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VEHICLE DYNAMICS

DJA50082

DEPARTMENT OF MECHANICAL ENGINEERING
POLITEKNIK PORT DICKSON

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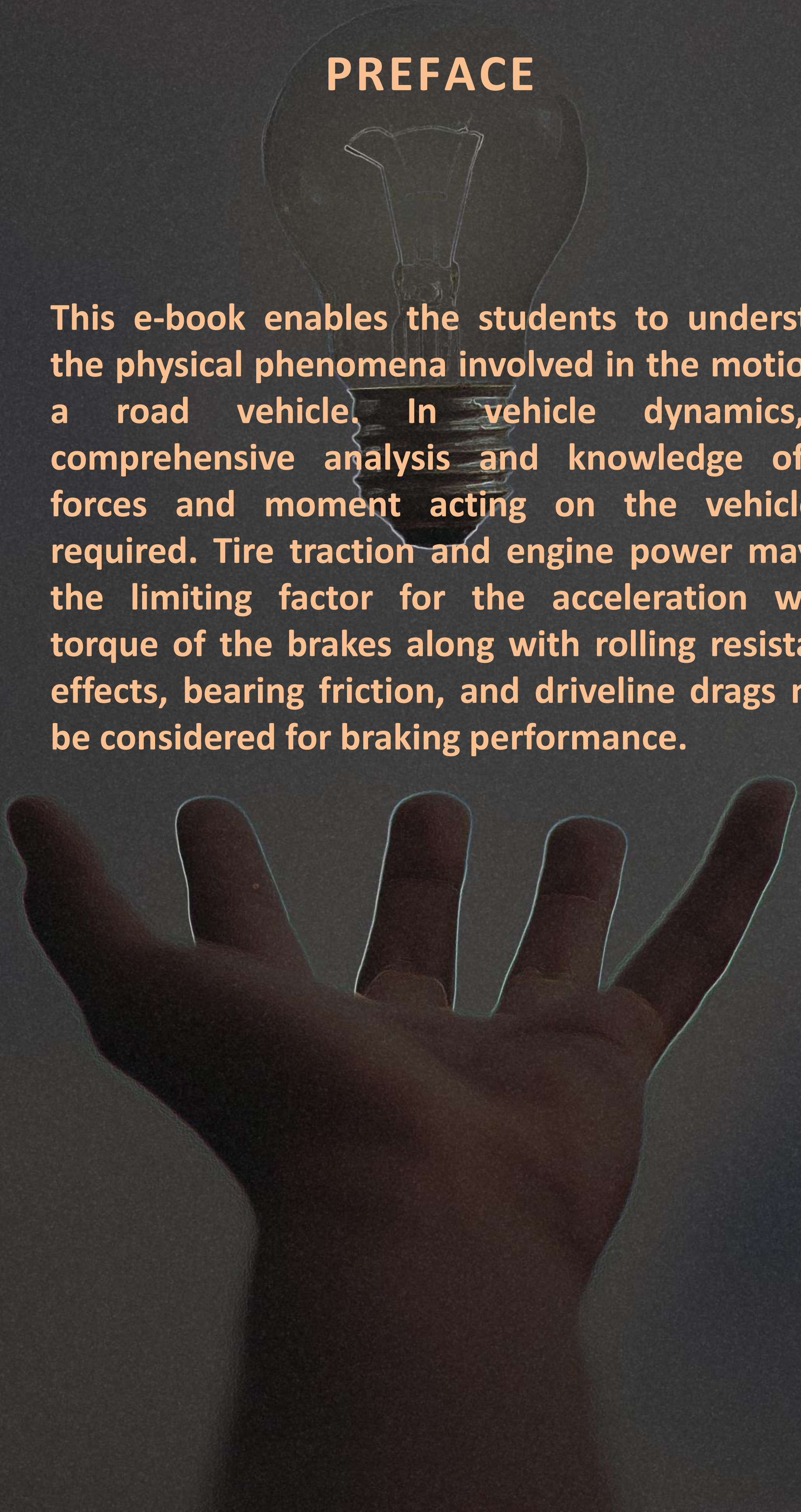
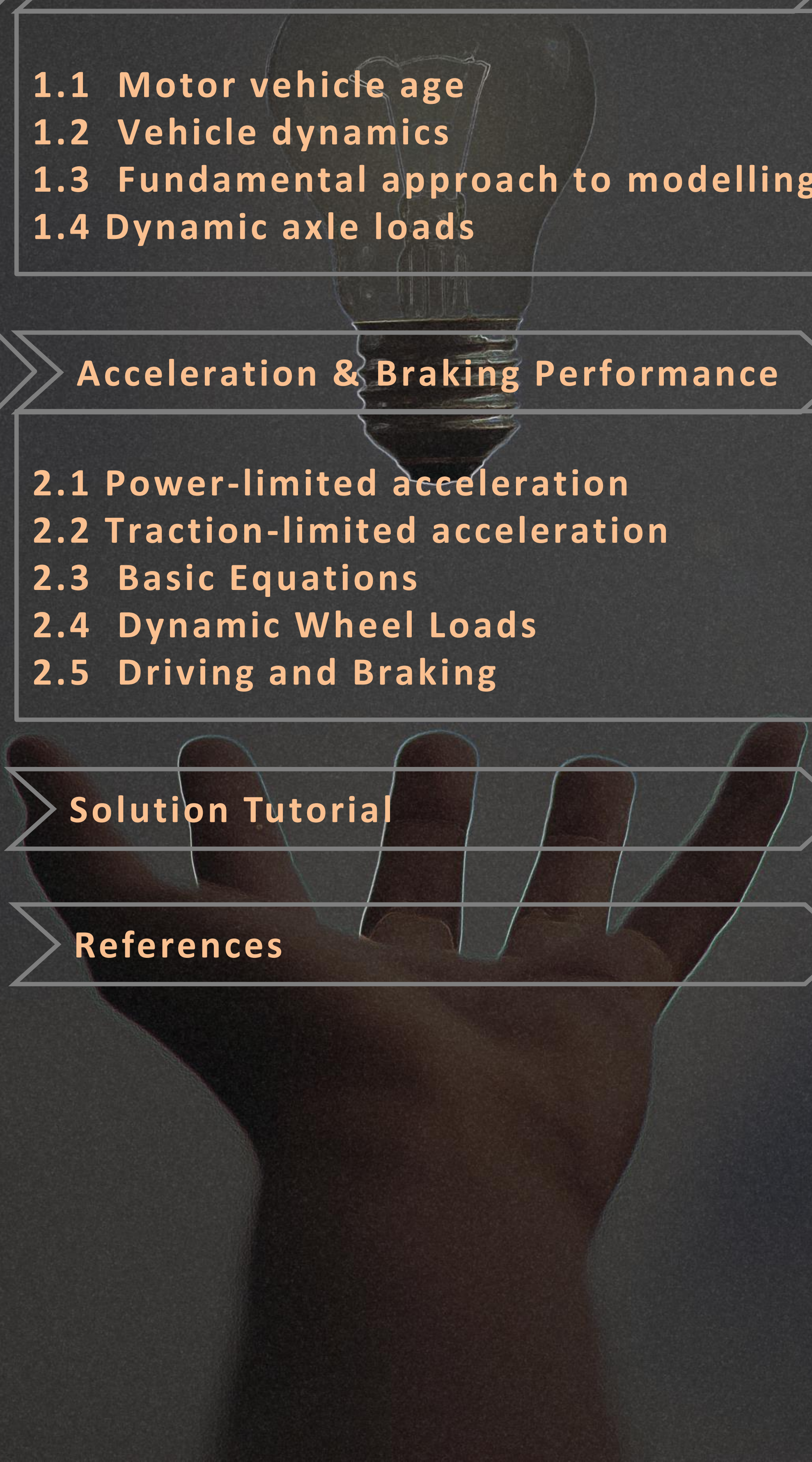
PREFACE



This e-book enables the students to understand the physical phenomena involved in the motion of a road vehicle. In vehicle dynamics, a comprehensive analysis and knowledge of all forces and moment acting on the vehicle is required. Tire traction and engine power may be the limiting factor for the acceleration whilst torque of the brakes along with rolling resistance effects, bearing friction, and driveline drags must be considered for braking performance.



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CHAPTER 1 – INTRODUCTION TO VEHICLE DYNAMICS

1. MOTOR VEHICLE AGE

2. VEHICLE DYNAMICS

3. FUNDAMENTAL APPROACH TO MODELLING

4. DYNAMIC AXLE LOADS

MOTOR VEHICLE AGE

1769

- French military engineer, Nicholas Joseph Cugnot built a three-wheeled steam-driven vehicle for pulling artillery pieces.

1784

- Steam-powered vehicle was built by the Scottish engineer, James Watt.

1802

- Richard Trevithick, an Englishman developed a steam coach that travelled from Cornwall to London.

1769

- Birth year of the modern car when German inventor, Karl Benz patented his Benz Patent-Motorwagen.

1902

- First cars accessible to the masses was the Model T manufactured by the Ford Motor Company.

1908

- One of the first engineer to write on automotive dynamics was Frederick Williams Lanchester.

1995

- The car Global Positioning System, or GPS, is introduced..

1997

- The first Toyota Prius is sold in Japan.

2010

- First commercially available plugin hybrid for sale by GM Motor Company.

Vehicle dynamics is a part of engineering primarily based on classical mechanics but it may also involved physics, electrical engineering, chemistry, communication, psychology etc. Here the focus will be laid on ground vehicles supported by wheels and tires that encompasses the interaction of:

DRIVER

- Driver can interfere with the vehicles by various means such as steering wheel, accelerator pedal, brake pedal, clutch gear shift and etc. Vehicle will provide the drivers with information such as vibrations, sounds, and instruments.

VEHICLE

- Following vehicles are listed in the ISO 3833 directive: Motorcycle, Passenger cars, Busses, Trucks, Agricultural tractors and Passenger cars with trailer.

LOAD

- Trucks are conceived for taking up load. Thus, their driving behaviour changes. In computer calculations problem occur at the determination of the inertias and the modelling of liquid loads

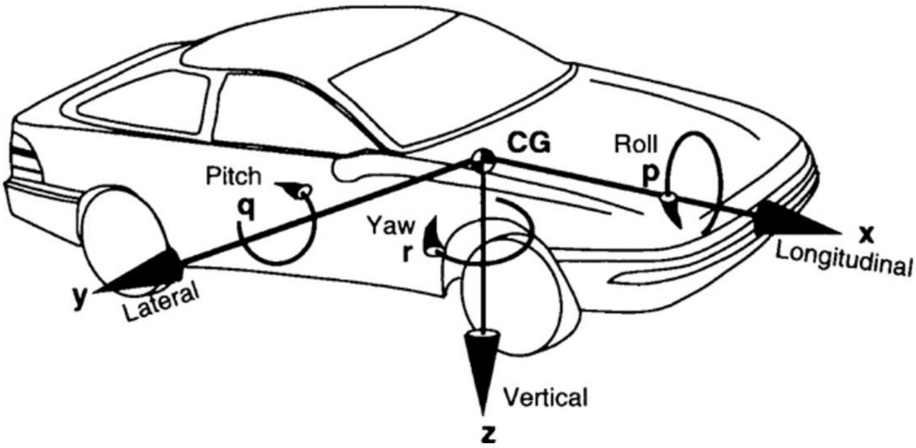
ENVIRONMENT

- Vehicle is primarily influenced by the environment. The influences can comes from road (irregularities, coefficient of friction) and air (resistance, cross wind) and can be transfer to vehicle.

The subject of vehicle dynamics is concerned with vehicles, truck, buses and special purpose vehicles on a road surface. The movement of interest are acceleration and braking, ride and turning. Dynamics behavior is determined by the forces imposed on the vehicle from the tires, gravity and aerodynamics. The car and its components are studied to determine what these sources will produce powers at a particular maneuver and trim condition and how the vehicle will respond to these forces. For this purpose, it is essential to establish a rigorous approach to modelling the systems and the conventions used to describe the motion.

