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STEP BY STEP

LINEAR EQUATION METHOD

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STEP BY STEP

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BIOGRAPHY

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ABSTRACT

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Linear equations represent a clear and versatile mathematical method for solving system of linear equations using matrix notation. This abstract provides an overview of the concept, where variables and constant are expressed in matrices, while the form of the equation is represented by the form of matrix multiplication. Solutions to linear equations can use various matrices operations, such as matrix addition, multiplication, inverse Cramers's Rule, Gauss's , Doolittle's , and Crout's elimination. These applications are widely used in various fields, including engineering, physics, computer science, and economics. In addition,this application also helps in testing efficient analysis and is suitable for solving a complex system with various variables ans constant. The understanding and application of linear equations plays an important role in solving critical mathematical problems asa well as various other scientific problems.

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INTRODUCTION

1 INTRODUCTION

The step-by-step linear equation method is a systematic approach used to solve linear equations, which are mathematical expressions involving variables raised to the power of 1 and constants. The method involves isolating the variable on one side of the equation by performing a series of operations that maintain the equation's balance. These operations include adding, subtracting, multiplying, and dividing both sides of the equation by constants, with the goal of simplifying the equation and determining the value of the variable. The method's step-by-step nature ensures that each operation is applied to both sides of the equation, allowing for the gradual transformation of the equation into a simplified form that reveals the solution for the variable.

Matrices are rectangular arrays of numbers, symbols, or expressions arranged in rows and columns. They play a fundamental role in mathematics, computer science, and various fields of science and engineering. Matrices are used to represent and manipulate data in linear algebra, providing a concise way to perform operations such as addition, subtraction, multiplication, and transformation of vectors and linear equations. They find applications in solving systems of equations, performing transformations in computer graphics, analyzing networks, optimizing operations, and numerous other areas where structured data manipulation is required. Matrices offer a powerful framework for representing and solving complex mathematical and real-world problems through their algebraic properties and operations.

HISTORY

Matrices have a rich historical background spanning various cultures and eras. Their origins can be traced back to ancient civilizations such as China, where matrix-like arrangements were used for calculations around 2,500 years ago. However, the modern formalization of matrices began in the 19th century with the works of mathematicians like Arthur Cayley and James Joseph Sylvester, who introduced the concept of matrix algebra and notation. The development of matrices gained momentum with the growth of linear algebra in the 20th century, playing a pivotal role in fields such as physics, engineering, computer science, and economics. Matrices have become fundamental tools for representing and solving systems of linear equations, transformations, and various mathematical operations, leading to their wide-ranging applications in diverse disciplines.

1.1 What is matrices

A matrix is a collection of numbers arranged into a fixed number of rows and columns and enclosed in brackets [ ] or ( ) . Usually the numbers are real numbers.

$$[A] = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

1.1.1 Characteristics of a matrices

- Row
  The m rows are horizontal numbers arranged.

m rows

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

- Column
  The n columns are vertical numbers arranged.

n columns

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

- Order
  The order of matrix is rows x columns = m x n

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

…. Order = 3 x 3

- Element
  Each number that makes up a matrix. The elements in a matrix have specific locations.

$$\begin{bmatrix} 4 & 6 & -6 \\ 8 & 5 & 3 \\ 2 & -1 & 1 \end{bmatrix} \qquad \begin{matrix} a_{11} = 4 & a_{12} = 6 & a_{13} = -6 \\ a_{21} = 8 & a_{22} = 5 & a_{23} = 3 \\ a_{31} = 2 & a_{32} = -1 & a_{33} = 1 \end{matrix}$$

1.1.2 **Special matrices.**

These are demonstrated for 3 x 3 matrix, but apply to all matrix sizes.

- Square matrix (m=n=3)

$$[A] = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

- Diagonal matrix

$$[A] = \begin{bmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{bmatrix}$$

- Identity matrix

$$[A] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- Transpose matrix

$$[A]^T = \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{bmatrix} \quad \text{(switch rows and columns)}$$

1.1.3 **Operation of matrices.**

- Addition of matrices
  Two matrices have to be conformable for addition if they have the same order of matrix.

$$[C] = [A] + [B]$$

$$\begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

$$= \begin{bmatrix} a_{11} + b_{11} & a_{12} + b_{12} \\ a_{21} + b_{21} & a_{22} + b_{22} \end{bmatrix}$$

- Subtraction of matrices

  Two matrices have to be conformable for subtraction if they have the same order of matrix.

$$[C] = [A] - [B]$$

$$\begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} - \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

$$= \begin{bmatrix} a_{11} - b_{11} & a_{12} - b_{12} \\ a_{21} - b_{21} & a_{22} - b_{22} \end{bmatrix}$$

- Multiplication of matrices

  There are two types of multiplication for matrices:

  I. Scalar multiplication

     Just use a regular number and multiply it on every entry in the matrix.

$$k[A] = k\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} ka_{11} & ka_{12} \\ ka_{21} & ka_{22} \end{bmatrix}$$

  II. Matrix multiplication

     Matrix A and B are to be conformable for the product AB if the number of columns in A is the same as the number of rows in B. That is, in a matrix multiplication product the number of columns *p* in the left-hand matrix must equal the number of rows *p* in the right-hand matrix. If this condition is not met, the matrix multiplication is undefined and cannot be done. The size of the resulting matrix [C] is from the number of rows m of the left-hand matrix and the number of columns n of the right-hand matrix, m x n.

[C]=[A][B]

(m x n) = (m x p) (p x n)

1.2 Example

1

If $A=\begin{pmatrix}1 & -3 & 2\\ 4 & 0 & 3\\ 1 & -1 & 2\end{pmatrix}$, $B=\begin{pmatrix}2 & 2 & -3\\ 1 & -3 & 4\\ 3 & -3 & 1\end{pmatrix}$

Solution: $A+B=\begin{pmatrix}1+2 & -3+2 & 2-3\\ 4+1 & 0-3 & 3+4\\ 1+3 & -1-3 & 2+1\end{pmatrix}=\begin{pmatrix}3 & -1 & -1\\ 5 & -3 & 7\\ 4 & -4 & 3\end{pmatrix}$

2

If $A=\begin{pmatrix}1 & 2\\ 3 & -1\\ 1 & -1\end{pmatrix}$, find 3A and -2A

Solution:

$$3A=3\begin{pmatrix}1 & 2\\ 3 & -1\\ 1 & -1\end{pmatrix}=\begin{pmatrix}3(1) & 3(2)\\ 3(3) & 3(-1)\\ 3(1) & 3(-1)\end{pmatrix}=\begin{pmatrix}3 & 6\\ 9 & -3\\ 3 & -3\end{pmatrix}$$

$$-2A=\begin{pmatrix}1 & 2\\ 3 & -1\\ 1 & -1\end{pmatrix}=\begin{pmatrix}-2(1) & -2(2)\\ -2(3) & -2(-1)\\ -2(1) & -2(-1)\end{pmatrix}=\begin{pmatrix}-2 & -4\\ -6 & 2\\ -2 & 2\end{pmatrix}$$

3

Given Matrix A and B, Find AB

$$\mathbf{A}=\begin{pmatrix}1 & 0 & 1\\ 2 & -7 & 8\\ 0 & 1 & -4\\ 6 & 2 & 1\end{pmatrix} \text{ dan } \mathbf{B}=\begin{pmatrix}2 & 0\\ -3 & -1\\ 4 & 5\end{pmatrix}$$

Solution:

$$\mathbf{AB}=\begin{pmatrix}1\cdot 2+0\cdot(-3)+1\cdot 4 & 1\cdot 0+0\cdot(-1)+1\cdot 5\\ 2\cdot 2+(-7)\cdot(-3)+8\cdot 4 & 2\cdot 0+(-7)\cdot(-1)+8\cdot 5\\ 0\cdot 2+1\cdot(-3)+(-4)\cdot 4 & 0\cdot 0+1\cdot(-1)+(-4)\cdot 5\\ 6\cdot 2+2\cdot(-3)+1\cdot 4 & 6\cdot 0+2\cdot(-1)+1\cdot 5\end{pmatrix}$$

$$=\begin{pmatrix}2+0+4 & 0+0+5\\ 4+21+32 & 0+7+40\\ 0-3-16 & 0-1-20\\ 12-6+4 & 0-2+5\end{pmatrix}$$

$$\mathbf{AB}=\begin{pmatrix}6 & 5\\ 57 & 47\\ -19 & -21\\ 10 & 3\end{pmatrix}$$

Video

HOW TO USE SCIENTIFIC CALCULATOR

https://www.youtube.com/watch?v=r53DE90rNXU

1.3 Exercises

Given ,

$$A = \begin{pmatrix} 2 & 1 & 3 \\ 4 & 2 & 1 \\ 5 & 3 & 4 \end{pmatrix} \quad \boldsymbol{B} = \begin{pmatrix} -1 & 5 & 6 \\ 2 & 4 & 3 \\ 3 & -5 & 7 \end{pmatrix} \quad \boldsymbol{C} = \begin{pmatrix} 2 & 4 \\ 3 & 6 \\ -1 & 5 \end{pmatrix}$$

a. Find $A + B$

b. Find $A - B$

c. Find $3C$

d. Find $A\mathbf{C}$

STEP BY STEP

CHAPTER 2
METHOD OF LINEAR EQUATION: INVERSE

2. METHOD OF LINEAR EQUATION: INVERSE

The inverse method refers to a mathematical technique employed to solve equations by reversing the roles of the dependent and independent variables. This method is particularly useful when attempting to find the solution for an equation that is not easily solvable directly. By rearranging the equation to isolate the desired variable on one side, the inverse method aims to reveal the relationship between the variables in a more manageable form, facilitating the determination of the unknown variable's value. This approach is widely utilized in various fields, including physics, engineering, and finance, to unravel complex relationships and find solutions that might otherwise be challenging to obtain.

HISTORY

The inverse matrix method dates back to ancient civilizations, where methods for solving systems of linear equations were developed. However, the formal concept of matrix inversion as a mathematical technique began to take shape in the 18th and 19th centuries, notably with the work of mathematicians like Carl Friedrich Gauss and Augustin-Louis Cauchy. In the mid-20th century, advancements in linear algebra, particularly with the development of computers, led to the practical implementation of the inverse matrix method for solving complex systems of equations. This method became a cornerstone in various scientific and engineering applications, from solving structural and electrical systems to computer graphics and cryptography, showcasing its widespread significance in modern problem-solving contexts.

2.1 Solving Inverse Matrix

- Transforming the system matrix form AX=B

$$\begin{aligned} a_{11}x + a_{12}y + a_{13}z &= b_1 \\ a_{21}x + a_{22}y + a_{23}z &= b_2 \\ a_{31}x + a_{32}y + a_{33}z &= b_3 \end{aligned} \longrightarrow \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

- Find the determinant $|A|$

If $[A] = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

$$|A| = a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{21} \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} + a_{31} \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}$$

$$|A| = a_{11}(a_{22}a_{33} - a_{32}a_{23}) - a_{21}(a_{12}a_{33} - a_{32}a_{13}) + a_{31}(a_{12}a_{23} - a_{22}a_{13})$$

- Find the minor

If $[A] = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ then, minor $[A] = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix}$

$$\begin{bmatrix} m_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} & m_{12} = \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} & m_{13} = \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} \\ m_{21} = \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} & m_{22} = \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} & m_{23} = \begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix} \\ m_{31} = \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} & m_{32} = \begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix} & m_{33} = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \end{bmatrix}$$

- Find the co-factor

Co-factor $A = \begin{bmatrix} (-1)^2 m_{11} & (-1)^3 m_{12} & (-1)^4 m_{13} \\ (-1)^3 m_{21} & (-1)^4 m_{22} & (-1)^5 m_{23} \\ (-1)^4 m_{31} & (-1)^5 m_{32} & (-1)^6 m_{33} \end{bmatrix}$

$$A = \begin{bmatrix} +m_{11} & -m_{12} & +m_{13} \\ -m_{21} & +m_{22} & -m_{23} \\ +m_{31} & -m_{32} & +m_{33} \end{bmatrix}$$

- Find the adjoin

  The adjoin is the transpose of the matrix that consists of co-factors.

$$A = \begin{bmatrix} +m_{11} & -m_{12} & +m_{13} \\ -m_{21} & +m_{22} & -m_{23} \\ +m_{31} & -m_{32} & +m_{33} \end{bmatrix}^T$$

$$A = \begin{bmatrix} +m_{11} & -m_{21} & +m_{31} \\ -m_{12} & +m_{22} & -m_{32} \\ +m_{13} & -m_{23} & +m_{33} \end{bmatrix}$$

- Find Inverse Matrix $A^{-1} = \frac{1}{|A|} Adj[A]$

$$A^{-1} = \frac{1}{|A|} \begin{bmatrix} +m_{11} & -m_{21} & +m_{31} \\ -m_{12} & +m_{22} & -m_{32} \\ +m_{13} & -m_{23} & +m_{33} \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \frac{+m_{11}}{|A|} & \frac{-m_{21}}{|A|} & \frac{+m_{31}}{|A|} \\ \frac{-m_{12}}{|A|} & \frac{+m_{22}}{|A|} & \frac{-m_{32}}{|A|} \\ \frac{+m_{13}}{|A|} & \frac{-m_{23}}{|A|} & \frac{+m_{33}}{|A|} \end{bmatrix}$$

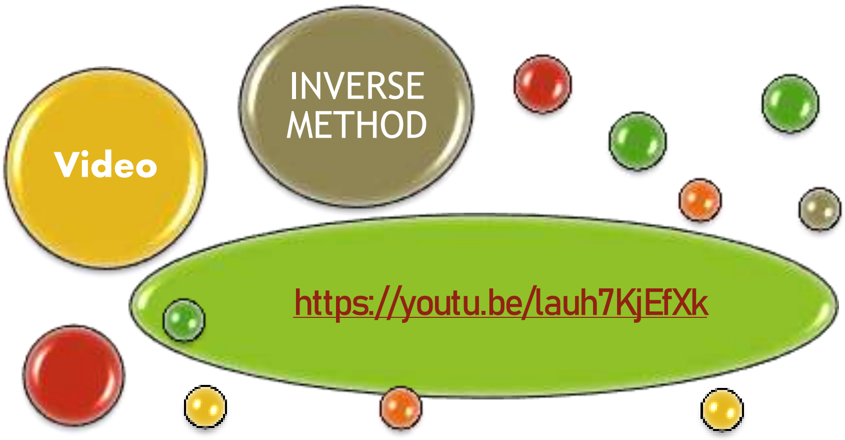
- Find $X = A^{-1}B$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{+m_{11}}{|A|} & \frac{-m_{21}}{|A|} & \frac{+m_{31}}{|A|} \\ \frac{-m_{12}}{|A|} & \frac{+m_{22}}{|A|} & \frac{-m_{32}}{|A|} \\ \frac{+m_{13}}{|A|} & \frac{-m_{23}}{|A|} & \frac{+m_{33}}{|A|} \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

Step by Step using the inverse method:

- step 1: Write linear equation to matrix form AX=B
- step 2: Find the determinant $|A|$
- step 3: Find the minor
- step 4: Find the co-factor
- step 5: Find the adjoin
- step 6: Find Inverse Matrix
- step 7: Find $X = A^{-1}B$

Remember, this method is specifically for linear equations. If you encounter non-linear equations, the process will differ based on the type of non-linearity.



