

SULIT



**BAHAGIAN PEPERIKSAAN DAN PENILAIAN
JABATAN PENGAJIAN POLITEKNIK
KEMENTERIAN PENDIDIKAN TINGGI**

JABATAN MATEMATIK, SAINS DAN KOMPUTER

PEPERIKSAAN AKHIR

SESI 2 : 2016/2017

**BBM3013: ADVANCED CALCULUS FOR ENGINEERING
TECHNOLOGY**

TARIKH : 5 JUN 2017

MASA : 9.00 AM – 12.00 PM

Kertas ini mengandungi **ENAM (6)** halaman bercetak.

Subjektif (4 soalan)

Dokumen sokongan yang disertakan : Formula

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIARAHKAN

(CLO yang tertera hanya sebagai rujukan)

SULIT

INSTRUCTION:

This section consists of **FOUR (4)** structured questions. Answer **ALL** questions.

ARAHAN:

Bahagian ini mengandungi **EMPAT (4)** soalan berstruktur. Jawab semua soalan.

QUESTION 1**SOALAN 1**

CLO 1
C1

- (a) Form the differential equations for the given general solutions.

Bentukkan persamaan pembezaan bagi penyelesaian yang diberi.

i. $y = Ax + x^2$ [5 marks]

[5 markah]

ii. $y = x^2 + \frac{A}{x}$ [5 marks]

[5 markah]

CLO 1
C2

- (b) Find the general solutions of the following differential equations.

Cari penyelesaian umum bagi persamaan pembezaan yang berikut.

contoh

i. $y' = \frac{e^{-x} + e^{2x} + e^{3x}}{e^x}$ [4 marks]

[4 markah]

ii. $x \frac{dy}{dx} = 4y + x^5$ [6 marks]

[6 markah]

CLO 2
C4

- (c) Solve the differential equation, $x \frac{dy}{dx} - 3 = 2(y + \frac{dy}{dx})$ which satisfies

the condition $y = 0$ when $x = 3$. [10 marks]

Selesaikan persamaan pembezaan $x \frac{dy}{dx} - 3 = 2(y + \frac{dy}{dx})$ yang

memenuhi syarat $y=0, x=3$. [10 markah]

QUESTION 2

SOALAN 2

CLO1
C2

- (a) Solve the following equations:

Selesaikan persamaan di bawah:

i. $y'' - y' - 6y = 0$

[3 marks]

[3 markah]

ii. $4\frac{d^2y}{dx^2} + 12\frac{dy}{dx} + 9y = 0$

[3 marks]

[3 markah]

CLO2
C3

- (b) Given a differential equation,

$$y'' - 3y' + 2y = 1.$$

Diberi satu persamaan pembezaan,

$$y'' - 3y' + 2y = 1.$$

- i. Find the complementary solution,
- y_c
- of the given differential equation.

Cari penyelesaian komplementari, y_c bagi persamaan pembezaan yang diberi.

[3 marks]

[3 markah]

- ii. Find the Wronskian determinant.

Cari penentu Wronskian.

[4 marks]

[4 markah]

- iii. Find the general solution for the given differential equation.

Cari penyelesaian umum bagi persamaan pembezaan yang diberi.

[7 marks]

[7 markah]

QUESTION 3

SOALAN 3

CLO1

C2

- (a) By using the Canonical form, classify each of the following equation as elliptic, hyperbolic or parabolic.

Dengan menggunakan bentuk Kanonikal, tentukan sama ada persamaan berikut adalah elliptik, hiperbolik atau parabolik.

i. $\frac{\partial^2 f}{\partial x^2} + 2 \frac{\partial^2 f}{\partial x \partial y} - 3 \frac{\partial^2 f}{\partial y^2} = 0$ [2marks]
[2 markah]

ii. $\frac{\partial^2 f}{\partial x^2} + 2 \frac{\partial^2 f}{\partial x \partial y} + \frac{\partial^2 f}{\partial y^2} = 0$ [2 marks]
[2 markah]

iii. $4 \frac{\partial^2 f}{\partial x^2} + 4 \frac{\partial^2 f}{\partial x \partial y} + 1 \frac{\partial^2 f}{\partial y^2} = 0$ [2 marks]
[2 markah]

CLO2
C4

- (b) Find the general solution for each of the following second order partial differential equation.

