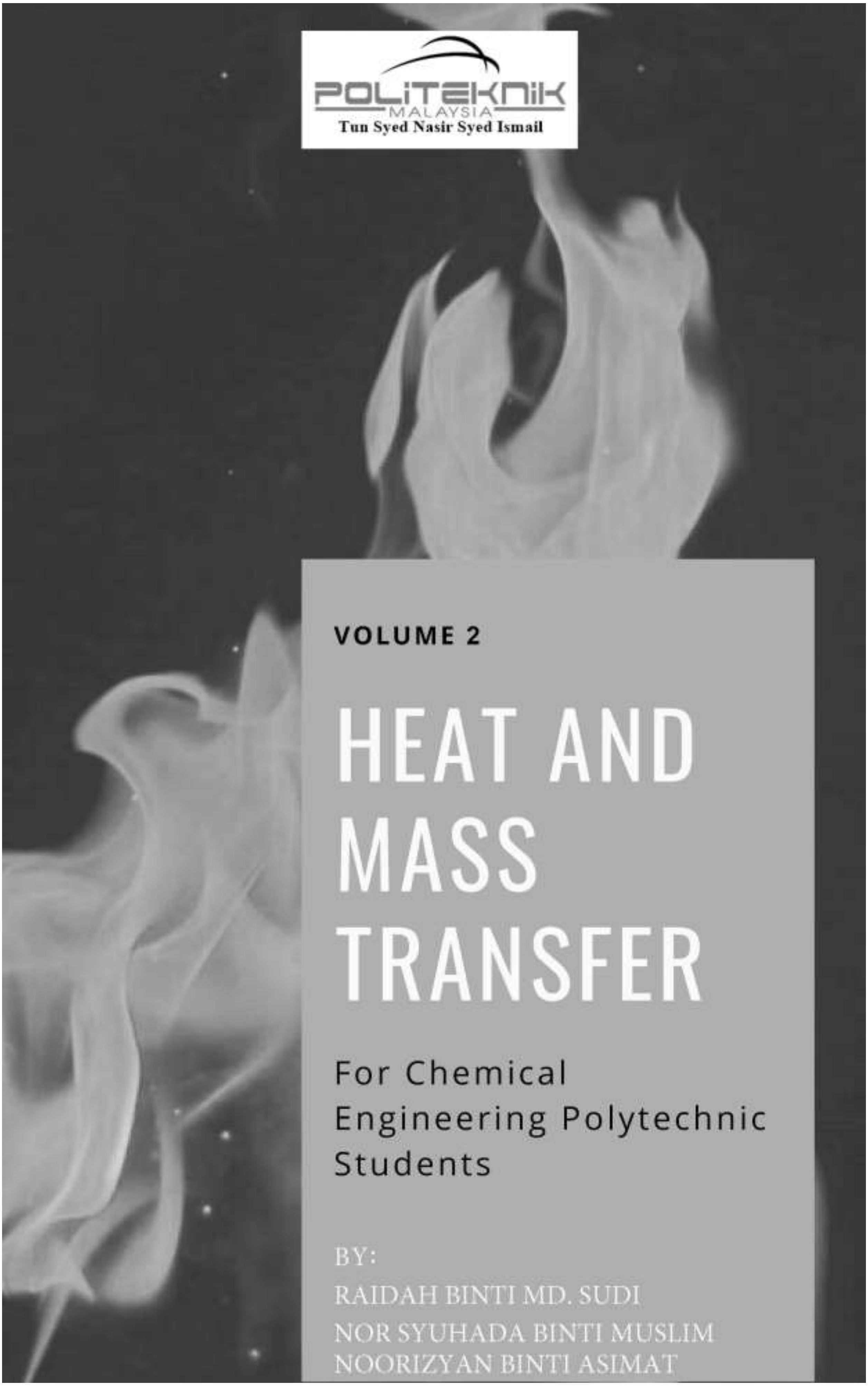
POLITEKNIK MALAYSIA
Tun Syed Nasir Syed Ismail

VOLUME 2

HEAT AND MASS TRANSFER

For Chemical Engineering Polytechnic Students

BY:
RAIDAH BINTI MD. SUDI
NOR SYUHADA BINTI MUSLIM
NOORIZYAN BINTI ASIMAT



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ENGINEERING POLYTECHNIC STUDENTS VOLUME 2

e ISBN 978-967-2736-33-2

9 789672 736332

Online

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Published in Malaysia by:
Politeknik Tun Syed Nasir Syed Ismail
Hab Pendidikan Tinggi Pagoh
KM 1 Jalan Panchor 84600 Pagoh, Johor
Tel: 06-974 2288 Faks: 06-974 2395
Website: www.ptsn.edu.my

Cataloguing-in-Publication Data

Perpustakaan Negara Malaysia

A catalogue record for this book is available from the National Library of Malaysia

eISBN 978-967-2736-33-2

PREFACE

HEAT AND MASS TRANSFER emphasize on the principles of the Heat Transfer in steady state by conduction, convection and radiation. Principles of steady-state and transient heat conduction in solid are investigated. Laminar and turbulent boundary layer flows are treated, as well as condensation and boiling phenomena, thermal radiation, and radiation heat transfer between surfaces. Students will be exposed to the procedure for general problem solving and its application on heat exchanger. Student will be also exposed to the mass transfer mechanisms and develop relations for mass transfer rate for situations commonly encountered in practice.

In this ebook, Heat and Mass Transfer Volume 2 will focused in the Steady State Heat Transfer in Radiation, Heat Exchanger and Mass Transfer. This book will help polytechnic student to refer following the course outline. The main objective of this book to help student more understand the content of the subject referring to the course outline with the excercises and examples.

Last but not least we would like to express our gratitude to our family, colleagues, e-learning unit, UIDM, library , management of Politeknik Tun Syed Nasir Syed Ismail and also Jabatan Pendidikan Polteknik and Kolej Komuniti for giving the chance to publish this Heat And Mass Transfer Volume 2 ebook.

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CONTENTS

CHAPTER 4

PRINCIPLES OF STEADY STATE HEAT TRANSFER IN RADIATION

At the end of this chapter, you should be able to:

- Apply the principles of thermal radiation.
- Discover the radiation properties

4.1 PRINCIPLE OF THERMAL RADIATION

So far, we have considered the conduction and convection modes of heat transfer, which are related to the nature of the materials involved and the presence of fluid motion, among other things. We now turn our attention to the third mechanism of heat transfer: radiation, which is characteristically different from the other two.

4.1.1 Principle of Radiation

Consider a hot object that is suspended in an evacuated chamber whose walls are at room temperature (Fig. 4.1). The hot object will eventually cool down and reach thermal equilibrium with its surroundings. That is, it will lose heat until its temperature reaches the temperature of the walls of the chamber. Heat transfer between the object and the chamber could not have taken place by conduction or convection, because these two mechanisms cannot occur in a vacuum. Therefore, heat transfer must have occurred through another mechanism that involves the emission of the internal energy of the object. This mechanism is radiation.

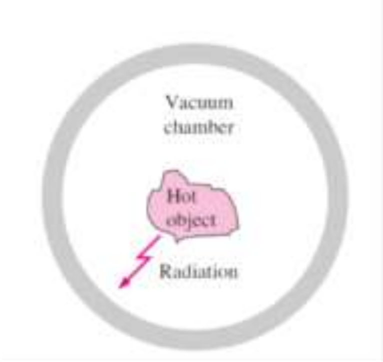


Figure 4.1 A hot object in a vacuum chamber loses heat by radiation only

Radiation is the energy emitted by matter in the form of electromagnetic waves (or photons) as a result of the changes in the electronic configurations of the atoms or molecules. Unlike conduction and convection, the transfer of energy by radiation does not require the presence of an intervening medium. In heat transfer studies we are interested in thermal radiation, which is the form of radiation emitted by bodies because of their temperature. A hot object that is suspended in an evacuated chamber whose walls are at room temperature. The hot object will eventually cool down and reach thermal equilibrium with its surroundings. Heat transfer between the object and the chamber could not have taken place by conduction or convection, because these two mechanisms cannot occur in a vacuum. Heat transfer must have occurred through another mechanism that involves the emission of the internal energy of the object.

Radiation differs from the other two heat transfer mechanisms in that it does not require the presence of a material medium to take place. In fact, energy transfer by radiation is fastest (at the speed of light) and it suffers no attenuation in a vacuum. Also, radiation transfer occurs in solids as well as liquids and gases. In most practical applications, all three modes of heat transfer occur concurrently at varying degrees. But heat transfer through an evacuated space can occur only by radiation. For example, the energy of the sun reaches the earth by radiation.

Simultaneous Heat Transfer Mechanisms

We mentioned that there are three mechanisms of heat transfer, but not all three can exist simultaneously in a medium. Although there are three mechanisms in heat transfer, a medium may involve only two of then simultaneously.

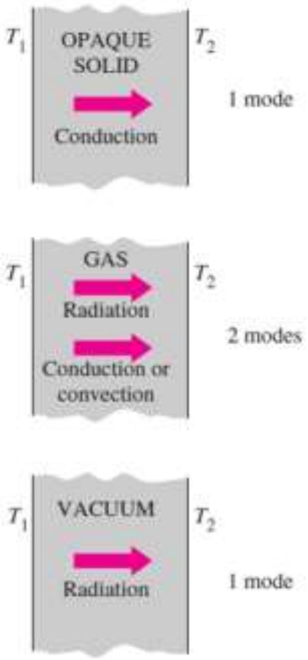


Figure 4.2: Mechanism of heat transfer

4.2.1 The Idealized Blackbody

Radiation-Emission

The maximum rate of radiation that can be emitted from a surface at a thermodynamic temperature T_s (in K or R) is given by the Stefan–Boltzmann law as

$$\dot{Q}_{emit,\max} = \sigma A_s T_s^4 \quad \text{(W)}$$

σ=5.670X10-8 W/m2·K4 is the Stefan–Boltzmann constant.

The idealized surface that emits radiation at this maximum rate is called a blackbody. The radiation emitted by all real surfaces is less than the radiation emitted by a blackbody at the same temperature, and is expressed as

$$\dot{Q}_{emit,\max} = \varepsilon \sigma A_s T_s^4 \quad \text{(W)}$$
$$0 \le \varepsilon \le 1$$

Where ε is the emissivity of the surface

Thermal Radiation

Electromagnetic radiation that is pertinent to heat transfer is the thermal radiation emitted as a result of energy transitions of molecules, atoms, and electrons of a substance. Temperature is a measure of the strength of these activities at the microscopic level, and the rate of thermal radiation emission increases with increasing temperature. Everything around us such as walls, furniture, and our friends constantly emits (and absorbs) radiation. Thermal radiation is also defined as the portion of the electromagnetic spectrum that extends from about 0.1 to 100 m, since the radiation emitted by bodies due to their temperature falls almost entirely into this wavelength range.

There are many types of electromagnetic radiation; thermal radiation is only one. Regardless of the type of radiation, we say that it is propagated at the spped of light, 3×10^8 m/s. This speed is equal to the product of the wavelength and frequency of the radiation,

$$c = \lambda v$$

where c = speed of light

λ = wavelength

v = frequency

The unit for λ may be centimetres, angstroms (1 Å = 10^{-8} cm), or micrometers (1 µm = 10^{-6}m). A portion of the electromagnetic sprectrum is shown in Figure 4.3.

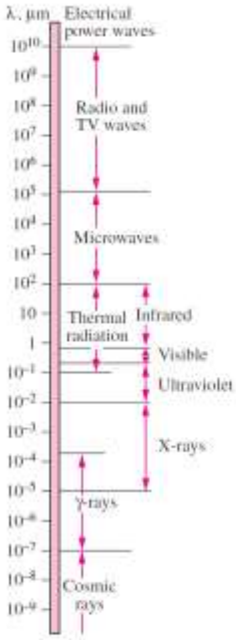
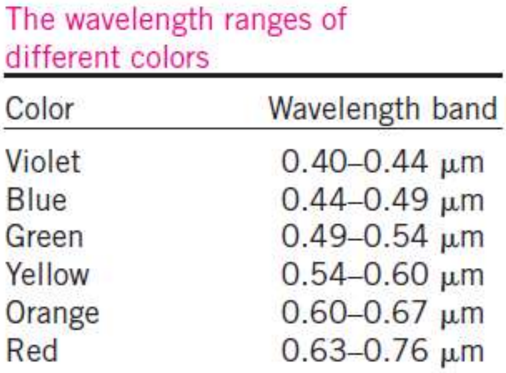


Figure 4.3 The electromagnetic wave spectrum

The electromagnetic radiation encountered in practice covers a wide range of wavelengths, varying from less than 10^{-10}µm for cosmic rays to more than 10^{10}µm for electrical power waves. The electromagnetic spectrum also includes gamma rays, X-rays, ultraviolet radiation, visible light, infrared radiation, thermal radiation, microwaves, and radio waves, as shown in Figure 4.4.

