

# **Statistics and Probability**

**ASMARINI MOHAMED**

**NURZAHIRA AHMAD ZABIDI**

**JABATAN MATEMATIK, SAINS DAN KOMPUTER**

**Editor**

Nurzahira binti Ahmad Zabidi

**Writer**

Asmarini binti Mohamed

**Designer**

Asmarini binti Mohamed

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## **ACKNOWLEDGEMENT**

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Our hope is, may this eBook help students out there, especially polytechnic students, conveniently easing them to understand the topics of Statistics and Probability. Students can study via this eBook about the fundamentals of Statistics and Probability, and able to understand by the help of some given examples, and test their understanding by attempting some exercises. Nevertheless, we value any beneficial feedbacks from the readers to help us improve on our next debut. We hope this eBook could assist everyone to understand Statistics and Probability and would fulfill everybody needs.

Thank you.

## ABSTRACT

The aim of producing this eBook is to fulfil the E-learning key performance indicator (KPI) as stipulated by The Instructional and Digital Learning Division, Department of Polytechnic and Community College Education, Ministry of Higher Education Malaysia. The publication of this eBook is solely intended for the betterment of polytechnic students with regards to their mastery and understanding in the realm of statistics and probability in Engineering Mathematics. This comprehensive eBook comprise of a wide range of notes, question examples with solution steps, formulas and exercises. This eBook also includes diagrams and figures which substantially help the readers to grasp the methods used in problem-solving. Additionally, to assist the readers even further, this eBook conveniently comes with the answer section located at the end of the eBook.

This eBook discusses about statistic terminology, for instance, the definition of statistics, grouped data and ungrouped data, discrete data, continuous data, population and others. This eBook also explains and show steps on how to construct distribution table, how to draw ogive and histogram, explaining and showing in greater details regarding how to properly find median and mode from the graph. Moreover, it describes solutions to problems by using formulas and graphs to find mean deviation, variance, standard deviation, quartiles, deciles and percentiles for ungrouped and grouped data.

In the topic of Probability, the matters discussed in this eBook are related to several types of probability events such as dependent event, independent event, conditional probability and expectation. The laws of probability, which include the Addition and Multiplication Law, are also discussed in this eBook by focusing through their application in the problem solving questions. In a nutshell, this Statistics and Probability eBook will definitely favor polytechnic students particularly towards bolstering their understanding in the topics of Statistics and Probability.

# CONTENTS

1.1	Demonstrate Statistical Data	
1.1.1	Define Statistical Terminology of Ungrouped Data and Grouped Data	1
	EXERCISE 1	8
1.2	Compute of Central Tendency and Dispersion	
1.2.1	Calculate Mean, Median and Mode for Ungrouped Data	10
	EXERCISE 2	13
1.2.2	Calculate Mean, Median and Mode for Grouped Data by Using Formula	15
	EXERCISE 3	20
1.2.3	Calculate Median and Mode for Grouped Data by Using Graph	23
	EXERCISE 4	29
1.2.4	Calculate Mean Deviation, Variance and Standard Deviation	32
	EXERCISE 5	40
1.2.5	Calculate Quartiles, Deciles and Percentiles for Grouped Data by Using Formula and Graph	43
	EXERCISE 6	54
1.3	Understand Probability	
1.3.1	Define the Following Types of Event: Expectation, Dependent Event, Independent Event and Conditional Probability	57
	EXERCISE 7	61
	EXERCISE 8	67
	EXERCISE 9	71
	EXERCISE 10	74

1.3.2	Use Laws of Probability: Addition Law of Probability and Multiplication Law of Probability	75
1.3.3	Solve Problems on Probability	75
	EXERCISE 11	77
	ANSWER	79
	REFERENCES	87

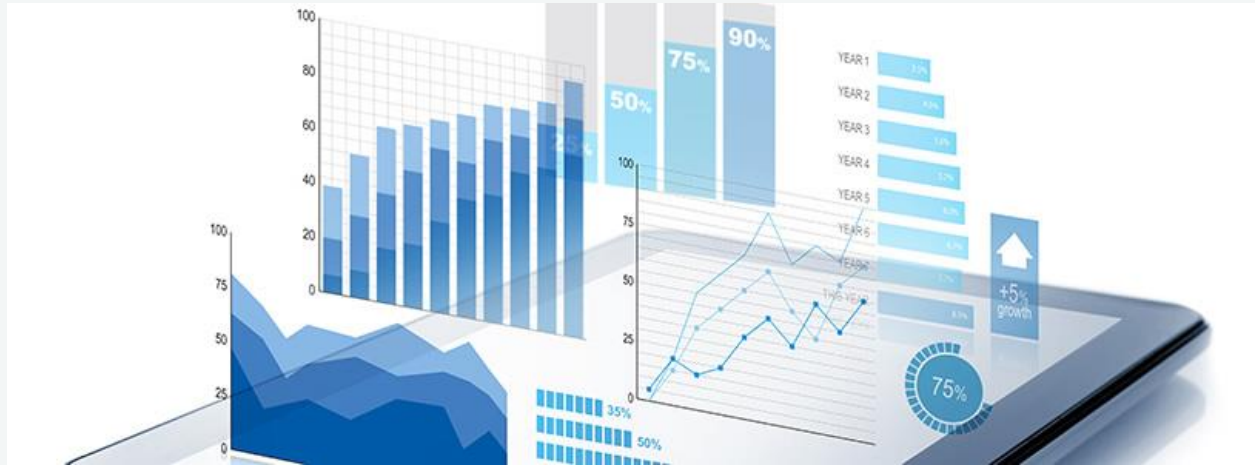
### 1.1 Demonstrate Statistical Data

#### 1.1.1 Define Statistical Terminology of Ungrouped Data and Grouped Data

---

#### STATISTICS

**Statistics** is the methodology for **collecting, organizing, analyzing, interpreting** and **presenting** the data to enable us for drawing conclusions from information.



#### STATISTICAL TERMINOLOGY

- ☞ **Data** is a collection of observations, measurements or information obtained from study that is carried out. Example: The marks obtained by students in mathematics test.
- ☞ Data are obtained by two methods:
  - By **counting** : number of patient enter to the clinic.  
**Discrete data** obtained by counting and only involves whole numbers.
  - By **measurement** : the weight of a group of polytechnic students.  
**Continuous data** is a measured data that can take any value within certain limits.
- ☞ **Population** is a set containing of all the members. Example, all the PTSS students in session of Jun 2021.
- ☞ **Sample** is some members selected at random from a population.
- ☞ **Frequency** is a number of times that the value of a member occurs in a data set.

## STATISTICS AND PROBABILITY

### UNGROUPEd DATA

- ☛ **Ungrouped data** is the data given as individual data points. It has not been classified or has not been subdivided in the form of groups. This type of data is totally the raw data.
- ☛ Ungrouped data can be representing by using bar chart as in Figure 1, line graph as in Figure 2, pie chart as in Figure 3 and pictogram / pictograph as in Figure 4. Normally, **bar chart** and **pie chart** are using to represent the ungrouped data.

Figure 1: Bar chart

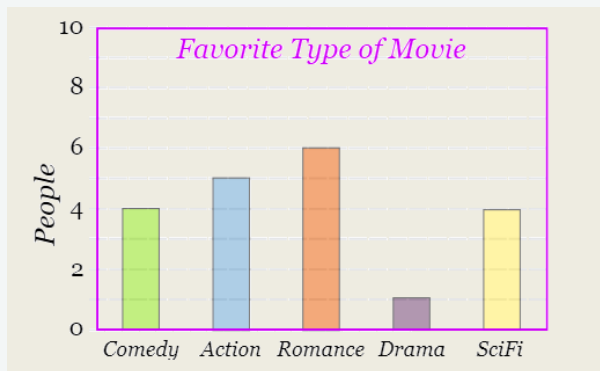


Figure 2: Line graph

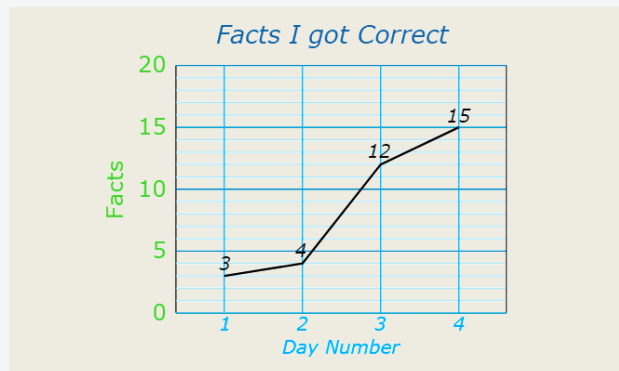


Figure 3: Pie chart

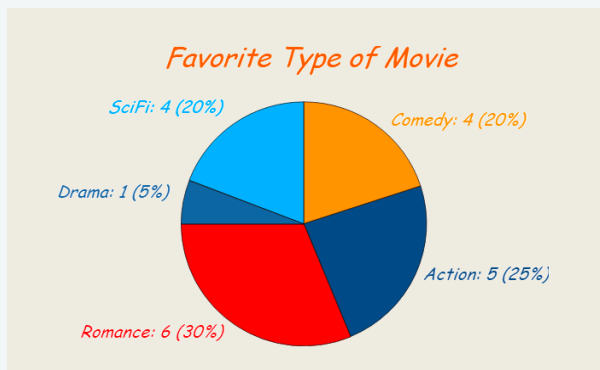
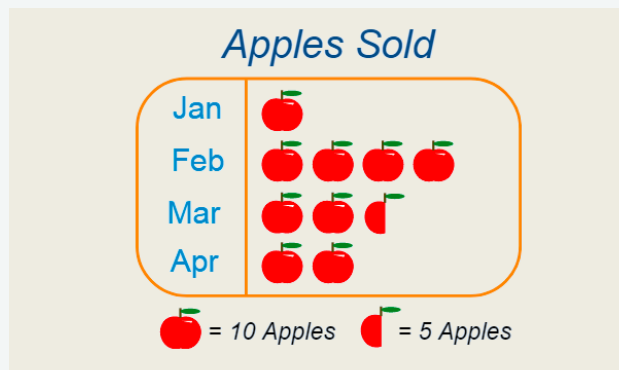


Figure 4: Pictogram / Pictograph





## GROUPED DATA

- ☛ **Grouped data** is data that has been organized into groups known as **classes** which means that the data is no longer raw.
- ☛ Grouped data can be representing by using **frequency distribution table** as in Table 1.

Table 1: Frequency distribution table

Class limits	Class boundaries	Tally marks	Frequency
100 – 104	99.5 – 104.5	//	2
105 – 109	104.5 – 109.5	///	8
110 – 114	109.5 – 114.5	/// ///	17
115 – 119	114.5 – 119.5	/// ///	14
120 – 124	119.5 – 124.5	///	4

## PROPERTIES OF FREQUENCY DISTRIBUTION TABLE

- ☛ **Lower class limit** – The smallest data value that can be included in the class. From the Table 2, lower class limits are 100, 105, 110.
- ☛ **Upper class limit** – The largest data value that can be included in the class. From the Table 2, upper class limits are 104, 109, 114.

Table 2: Example of class limits

Class limits
100 – 104
105 – 109
110 – 114

## STATISTICS AND PROBABILITY

- ☛ **Class boundary** – Numbers that do not occur in the sample data but can be finding by the midpoint between the upper limit of one class and the lower limit of the next class. Table 3 show the calculation in finding a class boundary.
- ☛ Two types of class boundary
  - **Lower boundary**
  - **Upper boundary**

Table 3: Class Boundary Calculation

Class limit	Calculation	Class boundary
100 – 104	$\frac{99+100}{2} = 99.5$ – $\frac{104+105}{2} = 104.5$	99.5 – 104.5
105 – 109	$\frac{104+105}{2} = 104.5$ – $\frac{109+110}{2} = 109.5$	104.5 – 109.5
110 – 114	$\frac{109+110}{2} = 109.5$ – $\frac{114+115}{2} = 114.5$	109.5 – 114.5

- ☛ **Class width / Class size** – The difference between the upper class boundary and lower class boundary. Table 4 show the calculation in finding a class width.

Table 4: Class Width Calculation

Class limit	Class boundary	Class width
100 – 104	99.5 – 104.5	$104.5 - 99.5 = 5$ or $104 - 100 + 1 = 5$
105 – 109	104.5 – 109.5	$109.5 - 104.5 = 5$ or $109 - 105 + 1 = 5$
110 – 114	109.5 – 114.5	$114.5 - 109.5 = 5$ or $114 - 110 + 1 = 5$

## STEPS FOR CONSTRUCT FREQUENCY DISTRIBUTION TABLE



**STEP 1** Determine the lowest and highest value.

**STEP 2** Find the range, R.

$$R = \text{highest value} - \text{lowest value}$$

**STEP 3** Determine the number of class by using Sturge's rule.

$$k = 1 + 3.33 \log n$$

$\rightarrow n = \text{total number of observations}$

**STEP 4** Determine the class width by using the following formula.

$$\frac{R}{\text{number of classes}}$$

**STEP 5** Determine the suitable class limits. Tally the data and find the frequencies from the tally.

**EXAMPLE 1**

Table 5 shows the amount of money earned weekly by 40 people working part-time in a factory, correct to the nearest RM. Construct a frequency distribution table for this data.

