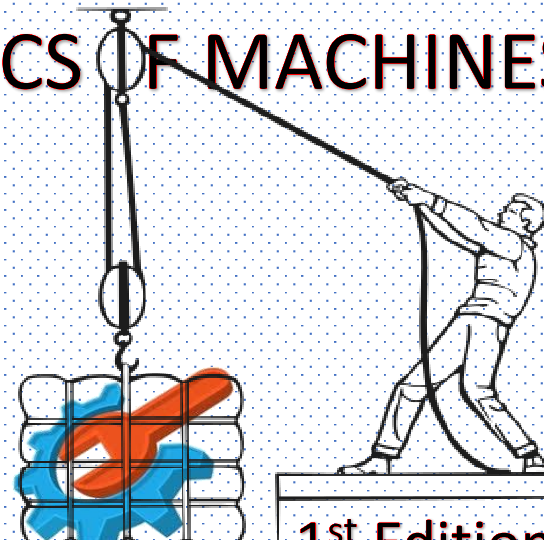
THEORY AND EXERCICE

MECHANICS OF MACHINES



1st Edition

Haswa-Sofilah

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THEORY AND EXERCISE MECHANICS OF MACHINES

1st Edition

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SYNOPSIS

This book introduces students to the fundamental techniques and concepts in Mechanics of Machines, emphasizing the analysis of velocity and acceleration diagrams, as well as balancing-related problems. It includes carefully structured exercises designed to guide students through problem-solving methods in a clear and engaging manner, promoting quick and effective understanding of key concepts.

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Table of Content

Velocity and Acceleration Diagram

1.0 Introduction

Velocity and acceleration of points in a mechanism can be obtain from the diagram

The diagram can give the velocity and acceleration of any point relative to any other point for one particular position of the mechanism

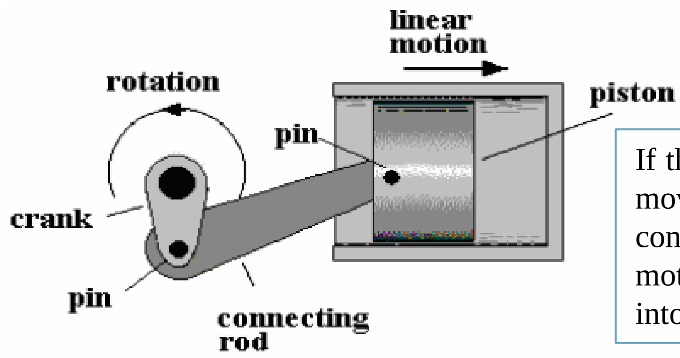
A mechanism is used to produce mechanical transformation in a machine such as to convert;

- one speed → another speed
- one force → another force
- one torque → another torque
- force → torque
- one angular motion → another angular motion
- angular motion → linear motion
- linear motion → angular motion

Example of mechanical transformation in a machine:

Crank ↔ Piston

Angular motion ↔ Linear motion



If the piston is forced to move, linear motion is converted into rotary motion and the force into torque

If the crank turned, angular motion is converted into linear motion of the piston and input torque is transformed into force on the piston

Angular motion ↔ Linear motion

Input Torque ↔ Piston Force

Velocity and acceleration of the mechanism

can be calculated by graphical method or analytical method

All diagram must be drawn to scale

by graphical method :

1. Sketch space diagram
2. Sketch velocity diagram
3. Sketch acceleration diagram

A: DIAGRAM SCALE

Example of scaling the diagrams:

1. SPACE DIAGRAM

Actual length : 1500 mm
Scale : 1 cm : 50 mm

✓ Length in space diagram (scale)

$$1500\ mm \times \frac{1\ cm}{50\ mm} = \mathbf{30\ cm}$$

**Draw 30 cm length in space diagram*

✓ Length in space diagram to actual length

$$30\ cm \times \frac{50\ mm}{1\ cm} = \mathbf{1500\ mm}$$

2. VELOCITY DIAGRAM

Actual velocity : 15 m/s
Scale : 1 cm : 2 m/s

✓ To draw the length of velocity in velocity diagram (scale)

$$15\ m/s \times \frac{1\ cm}{2\ m/s} = \mathbf{7.5\ cm}$$

**Draw 7.5 cm length for 15 m/s velocity*

✓ To find the velocity from velocity diagram

$$7.5\ cm \times \frac{2\ m/s}{1\ cm} = \mathbf{15\ m/s}$$

A: DIAGRAM SCALE

Examples of scaling the diagrams:

3. ACCELERATION DIAGRAM

Actual acceleration : 50 m/s^2
Scale : 1 cm : 10 m/s^2

✓ To draw the length of acceleration in acceleration diagram (scale)

$$50\ m/s^2 \times \frac{1\ cm}{10\ m/s^2} = \mathbf{5\ cm}$$

**Draw 5 cm length in acceleration diagram*

✓ To find acceleration from acceleration diagram

$$5\ cm \times \frac{10\ m/s^2}{1\ cm} = \mathbf{50\ m/s^2}$$

B: VELOCITY DIAGRAM

- There are **two types** of velocity :

ABSOLUTE VELOCITY

- velocity of a point measured from a fixed point (normally the ground or anything rigidly attached to the ground and not moving)

RELATIVE VELOCITY

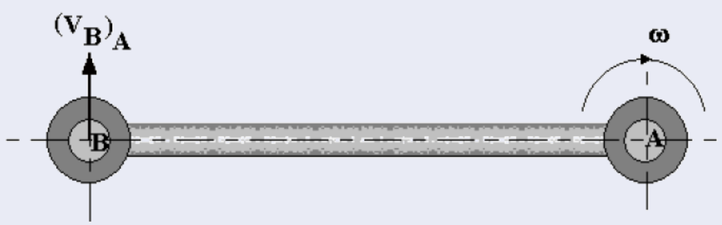
- velocity of a point measured relative to another that may itself be moving

B: VELOCITY DIAGRAM

- Due to movement of mechanism, the velocity need to be draw is relative velocity ***(any point relative to any point)***
- Velocity can be measured :–
  i. Tangential (for links),
  ii. Radial (for piston)

(i) TANGENTIAL VELOCITY

- ✓ Tangential velocity is measured for the velocity that act at the rigid link
- ✓ Velocity of a rigid link is the velocity of one point on a link relative to another and must be perpendicular (90^0) to the axis of the link
- ✓ Consider a link AB pinned at A and revolving about A at angular velocity (ω)

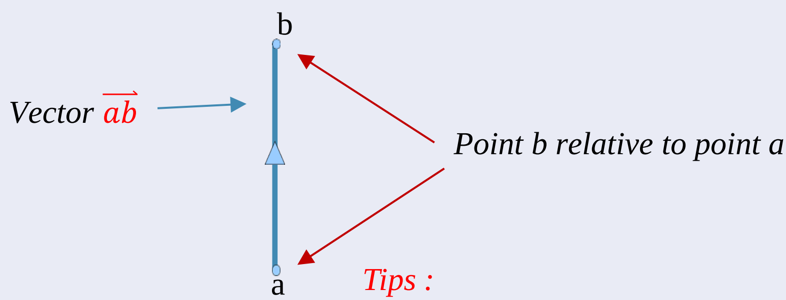


- ✓ Point B moves relative to point A but its velocity always tangential to the link.
- ✓ The denoting $(V_B)_A$ is the velocity of B relative to A
- ✓ Vector $\overrightarrow{ab}$ will represent the velocity in the velocity diagram

B: VELOCITY DIAGRAM

(i) TANGENTIAL VELOCITY

Figure below shows how the velocity diagram of the link is drawn;



Tips :
point b relative to point an arrow draw from point a to point b

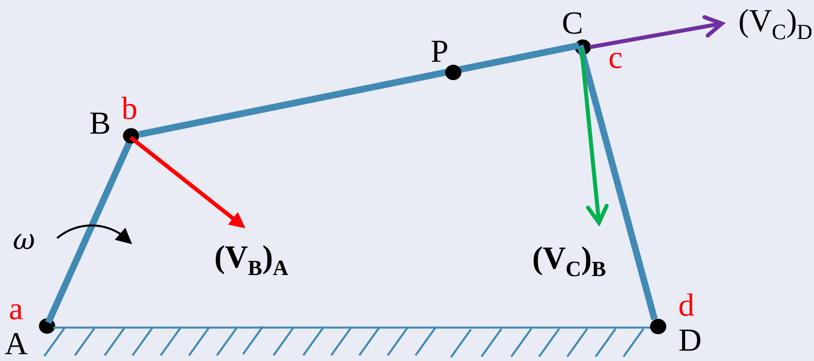
To find the velocity of $(V_B)_A$;

$$v_{ba} = \omega(AB)$$

*** We can calculate acceleration either from space diagram or velocity diagram*

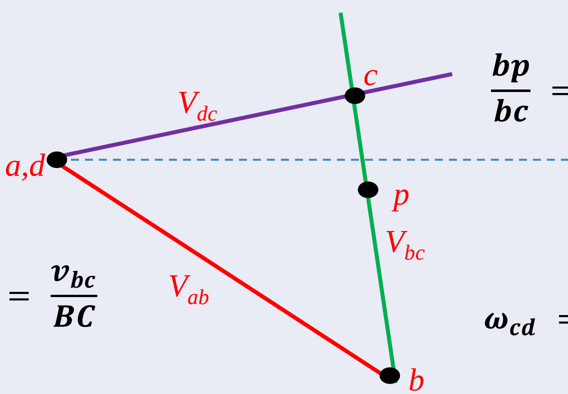
4-BAR LINKS MECHANISM

✓ Draw the space diagram with appropriate scale and then determine the velocity for each link.



✓ *Point C have 2 velocity that relative from point b and d. So that point c is the intersection for velocity of link bc and cd*

✓ Velocity Diagram of the 4-bar links mechanism:



$$\frac{bp}{bc} = \frac{BP}{BC}$$

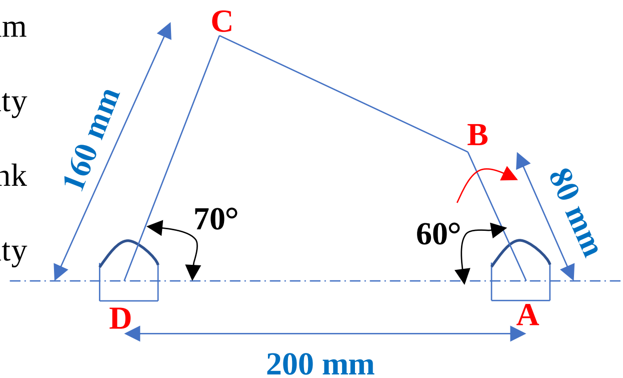
$$\omega_{bc} = \frac{v_{bc}}{BC}$$

$$\omega_{cd} = \frac{v_{cd}}{CD}$$

EXERCISE 1:

Diagram below shows a 4 bar-chain mechanism with A and D are the fixed points. The crank AB rotates at a constant speed of 480 rad/s in a clockwise direction. For the diagram shown :

i. draw the space diagram (1 cm : 25 mm)
ii. draw the velocity diagram (1 cm : 5 m/s)
iii. find the velocity for link BC
iv. find the angular velocity for link CD



Solution:

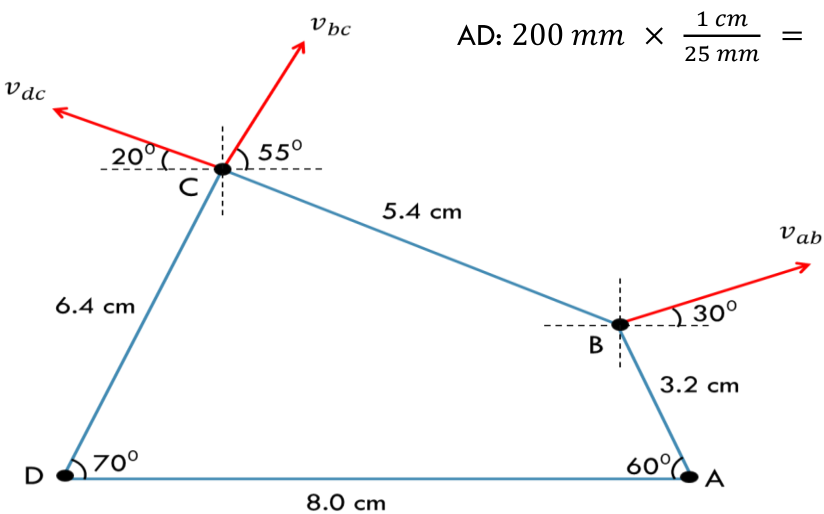
i) Space Diagram (Scale: 1 cm : 25 mm)

Space diagram scales

AB: $80\ mm \times \frac{1\ cm}{25\ mm} = \mathbf{3.2\ cm}$

DC: $160\ mm \times \frac{1\ cm}{25\ mm} = \mathbf{6.4\ cm}$

AD: $200\ mm \times \frac{1\ cm}{25\ mm} = \mathbf{8.0\ cm}$



EXERCISE 1:

Solution:

ii) **Velocity Diagram (Scale: 1 cm : 5 m/s)**

$$v_{ab} = \omega_{ab} \times AB$$
$$= 480 \times 0.08$$
$$= \mathbf{38.4 m/s}$$

Velocity diagram scales

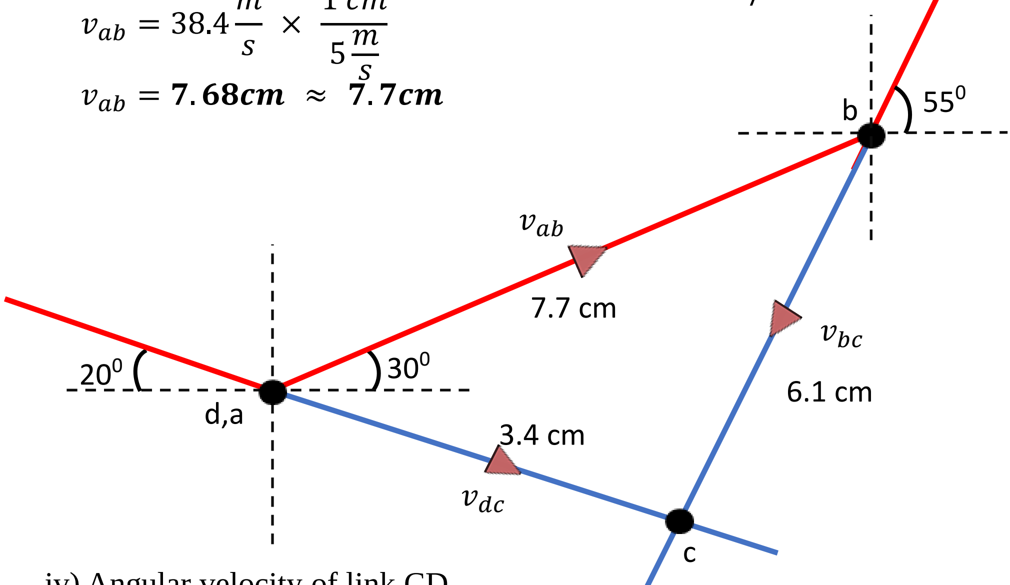
$$v_{ab} = 38.4\frac{m}{s} \times \frac{1\ cm}{5\frac{m}{s}}$$

$$v_{ab} = \mathbf{7.68cm} \approx \mathbf{7.7cm}$$

iii) Velocity of link BC

From velocity diagram

$$\therefore\ v_{bc} = 6.1\ cm \times \frac{5\ m/s}{1\ cm}$$
$$= \mathbf{30.5\ m/s}$$



iv) Angular velocity of link CD

From velocity diagram

$$v_{dc} = 3.4\ cm \times \frac{5\ m/s}{1\ cm} = 17\ m/s$$

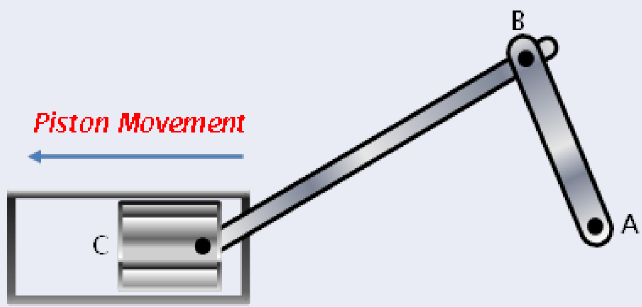
$$\therefore \omega_{dc} = \frac{v_{dc}}{OC} = \frac{17}{0.16}$$

$$= \mathbf{106.25\ rad/s}$$

B: VELOCITY DIAGRAM

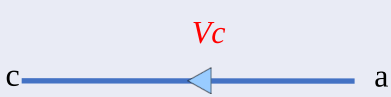
(ii) RADIAL VELOCITY

- ✓ Radial velocity is measured for the velocity that act at the piston
- ✓ Consider a piston C that can be move by the action of link AB as shown;



- ✓ If the link AB rotates about A at the same time then piston C will have radial velocity, denoted as $\mathbf{v_c}$

- ✓ Figure below shows how the velocity of piston is drawn;

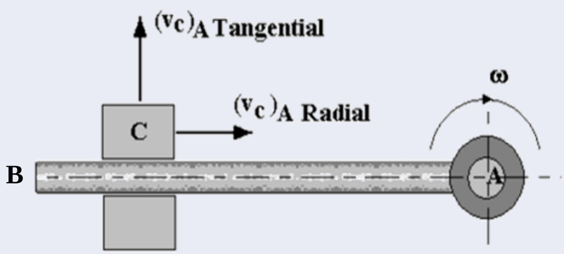


Velocity for piston always draw / measure from starting point (fixed point)

B: VELOCITY DIAGRAM

SLIDER CRANK MECHANISM

- ✓ Consider a slider C that can slide on link A;



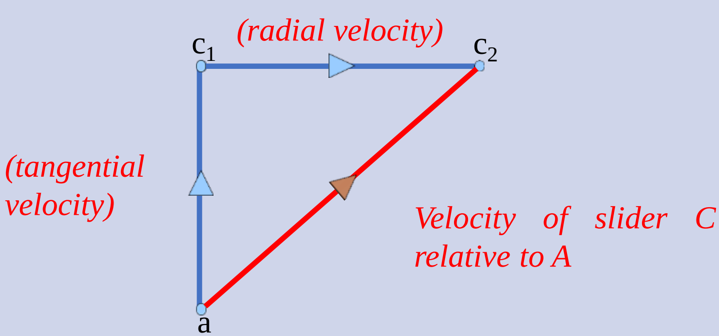
Note :
Tangential and radial velocities are denoted the same so that the tags radial and tangential are added

The velocities of slider C are denoted as;

(Vc)A – Tangential

(Vc)A – Radial

- ✓ Figure below shows how the velocity of slider C is drawn;



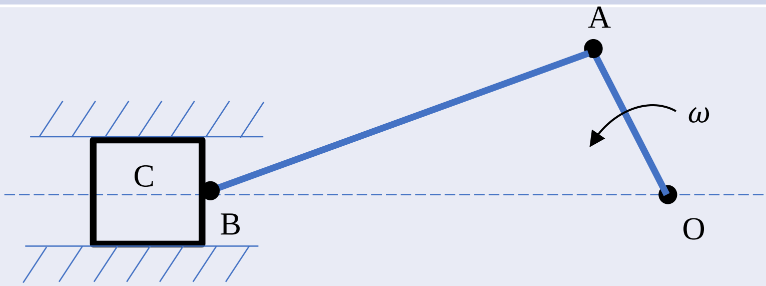
Note:
This diagram shows the velocity for slider C only. The actual diagram we must draw the velocity of link AB and slider C

- ✓ Tangential velocity is represent by vector ac_1 and radial velocity is represent by vector c_1c_2
- ✓ The velocity of slider C relative to A is represent by vector ac_2

B: VELOCITY DIAGRAM

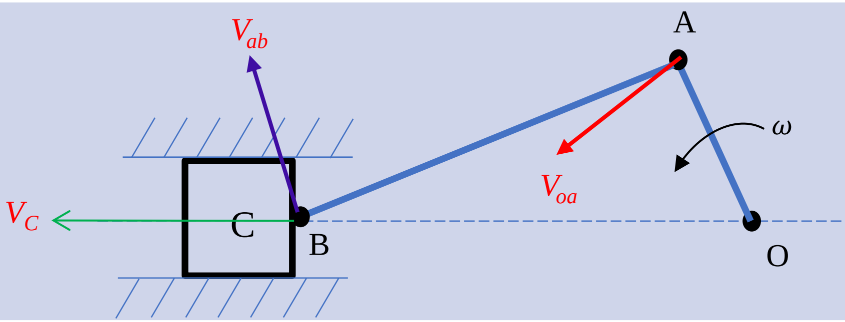
CRANK, CONNECTING ROD & PISTON

- ✓ Consider diagram below that consists of crank, connecting rod and piston;



- ✓ Crank and connecting rod is a link
- ✓ Velocity of piston must be relative to point O (fixed point)

- ✓ Find velocity of crank OA, velocity of connecting rod AB and velocity of piston C from the space diagram
- ✓ From space diagram we draw the related velocity (crank OA, connecting rod AB and piston C);

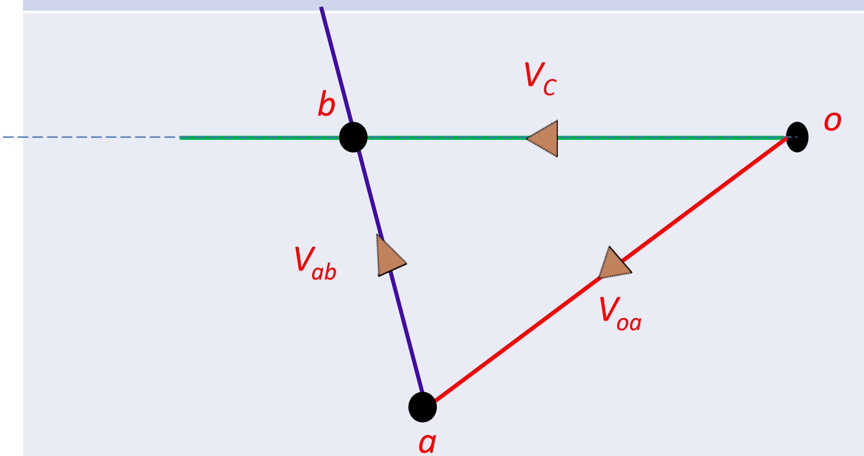


- ✓ Point B have 2 velocity. So that point b can be found by intersection for velocity of link AB and velocity of piston C
- ✓ All this velocities must be drawn in velocity diagram with appropriate scale

B: VELOCITY DIAGRAM

CRANK, CONNECTING ROD & PISTON

- ✓ Now we can draw the velocity diagram;



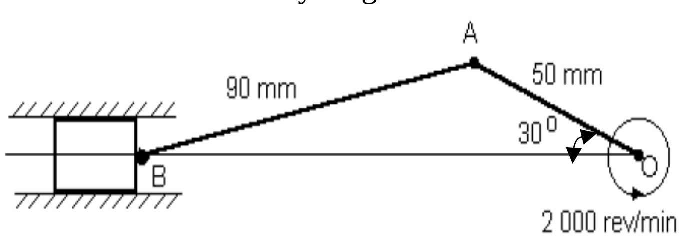
- ✓ Determine the piston's axis and then start drawing the diagram from a fix point

EXERCISE 2:

The mechanism shown has a crank 50 mm radius which rotates at 2000 rev/min. Determine the velocity of the piston for the position shown. Also determine the angular velocity of link AB about A.

Scale : Space diagram - 1 cm : 10 mm

Velocity diagram - 1 cm : 1 m/s



EXERCISE 2:

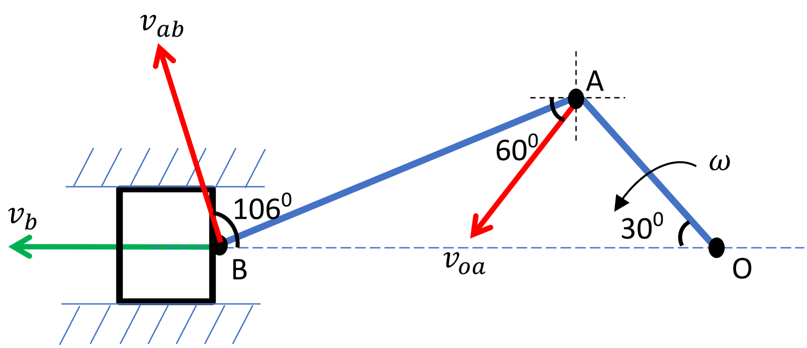
Solution:

Space Diagram (Scale: 1 cm : 10 mm)

Space diagram scales

OA: $50\ mm \times \frac{1\ cm}{10\ mm} = \mathbf{5\ cm}$

AB: $90\ mm \times \frac{1\ cm}{10\ mm} = 9\ cm$



Velocity Diagram (Scale: 1 cm : 1 m/s)

$$\omega_{oa} = \frac{2\pi N}{60} = \frac{2\pi(2000)}{60}$$

$$\omega_{oa} = \mathbf{209.44 rad/s}$$

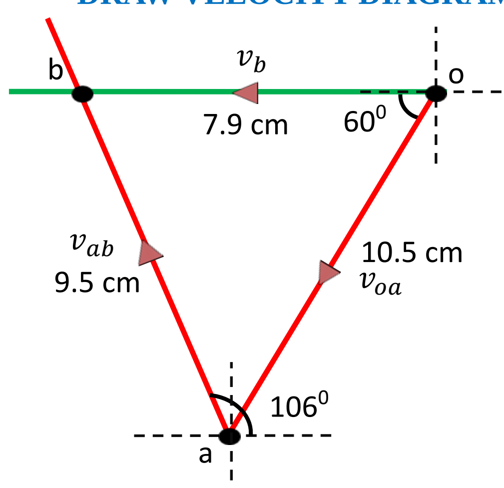
$$v_{oa} = \omega_{oa} \times OA$$
$$= 209.44 \times 0.05$$
$$= \mathbf{10.472\ m/s}$$

Velocity diagram scales

$$v_{oa} = 10.472\ m/s \times \frac{1\ cm}{1\ m/s}$$

$$v_{oa} = 10.47\ cm \approx \mathbf{10.5\ cm}$$

DRAW VELOCITY DIAGRAM



EXERCISE 2:

Solution:

Velocity of piston B

From velocity diagram

$$v_b = 7.9\ cm \times \frac{1\ m/s}{1\ cm} = \mathbf{7.9\ m/s}$$

Angular velocity of link AB

From velocity diagram

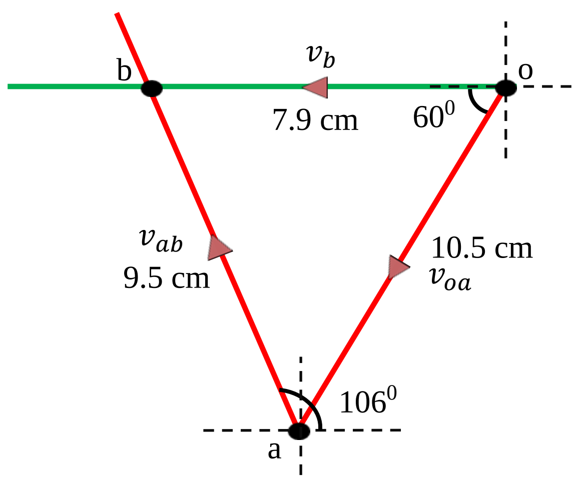
$$v_{ab} = 9.5\ cm \times \frac{1\ m/s}{1\ cm} = 9.5\ m/s$$

From velocity diagram

$$v_{ab} = 9.5\ cm \times \frac{1\ m/s}{1\ cm} = 9.5\ m/s$$

$$\therefore \omega_{ab} = \frac{v_{ab}}{AB} = \frac{9.5}{0.09} = 105.56\ rad/s$$

Velocity Diagram (Scale: 1 cm : 1 m/s)

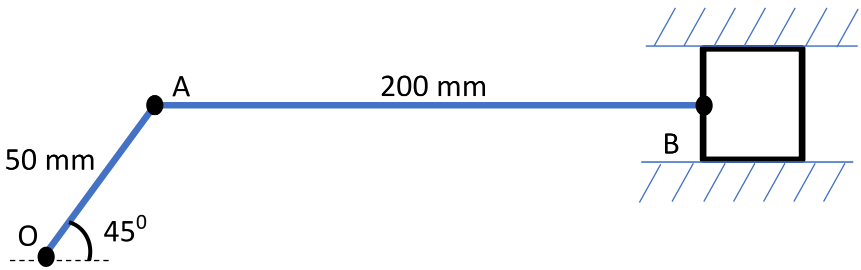


EXERCISE 3:

The crank OA revolves clockwise at 300 rev/min as shown in diagram below. Find the velocity of piston B and angular velocity of link AB.

Scale : Space diagram - 1 cm : 20 mm

Velocity diagram - 1 cm : 0.3 m/s



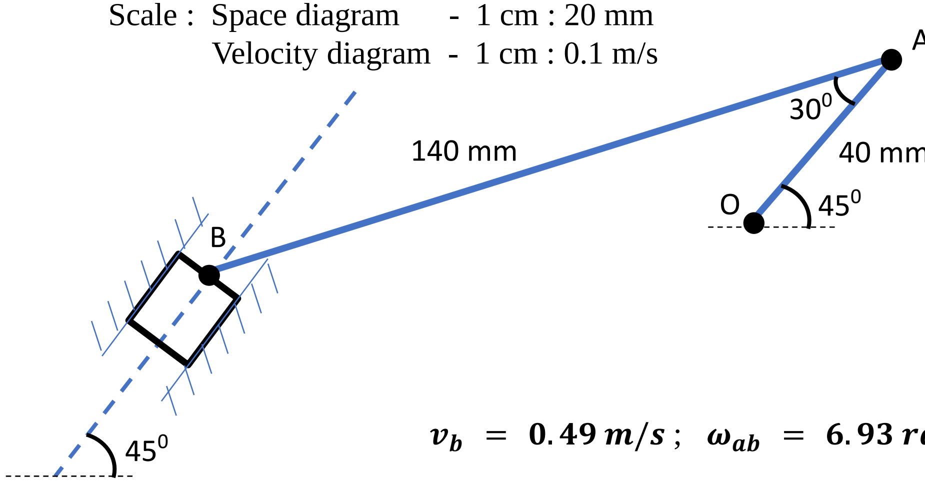
$$v_b = 1.11\, m/s\,;\quad \omega_{ab} = 5.55\, rad/s$$

EXERCISE 4:

The crank OA rotates at 200 rev/min clockwise at as shown in diagram below. Find the velocity of piston Band angular velocity of link AB.

Scale : Space diagram - 1 cm : 20 mm

Velocity diagram - 1 cm : 0.1 m/s



$$v_b = 0.49\, m/s\,;\quad \omega_{ab} = 6.93\, rad/s$$

C: ACCELARATION DIAGRAM:

- ✓ Acceleration of one point on a link relative to another has **two components :**

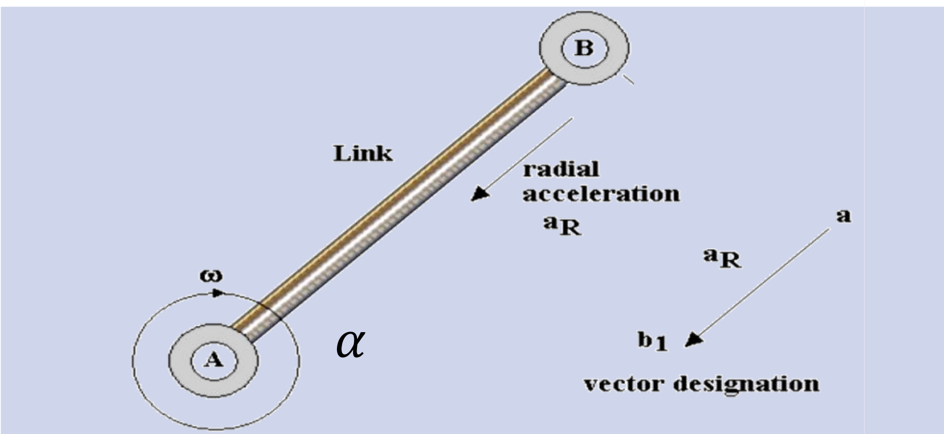
RADIAL COMPONENT

- due to the angular velocity of the link.
- Centripetal acceleration is an example of radial component

TANGENTIAL COMPONENT

- due to the angular acceleration of the link

ACCELARATION OF LINKS - 1. RADIAL ACCELERATION



- ✓ The velocity of B relative to A is tangential (VB)A (rotate ccw)
- ✓ It shows that the direction of centripetal acceleration of link AB is towards point A
- ✓ So that the radial/centripetal acceleration is of link AB is B relative to A

It is calculated using equation:

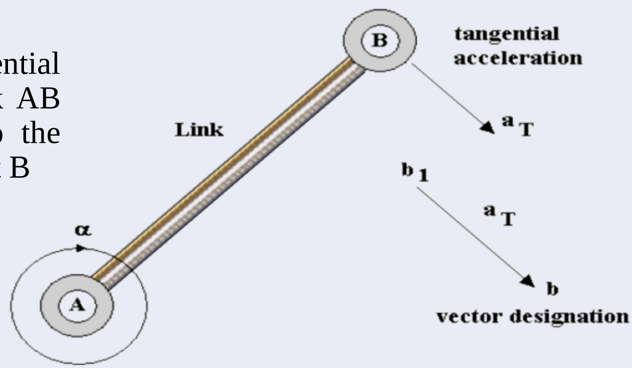
$$(a_R)_{ab} = \omega_{ab}^{\ 2}(AB) \quad or \quad (a_R)_{ab} = \frac{v_{ab}^{\ 2}}{AB}$$

Construction of vector for radial/centripetal acceleration is from a to b_1

ACCELARATION OF LINKS - 2. TANGENTIAL ACCELERATION

- Tangential acceleration only occurs if the link has an angular acceleration, $\alpha \ rad/s$
- Consider link AB with an angular acceleration about A

- It shows that tangential acceleration of link AB is perpendicular to the link and act at point B

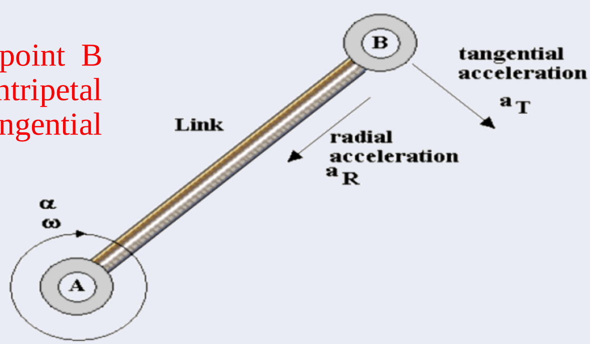


- It is calculated using equation;

$$(a_T)_{ab} = \alpha_{ab}(AB)$$

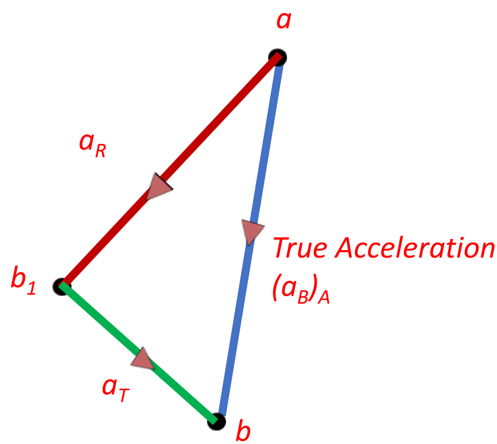
- From previous diagram we can see that point B will have both radial/centripetal acceleration and tangential acceleration relative to point A
- The construction of vector for tangential acceleration is from b_1 to b

- Diagram shows that point B will have both centripetal acceleration and tangential acceleration.

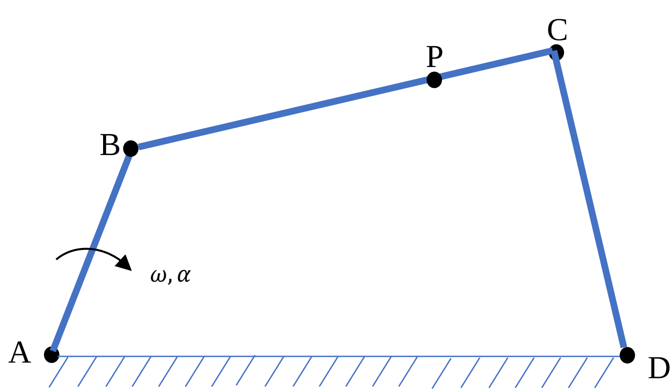


ACCELARATION OF LINKS :

- ✓ Draw both acceleration from point A (because B relative to A)
- ✓ Firstly, draw the centripetal acceleration and the represented vector is ab_1
- ✓ Then at point b1, draw the tangential acceleration perpendicular to vector ab1 and the represented vector is b_1b
- ✓ The resultant acceleration of B relative to A being given by vector ab

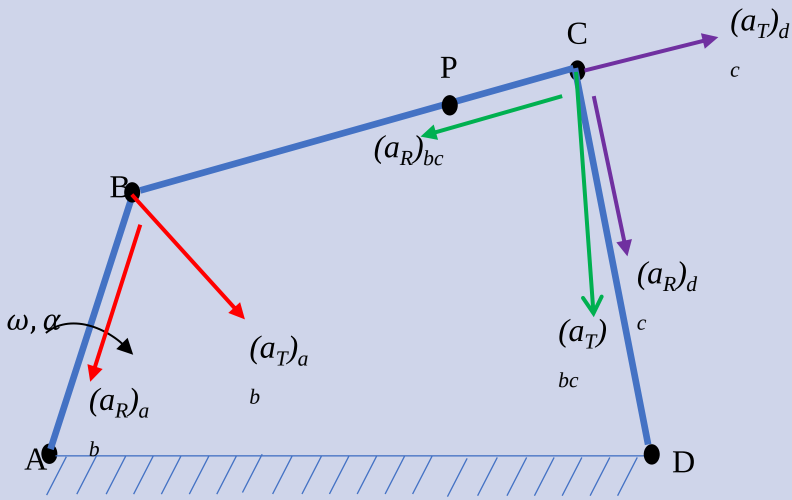


- ✓ For several of link that connect to each other, we need to draw each acceleration for each link.
- ✓ The acceleration diagram must be started at fix point
- ✓ Figure below shows four-bar mechanism, and the angular velocity and angular acceleration of link AB is given

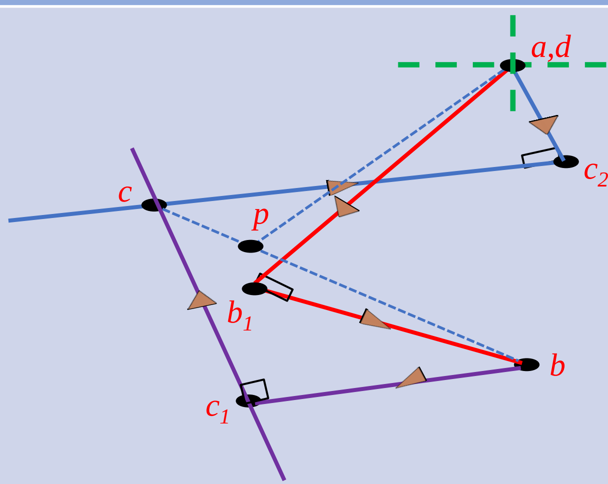


ACCELARATION OF LINKS :

- ✓ Draw the diagram with appropriate scale and then determine the acceleration for each link.