Thermal radiation lies in the range from about 0.1 to 100µm, since the radiation emitted by bodies due to their temperature falls almost entirely into this wavelength range. Thus, thermal radiation includes the entire visible and infrared (IR) radiation as well as a portion of the ultraviolet (UV) radiation.

What we call light is simply the visible portion of the electromagnetic spectrum that lies between 0.40 and 0.76 µm. Light is characteristically no different than other electromagnetic radiation, except that it happens to trigger the sensation of seeing in the human eye. Light, or the visible spectrum, consists of narrow bands of color from violet (0.40–0.44 µm) to red (0.63–0.76 µm), as shown in Table 4.1.



The wavelength ranges of different colors

Color	Wavelength band
Violet	0.40–0.44 µm
Blue	0.44–0.49 µm
Green	0.49–0.54 µm
Yellow	0.54–0.60 µm
Orange	0.60–0.67 µm
Red	0.63–0.76 µm

Table 4.1 : The wavelength ranges of different colours

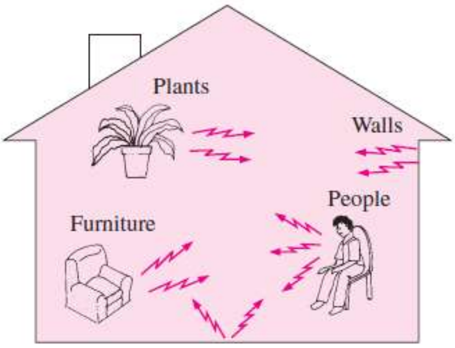


Figure 4.4 Everything around us constantly emits thermal radiation.

Blackbody Radiation

A **blackbody** is defined as *a perfect emitter and absorber of radiation.* At a specified temperature and wavelength, no surface can emit more energy than a blackbody. A blackbody absorbs *all* incident radiation, regardless of wavelength and direction. Also, a blackbody emits radiation energy uniformly in all directions per unit area normal to direction of emission. (Fig. 4.5). That is, a blackbody is a *diffuse* emitter. The term *diffuse* means “independent of direction.”

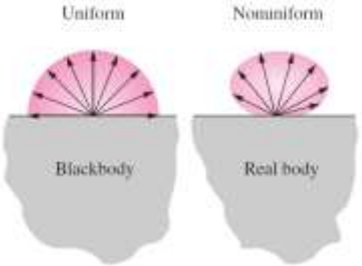


Figure 4.5 A blackbody is said to be a diffuse emitter since it emits radiation energy uniformly in all direction

The radiation energy emitted by a blackbody per unit time and per unit surface area was determined experimentally by Joseph Stefan in 1879 and expressed as

$$E_b(T) = \sigma T^4 \qquad (\mathrm{W/m^2})$$

where $\sigma = 5.67 \times 10^{-8}\ \mathrm{W/m^2 \cdot K^4}$ is the Stefan–Boltzmann constant and T is the absolute temperature of the surface in K. This relation was theoretically verified in 1884 by Ludwig Boltzmann. Equation above is known as the Stefan–Boltzmann law and E_b is called the *blackbody emissive power*. Note that the emission of thermal radiation is proportional to the fourth power of the absolute temperature.

We call this *blackbody radiation* because materials which obey this law appear black to the eye; they appear black because they do not reflect any radiation. Thus, a blackbody is also considered as one which absorbs all radiation incident upon it.

Sometimes we need to know the spectral blackbody emissive power, which is the amount of radiation energy emitted by a blackbody at an absolute temperature T per unit time, per unit surface area, and per unit wavelength about the wavelength λ. For example, we are more interested in the amount of radiation an incandescent light bulb emits in the visible wavelength spectrum than we are in the total amount emitted.

The relation for the spectral blackbody emissive power $E_{b\lambda}$ was developed by Max Planck in 1901 in conjunction with his famous quantum theory. This relation is known as Planck's law and is expressed as

$$E_{b\lambda}(\lambda, T) = \frac{C_1}{\lambda^5[\exp(C_2/\lambda T) - 1]} \qquad (\mathrm{W/m^2 \cdot \mu m})$$

Where

$$C_1 = 2\pi h c_0^2 = 3.742 \times 10^8\ \mathrm{W \cdot \mu m^4/m^2}$$
$$C_2 = hc_0/k = 1.439 \times 10^4\ \mathrm{\mu m \cdot K}$$

Also, T is the absolute temperature of the surface, λ is the wavelength of the radiation emitted, and $k = 1.38065 \times 10^{-23}$ J/K is *Boltzmann's constant*. This relation is valid for a surface in a vacuum or a gas.

The variation of the spectral blackbody emissive power with wavelength is plotted in Figure 4.5 for selected temperatures. Several observations can be made from this figure:

1. The emitted radiation is a continuous function of wavelength. At any specified temperature, it increases with wavelength, reaches a peak, and then decreases with increasing wavelength.

2. At any wavelength, the amount of emitted radiation increases with increasing temperature.

3. As temperature increases, the curves shift to the left to the shorter wavelength region. Consequently, a larger fraction of the radiation is emitted at shorter wavelengths at higher temperatures.

4. The radiation emitted by the sun, which is considered to be a blackbody at 5780 K (or roughly at 5800 K), reaches its peak in the visible region of the spectrum. Therefore, the sun is in tune with our eyes. On the other hand, surfaces at $T \leq$ 800 K emit almost entirely in the infrared region and thus are not visible to the eye unless they reflect light coming from other sources.

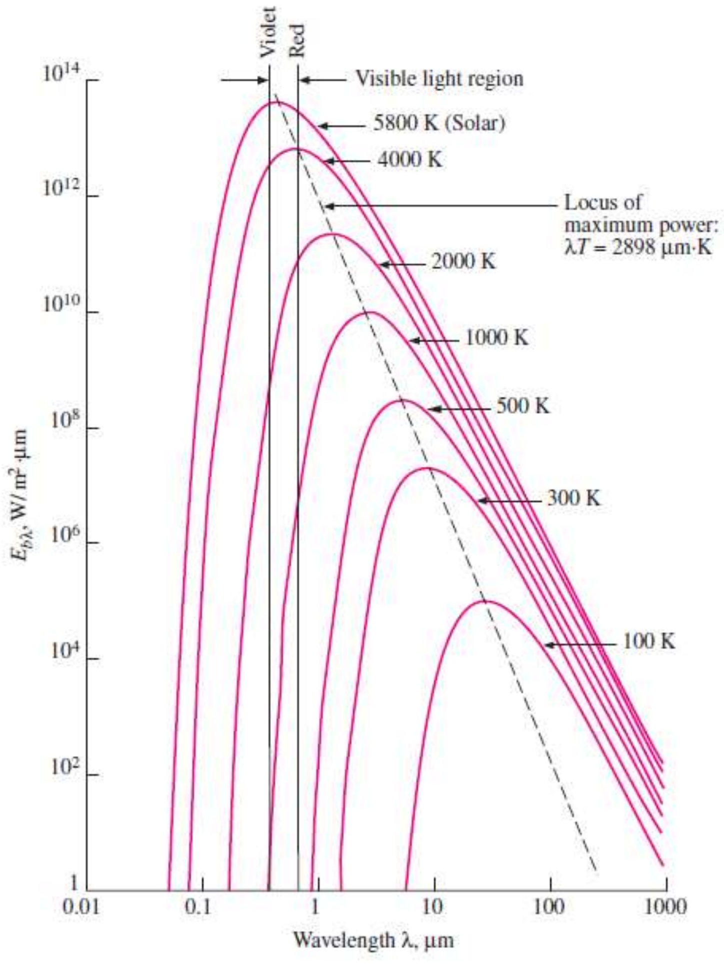


Figure 4.6 The variation of the blackbody emissive power with wavelength for several temperatures.

As the temperature increases, the peak of the curve in Figure 4.6 shifts toward shorter wavelengths. The wavelength at which the peak occurs for a specified temperature is given by Wien's displacement law as

$$(\lambda T)_{\text{max power}} = 2897.8\ \mu\text{m}\cdot\text{K}$$

A plot of Wien's displacement law, which is the locus of the peaks of the radiation emission curves, is also given in Figure 4.6. The peak of the solar radiation, for example, occurs at λ=2897.8/ 5780 = 0.50 µm, which is near the middle of the visible range. The peak of the radiation emitted by a surface at room temperature (T = 298 K) occurs at 9.72 µm, which is well into the infrared region of the spectrum.

It is left as an exercise to show that integration of the spectral blackbody emissive power $E_{b\lambda}$ over the entire wavelength spectrum gives the total blackbody emissive power E_b:

$$E_b(T) = \int_0^{\infty} E_{b\lambda}(\lambda, T)\, d\lambda = \sigma T^4 \qquad (\mathrm{W/m^2})$$

Thus, we obtained the Stefan–Boltzmann law by integrating Planck's law over all wavelengths. Note that on an $E_{b\lambda}$–λ chart, $E_{b\lambda}$ corresponds to any value on the curve, whereas E_b corresponds to the area under the entire curve for a specified temperature (Fig. 4.7). Also, the term total means "integrated over all wavelengths."

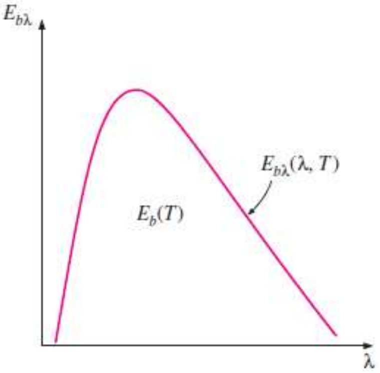
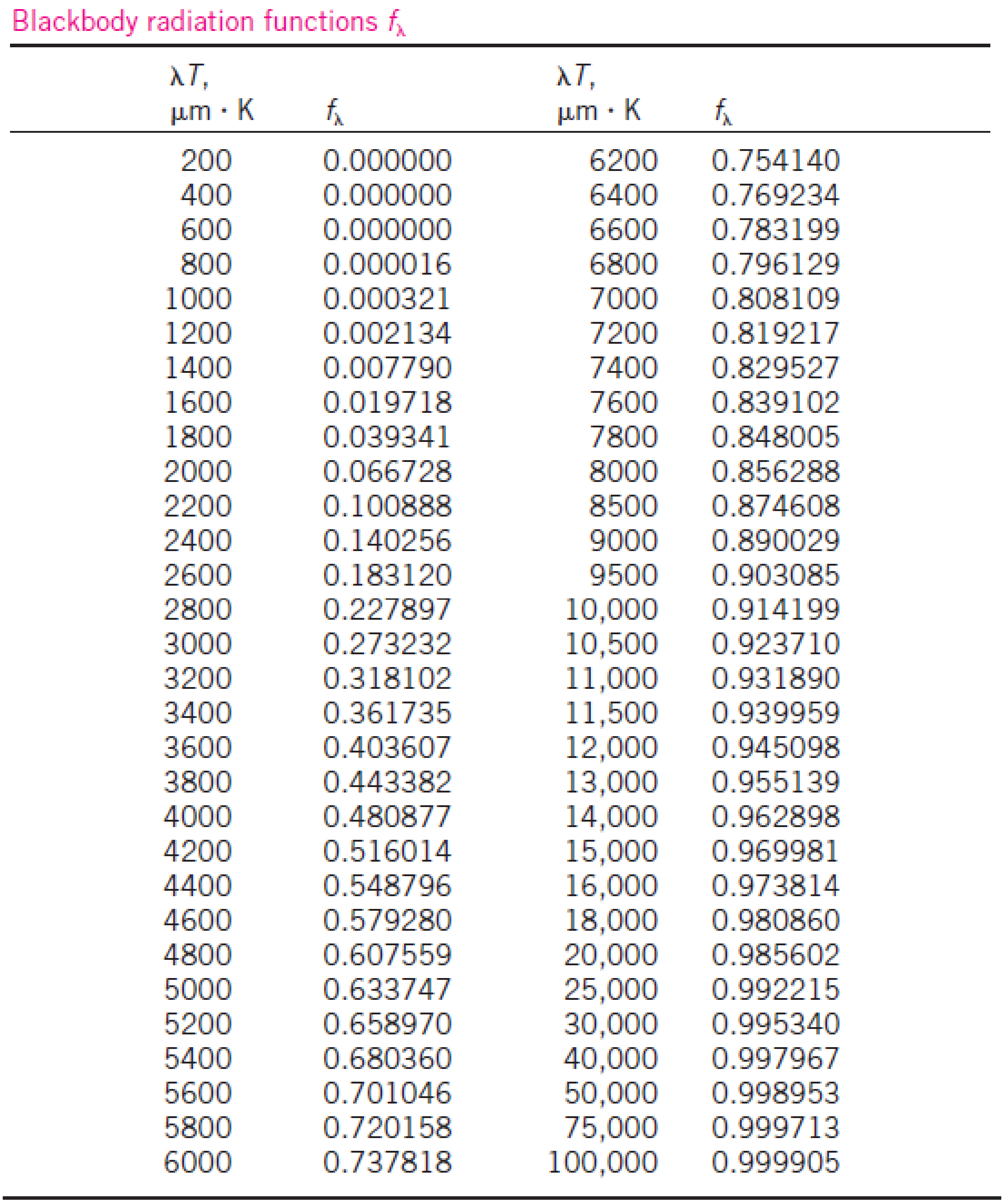