2.2 Example

1

Solve using inverse method for
$$4x+6y+8z=1$$
$$5x+5y+2z=3$$
$$x+3y-4z=8$$

Solution:

$$A=\begin{pmatrix}4&6&8\\5&5&2\\1&3&-4\end{pmatrix} X=\begin{pmatrix}x\\y\\z\end{pmatrix} \quad B=\begin{pmatrix}1\\3\\8\end{pmatrix}$$

Det $|A|=4\begin{vmatrix}5&2\\3&-4\end{vmatrix}-6\begin{vmatrix}5&2\\1&-4\end{vmatrix}+8\begin{vmatrix}5&5\\1&3\end{vmatrix}$

$|A|=5(-20-6)-6(-20-2)+8(15-5)$

$|A|=108$

$$\text{Minor}=\begin{bmatrix}\begin{vmatrix}5&2\\3&-4\end{vmatrix}&\begin{vmatrix}5&2\\1&-4\end{vmatrix}&\begin{vmatrix}5&5\\1&3\end{vmatrix}\\\begin{vmatrix}6&8\\3&-4\end{vmatrix}&\begin{vmatrix}4&8\\1&-4\end{vmatrix}&\begin{vmatrix}4&6\\1&3\end{vmatrix}\\\begin{vmatrix}6&8\\5&2\end{vmatrix}&\begin{vmatrix}4&8\\5&2\end{vmatrix}&\begin{vmatrix}4&6\\5&5\end{vmatrix}\end{bmatrix}$$

$$=\begin{pmatrix}-26&-22&10\\-48&-24&6\\-28&-32&-10\end{pmatrix}$$

$$Co-factor=\begin{bmatrix}+(-26)&-(-22)&+(10)\\-(-48)&(-24)&-(6)\\+(-28)&-(-32)&+(-10)\end{bmatrix}$$

$$=\begin{pmatrix}-26&22&10\\48&-24&-6\\-28&32&-10\end{pmatrix}$$

$$\text{Adjoin}=\begin{pmatrix}-26&48&-28\\22&-24&32\\10&-6&-10\end{pmatrix}$$

$$A^{-1}=\frac{1}{108}\begin{pmatrix}-26&48&-28\\22&-24&32\\10&-6&-10\end{pmatrix} \quad =\begin{pmatrix}-0.24&0.44&-0.26\\0.20&-0.22&0.3\\0.09&-0.06&0.09\end{pmatrix}$$

$$\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}-0.24&0.44&-0.26\\0.20&-0.22&0.3\\0.09&-0.06&0.09\end{pmatrix}\begin{pmatrix}1\\3\\8\end{pmatrix}$$

$$\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}-1\\1.91\\-0.8\end{pmatrix}$$

2

Solve using inverse method for
$$7x - y + 6z = 8$$
$$3x + 2y - z = 10$$
$$3x + 2z = 5$$

Solution:

$$A = \begin{pmatrix} 7 & -1 & 6 \\ 3 & 2 & -1 \\ 3 & 0 & 2 \end{pmatrix} X = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad B = \begin{pmatrix} 8 \\ 10 \\ 5 \end{pmatrix}$$

Det $|A| = 7\begin{vmatrix} 2 & -1 \\ 0 & 2 \end{vmatrix} - (-1)\begin{vmatrix} 3 & -1 \\ 3 & 2 \end{vmatrix} + 6\begin{vmatrix} 3 & 2 \\ 3 & 0 \end{vmatrix}$

$|A| = 7(4-0) - (-1)(6-(-3)) + 6(0-6)$

$|A| = 1$

$$\text{Minor} = \begin{bmatrix} \begin{vmatrix} 2 & -1 \\ 0 & 2 \end{vmatrix} & \begin{vmatrix} 3 & -1 \\ 3 & 2 \end{vmatrix} & \begin{vmatrix} 3 & 2 \\ 3 & 0 \end{vmatrix} \\ \begin{vmatrix} -1 & 6 \\ 0 & 2 \end{vmatrix} & \begin{vmatrix} 7 & 6 \\ 3 & 2 \end{vmatrix} & \begin{vmatrix} 7 & -1 \\ 3 & 0 \end{vmatrix} \\ \begin{vmatrix} -1 & 6 \\ 2 & -1 \end{vmatrix} & \begin{vmatrix} 7 & 6 \\ 3 & -1 \end{vmatrix} & \begin{vmatrix} 7 & -1 \\ 3 & 0 \end{vmatrix} \end{bmatrix} = \begin{pmatrix} 4 & 9 & -6 \\ -2 & -4 & 3 \\ -11 & -25 & 17 \end{pmatrix}$$

$$Co-factor = \begin{bmatrix} +(4) & -(9) & +(-6) \\ -(-2) & (-4) & -(3) \\ +(-11) & -(-25) & +(17) \end{bmatrix} = \begin{pmatrix} 4 & -9 & -6 \\ 2 & -4 & -3 \\ -11 & 25 & 17 \end{pmatrix}$$

$$\text{Adjoin} = \begin{pmatrix} 4 & 2 & -11 \\ -9 & -4 & 25 \\ -6 & -3 & 17 \end{pmatrix}$$

$$A^{-1} = \frac{1}{1}\begin{pmatrix} 4 & 2 & -11 \\ -9 & -4 & 25 \\ -6 & -3 & 17 \end{pmatrix} = \begin{pmatrix} 4 & 2 & -11 \\ -9 & -4 & 25 \\ -6 & -3 & 17 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 & 2 & -11 \\ -9 & -4 & 25 \\ -6 & -3 & 17 \end{pmatrix}\begin{pmatrix} 8 \\ 10 \\ 5 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -3 \\ 13 \\ 7 \end{pmatrix}$$

3

Solve using inverse method for

$$3x + 6z = 4$$
$$7x - 4y - z = 7$$
$$2x + 8y + 2z = 12$$

Solution:

$$A = \begin{pmatrix} 3 & 0 & 6 \\ 7 & -4 & -1 \\ 2 & 8 & 2 \end{pmatrix} X = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad B = \begin{pmatrix} 4 \\ 7 \\ 12 \end{pmatrix}$$

$$\text{Det } |A| = 3\begin{vmatrix} -4 & -1 \\ 8 & 2 \end{vmatrix} - (0)\begin{vmatrix} 7 & -1 \\ 2 & 2 \end{vmatrix} + 6\begin{vmatrix} 7 & -4 \\ 2 & 8 \end{vmatrix}$$

$$|A| = 3(-8 - (-8)) - (0)(14 - (-2)) + 6(56 - (-8))$$

$$|A| = 384$$

$$\text{Minor} = \begin{bmatrix} \begin{vmatrix} -4 & -1 \\ 8 & 2 \end{vmatrix} & \begin{vmatrix} 7 & -1 \\ 2 & 2 \end{vmatrix} & \begin{vmatrix} 7 & -4 \\ 2 & 8 \end{vmatrix} \\ \begin{vmatrix} 0 & 6 \\ 8 & 2 \end{vmatrix} & \begin{vmatrix} 3 & 6 \\ 2 & 2 \end{vmatrix} & \begin{vmatrix} 3 & 0 \\ 2 & 8 \end{vmatrix} \\ \begin{vmatrix} 0 & 6 \\ -4 & -1 \end{vmatrix} & \begin{vmatrix} 3 & 6 \\ 7 & -1 \end{vmatrix} & \begin{vmatrix} 3 & 0 \\ 7 & -4 \end{vmatrix} \end{bmatrix} = \begin{pmatrix} 0 & 16 & 64 \\ -48 & -6 & 24 \\ 24 & -45 & -12 \end{pmatrix}$$

$$Co - factor = \begin{bmatrix} +(0) & -(16) & +(64) \\ -(-48) & +(-6) & -(24) \\ +(24) & -(-45) & +(-12) \end{bmatrix} = \begin{pmatrix} 0 & -16 & 64 \\ 48 & -6 & -24 \\ 24 & 45 & -12 \end{pmatrix}$$

$$\text{Adjoin} = \begin{pmatrix} 0 & 48 & 24 \\ -16 & -6 & 45 \\ 64 & -24 & -12 \end{pmatrix}$$

$$A^{-1} = \frac{1}{384}\begin{pmatrix} 0 & 48 & 24 \\ -16 & -6 & 45 \\ 64 & -24 & -12 \end{pmatrix} = \begin{pmatrix} 0 & 0.13 & 0.06 \\ -0.04 & -0.02 & 0.12 \\ 0.17 & -0.06 & -0.03 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 & 0.13 & 0.06 \\ -0.04 & -0.02 & 0.12 \\ 0.17 & -0.06 & -0.03 \end{pmatrix}\begin{pmatrix} 8 \\ 10 \\ 5 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -3 \\ 13 \\ 7 \end{pmatrix}$$

4

Solve using inverse method for $\begin{matrix} 8x + 7y + 6z = 15 \\ -3x + 9y + 5z = 7 \\ 3x - 4y + 2z = 9 \end{matrix}$

Solution:

$$A = \begin{pmatrix} 8 & 7 & 6 \\ -3 & 9 & 5 \\ 3 & -4 & 2 \end{pmatrix} X = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad B = \begin{pmatrix} 15 \\ 7 \\ 9 \end{pmatrix}$$

$$\text{Det } |A| = 8\begin{vmatrix} 9 & 5 \\ -4 & 2 \end{vmatrix} - 7\begin{vmatrix} -3 & 5 \\ 3 & 2 \end{vmatrix} + 6\begin{vmatrix} -3 & 9 \\ 3 & -4 \end{vmatrix}$$

$$|A| = 8(18 - (-20)) - 7(-6 - 15) + 6(12 - 27)$$

$$|A| = 361$$

$$\text{Minor } = \begin{bmatrix} \begin{vmatrix} 9 & 5 \\ -4 & 2 \end{vmatrix} & \begin{vmatrix} -3 & 5 \\ 3 & 2 \end{vmatrix} & \begin{vmatrix} -3 & 9 \\ 3 & -4 \end{vmatrix} \\ \begin{vmatrix} 7 & 6 \\ -4 & 2 \end{vmatrix} & \begin{vmatrix} 8 & 6 \\ 3 & 2 \end{vmatrix} & \begin{vmatrix} 8 & 7 \\ 3 & -4 \end{vmatrix} \\ \begin{vmatrix} 7 & 6 \\ 9 & 5 \end{vmatrix} & \begin{vmatrix} 8 & 6 \\ -3 & 5 \end{vmatrix} & \begin{vmatrix} 8 & 7 \\ -3 & 9 \end{vmatrix} \end{bmatrix} = \begin{pmatrix} 38 & -21 & -15 \\ 38 & -2 & -53 \\ -19 & 58 & 93 \end{pmatrix}$$

$$Co - factor = \begin{bmatrix} +(38) & -(-21) & +(-15) \\ -(38) & +(-2) & -(-53) \\ +(-19) & -(58) & +(93) \end{bmatrix} = \begin{pmatrix} 38 & 21 & -15 \\ -38 & -2 & 53 \\ -19 & -58 & 93 \end{pmatrix}$$

$$\text{Adjoin} = \begin{pmatrix} 38 & -38 & -19 \\ 21 & -2 & -58 \\ -15 & 53 & 93 \end{pmatrix}$$

$$A^{-1} = \frac{1}{361}\begin{pmatrix} 38 & -38 & -19 \\ 21 & -2 & -58 \\ -15 & 53 & 93 \end{pmatrix} = \begin{pmatrix} 0.11 & -0.11 & -0.05 \\ 0.06 & -0.006 & -0.16 \\ -0.04 & 0.15 & 0.26 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0.11 & -0.11 & -0.05 \\ 0.06 & -0.006 & -0.16 \\ -0.04 & 0.15 & 0.26 \end{pmatrix}\begin{pmatrix} 15 \\ 7 \\ 9 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0.37 \\ -0.6 \\ 2.72 \end{pmatrix}$$

2.3 Exercises

(a) Identify AX = B from the equations below:

i.

$$2x + 5y = 9$$
$$x + 2y - z = 3$$
$$-3x - 4y + 7z = 1$$

ii.

$$3x + y = -1$$
$$2x + 4y + z = 7$$
$$2y + 5z = 9$$

(b) Solve the equations below by using Inverse Method.