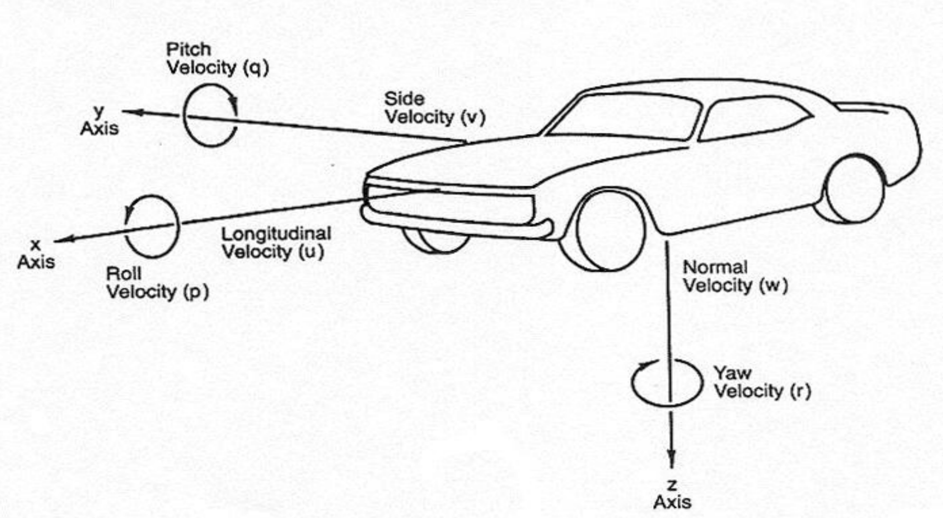
Lumped Mass

- Motor Vehicle has many components.
- When braking, the entire vehicle slows down as a unit-Represent as lumped mass at CG.
- We used braking, acceleration and most turning analyses.



Vehicle Fixed Coordinate System

- *Onboard, the vehicle motions are defined regarding a right-hand orthogonal coordinate system that originates at the CG and travels with the vehicle.*
- *x – Forward and on the Longitudinal plane of symmetry.*
- *y – Lateral out the right side of the vehicle.*
- *z – Downward with respect to the vehicle.*
- *p – Roll velocity about the x-axis.*
- *q – Pitch velocity about the y axis.*
- *r – Yaw velocity about the z-axis.*



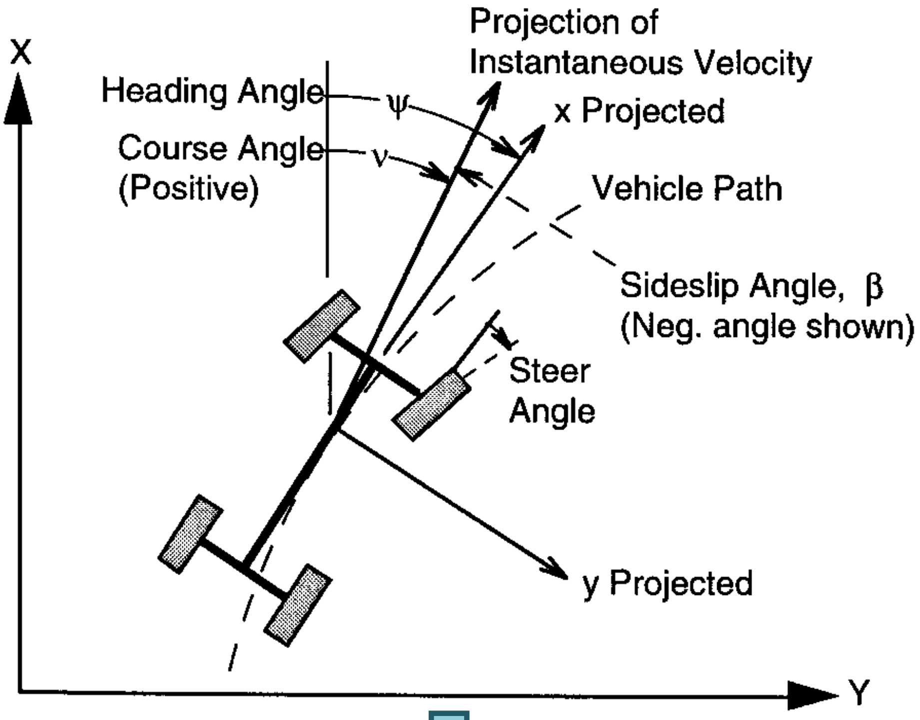
MOTION VARIABLES AND EARTH FIXED COORDINATE SYSTEM

Motion Variables

- The velocities (forward, lateral, vertical, roll, pitch and yaw) concerning the vehicle fixed coordinate system where the speeds are referenced to the earth fixed coordinate system.

Earth Fixed Coordinate System

- *Vehicle attitude and trajectory through the maneuver course are defined concerning a right-hand orthogonal axis system fixed on the earth.*
- *Usually selected to coincide with the vehicle fixed coordinate system at the point where the maneuver is started.*



X – Forward travel
Y – Travel to the right
Z – Vertical travel (positive downward)
ψ – Heading angle (angle between x and X in the ground plane)
ν – Course angle (angle between the vehicle's velocity vector and X axis)
β– Sideslip angle (angle between x axis and the vehicle velocity vector)

EULER ANGLES AND NEWTON SECOND LAW

Euler Angle

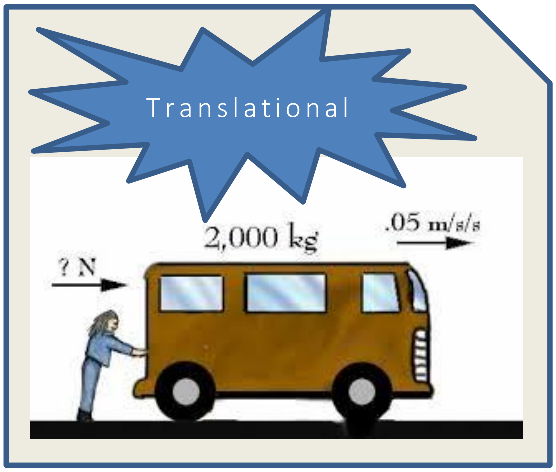
- Relation between vehicle fixed coordinate and earth fixed coordinate.
- Have 3 angles.
- Obtained by three angular rotation.
- z-axis (yaw), y-axis (pitch) and x-axis (roll).

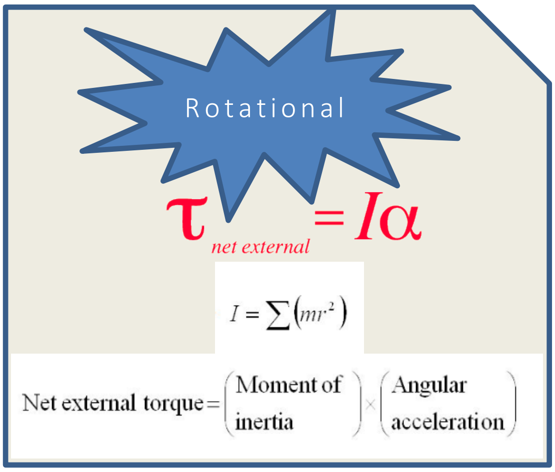
Newton Second Law

Fundamental for most vehicle dynamics analysis.

They are applied to both translational and rotational systems.

F = ma for translational, $\boldsymbol{\tau} = \boldsymbol{I\alpha}$ for rotational.





$$\tau_{net\ external} = I\alpha$$

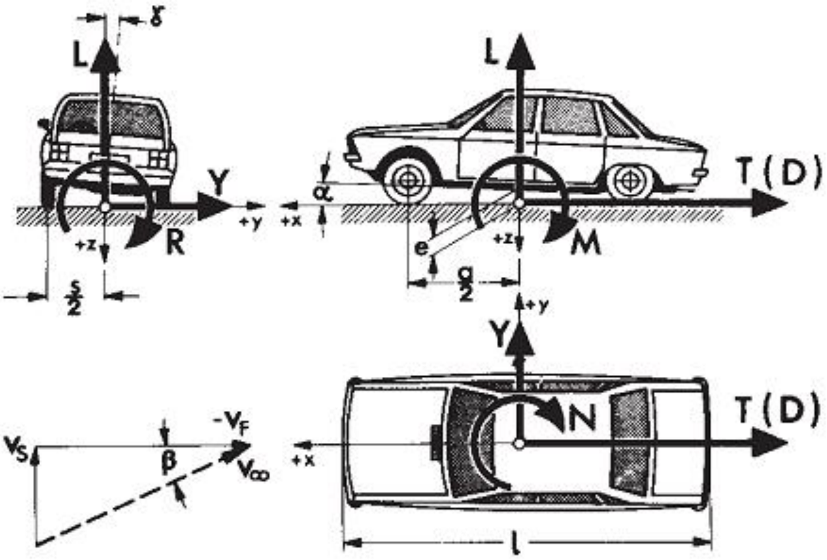
$$I = \sum (mr^2)$$

$$\text{Net external torque} = \begin{pmatrix}\text{Moment of}\\ \text{inertia}\end{pmatrix} \times \begin{pmatrix}\text{Angular}\\ \text{acceleration}\end{pmatrix}$$

FORCES AND MOMENTS

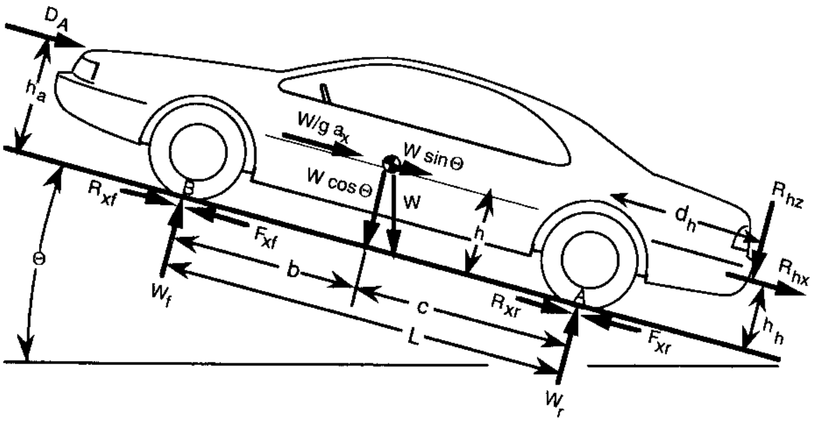
The reaction of forces and moments on a vehicle:

Direction	*Force*	*Moment*
Longitudinal (x-axis, +ve forward)	Drag	Rolling
Lateral (y-axis, +ve to the right)	Side force	Pitching
Vertical (z-axis, +ve downward)	Lift	Yawing

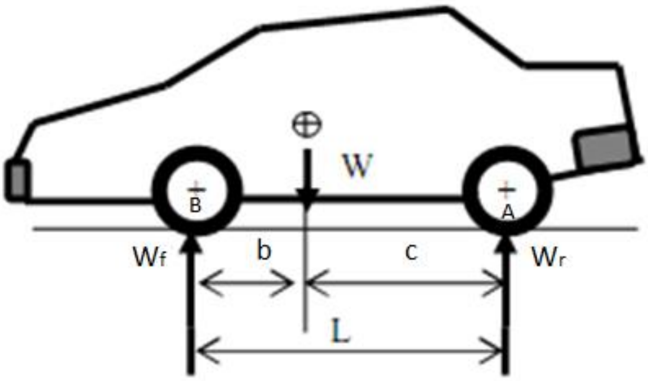


DYNAMIC AXLE LOADS

Determining axle loadings on vehicle under arbitrary conditions is simple application of Second Newton's Law. It is an important first step in analysis of acceleration and braking performance because the axle loads determine the tractive efforts obtained at each axle, affecting the acceleration, gradeability, maximum speed and drawbar effort.



- W is the weight of the vehicle acting at its CG. On a slope, it may have two components: a cosine component perpendicular to the road surface and a sine component parallel to the road.
- If the vehicle is accelerating, it is convenient to represent the effect by an equivalent inertial force known as a "d'Alembert force", acting at the center of gravity opposite to the direction of the acceleration.
- The tire will experience a force normal to the road, denoted by W_f and W_r representing the dynamics weights carried on the front and rear wheels.
- Tractive forces F_{xf} and F_{xr} or rolling resistance forces, R_{xf} and R_{xr} may act in the ground plane.
- D_A is the aerodynamic force acting on the body of the vehicle.
- R_{hz} and R_{hx} are vertical and longitudinal forces.