Figure 4.7 On an $Eb_{\lambda-\lambda}$ chart, the area under a curve for a given temperature represents the total radiation energy emitted by a blackbody at that temperature

$$E_{b,0-\lambda}(T) = \int_0^{\lambda} E_{b\lambda}(\lambda, T)\, d\lambda \qquad (\mathrm{W/m^2})$$

We define a dimensionless quantity f_λ called the blackbody function as



Blackbody radiation functions f_λ

λT, μm · K	f_λ	λT, μm · K	f_λ
200	0.000000	6200	0.754140
400	0.000000	6400	0.769234
600	0.000000	6600	0.783199
800	0.000016	6800	0.796129
1000	0.000321	7000	0.808109
1200	0.002134	7200	0.819217
1400	0.007790	7400	0.829527
1600	0.019718	7600	0.839102
1800	0.039341	7800	0.848005
2000	0.066728	8000	0.856288
2200	0.100888	8500	0.874608
2400	0.140256	9000	0.890029
2600	0.183120	9500	0.903085
2800	0.227897	10,000	0.914199
3000	0.273232	10,500	0.923710
3200	0.318102	11,000	0.931890
3400	0.361735	11,500	0.939959
3600	0.403607	12,000	0.945098
3800	0.443382	13,000	0.955139
4000	0.480877	14,000	0.962898
4200	0.516014	15,000	0.969981
4400	0.548796	16,000	0.973814
4600	0.579280	18,000	0.980860
4800	0.607559	20,000	0.985602
5000	0.633747	25,000	0.992215
5200	0.658970	30,000	0.995340
5400	0.680360	40,000	0.997967
5600	0.701046	50,000	0.998953
5800	0.720158	75,000	0.999713
6000	0.737818	100,000	0.999905

Table 4.2 Blackbody Radiation Function

The function f_λ represents the fraction of radiation emitted from a blackbody at temperature T in the wavelength band from λ = 0 to λ. The values of f_λ are listed in Table 4–2 as a function of λT, where λ is in μm and T is in K.

The fraction of radiation energy emitted by a blackbody at temperature T over a finite wavelength band from λ = λ1 to λ = λ2 is determined from (Fig. 4.8)

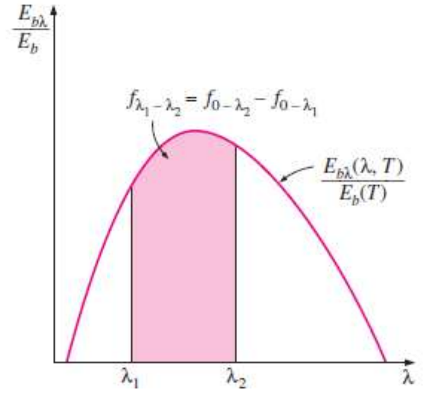


Figure 4.8 Graphical representation of the fraction of radiation emitted in the wavelength band from λ1 to λ2.

$$f_{\lambda_1-\lambda_2}(T) = f_{\lambda_2}(T) - f_{\lambda_1}(T)$$

where fλ1(T) and fλ2(T) are blackbody radiation functions corresponding to λ1T and λ2T, respectively

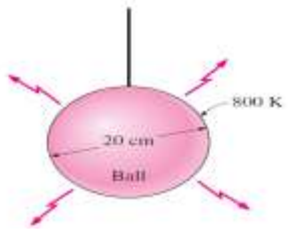
Example 4-1 : Radiation Emission from a Black Ball

Consider a 20-cm-diameter spherical ball at 800 K suspended in air as shown in Figure below. Assuming the ball closely approximates a blackbody, determine

(a) the total blackbody emissive power,

(b) the total amount of radiation emitted by the ball in 5 min, and

(c) the spectral blackbody emissive power at a wavelength of 3 μm.

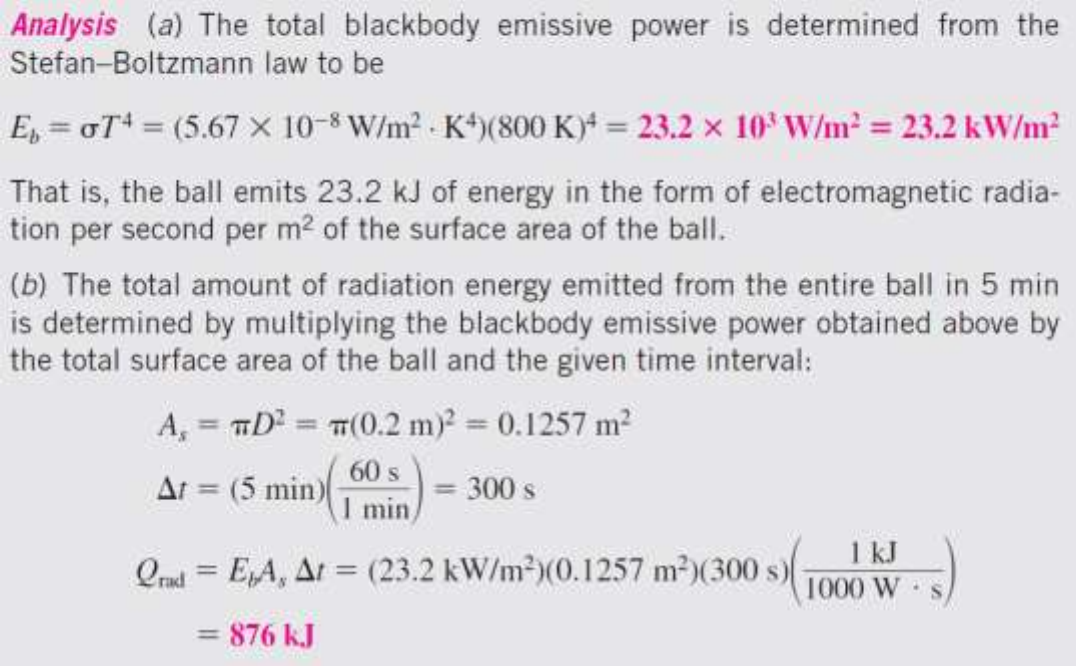


SOLUTION

An isothermal sphere is suspended in air. The total blackbody emissive power, the total radiation emitted in 5 minutes, and the spectral blackbody emissive power at 3 mm are to be determined.

Assumptions

The ball behaves as a blackbody.



Analysis (*a*) The total blackbody emissive power is determined from the Stefan–Boltzmann law to be

$$E_b = \sigma T^4 = (5.67 \times 10^{-8}\ \mathrm{W/m^2 \cdot K^4})(800\ \mathrm{K})^4 = \mathbf{23.2 \times 10^3\ W/m^2 = 23.2\ kW/m^2}$$

That is, the ball emits 23.2 kJ of energy in the form of electromagnetic radiation per second per m^2 of the surface area of the ball.

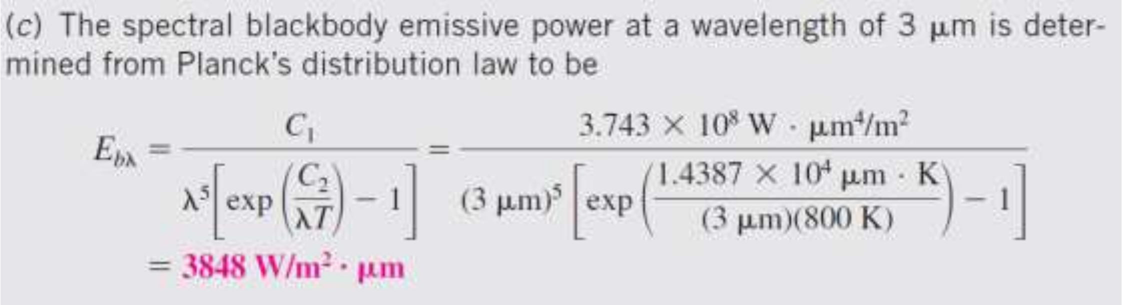
(*b*) The total amount of radiation energy emitted from the entire ball in 5 min is determined by multiplying the blackbody emissive power obtained above by the total surface area of the ball and the given time interval:

$$A_s = \pi D^2 = \pi(0.2\ \mathrm{m})^2 = 0.1257\ \mathrm{m^2}$$

$$\Delta t = (5\ \mathrm{min})\left(\frac{60\ \mathrm{s}}{1\ \mathrm{min}}\right) = 300\ \mathrm{s}$$

$$Q_{\mathrm{rad}} = E_b A_s\,\Delta t = (23.2\ \mathrm{kW/m^2})(0.1257\ \mathrm{m^2})(300\ \mathrm{s})\left(\frac{1\ \mathrm{kJ}}{1000\ \mathrm{W \cdot s}}\right)$$

$$= \mathbf{876\ kJ}$$



(*c*) The spectral blackbody emissive power at a wavelength of 3 μm is determined from Planck's distribution law to be

$$E_{b\lambda} = \frac{C_1}{\lambda^5\left[\exp\left(\dfrac{C_2}{\lambda T}\right) - 1\right]} = \frac{3.743 \times 10^8\ \mathrm{W \cdot \mu m^4/m^2}}{(3\ \mu\mathrm{m})^5\left[\exp\left(\dfrac{1.4387 \times 10^4\ \mu\mathrm{m \cdot K}}{(3\ \mu\mathrm{m})(800\ \mathrm{K})}\right) - 1\right]}$$

$$= \mathbf{3848\ W/m^2 \cdot \mu m}$$

Example 4-2 Emission of Radiation from a Lightbulb

The temperature of the filament of an incandescent lightbulb is 2500 K. Assuming the filament to be a blackbody, determine the fraction of the radiant energy emitted by the filament that falls in the visible range. Also, determine the wavelength at which the emission of radiation from the filament peaks.

SOLUTION

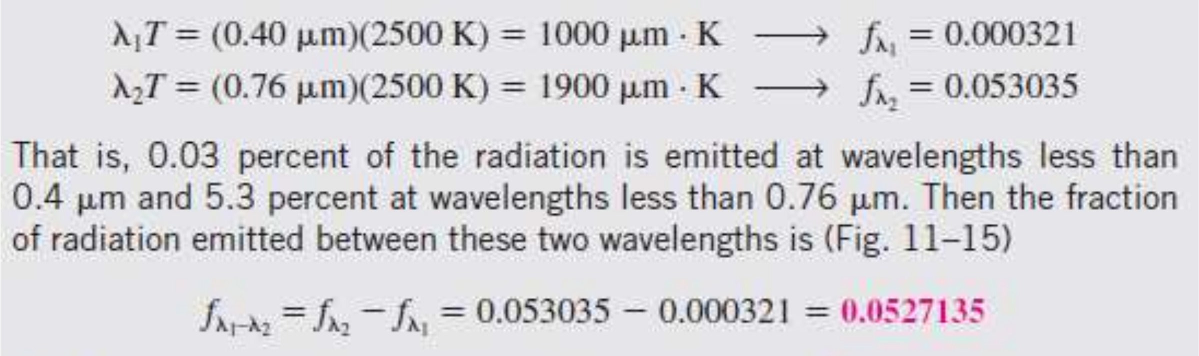
The temperature of the filament of an incandescent lightbulb is given. The fraction of visible radiation emitted by the filament and the wavelength at which the emission peaks are to be determined.

Assumptions

The filament behaves as a blackbody.