- ✓ Accelaration Diagram Of Links;



Tips:

- ✓ Centripetal acceleration can be obtained from velocity diagram
- ✓ Use equation to find **point P** that located on bc

EXERCISE 5:

A piston connecting rod and crank mechanism is shown in figure 1 below. The crank rotates at a constant speed of 300 rad/s counterclockwise direction. The figure is not drawn according to scale.

a) Draw the space and velocity diagram with scale of 1 cm : 20 mm and 1cm : 2 m/s respectively

b) Determine the velocity of link BC and piston C

c) Analyse the effect of the system if the angular velocity of crank AB decreased.

d) Draw the acceleration diagram with the scale 1 cm : 500 m/s^2

e) Determine the angular acceleration of link BC and piston C

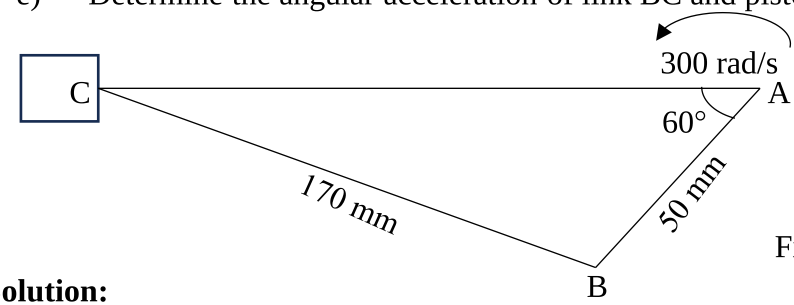


Figure 1

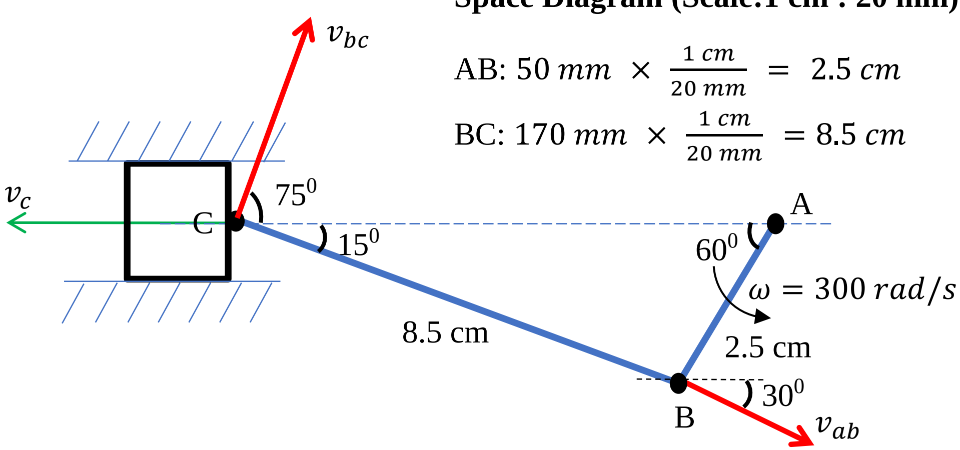
Solution:

a) Draw the space and velocity diagram with scale of 1 cm : 20 mm and 1cm : 2 m/s respectively

Space Diagram (Scale:1 cm : 20 mm);

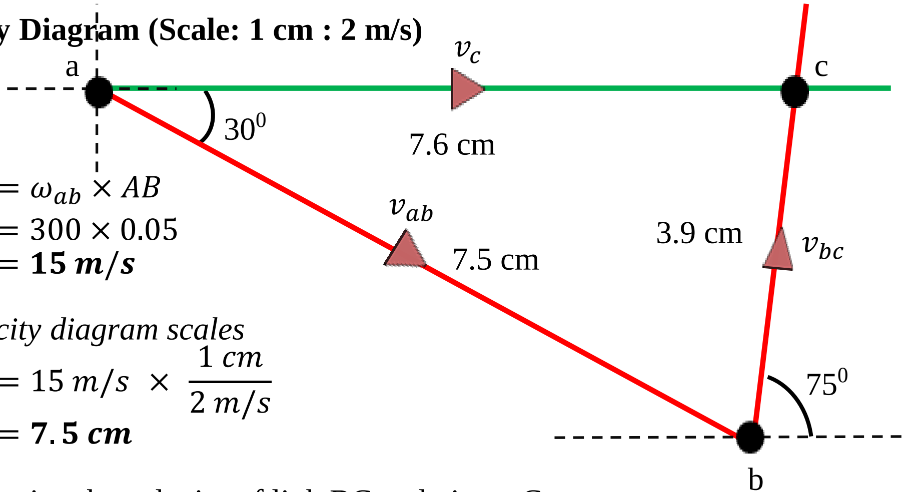
AB: $50\ mm \times \frac{1\ cm}{20\ mm} = 2.5\ cm$

BC: $170\ mm \times \frac{1\ cm}{20\ mm} = 8.5\ cm$



EXERCISE 5: Solution

Velocity Diagram (Scale: 1 cm : 2 m/s)



$$v_{ab} = \omega_{ab} \times AB$$
$$= 300 \times 0.05$$
$$= \mathbf{15\ m/s}$$

Velocity diagram scales

$$v_{ab} = 15\ m/s \ \times \frac{1\ cm}{2\ m/s}$$
$$v_{ab} = \mathbf{7.5\ cm}$$

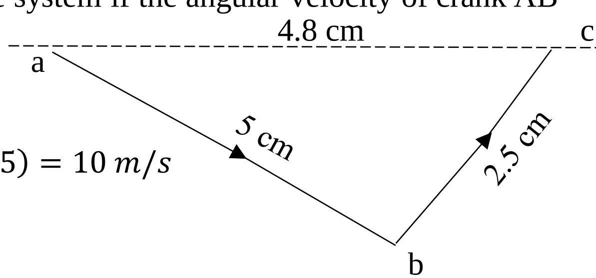
b) Determine the velocity of link BC and piston C

From velocity diagram;

$$v_{bc} = 3.9\ cm \times \frac{2\ m/s}{1\ cm} = \mathbf{7.8\ m/s}$$
$$v_c = \ 7.6\ cm \times \frac{2\ m/s}{1\ cm} = \mathbf{15.2\ m/s}$$

c) Analyse the effect of the system if the angular velocity of crank AB decreased

Scales: 1 cm : 2 m/s



Proof:

$$V_{BA} = \ \omega r = (200)(0.05) = 10\ m/s$$
$$V_{BA} = 10\ m/s \text{ x } \frac{1\ cm}{2\ m/s}$$
$$V_{BA} = 5\ cm$$

Reasoning:

V_{piston} for angular velocity of 300 rad/s is 15 m/s as the angular velocity decreased to 200 rad/s, it's velocity also decreased to 9.6 m/s. Therefore, when the angular velocity decreased, the velocity of piston also will decrease.

EXERCISE 5: Solution

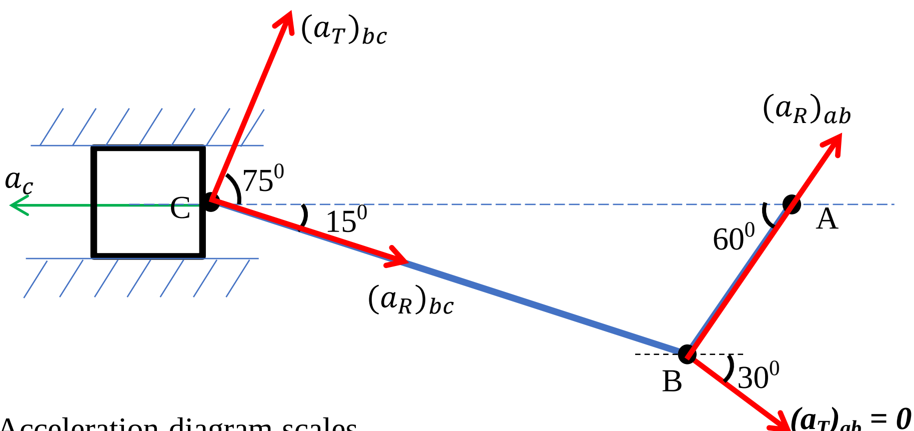
d) Draw the acceleration diagram with the scale 1 cm : 500 m/s^2

Space Diagram (Scale: 1 cm : 20 mm)

*Acceleration directions

$$(a_R)_{ab} = \frac{{v_{ab}}^2}{AB} = \frac{15^2}{0.05} = \mathbf{4500\ m/s^2}$$

$$(a_R)_{bc} = \frac{{v_{bc}}^2}{BC} = \frac{7.8^2}{0.17} = \mathbf{357.9\ m/s^2}$$



Acceleration diagram scales

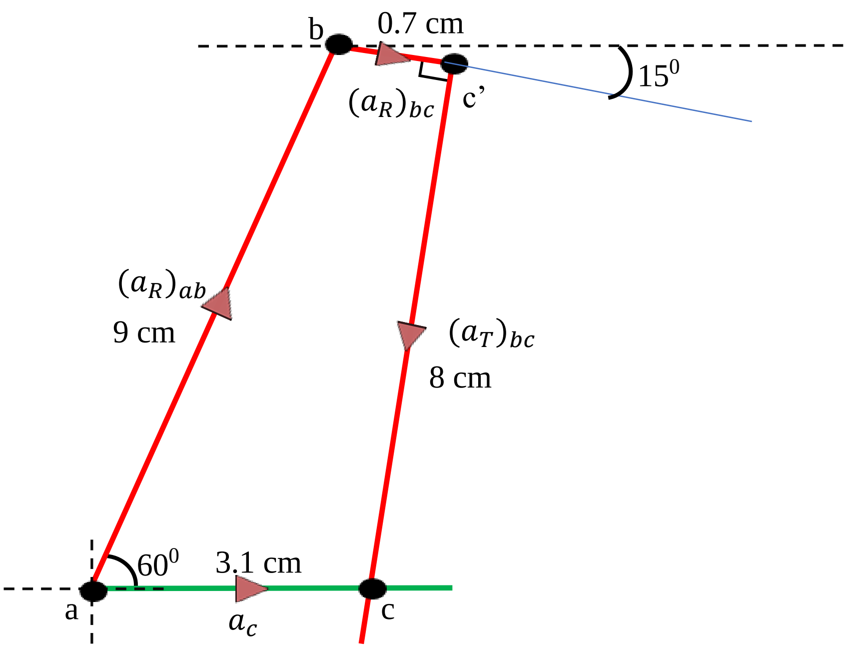
$$(a_R)_{ab} = 4500\ m/s^2 \times \frac{1\ cm}{500\ m/s^2} = \mathbf{9\ cm}$$

$$(a_R)_{bc} = 357.9\ m/s^2 \times \frac{1\ cm}{500\ m/s^2} = \mathbf{0.7\ cm}$$

EXERCISE 5: Solution

d) Draw the acceleration diagram with the scale 1 cm : 500 m/s^2

Acceleration diagram ; Scale: 1 cm : 500 m/s^2



e) Determine the angular acceleration of link BC and piston C

From acceleration diagram

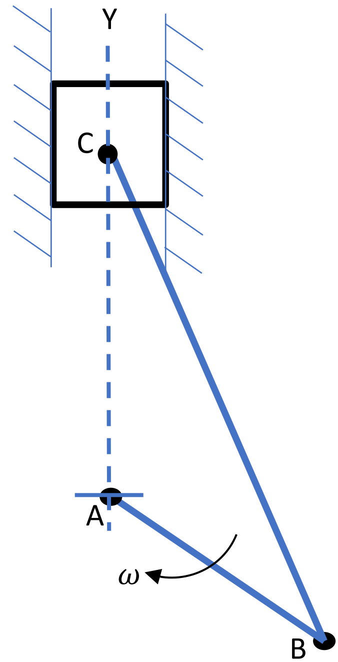
$$(a_T)_{bc} = 8\ cm \times \frac{500\ m/s^2}{1\ cm} = 4000\ m/s^2$$

$$\therefore \alpha_{bc} = \frac{(a_T)_{bc}}{BC} = \frac{4000}{0.17} = \mathbf{23529.41 rad/s^2}$$

$$\alpha_{piston\ C} = 3.1\ cm\ x\ \frac{500\ m/s^2}{1\ cm} = \mathbf{1550\ m/s^2}$$

EXERCISE 6:

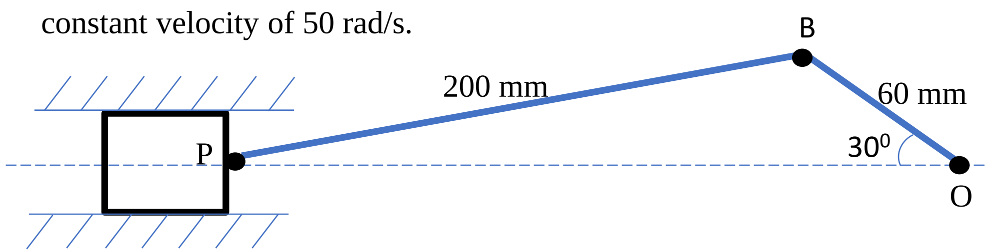
Figure below shows a link mechanism with crank AB rotating clockwise at a constant speed of 600 rpm. Piston C is connected to AB by link BC and it is sliding reciprocally along vertical AY. Angle BAC is 155^0. Given that AB = 80 mm, BC = 210 mm. For this configuration,



a) Draw the space diagram using a scale 1 cm : 20 mm
b) Draw the velocity diagram using a scale of 1 cm : 0.5 m/s
c) Draw the acceleration diagram using a scale 1 cm : 50 m/s^2
d) Determine the sliding acceleration of piston along AY
d) Find the angular acceleration of BC
e) Determine the direction of rotation for link BC

EXERCISE 7:

A piston, connecting rod and crank mechanism is shown in Figure below. The crank OB with a radius of 60 mm rotates clockwise at a constant velocity of 50 rad/s.



i. Draw a space diagram with a scale of 1 cm : 20 mm
ii. Draw a velocity diagram with a scale of 1 cm : 0.5 m/s
iii. Draw an acceleration diagram with a scale of 1 cm : 15 m/s^2
iv. Determine the velocity and the acceleration of the piston

Balancing

2.0 Introduction

The balancing of rotating bodies/part is important to avoid /reduced vibration and noise.

