Navigating the Complex:

A Guide to Mastering Complex Number

Azlina binti Morshidi
Ab Aziz Ikhwan bin Ab Wahab
Wida Yanti binti Mohammad Zen Umar

Politeknik Kuching Sarawak

Authors

AZLINA BINTI MORSHIDI
AB AZIZ IKHWAN BIN AB WAHAB
WIDA YANTI BINTI MOHAMMAD ZEN UMAR

POLITEKNIK KUCHING SARAWAK
MINISTRY OF HIGHER EDUCATION
KM22, JALAN MATANG,
93050 KUCHING, SARAWAK.

Phone No. : (082) 845596/7/8
Fax No. : (082) 845023
E-mail : poliku.info@poliku.edu.my
Website : http://www.poliku.edu.my/

e ISBN 978-629-7638-22-5

Published by:
Politeknik Kuching Sarawak
Ministry Of Higher Education

Cataloguing-in-Publication Data

Perpustakaan Negara Malaysia

A catalogue record for this book is available from the National Library of Malaysia

eISBN 978-629-7638-22-5

PREFACE

Navigating the Complex: A Guide to Mastering Complex Numbers is written based on the polythecnic Engineering Mathematics 1 syllabus. The main objectives of this eBook is to help students understanding the topic Complex Number and enhance their skills and techniques to master on solving the complex number calculation. Hence, this eBook will help student to achieve excellent results in their examination.

This eBook consists of four chapters which are Concept of Complex Number, Operation of Complex Number, Argand Digram and Complex Number Form. Each chapter contains ample work examples and video explaination that are geared to help students give the solution in the correct way.

This interactive eBook are flexible to be used by students for self-study in anytime and anywhere without any cost. Since the content of this eBook is simple and compact, students will find this eBook is useful for them to master the Complex Number's topic.

Azlina binti Morsidi
Ab Aziz Ikhwan bin Ab Wahab
Wida Yanti binti Mohammad Zen Umar

ABSTRACT

This eBook provides a comprehensive introduction to the concept of complex numbers, a fundamental component in engineering mathematics. It covers the basic definitions, Cartesian and polar representations, and fundamental operations such as addition, subtraction, multiplication, and division of complex numbers. Furthermore, the eBook explores advanced topics including the square roots of complex numbers, De Moivre's Theorem, and represented graphically using an Argand diagram. Each concept is accompanied by clear examples and practice questions to reinforce the student's understanding. With an interactive approach that includes videos and quizzes, this eBook is designed to help students master complex number concepts more deeply and effectively. This eBook is suitable for university students in engineering, mathematics, and physical sciences who wish to strengthen their understanding of complex numbers and their applications in engineering and science.

Table Of CONTENT

01

Concept of a Complex Number

Imaginary Number

Imaginary numbers produce negative real numbers when squared.

$\sqrt{-1} = ?$

$\sqrt{-16} = ?$

$\sqrt{-\frac{1}{7}} = ?$

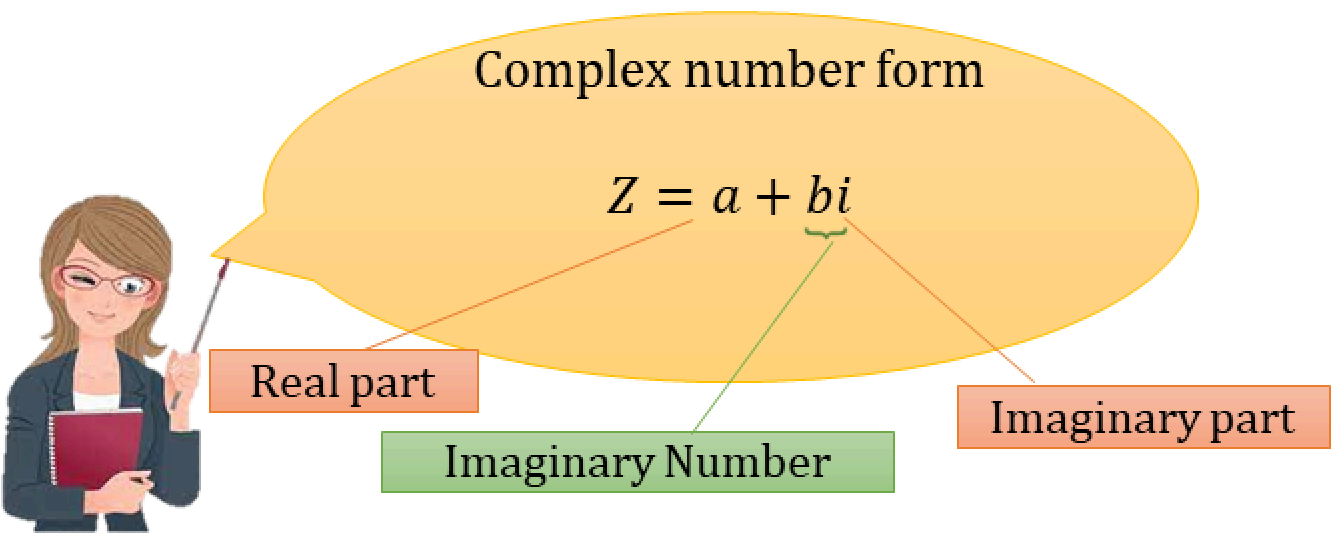
$\sqrt{-\frac{5}{81}} = ?$

$$x^2 = -36$$
$$x = \sqrt{-36}$$
$$= ???$$

So... how to do it!!!

What is Complex Number?

- A Complex number are helpful in finding the square root of negative numbers.
- A Complex numbers have applications in many scientific research, signal processing, electromagnetism, fluid dynamics, quantum mechanics and vibration analysis.
- A complex number is the sum of a real number and an imaginary number.



Imaginary Number

The Imaginary unit named i is the square root of -1.

$$i^2 = -1 \qquad i = \sqrt{-1}$$

So, we can solve things that need the square root of a negative number by simply using the i.

$$\sqrt{-r} = \sqrt{r} \times \sqrt{-1}$$
$$= \sqrt{r} \times i$$
$$= \sqrt{r}i$$

Imaginary Number

1. $\sqrt{-4} = \sqrt{4} \times \sqrt{-1}$

$= 2 \times i$

$= 2i$

2. $\sqrt{-9} = \sqrt{9} \times \sqrt{-1}$

$= 3 \times i$

$= 3i$

3. $\sqrt{-36} = \sqrt{36} \times \sqrt{-1}$

$= 6 \times i$

$= 6i$

4. $\sqrt{-64} = \sqrt{64} \times \sqrt{-1}$

$= 8 \times i$

$= 8i$

5. $\sqrt{-121} = \sqrt{121} \times \sqrt{-1}$

$= 11 \times i$

$= 11i$

6. $\sqrt{-9}\sqrt{-36} = \sqrt{9} \times \sqrt{-1} \times \sqrt{36} \times \sqrt{-1}$

$= 3 \times i \times 6 \times i$

$= 18i^2$

$= 18(-1)$

$= -18$

Imaginary Number

The first four powers of i establish an important pattern and should be memorized.

$$i^1 = i \qquad i^2 = -1 \quad i^3 = -i \quad i^4 = 1$$

Tips:

$$(-1)^{odd\ number} = -1$$
$$(-1)^{even\ number} = 1$$
$$*\ If\ power\ is\ even\ number,\ divide\ with\ 2$$

$$*\ If\ power\ is\ odd\ number:$$
$$a.\ Substract\ power\ of\ i\ with\ 1$$
$$b.\ Then\ divide\ with\ 2$$