Analysis

The visible range of the electromagnetic spectrum extends from λ1 = 0.4 µm to λ2 = 0.76 µm. Noting that T = 2500 K, the blackbody radiation functions corresponding to λ1T and λ2T are determined from Table 4–2 to be

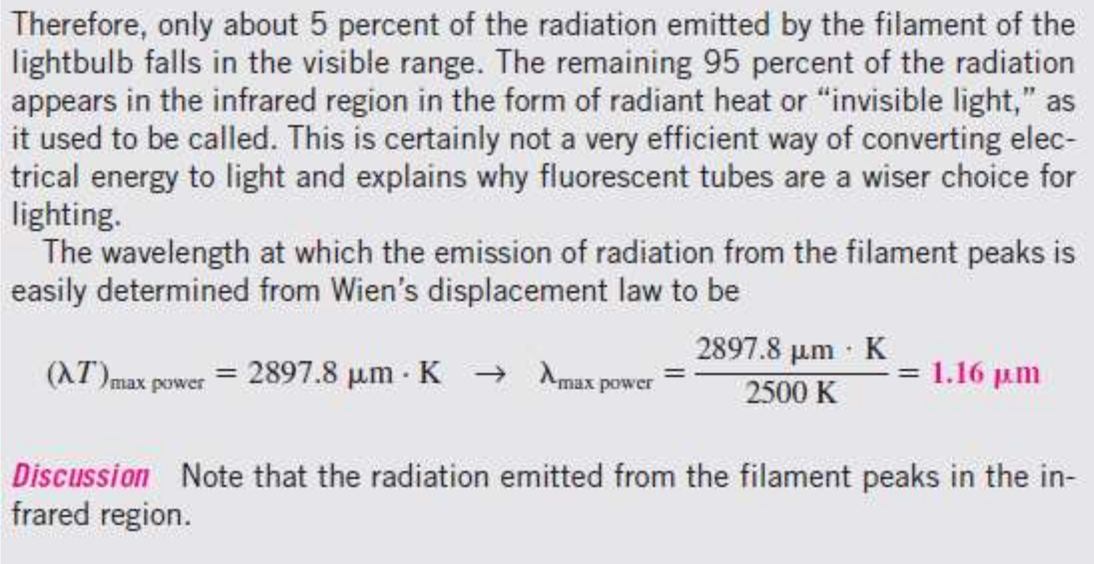


$$\lambda_1 T = (0.40\ \mu\text{m})(2500\ \text{K}) = 1000\ \mu\text{m}\cdot\text{K} \longrightarrow f_{\lambda_1} = 0.000321$$

$$\lambda_2 T = (0.76\ \mu\text{m})(2500\ \text{K}) = 1900\ \mu\text{m}\cdot\text{K} \longrightarrow f_{\lambda_2} = 0.053035$$

That is, 0.03 percent of the radiation is emitted at wavelengths less than 0.4 μm and 5.3 percent at wavelengths less than 0.76 μm. Then the fraction of radiation emitted between these two wavelengths is (Fig. 11–15)

$$f_{\lambda_1-\lambda_2} = f_{\lambda_2} - f_{\lambda_1} = 0.053035 - 0.000321 = 0.0527135$$



Therefore, only about 5 percent of the radiation emitted by the filament of the lightbulb falls in the visible range. The remaining 95 percent of the radiation appears in the infrared region in the form of radiant heat or "invisible light," as it used to be called. This is certainly not a very efficient way of converting electrical energy to light and explains why fluorescent tubes are a wiser choice for lighting.

The wavelength at which the emission of radiation from the filament peaks is easily determined from Wien's displacement law to be

$$(\lambda T)_{\text{max power}} = 2897.8\ \mu\text{m}\cdot\text{K} \rightarrow \lambda_{\text{max power}} = \frac{2897.8\ \mu\text{m}\cdot\text{K}}{2500\ \text{K}} = 1.16\ \mu\text{m}$$

Discussion Note that the radiation emitted from the filament peaks in the infrared region.

4.2 Radiation Properties

Most materials encountered in practice, such as metals, wood, and bricks, are opaque to thermal radiation, and radiation is considered to be a surface *phenomenon* for such materials. That is, thermal radiation is emitted or absorbed within the first few microns of the surface, and thus we speak of radiative properties of *surfaces* for opaque materials.

Some other materials, such as glass and water, allow visible radiation to penetrate to considerable depths before any significant absorption takes place. Radiation through such *semitransparent* materials obviously cannot be considered to be a surface phenomenon since the entire volume of the material interacts with radiation. On the other hand, both glass and water are practically opaque to infrared radiation. Therefore, materials can exhibit different behavior at different wavelengths, and the dependence on wavelength is an important consideration in the study of radiative properties such as emissivity, absorptivity, reflectivity, and transmissivity of materials.

In the preceding section, we defined a *blackbody* as a perfect emitter and absorber of radiation and said that no body can emit more radiation than a blackbody at the same temperature. Therefore, a blackbody can serve as a convenient *reference* in describing the emission and absorption characteristics of real surfaces.

Emmisivity

The emissivity of a surface represents the ratio of the radiation emitted by the surface at a given temperature to the radiation emitted by a blackbody at the same temperature. The emissivity of a surface is denoted by ε, and it varies between zero and one, $0 \leq \varepsilon \leq 1$. Emissivity is a measure of how closely a surface approximates a blackbody, for which $\varepsilon = 1$.

Absorptivity, Reflectivity, and Transmissivity

Everything around us constantly emits radiation, and the emissivity represents the emission characteristics of those bodies. This means that everybody, including our own, is constantly bombarded by radiation coming from all directions over a range of wavelengths. Recall that radiation flux incident on a surface is called irradiation and is denoted by G.

When radiation energy strikes a material surface, part of radiation is reflected, part is absorbed, and part is transmitted as shown in Figure 4-5. The fraction of irradiation absorbed by the surface is called the absorptivity α, the fraction reflected by the surface is called the reflectivity α, and the fraction transmitted is called the transmissivity τ.

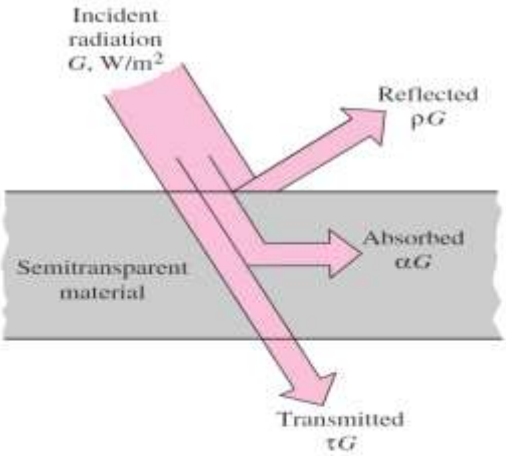


Figure 4.9 The absorption, reflection, and transmission of incident

That is,

$$\textit{Absorptivity:} \qquad \alpha = \frac{\text{Absorbed radiation}}{\text{Incident radiation}} = \frac{G_{\text{abs}}}{G}, \qquad 0 \leq \alpha \leq 1$$

$$\textit{Reflectivity:} \qquad \rho = \frac{\text{Reflected radiation}}{\text{Incident radiation}} = \frac{G_{\text{ref}}}{G}, \qquad 0 \leq \rho \leq 1$$

$$\textit{Transmissivity:} \qquad \tau = \frac{\text{Transmitted radiation}}{\text{Incident radiation}} = \frac{G_{\text{tr}}}{G}, \qquad 0 \leq \tau \leq 1$$

$$G_{\text{abs}} + G_{\text{ref}} + G_{\text{tr}} = G$$

Dividing each term of this relation by *G* yields

$$\alpha + \rho + \tau = 1$$

For opaque surfaces, $\tau=0$, and thus

$$\alpha + \rho = 1$$

4.2.2 Kirchoff’s Law

The total hemispherical emissivity of a surface at temperature T is equal to its total hemispherical absorptivity for radiation coming from a blackbody at the same temperature.

$$\varepsilon(T) = \alpha(T)$$

Consider a small body of surface area As, emissivity ε, and absorptivity ∝ at temperature T contained in a large isothermal enclosure at the same temperature, as shown in Figure 4.6

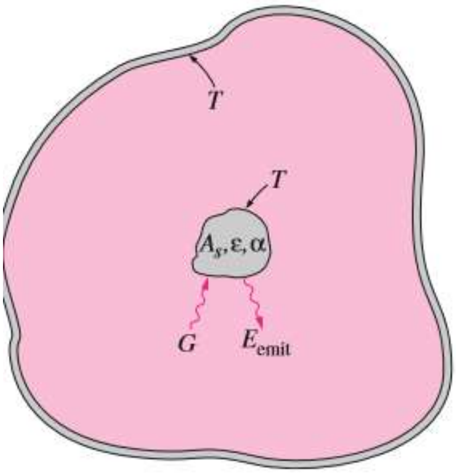


Figure 4.10 The small body contained in a large isothermal enclosure used in the development of Kirchoff’s law

The radiation absorbed by the small body per unit of its surface area is

$$G_{\text{abs}} = \alpha G = \alpha\sigma T^4$$

The radiation emitted by the small body is

$$E_{\text{emit}} = \varepsilon\sigma T^4$$

Example 4.3 Emissivity of a surface and Emissive Power

The spectral emmisivty function of an opaque surface at 800 K is approximates as (Figure 4.11)

$$\varepsilon_\lambda = \begin{cases} \varepsilon_1 = 0.3, & 0 \leq \lambda < 3\ \mu\text{m} \\ \varepsilon_2 = 0.8, & 3\ \mu\text{m} \leq \lambda < 7\ \mu\text{m} \\ \varepsilon_3 = 0.1, & 7\ \mu\text{m} \leq \lambda < \infty \end{cases}$$

Determine the average emissivity of the surface and its emissive power

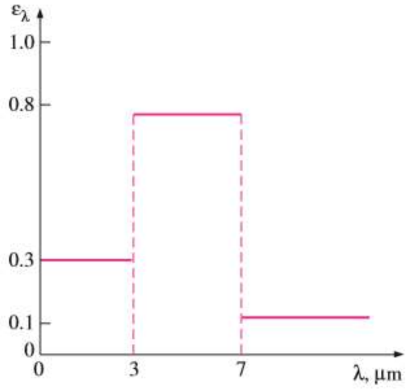
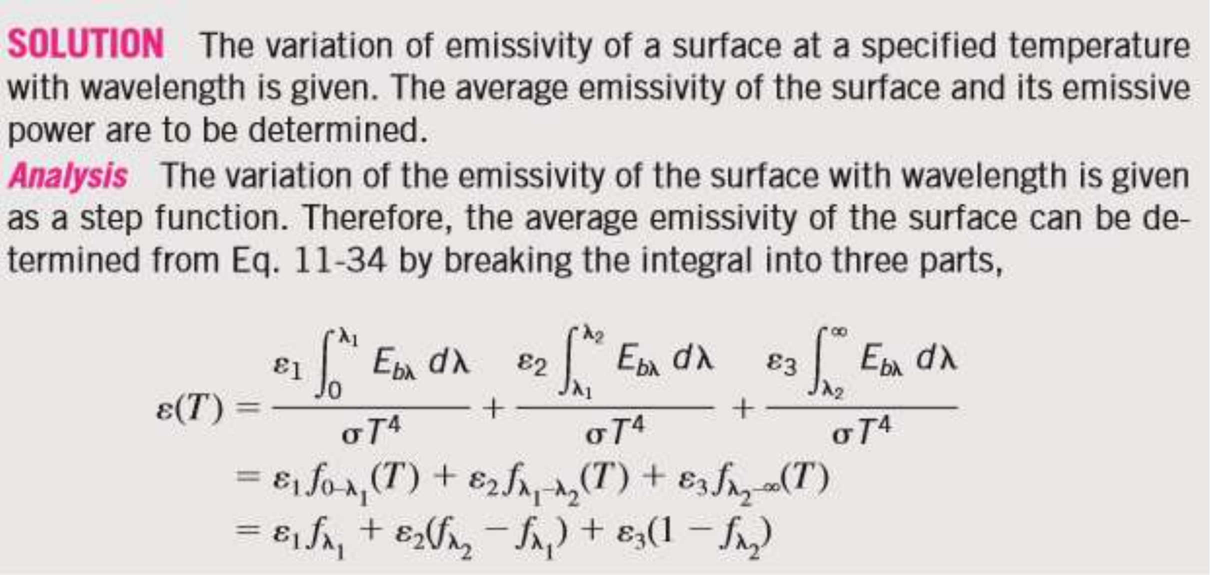


Figure 4.11 The spectral emissivity of the surface considered in Example 4.3



SOLUTION The variation of emissivity of a surface at a specified temperature with wavelength is given. The average emissivity of the surface and its emissive power are to be determined.