$$2x - 4y - 3z = -23$$
$$3x + 5y + z = 12$$
$$-2x + 4y + 7z = 83$$

ANS: x = - 4, y = 3, z = 9

(c) Solve the equations below by using Inverse Method.

$$x + 2y + 2z = 34$$
$$-3x - 4y + z = -25$$
$$4x + 7y + 3z = 84$$

ANS: x = 2, y = 7, z = 9

(d) Solve the equations below by using Inverse Method.

$$2x + 4y - 3z = -7$$
$$3x + 5y + z = -2$$
$$-2x + 4y + 7z = -13$$

ANS: x = 4, y = - 3, z = 1

STEP BY STEP

CHAPTER 3
METHOD OF LINEAR EQUATION: CRAMER'S RULE

3. METHOD OF LINEAR EQUATION: CRAMER'S RULE

Cramer's Rule is a mathematical theorem used to solve a system of linear equations by expressing the solutions as ratios of determinants. For a system with 'n' equations and 'n' variables, the rule involves constructing matrices by replacing the coefficient matrix with individual column vectors from the constants matrix one at a time. The solutions for each variable are then obtained by dividing the determinant of the resulting matrix with the variable's corresponding column vector determinant. Cramer's Rule provides an algebraic method to find unique solutions for each variable in a linear system, but it can be computationally expensive and numerically unstable for larger systems due to the reliance on determinants

HISTORY

Cramer's Rule, named after the Swiss mathematician Gabriel Cramer (1704–1752), has its origins in the 18th century. Cramer developed the rule as a method to solve systems of linear equations using determinants. He first introduced this technique in his book "Introduction à l'Analyse des Lignes Courbes Algébriques" published in 1750. Cramer's Rule provided an elegant and algebraic approach to solving linear systems by leveraging the properties of determinants, although it wasn't widely adopted in its early days due to the computational challenges of calculating determinants manually. Over time, with advancements in mathematics and computational techniques, Cramer's Rule became more accessible and applicable, contributing to the broader field of linear algebra and systems of equations solving

3.1 Solving Cramer's Rule

- Transforming the system matrix form AX=B

$$\begin{aligned} a_{11}x + a_{12}y + a_{13}z &= b_1 \\ a_{21}x + a_{22}y + a_{23}z &= b_2 \\ a_{31}x + a_{32}y + a_{33}z &= b_3 \end{aligned} \longrightarrow \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

- Find the determinant $|A|$

If $[A] = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

$$|A| = a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$|A| = a_{11}(a_{22}a_{33} - a_{32}a_{23}) - a_{12}(a_{21}a_{33} - a_{23}a_{31}) + a_{13}(a_{21}a_{32} - a_{22}a_{31})$$

- Find the determinant $|A_1|$, $|A_2|$, $|A_3|$

If$[A_1] = \begin{bmatrix} \boldsymbol{b_1} & a_{12} & a_{13} \\ \boldsymbol{b_2} & a_{22} & a_{23} \\ \boldsymbol{b_3} & a_{32} & a_{33} \end{bmatrix}$

$$|A_1| = b_1 \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12} \begin{vmatrix} b_2 & a_{23} \\ b_3 & a_{33} \end{vmatrix} + a_{13} \begin{vmatrix} b_2 & a_{22} \\ b_3 & a_{32} \end{vmatrix}$$

$$|A_1| = b_1(a_{22}a_{33} - a_{32}a_{23}) - a_{12}(a_{12}a_{33} - a_{32}a_{13}) + b_3(a_{12}a_{23} - a_{22}a_{13})$$

If $[A_2] = \begin{bmatrix} a_{11} & \boldsymbol{b_1} & a_{13} \\ a_{21} & \boldsymbol{b_2} & a_{23} \\ a_{31} & \boldsymbol{b_3} & a_{33} \end{bmatrix}$

$$|A_2| = a_{11} \begin{vmatrix} b_2 & a_{23} \\ b_3 & a_{33} \end{vmatrix} - b_1 \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13} \begin{vmatrix} a_{21} & b_1 \\ a_{31} & b_3 \end{vmatrix}$$

$$|A_2| = a_{11}(b_2a_{33} - b_3a_{23}) - b_1(a_{21}a_{33} - a_{23}a_{31}) + a_{13}(b_3a_{21} - b_1a_{31})$$

If $[A_3] = \begin{bmatrix} a_{11} & a_{12} & \boldsymbol{b_1} \\ a_{21} & a_{22} & \boldsymbol{b_2} \\ a_{31} & a_{32} & \boldsymbol{b_3} \end{bmatrix}$

$$|A_3| = a_{11}\begin{vmatrix} a_{22} & b_2 \\ a_{32} & b_3 \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & b_2 \\ a_{31} & b_3 \end{vmatrix} + b_1\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$|A_3| = a_{11}(a_{22}b_3 - a_{32}b_2) - a_{12}(a_{21}b_3 - a_{31}b_1) + b_1(a_{21}a_{32} - a_{22}a_{31})$$

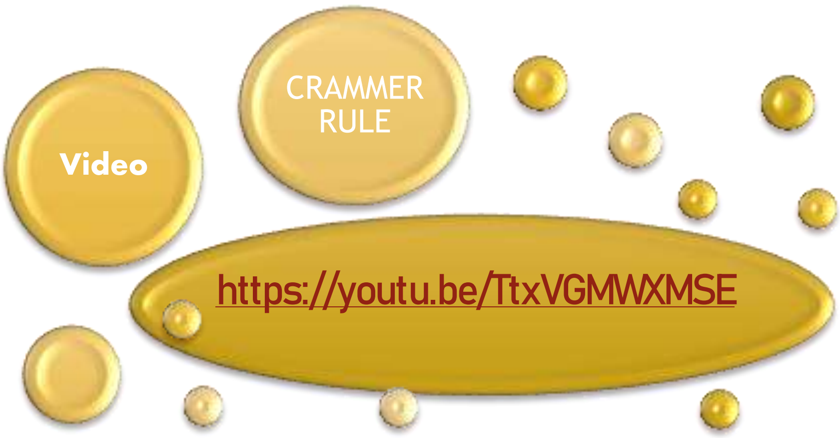
- Find the value $x = \frac{|A_1|}{|A|}$, $y = \frac{|A_2|}{|A|}$, $z = \frac{|A_3|}{|A|}$

$$x = \frac{\begin{bmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{bmatrix}}{\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}}, \quad y = \frac{\begin{bmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{bmatrix}}{\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}}, z = \frac{\begin{bmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{bmatrix}}{\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}}$$

Step by Step using the Cramer Rule

- step 1: Write linear equation to matrix form AX=B
- step 2: Find the determinant $|A|$
- step 3: Find the determinant $|A_1|$, $|A_2|$, $|A_3|$
- step 4: Find the value $x = \frac{|A_1|}{|A|}$, $y = \frac{|A_2|}{|A|}$, $z = \frac{|A_3|}{|A|}$

Remember that Cramer's Rule is most practical for small systems of equations due to its computational complexity and the potential for numerical instability with larger systems. For larger systems, numerical methods like Gaussian elimination are generally more efficient and stable.



3.2 Example

1

Solve the equations below by using Cramer's Rule.

$$\mathbf{4x + 6y + 8z = 1}$$
$$\mathbf{5x + 5y + 2z = 3}$$
$$\mathbf{x + 3y - 4z = 8}$$

Solution:

$$A = \begin{pmatrix} 4 & 6 & 8 \\ 5 & 5 & 2 \\ 1 & 3 & -4 \end{pmatrix} \quad X = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad B = \begin{pmatrix} 1 \\ 3 \\ 8 \end{pmatrix}$$

$$\text{Det } |A| = 4\begin{vmatrix} 5 & 2 \\ 3 & -4 \end{vmatrix} - 6\begin{vmatrix} 5 & 2 \\ 1 & -4 \end{vmatrix} + 8\begin{vmatrix} 5 & 5 \\ 1 & 3 \end{vmatrix}$$

$$|A| = 4(-20 - 6) - 6(-20 - 2) + 8(15 - 5)$$

$$|A| = 108$$

$$\text{Det } |A_1| = \begin{bmatrix} \mathbf{1} & 6 & 8 \\ \mathbf{3} & 5 & 2 \\ \mathbf{8} & 3 & -4 \end{bmatrix}$$

$$\text{Det } |A_1| = 1\begin{vmatrix} 5 & 2 \\ 3 & -4 \end{vmatrix} - 6\begin{vmatrix} 3 & 2 \\ 8 & -4 \end{vmatrix} + 8\begin{vmatrix} 3 & 5 \\ 8 & 3 \end{vmatrix}$$

$$|A_1| = 1(-20 - 6) - 6(-12 - 16) + 8(9 - 40)$$

$$|A_1| = -106$$

$$\text{Det } |A_2| = \begin{bmatrix} 4 & \mathbf{1} & 8 \\ 5 & \mathbf{3} & 2 \\ 1 & \mathbf{8} & -4 \end{bmatrix}$$

$$\text{Det } |A_2| = 4\begin{vmatrix} 3 & 2 \\ 8 & -4 \end{vmatrix} - 1\begin{vmatrix} 5 & 2 \\ 1 & -4 \end{vmatrix} + 8\begin{vmatrix} 5 & 3 \\ 1 & 8 \end{vmatrix}$$

$$|A_2| = 4(-12 - 16) - 1(-20 - 2) + 8(40 - 3)$$

$$|A_2| = 206$$

$$\text{Det } |A_3| = \begin{bmatrix} 4 & 6 & \mathbf{1} \\ 5 & 5 & \mathbf{3} \\ 1 & 3 & \mathbf{8} \end{bmatrix}$$

$$\text{Det } |A_3| = 4\begin{vmatrix} 5 & 3 \\ 3 & 8 \end{vmatrix} - 6\begin{vmatrix} 5 & 3 \\ 1 & 8 \end{vmatrix} + 1\begin{vmatrix} 5 & 5 \\ 1 & 3 \end{vmatrix}$$

$$|A_3| = 4(40 - 9) - 6(40 - 3) + 1(15 - 5)$$

$$|A_3| = -88$$

$$x = \frac{|A_1|}{|A|} = \frac{-106}{108} = -1$$

$$y = \frac{|A_2|}{|A|} = \frac{206}{108} = 1.9$$

$$z = \frac{|A_3|}{|A|} = \frac{-88}{108} = -1$$

2

Solve the equations below by using Cramer's Rule.

$$\mathbf{7x - y + 6z = 8}$$
$$\mathbf{3x + 2y - z = 10}$$
$$\mathbf{3x + 2z = 5}$$

Solution:

$$A = \begin{pmatrix} 7 & -1 & 6 \\ 3 & 2 & -1 \\ 3 & 0 & 2 \end{pmatrix} \quad X = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad B = \begin{pmatrix} 8 \\ 10 \\ 5 \end{pmatrix}$$

Det $|A| = 7\begin{vmatrix} 2 & -1 \\ 0 & 2 \end{vmatrix} - (-1)\begin{vmatrix} 3 & -1 \\ 3 & 2 \end{vmatrix} + 6\begin{vmatrix} 3 & 2 \\ 3 & 0 \end{vmatrix}$

$|A| = 7(4-0) - (-1)(6-(-3)) + 6(0-6)$

$|A| = 1$

Det $|A_1| = \begin{bmatrix} \mathbf{8} & -1 & 6 \\ \mathbf{10} & 2 & -1 \\ \mathbf{5} & 0 & 2 \end{bmatrix}$

Det $|A_1| = 8\begin{vmatrix} 2 & -1 \\ 0 & 2 \end{vmatrix} - (-1)\begin{vmatrix} 10 & -1 \\ 5 & 2 \end{vmatrix} + 6\begin{vmatrix} 10 & 2 \\ 5 & 0 \end{vmatrix}$

$|A_1| = 8(4-0) - (-1)(20-(-5)) + 6(0-10)$

$|A_1| = -3$

Det $|A_2| = \begin{bmatrix} 7 & \mathbf{8} & 6 \\ 3 & \mathbf{10} & -1 \\ 3 & \mathbf{5} & 2 \end{bmatrix}$

Det $|A_2| = 7\begin{vmatrix} 10 & -1 \\ 5 & 2 \end{vmatrix} - 8\begin{vmatrix} 3 & -1 \\ 3 & 2 \end{vmatrix} + 6\begin{vmatrix} 3 & 10 \\ 3 & 5 \end{vmatrix}$

$|A_2| = 7(20-(-5)) - 8(6-(-3)) + 6(15-30)$

$|A_2| = 13$

Det $|A_3| = \begin{bmatrix} 7 & -1 & \mathbf{8} \\ 3 & 2 & \mathbf{10} \\ 3 & 0 & \mathbf{5} \end{bmatrix}$

Det $|A_3| = 7\begin{vmatrix} 2 & 10 \\ 0 & 5 \end{vmatrix} - (-1)\begin{vmatrix} 3 & 10 \\ 3 & 5 \end{vmatrix} + 8\begin{vmatrix} 3 & 2 \\ 3 & 0 \end{vmatrix}$

$|A_3| = 7(10-0) - (-1)(15-30) + 8(0-6)$

$|A_3| = 7$

$$x = \frac{|A_1|}{|A|} = \frac{-3}{1} = -3$$

$$y = \frac{|A_2|}{|A|} = \frac{13}{1} = 13$$

$$z = \frac{|A_3|}{|A|} = \frac{7}{1} = 7$$

3

Solve the equations below by using Cramer's Rule.

$$3x + 6z = 4$$
$$7x - 4y - z = 7$$
$$2x + 8y + z = 12$$

Solution:

$$A = \begin{pmatrix} 3 & 0 & 6 \\ 7 & -4 & -1 \\ 2 & 8 & 2 \end{pmatrix} \quad X = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad B = \begin{pmatrix} 4 \\ 7 \\ 12 \end{pmatrix}$$