Imaginary Number

1. i^3

$= (i^2).i$

$= -1 \times i$

$= -i$

2. i^4

$= (i^2)^2$

$= (-1)^2$

$= 1$

3. i^6

$= (i^2)^3$

$= (-1)^3$

$= -1$

4. i^8

$= (i^2)^4$

$= (-1)^4$

$= 1$

5. i^{15}

$= (i^{14}).i^1$

$= (i^2)^7.i$

$= (-1)^7.i$

$= -i$

6. i^{90}

$= (i^2)^{45}$

$= (-1)^{45}$

$= -1$

7. i^{91}

$= (i)^{90}i^1$

$= (i^2)^{45}i$

$= (-1)^{45}i$

$= -i$

8. $i^4 - i^{10}$

$= (i^2)^2 - (i^2)^5$

$= (-1)^2-(-1)^5$

$= 1 + 1$

$= 2$

9. $i^6 - i^{13}$

$= (i^2)^3$

$- (i^{12})i^1$

$= (-1)^3-(i^2)^6i$

$= -1 - (-1)^6i$

$= -1 - i$

Imaginary Number

Simplify each of the following.

1. $\sqrt{-49} + 5$

$= \mathbf{7i + 5}$

2. $9 - \sqrt{-5}$

$= \mathbf{9 - \sqrt{5}i}$

3. $\sqrt{36} + \sqrt{-25}$

$= \mathbf{6 + 5i}$

4. $i^8 - 2i^3$

$= (i^2)^4 - 2i(i^2)$

$= 1 - 2i(-1)$

$= \mathbf{1 + 2i}$

5. $-6i^3 - 2i^4$

$= -6i(i^2) - 2(1)$

$= -6i(-1) - 2$

$= \mathbf{6i - 2}$

6. $i + 5i^7$

$= i + 5i(i^6)$

$= i + 5i(i^2)^3$

$= i + 5i(-1)$

$= \mathbf{i - 5i}$

$= \mathbf{-4i}$

7. $\sqrt{-9} \times \sqrt{-36}$

$= 3i \times 6i$

$= 18i^2$

$= 18(-1)$

$= \mathbf{-18}$

8. $\sqrt{(-9)(-25)}$

$= (\sqrt{-9})(\sqrt{-25})$

$= (3i)(5i)$

$= 15i^2$

$= \mathbf{-15}$

9. $\sqrt{-25} \times 2i^3$

$= 5i \times 2(-i)$

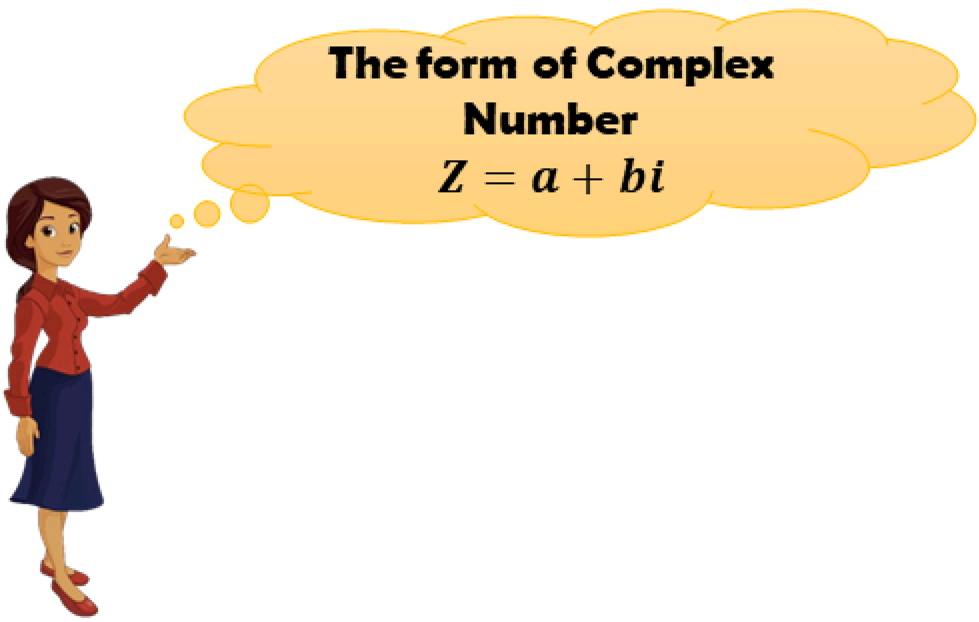
$= 5i \times -2i$

$= \mathbf{-10i^2}$

$= \mathbf{(-10)(-1)}$

$= \mathbf{10}$

Complex Number



Complex Number	Real Part	Imaginary Part
$1 + i$	1	1
$2 + 3i$	2	3
$-2 + 7i$	-2	7
$5 - 7i$	5	-7
$3i - 9$	-9	3

QUIZ TIME

Let's test your understanding by answering this quiz

START!

Concept of a Complex Number

The set of ______ is the set of all numbers written in the form ***a* + *bi*,** where a and b are real numbers. 25 points

- complex numbers
- real part of a complex number
- imaginary part of a complex number
- imaginary numbers

The term ***a*** is called the real part of a complex number, and the term ***bi*** is called the _______ . 25 points

- imaginary part of a complex number
- imaginary numbers
- pure imaginary number
- complex numbers

The set of _____ is the set of all numbers written in the form ***a* + *bi*,** where ***a*** and ***b*** are real numbers and ***b*** is not equal to 0. 25 points

- imaginary numbers
- complex numbers
- real part of a complex number
- imaginary part of a complex number

Quiz Activity

SCAN ME

https://forms.gle/bwhQKUyzMdGU6aex9

02

Operation of Complex Number

Addition and Subtraction

$\mathbf{1).\ 1 + 2i + 3 + 5i}$

$= 1 + 3 + (2 + 5)i$

$\mathbf{= 4 + 7i}$

$\mathbf{2).\ 3 + 8i - 3i}$

$= 3 + (8 - 3)i$

$\mathbf{= 3 + 5i}$

$\mathbf{3).\ 6i + 5 - i + 8i}$

$= 5 + (6 - 1 + 8)i$

$\mathbf{= 5 + 13i}$

$\mathbf{4).\ -4i - 5 - i + 8i}$

$= -5 + (-4 - 1 + 8)i$

$\mathbf{= -5 + 3i}$

$\mathbf{5).\ 10 - 5i - 12 + 3i - i}$

$= 10 - 12 + (-5 + 3 - 1)i$

$\mathbf{= -2 - 3i}$

$\mathbf{6).\ 20 - 5i - (4i + 3)}$

$= 20 - 5i - 4i - 3$

$= 20 - 3 + (-5 - 4)i$

$\mathbf{= 17 - 9i}$

Multiplication

$$(\boldsymbol{a}+\boldsymbol{bi})(\boldsymbol{c}+\boldsymbol{di}) = ac + adi + bci + bdi^2$$
$$= ac + adi + bci + bd(-1)$$
$$= (ac - bd) + (ad + bc)i$$

$$\mathbf{1).\ (2+3\boldsymbol{i})(4+5\boldsymbol{i})}$$
$$= (2)(4) + (2)(5i) + (3i)(4) + (3i)(5i)$$
$$= 8 + 10i + 12i + 15i^2$$
$$= 8 + 22i + 15(-1)$$
$$= 8 - 15 + 22i$$
$$= \mathbf{-7 + 22\boldsymbol{i}}$$

$$\mathbf{2).\ (6+7\boldsymbol{i})(3-2\boldsymbol{i})}$$
$$= (6)(3) + (6)(-2i) + (7i)(3) + (7i)(-2i)$$
$$= 18 - 12i + 21i - 14i^2$$
$$= 18 + 9i - 14(-1)$$
$$= 18 + 14 + 9i$$
$$= \mathbf{32 + 9\boldsymbol{i}}$$