Analysis The variation of the emissivity of the surface with wavelength is given as a step function. Therefore, the average emissivity of the surface can be determined from Eq. 11-34 by breaking the integral into three parts,

$$\begin{aligned}\varepsilon(T) &= \frac{\varepsilon_1 \int_0^{\lambda_1} E_{b\lambda}\, d\lambda}{\sigma T^4} + \frac{\varepsilon_2 \int_{\lambda_1}^{\lambda_2} E_{b\lambda}\, d\lambda}{\sigma T^4} + \frac{\varepsilon_3 \int_{\lambda_2}^{\infty} E_{b\lambda}\, d\lambda}{\sigma T^4} \\ &= \varepsilon_1 f_{0\text{-}\lambda_1}(T) + \varepsilon_2 f_{\lambda_1\text{-}\lambda_2}(T) + \varepsilon_3 f_{\lambda_2\text{-}\infty}(T) \\ &= \varepsilon_1 f_{\lambda_1} + \varepsilon_2(f_{\lambda_2} - f_{\lambda_1}) + \varepsilon_3(1 - f_{\lambda_2})\end{aligned}$$

where f_{λ_1} and f_{λ_2} are blackbody radiation functions corresponding to $\lambda_1 T$ and $\lambda_2 T$. These functions are determined from Table 11–2 to be

$$\lambda_1 T = (3\ \mu\text{m})(800\ \text{K}) = 2400\ \mu\text{m}\cdot\text{K} \quad \rightarrow \quad f_{\lambda_1} = 0.140256$$
$$\lambda_2 T = (7\ \mu\text{m})(800\ \text{K}) = 5600\ \mu\text{m}\cdot\text{K} \quad \rightarrow \quad f_{\lambda_2} = 0.701046$$

Note that $f_{0\text{-}\lambda_1} = f_{\lambda_1} - f_0 = f_{\lambda_1}$, since $f_0 = 0$, and $f_{\lambda_2\text{-}\infty} = f_\infty - f_{\lambda_2} = 1 - f_{\lambda_2}$, since $f_\infty = 1$. Substituting,

$$\begin{aligned}\varepsilon &= 0.3 \times 0.140256 + 0.8(0.701046 - 0.140256) + 0.1(1 - 0.701046)\\ &= \mathbf{0.521}\end{aligned}$$

That is, the surface will emit as much radiation energy at 800 K as a gray surface having a constant emissivity of $\varepsilon = 0.521$. The emissive power of the surface is

$$E = \varepsilon\sigma T^4 = 0.521(5.67 \times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(800\ \text{K})^4 = \mathbf{12{,}100\ W/m^2}$$

Discussion Note that the surface emits 12.1 kJ of radiation energy per second per m^2 area of the surface.

CHAPTER 5

HEAT EXCHANGER

At the end of this chapter, you should be able to:

- Classify the types of heat exchanger
- Apply the overall heat transfer coefficient
- Apply the use of log mean temperature difference (LMTD)
- Calculate the effectiveness of heat exchanger

5.1 HEAT EXCHANGER

Heat exchangers are devices that facilitate the exchange of heat between two fluids that are at different temperatures while keeping them from mixing with each other. Heat exchangers are commonly used in practice in a wide range of applications, from heating and air-conditioning systems in a household, to chemical processing and power production in large plants. Heat exchangers differ from mixing chambers in that they do not allow the two fluids involved to mix.

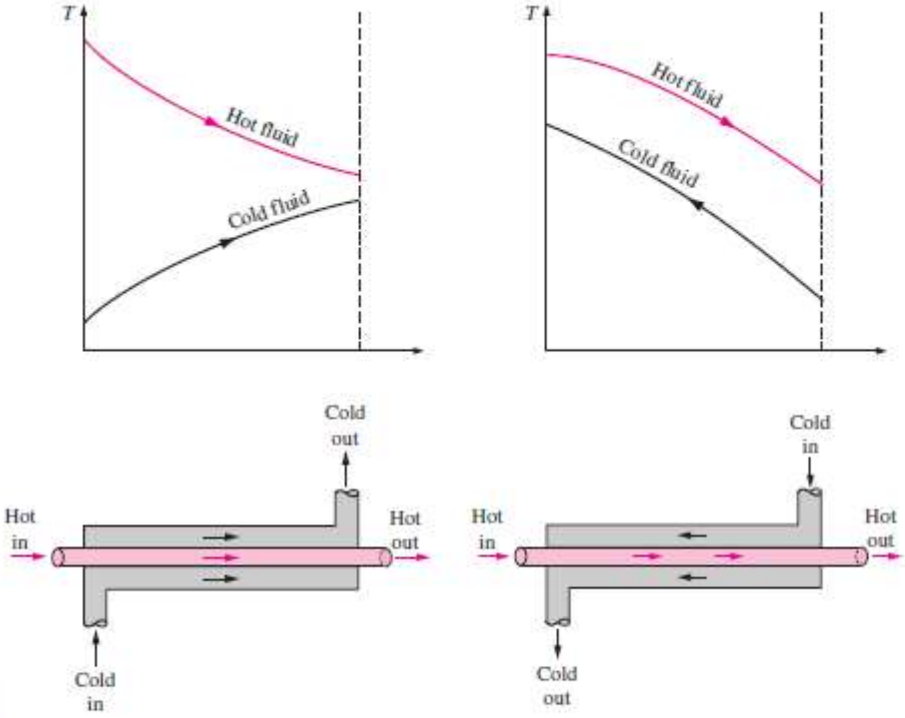
Heat transfer in a heat exchanger usually involves convection in each fluid and conduction through the wall separating the two fluids. In the analysis of heat exchangers, it is convenient to work with an overall heat transfer coefficient U that accounts for the contribution of all these effects on heat transfer. The rate of heat transfer between the two fluids at a location in a heat exchanger depends on the magnitude of the temperature difference at that location, which varies along the heat exchanger.

5.1.1 Types of Heat Exchangers

Different heat transfer applications require different types of hardware and different configurations of heat transfer equipment

A. Double Pipe Heat Exchanger/ Parallel and Counter Flow Heat Exchangers

The simplest type of heat exchanger consists of two concentric pipes of different diameters, as shown in Figure 5.1, called the double-pipe heat exchanger. One fluid in a double-pipe heat exchanger flows through the smaller pipe while the other fluid flows through the annular space between the two pipes. Two types of flow arrangement are possible in a double-pipe heat exchanger: in parallel flow, both the hot and cold fluids enter the heat exchanger at the same end and move in the same direction. In counter flow, on the other hand, the hot and cold fluids enter the heat exchanger at opposite ends and flow in opposite directions.



(*a*) Parallel flow

(*b*) Counter flow

Figure 5.1 Different flow regimes and associated temperature profiles in a double-pipe heat exchanger

B. Compact Heat Exchanger/ Cross Flow Heat Exchangers

Another type of heat exchanger, which is specifically designed to realize a large heat transfer surface area per unit volume, is the compact heat exchanger. Compact heat exchangers enable us to achieve high heat transfer rates between two fluids in small volume, and they are commonly used in applications with strict limitations on the weight and volume of heat exchangers (Fig. 5.2).

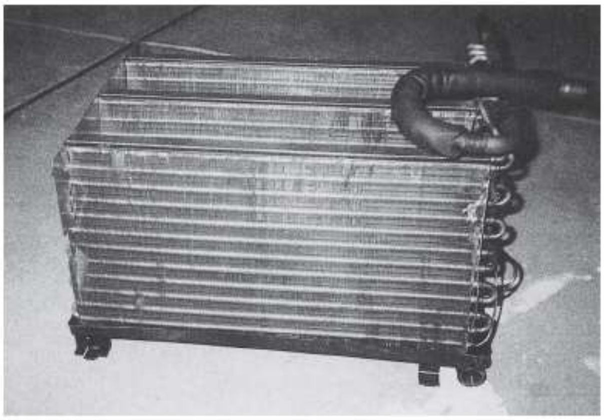


Figure 5.2 A gas-to-liquid compact heat exchanger for a residential air conditioning system

The large surface area in compact heat exchangers is obtained by attaching closely spaced thin plate or corrugated fins to the walls separating the two fluids. Compact heat exchangers are commonly used in gas-to-gas and gas-to liquid (or liquid-to-gas) heat exchangers to counteract the low heat transfer coefficient associated with gas flow with increased surface area. In a car radiator, which is a water-to-air compact heat exchanger, for example, it is no surprise that fins are attached to the air side of the tube surface.

In compact heat exchangers, the two fluids usually move perpendicular to each other, and such flow configuration is called cross-flow. The cross-flow is further classified as unmixed and mixed flow, depending on the flow configuration, as shown in Figure 5–3. In (a) the cross-flow is said to be unmixed since the plate fins force the fluid to flow through a particular interfin spacing and prevent it from moving in the transverse direction (i.e., parallel to the tubes). The cross-flow in (b) is said to be mixed since the fluid now is free to move in the transverse direction. Both fluids are unmixed in a car radiator. The presence of mixing in the fluid can have a significant effect on the heat transfer characteristics of the heat exchanger.

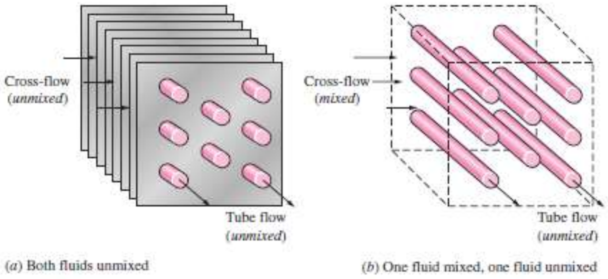


Figure 5.3 Different flow configurations in cross-flow heat exchangers

C. Shell and Tube Heat Exchangers

The most common type of heat exchanger in industrial applications is the shell-and-tube heat exchanger, shown in Figure 5.4.

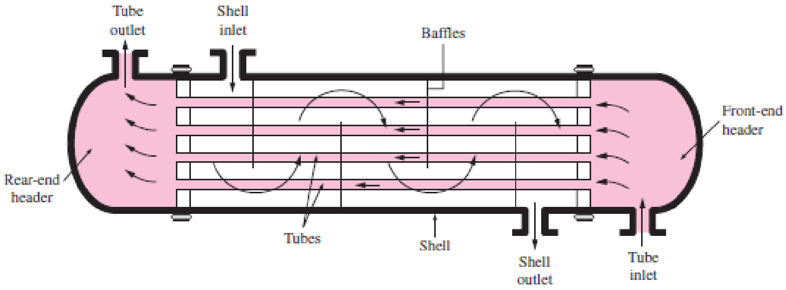
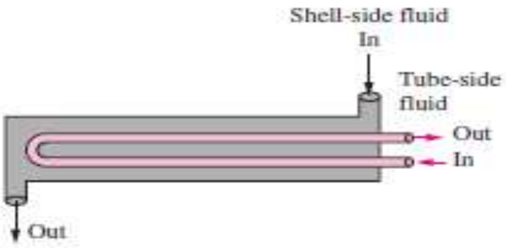


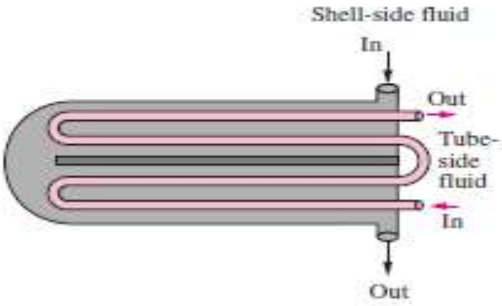
Figure 5.4 The schematic of shell-and-tube heat exchanger (one-shell pass and one-tube pass).

Shell-and-tube heat exchangers contain a large number of tubes (sometimes several hundred) packed in a shell with their axes parallel to that of the shell. Heat transfer takes place as one fluid flows inside the tubes while the other fluid flows outside the tubes through the shell. Baffles are commonly placed in the shell to force the shell-side fluid to flow across the shell to enhance heat transfer and to maintain uniform spacing between the tubes. Despite their widespread use, shell and tube heat exchangers are not suitable for use in automotive and aircraft applications because of their relatively large size and weight. Note that the tubes in a shell-and-tube heat exchanger open to some large flow areas called headers at both ends of the shell, where the tube-side fluid accumulates before entering the tubes and after leaving them.