Det $|A| = 3\begin{vmatrix} -4 & -1 \\ 8 & 2 \end{vmatrix} - 0\begin{vmatrix} 7 & -1 \\ 2 & 2 \end{vmatrix} + 6\begin{vmatrix} 7 & -4 \\ 2 & 8 \end{vmatrix}$

$|A| = 3(-8 - (-8)) - 0(14 - (-2) + 6(56 - (-8))$

$|A| = 384$

Det $|A_1| = \begin{bmatrix} \mathbf{4} & 0 & 6 \\ \mathbf{7} & -4 & -1 \\ \mathbf{12} & 8 & 2 \end{bmatrix}$

Det $|A_1| = 4\begin{vmatrix} -4 & -1 \\ 8 & 2 \end{vmatrix} - 0\begin{vmatrix} 7 & -1 \\ 12 & 2 \end{vmatrix} + 6\begin{vmatrix} 7 & -4 \\ 12 & 8 \end{vmatrix}$

$|A_1| = 4(-8 - (-8)) - 0(14 - (-12)) + 6(56 - (-48))$

$|A_1| = 624$

Det $|A_2| = \begin{bmatrix} 3 & \mathbf{4} & 6 \\ 7 & \mathbf{7} & -1 \\ 2 & \mathbf{12} & 2 \end{bmatrix}$

Det $|A_2| = 3\begin{vmatrix} 7 & -1 \\ 12 & 2 \end{vmatrix} - 4\begin{vmatrix} 7 & -1 \\ 2 & 2 \end{vmatrix} + 6\begin{vmatrix} 7 & 7 \\ 2 & 12 \end{vmatrix}$

$|A_2| = 3(14 - (-12)) - 4(14 - (-2)) + 6(84 - 14)$

$|A_2| = 434$

Det $|A_3| = \begin{bmatrix} 3 & 0 & \mathbf{4} \\ 7 & -4 & \mathbf{7} \\ 2 & 8 & \mathbf{12} \end{bmatrix}$

Det $|A_3| = 3\begin{vmatrix} -4 & 7 \\ 8 & 12 \end{vmatrix} - 0\begin{vmatrix} 7 & 7 \\ 2 & 12 \end{vmatrix} + 4\begin{vmatrix} 7 & -4 \\ 2 & 8 \end{vmatrix}$

$|A_3| = 3(-48 - 56) - 0(84 - 14) + 4(56 - (-8))$

$|A_3| = -56$

$$x = \frac{|A_1|}{|A|} = \frac{624}{384} = 1.6$$

$$y = \frac{|A_2|}{|A|} = \frac{434}{384} = 1.1$$

$$z = \frac{|A_3|}{|A|} = \frac{-56}{384} = -0.15$$

4

Solve the equations below by using Cramer's Rule.

$$8x + 7y + 6z = 15$$
$$-3x + 9y + 5z = 7$$
$$3x - 4y + 2z = 9$$

Solution:

$$A = \begin{pmatrix} 8 & 7 & 6 \\ -3 & 9 & 5 \\ 3 & -4 & 2 \end{pmatrix} \quad X = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad B = \begin{pmatrix} 15 \\ 7 \\ 9 \end{pmatrix}$$

$$\text{Det } |A| = 8\begin{vmatrix} 9 & 5 \\ -4 & 2 \end{vmatrix} - 7\begin{vmatrix} -3 & 5 \\ 3 & 2 \end{vmatrix} + 6\begin{vmatrix} -3 & 9 \\ 3 & -4 \end{vmatrix}$$

$$|A| = 8(18 - (-20)) - 7(-6 - 15) + 6(12 - 27)$$

$$|A| = 361$$

$$\text{Det } |A_1| = \begin{bmatrix} \mathbf{15} & 7 & 6 \\ \mathbf{7} & 9 & 5 \\ \mathbf{9} & -4 & 2 \end{bmatrix}$$

$$\text{Det } |A_1| = 15\begin{vmatrix} 9 & 5 \\ -4 & 2 \end{vmatrix} - 7\begin{vmatrix} 7 & 5 \\ 9 & 2 \end{vmatrix} + 6\begin{vmatrix} 7 & 9 \\ 9 & -4 \end{vmatrix}$$

$$|A_1| = 15(18 - (-20)) - 7(14 - 45) + 6(-28 - 81)$$

$$|A_1| = 133$$

$$\text{Det } |A_2| = \begin{bmatrix} 8 & \mathbf{15} & 6 \\ -3 & \mathbf{7} & 5 \\ 3 & \mathbf{9} & 2 \end{bmatrix}$$

$$\text{Det } |A_2| = 8\begin{vmatrix} 7 & 5 \\ 9 & 2 \end{vmatrix} - 15\begin{vmatrix} -3 & 5 \\ 3 & 2 \end{vmatrix} + 6\begin{vmatrix} -3 & 7 \\ 3 & 9 \end{vmatrix}$$

$$|A_2| = 8(14 - 45) - 15(-6 - 15) + 6(-27 - 21)$$

$$|A_2| = -221$$

$$\text{Det } |A_3| = \begin{bmatrix} 8 & 7 & \mathbf{15} \\ -3 & 9 & \mathbf{7} \\ 3 & -4 & \mathbf{9} \end{bmatrix}$$

$$\text{Det } |A_3| = 8\begin{vmatrix} 9 & 7 \\ -4 & 9 \end{vmatrix} - 7\begin{vmatrix} -3 & 7 \\ 3 & 9 \end{vmatrix} + 15\begin{vmatrix} -3 & 9 \\ 3 & -4 \end{vmatrix}$$

$$|A_3| = 8(81 - -28) - 7(-27 - 21) + 15(12 - 27)$$

$$|A_3| = 983$$

$$x = \frac{|A_1|}{|A|} = \frac{133}{361} = 0.4$$

$$y = \frac{|A_2|}{|A|} = \frac{-221}{361} = 0.61$$

$$z = \frac{|A_3|}{|A|} = \frac{983}{361} = 2.7$$

3.3 Exercises

a) Identify AX = B from the equations below:

i. $-2x + y + 9z = 12$

$-x - 4x + z = 1$

$4x - z = 3$

ii. $2x + 4y + z = 7$

$3x + y = -1$

$y + 3x - 5z = 9$

b) Solve the equations below by using Cramer's Rule.

$2x - 4y - 3z = -23$

$3x + 5y + z = 12$

$-2x + 4y + 7z = 83$ **ANS: x = - 4, y = 3, z = 9**

c) Solve the equations below by using Cramer's Rule.

$x + 2y + 2z = 34$

$-3x - 4y + z = -25$

$4x + 7y + 3z = 84$ **ANS: x = 2, y = 7, z = 9**

d) Solve the equations below by using Cramer's Rule.

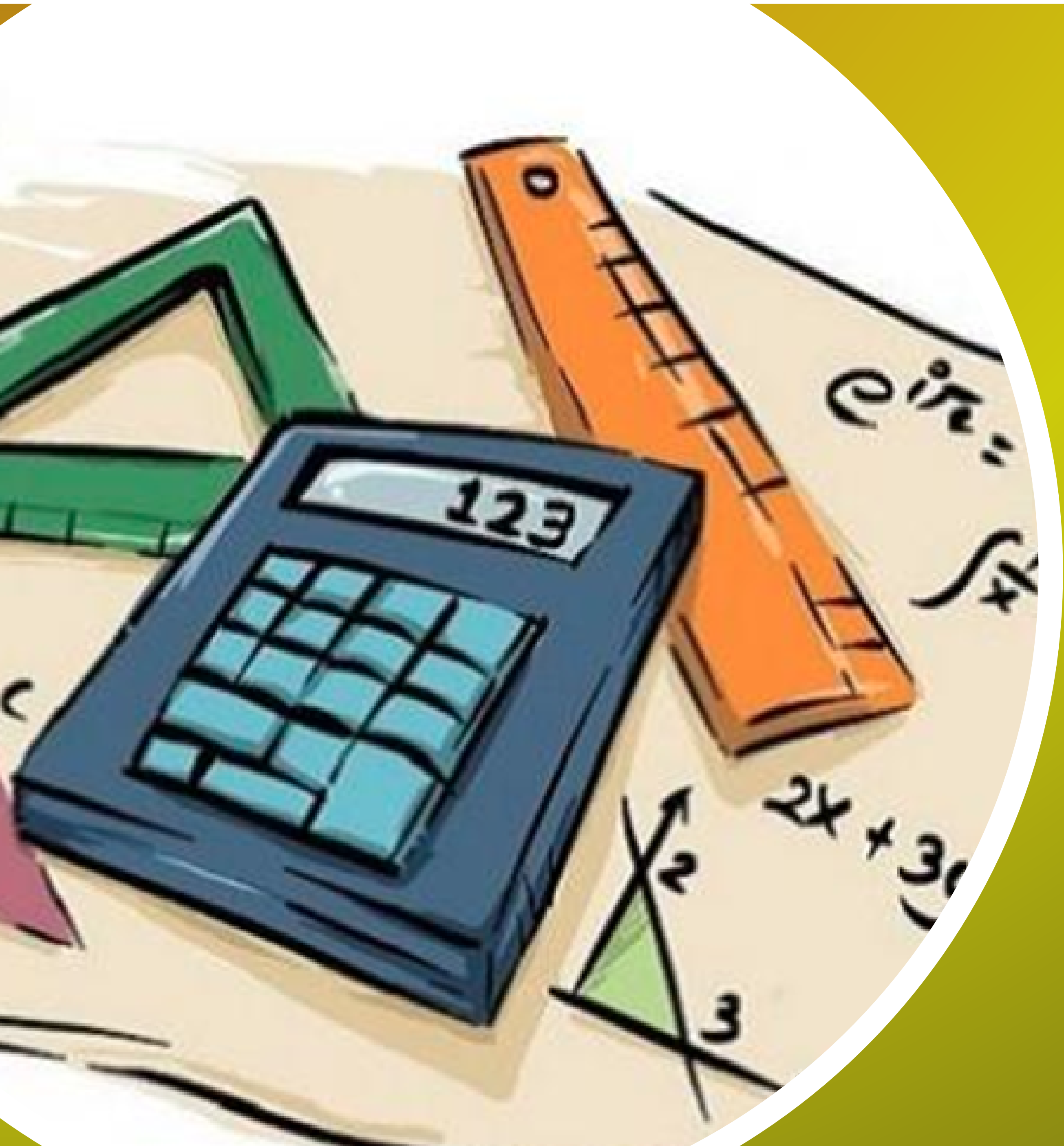
$2x + 4y - 3z = -7$

$3x + 5y + z = -2$

$-2x + 4y + 7z = -13$ **ANS: x = 4, y = - 3, z = 1**

STEP BY STEP

CHAPTER 4: METHOD OF LINEAR EQUATION: GAUSSIAN ELIMINATION



4 METHOD OF LINEAR EQUATION: GAUSSIAN ELIMINATION

Gaussian Elimination is a fundamental method in linear algebra used to solve systems of linear equations and manipulate matrices. The process involves transforming a matrix into row-echelon form through a series of elementary row operations, including adding or subtracting rows and multiplying rows by constants. This method simplifies the system of equations by eliminating variables one by one, ultimately yielding a triangular matrix that can be easily solved using back-substitution. Gaussian Elimination is a crucial tool for solving various mathematical and engineering problems involving linear equations and matrices.

HISTORY

Gaussian Elimination's origins trace back to ancient Chinese and Babylonian civilizations, where manual methods for solving linear equations were developed. However, the formalization and systematization of the method are attributed to the ancient Greek mathematician Euclid. Over the centuries, various mathematicians contributed to the development of the technique, but it was the German mathematician Carl Friedrich Gauss who significantly advanced the method in the late 18th century, leading to its recognition as Gaussian Elimination. Subsequent mathematicians, including Jordan and Gauss-Jordan, refined the method's understanding and application, solidifying its status as a cornerstone of linear algebra and matrix operations.

4.1 Solving Gaussian Elimination

- Transforming the system matrix form AX=B

$$a_{11}x + a_{12}y + a_{13}z = b_1$$
$$a_{21}x + a_{22}y + a_{23}z = b_2$$
$$a_{31}x + a_{32}y + a_{33}z = b_3$$

$$\longrightarrow \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

- First stage transformation
  Eliminate *x* in 2nd and 3rd equation using 1st equation as the pivot equation.

$$a'_{21} = a_{21} - a_{11}\left(\frac{a_{21}}{a_{11}}\right) = 0$$

$$a'_{22} = a_{22} - a_{12}\left(\frac{a_{21}}{a_{11}}\right)$$

$$a'_{23} = a_{23} - a_{13}\left(\frac{a_{21}}{a_{11}}\right)$$

$$b'_{2} = b_{2} - b_{1}\left(\frac{a_{21}}{a_{11}}\right)$$

$$a'_{31} = a_{31} - a_{11}\left(\frac{a_{31}}{a_{11}}\right) = 0$$

$$a'_{32} = a_{32} - a_{12}\left(\frac{a_{31}}{a_{11}}\right)$$

$$a'_{33} = a_{33} - a_{13}\left(\frac{a_{31}}{a_{11}}\right)$$

$$b'_{3} = b_{3} - b_{1}\left(\frac{a_{31}}{a_{11}}\right)$$

After the First Stage Transformation, we will get as below:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a'_{22} & a'_{23} \\ 0 & a'_{32} & a'_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} b_1 \\ b'_2 \\ b'_3 \end{bmatrix}$$

- Second stage transformation
  Eliminate y in the 3rd equation using equation 2 as the pivot equation.

$$a"_{32} = a'_{32} - a'_{22}\left[\frac{a'_{32}}{a'_{22}}\right] = 0$$

$$a"_{33} = a'_{33} - a'_{23}\left[\frac{a'_{32}}{a'_{22}}\right]$$

$$b"_{3} = b'_{3} - b'_{2}\left[\frac{a'_{32}}{a'_{22}}\right]$$

At the end of the transformation steps,

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a'_{22} & a'_{23} \\ 0 & 0 & a''_{33} \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} b_1 \\ b'_2 \\ b''_3 \end{bmatrix}$$

- Use the back-substitution method to solve the simultaneous linear equations.

$$z = \frac{b"_3}{a"_{33}}$$
$$y = \frac{b'_2 - a'_{23}\, z}{a'_{22}}$$
$$x = \frac{b_1 - a_{12}\, y - a_{13}\, z}{a_{11}}$$

Step by Step using the Gaussian Elimination method:

- step 1: Write linear equation to matrix form AX=B
- step 2: First stage transformation
- step 3: Second stage transformation
- step 4: Use the back-substitution method

Gaussian Elimination is a method used to solve systems of linear equations and find the reduced row echelon form (RREF) of a matrix.