Take moment at point A, summation of moments at point A

$$\sum M_A = 0$$

$$W_f \ x \ L - W \ x \ c = 0$$

$$W_f = \frac{W c}{L}$$

Take moment at point B, summation of moments at point B

$$\sum M_B = 0$$

$$W \ x \ b - W_r \ x \ L = 0$$

$$W_r = \frac{W b}{L}$$

Where;

- W_f = weight on the front axle
- W_r = weight on the rear axle
- L = wheelbase of the vehicle
- b = distance from front wheel to the center of gravity
- c = distance from rear wheel to the center of gravity

- Vehicle accelerate on level ground at low speed.
- Assume Aerodynamic Drag, D_A is zero.
- Assume no trailer hitch forces, the loads on the axle are:

$$W_f = W\left(\frac{c}{L} - \frac{a_x}{g}\frac{h}{L}\right) = W_{fs} - W\frac{a_x}{g}\frac{h}{L}$$

$$W_r = W\left(\frac{b}{L} + \frac{a_x}{g}\frac{h}{L}\right)$$

$$= W_{rs} + W\frac{a_x}{g}\frac{h}{L}$$

** When a vehicle is accelerating, the load is transferred from the front axle to the rear axle in proportion to acceleration (normalized by the gravitational acceleration) and the ratio of CG height to the wheelbase.*

LOADS ON GRADES

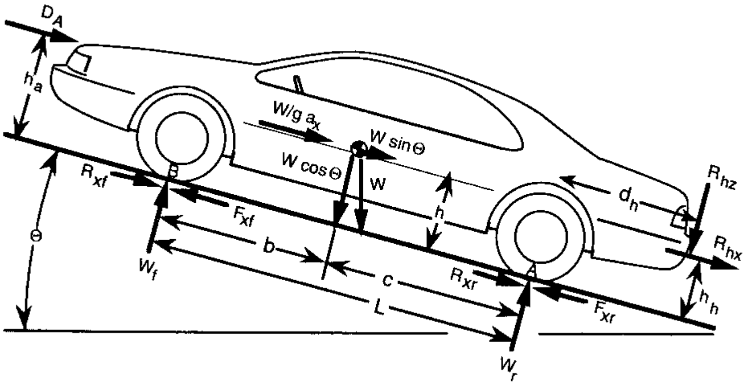
Static component

+

Load transfer from front to rear due to other forces

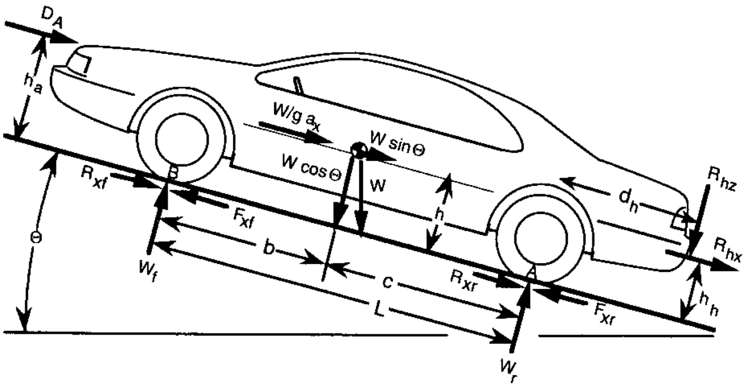
=

The loads carried on each axle



Summation of moment at point A

$$W_f L + D_A h_a + \frac{W}{g} a_x h + R_{hx} h_h + R_{hz} d_h + Wh\ sin\theta - Wc\ cos\theta = 0$$

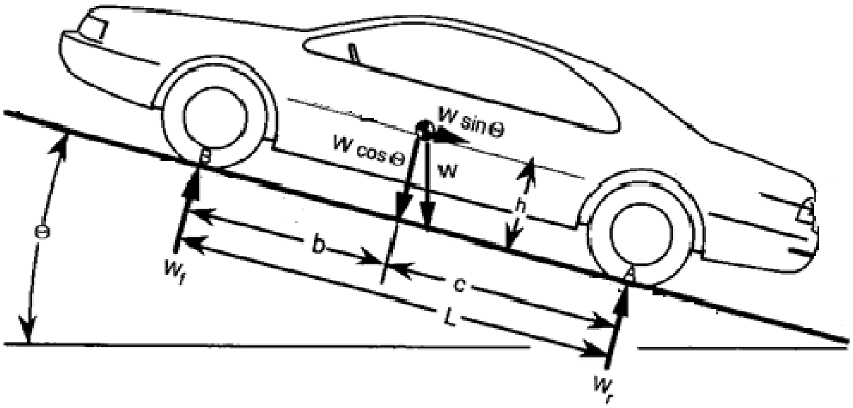


$$W_f L + D_A h_a + \frac{W}{g} a_x h + R_{hx} h_h + R_{hz} d_h + Wh\ sin\theta - Wc\ cos\theta = 0$$

- ❖ In static condition, R_{hx}, R_{hz}, a_x, D_A is zero
- ❖ sin 0 = 0 and cos 0 = 1
- ❖ Final equation:

$$W_f = \frac{Wc}{L}\ ; take\ moment\ at\ B\ so\ W_r = \frac{Wb}{L}$$

LOADS ON GRADES



Take moment at point A, summation of moments at point A,

$$\sum M_A = 0$$

$$W_f L - W\cos\theta c + W\sin\theta h = 0$$

$$W_f = \frac{Wc}{L}\cos\theta - \frac{Wh}{L}\sin\theta = W_{fs}\cos\theta - \frac{Wh}{L}\sin\theta$$

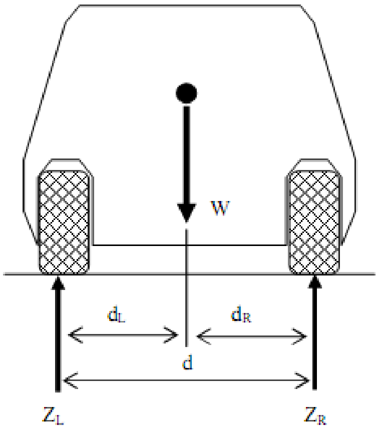
Take moment at point B, summation of moments at point B,

$$\mathbf{\Sigma} M_B = 0$$

$$W_r L - W\cos\theta b - W\sin\theta h = 0$$

$$W_r = \frac{Wb}{L}\cos\theta + \frac{Wh}{L}\sin\theta = W_{rs}\cos\theta + \frac{Wh}{L}\sin\theta$$

IDENTIFY CENTER OF GRAVITY FROM FRONT/REAR VIEW

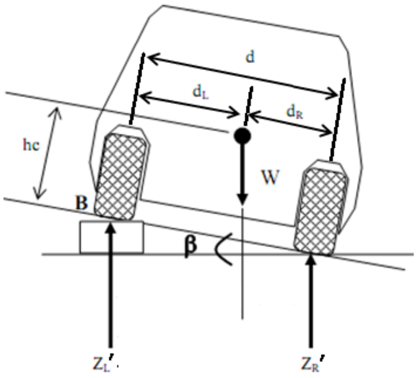


$$\sum M_B = 0$$

$$Z_R \mathrm{d} - W\, d_L = 0$$

$$d_L = \frac{Z_R\, d}{W}$$

$$d_R = \frac{Z_L\, d}{W}$$



$$\sum M_B = 0$$

$$W \cos\beta\, d_L + W \sin\beta\, h_c - Z_{R'} \cos\beta\, d = 0$$

$$W \sin\beta\, h_c = Z_{R'} \cos\beta\, d - W \cos\beta\, d_L$$

$$h_c = \frac{Z_{R'} \cos\beta\ d}{W \sin\beta} - \frac{W \cos\beta\, d_L}{W \sin\beta}$$

$$h_c = \frac{Z_{R'} \cos\beta\ d}{W \sin\beta} - \frac{\cos\beta\, d_L}{\sin\beta}$$

$$h_c = \frac{Z_{R'} d}{W} \cot\beta - d_L \cot\beta$$

Where;
Z_R = weight on the right wheel
Z_L = weight on the left wheel
d = distance *between right and left wheel*
W = *weight of the vehicle*
d_L = *distance from left wheel to the center of gravity*
d_R = *distance from right wheel to the center of gravity*
β = angle
h_c = center of gravity height

TUTORIAL 1

1) Define the center of gravity of a vehicle.

2) If the front wheel carried 65 % of the vehicle weight and the distance between the front and rear wheel is 2.1 m, calculate the distance of the gravity center to the front (b) and rear wheel (c).

3) Find the values of d_L and d_R if the load applied on the left wheel is 0.55N and the vehicle's weight is 1N. Given that distance between the left and right wheels are 2.2m.

CHAPTER 2 – ACCELERATION & BRAKING PERFORMANCE

1. POWER LIMITED ACCELERATION

2. TRACTION LIMITED ACCELERATION

3. GENERAL EQUATIONS

4. DYNAMIC WHEEL LOADS

5. DRIVING AND BRAKING

POWER LIMITED ACCELERATION

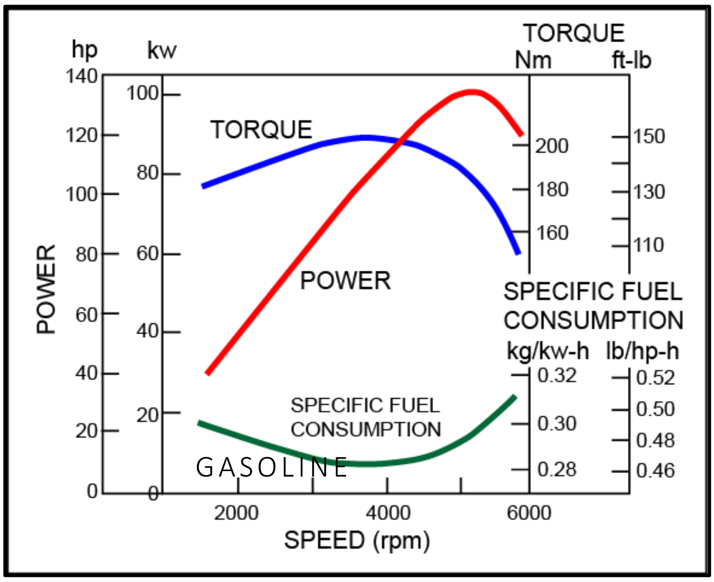
Acceleration performance can be considered as:

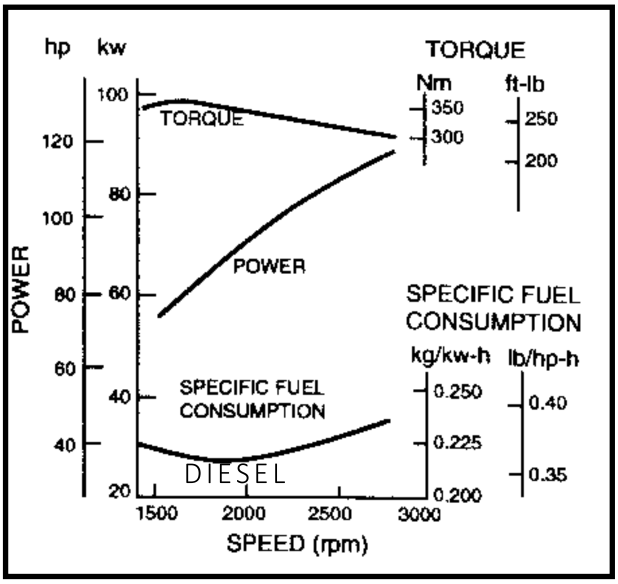
1. Engine Power Limited Acceleration
    - At higher speeds, engine power is the limiting factor.
2. Traction Limited Acceleration
    - At lower speeds, traction is the limiting factor

ACCELERATION PERFORMANCE ARE LIMIT BY

ENGINE POWER @ HIGH SPEED

TRACTION LIMIT @ LOW SPEED





<u>Engine Characteristic.</u>

1. Power limited acceleration involves an examination of engine characteristics and their interaction through power trains.
2. In a vehicle application, the engine output torque varies with speed.
3. The maximum torque is between 100 to 300 Nm at a 2500 to 4500 rpm speed for a typical passenger car engine.
4. The engine in a typical truck produces torque ten times the value at 1000 to 1500 rpm.
5. For heavier vehicle, diesel engines are usually used.