Shell-and-tube heat exchangers are further classified according to the number of shell and tube passes involved. Heat exchangers in which all the tubes make one U-turn in the shell, for example, are called one-shell-pass and two-tube-passes heat exchangers. Likewise, a heat exchanger that involves two passes in the shell and four passes in the tubes is called a two-shell-passes and four-tube-passes heat exchanger (Fig. 5.5).



(*a*) One-shell pass and two-tube passes



(*b*) Two-shell passes and four-tube passes

Figure 5.5 Multipass flow arrangements in shell-and-tube heat exchangers

D. Plate and Frame Heat Exchanger

This heat exchanger consists of a series of plates with corrugated flat flow passages. The hot and cold fluids flow in alternate passages. It is well suited for liquid-to-liquid heat exchange applications, provided that the hot and cold fluid streams are at about the same pressure.

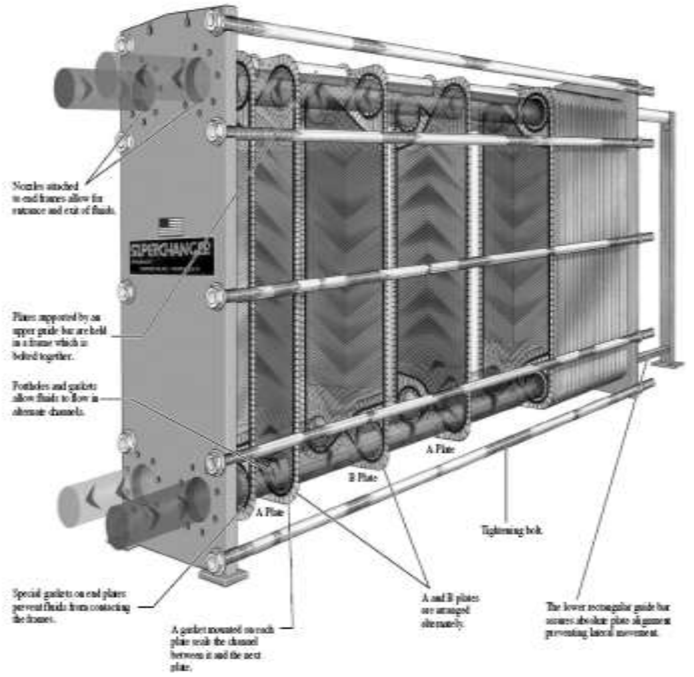


Figure 5.6 Plate and Frame Heat Exchanger

5.1.2 Flow Types of Heat Exchanger

There are three flow types of heat exchanger.

Parallel Flow – both the hot and cold fluids enter the heat exchanger at the same end and move in the same direction

Counter flow- the hot and cold fluids enter the heat exchanger at opposite ends and flow in opposite direction

Cross-flow – the hot and cold fluids stream move perpendicular to each other

5.2 THE OVERALL HEAT TRANSFER COEFFICIENT

A heat exchanger typically involves two flowing fluids separated by a solid wall. Heat is first transferred from the hot fluid to the wall by convection, through the wall by conduction, and from the wall to the cold fluid again by convection. Any radiation effects are usually included in the convection heat transfer coefficients.

The thermal resistance network associated with this heat transfer process involves two convection and one conduction resistances, as shown in Figure 5.7. Here the subscripts i and o represent the inner and outer surfaces of the inner tube. For a double-pipe heat exchanger, we have $A_i = \pi D_i L$ and $A_o = \pi D_o L$, and the thermal resistance of the tube wall in this case is

$$R_{wall} = \frac{\ln(\frac{D_o}{D_i})}{2\pi k L}$$

where,

k = thermal conductivity of the wall

L = length of the tube

Then the total thermal resistance becomes

$$R = R_{total} = R_i + R_{wall} + R_o = \frac{1}{h_i A_i} + \frac{\ln(D_o/D_i)}{2\pi k L} + \frac{1}{h_o A_o}$$

Where,

A_i = area of the inner surface of the wall that separates the two fluids

A_o = area of the outer surface of the wall.

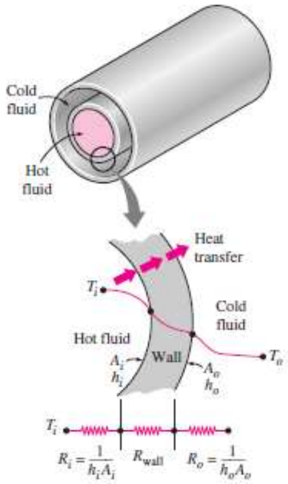


Figure 5.7 Thermal resistance network associated with heat transfer in a double-pipe heat exchanger.

In the analysis of heat exchangers, it is convenient to combine all the thermal resistances in the path of heat flow from the hot fluid to the cold one into a single resistance R, and to express the rate of heat transfer between the two fluids as

$$\dot{Q} = \frac{\Delta T}{R} = UA\Delta T = U_i A_i \Delta T = U_o A_o \Delta T$$

where U is the overall heat transfer coefficient, whose unit is W/m^2.^{o}C, which is identical to the unit of the ordinary convection coefficient h. Cancelling ΔT reduces to

$$R = \frac{1}{U_i A_i} = \frac{1}{U_o A_o} = \frac{1}{U A_s} = \frac{1}{h_i A_i} + R_{wall} + \frac{1}{h_o A_o}$$

When the wall thickness of the tube is small and the thermal conductivity of the tube material is high, as is usually the case, the thermal resistance of the tube is negligible ($R_{wall} \approx 0$) and the inner and outer surfaces of the tube are almost identical ($A_i \approx A_o \approx A_s$). Then for the overall heat transfer coefficient simplifies to

$$\frac{1}{U} \approx \frac{1}{h_i} + \frac{1}{h_o}$$

where $U \approx U_i \approx U_o$. The individual convection heat transfer coefficients inside and outside the tube, hi and ho, are determined using the convection relations discussed in earlier chapters.

When $h_i >> h_o$

$$\frac{1}{U} \approx \frac{1}{h_o}$$

When $h_i << h_o$

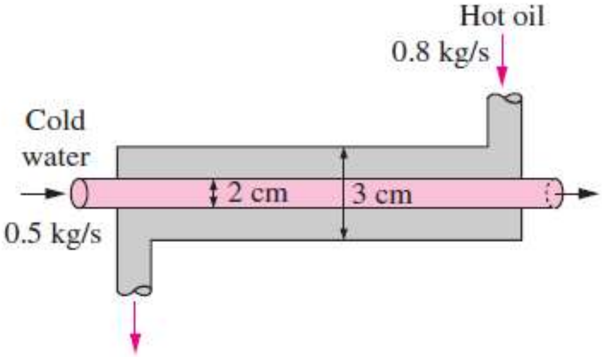
$$\frac{1}{U} \approx \frac{1}{h_i}$$

Under most practical conditions, the flow in a tube is laminar for Re<2300, fully turbulent for Re > 10000, and transitional in between.But should be kept in mind that in many cases the flow becomes fully turbulent for Re > 4000.

For fully develop turbulent flow in smooth pipe: $Nu = 0.023Re^{0.8}Pr^{1/3}$

EXAMPLE 5.1 Overall Heat Transfer Coefficient of a Heat Exchanger

Hot oil is to be cooled in a double-tube counter-flow heat exchanger. The copper inner tubes have a diameter of 2 cm and negligible thickness. The inner diameter of the outer tube (the shell) is 3 cm. Water flows through the tube at a rate of 0.5 kg/s, and the oil through the shell at a rate of 0.8 kg/s. Taking the average temperatures of the water and the oil to be 45°C and 80°C, respectively, determine the overall heat transfer coefficient of this heat exchanger. Given: $Nu = 0.023Re^{0.8}Pr^{0.4}$

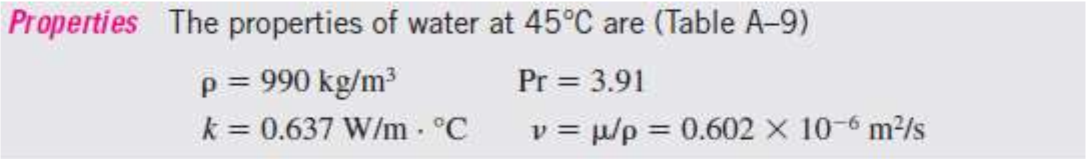


SOLUTION

Hot oil is cooled by water in a double-tube counter-flow heat exchanger. The overall heat transfer coefficient is to be determined.

Assumptions

1 The thermal resistance of the inner tube is negligible since the tube material is highly conductive and its thickness is negligible.
2 Both the oil and water flow are fully developed.
3 Properties of the oil and water are constant.



Properties The properties of water at 45°C are (Table A–9)

$\rho = 990\ \text{kg/m}^3$ $\quad$ $Pr = 3.91$

$k = 0.637\ \text{W/m}\cdot{}^\circ\text{C}$ $\quad$ $\nu = \mu/\rho = 0.602 \times 10^{-6}\ \text{m}^2/\text{s}$

The properties of oil at 80°C are (Table A–16).

$$\rho = 852 \text{ kg/m}^3 \qquad \text{Pr} = 490$$
$$k = 0.138 \text{ W/m} \cdot {}^\circ\text{C} \qquad \nu = 37.5 \times 10^{-6} \text{ m}^2\text{/s}$$

Analysis The schematic of the heat exchanger is given in Figure 13–10. The overall heat transfer coefficient U can be determined from Eq. 13-5:

$$\frac{1}{U} \approx \frac{1}{h_i} + \frac{1}{h_o}$$

where h_i and h_o are the convection heat transfer coefficients inside and outside the tube, respectively, which are to be determined using the forced convection relations.

The hydraulic diameter for a circular tube is the diameter of the tube itself, $D_h = D = 0.02$ m. The mean velocity of water in the tube and the Reynolds number are

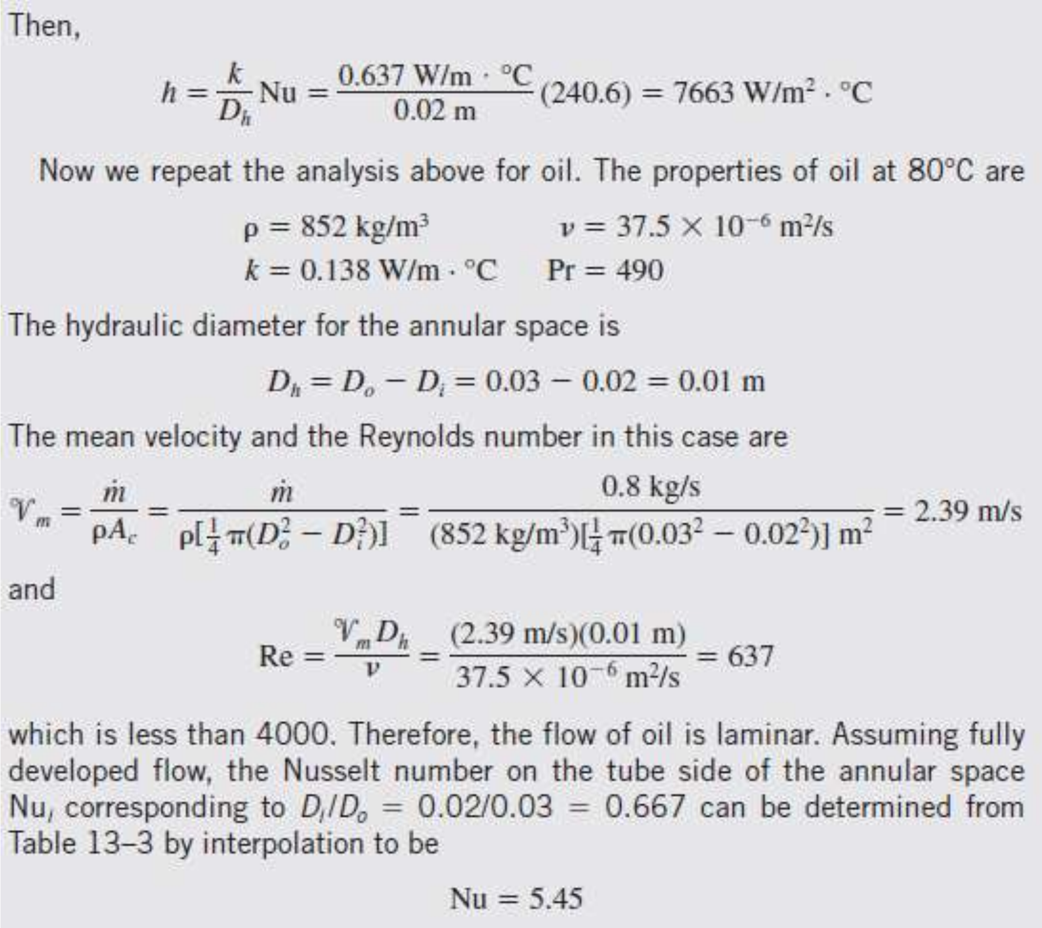
$$\mathcal{V}_m = \frac{\dot{m}}{\rho A_c} = \frac{\dot{m}}{\rho(\frac{1}{4}\pi D^2)} = \frac{0.5 \text{ kg/s}}{(990 \text{ kg/m}^3)[\frac{1}{4}\pi(0.02 \text{ m})^2]} = 1.61 \text{ m/s}$$

and

$$\text{Re} = \frac{\mathcal{V}_m D_h}{\nu} = \frac{(1.61 \text{ m/s})(0.02 \text{ m})}{0.602 \times 10^{-6} \text{ m}^2\text{/s}} = 53{,}490$$

which is greater than 4000. Therefore, the flow of water is turbulent. Assuming the flow to be fully developed, the Nusselt number can be determined from

$$\text{Nu} = \frac{hD_h}{k} = 0.023 \text{ Re}^{0.8}\text{Pr}^{0.4} = 0.023(53{,}490)^{0.8}(3.91)^{0.4} = 240.6$$