Multiplication

$$\mathbf{3).\ (-4+2i)(5-8i)}$$

$$= (-4)(5) + (-4)(-8i) + (2i)(5) + (2i)(-8i)$$

$$= -20 + 32i + 10i - 16i^2$$

$$= -20 + 42i - 16(-1)$$

$$= -20 + 16 + 42i$$

$$= \mathbf{-4 + 42i}$$

$$\mathbf{4).\ (2+3i)(2+7i)}$$

$$= (2)(2) + (2)(7i) + (3i)(2) + (3i)(7i)$$

$$= 4 + 14i + 6i + 21i^2$$

$$= 4 + 20i + 21(-1)$$

$$= 4 - 21 + 20i$$

$$= \mathbf{-17 + 20i}$$

Division

To divide complex numbers, we write the division as a fraction, then multiply the top and the bottom of the fraction by the conjugate of the denominator.

The conjugate of $z = c + di$ is $\bar{z} = c - di$

The conjugate of $z = c - di$ is $\bar{z} = c + di$

$$\frac{a+bi}{c+di} = \frac{a+bi}{c+di} \times \frac{c-di}{c-di}$$

$$\frac{(2+3i)}{(4+5i)}$$

$$= \frac{2+3i}{4+5i} \times \frac{4-5i}{4-5i}$$

$$= \frac{8 - 10i + 12i - 15i^2}{16 - 20i + 20i - 25i^2}$$

$$= \frac{8 + 2i - 15i^2}{16 - 25i^2}$$

$$= \frac{8 + 2i - 15(-1)}{16 - 25(-1)}$$

$$= \frac{8 + 2i + 15}{16 + 25}$$

$$= \frac{23 + 2i}{41} = \frac{23}{41} + \frac{2}{41}i$$

Complex number form : $a + bi$

Division

$$\frac{a+bi}{c+di} = \frac{a+bi}{c+di} \times \frac{c-di}{c-di}$$

$$1).\ \frac{(5+2i)}{(2-3i)}$$

$$= \frac{5+2i}{2-3i} \times \frac{2+3i}{2+3i}$$

$$= \frac{10+15i+4i+6i^2}{4+6i-6i-9i^2}$$

$$= \frac{10+19i+6i^2}{4-9i^2}$$

$$= \frac{10+19i+6(-1)}{4-9(-1)}$$

$$= \frac{10-6+19i}{4+9}$$

$$= \frac{4+19i}{13}$$

$$= \frac{4}{13} + \frac{19}{13}i$$

Division

$$\frac{a+bi}{c+di}=\frac{a+bi}{c+di}\times\frac{c-di}{c-di}$$

$$2).\frac{(2-3i)}{(4-5i)}$$

$$=\frac{2-3i}{4-5i}\times\frac{4+5i}{4+5i}$$

$$=\frac{8+10i-12i-15i^2}{16+20i-20i-25i^2}$$

$$=\frac{8-2i-15i^2}{16-25i^2}$$

$$=\frac{8-2i-15(-1)}{16-25(-1)}$$

$$=\frac{8+15-2i}{16+25}$$

$$=\frac{23-2i}{41}$$

$$=\frac{23}{41}-\frac{2}{41}i$$

Division

$$\frac{a+bi}{c+di}=\frac{a+bi}{c+di}\times\frac{c-di}{c-di}$$

$$3).\frac{(5+4i)}{(6-2i)}$$

$$=\frac{5+4i}{6-2i}\times\frac{6+2i}{6+2i}$$

$$=\frac{30+10i+24i+8i^2}{36+12i-12i-4i^2}$$

$$=\frac{30+34i+8i^2}{36-4i^2}$$

$$=\frac{30+34i+8(-1)}{36-4(-1)}$$

$$=\frac{30-8+34i}{36+4}$$

$$=\frac{22+34i}{40}$$

$$=\frac{22}{40}+\frac{34}{40}i$$

$$=\frac{11}{20}+\frac{17}{20}i$$

Division

$$\frac{a+bi}{c+di} = \frac{a+bi}{c+di} \times \frac{c-di}{c-di}$$

$$4).\ \frac{(10+3i)}{(4-3i)}$$

$$= \frac{10+3i}{4-3i} \times \frac{4+3i}{4+3i}$$

$$= \frac{40+30i+12i+9i^2}{16+12i-12i-9i^2}$$

$$= \frac{40+42i+9i^2}{16-9i^2}$$

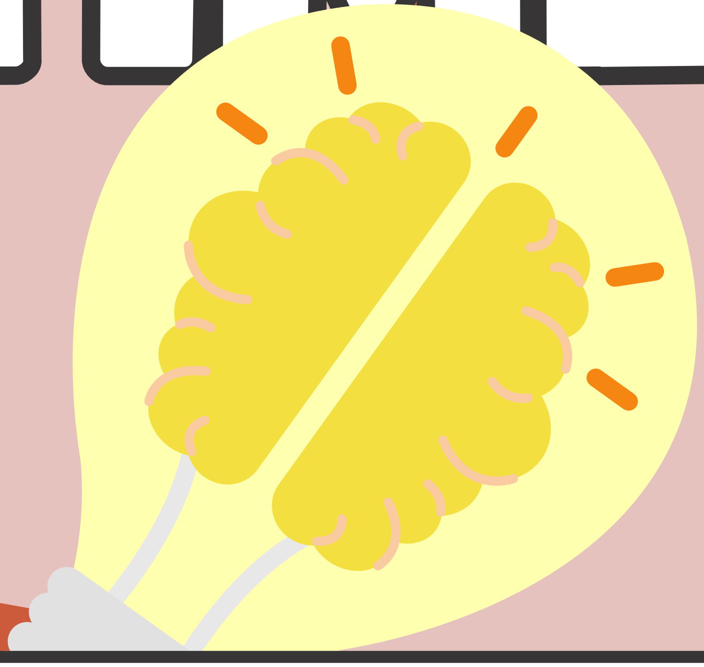
$$= \frac{40+42i+9(-1)}{16-9(-1)}$$

$$= \frac{40-9+42i}{16+9}$$

$$= \frac{31+42i}{25}$$

$$= \frac{31}{25} + \frac{42}{25}i$$

QUIZ TIME



Let's test your understanding by answering this quiz

START!

Complex Number

Sign in to Google to save your progress. Learn more

What is (3 − 5*i*) + (−4 + 7*i*)

20 points

- ◯ −1 + 2i
- ◯ 7 − 12i
- ◯ 1 + 2i
- ◯ −2 + 3i

What is (4 − 5i)(−2 + 7i)?

20 points

- ◯ 27 + 18i
- ◯ −43+ 38i
- ◯ −43 + 18i
- ◯ 27 + 38i

What is (−5 + 3i) − (4 + 7i)?