4.2 Example

1

Solve the equations below by using Gauss Method.

$$\mathbf{7x - y + 6z = 8}$$
$$\mathbf{3x + 2y - z = 10}$$
$$\mathbf{3x + 2z = 5}$$

$$A = \begin{pmatrix} 7 & -1 & 6 \\ 3 & 2 & -1 \\ 3 & 0 & 2 \end{pmatrix} \quad X = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad B = \begin{pmatrix} 8 \\ 10 \\ 5 \end{pmatrix}$$

First stage transformation

$a'_{21} = 3 - 7\left(\frac{3}{7}\right) = 0$	$a'_{31} = 3 - 7\left(\frac{3}{7}\right) = 0$
$a'_{22} = 2 - (-1)\left(\frac{3}{7}\right) = 2.43$	$a'_{32} = 0 - (-1)\left(\frac{3}{7}\right) = 0.43$
$a'_{23} = -1 - 6\left(\frac{3}{7}\right) = -3.57$	$a'_{33} = 2 - 6\left(\frac{3}{7}\right) = -0.57$
$b'_{2} = 10 - 8\left(\frac{3}{7}\right) = 6.57$	$b'_{3} = 5 - 8\left(\frac{3}{7}\right) = 1.5714$

After the First Stage Transformation, we will get as below:

$$\begin{bmatrix} 7 & -1 & 6 \\ 0 & 2.43 & -3.57 \\ 0 & 0.43 & -0.57 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 6.57 \\ 1.57 \end{bmatrix}$$

- Second stage transformation

$$a"_{32} = 0.43 - 2\left[\frac{0.4}{2}\right] = 0$$
$$a"_{33} = -0.57 - (-4)\left[\frac{0.4}{2}\right] = 0.06$$
$$b"_{3} = 1.57 - 7\left[\frac{0.4}{2}\right] = 0.41$$

At the end of the transformation steps,

$$\begin{bmatrix} 7 & -1 & 6 \\ 0 & 2.43 & -3.57 \\ 0 & 0 & 0.06 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 6.57 \\ 0.41 \end{bmatrix}$$

Solve

$0.06\,z = 0.4$ $z = 7$	$2.4\,y + (-3.57)z = 6.6$ $y = 13$	$7x + (-1)\,y + 6z = 8$ $x = -3$

2

Solve the equations below by using Gauss Method.

$$\mathbf{3x + 6z = 4}$$
$$\mathbf{7x - 4y - z = 7}$$
$$\mathbf{2x + 8y + 2z = 12}$$

$$A = \begin{pmatrix} 3 & 0 & 6 \\ 7 & -4 & -1 \\ 2 & 8 & 2 \end{pmatrix} \quad X = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad B = \begin{pmatrix} 4 \\ 7 \\ 12 \end{pmatrix}$$

First stage transformation

$a'_{21} = 7 - 3\left(\frac{7}{3}\right) = 0$ $a'_{22} = -4 - 0\left(\frac{7}{3}\right) = -4$ $a'_{23} = -1 - 6\left(\frac{7}{3}\right) = -15$ $b'_{2} = 7 - 4\left(\frac{7}{3}\right) = -2.33$	$a'_{31} = 2 - 3\left(\frac{2}{3}\right) = 0$ $a'_{32} = 8 - 0\left(\frac{2}{3}\right) = 8$ $a'_{33} = 2 - 6\left(\frac{2}{3}\right) = -2$ $b'_{3} = 12 - 4\left(\frac{2}{3}\right) = 9.33$

After the First Stage Transformation, we will get as below:

$$\begin{bmatrix} 3 & 0 & 6 \\ 0 & -4 & -15 \\ 0 & 8 & -32 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -2.3 \\ 9.33 \end{bmatrix}$$

- Second stage transformation

$$a"_{32} = 8 - (-4)\left[\frac{8}{-4}\right] = 0$$
$$a"_{33} = -2 - (-15)\left[\frac{8}{-4}\right] = -32$$
$$b"_{3} = 1.57 - 7\left[\frac{8}{-4}\right] = 0.467$$

At the end of the transformation steps,

$$\begin{bmatrix} 3 & 0 & 6 \\ 0 & -4 & -15 \\ 0 & 0 & -32 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -2.3 \\ 4.67 \end{bmatrix}$$

Solve

$-32\,z = 4.7$ $z = -0.1$	$-4\,y + (-15)z = -2.3$ $y = 1.1$	$3x + 0\,y + 6z = 4$ $x = 1.6$

3

Solve the equations below by using Gauss Method.

$$\mathbf{8x + 7y + 6z = 15}$$
$$\mathbf{-3x - 9y + 5z = 7}$$
$$\mathbf{3x - 4y + 2z = 9}$$

$$A = \begin{pmatrix} 8 & 7 & 6 \\ -3 & 9 & 5 \\ 3 & -4 & 2 \end{pmatrix} \quad X = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad B = \begin{pmatrix} 15 \\ 7 \\ 9 \end{pmatrix}$$

First stage transformation

$a'_{21} = -3 - 8\left(\frac{-3}{8}\right) = 0$	$a'_{31} = 3 - 8\left(\frac{3}{8}\right) = 0$
$a'_{22} = 9 - 7\left(\frac{-3}{8}\right) = 11.63$	$a'_{32} = -4 - 7\left(\frac{3}{8}\right) = -6.63$
$a'_{23} = 5 - 6\left(\frac{-3}{8}\right) = 7.25$	$a'_{33} = 2 - 6\left(\frac{3}{8}\right) = -0.25$
$b'_2 = 7 - 15\left(\frac{-3}{8}\right) = 12.63$	$b'_3 = 9 - 15\left(\frac{3}{8}\right) = 3.38$

After the First Stage Transformation, we will get as below:

$$\begin{bmatrix} 8 & 7 & 6 \\ 0 & 11.6 & 7.25 \\ 0 & -6.63 & -0.25 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 15 \\ 12.6 \\ 3.38 \end{bmatrix}$$

- Second stage transformation

$$a"_{32} = -6.63 - 12\left[\frac{-6.6}{12}\right] = 0$$
$$a"_{33} = -0.25 - 7\left[\frac{-6.6}{12}\right] = 3.88$$
$$b"_3 = 3.38 - 13\left[\frac{-6.6}{12}\right] = 10.57$$

- At the end of the transformation steps,

$$\begin{bmatrix} 8 & 7 & 6 \\ 0 & 11.6 & 7.25 \\ 0 & 0 & 3.88 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 15 \\ 12.6 \\ 10.6 \end{bmatrix}$$

Solve

$3.88\,z = 11$ $z = 2.7$	$12\,y + 7.25z = 13$ $y = -0.6$	$8x + 7\,y + 6z = 15$ $x = 0.4$

4 **Solve the equations below by using Gauss Method.**

$$\begin{aligned} 4x + 6y + 8z &= 1 \\ 5x + 5y + 2z &= 3 \\ x + 3y - 4z &= 8 \end{aligned}$$

$$A = \begin{pmatrix} 4 & 6 & 8 \\ 5 & 5 & 2 \\ 1 & 3 & -4 \end{pmatrix} \quad X = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad B = \begin{pmatrix} 1 \\ 3 \\ 8 \end{pmatrix}$$

First stage transformation

$a'_{21} = 5 - 4\left(\frac{5}{4}\right) = 0$	$a'_{31} = 1 - 4\left(\frac{1}{4}\right) = 0$
$a'_{22} = 5 - 6\left(\frac{5}{4}\right) = -2.5$	$a'_{32} = 3 - 6\left(\frac{1}{4}\right) = 1.5$
$a'_{23} = 2 - 8\left(\frac{5}{4}\right) = -8$	$a'_{33} = -4 - 8\left(\frac{1}{4}\right) = -6$
$b'_{2} = 3 - 1\left(\frac{5}{4}\right) = 1.75$	$b'_{3} = 8 - 1\left(\frac{1}{4}\right) = 7.75$

After the First Stage Transformation, we will get as below:

$$\begin{bmatrix} 4 & 6 & 8 \\ 0 & -6.63 & -6.63 \\ 0 & -6.63 & -0.25 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1.75 \\ 7.75 \end{bmatrix}$$

- Second stage transformation

$$a''_{32} = 1.5 - (-3)\left[\frac{1.5}{-3}\right] = 0$$

$$a''_{33} = -6 - (-8)\left[\frac{1.5}{-3}\right] = -10.8$$

$$b''_{3} = 7.75 - 2\left[\frac{1.5}{-3}\right] = 8.8$$

- At the end of the transformation steps,

$$\begin{bmatrix} 4 & 6 & 8 \\ 0 & -2.5 & -8 \\ 0 & 0 & -10.8 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1.75 \\ 8.8 \end{bmatrix}$$

Solve

$-10.8\,z = 8.8$ $z = -0.8$	$-2.5\,y - 8z = 1.8$ $y = 1.9$	$4x + 6\,y + 8z = 1$ $x = -1$

4.3 Exercises

a) Identify AX = B from the equations below:

i. $x + y + 9z = 3$

$-3x - 6x + z = 12$

$4x - z + 7y = 3$

ii. $2x + 4y + z = 7$

$2z + 3x + y = 5$

$3y - 3x + 5z = 9$

b) Solve the equations below by using Gauss Method.

$2x - 4y - 3z = -23$

$3x + 5y + z = 12$

$-2x + 4y + 7z = 83$ **ANS: x = - 4, y = 3, z = 9**

c) Solve the equations below by using Gauss Method.

$x + 2y + 2z = 34$

$-3x - 4y + z = -25$

$4x + 7y + 3z = 84$ **ANS: x = 2, y = 7, z = 9**

d) Solve the equations below by using Gauss Method.

$2x + 4y - 3z = -7$

$3x + 5y + z = -2$

$-2x + 4y + 7z = -13$ **ANS: x = 4, y = - 3, z = 1**

STEP BY STEP

CHAPTER 5: METHOD OF LINEAR EQUATION: DOOLITTLE

5 METHOD OF LINEAR EQUATION: DOOLITTLE

The Doolittle method, also known as Doolittle's algorithm, is a numerical technique used in linear algebra for performing LU decomposition of a square matrix. LU decomposition factors a matrix into the product of a lower triangular matrix (L) and an upper triangular matrix (U), allowing for efficient solutions to systems of linear equations and various matrix operations. The Doolittle method involves transforming the original matrix into its LU form by systematically eliminating entries in the lower and upper triangles through a series of row operations. This method is particularly useful in numerical simulations, scientific computing, and engineering applications where solving linear systems is a common task.

HISTORY

The Doolittle method, named after the American mathematician and physicist Wilbur D. Doolittle, emerged as an essential technique in numerical linear algebra during the mid-20th century. Doolittle, along with other mathematicians of his time, sought efficient ways to decompose matrices and solve systems of linear equations. His method, introduced in the context of LU decomposition, aimed to simplify complex mathematical operations by breaking down matrices into lower and upper triangular components. This advancement played a pivotal role in various scientific and engineering fields, enabling accurate simulations and computations that rely on solving intricate systems of linear equations. The Doolittle method's historical significance lies in its foundational contribution to numerical techniques that underpin many modern computational applications.

5.1 Solving Doolittle

- Transforming the system matrix form AX=B

$$a_{11}x + a_{12}y + a_{13}z = b_1$$
$$a_{21}x + a_{22}y + a_{23}z = b_2$$
$$a_{31}x + a_{32}y + a_{33}z = b_3$$

$$\longrightarrow \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

- Find L and U, A=LU

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

- Multiplying L and U.

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ l_{21}u_{11} & l_{21}u_{12} + u_{22} & l_{21}u_{13} + u_{23} \\ l_{31}u_{11} & l_{31}u_{12} + l_{32}u_{22} & l_{31}u_{13} + l_{32}u_{23} + u_{33} \end{bmatrix}$$

- Find the value of L and U.

$u_{11} = a_{11}$	$u_{12} = a_{12}$	$u_{13} = a_{13}$
$l_{21}u_{11} = a_{21}$	$l_{21}u_{12} + u_{22} = a_{22}$	$l_{21}u_{13} + u_{23} = a_{23}$
$l_{31}u_{11} = a_{31}$	$l_{31}u_{12} + l_{32}u_{22} = a_{32}$	$l_{31}u_{13} + l_{32}u_{23} + u_{33} = a_{33}$

- Find the matrix of y by using $Ly = B$

$$\begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$$y_1 = b_1$$
$$y_2 = b_2 - l_{21}y_1$$
$$y_3 = b_3 - l_{31}y_1 - l_{32}y_2$$

- Find the value of x from $Ux = y$

$$\begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$$\begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 - l_{21}y_1 \\ b_3 - l_{31}y_1 - l_{32}y_2 \end{bmatrix}$$

$$z = \frac{b_3 - l_{31}y_1 - l_{32}y_2}{u_{33}}$$

$$y = \frac{b_2 - l_{21}y_1 - u_{23}z}{u_{22}}$$

$$x = \frac{b_1 - u_{12}y - u_{13}z}{u_{11}}$$

Step by Step using the Doolittle method:

- step 1: Write linear equation to matrix form AX=B
- step 2: Find L and U, A=LU
- step 3: Multiplying L and U.
- step 4: Find the value of L and U.
- step 5: Find the matrix of y by using Ly=B
- step 6: Find the value of x from Ux=y

The Doolittle's method is an iterative technique used to solve systems of linear equations, particularly for solving linear systems where the coefficient matrix is lower triangular with ones on the main diagonal. Remember that Doolittle's method assumes that the coefficient matrix A can be decomposed into an L matrix (lower triangular) and a U matrix (upper triangular) without any pivoting.