Then,

$$h = \frac{k}{D_h}\,\text{Nu} = \frac{0.637\ \text{W/m}\cdot{}^\circ\text{C}}{0.02\ \text{m}}(240.6) = 7663\ \text{W/m}^2\cdot{}^\circ\text{C}$$

Now we repeat the analysis above for oil. The properties of oil at 80°C are

$$\rho = 852\ \text{kg/m}^3 \qquad \nu = 37.5 \times 10^{-6}\ \text{m}^2/\text{s}$$
$$k = 0.138\ \text{W/m}\cdot{}^\circ\text{C} \qquad \text{Pr} = 490$$

The hydraulic diameter for the annular space is

$$D_h = D_o - D_i = 0.03 - 0.02 = 0.01\ \text{m}$$

The mean velocity and the Reynolds number in this case are

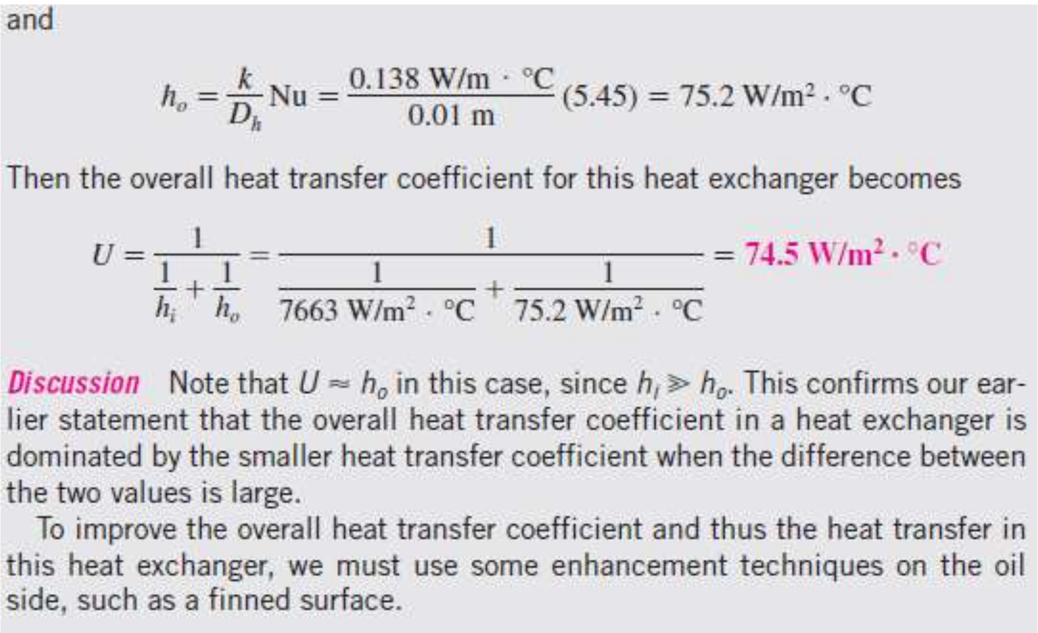
$$\mathcal{V}_m = \frac{\dot m}{\rho A_c} = \frac{\dot m}{\rho[\frac{1}{4}\pi(D_o^2 - D_i^2)]} = \frac{0.8\ \text{kg/s}}{(852\ \text{kg/m}^3)[\frac{1}{4}\pi(0.03^2 - 0.02^2)]\ \text{m}^2} = 2.39\ \text{m/s}$$

and

$$\text{Re} = \frac{\mathcal{V}_m D_h}{\nu} = \frac{(2.39\ \text{m/s})(0.01\ \text{m})}{37.5 \times 10^{-6}\ \text{m}^2/\text{s}} = 637$$

which is less than 4000. Therefore, the flow of oil is laminar. Assuming fully developed flow, the Nusselt number on the tube side of the annular space Nu_i corresponding to $D_i/D_o = 0.02/0.03 = 0.667$ can be determined from Table 13–3 by interpolation to be

$$\text{Nu} = 5.45$$



and

$$h_o = \frac{k}{D_h}\,\text{Nu} = \frac{0.138\ \text{W/m}\cdot{}^\circ\text{C}}{0.01\ \text{m}}(5.45) = 75.2\ \text{W/m}^2\cdot{}^\circ\text{C}$$

Then the overall heat transfer coefficient for this heat exchanger becomes

$$U = \frac{1}{\frac{1}{h_i} + \frac{1}{h_o}} = \frac{1}{\frac{1}{7663\ \text{W/m}^2\cdot{}^\circ\text{C}} + \frac{1}{75.2\ \text{W/m}^2\cdot{}^\circ\text{C}}} = 74.5\ \text{W/m}^2\cdot{}^\circ\text{C}$$

Discussion Note that $U \approx h_o$ in this case, since $h_i \gg h_o$. This confirms our earlier statement that the overall heat transfer coefficient in a heat exchanger is dominated by the smaller heat transfer coefficient when the difference between the two values is large.

To improve the overall heat transfer coefficient and thus the heat transfer in this heat exchanger, we must use some enhancement techniques on the oil side, such as a finned surface.

ANALYSIS OF HEAT EXCHANGER

Heat exchangers are commonly used in practice, and an engineer often finds himself or herself in a position to select a heat exchanger that will achieve a specified temperature change in a fluid stream of known mass flow rate, or to predict the outlet temperatures of the hot and cold fluid streams in a specified heat exchanger. In upcoming sections, we will discuss the two methods used in the analysis of heat exchangers. Of these, the log mean temperature difference (or LMTD) method is best suited for the first task and the effectiveness–NTU method for the second task as just stated. But first we present some general considerations.

Heat exchangers usually operate for long periods of time with no change in their operating conditions. Therefore, they can be modeled as steady-flow devices. As such, the mass flow rate of each fluid remains constant, and the fluid properties such as temperature and velocity at any inlet or outlet remain the same. Also, the fluid streams experience little or no change in their velocities and elevations, and thus the kinetic and potential energy changes are negligible. The specific heat of a fluid, in general, changes with temperature. But, in a specified temperature range, it can be treated as a constant at some average value with little loss in accuracy. Axial heat conduction along the tube is usually insignificant and can be considered negligible. Finally, the outer surface of the heat exchanger is assumed to be perfectly insulated, so that there is no heat loss to the surrounding medium, and any heat transfer occurs between the two fluids only. The idealizations stated above are closely approximated in practice, and they greatly simplify the analysis of a heat exchanger with little sacrifice of accuracy. Therefore, they are commonly used. Under these assumptions, the first law of thermodynamics requires that the rate of heat transfer from the hot fluid be equal to the rate of heat transfer to the cold one.

The transfer rate to the cold fluid is :

$$\dot{Q} = \dot{m}_c c_{pc} \left(T_{c,out} - T_{c,in}\right) = C_c \left(T_{c,out} - T_{c,in}\right) \; ; \; C_c = \dot{m}_c c_{pc}$$

The transfer rate to the hot fluid is :

$$\dot{Q} = \dot{m}_h c_{ph} \left(T_{h,in} - T_{h,out}\right) = C_h \left(T_{h,in} - T_{h,out}\right) \; ; \; C_h = \dot{m}_h c_{ph}$$

Two special types of heat exchangers commonly used in practice are condensers and boilers.

5.3 THE LOG MEAN TEMPERATURE DIFFERENT METHOD

The temperature difference between the hot and cold fluids varies along the heat exchanger, and it is convenient to have a mean temperature difference ΔT_m for use in the relation $Q = UA_s \Delta T_m$. In order to develop a relation for the equivalent average temperature difference between the two fluids, consider the parallel-flow double-pipe heat exchanger shown in Figure 5.8 Note that the temperature difference ΔT between the hot and cold fluids is large at the inlet of the heat exchanger but decreases exponentially toward the outlet. As you would expect, the temperature of the hot fluid decreases and the temperature of the cold fluid increases along the heat exchanger, but the temperature of the cold fluid can never exceed that of the hot fluid no matter how long the heat exchanger is. Assuming the outer surface of the heat exchanger to be well insulated so that any heat transfer occurs between the two fluids, and disregarding any changes in kinetic and potential energy, an energy balance on each fluid in a differential section of the heat exchanger

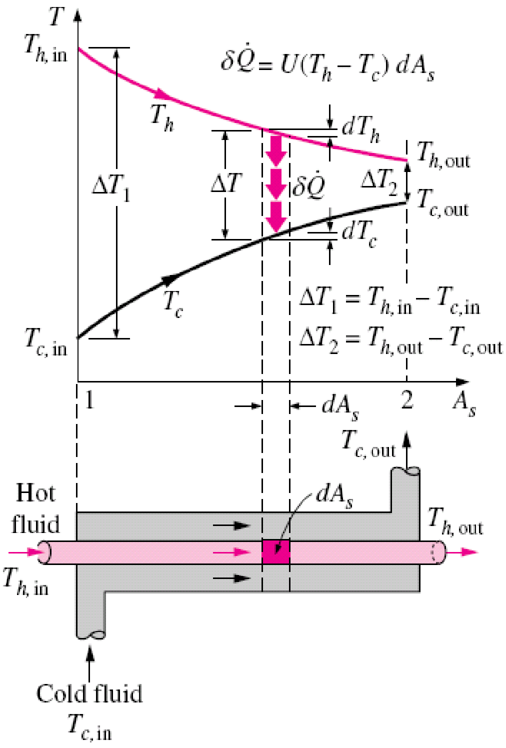


Figure 5.8 Variation of the fluid temperatures in a parallel-flow double-pipe heat exchanger.

An energy balance on each fluid in a differential section of the heat exchanger

$$\delta\dot{Q} = -\dot{m}_h c_{ph} dT_h$$

$$\delta\dot{Q} = \dot{m}_c c_{pc} dT_c$$

$$dT_h = -\frac{\delta\dot{Q}}{\dot{m}_h c_{ph}}$$

$$dT_c = \frac{\delta\dot{Q}}{\dot{m}_c c_{pc}}$$

Taking the difference, we get

$$dT_h - dT_c = d\left(T_h - T_c\right) = -\delta\dot{Q}\left(\frac{1}{\dot{m}_h c_{ph}} + \frac{1}{\dot{m}_c c_{pc}}\right)$$

Solving previous equation for $m_c c_{pc}$ and $m_h c_{ph}$ and substituting into above equation give

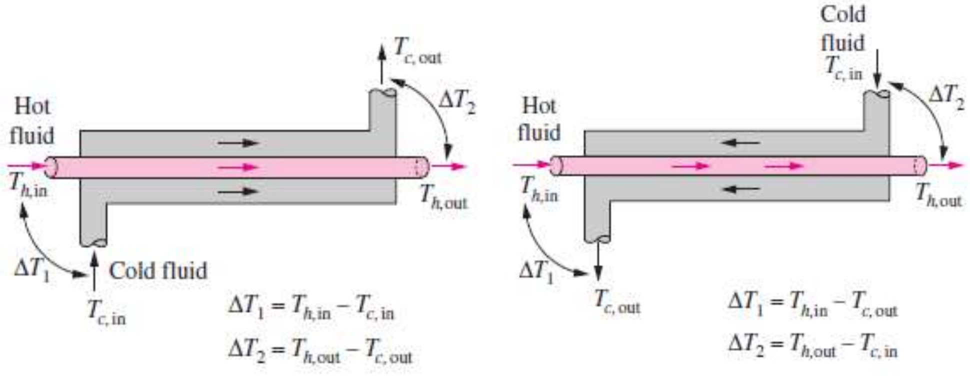
$$\dot{Q} = UA_s \Delta T_{lm}$$

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln\left(\Delta T_1 / \Delta T_2\right)}$$

Where

ΔT_{lm} is the log mean temperature difference

ΔT_1 and ΔT_2 are the temperature difference between the two fluids at the two ends (inlet and outlet). It makes no difference which end of the heat exchanger is designated as the inlet or the outlet. (Figure 5.9)



(*a*) Parallel-flow heat exchangers

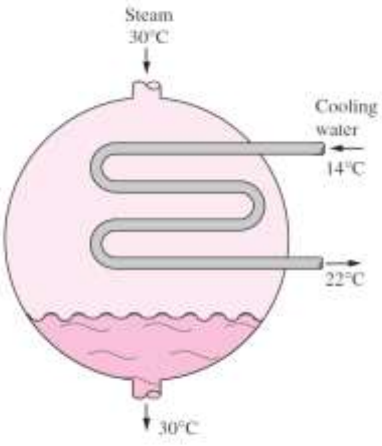
(*b*) Counter-flow heat exchangers

Figure 5.9 The ΔT_1 and ΔT_2 expressions in parallel-flow and counter flow heat exchangers

EXAMPLE 5.2 The Condensation of Steam in a Condenser

Steam in the condenser of a power plant is to be condensed at a temperature of 30°C with cooling water from a nearby lake, which enters the tubes of the condenser at 14°C and leaves at 22°C. The surface area of the tubes is 45 m2, and the overall heat transfer coefficient is 2100 W/m2 · °C.