20 points

- ◯ −1 − 4i
- ◯ −9 − 4i
- ◯ −1 + 4i
- ◯ −9 + 4i

Quiz Activity

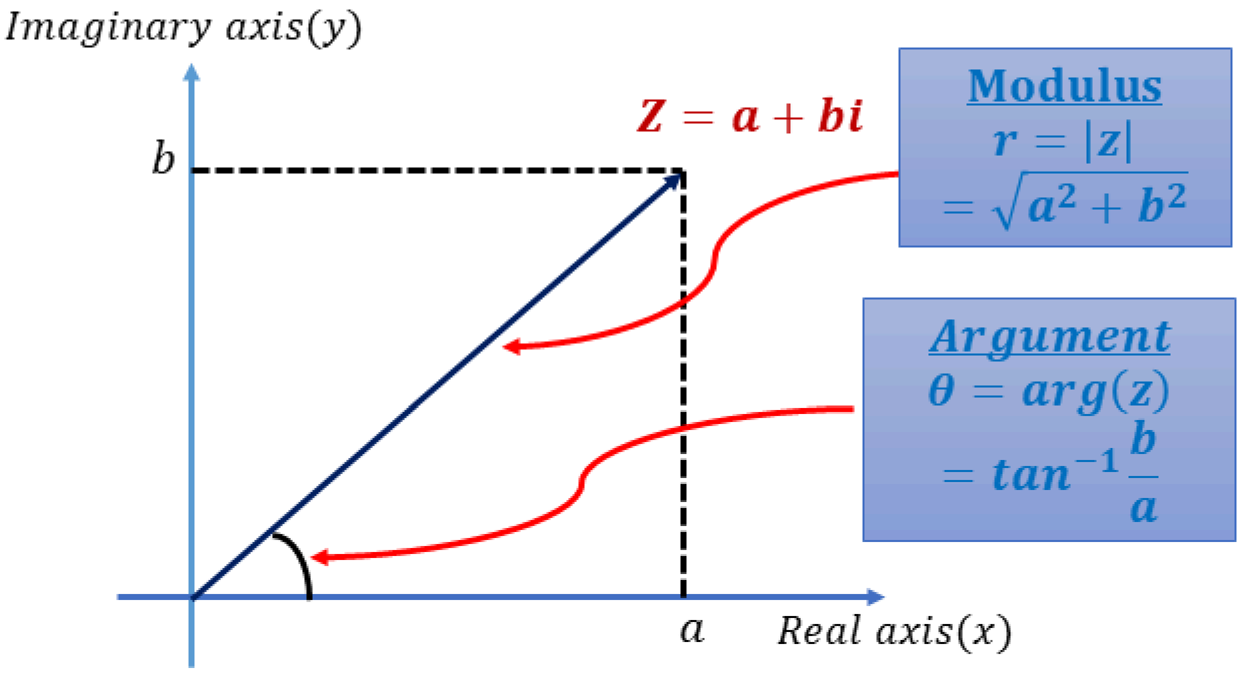
SCAN ME

https://forms.gle/bwhQKUyzMdGU6aex9

03

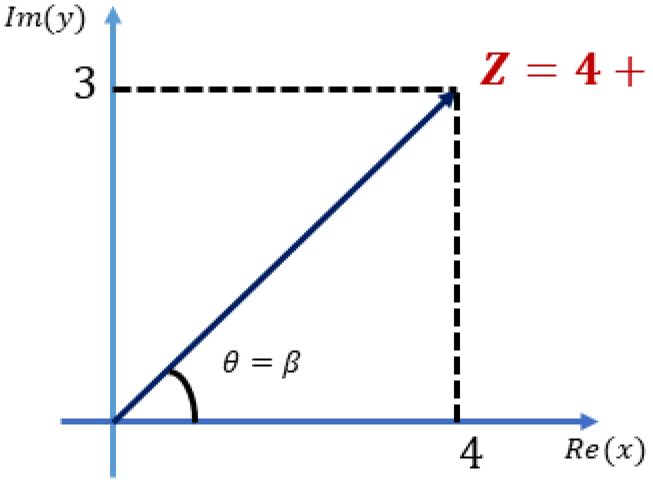
Argand Diagram

Graphical Representation



Graphical Representation

Represent $z = 4 + 3i$ in an Argand Diagram and find its modulus and argument.



Modulus:

$$r = |Z| = \sqrt{a^2 + b^2}$$

$$= \sqrt{4^2 + 3^2}$$

$$= \sqrt{16 + 9}$$

$$= \sqrt{25}$$

$$= \mathbf{5}$$

Argument:

$$\arg(z) = \theta$$

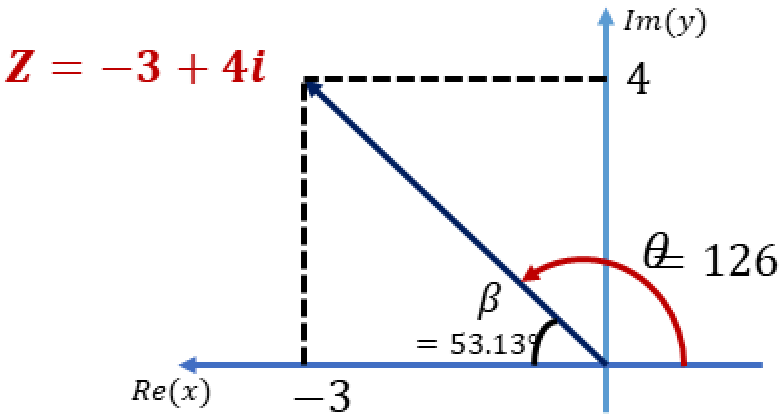
$$= tan^{-1}\frac{b}{a}$$

$$= tan^{-1}\frac{3}{4}$$

$$= \mathbf{36.87°}$$

Graphical Representation

Represent $z = -3 + 4i$ in an Argand Diagram and find its modulus and argument.



Modulus:

$$r = |Z| = \sqrt{a^2 + b^2}$$
$$= \sqrt{(-3)^2 + 4^2}$$
$$= \sqrt{9 + 16}$$
$$= \sqrt{25}$$
$$= \mathbf{5}$$

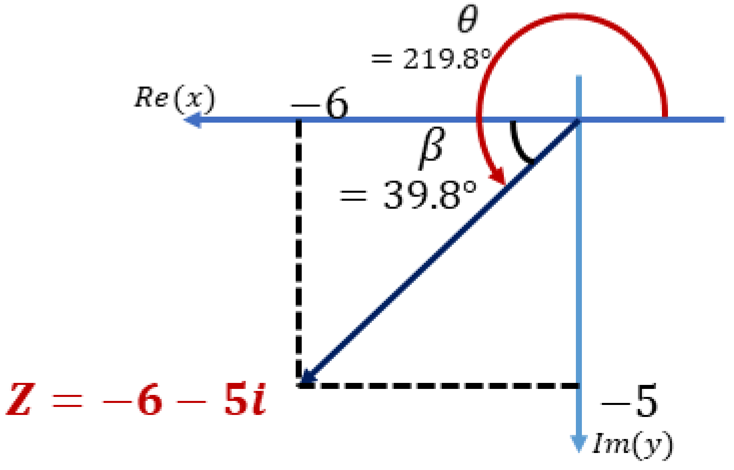
Argument:

$$\arg(z) = \beta$$
$$= tan^{-1}\frac{b}{a}$$
$$= tan^{-1}\frac{4}{3}$$
$$= \mathbf{53.13°}$$

$$\theta = (180° - 53.13°)$$
$$= \mathbf{126.87°}$$

Graphical Representation

Represent $z = -6 - 5i$ in an Argand Diagram and find its modulus and argument.