POWER LIMITED ACCELERATION – ENGINE

Difference between gasoline and diesel engine.

GASOLINE	DIESEL
Torque curve peak at midrange of engine speed	Torque curve flatter or even rise at decreasing speed
Govern by the induction system	Govern by the fuel injection system
Most efficient at 0.28 kg/kWh of specific fuel consumption.	Most efficient at 0.2i kg/kWh of specific fuel consumption.

Power to weight ratio

1. Power and torque are related by the speed,

Power = Torque x Rotational Speed

$$\mathbf{P} = \mathbf{T} \times \boldsymbol{\omega}_{\mathbf{e}}$$

Watt = N m x rad/sec

Note: Power (1 kW)=0.746 HP

Power to weight ratio is 1st order determinant of acceleration performance

2. At low to moderate speed, the Newton's Second law can be obtained by: (neglect all resistance force acted on vehicle)

$$\mathbf{F_x} = \mathbf{m}.\,\mathbf{a_x}$$

m = mass of the vehicle = W/g

a_x = acceleration in the forward direction

F_x = tractive force at the drive wheel

3. Since the drive power = Tractive force x forward speed

$$\mathbf{P = F_x V,}$$

therefore

$$a_x = \frac{1}{m}F_x = \frac{g}{W}\frac{P}{V}$$

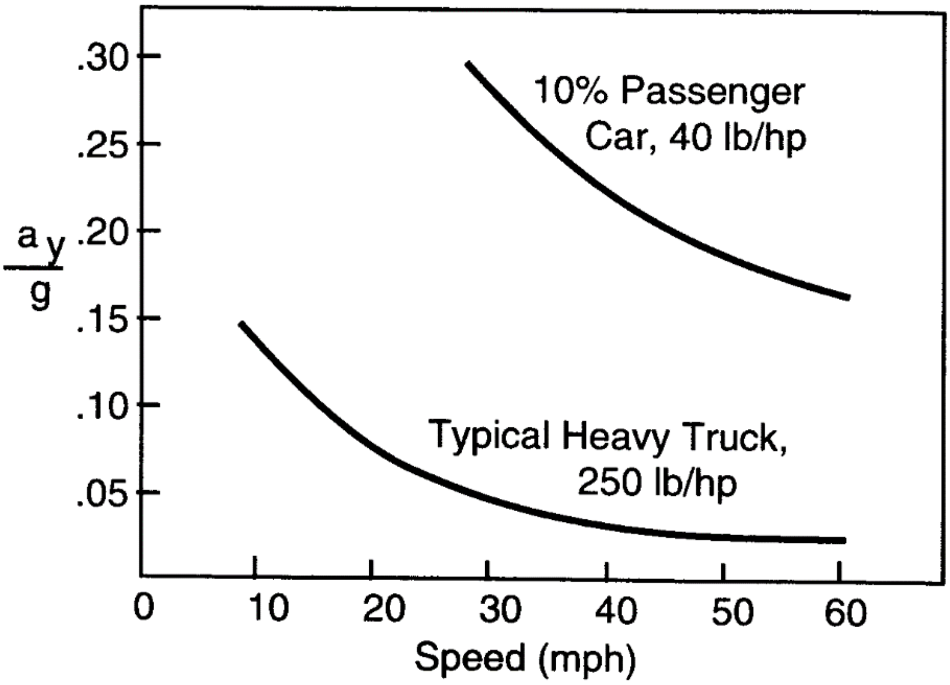
where;

g = Gravitational constant ($g = 9.81\ m/s^2$)

V = Forward speed (m/s)

P = Engine power (W)

W = weight of the vehicle (kg)

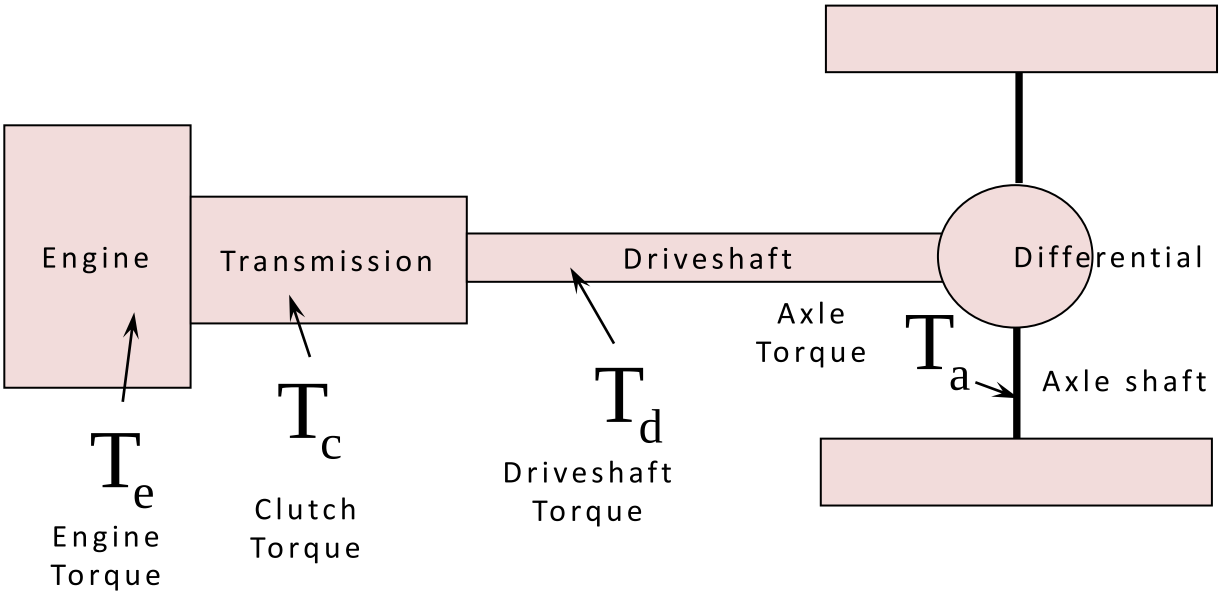


4. $a_x \alpha \frac{1}{V}$ shown that acceleration capability is decreasing with the increasing of speed

5. Based on the figure above, the heavy truck has lower acceleration performance compared to typical car because heavy truck has less power to weight ratio.

Effect of drive train on acceleration performance

1. Acceleration performance requires modelling the mechanical systems by which engine power is transmitted to the ground.

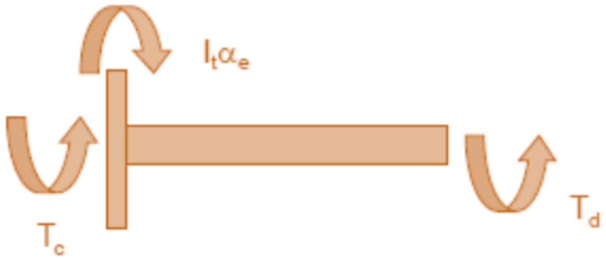


2. The engine provides the power from the combustion process to propel the vehicle via the drivetrain
3. The engine transfers the power to the clutch system (manual transmission) or torque converter (automatic transmission).
4. Either manual transmission or automatic transmission has gearsets that match engine speed to desired road speed.
5. Driveshaft transfers power from the transmission to the differential.
6. Differential turn the power flow 90 degrees and allow one wheel to rotate at a different speed
7. The axle shaft inside the axle tubing transfers power to rotate the rear wheel.

Are Engine Output Torque being greater than Actual Torque To Drivetrain?

Torque to driveshaft

1. Output torque at transmission amplified by the gear ratio of the transmission
2. However, inertial losses in the gears and shafts reduced the performance.
3. Suppose its value on the input side characterizes the transmission inertia. In that case, the output torque can be approximated by the expression:



$$T_d = (T_c - I_t\alpha_e)N_t$$

where ;

$T_d =$ Torque output to the driveshaft

$N_t =$ Numerical ratio of the transmission

$I_t =$ Rotational inertia of the transmission (as seen from the engine side)

$\alpha_e =$ Engine rotational acceleration

Torque to axle

1. Torque delivered to the axle which provides tractive force amplified by final drive ratio
2. But reduce by inertia between transmission and final drive;
3. The equation is:

$$T_a = F_x r + I_w\alpha_w = (T_d - I_d\alpha_d)N_f$$

where;

$T_a =$ Torque on the axles

$F_x =$ Tractive force at the ground

$r =$ Radius of the wheels

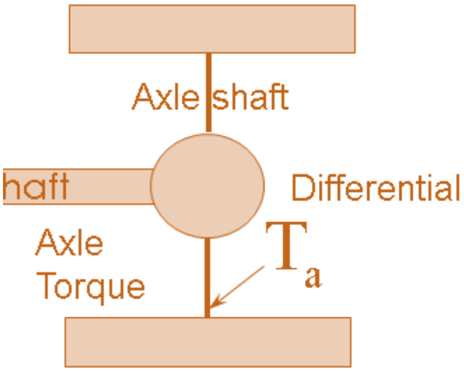
$I_w =$ Rotational inertia of the wheels and axles shafts

$\alpha_w =$ Rotational acceleration of the wheels

$I_d =$ Rotational inertia of the driveshaft

$\alpha_d =$ Rotational acceleration of the driveshaft

$N_f =$ Numerical ratio of the final drive



Steady state tractive force

If,

$\alpha_d = N_f \alpha_w$ and $\alpha_e = N_t \alpha_d = N_t N_f \alpha_w$

Solving all the equations,

$T_a = F_x r + I_w \alpha_w = (T_d - I_d \alpha_d) N_f$

T_d change to the term of T_c and T_e

All rotational acceleration substitute to α_w

The steady-state tractive force available at the ground to overcome the road load forces, to accelerate, to climb are:

$$F_x = \frac{T_e N_{tf}}{r} - \{(I_e + I_t) N_{tf}^2 + I_d N_f^2 + I_w\} \frac{a_x}{r^2}$$

where;

N_{tf} = Combined ratio of the transmission and final drive

$\{(I_e + I_t) N_{tf}^2 + I_d N_f^2 + I_w\} \frac{a_x}{r^2}$ are losses of tractive force due to inertia of the engine and driveline components

By including the inefficiencies due to mechanical and viscous losses,

$$F_x = \frac{T_e N_{tf} \eta_{tf}}{r} - \{(I_e + I_t) N_{tf}^2 + I_d N_f^2 + I_w\} \frac{a_x}{r^2}$$

where;

η_{tf} = Combined efficiency of transmission and final drive

From chapter 1,

$$Ma_x = W/g\, a_x = F_x - R_x - D_A - R_{hx} - W\sin\theta$$

$$Ma_x = \{\frac{T_e N_{tf} \eta_{tf}}{r} - \{(I_e + I_t) N_{tf}^2 + I_d N_f^2 + I_w\} \frac{a_x}{r^2}\} - R_x - D_A - R_{hx} - W\sin\theta$$

$$Ma_x + \{(I_e + I_t) N_{tf}^2 + I_d N_f^2 + I_w\} \frac{a_x}{r^2} = \frac{T_e N_{tf} \eta_{tf}}{r} - R_x - D_A - R_{hx} - W\sin\theta$$