Generally balancing is the process of eliminating or at least reducing the ground forces and or moments

It is achieved by changing the location of the mass centre link

The process of providing the second mass to counter at the effect of the centrifugal force of the first mass in rotating system is called **BALANCING**.

2.1 The Importance of Balancing

- ✓ Balancing plays a very important part in rotating bodies/part. Balancing in machines helps to rotating bodies to avoid vibrations; vibration in machines can lead to failure.
- ✓ Common failure occurs in generators and heavy machinery, so undertaking in balancing can help to avoid machines from breaking down.

2.2 Problems of Balancing

- ✓ Most of the serious problems encountered in high-speed engine/machinery are the direct result of unbalanced forces
- ✓ These force exerted on the frame by the moving machine members are time varying, impart, vibratory motion to the frame and produce noise. *(Automobile wheel, Grinding Wheel, Steam turbines, Compressor)*
- ✓ Also, there are human discomfort and detrimental effects on the machine performance and the structural integrity of the machine foundation
- ✓ Imbalanced – Increased loads on bearing & Stress → Effect of imbalance produced unpleasant & dangerous vibration, reduced efficiency, noise generation, structural damage, increased wear and tear, energy lose, reduced precision, and etc. *(Washing machine, aircraft propeller)*

2.3 Effects of unbalance masses on a rotating shaft.

1. **Vibration:** Imbalance leads to uneven distribution of mass, causing dynamic forces during rotation. This results in vibrations that can adversely affect the performance of machinery, lead to wear and tear, and compromise the structural integrity of components.
2. **Reduced Efficiency:** Imbalance causes additional forces and moments that can decrease the overall efficiency of the rotating system. This inefficiency can lead to increased energy consumption, decreased output, and a shorter lifespan for the equipment.
3. **Noise Generation:** The uneven distribution of mass can cause irregularities in the motion of rotating components, leading to noise generation. This not only affects the comfort of operators and nearby personnel but may also be indicative of potential damage to the machinery.
4. **Structural Damage:** Prolonged operation with imbalance can exert excessive stress on the bearings, shafts, and other supporting structures. This can result in accelerated wear and tear, leading to premature failure of components and, in extreme cases, catastrophic structural damage to the entire system.
5. **Increased Wear and Tear:** Uneven distribution of mass leads to additional stresses on bearings, shafts, and other components. This can accelerate wear and tear, reducing the overall lifespan of the machinery and increasing maintenance requirements.
6. **Energy Loss:** Imbalance can lead to increased friction and resistance in the system, causing energy loss. This results in reduced energy efficiency and higher operational costs over time.
7. **Reduced Precision:** Imbalance can compromise the precision of machinery, particularly in applications requiring high accuracy. This can impact the quality of the product or result in operational inefficiencies

2.4 The Principles of Balancing

- ✓ **Balancing Technique:** Balancing entails correcting or eliminating unwanted inertia force moments in rotating or reciprocating masses through a change in the location of the mass centers.
- ✓ **Machinery Design and Modification:** The process involves designing or modifying machinery to reduce unbalance, with the primary goal of achieving an acceptable level or complete elimination. A common approach is redistributing mass by adding or removing it from various machine members.
- ✓ **Crucial Role in Machinery Operation:** Balancing rotating parts is crucial for ensuring smooth and efficient machinery operation. This is achieved by minimizing vibrations, forces, and issues that could lead to wear and fatigue.
- ✓ **Principles and Precision:** The principles of balancing encompass both static and dynamic balancing. The magnitude and position of the balancing mass can be determined either analytically or graphically, allowing for precise adjustments. This precision is essential for achieving optimal balance in rotating systems.
- ✓ By using graphical method, we need to draw the mr/mrl polygon diagram.

i) FORCE POLYGON

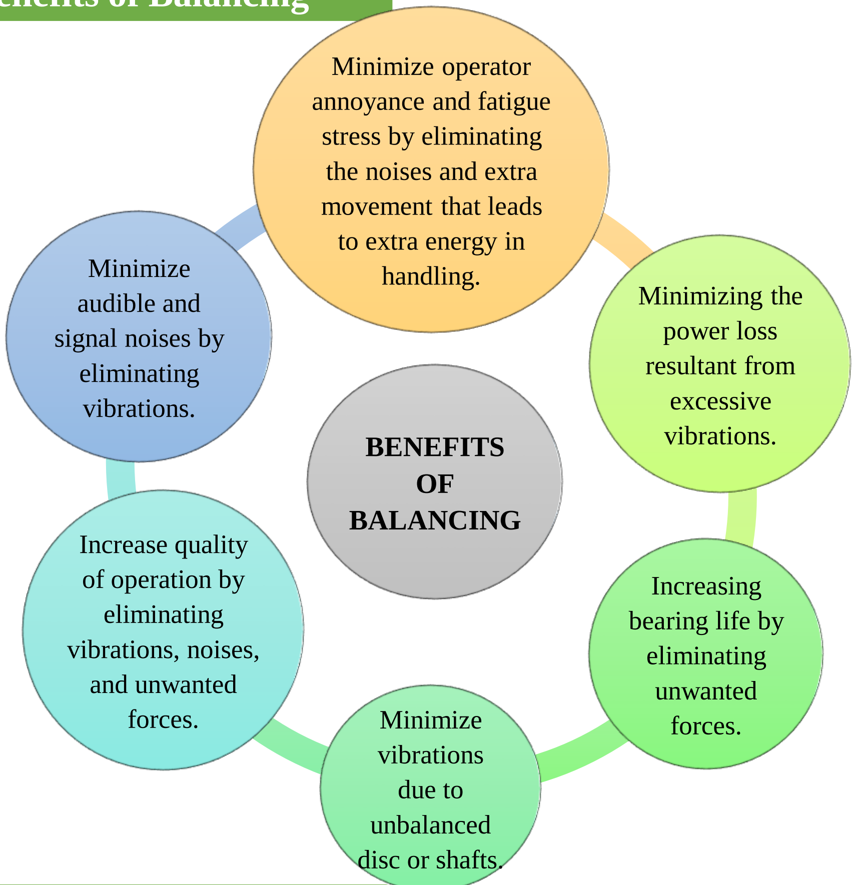
- ***mr*** polygon
- Rotating mass on *single plane*

ii) COUPLE POLYGON

- ***mrl*** polygon
- Rotating mass on *multiple plane*

- ✓ For a system to be incomplete balance both force and couple polygons should be **close** to prevent the effect of centrifugal force.
- ✓ If the system not incomplete balance, the polygon is **opened**.

2.5 Benefits of Balancing



2.6 Types of Balancing

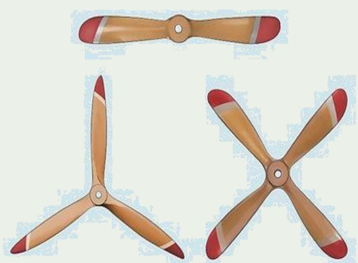
- ✓ Balancing of rotating massed can be;
  - i. Balancing of a single rotating mass by a single mass rotating in the same plane
  - ii. Balancing of a single rotating mass by two mass rotating in different planes
  - iii. Balancing of several masses rotating in the same plane
  - iv. Balancing of several masses rotating in different planes
- ✓ For a body to be completely balanced, the system must have both Static Balance and Dynamic Balance.

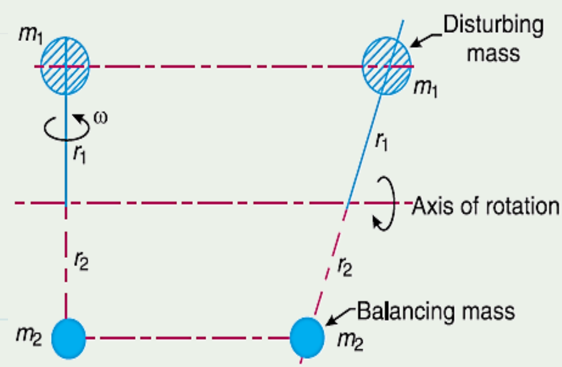
Table 2.1: Static Balancing vs Dynamic Balancing.

STATIC BALANCING	DYNAMIC BALANCING
Static balancing is the process of ensuring that an object is in equilibrium when stationary or at rest.	Dynamic balancing is the process of ensuring that an object is in equilibrium while in motion.
The main goal is to eliminate unbalanced forces or moments that would cause the object to tilt or vibrate when at rest.	The primary aim is to reduce or eliminate forces and moments that arise due to uneven mass distribution, preventing vibration during motion.
Typically achieved by redistributing mass within the object or adding counterweights to align the centre of mass with the axis of rotation.	Involves identifying specific locations where unbalanced forces occur during rotation and adding or removing material or counterweights to balance the system dynamically.
Mathematically, a rotor is claimed to be in static balance if the algebraic sum of centrifugal forces is Zero (0). e.g: $\sum m\omega^2 r = 0$; as ω is same for all masses ✓ It can be written as; $\sum \boldsymbol{mr} = \boldsymbol{0}$	Mathematically, a rotor is claimed to be in dynamic balance if the algebraic sum of centrifugal forces is zero as well as the algebraic sum of centrifugal couples is also equal to zero (0). ✓ It can be written as; $\sum \boldsymbol{mr} = \boldsymbol{0}$; Force Balance $\sum \boldsymbol{mrl} = \boldsymbol{0}$; Couple balance
Crucial in various engineering applications, such as designing rotating machinery like fans, crankshafts, and flywheels.	Essential in applications like automotive engines, industrial turbines, and rotating equipment to ensure smooth operation and extend machinery lifespan

2.6.1: BALANCING OF SEVERAL MASS ROTATING IN SAME PLANE

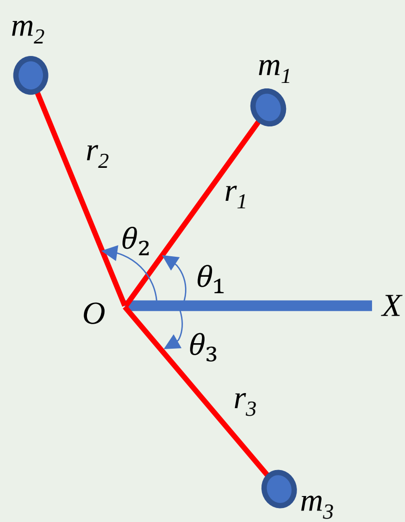
- Balancing of a single rotating mass by a single mass rotating in the same plane
- Example:





- Known as: FORCE POLYGON /mr POLYGON (on a single plane).
- 1 unknown [magnitude (mass) or angle (direction)].

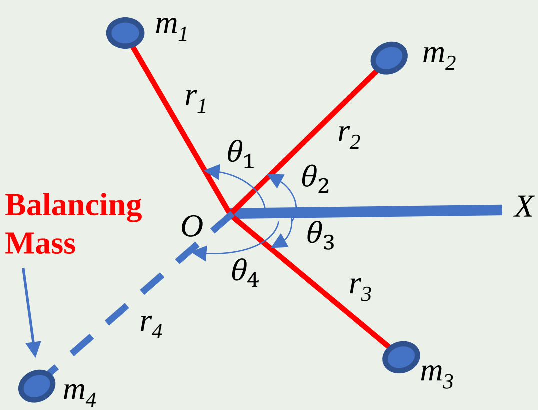
- Consider any number of *masses* of magnitude m_1, m_2 and m_3 at distances of r_1, r_2 and r_3 from the axis (X) of the rotating shaft. (Magnitude)
- Let θ_1, θ_2 and θ_3 be the *angles* of these masses with the horizontal line OX as shown in figure. (Direction)



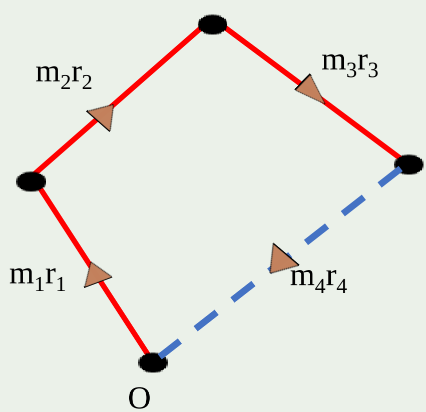
2.6.1a: BALANCING OF SEVERAL MASS ROTATING IN SAME PLANE (STATIC CONDITION)

- If the shaft in static condition (didn't rotate), the balancing is in static balance
- So that the centre of gravity of the system is on the axis of shaft which is O
- If the system is in complete balance, force polygon/mr polygon is closed.
- So; $\sum \boldsymbol{mr} = \mathbf{0}$ (When: m = mass, kg ; r = radius, m)