Table 5: The amount of money earned weekly by 40 people working part-time in a factory

80	90	70	110	90	160	110	80
140	30	90	50	100	110	60	100
80	90	110	80	100	90	120	70
130	169	80	120	100	110	40	110
50	100	110	90	100	70	110	80

**SOLUTION**

**STEP 1 :** Determine the lowest and highest value.

Lowest value = **30**

Highest value = **169**

**STEP 2 :** Determine the range.

Range (R) = Highest value – Lowest value

$$= 169 - 30$$

$$= 139$$

**STEP 3 :** Determine the number of class by using Sturge's rule.

$$k = 1 + 3.33 \log n$$

$$= 1 + 3.33 \log 40$$

$$= 6.33$$

$$\approx 7$$

## SOLUTION

**STEP 4 :** Find the class width.

$$\begin{aligned}
 \text{Class width} &= \frac{R}{k} \\
 &= \frac{139}{7} \\
 &= 19.86 \\
 &\approx \mathbf{20}
 \end{aligned}$$

**STEP 5 :** Construct the table with suitable class limit and tally the data.

Class limits	Class boundaries	Tally marks	Frequency
30 – 49	29.5 – 49.5	//	2
50 – 69	49.5 – 69.5	///	3
70 – 89	69.5 – 89.5	HHH ////	9
90 – 109	89.5 – 109.5	HHH HHH /	11
110 – 129	109.5 – 129.5	HHH HHH /	11
130 – 149	129.5 – 149.5	//	2
150 – 169	149.5 – 169.5	//	2

## STATISTICS AND PROBABILITY

### Exercise 1

1. The masses of 50 durian in kilograms are measured correct to the nearest 0.1kg and the results are as shown below. Construct a frequency distribution table for the data.

8.0	8.6	8.2	7.5	8.0	9.0	8.5	7.6	8.2	7.8
8.3	7.1	8.1	8.3	8.7	7.8	8.7	8.5	8.4	8.5
7.7	8.4	7.9	8.8	7.2	8.1	7.8	8.2	7.7	7.5
8.1	7.4	8.8	8.0	8.4	8.5	8.1	7.3	9.0	8.6
7.4	8.2	8.4	7.7	8.3	8.2	7.9	8.5	7.9	8.0

2. The mass in kilograms of 40 bars of metal are as shown. Construct a frequency distribution table by using 8 classes.

39.8	40.3	40.6	40.0	39.6	39.6	40.2	40.3	40.4	39.8
40.2	40.3	39.9	39.9	40.0	40.1	40.0	40.1	40.1	40.2
39.7	40.4	39.9	40.4	39.9	39.5	40.0	39.8	39.5	39.9
40.1	40.0	39.7	40.4	39.3	40.7	39.9	40.2	39.9	40.0

3. The time taken in hours to the failure of 50 specimens of a metal subjected to fatigue failure tests are as shown. Based on the data, construct the frequency distribution table with 6 class width.

28	22	23	20	12	24	37	28	21	25
21	14	30	23	27	13	23	7	26	19
24	22	26	3	21	24	28	40	27	24
20	25	23	26	47	21	29	26	22	33
27	9	13	35	20	16	20	25	18	22

4. The ages (in years) of 30 mothers giving birth to her first child are recorded as follows. Construct a frequency distribution table using 15 – 18 as a first class.

16	20	27	29	38	18	22	25	30	34
19	21	21	26	18	31	34	28	36	35
28	30	27	23	26	21	26	19	25	22

# Exercise 1

5. The height of 40 students in cm is shown as below. Construct a frequency distribution table if the number of classes required is 6.

131	145	132	141	125	150	146	136
146	135	128	155	166	148	133	160
134	160	158	151	146	156	148	164
135	146	148	137	158	140	136	156

6. The following are the number of quality eggs collected in a poultry farm in the month of December. With 50 class width, construct a frequency distribution table for the data.

658	253	541	357	643	450
387	552	346	637	248	582
624	394	487	365	576	492
612	293	547	338	295	535

Try your best!

## 1.2 Compute of Central Tendency and Dispersion

### 1.2.1 Calculate Mean, Median and Mode for Ungrouped Data

---

#### MEAN

**Mean** is the **average of the scores** in the population. Numerically, it equals the sum of the scores divided by the number of scores.

$$\bar{x} = \frac{\sum x}{n}$$

$x$  = data

$n$  = total number of observation

#### MEDIAN

Median is the value in the center of the data. Half of the data values are less than the median and half of the data values are more than the median when the data are arranged in ascending order.

$$\text{Median} = \left(\frac{n+1}{2}\right)^{th} \text{ observation (Odd data)}$$

$$\text{Median} = \frac{\left(\frac{n}{2}\right)^{th} \text{ observation} + \left(\frac{n}{2}+1\right)^{th} \text{ observation}}{2} \text{ (Even data)}$$

$n$  = number of data

#### MODE

- ☞ **Mode** is the data value that appears most often.
- ☞ If no value appears more than any other, then there is **no mode**.
- ☞ If two or more values appear more than the others, then the data is **bimodal or multimodal**.



**EXAMPLE 2**

Determine the mean, median and mode for the following set of data.

2, 3, 7, 5, 5, 13, 1, 7, 4, 8, 3, 4

**SOLUTION****MEAN**

**STEP 1 :** Write the correct formula for mean.

$$\text{Mean, } \bar{x} = \frac{\sum x}{n}$$

**STEP 2 :** Add up all the data and divide with the number of data.

$$\begin{aligned}\text{Mean, } \bar{x} &= \frac{2+3+7+5+5+13+1+7+4+8+3+4}{12} \\ &= \frac{62}{12} \\ &= 5.167\end{aligned}$$

## SOLUTION

### MEDIAN

**STEP 1 :** Rearrange the data in ascending order.

1, 2, 3, 3, 4, 4, 5, 5, 7, 7, 8, 13

**STEP 2 :** Determine the suitable formula by looking the number of data either odd or even and find the answer.

$$\begin{aligned}
 \text{Median} &= \frac{\left(\frac{12}{2}\right)^{\text{th}} \text{ observation} + \left(\frac{12}{2} + 1\right)^{\text{th}} \text{ observation}}{2} \\
 &= \frac{6^{\text{th}} \text{ observation} + 7^{\text{th}} \text{ observation}}{2} \\
 &= \frac{4 + 5}{2} \\
 &= \frac{9}{2} \\
 &= 4.5
 \end{aligned}$$

### MODE

**STEP 1 :** Determine the repetitive data.

Mode = 3, 4, 5, 7

## Exercise 2

- Determine the mean, median and mode for the following sets of data.
  - 3, 8, 10, 7, 5, 14, 2, 9
  - 26, 31, 21, 29, 32, 26, 25, 28
  - 4.72, 4.71, 4.74, 4.73, 4.72, 4.71, 4.73, 4.72, 4.74
- The following frequency distribution table shows the sizes of blouse sold in a mall in a week. Calculate the mean, median and mode of the blouse sold.

Size of blouse	1	2	3	4	5
Frequency	24	46	18	16	11

- The following frequency distribution table shows the number of pencils each student has for a group of students. Given that the mean number of pencils is 2.5, calculate the value of P.

Number of pencils	1	2	3	4	5
Number of students	4	7	5	P	1

- Given that the mean of the set of data 5, 7, 9, 12, M, 19, 9, 10 is 11. Find the value of M. Hence find median and mode.
- A set of numbers has eight numbers. The mean of the set of number is 11.
  - Find  $\sum x$
  - When a number k is added to the set of numbers, the mean becomes 10. Find the value of k.

## Exercise 2

6. A set of numbers 3, 6, 8,  $h$ , 12,  $k$ , 15, 20 that are arranged in ascending order has a mean and a median of 11. Find the values of  $h$  and  $k$ .
7. The table shows the frequency distribution of children in a nursery according to age. Find
- The maximum value of  $y$  if the mode age is 2 years.
  - The minimum value of  $y$  if the mean age is more than 3 years.

Age (years)	1	2	3	4	5
Frequency	3	5	2	$y$	4

Try your best!

### 1.2.2 Calculate Mean, Median and Mode for Grouped Data by Using Formula

#### MEAN

- ☞ **Mean** is the **average of the scores** in the population.
- ☞ The procedure for finding mean for grouped data is uses the midpoints of the classes.

$$\text{Midpoint, } x = \frac{\text{lower class limit} + \text{upper class limit}}{2}$$

- ☞ The formula we use to find mean for grouped data is:

$$\bar{x} = \frac{\sum fx}{\sum f}$$

$x$  = midpoint

$f$  = frequency

#### MEDIAN

- ☞ **Median** is the value in the center of the data.
- ☞ Median can be found by using the following steps:
  - Construct the cumulative frequency distribution table.
  - Decide the class that contain the median by using the formula  $\frac{n}{2}$  which  $n$  is the total of frequency.

## STATISTICS AND PROBABILITY

Find the median by using the following formula.

$$\text{Median} = L_m + \left( \frac{\frac{n}{2} - F}{f_m} \right) c$$

$L_m$  = lower boundary of the class median

$F$  = cumulative frequency before class median

$f_m$  = frequency of the class median

$n$  = total frequency

$c$  = class width

### MODE

**Mode** can be obtained by using the following formula.

$$\text{Mode} = L_m + \left( \frac{d_1}{d_1 + d_2} \right) c$$

$L_m$  = lower boundary of the class mode

$d_1$  = the difference between the frequency of the class mode and the frequency of the class before the class mode

$d_2$  = the difference between the frequency of the class mode and the frequency of the class after the class mode

$c$  = class width

### EXAMPLE 3

The following table shows the daily allowance, correct to the nearest RM1.00, of 60 workers of Eisham's Company. Calculate mean, median and mode for the data.

Daily allowance (RM)	50 – 59	60 – 69	70 – 79	80 – 89	90 – 99	100 – 109
Number of workers	6	10	16	14	10	4

### SOLUTION

#### MEAN

**STEP 1 :** Write the correct formula for mean.

$$\text{Mean, } \bar{x} = \frac{\sum fx}{\sum f}$$

**STEP 2 :** Based on the formula, determine the column needed to put in the frequency distribution table.

Daily Allowance (RM)	Frequency ( <i>f</i> )	Midpoint ( <i>x</i> )	<i>fx</i>
50 – 59	6	54.5	327
60 – 69	10	64.5	645
70 – 79	16	74.5	1192
80 – 89	14	84.5	1183
90 – 99	10	94.5	945
100 – 109	4	104.5	418
<b>Total</b>	<b>60</b>		<b>4710</b>

## SOLUTION

**STEP 3 :** Replace the required value into the formula and get the answer.

$$\begin{aligned}\text{Mean, } \bar{x} &= \frac{\sum fx}{\sum f} \\ &= \frac{4710}{60} \\ &= 78.5\end{aligned}$$

### MEDIAN

**STEP 1 :** Write the median formula.

$$\text{Median} = L_m + \left( \frac{\frac{n}{2} - F}{f_m} \right) c$$

**STEP 2 :** Based on the formula, determine the column needed to add in the previous frequency distribution table. (Use the previous table).

Daily Allowance (RM)	Frequency (f)	Midpoint (x)	fx	Lower boundary	Cumulative frequency (F)
50 – 59	6	54.5	327	49.5	6
60 – 69	10	64.5	645	59.5	16
70 – 79	16	74.5	1192	69.5	32
80 – 89	14	84.5	1183	79.5	46
90 – 99	10	94.5	945	89.5	56
100 – 109	4	104.5	418	99.5	60
<b>Total</b>	<b>60</b>		<b>4710</b>		

**STEP 3 :** Determine the median class by using formula  $\frac{n}{2}$ .

$$\text{Median class} = \left( \frac{n}{2} \right)^{th} = \left( \frac{60}{2} \right)^{th} = 30^{th}$$



## SOLUTION

**STEP 4 :** Replace the required value into the formula and get the answer.

$$\begin{aligned}
 \text{Median} &= L_m + \left( \frac{\frac{n}{2} - F}{f_m} \right) c \\
 &= 69.5 + \left( \frac{\frac{60}{2} - 16}{16} \right) 10 \\
 &= 69.5 + 8.75 \\
 &= \mathbf{78.25}
 \end{aligned}$$

**MODE**

**STEP 1 :** Write the mode formula.

$$\text{Mode} = L_m + \left( \frac{d_1}{d_1 + d_2} \right) c$$

**STEP 2 :** Based on the formula, determine the column needed to add in the previous frequency distribution table. (Use the previous table)

\*No need to add any other column.

**STEP 3 :** Determine the mode class.

Mode class = third class

**STEP 4 :** Replace the required value into the formula and get the answer.

$$\begin{aligned}
 \text{Mode} &= L_m + \left( \frac{d_1}{d_1 + d_2} \right) c \\
 &= 69.5 + \left( \frac{16 - 10}{[16 - 10] + [16 - 14]} \right) 10 \\
 &= 69.5 + 7.5 \\
 &= \mathbf{77}
 \end{aligned}$$

# Exercise 3

1. The length of a sample of 90 leaves from a tree is given in the table. Determine mean, median and mode for the data.

Length (cm)	Frequency
4 – 5	3
6 – 7	5
8 – 9	14
10 – 11	31
12 – 13	30
14 – 15	7

2. The height for 250 students were collected and grouped in the table below. Based on given data, find mean, median and mode.

Height (cm)	Number of students
150 – 154	19
155 – 159	36
160 – 164	54
165 – 169	55
170 – 174	41
175 – 179	23
180 – 184	14
185 – 189	8

## STATISTICS AND PROBABILITY

### Exercise 3

3. The table shows the number of books published in a year by 30 publishers. Find the median and mode for the data.

Number of books	50 – 99	100 – 149	150 – 199	200 – 249	250 – 299
Frequency	4	8	11	5	2

4. The following table shows the age range of teachers in a school.

Age (year)	Number of teacher
25 – 29	5
30 – 34	8
35 – 39	k
40 – 44	15
45 – 49	8
50 – 54	4

- (a) If the mean teacher's age is 39.5, find the value k.
- (b) Find the median and mode.
5. The table shows the marks obtained by 40 students in a test. Given that the median mark is 52.5. Find the value of x and y. Hence find the mode.