5.2 Example

1

Solve the equations below by using Doolittle Method.

$$4x + 6y + 8z = 1$$
$$5x + 5y + 2z = 3$$
$$x + 3y - 4z = 8$$

$$A = \begin{pmatrix} 4 & 6 & 8 \\ 5 & 5 & 2 \\ 1 & 3 & -4 \end{pmatrix} \quad X = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad B = \begin{pmatrix} 1 \\ 3 \\ 8 \end{pmatrix}$$

- Find L and U, A=LU

$$\begin{bmatrix} 4 & 6 & 8 \\ 5 & 5 & 2 \\ 1 & 3 & -4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

- Multiplying L and U.

$$\begin{bmatrix} 4 & 6 & 8 \\ 5 & 5 & 2 \\ 1 & 3 & -4 \end{bmatrix} = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ l_{21}u_{11} & l_{21}u_{12} + u_{22} & l_{21}u_{13} + u_{23} \\ l_{31}u_{11} & l_{31}u_{12} + l_{32}u_{22} & l_{31}u_{13} + l_{32}u_{23} + u_{33} \end{bmatrix}$$

- Find the value of L and U.

$u_{11} = 4$	$u_{12} = 6$	$u_{13} = 8$
$l_{21}u_{11} = 5$ $l_{21}(4) = 5$ $l_{21} = \frac{5}{4}$ $l_{21} = 1.3$	$l_{21}u_{12} + u_{22} = 5$ $(\frac{1}{4})(6) + u_{22} = 5$ $u_{22} = 5 - (\frac{1}{4})(6)$ $u_{22} = -2.5$	$l_{21}u_{13} + u_{23} = 2$ $(\frac{5}{4})(8) + u_{23} = 2$ $u_{23} = 2 - (\frac{5}{4})(8)$ $u_{23} = -8$
$l_{31}u_{11} = 1$ $l_{31}(4) = 1$ $l_{31} = \frac{1}{4}$ $l_{31} = 0.25$	$l_{31}u_{12} + l_{32}u_{22} = 3$ $(0.25)(6) + l_{32}(-2.5) = 3$ $l_{32}(-2.5) = 3 - (0.25)(6)$ $l_{32} = \frac{3 - 1.5}{-2.5}$ $l_{32} = -0.6$	$l_{31}u_{13} + l_{32}u_{23} + u_{33} = -4$ $(0.25)(8) + (-0.6)(-8) + u_{33} = -4$ $u_{33} = -4 - \ 2 - 4.8$ $u_{33} = -10.8$

- Find the matrix of y by using Ly=B

$$\begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 8 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1.3 & 1 & 0 \\ 0.25 & -0.6 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 8 \end{bmatrix}$$

$y_1 = b_1$ $y_1 = 1$	$y_2 = b_2 - l_{21}y_1$ $y_2 = 3 - (1.3)(1)$ $y_2 = 1.7$	$y_3 = b_3 - l_{31}y_1 - l_{32}y_2$ $y_3 = 8 - 0.25(1)$ $- 0.6(1.7)$ $y_3 = 8.77$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1.7 \\ 8.77 \end{bmatrix}$$

- Find the value of x from Ux=y

$$\begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 6 & 8 \\ 0 & -2.5 & -8 \\ 0 & 0 & -10.8 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1.7 \\ 8.77 \end{bmatrix}$$

$$\begin{bmatrix} 4x + 6y + 8z \\ -2.5y - 8z \\ -10.8z \end{bmatrix} = \begin{bmatrix} 1 \\ 1.7 \\ 8.77 \end{bmatrix}$$

$-10.8z$ $= 8.77$ $z = \frac{8.77}{-10.8}$ $z = -0.81$	$-2.5y - 8z = 1.7$ $-2.5y - 8(-0.81) = 1.7$ $-2.5y = 1.7 - 6.48$ $y = \frac{-4.78}{-2.5}$ $y = 1.912$	$4x + 6y + 8z$=1 $4x + 6(1.912) + 8(-0.81)$=1 $4x$=$1 - 11.47 + 6.48$ $4x$=-3.992 x=$\frac{-3.992}{4}$ x=-1

2

Solve the equations below by using Doolittle Method.

$$\mathbf{7x - y + 6z = 8}$$
$$\mathbf{3x + 2y - z = 10}$$
$$\mathbf{3x + 2z = 5}$$

$$A = \begin{pmatrix} 7 & -1 & 6 \\ 3 & 2 & -1 \\ 3 & 0 & 2 \end{pmatrix} \quad X = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad B = \begin{pmatrix} 8 \\ 10 \\ 5 \end{pmatrix}$$

- Find L and U, A=LU

$$\begin{bmatrix} 7 & -1 & 6 \\ 3 & 2 & -1 \\ 3 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

- Multiplying L and U.

$$\begin{bmatrix} 7 & -1 & 6 \\ 3 & 2 & -1 \\ 3 & 0 & 2 \end{bmatrix} = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ l_{21}u_{11} & l_{21}u_{12} + u_{22} & l_{21}u_{13} + u_{23} \\ l_{31}u_{11} & l_{31}u_{12} + l_{32}u_{22} & l_{31}u_{13} + l_{32}u_{23} + u_{33} \end{bmatrix}$$

- Find the value of L and U.

$u_{11} = 7$	$u_{12} = -1$	$u_{13} = 6$
$l_{21}u_{11} = 3$ $l_{21}(7) = 3$ $l_{21} = \frac{3}{7}$ $l_{21} = 0.43$	$l_{21}u_{12} + u_{22} = 2$ $(0.43)(-1) + u_{22} = 2$ $u_{22} = 2 + (0.43)$ $u_{22} = 2.43$	$l_{21}u_{13} + u_{23} = -1$ $(0.43)(6) + u_{23} = -1$ $u_{23} = -1 - 2.58$ $u_{23} = -3.58$
$l_{31}u_{11} = 3$ $l_{31}(7) = 3$ $l_{31} = \frac{3}{7}$ $l_{31} = 0.43$	$l_{31}u_{12} + l_{32}u_{22} = 0$ $(0.43)(-1) + l_{32}(2.43) = 0$ $l_{32}(2.43) = 0 + (0.43)$ $l_{32} = \frac{0.43}{2.43}$ $l_{32} = 0.177$	$l_{31}u_{13} + l_{32}u_{23} + u_{33} = 2$ $(0.43)(6) + (0.177)(-3.58)$ $+ u_{33} = 2$ $u_{33} = 2 - 1.946$ $u_{33} = 0.054$

- Find the matrix of y by using Ly=B

$$\begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 8 \\ 10 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0.43 & 1 & 0 \\ 0.43 & 0.177 & 1 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 8 \\ 10 \\ 5 \end{bmatrix}$$

$y_1 = b_1$ $y_1 = 8$	$y_2 = b_2 - l_{21}y_1$ $y_2 = 3 - (0.43)(8)$ $y_2 = 6.56$	$y_3 = b_3 - l_{31}y_1 - l_{32}y_2$ $y_3 = 5 - 0.43(8) - 0.177(6.56)$ $y_3 = 0.4$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 8 \\ 6.56 \\ 0.4 \end{bmatrix}$$

- Find the value of x from Ux=y

$$\begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$$\begin{bmatrix} 7 & -1 & 6 \\ 0 & 2.43 & -3.6 \\ 0 & 0 & 0.054 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 6.56 \\ 0.4 \end{bmatrix}$$

$$\begin{bmatrix} 7x - y + 6z \\ 2.43y - 3.6z \\ 0.054z \end{bmatrix} = \begin{bmatrix} 8 \\ 6.56 \\ 0.4 \end{bmatrix}$$

$0.054z = 0.4$ $z = \frac{0.4}{0.054}$ $z = 7.4$	$2.43y - 3.6z = 6.56$ $2.43y - 3.6(7.4) = 6.56$ $2.43y = 6.56 + 26.64$ $y = \frac{33.2}{2.43}$ $y = 13.66$ $7x - y + 6z$=8	$7x - 13.66 + 6(7.4)$ =8 $7x$=$8 + 13.66 - 44.4$ $7x$=-22.74 x=$\frac{-22.74}{7}$ x=-3.25

3

Solve the equations below by using Doolittle Method.

$$\mathbf{3x + 6z = 4}$$
$$\mathbf{7x - 4y - z = 7}$$
$$\mathbf{2x + 8y + 2z = 12}$$

$$A = \begin{pmatrix} 3 & 0 & 6 \\ 7 & -4 & -1 \\ 2 & 8 & 2 \end{pmatrix} \quad X = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad B = \begin{pmatrix} 4 \\ 7 \\ 12 \end{pmatrix}$$

- Find L and U, A=LU

$$\begin{bmatrix} 3 & 0 & 6 \\ 7 & -4 & -1 \\ 2 & 8 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

- Multiplying L and U.

$$\begin{bmatrix} 3 & 0 & 6 \\ 7 & -4 & -1 \\ 2 & 8 & 2 \end{bmatrix} = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ l_{21}u_{11} & l_{21}u_{12} + u_{22} & l_{21}u_{13} + u_{23} \\ l_{31}u_{11} & l_{31}u_{12} + l_{32}u_{22} & l_{31}u_{13} + l_{32}u_{23} + u_{33} \end{bmatrix}$$

- Find the value of L and U.

$u_{11} = 3$	$u_{12} = 0$	$u_{13} = 6$
$l_{21}u_{11} = 7$ $l_{21}(3) = 7$ $l_{21} = \frac{7}{3}$ $l_{21} = 2.3$	$l_{21}u_{12} + u_{22} = -4$ $(2.3)(0) + u_{22} = -4$ $u_{22} = -4$	$l_{21}u_{13} + u_{23} = -1$ $(2.3)(6) + u_{23} = -1$ $u_{23} = -1 - 13.8$ $u_{23} = -14.8$
$l_{31}u_{11} = 2$ $l_{31}(3) = 2$ $l_{31} = \frac{2}{3}$ $l_{31} = 0.67$	$l_{31}u_{12} + l_{32}u_{22} = 8$ $(0.67)(0) + l_{32}(-4) = 8$ $l_{32}(-4) = 8$ $l_{32} = \frac{8}{-4}$ $l_{32} = -2$	$l_{31}u_{13} + l_{32}u_{23} + u_{33} = 2$ $(0.67)(6) + (-2)(-14.8)$ $+ u_{33} = 2$ $u_{33} = 2 - 33.62$ $u_{33} = -31.62$

- Find the matrix of y by using Ly=B

$$\begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 7 \\ 12 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2.3 & 1 & 0 \\ 0.67 & -2 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 7 \\ 12 \end{bmatrix}$$

$y_1 = b_1$ $y_1 = 4$	$y_2 = b_2 - l_{21}y_1$ $y_2 = 7 - (2.3)(4)$ $y_2 = -2$	$y_3 = b_3 - l_{31}y_1 - l_{32}y_2$ $y_3 = 12 - 0.67(4) + 2(-2)$ $y_3 = 5.32$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \\ 5.32 \end{bmatrix}$$

- Find the value of x from Ux=y

$$\begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 0 & 6 \\ 0 & -4 & -14.8 \\ 0 & 0 & -31.62 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \\ 5.32 \end{bmatrix}$$

$$\begin{bmatrix} 3x + 6z \\ -4y - 14.8z \\ -31.62z \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \\ 5.32 \end{bmatrix}$$

$-31.62z = 5.32$ $z = \frac{5.32}{-31.62}$ $z = -0.17$	$-4y - 14.8z = -2$ $-4y - 14.8(-0.17)$ $= -2$ $-4y = -2 - 2.516$ $y = \frac{-4.516}{-4}$ $y = 1.13$	$7x - y + 6z$=8 $3x + 6z = 4$ $3x$=4 – 6(−0.17) $3x$=5.02 x=$\frac{5.02}{3}$ x=1.67

4

Solve the equations below by using Doolittle Method

$$\mathbf{8x + 7y + 6z = 15}$$
$$\mathbf{-3x + 9y + 5z = 7}$$
$$\mathbf{3x - 4y + 2z = 9}$$

$$A = \begin{pmatrix} 8 & 7 & 6 \\ -3 & 9 & 5 \\ 3 & -4 & 2 \end{pmatrix} \quad X = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad B = \begin{pmatrix} 15 \\ 7 \\ 9 \end{pmatrix}$$

- Find L and U, A=LU

$$\begin{bmatrix} 8 & 7 & 6 \\ -3 & 9 & 5 \\ 3 & -4 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

- Multiplying L and U.

$$\begin{bmatrix} 8 & 7 & 6 \\ -3 & 9 & 5 \\ 3 & -4 & 2 \end{bmatrix} = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ l_{21}u_{11} & l_{21}u_{12} + u_{22} & l_{21}u_{13} + u_{23} \\ l_{31}u_{11} & l_{31}u_{12} + l_{32}u_{22} & l_{31}u_{13} + l_{32}u_{23} + u_{33} \end{bmatrix}$$

- Find the value of L and U.