System (Space Diagram)



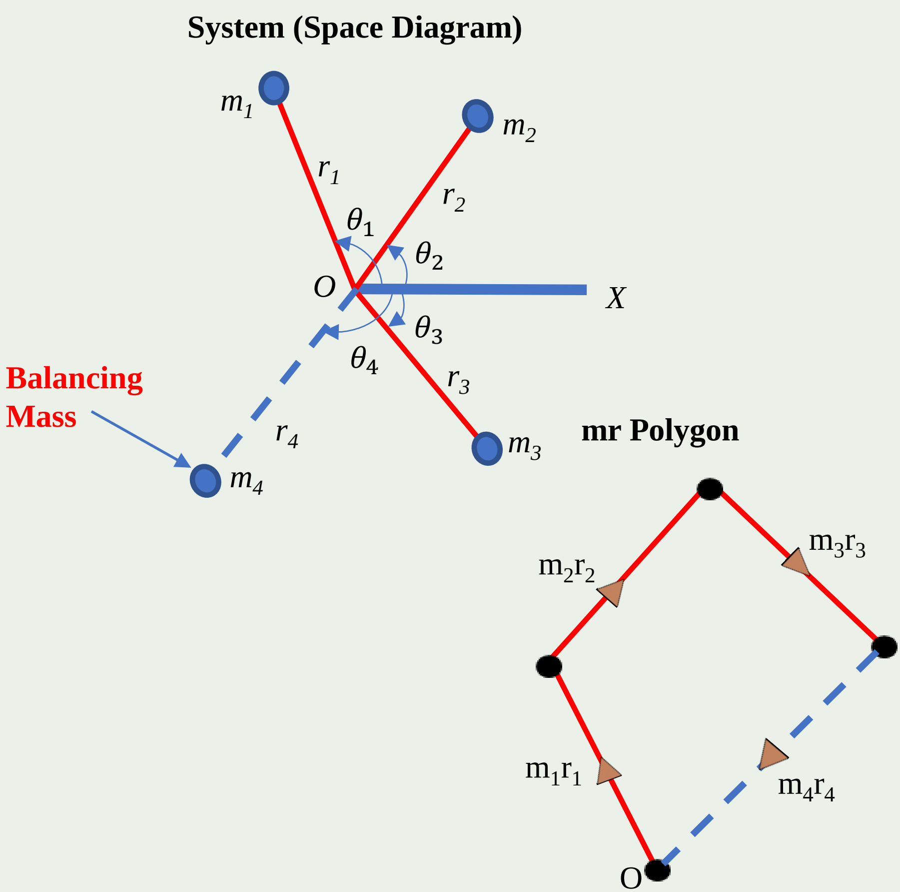
mr polygon



2.6.1b: BALANCING OF SEVERAL MASS ROTATING IN SAME PLANE (DYNAMIC CONDITION)

- If the shaft is rotating, the balancing is in dynamic balance
- Centrifugal force occur at each masses
- If the system is in complete balance, mr polygon is closed
- So; $\sum mr\omega^2 = 0$

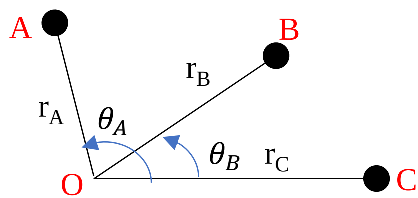
(When: m = mass, kg ; r = radius, m ; ω = angular velocity, rad/s)



2.6.1c: HOW to Construct mr POLYGON (FORCE POLYGON)

STEP TO DO:

Step 1 : Sketch **SPACE DIAGRAM** with position of several masses



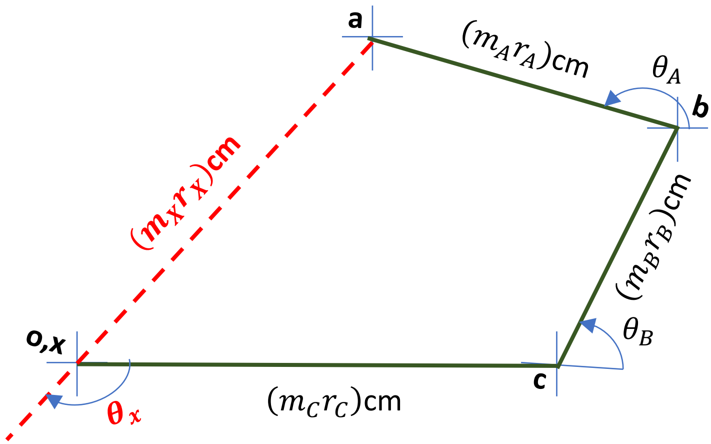
Step 2 : Construct **mr TABLE** – obtain centrifugal force (or mr value)

mr TABLE;

Mass	**m (kg)**	**r (m)**	**mr (kgm)**	**SCALE 1 cm: ___ kgm**
A	m_A	r_A	$m_A r_A$	$(m_A r_A)$ cm
B	m_B	r_B	$m_B r_B$	$(m_B r_B)$ cm
C	m_C	r_C	$m_C r_C$	$(m_C r_C)$ cm
X	m_X	r_X	$m_X r_X$	$(m_X r_X)$ cm

***Convert kgm to cm (refer scale)**

Step 3 : Sketch the **VECTOR DIAGRAM** with appropriate scale. This several Vector Diagrams will construct the **FORCE POLYGON /mr POLYGON**.



Step 4 : Convert balancing force from cm to kgm (refer scale), **determine;**

i. the magnitude (**mass, m kg**) and direction (**angle, θ° → ccw/cw**) of the balancing mass.

Example (refer figure above):

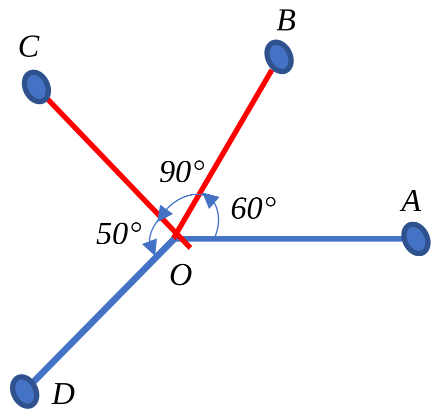
$$mass = m_x\,kg \quad ; \quad angle = \theta_x^{\circ}\ cw\ from\ C$$

ii. For **static balanced:** magnitude of balancing force or unbalanced force; $\sum mr = 0$

iii. For **dynamic balanced:** magnitude of balancing force or unbalanced force; $\sum mr\omega^2 = 0$

EXERCISE 1:

Four masses rotates at the same plane was attached on the shaft as shown on figure below. Mass A = 34 kg and the radius = 250 mm; Mass B = 23 kg and the radius = 300 mm; Mass C = 10 kg and the radius = 350 mm; Mass D = 15 kg and the radius = 170 mm. Calculate :

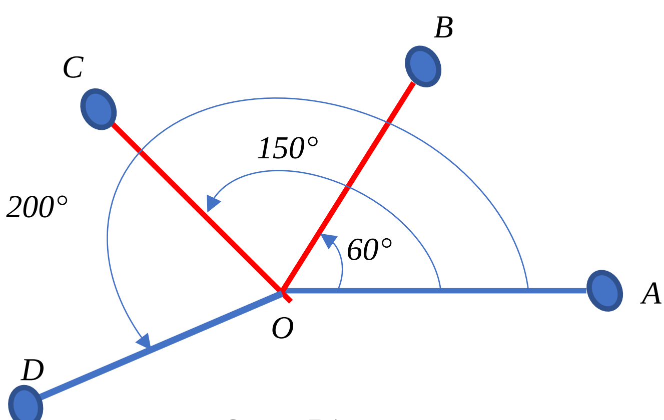


a) Unbalanced force act on the shaft when the system rotates at 18 rotation per second

b) Magnitude of balancing mass that located at radius 400 mm and the angle relative to mass A

Solution:

Step 1 : Construct Space Diagram



Space Diagram

Solution:

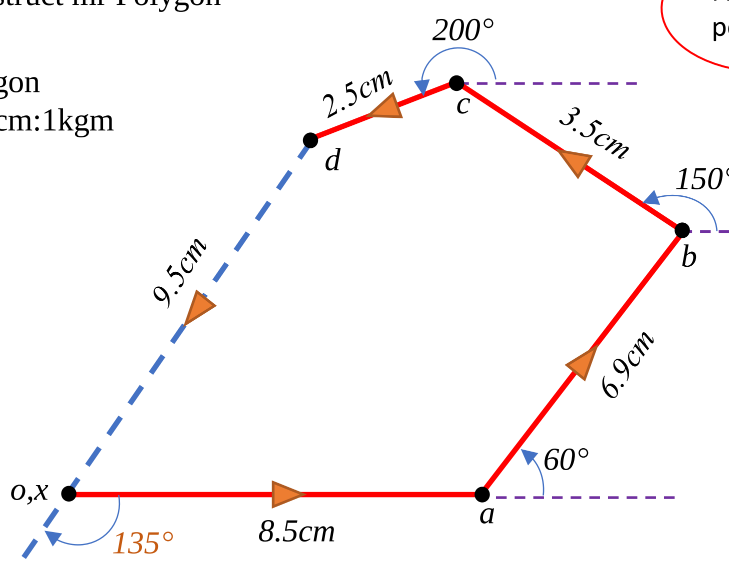
Step 2 : Construct *mr* table

Mass	**m(kg)**	**r(m)**	**mr(kgm)**	**Scale (1cm:1kgm)**
A	34	0.25	8.5	8.5 cm
B	23	0.30	6.9	6.9 cm
C	10	0.35	3.5	3.5 cm
D	15	0.17	2.55	2.6 cm
X	**m_x**	**0.40**	**$0.4m_x$**	**9.5 cm**

From mr polygon

Step 3 : Construct mr Polygon

mr Polygon
Scale: 1cm:1kgm



Solution:

Step 4 : Convert balancing force from cm to kgm

From the mr polygon, '***dx***' represents the balancing force for rotating masses A, B, C & D;

$$dx = m_X r_X = 9.5\ cm \times \frac{1\ kgm}{1\ cm} = 9.5\ kgm$$

$$\therefore \boldsymbol{m_X\ r_X = 9.5\ kgm}$$

a) Unbalance force acting on the shaft when the system rotates at 18 rotation per second;

$$\omega = 2\pi N = 2\pi\ (18) = 113.1\ rad/s$$

$$\therefore Unbalance\ force\ = (m_X r_X)\omega^2$$

$$F_U = (9.5)(113.1)^2 = \mathbf{121514.57\ N}$$

b) Magnitude and angle of balancing mass, m_X

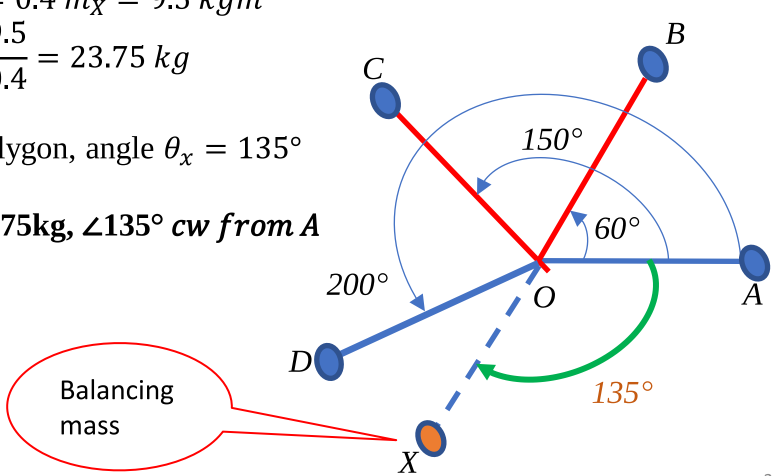
$$m_X r_X = 0.4\ m_X = 9.5\ kgm$$

$$m_X = \frac{9.5}{0.4} = 23.75\ kg$$

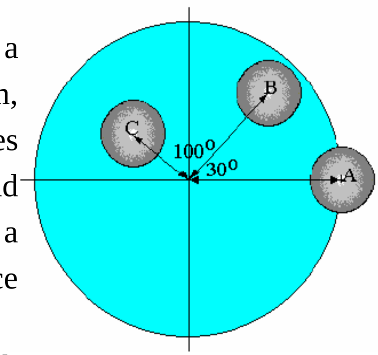
From mr polygon, angle $\theta_x = 135°$

$\therefore$ $\boldsymbol{m_X}$ **= 23.75kg, ∠135°** $\boldsymbol{cw\ from\ A}$

Space Diagram



EXERCISE 2:



Three masses A, B and C are placed in a balanced disc as shown at radii of 120 mm, 100 mm and 80 mm respectively. The masses are 1 kg, 0.5 kg and 0.7 kg respectively. Find the 4th masses which should be added at a radius of 60 mm in order to statically balance the system.

$$\textit{Ans:}\ m_X = 2.4\ kg\ \angle 152°\ cw\ from\ A$$

EXERCISE 3:

Four masses of P, Q, R and S are placed on a shaft and rotated in the same plane. The position angle Q, R and S in measured clockwise from P is 70°, 130° and 210°, respectively. The magnitude of the mass and distance from the axis of rotation for the P, Q, R and S are as follows; P : 16 kg, 500 mm; Q : 20 kg, 300 mm; R : 12 kg, 150 mm and S : 35 kg, 400 mm. Find the magnitude and angular position of a mass balance that has a radius of rotation at 520 mm.