Marks	1 – 20	21 – 40	41 – 60	61 – 80	81 – 100
Frequency	6	x	y	9	7

## Exercise 3

6. A total of 100 candidates sat for a Mathematics test which carried a total mark of 140. The results are as shown in the following frequency distribution table. Determine mean, median and mode.

Marks	Number of candidates
1 – 20	2
21 – 40	9
41 – 60	15
61 – 80	28
81 – 100	27
101 – 120	14
121 – 140	5

7. The following frequency distribution table shows the amount of pocket money received in a day by 40 students. Determine mean, median and mode.

Pocket money (RM)	Number of students
1.00 – 1.90	6
2.00 – 2.90	12
3.00 – 3.90	13
4.00 – 4.90	5
5.00 – 5.90	3
6.00 – 6.90	1

Try your best!

### 1.2.3 Calculate Median and Mode for Grouped Data by Using Graph

#### MEDIAN

- ☞ Other than using the formula, median for grouped data can also be found by using **ogive**.
- ☞ There are a few steps that must follow when draw an ogive.
  - On the **vertical axis (y-axis)**, place **cumulative frequencies**. Label this axis "Cumulative Frequency".
  - On the **horizontal axis (x-axis)**, place the **upper boundary value** of each interval. Label this axis with the type of data shown.
  - Plot the cumulative frequencies of each class.

Figure 5: Less than ogive

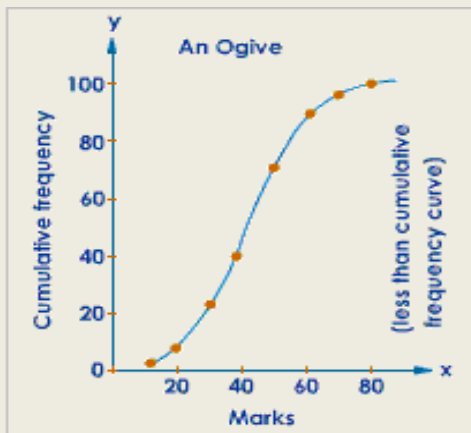
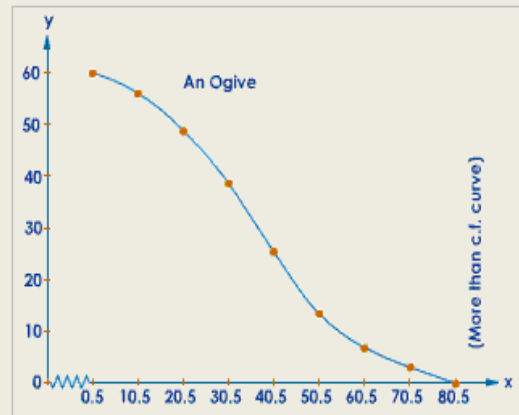
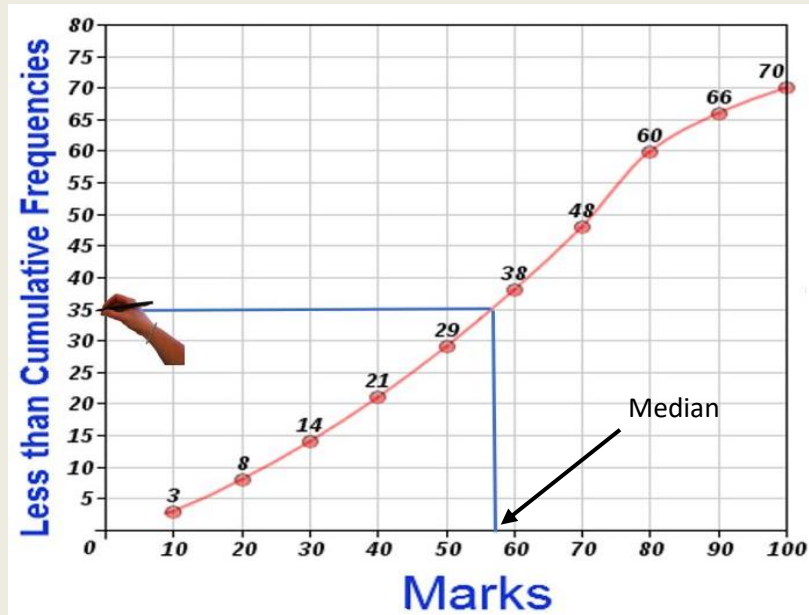


Figure 6: More than ogive



- ☞ After draw an ogive, find  $\frac{n}{2}$  value which  $n$  is a total of frequency.
- ☞ Based on the ogive, read the median at the  $\frac{n}{2}$  value. As example, total frequency is equal to 70. Then,  $\frac{n}{2}$  value is  $\frac{70}{2} = 35$ . At the ogive, draw a line at value 35, and read the value of median at the x axis.

Figure 7: Finding median of less than ogive graph

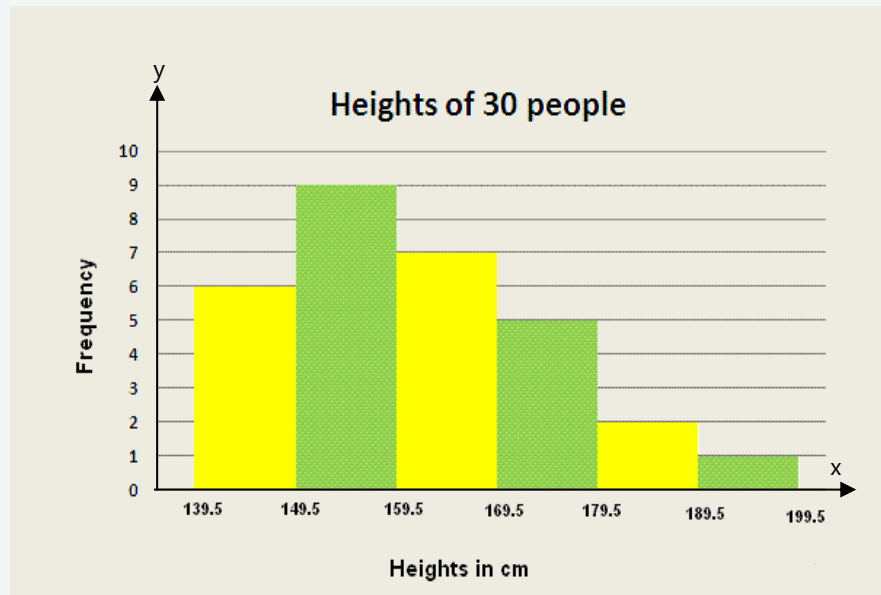


## MODE

- ☞ Other than using the formula, mode for grouped data can also be found by using **histogram**. Refer figure 8.
- ☞ There are a few steps that must follow when draw a histogram.
  - On the **vertical axis**, place **frequencies**. Label this axis "Frequency".
  - On the **horizontal axis**, place the **boundary value** of each interval. Label this axis with the type of data shown or class boundary.
  - Draw a bar based on the frequency. Make sure there has no gap without each bar.

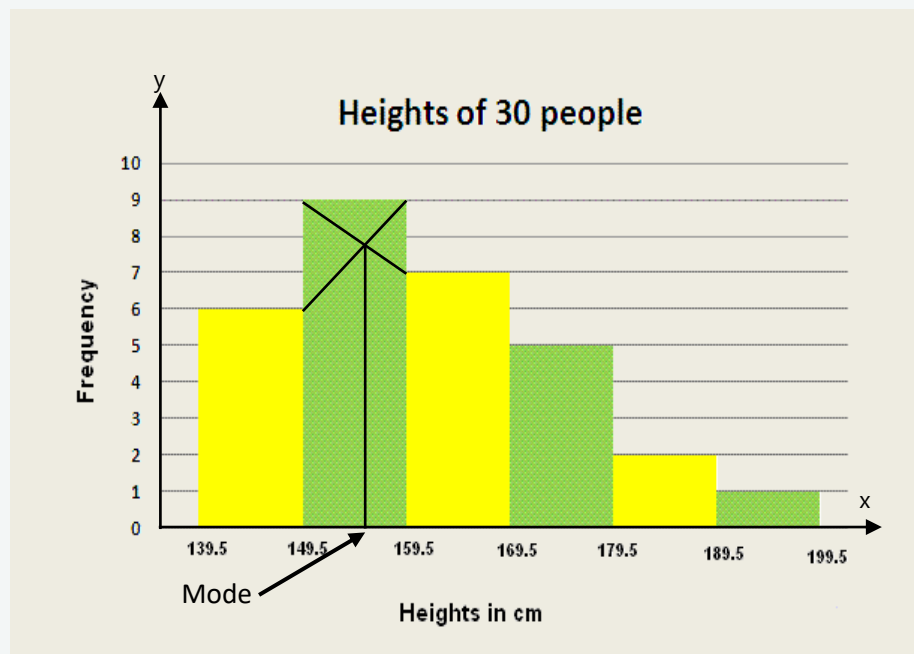
## STATISTICS AND PROBABILITY

Figure 8: Histogram



- Based on the histogram, choose the highest bar and draw a line as in the figure 9. Find the mode value through its intersection point.

Figure 9: Finding mode of a histogram



### EXAMPLE 4

The table below shows the marks obtained by a number of students in a recent Mathematics test.

Marks	Number of students
10 – 19	5
20 – 29	8
30 – 39	10
40 – 49	18
50 – 59	12
60 – 69	7

- (a) Draw a histogram to represent the data and estimate the mode.
- (b) Draw an ogive to illustrate the data and estimate the median.

### SOLUTION

#### HISTOGRAM

**STEP 1 :** Construct a frequency distribution table.

Marks	Number of students	Class boundary
10 – 19	5	9.5 – 19.5
20 – 29	8	19.5 – 29.5
30 – 39	10	29.5 – 39.5
40 – 49	18	39.5 – 49.5
50 – 59	12	49.5 – 59.5
60 – 69	7	59.5 – 69.5

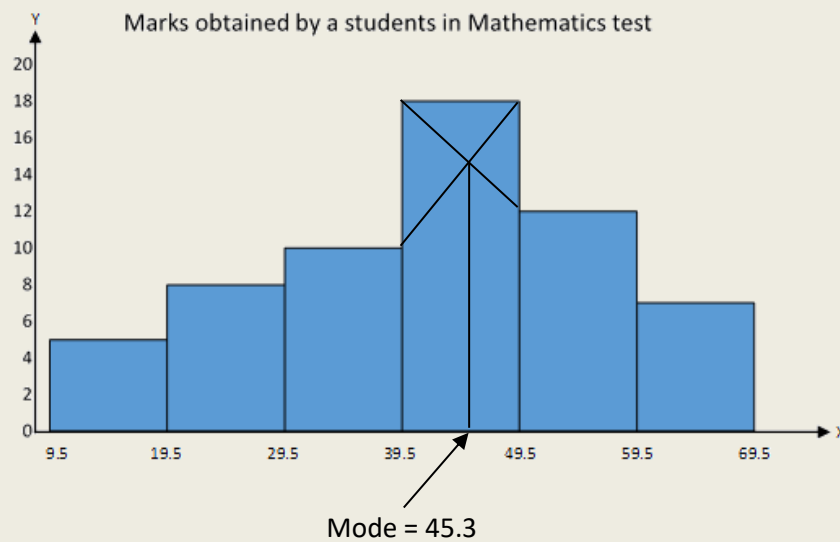


## SOLUTION

**STEP 2 :** Draw x-axis and y-axis with suitable scale & label the axis.

**STEP 3 :** Draw a bar based on the frequency and write a title.

**STEP 4 :** From the histogram, determine the highest bar and find the mode.



## OGIVE

**STEP 1 :** Construct a cumulative frequency distribution table by adding a row with 0 frequency at the top row of the data.

Marks	Number of students	Class boundary	Cumulative frequency
< 10	0	$\leq 9.5$	0
10 – 19	5	9.5 – 19.5	5
20 – 29	8	19.5 – 29.5	13
30 – 39	10	29.5 – 39.5	23
40 – 49	18	39.5 – 49.5	41
50 – 59	12	49.5 – 59.5	53
60 – 69	7	59.5 – 69.5	60

## SOLUTION

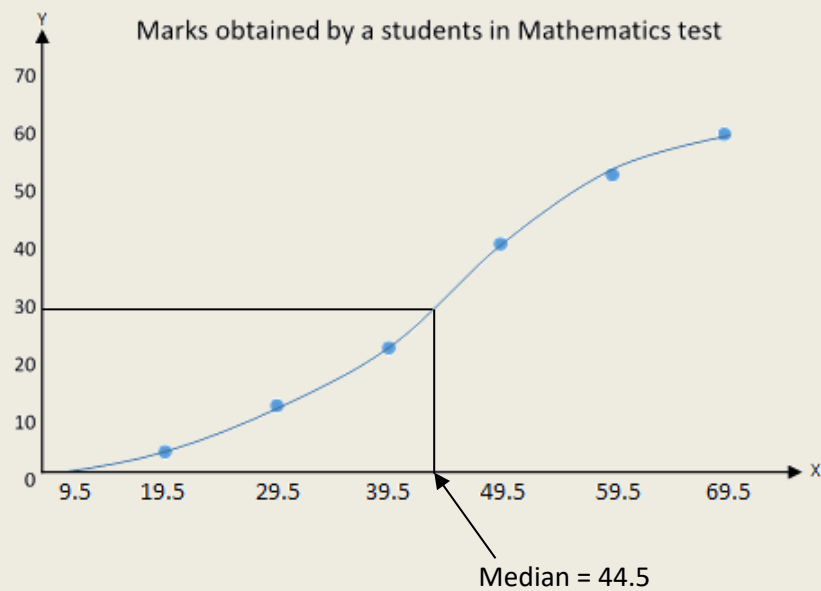
**STEP 2 :** Draw x-axis and y-axis with suitable scale and label the axis.