Modulus:

$$r = |Z| = \sqrt{a^2 + b^2}$$

$$= \sqrt{(-6)^2 + (-5)^2}$$

$$= \sqrt{36 + 25}$$

$$= \sqrt{61}$$

Argument:

$$\arg(z) = \beta$$

$$= tan^{-1}\frac{b}{a}$$

$$= tan^{-1}\frac{5}{6}$$

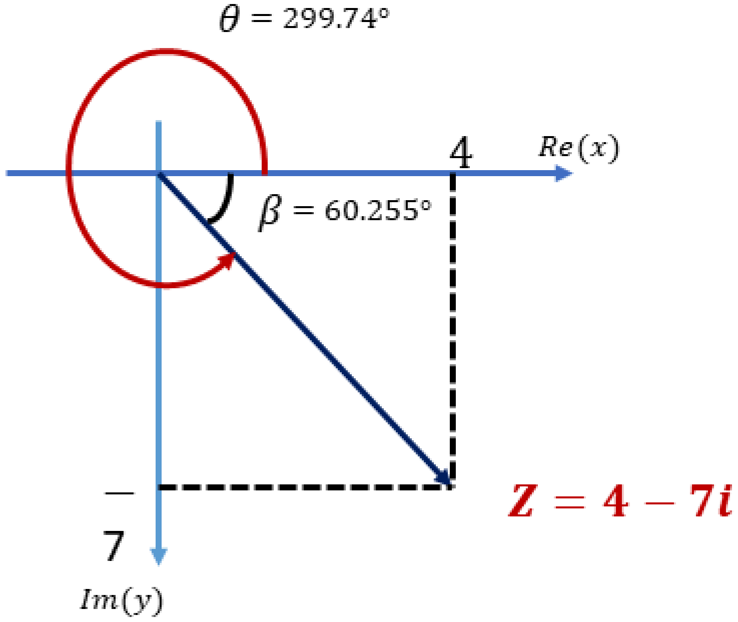
$$= 39.8°$$

$$\theta = (180° + 39.8°)$$

$$= 219.8°$$

Graphical Representation

Represent $\boldsymbol{z = 4 - 7i}$ in an Argand Diagram and find its modulus and argument.



Modulus:

$$r = |Z| = \sqrt{a^2 + b^2}$$

$$= \sqrt{(-7)^2 + (4)^2}$$

$$= \sqrt{49 + 16}$$

$$= \sqrt{65}$$

Argument:

$$\arg(z) = \beta$$

$$= tan^{-1}\frac{b}{a}$$

$$= tan^{-1}\frac{7}{4}$$

$$= 60.255°$$

$$\theta = (360° - 60.26°)$$

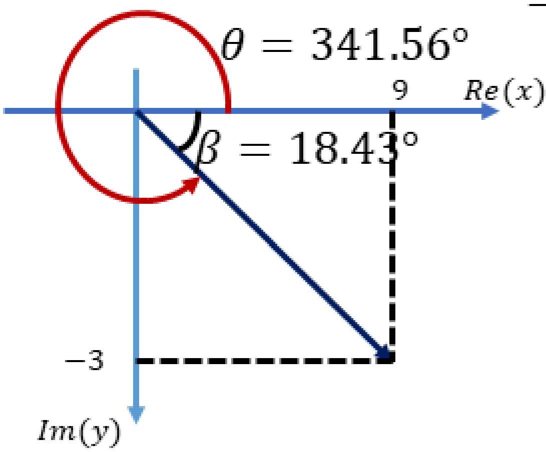
$$= 299.74°$$

Graphical Representation

Example:

Given $Z_1 = 5 + 3i$ and $Z_2 = 4 - 6i$, represent each of the following in an Argand Diagram and find its modulus and argument.

$Z_1 + Z_2$
$= 5 + 3i + 4 - 6i$
$= 9 - 3i$



Modulus:

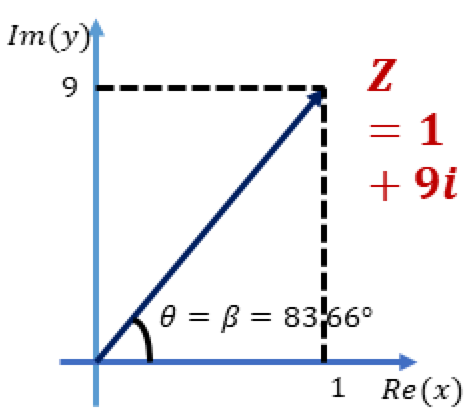
$r = |Z_1 + Z_2|$
$= \sqrt{9^2 + (-3)^2}$
$= \sqrt{81 + 9}$
$= \sqrt{90}$

Argument:

$\arg(z) = \beta$
$= tan^{-1}\frac{b}{a}$
$= tan^{-1}\frac{3}{9}$
$= 18.43°$

θ
$= (360°$
$- 18.43°)$
$= 341.56°$

$Z_1 - Z_2$
$= 5 + 3i - (4 - 6i)$
$= 1 + 9i$



Modulus:

$r = |Z|$
$= \sqrt{1^2 + (9)^2}$
$= \sqrt{1 + 81}$
$= \sqrt{82}$

Argument:

$\arg(z) = \beta$
$= tan^{-1}\frac{b}{a}$
$= tan^{-1}\frac{9}{1}$
$= 83.66°$

Graphical Representation

Example:

Represent $\boldsymbol{Z_1 = 5 + 3i}$ and $\boldsymbol{Z_2 = 4 - 6i}$ in an Argand Diagram and find its modulus and argument.

a. $Z_1 Z_2$

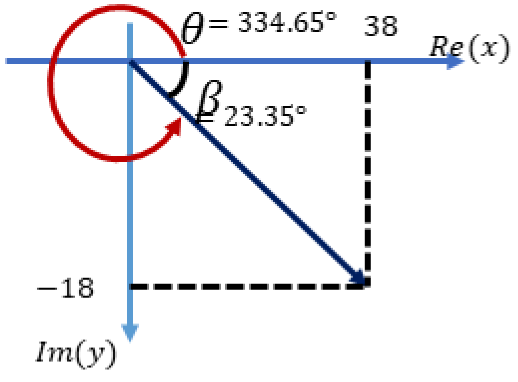
$$= (5 + 3i)(4 - 6i)$$
$$= 20 - 30i + 12i - 18i^2$$
$$= 20 - 18i - 18(-1)$$
$$= 20 + 18 - 18i$$
$$= 38 - 18i$$

Modulus:

$$r = |Z|$$
$$= \sqrt{38^2 + (-18)^2}$$
$$= \sqrt{1444 + 324}$$
$$= \sqrt{1768}$$
$$= 42.05$$

Argument:

$$\arg(z) = \beta = tan^{-1}\frac{b}{a}$$
$$= tan^{-1}\frac{18}{38}$$
$$= 25.35°$$



$$\theta = (360° - 25.35°)$$
$$= 334.65°$$

Graphical Representation

Example:

Represent $Z_1 = 5 + 3i$ and $Z_2 = 4 - 6i$ in an Argand Diagram and find its modulus and argument.