Lumped the rotational inertias in with the mass of the vehicle,

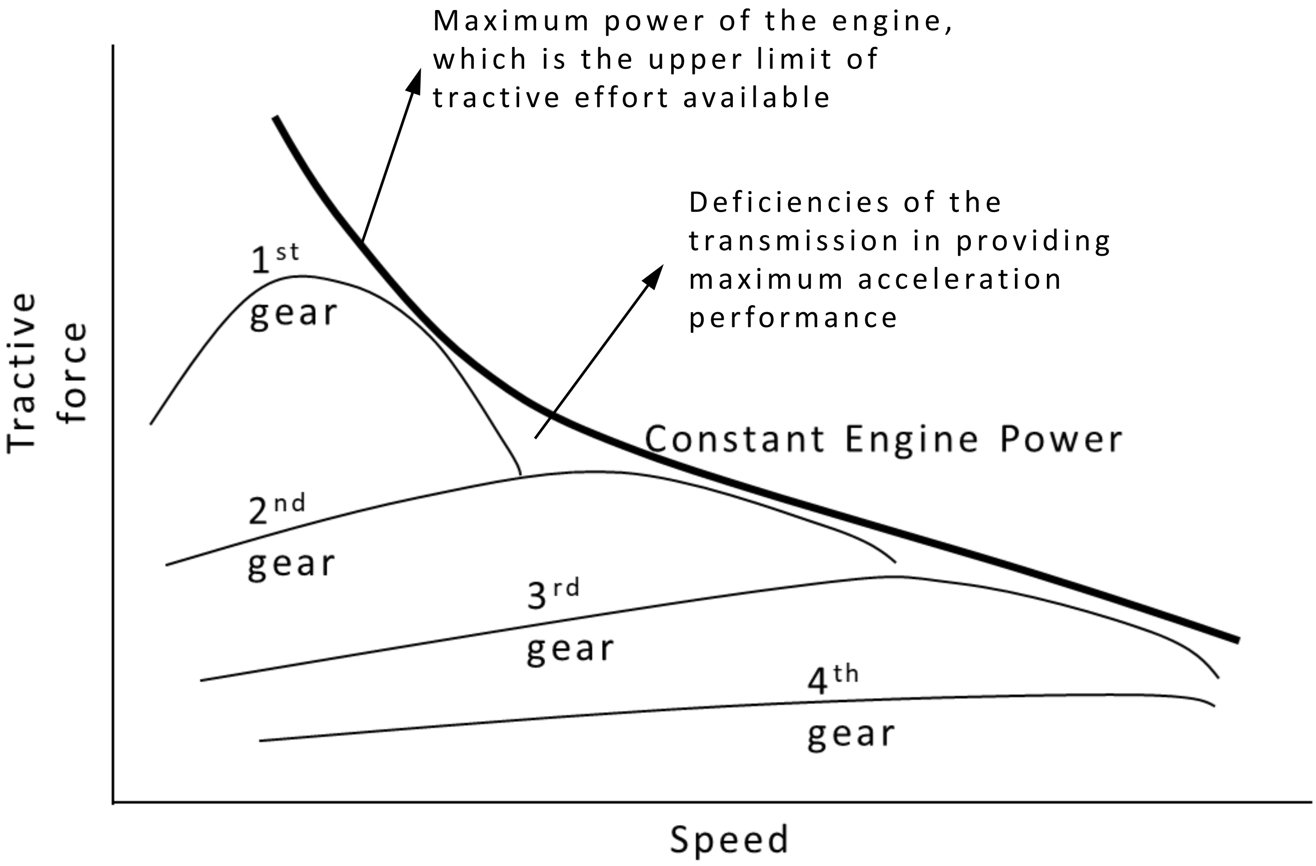
$$M_r = [(I_e + I_t) N_{tf}^2 + I_d N_f^2 + I_w]/r^2$$

Effect of drive train on acceleration are

$$a_x = [\frac{T_e N_{tf} \eta_{tf}}{r} - R_x - D_A - R_{hx} - W\sin\theta] / M + M_r$$

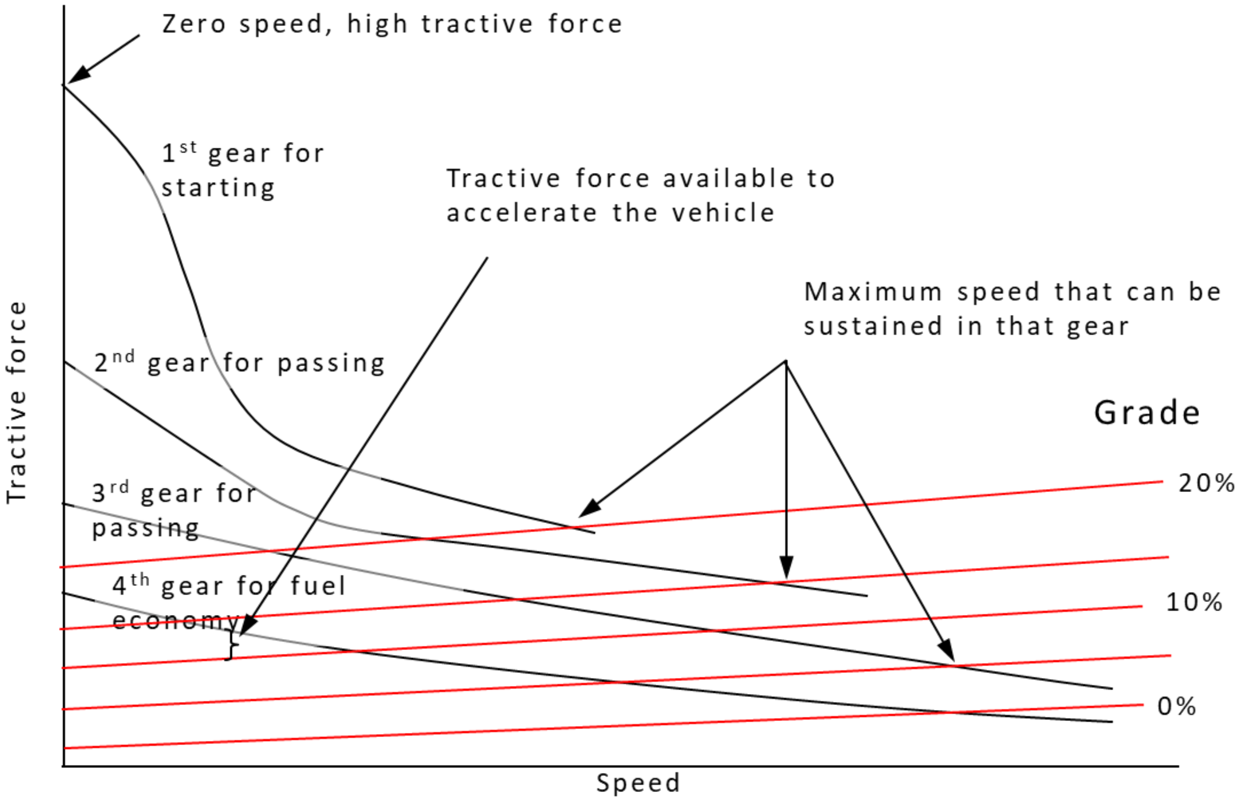
Manual Transmission

1. Power available at the wheel is the product of tractive force and velocity.
2. Tractive effort is proportional to the power and inversely proportional to the speed.

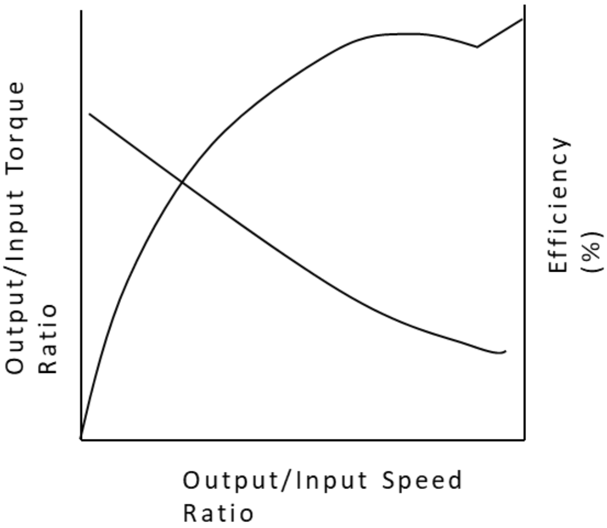


3. The graph above shows the tractive force versus speed characteristic for manual transmission
4. Under gearing and over gearing condition is provided based on the location of the curves relative to each other.
5. The point where the power available at the road wheels represents maximum road speed.
6. The maximum road speed is reduced in a minimal amount by slight under gear.

1. Automatic transmission provide torque more closely matching to the ideal torque because of the torque converter on the input
2. The graph below shows the tractive force versus speed characteristic for automatic transmission
3. At zero speed the tractive force is the highest for every gear
4. These conditions are providing good off-the-line acceleration performance.



3. As the speed increase, the transmission will be approaching the engine speed and the torque drop



1. Suppose there is sufficient power from the engine. In that case, the acceleration may be limited by the coefficient of friction between the tire and the road.
2. For this case, F_x is limited by;

$$F_x = \mu W$$

where;

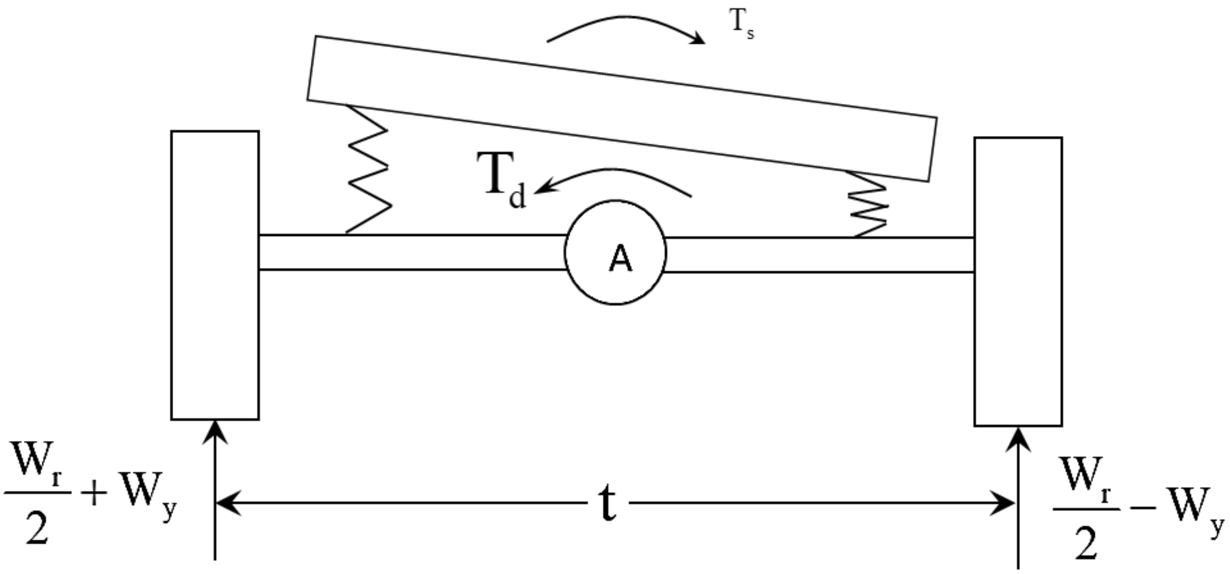
μ = Peak coefficient of friction

W = Weight on drive wheels

3. The weight on the drive wheel is depends on the static and the dynamic load due to acceleration and on transverse shift of load due to drive torque.