Cari penyelesaian umum bagi setiap persamaan separa peringkat kedua yang berikut.

i. $\frac{\partial^2 u}{\partial x \partial y} = 3x - 5y$ [2 marks]
[2 markah]

ii. $\frac{\partial^2 f}{\partial u \partial v} = 2u^3 v^2 - 3u^2 v - 4u$ [3 marks]
[3 markah]

iii. $U_{xy} = e^x + \sin y$ [3 marks]
[3 markah]

CLO2
C4

(c) Solve each of the following second order partial differential equation.

Give your answers in the form of $u(x, y) = F(mx + y)$.

Selesaikan setiap persamaan separa peringkat kedua berikut. Beri jawapan anda dalam bentuk $u(x, y) = F(mx + y)$.

i.
$$25 \frac{\partial^2 u}{\partial x^2} - 20 \frac{\partial^2 u}{\partial x \partial y} + 4 \frac{\partial^2 u}{\partial y^2} = 0$$

[3 marks]

[3 markah]

ii.
$$6 \frac{\partial^2 u}{\partial x^2} + 3 \frac{\partial^2 u}{\partial x \partial y} - 30 \frac{\partial^2 u}{\partial y^2} = 0$$

[3 marks]

[3 markah]

QUESTION 4**SOALAN 4**CLO1
C1

- (a) By using Laplace Transform Table, determine the Laplace transform for each of the following function:

Dengan menggunakan Jadual Jelmaan Laplace, tentukan jelmaan Laplace bagi fungsi berikut:

i. $f(t) = 5$ [1 mark]

[1 markah]

ii. $f(t) = e^{2t} + e^{-3t}$ [2 marks]

[2 markah]

iii. $f(t) = t - \frac{t^3}{2}$ [2 marks]

[2 markah]

CLO1
C3

- (b) Find the following inverse Laplace transforms.

Cari songsangan bagi jelmaan Laplace berikut.

i. $L^{-1}\left\{\frac{20}{s^2 + 4}\right\}$ [3 marks]

[3 markah]

ii. $L^{-1}\left\{\frac{5}{s} - \frac{10}{2s - 4}\right\}$ [3 marks]

[3 markah]

iii. $L^{-1}\left\{\frac{6}{s^2 + 4} - \frac{2s}{s^2 + 9} - \frac{1}{s^4}\right\}$ [4 marks]

[4 markah]

CLO2
C5

- (c) Use the Laplace transform to solve the given initial value problem.

Dengan menggunakan jelmaan Laplace, selesaikan masalah nilai awal yang diberi.

$$y'' - 3y' + 2y = 0; y(0) = 8, y'(0) = 11$$
 [15 marks]

[15 markah]

SOALAN TAMAT

FORMULA

Basic Differentiation	Basic Integration
$\frac{dy}{dx} = \frac{du}{dx}v + u \frac{dv}{dx}$ $\frac{dy}{dx} = \frac{\frac{du}{dx}v - u \frac{dv}{dx}}{v^2}$ $\frac{d}{dx}(e^{ax}) = ae^{ax}$ $\frac{d}{dx}(\ln x) = \frac{1}{x}$ $\frac{d}{dx}[\sin(ax)] = a \cos(ax)$ $\frac{d}{dx}[\cos(ax)] = -a \sin(ax)$ $\frac{d}{dx}[\tan(ax)] = a \sec^2(ax)$	$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$ $\int u dv = uv - \int v du$ $\int e^{ax} du = \frac{1}{a} e^{ax} + C$ $\int \frac{1}{x} dx = \ln x + C$ $\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + C$ $\int \cos(ax) dx = \frac{1}{a} \sin(ax) + C$ $\int \sec^2(ax) dx = \frac{1}{a} \tan(ax) + C$
First Order Differential	
Separable: $\frac{dy}{dx} = f(x) \cdot g(y)$ Homogeneous: $P(x, y)dx + Q(x, y)dy = 0, \quad P \text{ and } Q \text{ have same degree.}$ Exact: $P(x, y)dx + Q(x, y)dy = 0, \quad \frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$	Linear: $\frac{dy}{dx} + P(x)y = Q(x)$ $ye^{\int P(x) dx} = \int Q(x) \cdot e^{\int P(x) dx} dx + c$ Bernoulli: $\frac{dy}{dx} + P(x)y = Q(x)y^n$ $y^{1-n} e^{\int (1-n)P(x) dx} = \int (1-n) \cdot Q(x) \cdot e^{\int (1-n)P(x) dx} dx$
Second Order Differential	
Quadratic Equation: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ Roots real and different $m = m_1$ and $m = m_2$ $\therefore$ Solution is $y = Ae^{m_1 x} + Be^{m_2 x}$	