b. $\frac{Z_1}{Z_2}$

$$= \frac{(5+3i)}{(4-6i)}$$

$$= \frac{(5+3i)}{(4-6i)} \times \frac{(4+6i)}{(4+6i)}$$

$$= \frac{20 + 30i + 12i + 18i^2}{16 - 36i^2}$$

$$= \frac{20 + 42i + 18i^2}{16 - 36i^2}$$

$$= \frac{20 + 42i + 18(-1)}{16 - 36(-1)}$$

$$= \frac{2 + 42i}{52} = \frac{2}{52} + \frac{42}{52}i$$

$$= \frac{1}{26} + \frac{21}{26}i$$

Modulus:

$$r = |Z|$$

$$= \sqrt{(\frac{1}{26})^2 + (\frac{21}{26})^2}$$

$$= \sqrt{\frac{442}{676}}$$

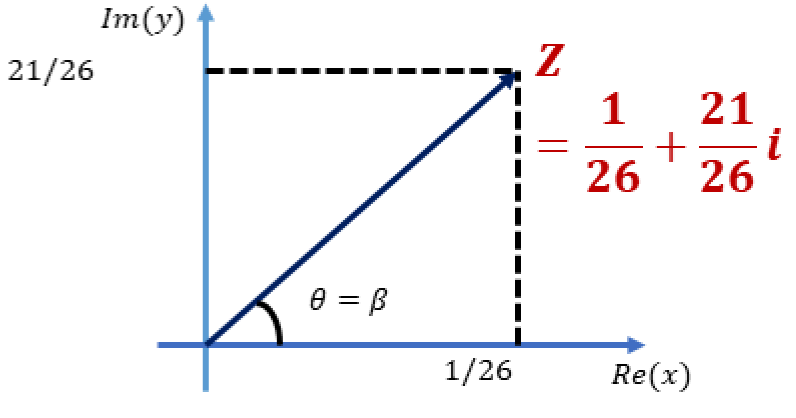
$$= \mathbf{0.8086}$$

Argument:

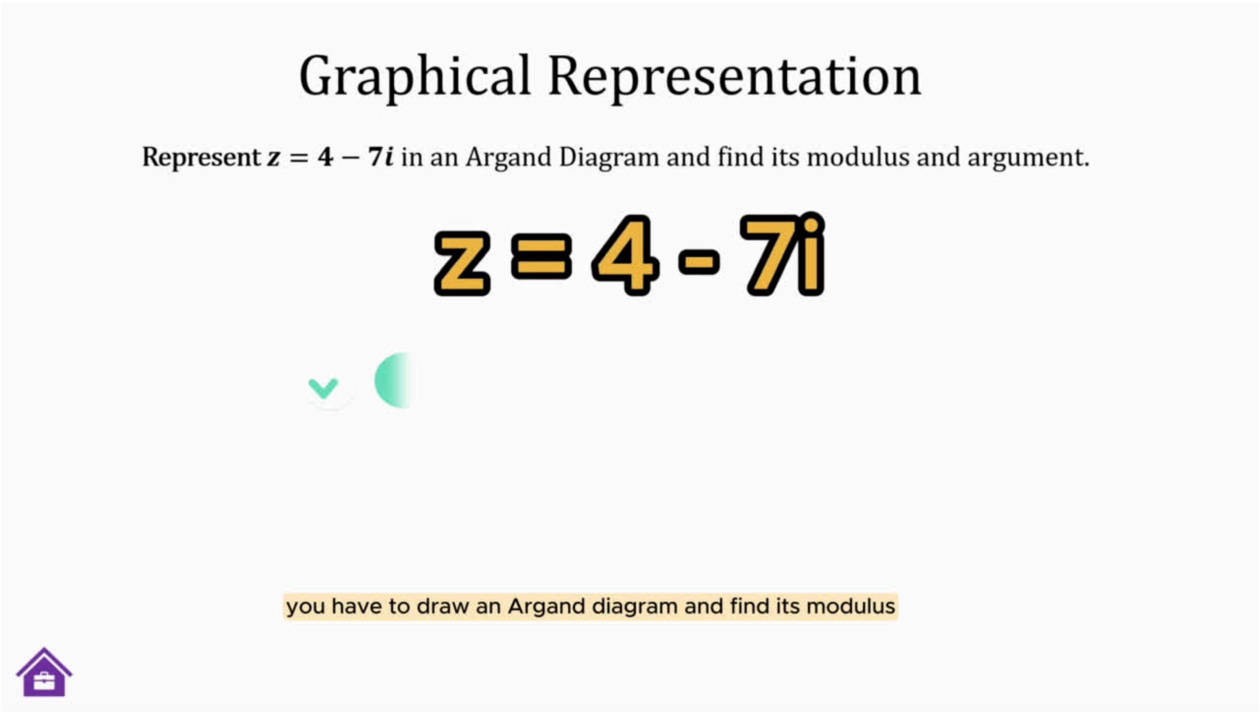
$$\arg(z) = \beta = tan^{-1}\frac{b}{a}$$