The transverse shift of weight due to driving torque

1. When driveshaft torque, Td react between the frame-mounted engine and the axle, the transverse weight transfer occurred. The engine, which is a component of the sprung weight, produced torque through the transmission. The rear axle must counter that torque at the rear tire patch (rear-wheel-drive, RWD situation)
2. This transverse weight transfer then adds cross weight under acceleration and removes cross weight during deceleration.
3. The magnitude of transverse weight is a function of the instantaneous engine torque and the roll stiffness, K_φ



4. The figure above shows that the driveshaft imposes a torque T_d on the axle.
5. Due to suspension roll stiffness, T_s is produced as the chassis roll, compressing and extending spring on opposite sides of the vehicle.
6. At equilibrium,

$$\sum T_0 = (W_r/2 + W_y - W_r/2 + W_y)t/2 + T_s - T_d = 0$$

or

$W_y = (T_d - T_s)/t$

and

$T_d = F_x r/N_f$

where;

$F_x =$ Total drive force from the two rear wheels

r = Tire radius

$N_f =$ Final drive ratio

Roll torque, T_s

1. Drive torque reaction at engine and transmission is transferred and distributed between the front and rear suspension.
2. Roll torque produced by suspension is proportional to roll angle.
3. According to Hooke's Law f the chassis,

$$T_{sf} = K_{\phi f}\phi$$

$$T_{sr} = K_{\phi r}\phi$$

$$K_{\phi} = K_{\phi f} + K_{\phi r}$$

Where;

T_{sf} = Roll torque on the front suspension

T_{sr} = Roll torque on the rear suspension

$K_{\phi f}$ = Front suspension roll stiffness

$K_{\phi r}$ = Rear suspension roll stiffness

K_{ϕ} = Total roll stiffness

4. Roll torque can be related to roll angle. The roll angle is simply the drive torque divided by the total roll stiffness.

$$\phi = \frac{T_d}{K_\phi} = \frac{T_d}{K_{\phi f} + K_{\phi r}}$$

5. From equation $T_{sr} = K_{\phi r}\phi$,

$$T_{sr} = K_{\phi r}\frac{T_d}{K_{\phi f}+K_{\phi r}}$$

6. From equation $W_y = (T_d - T_s)/t$ and $T_d = F_x r/N_f$ we get,

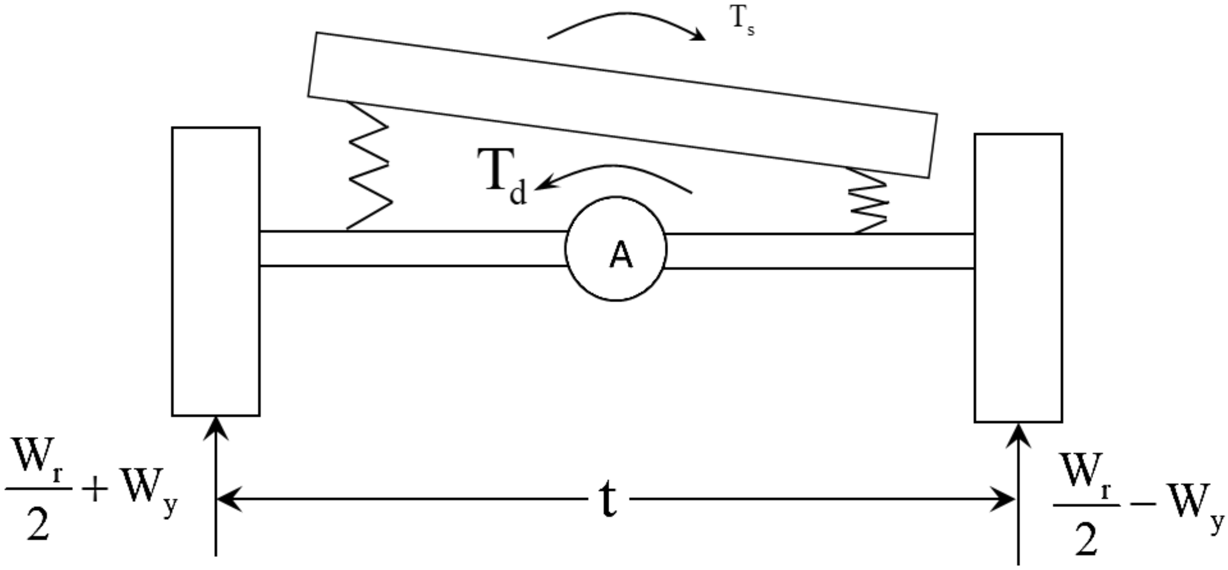
$$W_y = \frac{F_x r}{N_f t}\left[1 - \frac{K_{\phi r}}{K_{\phi r}+K_{\phi f}}\right] \quad \text{or } W_y = \frac{F_x r}{N_f t}\left[\frac{K_{\phi f}}{K_\phi}\right]$$

7. During acceleration, the load on the rear axle will be static load plus dynamic load, which are,

$$W_r = W\left(\frac{b}{L} + \frac{a_x h}{gL}\right)$$

8. As acceleration is simply the tractive force divided by mass, neglecting rolling resistance and aerodynamic drag,

$$W_r = W(\frac{b}{L} + \frac{F_x h}{MgL})$$



9. Therefore, the weight on the right rear wheel will be,

$$W_{rr} = \frac{W_r}{2} - W_y$$

Or

$$W_{rr} = W\left(\frac{b}{2L} + \frac{F_x h}{2MgL}\right) - \frac{F_x r}{N_f t}[\frac{K_{\phi f}}{K_\phi}]$$

10. The weight on the left rear wheel will be,

$$W_{rr} = \frac{W_r}{2} + W_y$$

Or

$$W_{rr} = W\left(\frac{b}{2L} + \frac{F_x h}{2MgL}\right) + \frac{F_x r}{N_f t}[\frac{K_{\phi f}}{K_\phi}]$$

TRACTION LIMITS – REAR-WHEEL DRIVE

1. Solving for Fx from the previous equation, based on the relationship and assumptions, a maximum tractive force that can be developed by solid rear axle with a non-locking differential are:

$$F_x\text{max} = \frac{\mu \frac{Wb}{L}}{1 - \frac{h}{L}\mu + \frac{2\mu r}{N_f t}\frac{K_{\o f}}{K_{\o}}}$$

Where;

μ = coefficient of friction of the driving wheel

$\frac{Wb}{L}$ = static load distribution

$1 - \frac{h}{L}\mu$ = dynamic transfer load

$\frac{2\mu r}{N_f t}\frac{K_{\o f}}{K_{\o}}$ = lateral load transfer for lower traction force limited wheel

2. When a solid rear axle with a locking differential is used, the additional tractive force can be obtained from the other wheel up to its traction limits such that the last term in the denominator of the above equation dropout.

$$F_x\text{max} = \frac{\mu \frac{Wb}{L}}{1 - \frac{h}{L}\mu + \cancel{\frac{2\mu r}{N_f t}\frac{K_{\phi f}}{K_{\varphi}}}} = \frac{\mu \frac{Wb}{L}}{1 - \frac{h}{L}\mu}$$

3. The above equation is also valid for independent rear suspension because the driveline torque reaction is picked up by the chassis-mounted differential

TRACTION LIMITS – FRONT-WHEEL DRIVE

1. The maximum tractive force that a solid front axle can develop with a non-locking differential are:

$$F_x max = \frac{\mu \frac{Wc}{L}}{1 + \frac{h}{L}\mu + \frac{2\mu r}{N_f t}\frac{K_{\emptyset f}}{K_{\emptyset}}}$$

Where;

μ = coefficient of friction of the driving wheel

$\frac{Wb}{L}$ = static load distribution

$1-\frac{h}{L}\mu$ = dynamic transfer load

$\frac{2\mu r}{N_f t}\frac{K_{\emptyset f}}{K_{\emptyset}}$ = lateral load transfer for lower traction force limited wheel

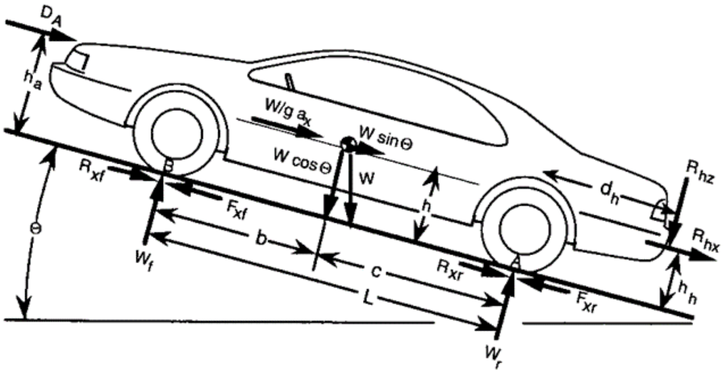
2. When a solid front axle with locking differential or independent front suspension is used,

$$F_x max = \frac{\mu \frac{Wc}{L}}{1 + \frac{h}{L}\mu}$$

3. In this case, the load on the front axle is higher than the rear axle. However, during acceleration, the loads get transfer to the rear axle.
4. Although the numerical values of acceleration performance are higher for front-wheel drive, it will not be proportionately better than rear-wheel-drive vehicles.

GENERAL EQUATIONS FOR BRAKING

1. Based on the Newton Second Law written in the x – direction, the general equation for braking may be obtained.



$$Ma_x = -\frac{W}{g}D_x = -F_{xf} - F_{xr} - D_A - W\sin\Theta \quad \text{or} \quad D_x = \frac{F_{xf}+F_{xr}+D_A+W\sin\theta}{M} = \frac{F_{xt}}{M}$$

where;

W = Vehicle weight

g = Gravitational acceleration

$D_x = -a_x$ = Linear deceleration

F_{xf} = Front axle braking force

F_{xr} = Rear axle braking force

D_A = Aerodynamic drag

Θ = Uphill grade

Constant Deceleration

1. Throughout the braking application, assume that the forces acting on the will is constant.

$$D_x = \frac{F_{xt}}{M} = -\frac{dV}{dt}$$

where;

F_{xt} = The total of all longitudinal deceleration forces on the vehicle

V = Forward velocity

2. By integrating the equation from initial velocity V_0 to final velocity $V_{f,}$

$$\int_{V_0}^{V_f} dV = -\frac{F_{xt}}{M}\int_0^{t_s} dt$$

$$V_0 - V_f = \frac{F_{xt}}{M} t_s$$

where;

t_s = Time for the velocity change

3. Since V = dx/dt,

$$\frac{V_0^2 - V_f^2}{2} = \frac{F_{xt}}{M} X$$

where;

X = Distance traveled during the deceleration

4. In the case where the deceleration is a complete stop, then V_f is zero, then the stopping distance, SD and time have taken to stop are,

$$SD = \frac{V_0^2}{2\frac{F_{xt}}{M}} = \frac{V_0^2}{2D_x}$$

$$t_s = \frac{V_0}{\frac{F_{xt}}{M}} = \frac{V_0}{D_x}$$

5. From the equation, we know that stopping distance is directly proportional to ${V_0}^2$ and stopping time is directly proportional to the initial velocity.

$SD \propto V_0^2$

and

$t_s \propto V_0$

1. When the wind resistance is involved, the total braking force is the summation of total brake force from the axles and the drag force from the wind resistance

$$\sum F_x = F_b + CV^2$$

where;

F_b = Total brake force of front and rear wheels

C = Aerodynamic drag factor

2. Therefore,

$$\int_0^{SD} dx = M \int_{V_0}^{0} \frac{VdV}{F_b + CV^2}$$

3. Integrating the above equation, the stopping distance with wind resistance is,

$$SD = \frac{M}{2C} \ln \left[\frac{F_b + CV_0^2}{F_b}\right]$$

ENERGY/POWER ABSORPTION

1. During a typical maximum effort stop, either energy or power absorbed by a brake system can be significant.
2. As the energy absorbed is the vehicle's kinematic energy of motion, it depends on the mass.

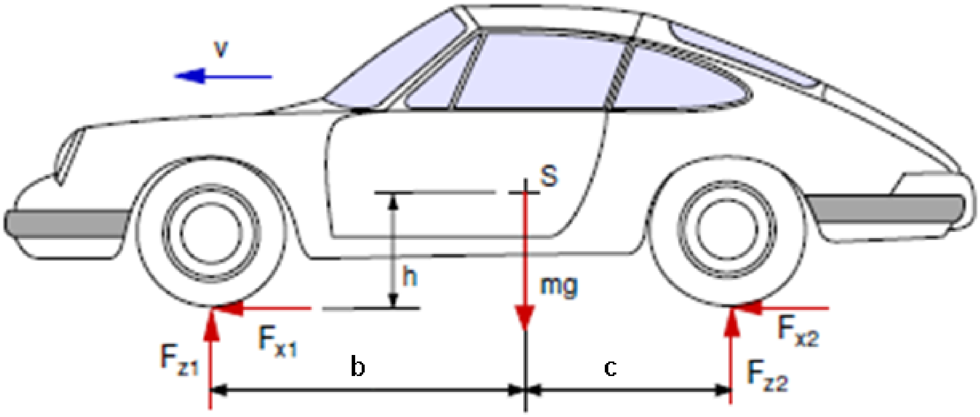
$$\text{Energy} = \frac{M}{2}\left(V_0^2 - V_f^2\right)$$

3. The power absorption will vary with the speed at any instant, equivalent to the braking force times the speed.
4. At the beginning of the stopping process, the power dissipation is the greatest when the highest speed.
5. The average power absorption over the entire stopping process is the energy divided by the time to stop.
6. Thus,

$$\text{Power} = \frac{M}{2}\frac{V_0^2}{t_s}$$

Simple Vehicle Model

1. When a car is speeding with acceleration $\dot{v}$ on the level road as shown in the figure below, neglecting the aerodynamic drag, the equations are,



I. Summation of force at x-plane are is, $F_{x1} + F_{x2} = m\dot{v}$
II. Summation of force at z-plane is, $F_{z1} + F_{z2} - mg = 0$
III. Moment at center of gravity, CG is, $F_{z1}b - F_{z2}c + (F_{x1} + F_{x2})h = 0$

2. Substitute equation I into equation III, we have the equation,

IV. $F_{z1}b - F_{z2}c + m\dot{v}h = 0$

3. From equation II, $F_{z1} + F_{z2} - mg = 0$, we have equation

V. $F_{z2} = mg - F_{z1}$

4. Substitute equation V into equation IV:

$$F_{z1}b - (mg - F_{z1})c + m\dot{v}h = 0$$
$$F_{z1}b + F_{z1}c = mgc - m\dot{v}h$$
$$F_{z1}(b + c) = mgc - m\dot{v}h$$

5. Therefore, the normal force of front-wheel F_{z1} would be,

$$F_{z1} = mg(\frac{c}{b+c}) - m\dot{v}(\frac{h}{b+c})$$

6. From equation II, $F_{z1} + F_{z2} - mg = 0$, we have equation

   VI. $F_{z1} = mg - F_{z2}$

7. Substitute equation V into equation VI:

   $$(mg - F_{z2})b - F_{z2}c + m\dot{v}h = 0$$

   $$mgb - F_{z2}b - F_{z2}c = -m\dot{v}h$$

   $$F_{z2}(b + c) = mgb + m\dot{v}h$$

8. Therefore, the normal force rear-wheel F_{z2} would be,

   $$F_{z2} = mg(\frac{b}{b + c}) + m\dot{v}(\frac{h}{b + c})$$

7. From the equation, the weight distribution for the stationary car depends on the mass center's horizontal position, and the weight distribution for accelerating vehicle depend on the vertical position of the mass center.