**STEP 3 :** Plot the point based on the cumulative frequency.

**STEP 4 :** Connect the point with smooth curve and write a title.

**STEP 5 :** Calculate the value of  $\frac{n}{2}$ . Based on the ogive, find the median at the  $\frac{n}{2}$  value.

$$\frac{n}{2} = \frac{60}{2} = 30$$



# Exercise 4

1. The following frequency table shows the distribution of the numbers of books read by 40 students in a particular month. Find median and mode by using graph.

Number of books	Frequency
1 – 5	5
6 – 10	8
11 – 15	12
16 – 20	10
21 – 25	5

2. The frequency table below shows the amount of year-end bonus received by 40 employees. Based on the data given in the table, find the mode and median.

Bonus (RM)	Number of employees
100 – 199	4
200 – 299	13
300 – 399	12
400 – 499	6
500 – 599	3
600 – 699	2

# Exercise 4

3. The following frequency distribution table shows the prices of ballpoint pens sold in a stationary shop. Find the mode and median by using graph.

Pocket money (RM)	Number of ballpoint pen
0.50 – 0.99	26
1.00 – 1.49	40
1.50 – 1.99	48
2.00 – 2.49	30
2.50 – 2.99	20
3.00 – 3.49	12

4. The following table shows the distribution of marks in a Science test for 50 students in a class. Determine the mode and median by using graph.

Marks	Number of students
31 – 40	2
41 – 50	8
51 – 60	15
61 – 70	12
71 – 80	10
81 – 90	3

## Exercise 4

5. The table shows the frequency distribution of the masses of a group of students.

Mass (kg)	Number of students
41 – 45	18
46 – 50	20
51 – 55	40
56 – 60	30
61 – 65	12
66 – 70	10

- (a) Draw a histogram and determine the mode of the data.  
 (b) Draw an ogive and determine the median of the data.
6. The table shows the frequency distribution of the pipes bought by a contractor based on their lengths. Determine the mode and median by using graph.

Length (m)	Frequency
1.0 – 1.4	4
1.5 – 1.9	5
2.0 – 2.4	6
2.5 – 2.9	7
3.0 – 3.4	3

Try your best!

### 1.2.4 Calculate Mean Deviation, Variance and Standard Deviation

#### MEAN DEVIATION

☞ **Mean deviation** (mean absolute deviation) of a data set is the average distance between each data value and the mean.

☞ The mean deviation formula for ungrouped data:

$$D_{\bar{x}} = \frac{|x_1 - \bar{x}| + |x_2 - \bar{x}| + \dots + |x_n - \bar{x}|}{N}$$

$$D_{\bar{x}} = \frac{\sum_{i=1}^n |x_i - \bar{x}|}{N}$$

$x_i$  = data

$\bar{x}$  = mean

$N$  = number of data

☞ The mean deviation formula for grouped data:

$$D_{\bar{x}} = \frac{|x_1 - \bar{x}|f_1 + |x_2 - \bar{x}|f_2 + \dots + |x_n - \bar{x}|f_n}{\sum f}$$

$$D_{\bar{x}} = \frac{\sum_{i=1}^n |x_i - \bar{x}|f_i}{\sum f}$$

$x_i$  = midpoint

$\bar{x}$  = mean

$f$  = frequency

## VARIANCE

☞ **Variance** is defined as the average of the **squared** differences from the mean.

☞ The variance formula for ungrouped data:

$$\sigma^2 = \frac{\sum |x - \bar{x}|^2}{N} \quad \text{or}$$

$$\sigma^2 = \frac{\sum x^2}{N} - \bar{x}^2$$

$x$  = data

$\bar{x}$  = mean

$N$  = number of data

☞ The variance formula for grouped data:

$$\sigma^2 = \frac{\sum f|x - \bar{x}|^2}{\sum f} \quad \text{or}$$

$$\sigma^2 = \frac{\sum fx^2}{\sum f} - \bar{x}^2$$

$x$  = midpoint

$\bar{x}$  = mean

$f$  = frequency

## STANDARD DEVIATION

☞ **Standard deviation** is the **square root** of the **variance**.

$$\sigma = \sqrt{\text{variance}}$$

**EXAMPLE 5**

Determine the mean deviation, variance and standard deviation for the following data.

12      6      3      7      8      10      11

**SOLUTION****MEAN DEVIATION**

**STEP 1 :** State the mean deviation formula.

$$\text{Mean deviation, } D_{\bar{x}} = \frac{\sum_{i=1}^n |x_i - \bar{x}|}{N}$$

**STEP 2 :** From the formula, the first value require to find mean deviation is mean. Find that value.

$$\begin{aligned} \text{Mean, } \bar{x} &= \frac{\sum x}{n} \\ &= \frac{12+6+3+7+8+10+11}{7} \\ &= \frac{57}{7} \\ &= 8.14 \end{aligned}$$

**Step 3 :** Based on the formula, construct a table with the column needed and complete the table.

$x$	$ x - \bar{x} $
12	3.86
6	2.14
3	5.14
7	1.14
8	0.14
10	1.86
11	2.86
Total	17.14



## SOLUTION

**STEP 4 :** Substitute all the value needed into the formula and do the calculation.

$$\begin{aligned}\text{Mean deviation, } D_{\bar{x}} &= \frac{\sum_{i=1}^n |x_i - \bar{x}|}{N} \\ &= \frac{17.14}{7} \\ &= \mathbf{2.45}\end{aligned}$$

### VARIANCE

**STEP 1 :** Write the variance formula. Based on the formula, determine the column needed to add into the previous table and complete it.

$$\text{Variance, } \sigma^2 = \frac{\sum |x - \bar{x}|^2}{N}$$

$x$	$ x - \bar{x} $	$ x - \bar{x} ^2$
12	3.86	14.8996
6	2.14	4.5796
3	5.14	26.4196
7	1.14	1.2996
8	0.14	0.0196
10	1.86	3.4596
11	2.86	8.1796
Total	17.14	58.8572

**Step 2 :** Substitute all the value needed into the formula and do the calculation.

$$\begin{aligned}\text{Variance, } \sigma^2 &= \frac{\sum |x - \bar{x}|^2}{N} \\ &= \frac{58.8572}{7} \\ &= \mathbf{8.41}\end{aligned}$$

## SOLUTION

### STANDARD DEVIATION

**STEP 1 :** Find standard deviation using the suitable formula.

$$\begin{aligned}\text{Standard deviation, } \sigma &= \sqrt{\text{variance}} \\ &= \sqrt{8.41} \\ &= 2.9\end{aligned}$$

## EXAMPLE 6

Determine the mean deviation, variance and standard deviation for the following data.

Number of students	27 - 29	30 - 32	33 - 35	36 - 38	39 - 41	42 - 44
Number of classes	5	7	5	6	4	3

## SOLUTION

### MEAN DEVIATION

**STEP 1 :** State the mean deviation formula.

$$\text{Mean deviation, } D_{\bar{x}} = \frac{\sum_{i=1}^n |x_i - \bar{x}| f_i}{\sum f}$$

**SOLUTION**

**Step 2 :** From the formula, the first value require to find mean deviation is a mean. Find that value.

No. of students	$f$	$x$	$fx$
27 – 29	5	28	140
30 – 32	7	31	217
33 – 35	5	34	170
36 – 38	6	37	222
39 – 41	4	40	160
42 – 44	3	43	129
Total	30		1038

$$\begin{aligned}
 \text{Mean, } \bar{x} &= \frac{\sum fx}{\sum f} \\
 &= \frac{1038}{30} \\
 &= 34.6
 \end{aligned}$$

**Step 3 :** Based on the mean deviation formula, determine the next column needed and add into the previous table.

No. of students	$f$	$x$	$fx$	$ x - \bar{x} $	$f x - \bar{x} $
27 – 29	5	28	140	6.6	33
30 – 32	7	31	217	3.6	25.2
33 – 35	5	34	170	0.6	3
36 – 38	6	37	222	2.4	14.4
39 – 41	4	40	160	5.4	21.6
42 – 44	3	43	129	8.4	25.2
Total	30		1038		122.4

## SOLUTION

**Step 4 :** Substitute all the value needed into the formula and do the calculation.

$$\begin{aligned}\text{Mean deviation, } D_{\bar{x}} &= \frac{\sum_{i=1}^n |x_i - \bar{x}| f_i}{\sum f} \\ &= \frac{122.4}{30} \\ &= \mathbf{4.08}\end{aligned}$$

### VARIANCE

**STEP 1 :** State the variance formula.

$$\text{Variance, } \sigma^2 = \frac{\sum f |x - \bar{x}|^2}{\sum f}$$

**Step 2 :** Based on the formula, determine the next column needed and add into the previous table.

No. of students	$f$	$x$	$fx$	$ x - \bar{x} $	$f x - \bar{x} $	$ x - \bar{x} ^2$	$f x - \bar{x} ^2$
27 – 29	5	28	140	6.6	33	43.56	217.8
30 – 32	7	31	217	3.6	25.2	12.96	90.72
33 – 35	5	34	170	0.6	3	0.36	1.8
36 – 38	6	37	222	2.4	14.4	5.76	34.56
39 – 41	4	40	160	5.4	21.6	29.16	116.64
42 – 44	3	43	129	8.4	25.2	70.56	211.68
Total	30		1038		122.4		673.2

**SOLUTION**

**Step 3** : Substitute all the value needed into the formula and do the calculation.

$$\begin{aligned}\text{Variance, } \sigma^2 &= \frac{\sum f|x-\bar{x}|^2}{\sum f} \\ &= \frac{673.2}{30} \\ &= \mathbf{22.44}\end{aligned}$$

**STANDARD DEVIATION**

**Step 1** : Find standard deviation using the suitable formula.

$$\begin{aligned}\text{Standard deviation, } \sigma &= \sqrt{\text{variance}} \\ &= \sqrt{22.44} \\ &= \mathbf{4.74}\end{aligned}$$

# Exercise 5

1. Given the data as below. Find mean deviation, variance and standard deviation.

(a) 17, 12, 5, 19, 8, 13, 43, 6, 11, 24

(b) 21, 12, 15, 14, 9, 18, 34, 16, 24, 42

2. Mean given by a set of numbers  $k-1$ ,  $k+4$ ,  $3k+5$ ,  $3k-1$ ,  $3k+2$  and  $k-3$  is 11. Calculate mean deviation, variance and standard deviation.

3. Given the data below

Frequency $f$	$ x - \bar{x} $
5	17.5
2	13.2
1	9.2
9	5.2
2	1.2
6	2.8
8	6.8
2	10.8
4	14.8
1	18.8
$\Sigma f = n = 40$	

Calculate mean deviation, variance and standard deviation.

# Exercise 5

4. Find the mean deviation, variance and standard deviation of the following frequency distribution.

Class interval	Frequency
0 – 5	2
6 – 11	4
12 – 17	10
18 – 23	12
24 – 29	8
30 – 36	4

5. The table shows the distribution of time taken by 30 typists to type a document. Calculate the mean deviation, variance and standard deviation of time to type the document.

Time (minute)	Frequency
10	6
11	10
12	8
13	4
14	2

6. If the mean and standard deviation of the numbers below are 10 and 5, respectively, calculate the value of  $x$  and the values of  $y$ .

$$9, 11, x + y, x - y$$

# Exercise 5

7. Determine the standard deviation of the following data.

(a) 4, 6, 3, 2, 9, 10, 13, 5

(b)

Score	0	1	2	3	4
Number of attempts	5	8	9	5	3

8. The sum of 12 numbers is 156 and the sum of the squares of these 12 numbers is 2496. For this set of numbers, find:

(a) the variance

(b) the standard deviation

9. The table below shows the frequency distribution of the ages of workers in a factory.

Age (years)	Frequency
25 – 29	5
30 – 34	8
35 – 39	$x$
40 – 44	15
45 – 49	8
50 – 54	4

(a) If the mean age of the workers is 39.5 years, find the value of  $x$ .

(b) Find mean deviation, variance and standard deviation.





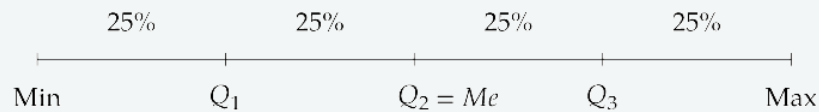
### 1.2.5 Calculate Quartiles, Deciles and Percentiles for Grouped Data by Using Formula and Graph

#### QUARTILE

☞ **Quartile** is **3** values of a variable that divide the distribution into **4** intervals with equal probability.

- First quartile ( $Q_1$ ) = lower quartile
- Second quartile ( $Q_2$ ) = median = 50th percentile
- Third quartile ( $Q_3$ ) = upper quartile

☞ The difference between the upper and lower quartiles is called the **interquartile range**.