$$\textit{Ans:}\ m_X = 6.1538kg\ , \angle 3°\ cw\ from\ P$$

EXERCISE 4:

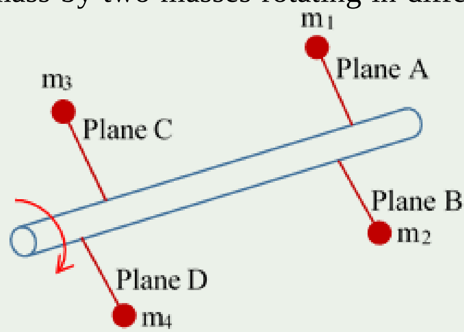
A, B, C and D are four rotating masses in the same plane on an axis O. The position angle of the mass are as follows: Angle AOB = 45°, angle BOC = 45° and angle COD = 100°. The mass and radius of rotation A : 3 kg, 700mm; B : 9 kg, 300mm; C : 4 kg, 1100 mm and D : 8 kg, 900 mm. Find :

a) an unbalanced force of system when it is speed at 500 rpm
b) the radius of rotation and the angular position of a mass balance at 7.5 kg

Ans: **a)** F**=15.901 k**N ; $b)\ r_X = 0.7733m\ ; \angle 58°\ cw\ from\ A$

2.6.2 BALANCING OF SEVERAL MASS ROTATING IN DIFFERENT PLANE

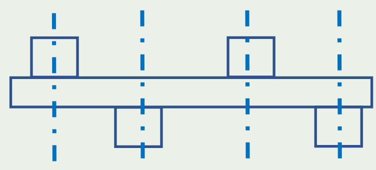
- Balancing of a single rotating mass by two masses rotating in different planes/multiple planes



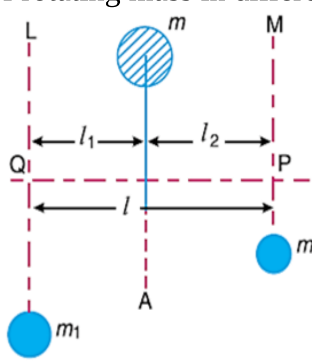
- Multiple Planes;

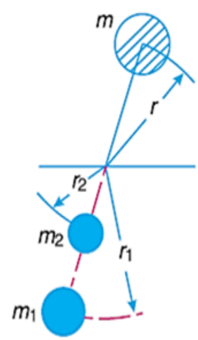
  1) Force/mr polygon
  2) Couple/mrl polygon

     ✓ 2 unknown [magnitude (mass) and angle (direction)]



- Example of rotating mass in different plane:





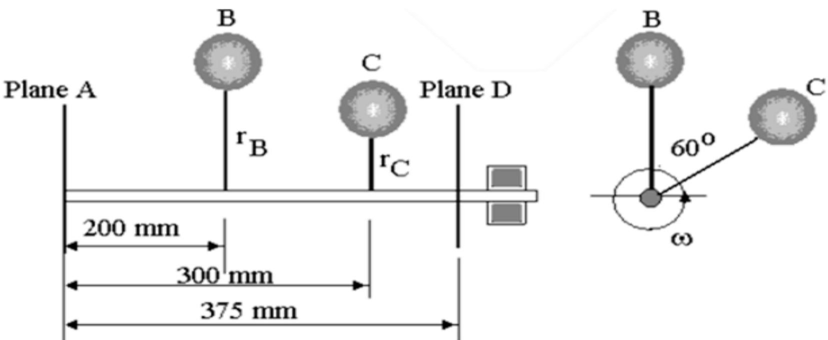
- For static balance, unbalanced force: $\sum mrl = 0$
- For dynamic balance, unbalanced couple: $\sum mrl\omega^2 = 0$
- All these unbalanced force and unbalanced couple must be eliminated to balance the system.

EXERCISE 5:

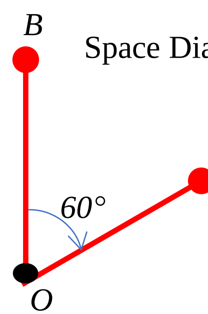
Find the mass and the angle at which it should be positioned in planes A and D at a radius of 60 mm to produce complete balance of system shown;

Radius B is 75 mm   Radius C is 50 mm

Mass of B is 5 kg   Mass of C is 2 kg



Solution:



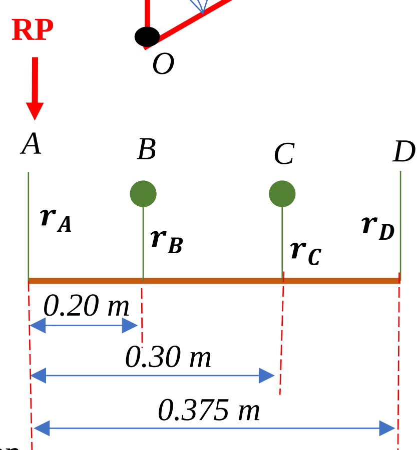
Space Diagram

Step 1 :
Draw the space diagram with position of several masses. Construct Space Diagram.

Step 2 : Sketch mass position:

i. Choose **References Plane (RP)**. Take one of the plane as the RP (**unknown value**).

ii. Insert distance, *l*. (**measure refer to RP)**



Mass Position

EXERCISE 5: Solution

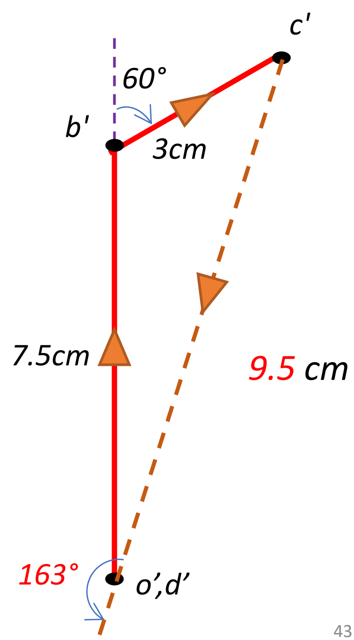
Step 3 : Construct *mrl* table – obtained unbalanced couple. The couple can be represented by the couple polygon (mrl polygon).

Plane	*m (kg)*	*r (m)*	*mr (kgm)*	*l (m)*	*mrl (kgm^2)*	*Scale : mr 1cm:0.05kgm*	*Scale : mrl 1cm:0.01kgm^2*
RP →A	m_A	0.06	$0.06m_A$	0	0	3.7 cm	0
B	5	0.075	0.375	0.2	0.075	7.5 cm	7.5 cm
C	2	0.05	0.1	0.3	0.03	2.0 cm	3.0 cm
D	m_D	0.06	$0.06m_D$	0.375	$0.0225m_D$	5.1 cm	9.5 cm

i. Sketch *mr* Polygon or *mrl* polygon. **How to choose?**
ii. If the polygon closed, the system is in balanced condition
iii. If the polygon opened, the system is in unbalanced condition. Need to draw the balancing force. So that the polygon become closed. The direction of balancing force in opposite direction

Step 4:
i. construct mr Polygon & mrl polygon.
ii. Find m & θ

Draw Couple/*mrl* Polygon (1 cm: 0.01 kgm^2) (Graph Paper)



EXERCISE 5: Solution

From *mrl* polygon, ***c'd'*** = $m_D\, r_D\, l_D$ = 9.5cm

Convert cm to $\boldsymbol{kgm^2}$

$$m_D r_D l_D = 9.5\ cm \times \frac{0.01\ kgm^2}{1\ cm}$$

$$\therefore m_D\ r_D l_D = 0.095 kgm^2$$

Find mass & Angle of balancing mass

From *mrl* table;

$$m_D r_D l_D = 0.0225 m_D = 0.095 kgm^2$$

$$m_D = 0.095/0.0225 = 4.2222 \text{ kg}$$

$\therefore$ $\boldsymbol{m_D}$ **= 4.2222 kg ,** $\boldsymbol{\angle 163° \ ccw\ from\ B}$

Now, we are able to calculate force (mr) for m_D (from *mrl* table);

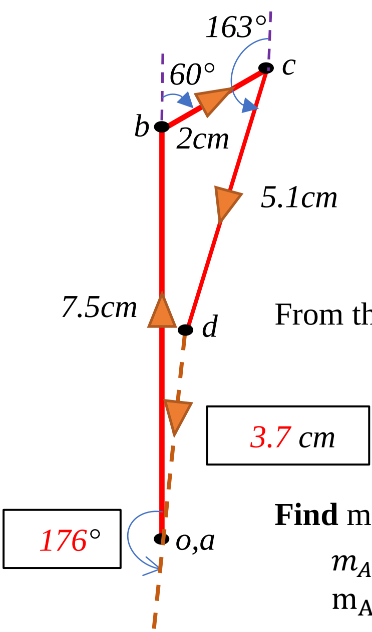
$$m_D r_D = 0.06\text{mD} = 0.06(4.2222) = 0.2533\ kgm$$

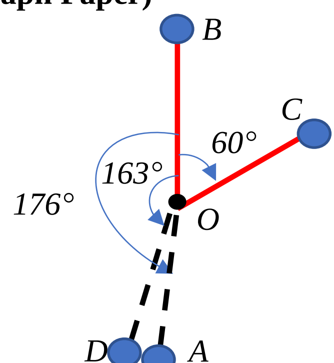
Convert kgm to cm

$$m_D r_D = 0.2533\ kgm\ x\ \frac{1\ cm}{0.05\ kgm}$$

= 5.066 cm = **5.1cm**

mr Polygon (Scale: 1 cm: 0.05 kgm) (Graph Paper)





From the force/mr polygon;

$$\boldsymbol{da} = m_A r_A = 3.7 cm \times \frac{0.05\ kgm}{1\ cm}$$

$$\therefore m_A\ r_A = 0.185 kgm$$

Find mass & Angle of balancing mass : From *mrl* table;

$$m_A r_A = 0.06 m_A = 0.185 kgm$$

$$m_A = 3.0833 \text{ kg}$$

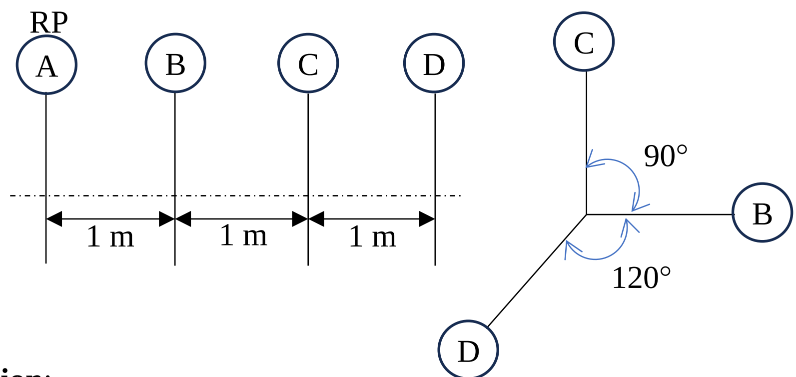
$\therefore$ $\boldsymbol{m_A}$ **= 3.0833kg ,** $\boldsymbol{\angle 176° \ ccw\ from\ B}$

EXERCISE 6:

A rotating shaft has four masses A, B, C and D with a radius 100 cm. The mass B is 7 kg with an angle BOC 90° and BOD as shown figure below. Find;

a) the masses of C and D.

b) the magnitude and the angular position of A, so the system is balanced.



Solution:

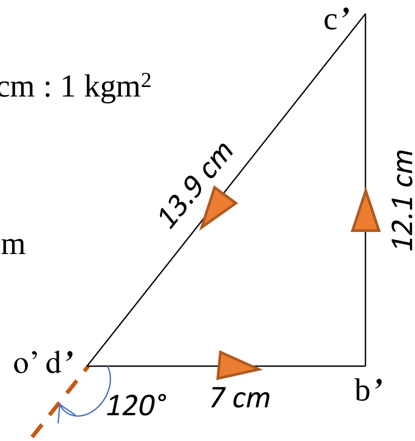
a) The masses of C and D.