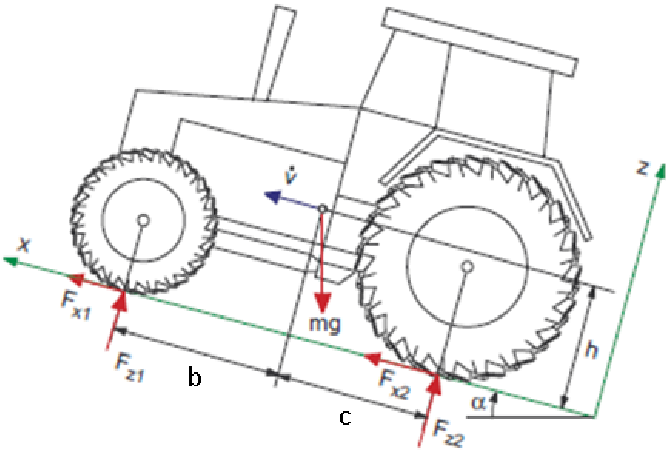
   $$F_{z1\ \mathrm{static}} = mg\left(\frac{c}{b+c}\right) \quad F_{z2\ \mathrm{static}} = mg(\frac{b}{b+c})$$

   $$F_{z1\ \mathrm{dynamic}} = -m\dot{v}\left(\frac{h}{b + c}\right) \quad F_{z2\ \mathrm{dynamic}} = m\dot{v}(\frac{h}{b + c})$$

INFLUENCE OF GRADIENT

1. A vehicle accelerates on the inclined pavement with the angle, α as shown in the figure below.

   The equations are,



   I. Summation of force at x-plane is, $F_{x1} + F_{x2} = m\dot{v} + mg\sin\alpha$
   II. Summation of force at z-plane is, $F_{z1} + F_{z2} - mg\cos\alpha = 0$
   III. Moment at center of gravity, CG is, $F_{z1}b - F_{z2}c + (F_{x1} + F_{x2})h = 0$

2. Substitute equation I into equation III:

   IV. $F_{z1}b - F_{z2}c + m\dot{v}h + mgh\sin\alpha = 0$

3. From equation II, $F_{z1} + F_{z2} - mg\cos\alpha = 0$,

   V. $F_{z2} = mg\cos\alpha - F_{z1}$

4. Substitute equation V into equation IV:

$$F_{z1}b - (mg\cos\alpha - F_{z1})c + m\dot{v}h + mgh\sin\alpha = 0$$

$$F_{z1}(b + c) - mgc\cos\alpha + m\dot{v}h + mgh\sin\alpha = 0$$

$$\frac{F_{z1}(b + c)}{\cos\alpha} = \frac{mg(c\cos\alpha - h\sin\alpha)}{\cos\alpha} - \frac{m\dot{v}h}{\cos\alpha}$$

5. Therefore, the normal force of front-wheel F_{z1} would be,

$$F_{z1} = mg\cos\alpha\,(\frac{c - h\tan\alpha}{b + c}) - m\dot{v}(\frac{h}{b + c})$$

6. From equation II, $\mathrm{F_{z1} + F_{z2} - mg\cos\alpha = 0}$, we have equation

   $$VI.\quad \mathrm{F_{z1} = mg\cos\alpha - F_{z2}}$$

7. Substitute equation V into equation VI:

   $$mg\cos\alpha - F_{z2})b - F_{z2}c + m\dot{v}h + mgh\sin\alpha = 0$$

   $$-F_{z2}(b+c) + mgb\cos\alpha + m\dot{v}h + mgh\sin\alpha = 0$$

   $$\frac{F_{z2}(b+c)}{\cos\alpha} = \frac{mg(b\cos\alpha + h\sin\alpha)}{\cos\alpha} + \frac{m\dot{v}h}{\cos\alpha}$$

8. Therefore, the normal force rear-wheel $\mathrm{F_{z2}}$ would be,

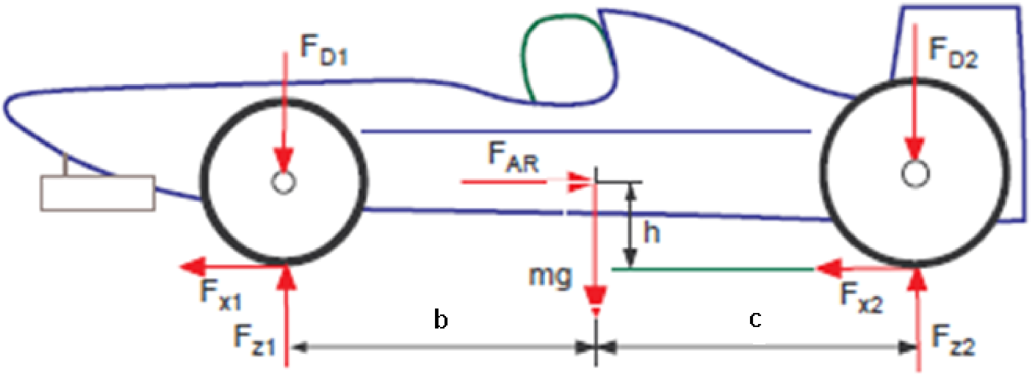
   $$\mathrm{F_{z2} = mg\cos\alpha\,(\frac{b + h\tan\alpha}{b+c}) + m\dot{v}(\frac{h}{b+c})}$$

9. From the equation, the dynamic part is unchanged from the previous case, dependent on the mass center’s height and acceleration. However, the static part are influenced by the slope angle and the height of mass center.

   $$\mathrm{F_{z1\,static} = mg\cos\alpha\left(\frac{c - h\tan\alpha}{b+c}\right)} \qquad \mathrm{F_{z2\,static} = mg\cos\alpha\,(\frac{b + h\tan\alpha}{b+c})}$$

   $$\mathrm{F_{z1\,dynamic} = -m\dot{v}\left(\frac{h}{b+c}\right)} \qquad \mathrm{F_{z2\,dynamic} = m\dot{v}(\frac{h}{b+c})}$$

1. When a car is speeding with acceleration $\dot{v}$ on the level road as shown in the figure below, with the aerodynamic resistance force, F_{AR} and downforce, F_D is considered, the equations are,



I. Summation of force at x-plane are is, $F_{x1}+F_{x2} = m\dot{v} + F_{AR}$

II. Summation of force at z-plane is, $F_{z1} - F_{D1} + F_{z2} - F_{D2} - mg = 0$

III. Moment at center of gravity, CG is, $(F_{z1} - F_{D1})b - (F_{z2} - F_{D2})c + (F_{x1} + F_{x2})h = 0$

2. Substitute equation I into equation III, we have equation,

IV. $(F_{z1} - F_{D1})b - (F_{z2} - F_{D2})c + (m\dot{v} + F_{AR})h = 0$

3. From equation II, $F_{z1} - F_{D1} + F_{z2} - F_{D2} - mg = 0$, we have equation

V. $F_{z2} - F_{D2} = F_{D1} - F_{z1} + mg$

4. Substitute equation V into equation IV:

$$(F_{z1} - F_{D1})b - (F_{D1} - F_{z1} + mg)c + (m\dot{v} + F_{AR})h = 0$$
$$F_{z1}(b + c) - F_{D1}(b + c) - mga_2 + (m\dot{v} + F_{AR})h = 0$$
$$F_{z1}(b + c) = F_{D1}(b + c) + mgc - h(m\dot{v} + F_{AR})$$

5. Therefore, the normal force of front-wheel F_{z1} would be,

$$F_{z1} = F_{D1} + mg(\frac{c}{b + c}) - \frac{h}{b + c}(m\dot{v} + F_{AR})$$

6. From equation II, $\mathrm{F_{z1} - F_{D1} + F_{z2} - F_{D2} - mg = 0}$, we have equation

$$\mathit{VI.}\quad \mathrm{F_{z1} - F_{D1} = F_{D2} - F_{z2} + mg}$$

7. Substitute equation V into equation VI:

$$(F_{D2} - F_{z2} + mg)b - (F_{z2} - F_{D2})c + (m\dot{v} + F_{AR})h = 0$$
$$-F_{z2}(b + c) + F_{D2}(b + c) + mgb + (m\dot{v} + F_{AR})h = 0$$
$$F_{z2}(b + c) = F_{D2}(b + c) + mgb + h(m\dot{v} + F_{AR})$$

8. Therefore, the normal force rear-wheel $\mathrm{F_{z2}}$ would be,

$$\mathrm{F_{z2} = F_{D2} + mg(\frac{b}{b + c}) + \frac{h}{b + c}(m\dot{v} + F_{AR})}$$

Single Axle Rear Wheel Drive RWD

1. When a car is a rear-wheel drive, then F_{x1} is equal to zero, and the required force to achieve acceleration $\dot{v}$ must be provided only by the rear wheel. Therefore, from simple vehicle model equations,

$$F_{x1} + F_{x2} = m\dot{v}\,,$$

$$F_{z2} = mg(\frac{b}{b+c}) + m\dot{v}(\frac{h}{b+c})$$

2. With rear axle are driven in limit situation, $F_{x1} = 0$, $F_{x2} = \mu F_{z2}$;

$$0 + \mu F_{z2} = m\dot{v}_{RWD}$$

$$F_{z2} = \frac{m\dot{v}_{RWD}}{\mu}$$

3. With the normal force acting on the rear wheel also,

$$F_{z2} = mg(\frac{b}{b+c}) + m\dot{v}_{RWD}(\frac{h}{b+c})$$

4. Therefore, the maximum acceleration for rear-wheel drive (RWD) is,

$$m\dot{v}_{RWD} = \mu mg\frac{b}{b+c} + \mu m\dot{v}_{RWD}(\frac{h}{b+c})$$

$$\dot{v}_{RWD} - \mu\dot{v}_{RWD}(\frac{h}{b+c}) = \mu g\frac{b}{b+c}$$

$$\dot{v}_{RWD}[1 - \mu(\frac{h}{b+c})] = \mu g\frac{b}{b+c}$$

$$\frac{\dot{v}_{RWD}}{g} = \frac{\mu}{1 - \mu\frac{h}{b+c}}(\frac{b}{b+c})$$

Single Axle Front Wheel Drive FWD

1. When a car is front-drive, then F_{x2} is equal to zero, and the required force to achieve acceleration $\dot{v}$ must be provided only by the front wheel. Therefore, from simple vehicle model equations,

$$F_{x1} + F_{x2} = m\dot{v}\,,$$

$$F_{z1} = mg(\frac{c}{b + c}) - m\dot{v}(\frac{h}{b + c})$$

2. With front axle are driven in limit situation, $F_{x1} = \mu F_{z1}$ and $F_{x2} = 0$;

$$\mu F_{z1} + 0 = m\dot{v}_{FWD}$$

$$F_{z1} = \frac{m\dot{v}_{FWD}}{\mu}$$

3. With the normal force acting on the front-wheel also,

$$F_{z1} = mg(\frac{c}{b + c}) - m\dot{v}_{FWD}(\frac{h}{b + c})$$

4. Hence, the maximum acceleration for front-wheel drive (FWD) is,

$$m\dot{v}_{FWD} = \mu mg\frac{c}{b + c} - \mu m\dot{v}_{FWD}(\frac{h}{b + c})$$

$$\dot{v}_{FWD} + \mu\dot{v}_{FWD}(\frac{h}{b + c}) = \mu g\frac{c}{b + c}$$

$$\dot{v}_{FWD}[1 + \mu(\frac{h}{b + c})] = \mu g\frac{c}{b + c}$$

$$\frac{\dot{v}_{FWD}}{g} = \frac{\mu}{1 + \mu\frac{h}{b + c}}(\frac{c}{b + c})$$