Roots real and equal $m_1 = m_2$

$\therefore$ Solution is $y = e^{m_1 x} (A + Bx)$

Complex roots $m = \alpha \pm \beta i$

$\therefore$ Solution is $y = e^{\alpha x} (A \cos \beta x + B \sin \beta x)$

$$\frac{d^2 y}{dx^2} + n^2 y = 0 \quad m = \pm n i$$

$\therefore$ Solution is $y = A \cos nx + B \sin nx$

$$\frac{d^2 y}{dx^2} - n^2 y = 0 \quad m = \pm n$$

$\therefore$ Solution is $y = A \cosh nx + B \sinh nx$

Particular Integral

If $G(x)$	Assume (y_p)
k (constant)	A
kx	$Ax + B$
kx^2	$Ax^2 + Bx + C$
$k \sin \alpha x$ or $k \cos \alpha x$	$A \cos \alpha x + B \sin \alpha x$
$k \sinh \alpha x$ or $k \cosh \alpha x$	$A \cosh \alpha x + B \sinh \alpha x$
e^{kx}	Ae^{kx}

Wronskian Determinant

$$W(x) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

$$= y_1 y_2' - y_2 y_1'$$

Particular solution

$$y_p = -y_1 \int \frac{y_2(G(x))}{W} dx + y_2 \int \frac{y_1(G(x))}{W} dx$$

Second Order Partial Differential					
$B^2 - AC > 0$ Hyperbolic equation					
$B^2 - AC < 0$ Elliptic equation					
$B^2 - AC = 0$ Parabolic equation					
Laplace Transform Table					
No.	$f(t)$	$F(s)$	No.	$f(t)$	$F(s)$
1.	a	$\frac{a}{s}$	13.	$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2 + \omega^2}$
2.	at	$\frac{a}{s^2}$	14.	$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2 + \omega^2}$
3.	$t^n, n=1,2,3...$	$\frac{n!}{s^{n+1}}$	15.	$\sinh \alpha t$	$\frac{\omega}{s^2 - \omega^2}$
4.	$\frac{t^{n-1}}{(n-1)!}$	$\frac{1}{s^n}$	16.	$\cosh \alpha t$	$\frac{s}{s^2 - \omega^2}$
5.	e^{-at}	$\frac{1}{s+a}$	17.	$e^{at} \sinh \omega t$	$\frac{\omega}{(s-a)^2 - \omega^2}$
6.	te^{-at}	$\frac{1}{(s+a)^2}$	18.	$e^{-at} \sinh \omega t$	$\frac{\omega}{(s+a)^2 - \omega^2}$
7.	$t^n \cdot e^{at} \quad n=1,2,3...$	$\frac{n!}{(s-a)^{n+1}}$	19.	$e^{-at} \cosh \omega t$	$\frac{s+a}{(s+a)^2 - \omega^2}$
8.	$t^n \cdot f(t)$	$(-1)^n \frac{d^n}{ds^n} [F(s)]$	20.	$f_1(t) + f_2(t)$	$F_1(s) + F_2(s)$
9.	$\sin \alpha t$	$\frac{\omega}{s^2 + \omega^2}$	21.	$\int_0^t f(u) du$	$\frac{F(s)}{s}$
10.	$\cos \alpha t$	$\frac{s}{s^2 + \omega^2}$	22.	$f(t-a)u(t-a)$	$e^{-as} F(s)$
11.	$t \sin \alpha t$	$\frac{2\omega s}{(s^2 + \omega^2)^2}$	23.	First derivative $\frac{dy}{dt}, y'(t)$	$sY(s) - y(0)$
12.	$t \cos \alpha t$	$\frac{s^2 - \omega^2}{(s^2 + \omega^2)^2}$	24.	Second derivative $\frac{d^2 y}{dt^2}, y''(t)$	$s^2 Y(s) - sy(0) - y'(0)$