$$= tan^{-1}(\frac{21}{26} \div \frac{1}{26})$$

$$= \mathbf{87.27°}$$



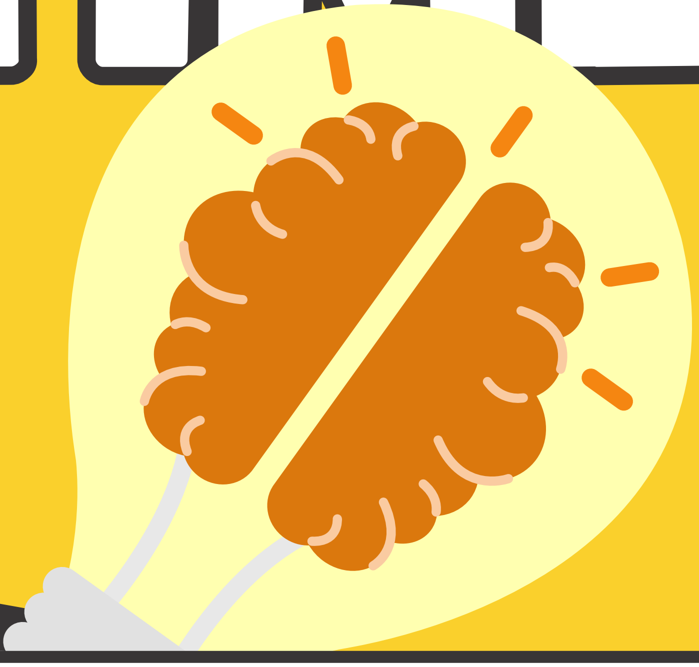
Video Explanation



SCAN ME

https://youtu.be/guxr4PtU0ic

QUIZ TIME



Let's test your understanding by answering this quiz

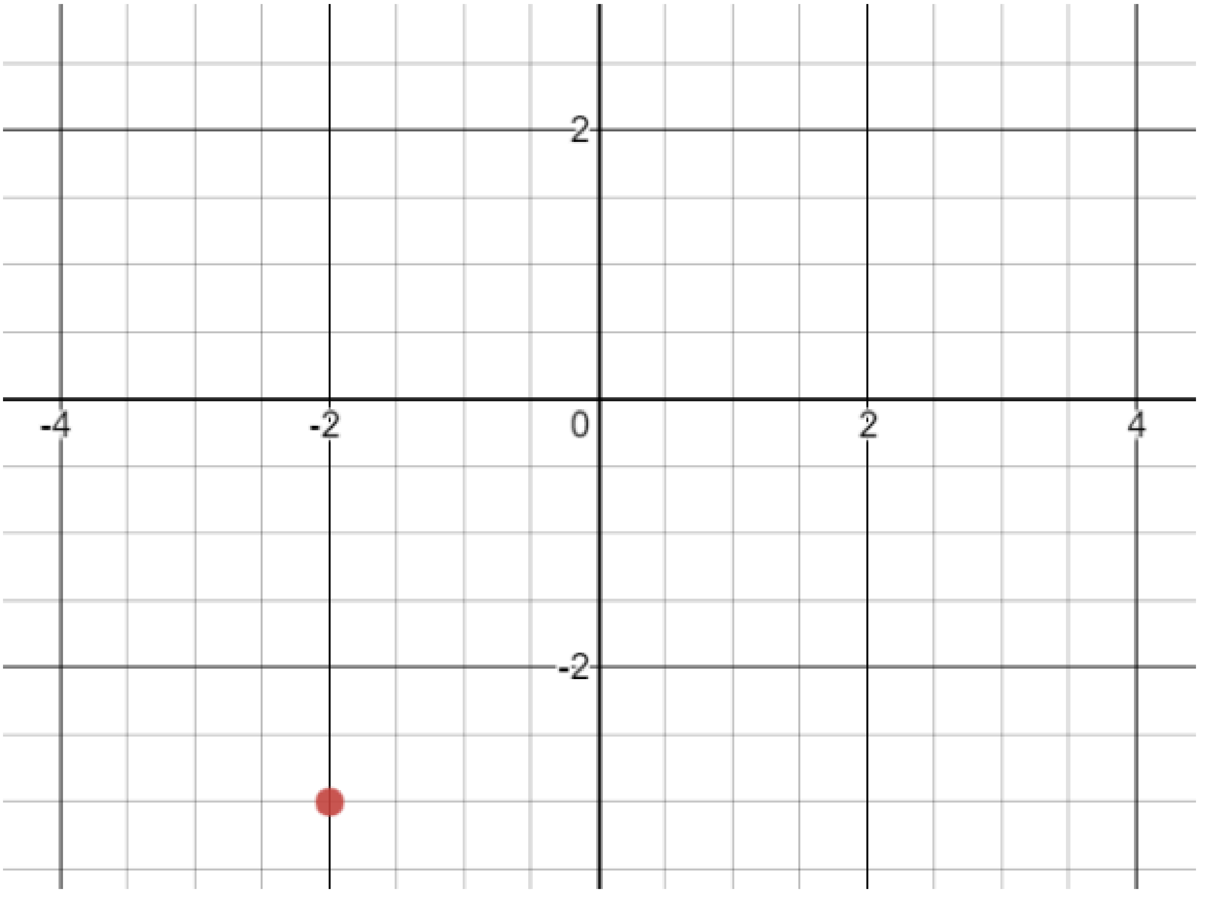
START!

The Argand Diagram

Sign in to Google to save your progress. Learn more

What complex number is shown on the graph?

25 points



- (-2, -3)
- -2 - 3i
- 2 - 3i
- 3i -2

Find the Argument for $z = -6 + 9i$.

25 points

- 13.94°
- 166.06°

Quiz Activity

SCAN ME

https://forms.gle/bwhQKUyzMdGU6aex9

04
Complex Number Form

Form of Complex Number

Cartesian Form/Rectangular Form

$$Z = a + bi$$

Polar Form

$$Z = |z|\angle\theta$$
@
$$Z = r\angle\theta$$

Trigonometric Form

$$Z = |z|(\cos\theta + i\sin\theta)$$
@
$$Z = r(\sin\theta + i\sin\theta)$$

Exponential Form

$$Z = |z|e^{\theta i}(\theta\ in\ radius)$$
@
$$Z = re^{\theta i}$$

Convert Form

Example:

Convert $z = 3 + 5i$ into trigonometric form, polar form and exponential form.

Step 1: **Find the values of modulus and argument.**

$$r = \sqrt{3^2 + 5^2} = \sqrt{9 + 25} = \sqrt{34}$$

$$\theta = tan^{-1}\frac{5}{3} = 59.0362°$$

Step 2: **Substitute the values of r and θ into the formula.**

Trigonometric Form

$$z = r(\cos\theta + i\sin\theta)$$

$$= \sqrt{34}(\cos 59.0362° + i\sin 59.0362°)$$

Polar Form

$$z = r\angle\theta$$

$$= \sqrt{34}\angle 59.0362°$$

Exponential Form

$$\theta_0 = 59.0362 \times \frac{\pi}{180°} = 1.0304\ rad$$

$$z = re^{\theta i}$$

$$= \sqrt{34}e^{1.0304i}$$

Must change to radian

Convert Form

Example:

Convert $z = 4 + 6i$ into trigonometric form, polar form and exponential form

Step 1: Find the values of modulus and argument.

$$r = \sqrt{4^2 + 6^2} = \sqrt{16 + 36} = \sqrt{52}$$

$$\theta = tan^{-1}\frac{6}{4} = 56.3099°$$

Step 2: Substitute the values of r and θ into the formula.

Trigonometric Form

$$z = r(\cos\theta + i\sin\theta)$$

$$= \sqrt{52}(\cos 56.3099° + i\sin 56.3099°)$$

Polar Form

$$z = r\angle\theta$$

$$= \sqrt{52}\angle 56.3099°$$

Exponential Form

$$\theta_0 = 56.3099 \times \frac{\pi}{180°} = 0.9828\ rad$$

$$z = re^{\theta i}$$

$$= \sqrt{52}e^{0.9828i}$$

Convert Form

Example:

Convert $z = 5\angle 48°$ into trigonometric form, cartesian form and exponential form.

Trigonometric Form

$$z = r(\cos\theta + i\sin\theta)$$
$$= \mathbf{5(\cos 48° + i\sin 48°)}$$

Cartesian Form

$$z = r(\cos\theta + i\sin\theta)$$
$$= 5(\cos 48° + i\sin 48°)$$
$$= \mathbf{3.346 + 3.716i}$$

Exponential Form

$$\theta_0 = 48 \times \frac{\pi}{180°} = 0.83778\ rad$$
$$z = re^{\theta i}$$
$$= \mathbf{5e^{0.8378i}}$$

Convert Form

Exercise:

1. **Given that $\boldsymbol{R = 5 - 10i\ and\ S = -8 + 2i}$.** Calculate the modulus and the argument for R+S. Then sketch the Argand diagram for R+S.

2. **Solve the following expression in an exponential form.** (6 marks)

$$\frac{10(\cos 200° + i\sin 200°) \times 6(\cos 10° + i\sin 10°)}{20(\cos 70° + i\sin 70°)}$$

3. **Given that**

$$\boldsymbol{Z_1} = 10(\cos 12° + i\sin 12°)\ and\ Z_2 = 20\angle 125°.$$

Solve $\frac{Z_2}{Z_1}$ in trigonometric form.

Convert Form

Answer:

1. **Given that $R = 5 - 10i$ and $S = -8 + 2i$.** Calculate the modulus and the argument for R+S. Then sketch the Argand diagram for R+S.

$$R + S$$

$$= (5 - 10i) + (-8 + 2i)$$

$$= 5 - 10i - 8 + 2i$$

$$= -3 - 8i$$

Argument:

$$\arg(z) = \beta = tan^{-1}\frac{b}{a}$$

$$= tan^{-1}\frac{8}{3}$$

$$= \mathbf{69.44^\circ}$$

Modulus:

$$r = |Z| = \sqrt{a^2 + b^2}$$

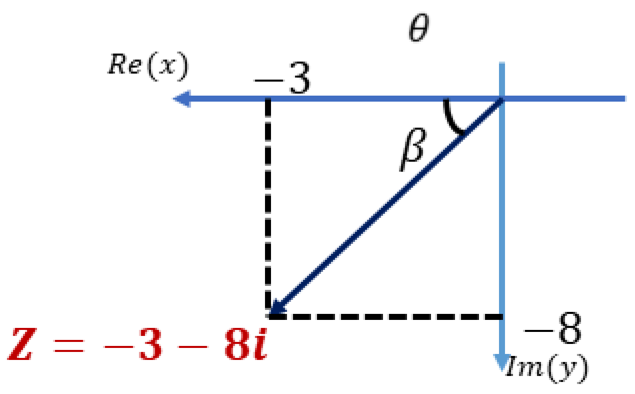
$$= \sqrt{(-3)^2 + (-8)^2}$$

$$= \sqrt{9 + 64}$$

$$= \sqrt{\mathbf{73}}$$

$$\theta = (180^\circ + 69.44^\circ)$$

$$= \mathbf{249.44^\circ}$$



Convert Form

Answer:

Solve the following expression in an exponential form.

$$\frac{10(\cos 200° + i \sin 200°) \times 6(\cos 10° + i \sin 10°)}{20(\cos 70° + i \sin 70°)}$$

$$= \frac{60(\cos 210° + i \sin 210°)}{20(\cos 70° + i \sin 70°)}$$

$$= 3(\cos 140° + i \sin 140°)$$

$$Convert\ 140°\ to\ radian$$

$$140° \times \frac{\pi}{180} = 2.443$$

$$\boldsymbol{Exponential\ form = 3e^{2.443i}}$$

Tips:

Multiply:
Multiply the modulus dan add the argument.

Division:
Divide the modulus and subtract the argument.

Convert Form

Answer:

Given that

$$\boldsymbol{Z_1} = 10(\cos 12° + i\sin 12°)\ and\ Z_2 = 20\angle 125°.$$

Solve $\frac{Z_2}{Z_1}$ in trigonometric form.

$$Trigonometric\ form: Z_2 = 20(\cos 125° + i\sin 125°)$$

$$\frac{Z_2}{Z_1} = \frac{20(\cos 125° + i\sin 125°)}{10(\cos 12° + i\sin 12°)}$$

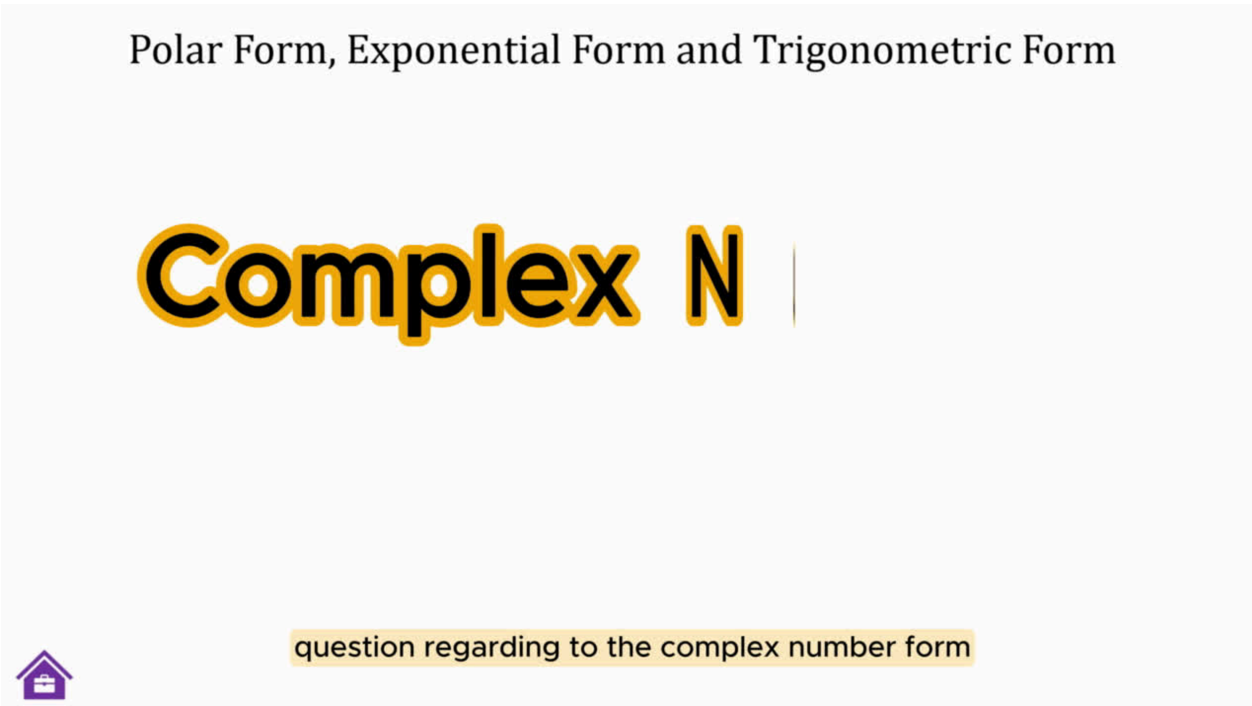
$$= \mathbf{2(\cos 113° + \boldsymbol{i}\sin 113°)}$$

Tips:

Multiply:
Multiply the modulus dan add the argument.

Division:
Divide the modulus and subtract the argument.

Video Explanation



SCAN ME

https://youtu.be/0UcHBxKkdOw

QUIZ TIME

Let's test your understanding by answering this quiz

START!

Complex Number Form

Sign in to Google to save your progress. Learn more

Express ***z = 6 − 9i*** in polar form. 25 points

- z=166.06∠10.81°
- z=13.95∠10.81°
- z=10.81∠303.69°
- z=10.81∠13.95°

The product of the following complex number in polar form ***√5(cos130° + i sin130°) · √45(cos15° + i sin15°)*** is... 25 points

- 50(cos145° + i sin145°)
- 15(cos145° + i sin145°)
- 81(cos100° + i sin100°)
- All Options are Correct

z = 3e5.2134i Express z in polar form 25 points

- z=3∠5.213°
- z=3∠298.71°
- z=5∠298.71°
- 3∠298.75°
- Option 5

z1 = 2(cos60°+isin60°), z2 = 3∠75° find ***z1z2*** in trigonometric form 25 points

- 6(cos125°+isin125°)
- 6(cos135°+isin135°)
- 5(cos135°+isin135°)

Quiz Activity

SCAN ME

https://forms.gle/bwhQKUyzMdGU6aex9

Conclusion

In conclusion, this eBook has provided a thorough exploration of complex numbers, an essential topic in engineering mathematics. By delving into the fundamental concepts, such as the definition and representation of complex numbers, as well as performing basic operations like addition, subtraction, multiplication, and division, we have laid a strong foundation for understanding this critical area of mathematics. The interactive elements incorporated throughout this eBook, such as videos explanation and online quiz, have been designed to enhance engagement and facilitate a better grasp of complex concepts. By actively participating in the learning process, students are better equipped to apply their knowledge in practical engineering contexts. As you advance in your studies and professional career, the understanding of complex numbers gained from this eBook will serve as a vital tool in tackling a wide range of engineering problems. We hope this resource has been both informative and inspiring, and that it will contribute significantly to your success in the field of engineering mathematics.

References

Bird, J. (2017). *Higher Engineering Mathematics. (8th Edition).* New York, NY: Routledge.

Croft A. And Davison R. (2019). *Mathematics for Engineers (5th Edittion).* New York, NY: Routledge.

Zuraini Ibrahim, Suria Masnin, Fatin Hamimah, Mohamed Salleh, Myzatul Mansor dan Baharudin Azit. (2018). *Engineering Mathematics 2*. Shah Alam: Oxford Fajar Sdn. Bhd.

Explore our interactive version!

SCAN ME!

Navigating the Complex: A Guide to Mastering Complex Numbers

e ISBN 978-629-7638-22-5

9 786297 638225

Politeknik Kuching Sarawak

(online)