☞ To finding the quartile value, firstly we must find the quartile class by using the following formula:

$$Q_k = \frac{k}{4} \sum f; k = 1, 2, 3$$

$\sum f$  = total frequency

Then **Quartile** can be obtained by using the following formula:

$$Q_k = L_{Q_k} + \left( \frac{\frac{kN}{4} - F}{f_{Q_k}} \right) c ; k = 1, 2, 3$$

$L_{Q_k}$  = lower boundary of the quartile class

$f_{Q_k}$  = frequency of quartile class

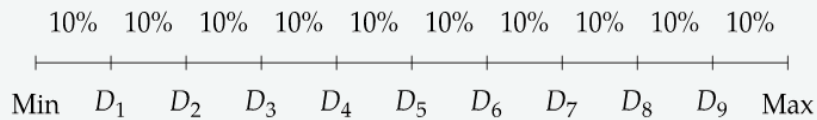
$N$  = total frequency

$F$  = cumulative frequency preceding quartile class

$c$  = class width

## DECILE

**Deciles** are the 9 values that split the data set into **10 equal parts**.



To finding the decile value, firstly we must find the decile class by using the following formula:

$$D_k = \frac{k}{10} \sum f ; k = 1, 2, \dots, 9$$

$\sum f$  = total frequency

Then **Decile** can be obtained by using the following formula:

$$D_k = L_{D_k} + \left( \frac{\frac{kN}{10} - F}{f_{D_k}} \right) c ; k = 1, 2, \dots, 9$$

$L_{D_k}$  = lower boundary of the decile class

$f_{D_k}$  = frequency of decile class

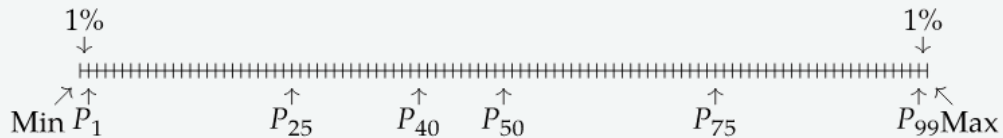
$N$  = total frequency

$F$  = cumulative frequency preceding decile class

$c$  = class width

## PERCENTILE

**Percentiles** are the **99** values that split the data set into **100 equal parts**.



To finding the percentile value, firstly we must find the percentile class by using the following formula:

$$P_k = \frac{k}{100} \sum f ; k = 1, 2, \dots, 99$$

$\sum f$  = total frequency

Then **Percentile** can be obtained by using the following formula:

$$P_k = L_{P_k} + \left( \frac{\frac{kN}{100} - F}{f_{P_k}} \right) c ; k = 1, 2, \dots, 99$$

$L_{P_k}$  = lower boundary of the percentile class

$f_{P_k}$  = frequency of percentile class

$N$  = total frequency

$F$  = cumulative frequency preceding percentile class

$c$  = class width

### QUARTILE, DECILE & PERCENTILE USING GRAPH

**Quartile, Decile, Percentile** can also be finding by using a graph which is **ogive**.

After draw ogive, quartile class, decile, percentile should be find first by using the following formula.

$$Q_k = \frac{k}{4} \sum f ; k = 1, 2, 3$$

$\sum f$  = total frequency

$$D_k = \frac{k}{10} \sum f ; k = 1, 2, \dots, 9$$

$\sum f$  = total frequency

$$P_k = \frac{k}{100} \sum f ; k = 1, 2, \dots, 99$$

$\sum f$  = total frequency

### EXAMPLE 7

Table below shows a frequency distribution of the weight of the 40 female students at one college. Estimate the following by using formula and graph.

Weight (kg)	Frequency
118 – 126	2
127 – 135	6
136 – 144	8
145 – 153	13
154 – 162	5
163 – 171	4
172 – 180	2

- (a) The lower quartile
- (b) The interquartile range
- (c) The 75<sup>th</sup> percentile
- (d) The 6<sup>th</sup> decile

### SOLUTION

#### (a) Lower Quartile

**Step 1** : Construct a cumulative frequency distribution table.

Weight (kg)	$f$	Class boundary	$F$
118 – 126	2	117.5 – 126.5	2
127 – 135	6	126.5 – 135.5	8
136 – 144	8	135.5 – 144.5	16
145 – 153	13	144.5 – 153.5	29
154 – 162	5	153.5 – 162.5	34
163 – 171	4	162.5 – 171.5	38
172 – 180	2	171.5 – 180.5	40

## SOLUTION

**Step 2 :** Determine the lower quartile class.

$$\frac{1}{4} \sum f = \frac{1}{4}(40) = 10$$

$\therefore$  lower quartile class = 136 – 144

**Step 3 :** Write the lower quartile formula. Substitute all the value needed into the formula and get the answer.

$$\begin{aligned} \text{Lower quartile, } Q_1 &= L_{Q_1} + \left( \frac{\frac{1(N)}{4} - F}{f_{Q_1}} \right) c \\ &= 135.5 + \left( \frac{\frac{1(40)}{4} - 8}{8} \right) 9 \\ &= 137.75 \end{aligned}$$

**(b) Interquartile Range**

**Step 1 :** Construct a cumulative frequency distribution table.

Weight (kg)	$f$	Class boundary	$F$
118 – 126	2	117.5 – 126.5	2
127 – 135	6	126.5 – 135.5	8
136 – 144	8	135.5 – 144.5	16
145 – 153	13	144.5 – 153.5	29
154 – 162	5	153.5 – 162.5	34
163 – 171	4	162.5 – 171.5	38
172 – 180	2	171.5 – 180.5	40

**Step 2 :** Determine the upper quartile class.

$$\frac{3}{4} \sum f = \frac{3}{4}(40) = 30$$

$\therefore$  upper quartile class = 154 – 162

## SOLUTION

**Step 3 :** Write the upper quartile formula. Substitute all the value needed into the formula and get the answer.

$$\begin{aligned}\text{Upper quartile, } Q_3 &= L_{Q_3} + \left( \frac{\frac{3(N)}{4} - F}{f_{Q_3}} \right) c \\ &= 153.5 + \left( \frac{\frac{3(40)}{4} - 29}{5} \right) 9 \\ &= 155.3\end{aligned}$$

**Step 4 :** Find the interquartile range by using the correct formula.

$$\begin{aligned}\text{Interquartile range} &= Q_3 - Q_1 \\ &= 155.3 - 137.75 \\ &= 17.55\end{aligned}$$

**(c) Percentile**

**Step 1 :** Construct a cumulative frequency distribution table.

Weight (kg)	$f$	Class boundary	$F$
118 – 126	2	117.5 – 126.5	2
127 – 135	6	126.5 – 135.5	8
136 – 144	8	135.5 – 144.5	16
145 – 153	13	144.5 – 153.5	29
154 – 162	5	153.5 – 162.5	34
163 – 171	4	162.5 – 171.5	38
172 – 180	2	171.5 – 180.5	40

## SOLUTION

**Step 2** : Determine the 75<sup>th</sup> percentile class.

$$\frac{75}{100} \sum f = \frac{75}{100} (40) = 30$$

$\therefore$  75<sup>th</sup> percentile class = 154 – 162

**Step 3** : Write the percentile formula. Substitute all the value needed into the formula and get the answer.

$$\begin{aligned} 75^{\text{th}} \text{ percentile, } P_{75} &= L_{P_{75}} + \left( \frac{\frac{75(N)}{100} - F}{f_{P_{75}}} \right) c \\ &= 153.5 + \left( \frac{\frac{75(40)}{100} - 29}{5} \right) 9 \\ &= \mathbf{155.3} \end{aligned}$$

**(d) Decile**

**Step 1** : Construct a cumulative frequency distribution table.

Weight (kg)	$f$	Class boundary	$F$
118 – 126	2	117.5 – 126.5	2
127 – 135	6	126.5 – 135.5	8
136 – 144	8	135.5 – 144.5	16
145 – 153	13	144.5 – 153.5	29
154 – 162	5	153.5 – 162.5	34
163 – 171	4	162.5 – 171.5	38
172 – 180	2	171.5 – 180.5	40



## SOLUTION

**Step 2 :** Determine the 6<sup>th</sup> decile class.

$$\frac{6}{10} \sum f = \frac{6}{10} (40) = 24$$

$$\therefore 6^{\text{th}} \text{ decile class} = 145 - 153$$

**Step 3 :** Write the percentile formula. Substitute all the value needed into the formula and get the answer.

$$\begin{aligned} 6^{\text{th}} \text{ decile, } D_6 &= L_{D_6} + \left( \frac{\frac{6(N)}{10} - F}{f_{D_6}} \right) c \\ &= 144.5 + \left( \frac{\frac{6(40)}{10} - 16}{13} \right) 9 \\ &= \mathbf{150.04} \end{aligned}$$

## SOLUTION

Using Graph (Ogive)

**Step 1 :** Construct a cumulative frequency distribution table by adding a row with 0 frequency at the top row of the data.

Weight (kg)	$f$	Upper boundary	$F$
< 118	0	$\leq 117.5$	0
118 – 126	2	126.5	2
127 – 135	6	135.5	8
136 – 144	8	144.5	16
145 – 153	13	153.5	29
154 – 162	5	162.5	34
163 – 171	4	171.5	38
172 – 180	2	180.5	40

**Step 2 :** Draw an ogive like the previous subtopic.

**Step 3 :** Find lower quartile class, upper quartile class, 75<sup>th</sup> percentile class and 6<sup>th</sup> decile class.

$$\text{Lower quartile class} = \frac{1}{4} \sum f = \frac{1}{4}(40) = 10$$

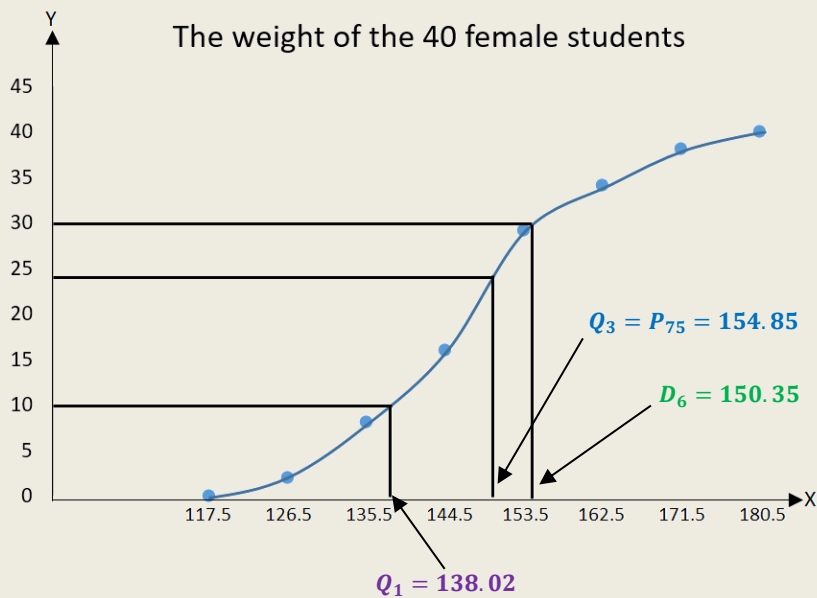
$$\text{Upper quartile class} = \frac{3}{4} \sum f = \frac{3}{4}(40) = 30$$

$$75^{\text{th}} \text{ percentile class} = \frac{75}{100} \sum f = \frac{75}{100}(40) = 30$$

$$6^{\text{th}} \text{ decile class} = \frac{6}{10} \sum f = \frac{6}{10}(40) = 24$$

**SOLUTION**

**Step 4** : Based on the value obtained, find lower quartile, upper quartile, interquartile range, 75<sup>th</sup> percentile and 6<sup>th</sup> decile through the ogive drawn.



$$\begin{aligned}
 \text{Interquartile range} &= Q_3 - Q_1 \\
 &= 154.85 - 138.02 \\
 &= \mathbf{16.83}
 \end{aligned}$$

## Exercise 6

1. The frequency distribution table shows the lengths recorded to the nearest mm, of 250 leaves collected by a group of botanist from a particular rainforest.

Length (mm)	Number of leaves
98 – 99	12
100 – 101	36
102 – 103	106
104 – 105	58
106 – 107	30
108 – 109	8

Calculate

- (a) First quartile
- (b) 45<sup>th</sup> persentile

By using formula and graph.

2. In a mathematics exam, this distribution was obtained from 25 students.

Score	40 – 45	46 – 51	52 – 57	58 – 63	64 – 69
Number of students	3	8	10	3	1

Calculate

- (a) Interquartile range
- (b) 9<sup>th</sup> decile

By using formula and graph.

## Exercise 6

3. The distribution of the number of errors that 10 students made on a typing test is shown below.

Error	0 – 2	3 – 5	6 – 8	9 – 11	12 – 14
Number of students	1	3	4	1	1

Calculate

- (a) Third quartile
- (b) 7<sup>th</sup> decile
- (c) 33<sup>rd</sup> percentile

By using formula and graph.

4. The following data indicates the number of pineapples that have been sold by the sellers near Juru Industrial area.

Number of Pineapples	Frequency
1 – 10	3
11 – 20	5
21 – 30	10
31 – 40	21
41 – 50	13
51 – 60	6
61 – 70	2

By using formula and graph, calculate

- (a) First quartile
- (b) 70<sup>th</sup> percentile

## Exercise 6

5. The table below shows the frequency of 250 producers who use Milo as an additional ingredient for chocolate cakes.

Weight (g)	Number of Producers
1.45 – 2.44	12
2.45 – 3.44	15
3.45 – 4.44	40
4.45 – 5.44	59
5.45 – 6.44	68
6.45 – 7.44	30
7.45 – 8.44	14
8.45 – 9.44	12

Calculate

- (a) Interquartile range
- (b) 8<sup>th</sup> decile

By using formula and graph.



