is added by 1
2. Denominator : new POWER
3. '+ c' at the end

Formula

$$\int x^n dx$$

$$= \frac{x^{n+1}}{n+1} + c$$

1

2

3

27

Basic of Integration Function

Constant Multiple Rule

$$\frac{dy}{dx} = \int ax\,dx \qquad n \neq -1$$

$$= \frac{ax^{1+1}}{1+1} + c$$

Power Function Rule

$$\frac{dy}{dx} = \int x^n\,dx \qquad n \neq -1$$

$$= \frac{x^{n+1}}{n+1} + c$$

$$\frac{dy}{dx} = \int ax^n\,dx \qquad n \neq -1$$

$$= \frac{ax^{n+1}}{n+1} + c$$

28

Basic of Integration Function

Example 1

$$\int 5x\,dx$$

$$= \frac{5x^{1+1}}{1+1} + c$$

$$= \frac{5x^2}{2} + c$$

Example 2

$$\int x^5\,dx$$

$$= \frac{x^{5+1}}{5+1} + c$$

$$= \frac{x^6}{6} + c$$

Example 3

$$\frac{dy}{dx} = \int 3x^5\,dx$$

$$= 3 \cdot \frac{x^{5+1}}{5+1} + c$$

$$= \frac{3x^6}{6} + c$$

$$= \frac{x^6}{2} + c$$

Basic of Integration Function

Addition & Subtraction

$$f'(x) = \int f(x) \pm g(x)\, dx$$

$$= \int f'(x)\, dx \pm \int g'(x)\, dx$$

Example 1

$$\int (x+3)\, dx$$

$$= \frac{x^{1+1}}{1+1} + 3x + c$$

$$= \frac{x^2}{2} + 3x + c$$

30

Example 2

$$\int 2x^3 + 5x + 3\,dx$$

$$= \frac{2x^{3+1}}{3+1} + \frac{5x^{1+1}}{1+1} + 3x + c$$

$$= \frac{2x^4}{4} + \frac{5x^2}{2} + 3x + c$$

$$= \frac{x^4}{2} + \frac{5x^2}{2} + 3x + c$$

Example 3

$$\int (3x+2)(x-7)\,dx$$

expand

$$= \int 3x^2 - 21x + 2x - 14\,dx$$

$$= \int 3x^2 - 19x - 14\,dx$$

$$= \frac{3x^{2+1}}{2+1} - \frac{19x^{1+1}}{1+1} - 14x + c$$

$$= \frac{3x^3}{3} - \frac{19x^2}{2} - 14x + c$$

$$= x^3 - \frac{19x^2}{2} - 14x + c$$

31

Basic of Integration Function

Composite Function

$$\int (ax+b)^n \, dx, \qquad n \neq -1$$

$$= \frac{(ax+b)^{n-1}}{a(ax+b)} + c$$

Substitution Method

$$\int f[g(x)] \times g'(x) \, dx$$

OR

$$\int \frac{g\prime(x)}{f[g(x)]} \, dx$$

32

Basic of Integration Function

Composite Function

$$\frac{dy}{dx} = \int (ax+b)^k \, dx$$

$$= \frac{(ax+b)^{\overset{1}{k+1}}}{\underset{2}{(a)(k+1)}} \overset{3}{+ c}$$

Example 1

$$\int (7-6x)^8$$

$$= \frac{\boxed{(7-6x)^{8+1}}^{\,1}}{\underset{2}{\boxed{(-6)(8+1)}}} \boxed{+c}^{\,3}$$