$u_{11} = 8$	$u_{12} = 7$	$u_{13} = 6$
$l_{21}u_{11} = -3$ $l_{21}(8) = -3$ $l_{21} = \frac{-3}{8}$ $l_{21} = -0.38$	$l_{21}u_{12} + u_{22} = 9$ $(-0.38)(7) + u_{22} = 9$ $u_{22} = 9 + 2.66$ $u_{22} = 11.66$	$l_{21}u_{13} + u_{23} = 5$ $(-0.38)(6) + u_{23} = 5$ $u_{23} = 5 + 2.28$ $u_{23} = 7.28$
$l_{31}u_{11} = 3$ $l_{31}(8) = 3$ $l_{31} = \frac{3}{8}$ $l_{31} = 0.38$	$l_{31}u_{12} + l_{32}u_{22} = -3$ $(0.38)(7) + l_{32}(11.66) = -4$ $l_{32}(11.66) = -6.66$ $l_{32} = \frac{-6.66}{11.66}$ $l_{32} = -0.57$	$l_{31}u_{13} + l_{32}u_{23} + u_{33} = 2$ $(0.38)(6) + (-0.57)(7.28) + u_{33} = 2$ $u_{33} = 2 + 1.87$ $u_{33} = 3.87$

- Find the matrix of y by using Ly=B

$$\begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 15 \\ 7 \\ 9 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ -0.38 & 1 & 0 \\ 0.38 & -0.57 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 15 \\ 7 \\ 9 \end{bmatrix}$$

$y_1 = b_1$ $y_1 = 15$	$y_2 = b_2 - l_{21}y_1$ $y_2 = 7 - (-0.38)(15)$ $y_2 = 12.7$	$y_3 = b_3 - l_{31}y_1 - l_{32}y_2$ $y_3 = 9 - 0.38(15) + (-0.57)(12.7)$ $y_3 = 10.54$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 15 \\ 12.7 \\ 10.54 \end{bmatrix}$$

- Find the value of x from Ux=y

$$\begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$$\begin{bmatrix} 8 & 7 & 6 \\ 0 & 11.66 & 7.28 \\ 0 & 0 & 3.87 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 15 \\ 12.7 \\ 10.54 \end{bmatrix}$$

$$\begin{bmatrix} 8x + 7y + 6z \\ 11.66y + 7.28z \\ 3.87z \end{bmatrix} = \begin{bmatrix} 15 \\ 12.7 \\ 10.54 \end{bmatrix}$$

$3.87z = 10.54$ $z = \frac{10.54}{3.87}$ $z = 2.72$	$11.66y + 7.28z = 12.7$ $11.66y + 7.28(2.72) = 12.7$ $11.66y = -7.1$ $y = \frac{-7.1}{11.66}$ $y = -0.61$	$8x + 7y + 6z = 15$ $8x + 7(-0.61) + 6(2.72) = 15$ $8x = 15 - 12.05$ $3x = 2.95$ $x = \frac{2.95}{8}$ $x = 0.37$

5.3 Exercises

a) Identify AX = B from the equations below:

i. $x + 2y - 5z = -6$

$4x + 4x - 3z = 7$

$5x + 7y - 13z = -9$

ii. $3x + 2y - z = 10$

$\boldsymbol{6z + 7x - y} = 8$

$3x + 2z = 5$

b) Solve the equations below by using Doolittle Method.

$2x - 4y - 3z = -23$

$3x + 5y + z = 12$

$-2x + 4y + 7z = 83$ **ANS: x = - 4, y = 3, z = 9**

c) Solve the equations below by using DoolittleMethod.

$x + 2y + 2z = 34$

$-3x - 4y + z = -25$

$4x + 7y + 3z = 84$ **ANS: x = 2, y = 7, z = 9**

d) Solve the equations below by using Doolittle Method.

$2x + 4y - 3z = -7$

$3x + 5y + z = -2$

$-2x + 4y + 7z = -13$ **ANS: x = 4, y = - 3, z = 1**

STEP BY STEP

CHAPTER 6: METHOD OF LINEAR EQUATION: CROUT

6 METHOD OF LINEAR EQUATION: CROUT

The CROUT method, also known as Crout's decomposition, is a numerical technique used for solving systems of linear equations through matrix factorization. It involves decomposing a square matrix into the product of a lower triangular matrix and an upper triangular matrix, where the diagonal elements of the lower triangular matrix are set to 1. This decomposition simplifies the process of solving linear systems, as it transforms the original system into two triangular systems that are easier to solve. The CROUT method is particularly useful in numerical linear algebra and computational mathematics for

HISTORY

The CROUT method, named after its developer David G. Crout, is a significant contribution to numerical linear algebra. It was introduced in the mid-20th century as a variation of the LU decomposition technique, which involves factoring a matrix into a lower triangular matrix and an upper triangular matrix. Crout's specific innovation was to modify the decomposition process by setting the diagonal elements of the lower triangular matrix to 1, leading to improved numerical stability in solving linear systems. This method became an essential tool in various scientific and engineering fields, aiding in solving complex systems of equations efficiently and accurately, and it continues to be a cornerstone of modern numerical computinglinear equation method.

6.1 Solving Crout

- Transforming the system matrix form AX=B

$$a_{11}x + a_{12}y + a_{13}z = b_1$$
$$a_{21}x + a_{22}y + a_{23}z = b_2$$
$$a_{31}x + a_{32}y + a_{33}z = b_3$$

$$\longrightarrow \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

- Find L and U, A=LU

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

- Multiplying L and U.

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} l_{11} & l_{11}u_{12} & l_{11}u_{13} \\ l_{21} & l_{21}u_{12} + l_{22} & l_{21}u_{13} + l_{22}u_{23} \\ l_{31} & l_{31}u_{12} + l_{32} & l_{31}u_{13} + l_{32}u_{23} + l_{33} \end{bmatrix}$$

- Find the value of L and U.

$l_{11} = a_{11}$	$l_{11}u_{12} = a_{12}$	$l_{11}u_{13} = a_{13}$
$l_{21} = a_{21}$	$l_{21}u_{12} + l_{22} = a_{22}$	$l_{21}u_{13} + l_{22}u_{23} = a_{23}$
$l_{31} = a_{31}$	$l_{31}u_{12} + l_{32} = a_{32}$	$l_{31}u_{13} + l_{32}u_{23} + l_{33} = a_{33}$

- Find the matrix of y by using Ly=B

$$\begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$$y_1 = \frac{b_1}{l_{11}}$$

$$y_2 = \frac{b_2 - l_{21}y_1}{l_{22}}$$

$$y_3 = \frac{b_3 - l_{31}y_1 - l_{32}y_2}{l_{23}}$$

- Find the value of x from Ux=y

$$\begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{b_1}{l_{11}} \\ \frac{b_2 - l_{21} y_1}{l_{22}} \\ \frac{b_3 - l_{31} y_1 - l_{32} y_2}{l_{23}} \end{bmatrix}$$

$$z = \frac{b_3 - l_{31} y_1 - l_{32} y_2}{l_{23}}$$

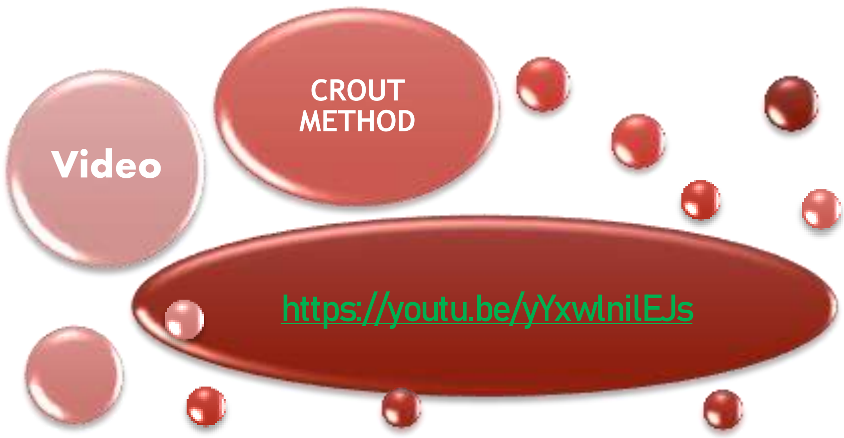
$$y = \frac{b_2 - l_{21} y_1}{l_{22}} - u_{23} z$$

$$x = \frac{b_1}{l_{11}} - u_{12} y - u_{13} z$$

Step by Step using the Crout method:

- step 1: Write linear equation to matrix form AX=B
- step 2: Find L and U, A=LU
- step 3: Multiplying L and U
- step 4: Find the value of L and U
- step 5: Find the matrix of y by using Ly=B
- step 6: Find the value of x from Ux=y

The Crout's elimination method, which is used for solving systems of linear equations by using LU decomposition. Remember that Crout's method can fail if any of the diagonal elements of matrix U become zero, indicating that LU decomposition isn't possible for the given matrix. This method also more computationally intensive compared to other methods for solving linear systems, but it can be advantageous for certain cases where matrix decomposition is useful.



6.2 Example

1

Solve the equations below by using Crout Method.

$$\begin{aligned} 4x + 6y + 8z &= 1 \\ -3x + 9y + 5z &= 3 \\ 3x - 4y + 2z &= 8 \end{aligned}$$

$$A = \begin{pmatrix} 4 & 6 & 8 \\ 5 & 5 & 2 \\ 1 & 3 & -4 \end{pmatrix} \quad X = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad B = \begin{pmatrix} 1 \\ 3 \\ 8 \end{pmatrix}$$

- Find L and U, A=LU
- $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix}$
- Multiplying L and U.

$$\begin{bmatrix} 4 & 6 & 8 \\ 5 & 5 & 2 \\ 1 & 3 & -4 \end{bmatrix} = \begin{bmatrix} l_{11} & l_{11}u_{12} & l_{11}u_{13} \\ l_{21} & l_{21}u_{12} + l_{22} & l_{21}u_{13} + l_{22}u_{23} \\ l_{31} & l_{31}u_{12} + l_{32} & l_{31}u_{13} + l_{32}u_{23} + l_{33} \end{bmatrix}$$

- Find the value of L and U.

$l_{11} = 4$	$l_{11}u_{12} = 6$ $(4)u_{12} = 6$ $u_{12} = \frac{6}{4}$ $u_{12} = 1.5$	$l_{11}u_{13} = 8$ $(4)u_{13} = 8$ $u_{13} = \frac{8}{4}$ $u_{13} = 2$
$l_{21} = 5$	$l_{21}u_{12} + l_{22} = 5$ $(5)(1.5) + l_{22} = 5$ $7.5 + l_{22} = 5$ $l_{22} = 5 - 7.5$ $l_{22} = -2.5$	$l_{21}u_{13} + l_{22}u_{23} = 2$ $(5)(2) + (-2.5)u_{23} = 2$ $(-2.5)u_{23} = 2 - 10$ $u_{23} = \frac{-8}{-2.5}$ $u_{23} = 3.2$
$l_{31} = 1$	$l_{31}u_{12} + l_{32} = 3$ $(1)(1.5) + l_{32} = 3$ $l_{32} = 3 - 1.5$ $l_{32} = 1.5$	$l_{31}u_{13} + l_{32}u_{23} + l_{33} = -4$ $(1)(2) + (1.5)(3.2) + l_{33} = -4$ $l_{33} = -4 - 6.8$ $l_{33} = -10.8$

- Find the matrix of y by using $Ly = B$

$$\begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 8 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 0 & 0 \\ 5 & -2.5 & 0 \\ 1 & 1.5 & -10.8 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 8 \end{bmatrix}$$

$l_{11}y_1 = b_1$ $4y_1 = 1$ $\boldsymbol{y_1 = 0.25}$	$l_{21}y_1 + l_{22}y_2 = b_2$ $5y_1 + (-2.5)y_2 = 3$ $5(0.25) + (-2.5)y_2 = 3$ $(-2.5)y_2 = 3 - 1.25$ $y_2 = \frac{1.75}{-2.5}$ $\boldsymbol{y_2 = -0.7}$	$l_{31}y_1 + l_{32}y_2 + l_{23}y_3 = l_{23}$ $1y_1 + 1.5y_2 + (-10.8)y_3 = 8$ $0.25 + (1.5)(-0.7) + (-10.8)y_3 = 8$ $(-10.8)y_3 = 8 + 0.8$ $y_3 = \frac{8.8}{-10.8}$ $\boldsymbol{y_3 = -0.815}$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 0.25 \\ -0.7 \\ -0.815 \end{bmatrix}$$

- Find the value of x from $Ux = Y$

$$\begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1.5 & 2 \\ 0 & 1 & 3.2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0.25 \\ -0.7 \\ -0.815 \end{bmatrix}$$

$\boldsymbol{z = -0.815}$	$y + 3.2(-0.815) = -0.7$ $y = -0.7 + 2.608$ $\boldsymbol{y = 1.908}$	$x + 1.5y + 2z = 0.25$ $x + 1.5(1.908) + 2(-0.815) = 0.25$ $x + 2.862 - 1.63 = 0.25$ $x = 0.25 - 1.232$ $\boldsymbol{x = -0.982}$

2 **Solve the equations below by using Crout Method.**

$$7x - y + 6z = 8$$
$$3x + 2y - z = 10$$
$$3x + 2z = 5$$

$$A = \begin{pmatrix} 7 & -1 & 6 \\ 3 & 2 & -1 \\ 3 & 0 & 2 \end{pmatrix} \quad X = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad B = \begin{pmatrix} 8 \\ 10 \\ 5 \end{pmatrix}$$

- Find L and U, A=LU

- $$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

- Multiplying L and U.

$$\begin{bmatrix} 7 & -1 & 6 \\ 3 & 2 & -1 \\ 3 & 0 & 2 \end{bmatrix} = \begin{bmatrix} l_{11} & l_{11}u_{12} & l_{11}u_{13} \\ l_{21} & l_{21}u_{12} + l_{22} & l_{21}u_{13} + l_{22}u_{23} \\ l_{31} & l_{31}u_{12} + l_{32} & l_{31}u_{13} + l_{32}u_{23} + l_{33} \end{bmatrix}$$

- Find the value of L and U.