## 1.3 Understand Probability

### 1.3.1 Define the Following Types of Event: Expectation, Dependent Event, Independent Event and Conditional Probability

---

#### EXPERIMENT

**Experiment** is a process involving chance that leads to one of several possible outcomes. Example, tossing coins, rolling dice and testing product for defects.

#### OUTCOME

**Outcome** is a result of all possible results of an experiment.

#### SAMPLE SPACE

**Sample space, S** is the set of all possible outcomes of an experiment.

Experiment	Sample space
Roll a die	1, 2, 3, 4, 5, 6
Toss two coins	HH, HT, TT, TH

#### PROBABILITY OF EVENT

The probability of any event  $E$  is defined as:

$$P(E) = \frac{n(E)}{n(S)}$$

$$= \frac{\text{Number of possible outcomes in } E}{\text{Number of possible outcomes in sample space } S}$$

### PROPERTIES OF PROBABILITY

The **probability** of any event  $E$  is defined as:

- For any event  $A$ , the probability is  $0 \leq P(A) \leq 1$ .
- If an event cannot occur, its probability is 0.
- The sum of the probabilities of the outcomes in the sample space is 1.

### COMPLEMENTARY EVENT

- The **complement of an event**  $E$  is the set of outcomes in the sample space that are not included in the outcomes of event  $E$ .
- The complement of  $E$  is denoted by  $\bar{E}$ .

$$P(\bar{E}) = 1 - P(E)$$



**EXAMPLE 8**

A fair coin is tossed before a fair dice is rolled. State the outcomes of the following events.

- (a) Heads is obtained in the toss.
- (b) The number obtained is an odd number.
- (c) Hence find the probability for question a) and b).

**SOLUTION**

**Step 1 :** State the sample space,  $S$  & determine  $n(S)$ .

$$S = \{(H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6), (T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6)\}$$

$$n(S) = 12$$

**Step 2 :** Declare the events using suitable symbol.

$H$  = heads is obtained in the toss

$O$  = number obtained is an odd number

**Step 3 :** State the outcomes for each event & determine the number of outcomes.

a)  $H = \{(H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6)\}$

$$n(H) = 6$$

b)  $O = \{(H, 1), (H, 3), (H, 5), (T, 1), (T, 3), (T, 5)\}$

$$n(O) = 6$$

## SOLUTION

**Step 4** : Use the probability formula for an event and find the answer.

$$\begin{aligned}\text{c) } P(H) &= \frac{n(H)}{n(S)} \\ &= \frac{6}{12} \\ &= \frac{1}{2}\end{aligned}$$

$$\begin{aligned}P(O) &= \frac{n(O)}{n(S)} \\ &= \frac{6}{12} \\ &= \frac{1}{2}\end{aligned}$$

# Exercise 7

1. A bag has 20 cards. The cards are numbered 1 to 20 respectively. If a card is chosen at random, find the probability of obtaining:
  - (a) An even number
  - (b) An odd number greater than 4
  
2. A box contains 20 oranges, 15 apples and 5 lemons. If a fruit is selected at random from the box, find the probability of getting:
  - (a) An apple
  - (b) A lemon
  
3. In year 2015, there are 120 male workers and 180 female workers in a factory. In year 2018, another 20 female workers joined the factory. A worker is randomly selected from the factory in year 2018. Find the probability that the selected worker is female.
  
4. The following table shows the number of chalks in two boxes, A and B.

Box	Number of chalks	
	White	Red
A	64	36
B	86	$x$

The probability of choosing a white chalk from the total number of chalks in the two boxes is  $\frac{3}{5}$ . Find the value of  $x$ .

## Exercise 7

5. A letter is chosen at random from the word 'PROBLEMATIC'. Find the probability of choosing
- (a) a vowel
  - (b) a consonant
6. A number is chosen at random from the set of numbers  $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ . Determine the probability of choosing
- (a) an odd number
  - (b) an even number
  - (c) a prime number
  - (d) a number that is less than 6
  - (e) a number that is greater than 3

Try your best!

### DEPENDENT EVENT

Two events are **dependent** if the outcome or occurrence of the first event **affects** the outcome of the second event so that the probability is changed.

$$P(A \cap B) \neq P(A) \times P(B)$$

### INDEPENDENT EVENT

Two events A and B are **independent** if the outcome of A does **not affects** the outcome or occurrence of B and vice versa.

$$P(A \cap B) = P(A) \times P(B)$$

## EXAMPLE 9

Five yellow marbles and seven black marbles are placed in a bag. A marble is drawn from the bag and not replaced. If the process is repeated two times, what is the probability that all two draws are black marbles?

### SOLUTION

**Step 1 :** Declare the events using suitable symbol.

$Y$  = Drawn a yellow marble

$B$  = Drawn a black marble

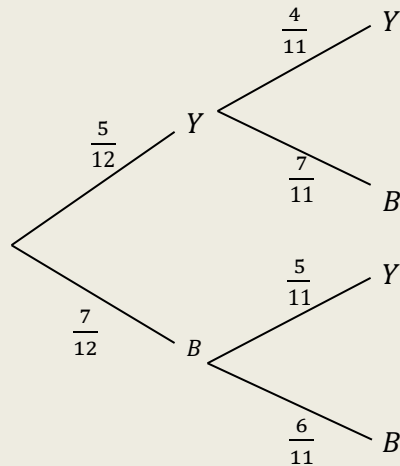
**Step 2 :** State the sample space,  $S$  and determine  $n(S)$ .

$S = \{Y, Y, Y, Y, Y, B, B, B, B, B, B, B\}$

$n(S) = 12$

**SOLUTION**

**Step 3 :** Draw a tree diagram to show the probability.



**Step 4 :** Use the probability formula for an event & find the answer.

$$P(\text{all two draws are black marbles}) = P(B) \times P(B)$$

$$= \frac{7}{12} \times \frac{6}{11}$$

$$= \frac{7}{22}$$

**EXAMPLE 10**

A box contains 3 red balls, 2 blue balls and 5 white balls. A ball is selected and its color noted. Then it is replaced. A second ball is selected and its color noted. Find the probability of each of these.

- (a) Selecting 2 blue balls
- (b) Selecting a blue ball and then a white ball
- (c) Selecting a red ball and then a blue ball

**SOLUTION**

**Step 1 :** State the sample space,  $S$  and determine  $n(S)$ .

$S = \{\text{red, red, red, blue, blue, white, white, white, white, white}\}$

$$n(S) = 10$$

**Step 2 :** Declare the events using suitable symbol.

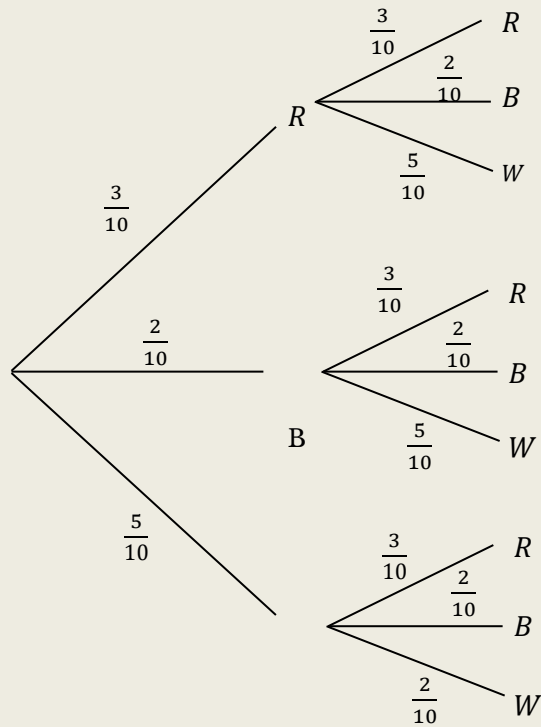
$R$  = Select a red ball

$B$  = Select a blue ball

$W$  = Select a white ball

**SOLUTION**

**Step 3 :** Draw a tree diagram to show the probability.



**Step 4 :** Use the probability formula for an event and find the answer.

(a)  $P(\text{selecting 2 blue balls}) = P(B) \times P(B)$

$$= \frac{2}{10} \times \frac{2}{10}$$

$$= \frac{1}{25}$$

(b)  $P(\text{Selecting a blue ball and then a white ball}) = P(B) \times P(W)$

$$= \frac{2}{10} \times \frac{5}{10}$$

$$= \frac{1}{10}$$

(c)  $P(\text{Selecting a red ball and then a blue ball}) = P(R) \times P(B)$

$$= \frac{3}{10} \times \frac{2}{10}$$

$$= \frac{3}{50}$$





## Exercise 8

1. A bag has 3 blue marbles and 5 red marbles. A marble is drawn at random from the bag and its color is recorded. It is then returned to the bag. After that, another marble is drawn at random from the bag and its color is also recorded. Find the probability
  - (a) Both the marbles are of the same colors.
  - (b) Both the marbles are of the different colors.
  
2. A bag has 12 coloured disc; 7 green and 5 yellow. Two discs are drawn at random, one after the other, without replacement. Find the probability
  - (a) A green disc and a yellow disc are drawn
  - (b) The two discs drawn are either both green or both yellow.
  
3. Box A contains 2 cubes and 4 spheres. Box B contains 3 cubes and 1 sphere. An object is drawn at random from box A and put into box B. Then, an object is drawn at random from box B. Calculate the probability that a sphere is drawn from box B.
  
4. On a certain day, three babies were born at a hospital in Kuala Lumpur. Construct a tree diagram to show all the possible outcomes of the genders of the babies. Hence, find the probability that
  - (a) 2 baby boys and 1 baby girl were born.
  - (b) at least one baby boy was born.

## Exercise 8

5. A company gives away 10 towels as souvenir to the participants of a public speaking competition. It is known that there are 7 blue towels and the rest are red. If Siti and Mariam are the first and second participants to receive to souvenir, find the probability that Siti and Mariam receive towels that have the same colors.
  
6. The shelf displays 50 packets of chocolate where 15 packets are of brand A, 17 packets are of brand B and the rest are brand C. One packet of chocolate is taken and not replaced by a customer. Then, a second packet of chocolate is taken by another customer. Determine the probability that:
  - (a) Both packets of chocolate are brand C.
  - (b) The first packet of chocolate from brand A and the second packet of chocolate from brand B.
  
7. A bag has 36 transistors. 4 of them are faulty transistors. 2 transistors are taken one by one at random from the bag without replacement. Find the probability if:
  - (a) Both transistors taken are faulty.
  - (b) Both transistors taken are not faulty.
  
8. Shafiq, Chua and Dolly take an exam and the probabilities that they pass are  $\frac{1}{2}$ ,  $\frac{2}{3}$  and  $\frac{3}{4}$  respectively. Calculate the probability that
  - (a) Only one of them passes the exam.
  - (b) At least two of them pass the exam.



**CONDITIONAL PROBABILITY**

**Conditional probability** of event B occurring given that event A has occurred is given by:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

**EXAMPLE 11**

A survey is conducted among 100 students who have graduated from a university, 20 of them are employed and 80 are not employed. 15 of the employed students are female and 55 of the unemployed students are male. If a student is selected at random, find the probability of selecting the following:

- (a) Students are female given that the students are not employed.
- (b) Students are employed given that the students are female.

**SOLUTION**

**Step 1** : Declare the events using suitable symbol.

$E$  = Employed student

$N$  = Not employed student

$F$  = Female student

$M$  = Male student

**SOLUTION**

**Step 2 :** Draw a table to present the information given.

	<i>E</i>	<i>N</i>	Total
<i>F</i>	15	25	40
<i>M</i>	5	55	60
Total	20	80	100

**Step 3 :** Use the conditional probability formula and find the answer.

$$\begin{aligned}
 \text{(a)} \quad P(F|N) &= \frac{P(F \cap N)}{P(N)} \\
 &= \frac{\frac{25}{100}}{\frac{80}{100}} \\
 &= \frac{5}{16}
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad P(E|F) &= \frac{P(E \cap F)}{P(F)} \\
 &= \frac{\frac{15}{100}}{\frac{40}{100}} \\
 &= \frac{3}{8}
 \end{aligned}$$

## Exercise 9

1. A company has 100 salespersons, 40% of them are males and the rest are females. Sixteen of the male salespersons are bachelors. From all the female salespersons, 36 of them are married. A salesperson is selected at random from the company. If it is known that the salesperson selected is single, what is the probability that the salesperson is a male?
2. A recent survey asked 100 people if they thought women in the armed forces should be permitted to participate in combat. The results of the survey are shown.

Gender	Yes	No	Total
Male	32	18	50
Female	8	42	50
Total	40	60	100

Find the probabilities:

- (a) The respondent answered yes, given that the respondent was a female.
  - (b) The respondent was a male, given that the respondent answered no.
3. An electrical appliances shop purchased 30%, 20% and 50% of the total sum of an electrical item from factories A, B and C respectively. The defective items produced by the factories A, B and C are known with probabilities 0.02, 0.03 and 0.04 respectively. An electrical item is selected at random. Calculate the probability that
    - (a) The item is defective
    - (b) The defective item is from factory C
  4. In a university, 48% of the students are females and 17.5% of the students are taking business programs. 4.7% of the university students are female students who study business programs. A student is selected randomly. Events A and B are defined as follows. Find  $P(A|B)$ .
 