Front Wheel Braking (FWB)

1. When a car is on front-wheel braking, then F_{x2} is equal to zero and the required braking force to perform deceleration -$\dot{v}$ is provided only the by front wheel. Therefore, from simple vehicle model equations,

$$F_{x1} + F_{x2} = m\dot{v} \,,$$

$$F_{z1} = mg(\frac{c}{b+c}) - m\dot{v}(\frac{h}{b+c})$$

2. With only front axle is braked in limit situation, F_{x1} = -μF_{z1} and F_{x2} = 0 ;

$$-\mu F_{z1} + 0 = m\dot{v}_{FWB}$$

$$F_{z1} = -\frac{m\dot{v}_{FWB}}{\mu}$$

3. With the normal force acting on the front wheel is,

$$F_{z1} = mg(\frac{c}{b+c}) - m\dot{v}_{FWD}(\frac{h}{b+c})$$

4. Hence, the maximum deceleration for front-wheel braking (FWB) is,

$$-m\dot{v}_{FWD} = \mu mg\frac{c}{b+c} - \mu m\dot{v}_{FWD}(\frac{h}{b+c})$$

$$\dot{v}_{FWD} - \mu\dot{v}_{FWD}\left(\frac{h}{b+c}\right) = -\mu g\frac{c}{b+c}$$

$$\dot{v}_{FWD}\left[1 - \mu\left(\frac{h}{b+c}\right)\right] = -\mu g\frac{c}{b+c}$$

$$\frac{\dot{v}_{FWB}}{g} = -\frac{\mu}{1-\mu\frac{h}{b+c}}(\frac{c}{b+c})$$

Rear Wheel Braking (RWB)

1. When a car is on rear wheel braking, then F_{x1} is equal to zero and the required braking force to perform deceleration -$\dot{v}$ is provided only by the rear wheel. Therefore, from simple vehicle model equations,

$$F_{x1} + F_{x2} = m\dot{v}\,,$$

$$F_{z2} = mg(\frac{b}{b+c}) + m\dot{v}(\frac{h}{b+c})$$

2. With only rear axle is braked in limit situation, $F_{x1} = 0$ and $F_{x2} = -\mu F_{z2}$;

$$0 - \mu F_{z2} = m\dot{v}_{RWB}$$

$$F_{z2} = -\frac{m\dot{v}_{RWB}}{\mu}$$

3. With the normal force acting on the rear wheel is,

$$F_{z2} = mg(\frac{b}{b+c}) + m\dot{v}(\frac{h}{b+c})$$

4. Hence, the maximum deceleration for rear-wheel braking (RWB) is,

$$-m\dot{v}_{RWB} = \mu mg\frac{b}{b+c} + \mu m\dot{v}_{RWB}(\frac{h}{b+c})$$
$$\dot{v}_{RWB} + \mu\dot{v}_{RWB}(\frac{h}{b+c}) = -\mu g\frac{b}{b+c}$$
$$\dot{v}_{RWB}[1 + \mu(\frac{h}{b+c})] = -\mu g\frac{b}{b+c}$$

$$\frac{\dot{v}_{RWB}}{g} = -\frac{\mu}{1+\mu\frac{h}{b+c}}(\frac{b}{b+c})$$

1. We are given the following information about the engine and drivetrain components for a passenger car:

Engine inertia = 0.0904kgm2

Final drive inertia = 0.1356 kgm2

Final Drive Ratio = 2.92

Efficiency = 0.99

Wheel inertia's Drive = 1.243kgm2

Wheel inertia's Non-drive = 1.243kgm2

Wheel size radius = 31.98 cm

Wheel circumference = 497.7 rev/km

RPM	1200	2000	2400	3200	3600	4800	5200
TORQUE (Nm)	179.0	217.0	237.4	257.7	268.5	272.6	244.1

Transmission Data – Gear	1	2	3	4	5
Inertia (kgm^2)	0.1469	0.1017	0.091	0.0565	0.0339
Ratios	4.28	2.79	1.83	1.36	1.00
Efficiencies	0.966	0.967	0.972	0.973	0.970

a) Calculate the effective inertia of the drive-train components in the first gear.

b) Calculate the maximum tractive effort and corresponding road speed in the first and fifth gears of the car described above when inertial losses are neglected.

2. Find the traction-limited acceleration for the rear-drive passenger car with and without a locking differential on a surface of moderate friction level. The information that will be needed is as follows:

   Weights Front = 952 kg
   Weight Rear = 839 kg
   Weight Total = 1791 kg
   CG height = 53.34 cm
   Coefficient of friction = 0.62
   Wheelbase = 274.3 cm
   Tread = 149.86 cm
   Tire size = 33 cm
   Final drive ratio = 2.90
   Roll Stiffness Front = 1559.7 Nm/deg
   Roll Stiffness Rear = 379.8 Nm/deg

3. Consider a light truck weighing 1672.1 kg, performing a complete stop from 96.6 km/h on a level surface with a brake application that develops a steady-state force of 920kg. Determine the deceleration, stopping distance, stopping time, energy dissipated and the brake horsepower at initial application. Neglect aerodynamic and rolling resistance forces.

SOLUTIONS TUTORIAL

TUTORIAL 1 (ANSWER)

1) Define the center of gravity of a vehicle.

   Center of gravity is the point that sums up the vehicle's mass in one central point. To put it simply, a center of gravity is the average location of the weight of an object.

2) If the front wheel carried 65 % of the vehicle weight and the distance between front and rear wheel is 2.1 m, calculate the distance of the gravity center to the front (b) and rear wheel (c).

$$\boldsymbol{W_f = \frac{Wc}{L}}$$

$$\boldsymbol{W_f = 0.65W}$$

$$\boldsymbol{0.65W = \frac{Wc}{2.1}}$$

$$\boldsymbol{c = 0.65\ x\ 2.1}$$

$$\boldsymbol{c = 1.365\ m}$$

$$\boldsymbol{W_r = \frac{Wb}{L}}$$

$$\boldsymbol{W_r = 0.35W}$$

$$\boldsymbol{0.35W = \frac{Wb}{2.1}}$$

$$\boldsymbol{c = 0.35\ x\ 2.1}$$

$$\boldsymbol{c = 0.735\ m}$$

3) Find the values of d$_L$ and d$_R$ if the load applied on the left wheel is 0.55N and the vehicle’s weight is 1N. Given that distance between the left and right wheels is 2.2m.

$$\boxed{d_R = \frac{Z_L d}{W}}$$

$$\boxed{d_L = \frac{0.55N\ x\ 2.2}{1N}}$$

$$\boxed{d_L = 0.55\ x\ 2.2}$$

$$\boxed{d_L = 1.21\ m}$$

$$\boxed{d_L = \frac{Z_R d}{W}}$$

$$\boxed{d_L = \frac{0.45N\ x\ 2.2}{1N}}$$

$$\boxed{d_L = 0.45\ x\ 2.2}$$

$$\boxed{d_L = 0.99\ m}$$

TUTORIAL 2 (ANSWER)

1a. Calculate the effective inertia of the drive-train components in first gear.

$$I_{eff} = [(I_e + I_t)(N_{tf})^2 + I_d N_f{}^2 + 2I_w$$

$$I_{eff} = [(0.0904 + 0.1469)(4.28 \text{ x } 2.92)^2 + 0.1356(2.92)^2 + 2(1.243)$$

$$I_{eff} = 0.2373(156.19) + 1.156 + 2.486$$

$$I_{eff} = 40.706 \text{ kgm}^2$$

1b. Calculate the maximum tractive effort and corresponding road speed in first and fifth gears of the car described above when inertial losses are neglected.

$$F_x = \frac{T_e N_{tf} \eta_{tf}}{r} - \cancelto{0}{\{(I_e + I_t)\ N_{tf}{}^2 + I_d N_f{}^2 + I_w\} \frac{a_x}{r^2}}$$

$$F_x = \frac{T_e N_{tf} \eta_{tf}}{r}$$

First gear

$$F_x = \frac{T_e N_{tf} \eta_{tf}}{r}$$

$$F_x = \frac{(272.6)(4.28\text{x}2.92)(0.966\text{x}0.99)}{0.3198}$$

$$F_x = \frac{3237.87}{0.3198}$$

$$F_x = 10124.67 \text{ N}$$

$$W_w = \frac{W_e}{N_t N_f}$$

$$W_w = \frac{4800 \text{ x } \frac{2\pi}{60}}{4.28\text{x}2.92}$$

$$W_w = 40.22 \text{ rad/s}$$

$$v = W_w \text{x} r_w$$

$$v = 40.22 \text{ x } 0.3198$$

$$v = 12.86 \text{ m/s}$$

Fifth gear

$$F_x = \frac{T_e N_{tf} \eta_{tf}}{r}$$

$$F_x = \frac{(272.6)(1.00x2.92)(0.970x0.99)}{0.3198}$$

$$F_x = \frac{764.39}{0.3198}$$

$$F_x = 2390.22 \text{ N}$$

$$W_w = \frac{W_e}{N_t N_f}$$

$$W_w = \frac{4800 \text{ x } \frac{2\pi}{60}}{1.00x2.92}$$

$$W_w = 172.14 \text{ rad/s}$$

$$v = W_w x r_w$$

$$v = 172.14 \text{ x } 0.3198$$

$$v = 55.05 \text{ m/s}$$

2.

$$F_{xmax} = \frac{\mu \frac{Wb}{L}}{1 - \frac{h}{L}\mu + \frac{2\mu r}{N_{tf}} \frac{k_{\varphi f}}{k_{\varphi}}}$$

$$F_{xmax} = \frac{\mu W_r}{1 - \frac{h}{L}\mu + \frac{2\mu r}{N_f t} \frac{k_{\varphi f}}{k_{\varphi}}}$$

$$F_{xmax} = \frac{0.62x839(9.81)}{1 - \frac{0.5334}{2.743}(0.62) + \frac{2(0.62)(0.33)}{2.90\ (1.4986)} \frac{(1559.7)}{(1939.5)}}$$

$$F_{xmax} = \frac{5102.97}{1 - 0.1206 + 0.07572}$$

$$F_{xmax} = \frac{5102.97}{0.95512}$$

$$F_{xmax} = 5342.75 \text{ N}$$

3.

$$v = \frac{96.6\ km}{1h} x \frac{1000m}{1km} x \frac{1h}{3600s}$$
$$v = 26.83\ m/s$$

Deceleration

$$D_x = \frac{F_x}{M}$$

$$D_x = \frac{920x9.81}{1672.1}$$

$$D_x = 5.398\ m/s^2$$

Stopping distance

$$SD = \frac{v_0{}^2}{2D_x}$$

$$SD = \frac{(26.83)^2}{2(5.398)}$$

$$SD = 66.67\ m$$

Time to stop

$$t_s = \frac{V_0}{D_x}$$

$$t_s = \frac{26.83}{5.398}$$

$$t_s = 4.97\ s$$

Energy dissipated

$$E = \frac{M}{2}(V_0{}^2 - V_f{}^2)$$

$$E = \frac{1672.1}{2}(26.83)^2$$

$$E = 601.83\ kJ$$

Brake horsepower

$$P = \frac{MV_0{}^2}{2t_s}$$

$$P = \frac{1672.1(26.83)^2}{2(4.97)}$$

$$P = 121.09kW$$

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THANK YOU