$l_{11} = 7$	$l_{11}u_{12} = -1$ $(7)u_{12} = -1$ $u_{12} = \frac{-1}{7}$ $u_{12} = -0.14$	$l_{11}u_{13} = 6$ $(7)u_{13} = 6$ $u_{13} = \frac{6}{7}$ $u_{13} = 0.9$
$l_{21} = 3$	$l_{21}u_{12} + l_{22} = 2$ $(3)(-0.14) + l_{22} = 2$ $-0.42 + l_{22} = 2$ $l_{22} = 2 + 0.42$ $l_{22} = 2.4$	$l_{21}u_{13} + l_{22}u_{23} = -1$ $(3)(0.9) + (2.4)u_{23} = -1$ $(2.4)u_{23} = -1 - 2.7$ $u_{23} = \frac{-3.7}{2.4}$ $u_{23} = 1.542$
$l_{31} = 3$	$l_{31}u_{12} + l_{32} = 0$ $(3)(-0.14) + l_{32} = 0$ $l_{32} = 0 + 0.4$ $l_{32} = 0.4$	$l_{31}u_{13} + l_{32}u_{23} + l_{33} = 2$ $(3)(0.9) + (0.4)(-1.542)$ $+ l_{33} = 2$ $l_{33} = 2 - 1.9412$ $l_{33} = 0.0588$

- Find the matrix of y by using $Ly = B$

$$\begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 8 \\ 10 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} 7 & 0 & 0 \\ 3 & 2.4 & 0 \\ 3 & 0.4 & 0.0588 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 8 \\ 10 \\ 5 \end{bmatrix}$$

$l_{11}y_1 = b_1$ $7y_1 = 8$ $\boldsymbol{y_1 = 1.1429}$	$l_{21}y_1 + l_{22}y_2 = b_2$ $3y_1 + 2.4y_2 = 10$ $3(1.1429) + 2.4y_2 = 10$ $2.4y_2 = 6.5713$ $y_2 = \frac{6.5713}{2.4}$ $\boldsymbol{y_2 = 2.73}$	$l_{31}y_1 + l_{32}y_2 + l_{23}y_3 = l_{23}$ $3y_1 + 0.4y_2 + 0.0588y_3 = 5$ $3(1.1429) + (0.42)(2.73) + (0.0588)y_3 = 5$ $(0.0588)y_3 = 5 - 4.569$ $y_3 = \frac{0.0431}{0.0588}$ $\boldsymbol{y_3 = 7.33}$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1.1429 \\ 2.73 \\ 7.33 \end{bmatrix}$$

- Find the value of x from $Ux = Y$

$$\begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -0.14 & 0.9 \\ 0 & 1 & -1.542 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1.1429 \\ 2.73 \\ 7.33 \end{bmatrix}$$

$\boldsymbol{z = 7.33}$	$y + (-1.542)z = 2.73$ $y + (-1.542)(7.33) = 2.73$ $y = 2.7 + 11.3$ $\boldsymbol{y = 14}$	$x + (-0.14)y + 0.9z = 1.1429$ $x + (-0.14)(14) + 0.9(7.33) = 1.1429$ $x - 1.96 + 6.597 = 1.1429$ $x = 1.1429 - 4.637$ $\boldsymbol{x = -3.497}$

3

Solve the equations below by using Crout Method.

$$\mathbf{3x + 6z = 4}$$
$$\mathbf{7x - 4y - z = 7}$$
$$\mathbf{2x + 8y + 2z = 12}$$

$$A = \begin{pmatrix} 3 & 0 & 6 \\ 7 & -4 & -1 \\ 2 & 8 & 2 \end{pmatrix} \quad X = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad B = \begin{pmatrix} 4 \\ 7 \\ 12 \end{pmatrix}$$

- Find L and U, A=LU

- $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix}$

- Multiplying L and U.

$$\begin{bmatrix} 3 & 0 & 6 \\ 7 & -4 & -1 \\ 2 & 8 & 2 \end{bmatrix} = \begin{bmatrix} l_{11} & l_{11}u_{12} & l_{11}u_{13} \\ l_{21} & l_{21}u_{12} + l_{22} & l_{21}u_{13} + l_{22}u_{23} \\ l_{31} & l_{31}u_{12} + l_{32} & l_{31}u_{13} + l_{32}u_{23} + l_{33} \end{bmatrix}$$

- Find the value of L and U.

$l_{11} = 3$	$l_{11}u_{12} = 0$ $(3)u_{12} = 0$ $u_{12} = 0$	$l_{11}u_{13} = 6$ $(3)u_{13} = 6$ $u_{13} = \frac{6}{3}$ $u_{13} = 2$
$l_{21} = 7$	$l_{21}u_{12} + l_{22} = -4$ $(7)(0) + l_{22} = -4$ $l_{22} = -4$	$l_{21}u_{13} + l_{22}u_{23} = -1$ $(7)(2) + (-4)u_{23} = -1$ $(-4)u_{23} = -1 - 14$ $u_{23} = \frac{-15}{-4}$ $u_{23} = 3.75$
$l_{31} = 2$	$l_{31}u_{12} + l_{32} = 8$ $(2)(0) + l_{32} = 8$ $l_{32} = 8$	$l_{31}u_{13} + l_{32}u_{23} + l_{33} = 2$ $(2)(2) + (8)(3.75) + l_{33} = 2$ $l_{33} = 2 - 34$ $l_{33} = -32$

$$\begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 7 \\ 12 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 0 & 0 \\ 7 & -4 & 0 \\ 2 & 8 & -32 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 7 \\ 12 \end{bmatrix}$$

$l_{11}y_1 = b_1$ $3y_1 = 4$ $\boldsymbol{y_1 = 1.33}$	$l_{21}y_1 + l_{22}y_2 = b_2$ $5y_1 + (-4)y_2 = 7$ $5(1.333) + (-4)y_2 = 7$ $(-4)y_2 = 7 - 9.331$ $y_2 = \frac{-2.331}{-4}$ $\boldsymbol{y_2 = 0.583}$	$l_{31}y_1 + l_{32}y_2 + l_{23}y_3 = l_{23}$ $2y_1 + 8y_2 + (-32)y_3 = 12$ $2(1.333)$ $+ (8)(0.583) + (-32)y_3 = 12$ $(-32)y_3 = 12 - 7.33$ $y_3 = \frac{4.67}{-32}$ $\boldsymbol{y_3 = -0.1459}$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1.33 \\ 0.583 \\ -0.1459 \end{bmatrix}$$

- Find the value of x from $Ux = Y$

$$\begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3.75 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1.33 \\ 0.583 \\ -0.1459 \end{bmatrix}$$

$\boldsymbol{z = -0.1459}$	$y + (3.75)z = 0.583$ $y + 3.75(\boldsymbol{-0.1459}) = 0.583$ $y = 0.583 + 0.5471$ $\boldsymbol{y = 1.1301}$	$x + 0y + 2z = 1.33$ $x + 2(\boldsymbol{-0.1459}) = 1.33$ $x = 1.33 + 0.298$ $\boldsymbol{x = 1.6218}$

4

Solve the equations below by using Crout Method.

$$\mathbf{8x + 7y + 6z = 15}$$
$$\mathbf{-3x + 9y + 5z = 7}$$
$$\mathbf{3x - 4y + 2z = 9}$$

$$A = \begin{pmatrix} 8 & 7 & 6 \\ -3 & 9 & 5 \\ 3 & -4 & 2 \end{pmatrix} \quad X = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad B = \begin{pmatrix} 15 \\ 7 \\ 9 \end{pmatrix}$$

- Find L and U, A=LU

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

- Multiplying L and U.

$$\begin{bmatrix} 8 & 7 & 6 \\ -3 & 9 & 5 \\ 3 & -4 & 2 \end{bmatrix} = \begin{bmatrix} l_{11} & l_{11}u_{12} & l_{11}u_{13} \\ l_{21} & l_{21}u_{12} + l_{22} & l_{21}u_{13} + l_{22}u_{23} \\ l_{31} & l_{31}u_{12} + l_{32} & l_{31}u_{13} + l_{32}u_{23} + l_{33} \end{bmatrix}$$

- Find the value of L and U.

$l_{11} = 8$	$l_{11}u_{12} = 7$ $(8)u_{12} = 7$ $u_{12} = \frac{7}{8}$ $u_{12} = 0.875 \approx 0.9$	$l_{11}u_{13} = 6$ $(8)u_{13} = 6$ $u_{13} = \frac{6}{8}$ $u_{13} = 0.75 \approx 0.8$
$l_{21} = -3$	$l_{21}u_{12} + l_{22} = 9$ $(-3)(0.9) + l_{22} = 9$ $-2.7 + l_{22} = 9$ $l_{22} = 9 + 2.7$ $l_{22} = 11.7 \approx 12$	$l_{21}u_{13} + l_{22}u_{23} = 5$ $(-3)(0.8) + (12)u_{23} = 5$ $(12)u_{23} = 5 + 2.4$ $u_{23} = \frac{7.4}{12}$ $u_{23} = 0.616 \approx 0.6$
$l_{31} = 3$	$l_{31}u_{12} + l_{32} = -4$ $(3)(0.9) + l_{32} = -4$ $l_{32} = -4 - 2.7$ $l_{32} = -6.625 \approx -6.6$	$l_{31}u_{13} + l_{32}u_{23} + l_{33} = 2$ $(3)(0.75) + (-6.6)(0.616)$ $+ l_{33} = 2$ $l_{33} = -4 + 1.56$ $l_{33} = -3.831 \approx 3.9$

- Find the matrix of y by using $Ly = B$

$$\begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 15 \\ 7 \\ 9 \end{bmatrix}$$

$$\begin{bmatrix} 8 & 0 & 0 \\ -3 & 12 & 0 \\ 3 & -6.6 & 3.831 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 15 \\ 7 \\ 9 \end{bmatrix}$$

$l_{11}y_1 = b_1$ $8y_1 = 15$ $\boldsymbol{y_1 = 1.875}$	$l_{21}y_1 + l_{22}y_2 = b_2$ $-3y_1 + (12)y_2 = 7$ $-3(1.875) + (12)y_2 = 7$ $(12)y_2 = 7 + 5.625$ $y_2 = \frac{12.625}{12}$ $\boldsymbol{y_2 = 1.052}$	$l_{31}y_1 + l_{32}y_2 + l_{23}y_3 = l_{23}$ $3y_1 + (-6.6)y_2 + (3.831)y_3 = 9$ $3(1.875)$ $+ (-6.6)(1.052) + (3.831)y_3 = 9$ $(3.831)y_3 = 9 + 1.3182$ $y_3 = \frac{10.3182}{3.831}$ $\boldsymbol{y_3 = 2.693}$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1.875 \\ 1.052 \\ 2.693 \end{bmatrix}$$

- Find the value of x from $Ux = Y$

$$\begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0.9 & 0.8 \\ 0 & 1 & 0.6 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1.875 \\ 1.052 \\ 2.693 \end{bmatrix}$$

$\boldsymbol{z = 2.693}$ $\boldsymbol{\approx 2.7}$	$y + 0.6z = 1.052$ $y + 0.6(\boldsymbol{2.693}) = 1.052$ $y = 1.052 - 1.6158$ $\boldsymbol{y = -0.5638 \approx -0.6}$	$x + 0.9y + 0.8z = 1.875$ $x + 0.9(\boldsymbol{-0.5638}) + 0.8(\boldsymbol{2.693})$ $= 1.875$ $x - 0.50742 + 2.1544 = 1.875$ $x = 1.875 - 1.64698$ $\boldsymbol{x = 0.223 \approx 0.2}$

6.3 Exercises

a) Identify AX = B from the equations below:

i. $3x - 5z = 2$

$x + 4x - z = 7$

$5x + y - 13z = 4$

ii. $3x + 2y - z = 4$

$\boldsymbol{z + 7x - y} = 3$

$3x + 2y + z = 5$

b) Solve the equations below by using Crout Method.

$2x - 4y - 3z = -23$

$3x + 5y + z = 12$

$-2x + 4y + 7z = 83$ **ANS: x = - 4, y = 3, z = 9**

c) Solve the equations below by using Crout Method.

$x + 2y + 2z = 34$

$-3x - 4y + z = -25$

$4x + 7y + 3z = 84$ **ANS: x = 2, y = 7, z = 9**

d) Solve the equations below by using Crout Method.

$2x + 4y - 3z = -7$

$3x + 5y + z = -2$

$-2x + 4y + 7z = -13$ **ANS: x = 4, y = - 3, z = 1**

CONCLUSION

CONCLUSION

As a conclusion, the linear equations method is the best method in solving linear equations.By expressing these equations in matrix form, we can efficiently handle multiple equations and variables simultaneously, making it particularly useful in various fields, including physics, engineering, economics, computer graphics, and more.

Using matrix notation, the system of linear equations can be represented as "AX = B," where "A" is the coefficient matrix, "X" is the column vector of unknown variables, and "B" is the column vector of constants on the right-hand side. Solving such equations involves finding the values of the unknown variables "X" that satisfy the given conditions.

Matrix methods, such as Gaussian elimination, LU decomposition, and matrix inverses, offer efficient and systematic ways to solve these systems, even when dealing with large sets of equations. Additionally, advanced techniques like eigenvalue and eigenvector analysis come into play for more complex problems involving matrices.

Linear equation methods find practical applications in various areas, such as solving networks and circuits, optimizing production processes, analyzing economic systems, and handling transformations in computer graphics and computer vision.

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