A : A female student of the university is selected

B : A student of the university, who studies the business program is selected

## Exercise 9

5. A survey is done on 50 customers of a supermarket. The results show that 30 of the 50 customers say that they visit the supermarket because they have read the advertisement in the local newspaper while the remainders (20 people) have not read the advertisement. From the 28 respondents who make a purchase, 20 of them say that they have read the advertisement.
  - (a) Display the information in a  $2 \times 2$  table.
  - (b) A customer is randomly selected from the 50 respondents. What is the probability that
    - i. the customer makes a purchase although he/she has not read the advertisement?
    - ii. the customer makes a purchase after reading the advertisement?
  
6. Analysis of 80 applications for a vacancy shows that  $\frac{3}{4}$  of the applicants are males and from all the male applicants,  $\frac{1}{3}$  are degree holders. From all the female applicants, half are degree holders. Assume that each applicant has an equal chance of getting the job.
  - (a) Find the probability that the successful applicant is a male with a degree.
  - (b) If it is known that the successful applicant is a degree holder, what is the probability that the applicant is a female?



### EXPECTATION

Mathematical **expectation** or the expected value or the mean is defined as:

$$\text{Mean} = E(x) = \sum_{i=1}^n x_i P(x_i)$$

### EXAMPLE 12

In a family with two children, find the mean of the number of children who will be girls.

Number of girls, x	0	1	2
$P(X = x)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

### SOLUTION

**Step 1** : Use the expectation formula and find the answer.

$$\begin{aligned}
 \text{Mean} = E(x) &= \sum_{i=1}^n x_i P(x_i) \\
 &= (0) \left(\frac{1}{4}\right) + (1) \left(\frac{1}{2}\right) + (2) \left(\frac{1}{4}\right) \\
 &= 0 + \frac{1}{2} + \frac{1}{2} \\
 &= 1
 \end{aligned}$$

## Exercise 10

1. A fair die is thrown. Find out the expected value of its outcomes.
2. Six men and five women apply for an executive position in a small company. Two of the applicants are selected for an interview. Let  $X$  represent the number of women in the interview. We have found the probability mass function of  $X$ . How many women do you expect in the interview?

$X = x$	0	1	2
$P(x)$	$\frac{2}{11}$	$\frac{5}{11}$	$\frac{4}{11}$

3. Determine the expected value of a discrete random variable, given its distribution as follows:

$X = x$	1	2	3	4	5	6
$P(x)$	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{3}{6}$	$\frac{4}{6}$	$\frac{5}{6}$	1

4. Let  $X$  be a random variable defining number of students getting A grade. Find the expected value of  $X$  from the given table.

$X = x$	0	1	2	3
$P(x)$	0.2	0.1	0.4	0.3

Try your best!



### 1.3.2 Use Laws of Probability: Addition Law of Probability and Multiplication Law of Probability

#### 1.3.3 Solve Problems on Probability

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##### ADDITION LAW

Two events are **mutually exclusive** if they cannot occur at the same time.

##### Addition Rule 1:

When two events **A and B are mutually exclusive**, the probability that **A or B** will occur is

$$P(A \cup B) = P(A) + P(B)$$

##### Addition Rule 2:

When two events **A and B are not mutually exclusive**, the probability that **A or B** will occur is

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

##### MULTIPLICATION LAW

Multiplication law / rule can be used to find the probability of two or more events that occur in sequence.

##### Multiplication Rule 1:

When two events are **independent**, the probability of both occurring is

$$P(A \cap B) = P(A) \times P(B)$$

##### Multiplication Rule 2:

When two events are **dependent**, the probability of both occurring is

$$P(A \cap B) = P(A) \times P(B|A)$$

**EXAMPLE 13**

An experiment is carried out by randomly choosing a card from a set of cards numbered 1 to 8. Calculate the probability that the card chosen has a number less than 5 or an even number.

**SOLUTION**

**Step 1 :** State the sample space,  $S$  and determine  $n(S)$ .

$$S = \{1, 2, 3, 4, 5, 6, 7, 8\}$$

$$n(S) = 8$$

**Step 2 :** Declare the events using suitable symbol.

$F$  = choose a number less than 5

$E$  = choose an even number

**Step 3 :** State the outcomes for each event and determine the number of outcomes.

$$F = \{1, 2, 3, 4\}$$

$$E = \{2, 4, 6, 8\}$$

$$F \cap E = \{2, 4\}$$

$$n(F) = 4$$

$$n(E) = 4$$

$$n(F \cap E) = 2$$

**Step 4 :** Use the suitable probability formula and find the answer.

$$P(F \cup E) = P(F) + P(E) - P(F \cap E)$$

$$= \frac{4}{8} + \frac{4}{8} - \frac{2}{8}$$

$$= \frac{3}{4}$$

## Exercise 11

1. A number is chosen at random from set  $A = \{1, 2, 3, 4, 9, 11, 12, 13\}$ . Calculate the probability that the chosen number is less than 2 or a prime number.
2. A fair dice is rolled. Find the probability of getting
  - (a) A prime number or an odd number
  - (b) A prime number and an odd number
3. A total of 24 students took a Chemistry test and found that 8 of them received A grade, 12 were grade B and the other received C. If a student is randomly selected among the student, determine the probability that the selected student gets
  - (a) Grade A or Grade B
  - (b) Grade B or grade C

4.

Group	Number of boys	Number of girls
A	3	2
B	4	4
C	4	5

Table shows number of boys and girls in three groups, A, B and C. A kid is chosen randomly from each group. Determine the probability of choosing three kids that:

- (a) All of them are girls
  - (b) All of them are boys
  - (c) Consists of a girl and two boys
5. A total of 655 persons have multiple jobs in a city. Of them, 381 are male, 299 are married and 168 are male and married. Find the probability that a randomly selected person with multiple jobs is a male or married.

## Exercise 11

6. Three shooters are shooting once at a specific target point. The probability of participant P hits the target is  $\frac{1}{5}$ . The probability of participant Q hits the target is  $\frac{1}{3}$ . The probability of participant R hits the target is  $\frac{1}{2}$ . If they shoot at the same time, calculate the probability of:
- Only participant R hits the target.
  - Only one shooting hits the target.
  - Only two shootings hit the target.
7. In a mathematics class of 30 students, 19 students are boys and 11 students are girls. On a test, 4 boys and 5 girls get A grade. If a student is chosen randomly from the class, what is the probability to choose a girl or an A student?
8. The following table, based on a survey, shows the numbers of male and female viewers who prefer either documentary or drama programs on television. A television viewer involved in the survey is selected at random. A is the event that a female viewer is selected and B is the event that a viewer prefers documentary programs.

	Documentary	Drama
Male	96	24
Female	45	85

- Find  $P(A \cap B)$  and  $P(A \cup B)$
- Determine whether A and B are independent and whether A and B are mutually exclusive.

Try your best!

## ANSWER

### Exercise 1 (page 8)

1.

Mass of guava (kg)	Class boundary	Tally marks	Frequency
7.0 – 7.2	6.95 – 7.25	//	2
7.3 – 7.5	7.25 – 7.55	HHH	5
7.6 – 7.8	7.55 – 7.85	HHH //	7
7.9 – 8.1	7.85 – 8.15	HHH HHH /	11
8.2 – 8.4	8.15 – 8.45	HHH HHH //	12
8.5 – 8.7	8.45 – 8.75	HHH HHH	9
8.8 – 9.0	8.75 – 9.05	////	4

2.

Mass of metal (kg)	Class boundary	Tally marks	Frequency
39.3 – 39.4	39.25 – 39.45	/	1
39.5 – 39.6	39.45 – 39.65	////	4
39.7 – 39.8	39.65 – 39.85	HHH	5
39.9 – 40.0	39.85 – 40.05	HHH HHH ///	13
40.1 – 40.2	40.05 – 40.25	HHH ///	8
40.3 – 40.4	40.25 – 40.45	HHH //	7
40.5 – 40.6	40.45 – 40.65	/	1
40.7 – 40.8	40.65 – 40.85	/	1

3.

Time (hour)	Class boundary	Tally marks	Frequency
3 – 8	2.5 – 8.5	//	2
9 – 14	8.5 – 14.5	HHH	5
15 – 20	14.5 – 20.5	HHH //	7
21 – 26	20.5 – 26.5	HHH HHH HHH HHH ///	23
27 – 32	26.5 – 32.5	HHH ///	8
33 – 38	32.5 – 38.5	///	3
39 – 44	38.5 – 44.5	/	1
45 – 50	44.5 – 50.5	/	1

4.

Age (year)	Class boundary	Tally marks	Frequency
15 – 18	14.5 – 18.5	///	3
19 – 22	18.5 – 22.5	HHH ///	8
23 – 26	22.5 – 26.5	HHH /	6
27 – 30	26.5 – 30.5	HHH //	7
31 – 34	30.5 – 34.5	///	3
35 – 38	34.5 – 38.5	///	3

5.

Height (cm)	Class boundary	Tally marks	Frequency
125 – 131	124.5 – 131.5	////	4
132 – 138	131.5 – 138.5	HHH ///	9
139 – 145	138.5 – 145.5	////	4
146 – 152	145.5 – 152.5	HHH HHH	10
153 – 159	152.5 – 159.5	HHH //	7
160 – 166	159.5 – 166.5	HHH /	6

6.

Number of egg	Class boundary	Tally marks	Frequency
248 – 297	247.5 – 297.5	////	4
298 – 347	297.5 – 347.5	///	3
348 – 397	347.5 – 397.5	////	4
398 – 447	397.5 – 447.5	/	1
448 – 497	447.5 – 497.5	HHH	5
498 – 547	497.5 – 547.5	////	4
548 – 597	547.5 – 597.5	////	4
598 – 647	597.5 – 647.5	////	4
648 – 697	647.5 – 697.5	/	1

### Exercise 2 (page 13)

1. (a) Mean = 7.25

Median = 7.5

Mode = no mode

(b) Mean = 27.25

Median = 27

Mode = 26

(c) Mean = 4.724

Median = 4.72

Mode = 4.72

2. Mean = 2.513

Median = 2

Mode = 2

## ANSWER

3.  $P = 3$

4.  $M = 17$   
Median = 9.5  
Mode = 9

5. (a)  $\sum x = 88$   
(b)  $k = 2$

6.  $h = 10$   
 $k = 14$

7. (a) 4  
(b) 4

### Exercise 3 (page 20)

1. Mean = 10.744  
Median = 10.984  
Mode = 11.389

2. Median = 163.136  
Mode = 166.167

3. Mean = 166.56  
Median = 165.955  
Mode = 164.833

4. (a)  $k = 10$   
(b) Median = 40.167  
Mode = 41.583

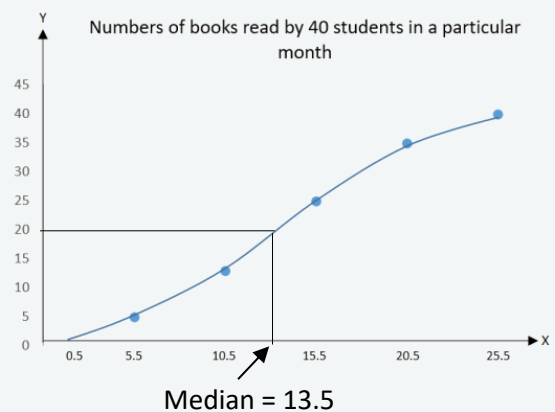
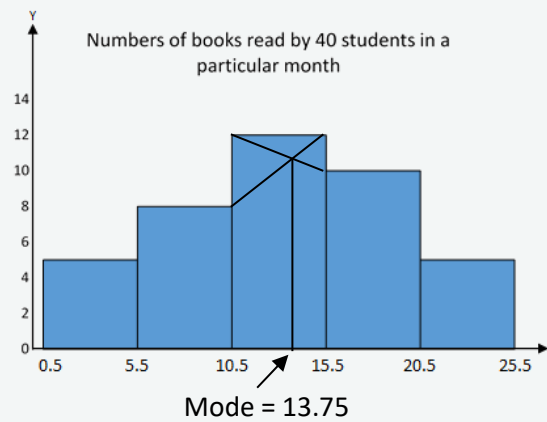
5. Mean = 76.7  
Median = 77.643  
Mode = 79.071

6. Mean = 3.2  
Median = 3.104  
Mode = 3.061

7.  $x = 8$   
 $y = 10$   
Mode = 53.833

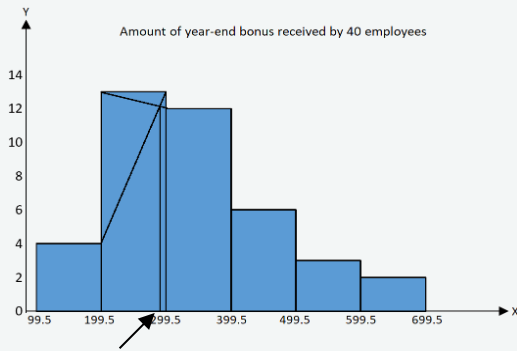
### Exercise 4 (page 29)

1.

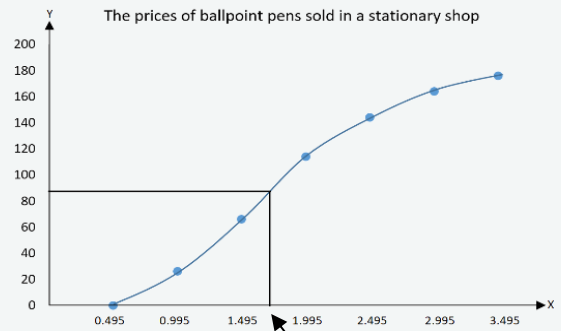


# ANSWER

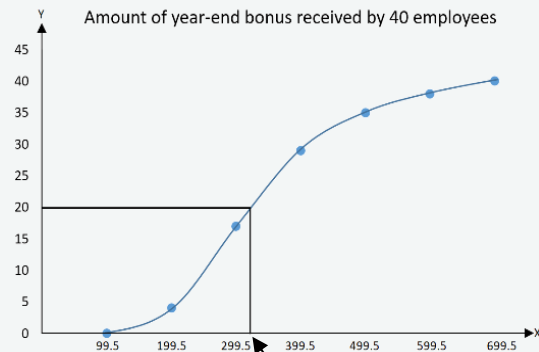
2.