$$= \frac{(7-6x)^9}{-54} + c$$

1. POWER is added by 1
2. Denominator : $(a) \times (new\ POWER)$
3. '+ c' at the end

Example 2

$$\int \frac{7}{\sqrt[4]{1-4x}}\,dx$$

change the form

$$= \int 7(1-4x)^{-\frac{1}{4}}\,dx$$

$$= \frac{7(1-4x)^{-\frac{1}{4}+1}}{(-4)\left(-\frac{1}{4}+1\right)} + c$$

$$= \frac{7(1-4x)^{\frac{3}{4}}}{-3} + c$$

Basic of Integration Function

Substitution Method

$$\frac{dy}{dx} = \int f[g(x)] \times g'(x)dx$$

Need to find 'u'

i. In the bracket ➔ $\int 5x(2x^2+5)^3 dx$

ii. Denominator ➔ $\int \frac{2x}{(1-x^2)} dx$

iii. Highest power ➔ $\int sin3x\, cos^4 3xdx$

How To Integrate ?

STEPS:

1. Substitute $u = g(x)$
2. Differentiate $u = \frac{du}{dx} = g'(x)$
   - Then make $dx = \frac{du}{g'(x)}$
3. Substitute $g(x) = u$ and $dx = \frac{du}{g'(x)}$ into the original function
4. Solve the integral
5. Substitute $u = g(x)$ → for the final answer

STEPS:

1. Substitute $u = g(x)$
2. Differentiate $u = \frac{du}{dx} = g'(x)$
   - Then make $dx = \frac{du}{g\prime(x)}$
3. Substitute $g(x) = u$ and $dx = \frac{du}{g\prime(x)}$ into the original function
4. Solve the integral
5. Substitute $u = g(x)$ → for the final answer

Example 1

$$\int (7-6x)^8\, dx$$

STEP 1

$$u = 7 - 6x$$

STEP 2

$$\frac{du}{dx} = -6$$

$$dx = \frac{du}{-6}$$

STEP 3

$$\frac{dy}{dx} = \int u^8 \cdot \frac{du}{-6}$$

STEP 4

$$\frac{dy}{dx} = \frac{1}{-6}\int u^8\, du$$

$$= \frac{1}{-6} \cdot \frac{u^{8+1}}{8+1} + c$$

$$= \frac{u^9}{-54} + c$$

STEP 5

$$= \frac{(7-6x)^9}{-54} + c$$

Example 2

$$\int \frac{4x}{(x^2+1)^5}\,dx$$

change the form

$$= 4x\int (x^2+1)^{-5}\,dx$$

$$u = x^2 + 1$$

$$\frac{du}{dx} = 2x$$

$$dx = \frac{du}{2x}$$

$$= 4x\int u^{-5}\cdot\frac{du}{2x}$$

$$= 2\int u^{-5}\,du$$

$$= 2\cdot\frac{u^{-4}}{-4} + c$$

$$= \frac{1}{2u^2}\cdot +c$$

$$= \frac{1}{2(x^2+1)^2}\cdot +c$$

Example 3

$$\int 3\sqrt{\left(\frac{x}{4}+5\right)}\,dx$$

change the form

$$= \int 3\left(\frac{x}{4}+5\right)^{\frac{1}{2}} dx$$

Composite Function

$$= \int 3\left(\frac{x}{4}+5\right)^{\frac{1}{2}} dx$$

$$= \frac{3\left(\frac{x}{4}+5\right)^{\frac{1}{2}+1}}{\left(\frac{1}{4}\right)\left(\frac{1}{2}+1\right)} + c$$

$$= 8\left(\frac{x}{4}+5\right)^{\frac{1}{2}} + c$$

Substitution Method

$$= \int 3\left(\frac{x}{4}+5\right)^{\frac{1}{2}} dx$$

$$u = \frac{x}{4}+5$$

$$\frac{du}{dx} = \frac{1}{4}$$

$$dx = 4\,du$$

$$= 3\int u^{\frac{1}{2}} \cdot 4du$$

$$= 3 \cdot \frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1} + c$$

$$= 8u^{\frac{1}{2}} \cdot +c$$

$$= 8\left(\frac{x}{4}+5\right)^{\frac{1}{2}} + c$$

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