Plane	*m (kg)*	*r (m)*	*mr (kgm)*	*l (m)*	*mrl (kgm^2)*	*Scale : mr 1cm: 1 kgm*	*Scale : mrl 1cm: 1 kgm^2*
RP A	m_A	1	m_A	0	0	5.3 cm	0
B	7	1	7	1	7	7 cm	7 cm
C	m_C	1	m_C	2	$2m_C$	6.1 cm	12.1 cm
D	m_D	1	m_D	3	$3m_D$	4.6 cm	13.9 cm

EXERCISE 6: Solution

a) The masses of C and D.

mrl polygon: 1cm : 1 kgm^2



From *mrl* polygon, b'***c'*** = $m_C\, r_C\, l_C$ = 10.8 cm

Convert cm to $\boldsymbol{kgm^2}$

$$m_C\, rC\, l_C = 12.1\ cm\ \times \frac{1\ kgm^2}{1\ cm}$$

$$m_C\, rC\, l_C = 12.1\ kgm^2$$

Find mass & Angle of balancing mass

From *mrl* table;

$m_C\, rC\, l_C = 2m_c = 12.1\ kgm^2$

$\boldsymbol{m_C = 12.1/2 = 6.05}$**kg**

From *mrl* polygon, c'***d'*** = $m_D\, r_D\, l_D$ = 13.9 cm

Convert cm to $\boldsymbol{kgm^2}$

$$m_D\, rD\, l_D = 13.9cm\ \times \frac{1\ kgm^2}{1\ cm}$$

$$\therefore\ m_D\, rD\, l_D = 13.9\ kgm^2$$

Find mass & Angle of balancing mass

From *mrl* table;

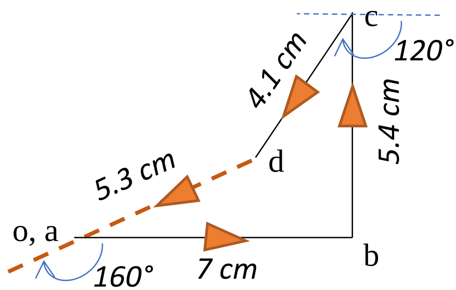
$m_D\, rD\, l_D = 3m_D = 13.9\ kgm^2$

$\boldsymbol{m_D = 13.9/3 = 4.63}$ **kg** $(\angle 120°$ CW From B)

EXERCISE 6: Solution

b) the magnitude and the angular position of A, so the system is balanced

mr polygon: 1cm : 1 kgm



From *mr* polygon, ***d, a*** = $m_A\, r_A$ = 5.3 cm

Convert cm to kgm

$$m_A r_A = 5.3\ cm\ x\ \frac{1kgm}{1cm} = 5.3kgm$$

Find mass & Angle of balancing mass

From *mrl* table; $m_A r_A = \ m_A\,(1m) = 5.3\ kgm$

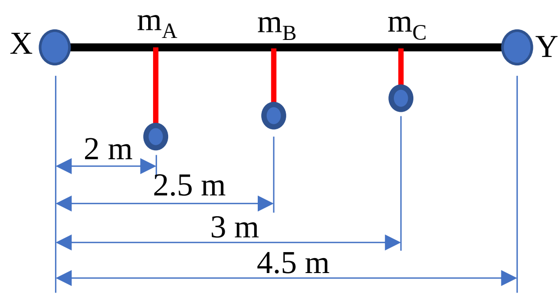
$m_A\ = 5.3kg/1m = 5.3kg$

$\therefore$ $\boldsymbol{m_A}$ **= 5.3 kg ,** $\boldsymbol{\angle 160°\ ccw\ from\ B}$

EXERCISE 7:

Figure below shows three masses attached to a shaft. The shaft is supported by bearings at both ends. The system is in static equilibrium.

m_A = 7 kg radius = 0.2 m
m_B = 12 kg radius = 0.18 m
m_C = 15 kg radius = 0.15 m



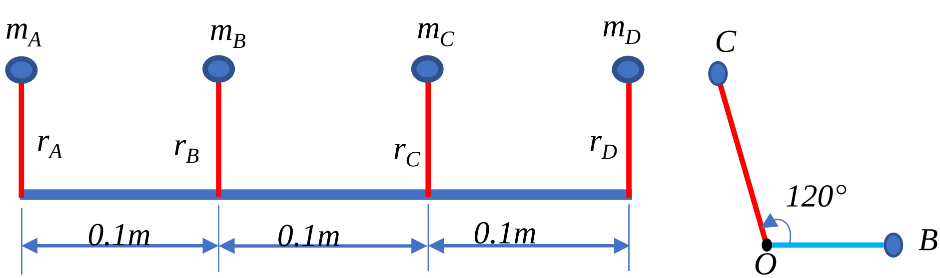
By using the data given :

a) Complete the mrl table
b) Draw the space diagram, the mr and mrl polygon
c) Determine the unbalance couple for the shaft when it rotates at 180 rpm

Ans; ***c***) $\tau = 532.9554\ Nm$

EXERCISE 8:

A shaft has 4 discs A, B, C and D along its length 100 mm apart. A mass of 0.8 kg is placed on B at a radius of 20 mm. A mass of 2 kg is placed on C at a radius of 30 mm and rotated 120° from the mass on B. Find the masses to be placed on A and D at a radius of 25 mm that will produce total balance.



Ans;

$m_D = 1.5067\ kg\ \angle 67°\ cw\ from\ B$

$m_A = 0.7\ kg\ \angle 95°\ cw\ from\ B$

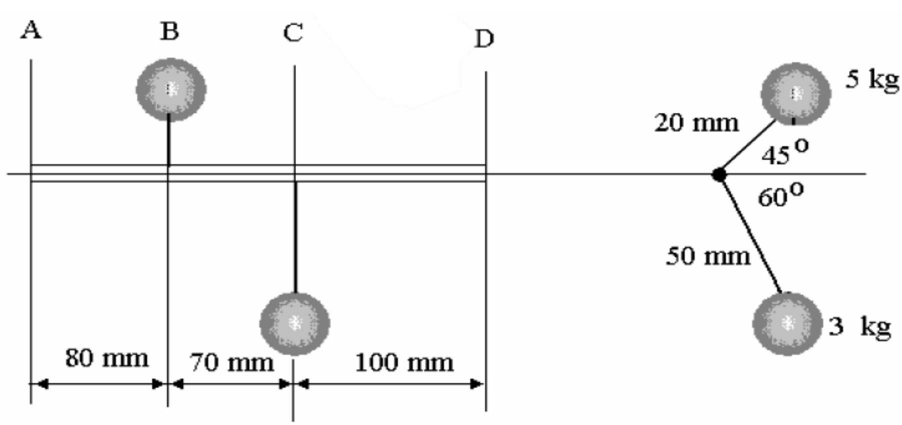
EXERCISE 9:

A shaft carries four masses in parallel planes A, B, C and D in this order along its length. The masses at B and C are 18 kg and 12.5 kg respectively, and each has an eccentricity of 60 mm. The masses at A and D have an eccentricity of 80 mm. The angle between the masses at B and C is 100° and that between the masses at B and A is 190°, both being measured in the same direction. The axial distance between the planes A and B is 100 mm and that between B and C is 200 mm. If the shaft is in complete dynamic balance, determine :

a) The magnitude of the masses at A and D
b) The distance between planes A and D
c) The angular position of the mass D

EXERCISE 10:

The diagram below shows masses on two rotors in planes B and C. Determine the masses to be added on the rotors in planes A and D at a radius of 40 mm which will produce static and dynamic balance.



FORMATIVE EVALUATION

Velocity and Acceleration Diagram

QUESTION 1:

In the mechanism, as shown in figure 1, the crank OA rotates at 20 rpm anticlockwise and gives motion to the sliding blocks B and D. The dimensions of the various links are OA = 300 mm; AB = 1200 mm; BC = 450 mm and CD = 450 mm. For the given configuration;

i. Sketch Space diagram with scale of 1 cm : 100 mm.
ii. Sketch velocity diagram with scale of 1 cm : 0.1 m/s.
iii. Find velocity of link AB.
iv. Find the velocity of piston D.
v. Find the angular velocity of link CD.

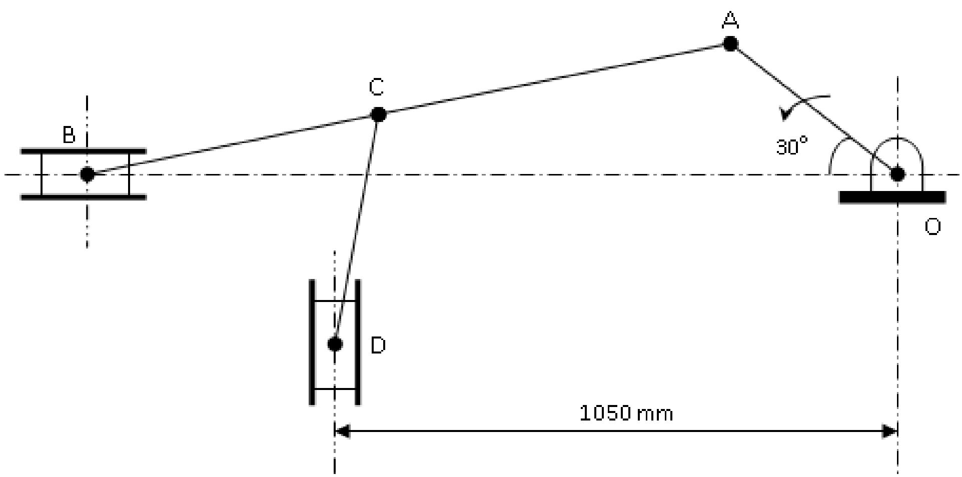


Figure 1

Ans;

$v_{ab} = 0.55\, m/s,\ \ v_d = 0.26\, m/s$

$\omega_{cd} = 0.8\, rad/s$

FORMATIVE EVALUATION

Velocity and Acceleration Diagram

QUESTION 2:

The dimensions of the various links of a mechanism, as shown in figure 2, are as follows: OA = 80 mm; AC = CB = CD = 120 mm.

If the crank OA rotates with constant speed at 150 rpm in the anti-clockwise direction and link CD perpendicular with link AB, find for the given configuration:

i. Draw the space diagram by scale of 1cm : 20mm
ii. Draw the velocity diagram by scale of 1cm : 0.2m/s
iii. Draw the accelerating diagram by scale of 1cm : 3m/s2
iv. velocity and acceleration of piston B and piston D
v. angular acceleration of the links AB and CD

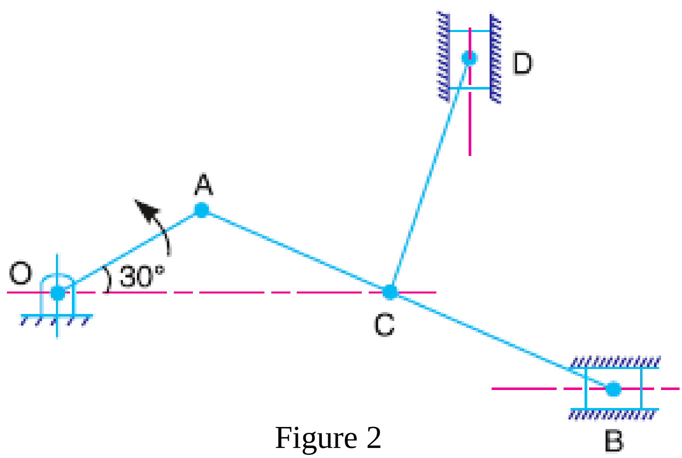


Figure 2

***Ans*;**

$a_b = 19.5\, m/s^2, a_d = 18\, m/s^2$

$v_b = 1.02\, m/s, v_d = 0.24\, m/s$

$\alpha_{ab} = 500\, rad/s^2, \alpha_{cd} = 550\, rad/s^2$

FORMATIVE EVALUATION

Balancing

QUESTION 3:

A shaft carries four masses A, B, C and D of magnitude 200 kg, 300 kg, 400 kg and 200 kg respectively and revolving at radius 80 mm, 70 mm, 60 mm and 80 mm in planes measured from A at 300 mm, 400 mm and 700 mm. The angles between the cranks measured anticlockwise are A to B is 45°, B to C is 70° and C to D is 120°. The balancing masses are to be placed in planes X and Y. The distance between the planes A and X is 100 mm, between X and Y is 400 mm and between Y and D is 200 mm. If the balancing masses revolve at a radius of 100 mm, find their magnitudes and angular positions.

$\boldsymbol{Ans;}$

$\boldsymbol{m_Y} = 182.5\,kg\ \angle 12°\ cw\ from\ A$

$\boldsymbol{m_X} = 355\,kg\ \angle 145°\ cw\ from\ A$

QUESTION 4:

A shaft carries three masses A, B and C of magnitude 200 kg, 300 kg and 400 kg respectively and revolving at radius 80 mm, 70 mm and 60 mm. The distances from A to B and C are 300 mm and 400 mm respectively. The angles between the cranks measured anticlockwise for A to B is 45°, B to C is 70°. The balancing masses are to be placed in planes X and Y at the end of shaft respectively. The distance between the planes X and A is 100 mm and between C and Y is 200 mm. If the balancing masses revolve at a radius of 100 mm, find :

a) Construct the balancing table for the system
b) Balancing mass and angular position of Y
c) Balancing mass and angular position of X

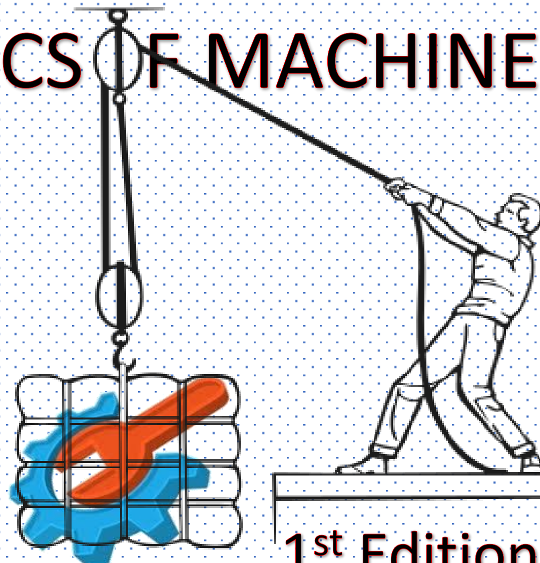
$\boldsymbol{Ans;}$

$\boldsymbol{m_Y} = 242.857 kg\ \angle 98°\ cw\ from\ A$

$\boldsymbol{m_X} = 215\,kg\ \angle 144°\ cw\ from\ A$

THEORY AND EXERCISE

MECHANICS OF MACHINES



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