Mode = 289.5

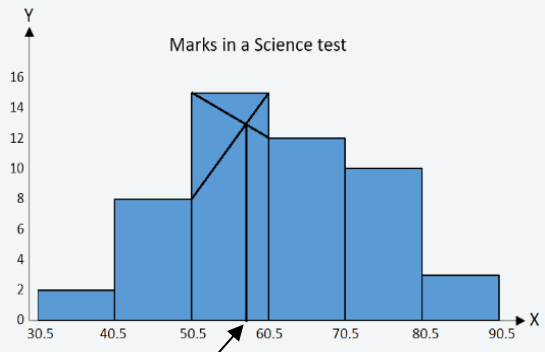


Median = 1.72



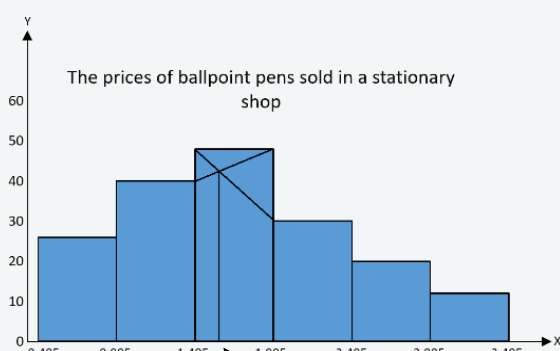
Median = 319.5

4.

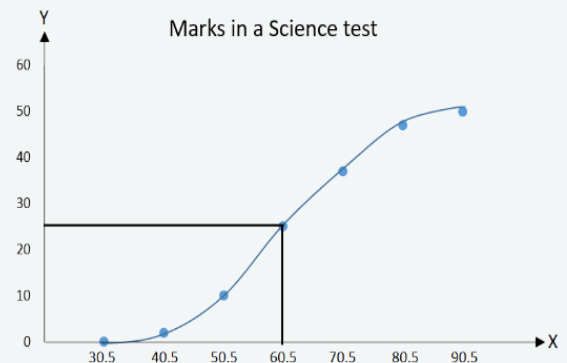


Mode = 57.5

3.



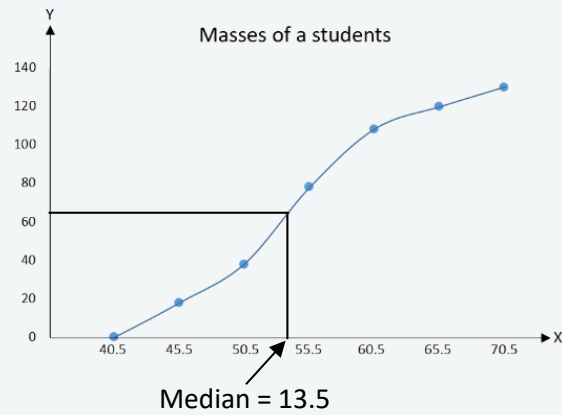
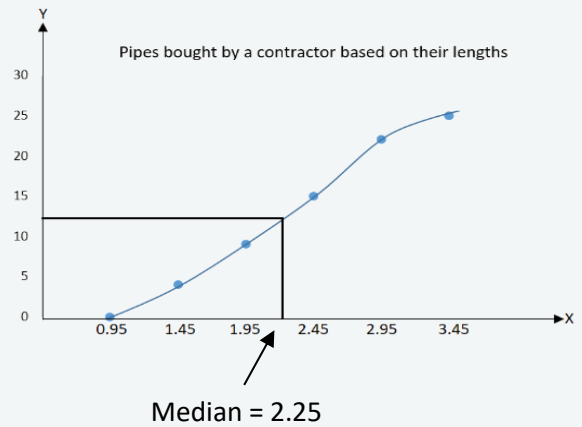
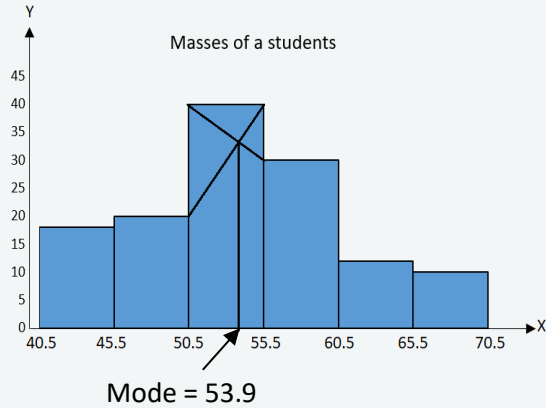
Mode = 1.645



Median = 60.5

## ANSWER

5.



### Exercise 5 (page 40)

1. (a) Mean deviation = 7.96

Variance = 113.76

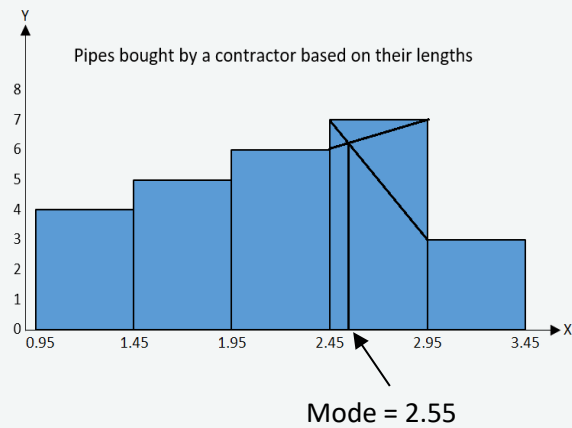
Standard deviation = 10.666

(b) Mean deviation = 7.8

Variance = 96.05

Standard deviation = 9.801

6.



2. Mean deviation = 6

Variance = 43.333

Standard deviation = 6.583

3. Mean deviation = 8.578

Variance = 102.261

Standard deviation = 10.112

4. Mean deviation = 6.24

Variance = 59.76

Standard deviation = 7.730



## ANSWER

5. Mean deviation = 0.969  
 Variance = 1.316  
 Standard deviation = 1.147

6.  $x = 10$   
 $y = 7$

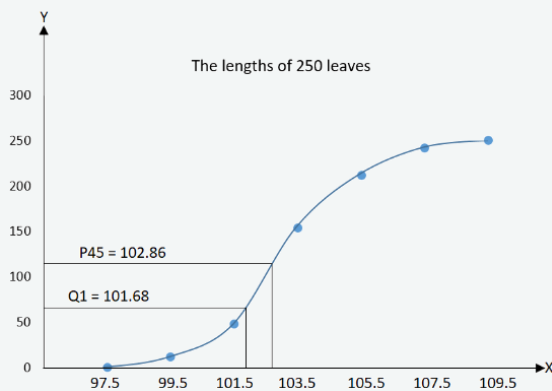
7. (a) 3.717  
 (b) 1.202

8. Variance = 39  
 Standard deviation = 6.245

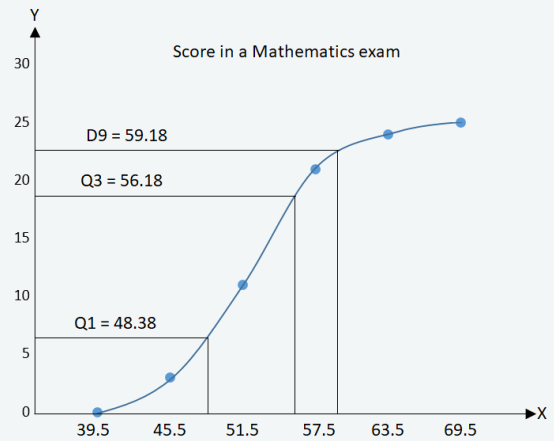
9. (a)  $x = 10$   
 (b) Mean deviation = 5.9  
 Variance = 49.25  
 Standard deviation = 7.018

### Exercise 6 (page 54)

1. (a)  $Q_1 = 101.774$  (formula)  
 (b)  $P_{45} = 102.717$  (formula)

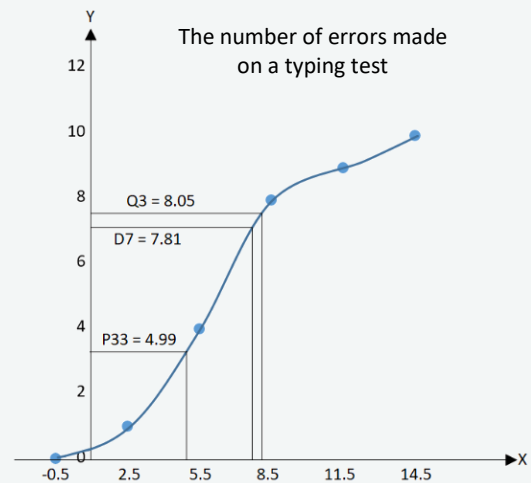


2. (a) Interquartile range = 8.213 (formula)  
 (b)  $D_9 = 60.5$  (formula)



Interquartile range = 7.8 (graph)

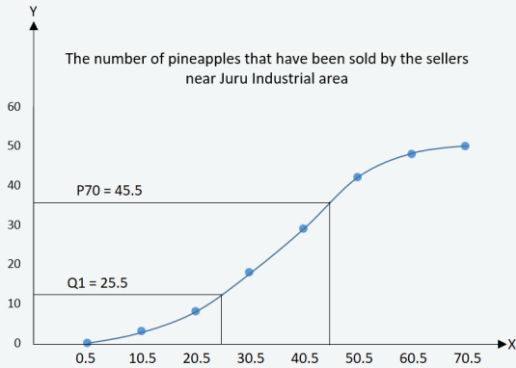
3. (a)  $Q_3 = 8.125$  (formula)  
 (b)  $D_7 = 7.75$  (formula)  
 (c)  $P_{33} = 4.8$  (formula)



## ANSWER

4. (a)  $Q_1 = 25$  (formula)

(b)  $P_{70} = 45.115$  (formula)



4.  $x = 64$

5. (a)  $\frac{4}{11}$

(b)  $\frac{7}{11}$

6. (a)  $\frac{5}{9}$

(b)  $\frac{4}{9}$

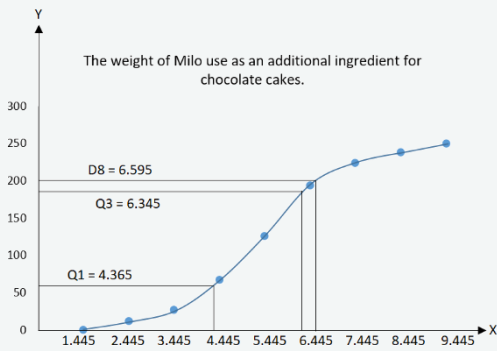
(c)  $\frac{4}{9}$

(d)  $\frac{5}{9}$

(e)  $\frac{2}{3}$

5. (a) Interquartile range = 2.016

(b)  $D_8 = 6.645$



Interquartile range = 1.98 (graph)

### Exercise 8 (page 67)

1. (a)  $\frac{17}{32}$

(b)  $\frac{15}{32}$

2. (a)  $\frac{35}{66}$

(b)  $\frac{31}{66}$

3.  $\frac{1}{3}$

4. (a)  $\frac{3}{8}$

(b)  $\frac{7}{8}$

5.  $\frac{8}{15}$

### Exercise 7 (page 61)

1. (a)  $\frac{1}{2}$

(b)  $\frac{2}{5}$

2. (a)  $\frac{3}{8}$

(b)  $\frac{1}{8}$

3.  $\frac{5}{8}$

# ANSWER

6. (a)  $\frac{153}{1225}$

(b)  $\frac{51}{490}$

7. (a)  $\frac{1}{105}$

(b)  $\frac{248}{315}$

8. (a)  $\frac{1}{4}$

(b)  $\frac{17}{24}$

## Exercise 9 (page 71)

1.  $\frac{2}{5}$

2. (a)  $\frac{4}{25}$

(b)  $\frac{3}{10}$

3. (a)  $\frac{4}{125}$

(b)  $\frac{5}{8}$

4.  $\frac{47}{175}$

5. (a)

	Read	Not read	
Purchase	20	8	28
Not purchase	10	12	22
	30	20	50

(b)(i)  $\frac{2}{5}$

(b)(ii)  $\frac{2}{3}$

6. (a)  $\frac{2}{3}$

(b)  $\frac{1}{3}$

## Exercise 10 (page 74)

1.  $\frac{7}{2}$

2.  $\frac{13}{11}$

3.  $\frac{91}{6}$

4.  $\frac{9}{5}$

## Exercise 11 (page 77)

1.  $\frac{5}{8}$

2. (a)  $\frac{2}{3}$

(b)  $\frac{1}{4}$

3. (a)  $\frac{5}{6}$

(b)  $\frac{2}{3}$

4. (a)  $\frac{1}{9}$

(b)  $\frac{2}{15}$

(c)  $\frac{7}{18}$

## ANSWER

5. (a)  $\frac{4}{15}$

(b)  $\frac{7}{15}$

(c)  $\frac{7}{30}$

6.  $\frac{1}{2}$

7. (a)  $\frac{9}{50}, \frac{113}{125}$

(c) not independent,  
not mutually exclusive

8.  $\frac{512}{655}$

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