Determine the mass flow rate of the cooling water needed and the rate of condensation of the steam in the condenser.



SOLUTION

Steam is condensed by cooling water in the condenser of a power plant. The mass flow rate of the cooling water and the rate of condensation are to be determined.

Assumptions

1 Steady operating conditions exist.
2 The heat exchanger is well insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid.
3 Changes in the kinetic and potential energies of fluid streams are negligible.
4 There is no fouling.
5 Fluid properties are constant.

Properties

The heat of vaporization of water at 30°C is h_{fg} = 2431 kJ/kg and the specific heat of cold water at the average temperature of 18°C is C_p = 4184 J/kg · °C (Table A–9).

Analysis

The schematic of the condenser is given in figure above. The condenser can be treated as a counter-flow heat exchanger since the temperature of one of the fluids (the steam) remains constant.

The temperature difference between the steam and the cooling water at the two ends of the condenser is

$$\Delta T_1 = T_{h,\text{in}} - T_{c,\text{out}} = (30 - 22)^\circ\text{C} = 8^\circ\text{C}$$
$$\Delta T_2 = T_{h,\text{out}} - T_{c,\text{in}} = (30 - 14)^\circ\text{C} = 16^\circ\text{C}$$

That is, the temperature difference between the two fluids varies from 8°C at one end to 16°C at the other. The proper average temperature difference between the two fluids is the logarithmic mean temperature difference (not the arithmetic), which is determined from

$$\Delta T_{\text{lm}} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} = \frac{8 - 16}{\ln(8/16)} = 11.5^\circ\text{C}$$

Then the heat transfer rate in the condenser is determined from

$$\dot{Q} = UA_s\,\Delta T_{\text{lm}} = (2100\ \text{W/m}^2\cdot{}^\circ\text{C})(45\ \text{m}^2)(11.5^\circ\text{C}) = 1.087 \times 10^6\ \text{W} = 1087\ \text{kW}$$

Therefore, the steam will lose heat at a rate of 1,087 kW as it flows through the condenser, and the cooling water will gain practically all of it, since the condenser is well insulated.

The mass flow rate of the cooling water and the rate of the condensation of the steam are determined from

$$[\dot{m}C_p(T_{\text{out}} - T_{\text{in}})]_{\text{cooling water}} = (\dot{m}h_{fg})_{\text{steam}}$$

$$\begin{aligned}\dot{m}_{\text{cooling water}} &= \frac{\dot{Q}}{C_p(T_{\text{out}} - T_{\text{in}})}\\ &= \frac{1{,}087\ \text{kJ/s}}{(4.184\ \text{kJ/kg}\cdot{}^\circ\text{C})(22 - 14)^\circ\text{C}} = 32.5\ \text{kg/s}\end{aligned}$$

5.4 THE EFFECTIVENESS-NTU METHOD

The LMTD approach to heat exchanger analysis is useful when the inlet and outlet temperatures are known or are easily determined. The LMTD is then easily calculated, and the heat flow, surface area, or overall heat transfer coefficient may be determined. When the inlet or exit temperatures are to be evaluated for a given heat exchanger, the analysis frequently involves an iterative procedure because of the logarithmic function in the LMTD. In these cases the analysis is performed more easily by utilizing a method based on the effectiveness of the heat exchanger in transferring a given amount of heat. The effectiveness method also offers many advantages for analysis of problems in which a comparison between various types of heat exchangers must be made for purposes of selecting the type best suited to accomplish a particular heat transfer objective.

We define the heat exchanger effectiveness as

$$effectiveness = \frac{\dot{Q}}{Q_{max}} = \frac{Actual\ heat\ transfer\ rate}{Maximum\ possible\ heat\ transfer\ rate}$$

The actual heat transfer rate in a heat exchanger can be determined from an energy balance on the hot or cold fluids and can be expressed as

$$\dot{Q} = C_c\left(T_{c,out} - T_{c,in}\right) = C_h(T_{h,in} - T_{h,out})$$

where $C_c = \dot{m}_c C_{pc}$ and $C_h = \dot{m}_c C_{ph}$ are the heat capacity rates of the cold and the hot fluids, respectively.

To determine the maximum possible heat transfer rate in a heat exchanger, we first recognize that the maximum temperature difference in a heat exchanger is the difference between the inlet temperatures of the hot and cold fluids. That is,

$$\Delta T_{max} = T_{h,in} - T_{c,in}$$

The heat transfer in a heat exchanger will reach its maximum value when (1) the cold fluid is heated to the inlet temperature of the hot fluid or (2) the hot fluid is cooled to the inlet temperature of the cold fluid. These two limiting conditions will not be reached simultaneously unless the heat capacity rates of the hot and cold fluids are identical (i.e.,$C_c = C_h$). When $C_c \neq C_h$, which is usually the case, the fluid with the smaller heat capacity rate will experience a larger temperature change, and thus it will be the first to experience the maximum temperature, at which point the heat transfer will come to a halt.

Therefore, the maximum possible heat transfer rate in a heat exchanger is (Fig. 5.10)

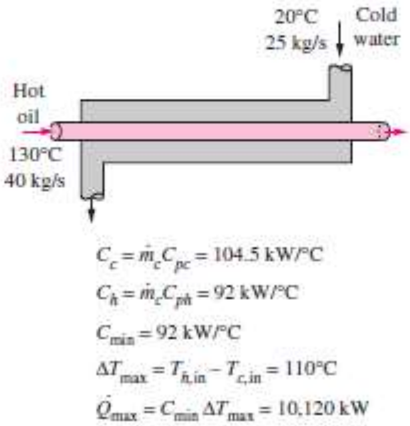


Figure 5.10 The determination of the maximum rate of heat transfer in a heat exchanger

$$\dot{Q}_{max} = C_{min}(T_{h,in} - T_{c,in})$$

where C_{min} is the smaller of C_h and C_c.

The determination of $\dot{Q}_{max}$requires the availability of the inlet temperature of the hot and cold fluids and their mass flow rates, which are usually specified. Then, once the effectiveness of the heat exchanger is known, the actual heat transfer rate $\dot{Q}$ can be determined from

$$Q = \varepsilon\dot{Q}_{max} = \varepsilon C_{min}(T_{h,in} - T_{c,in})$$

Therefore, the effectiveness of a heat exchanger enables us to determine the heat transfer rate without knowing the outlet temperatures of the fluids. The effectiveness of a heat exchanger depends on the geometry of the heat exchanger as well as the flow arrangement. Therefore, different types of heat exchangers have different effectiveness relations. Effectiveness relations of the heat exchangers typically involve the dimensionless group UA_s/C_{min}. This quantity is called the number of transfer units NTU and is expressed as

$$NTU = \frac{UA_s}{C_{min}} = \frac{UA_s}{(\dot{m}c_p)_{min}}$$

where U is the overall heat transfer coefficient and As is the heat transfer surface area of the heat exchanger. Note that NTU is proportional to A_s. Therefore, for specified values of U and C_{min}, the value of NTU is a measure of the heat transfer surface area A_s . Thus, the larger the NTU, the larger the heat exchanger. In heat exchanger analysis, it is also convenient to define another dimensionless quantity called the capacity ratio c as

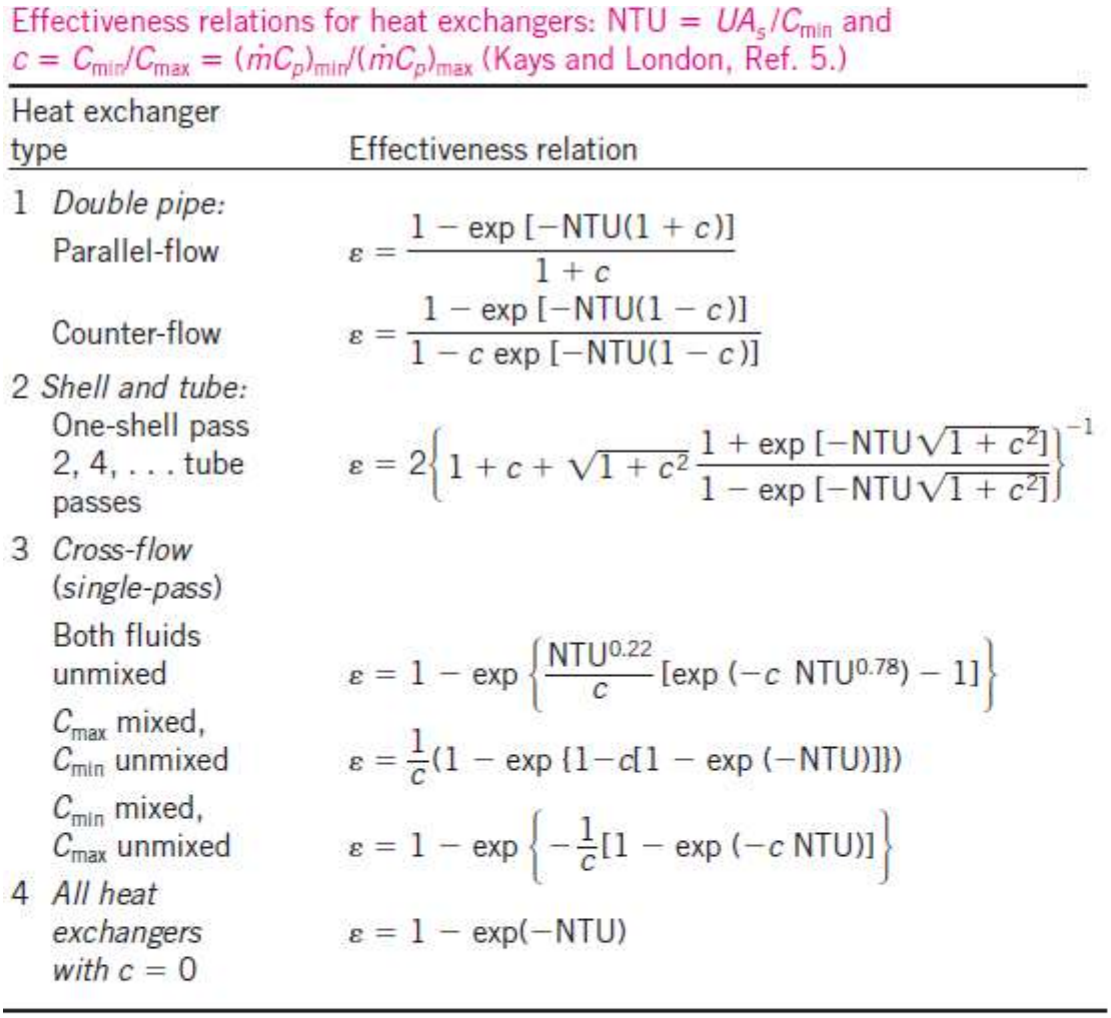
$$c = \frac{C_{min}}{C_{max}}$$

It can be shown that the effectiveness of a heat exchanger is a function of the number of transfer units NTU and the capacity ratio c. That is,

$$\varepsilon = function(UA_s/C_{min,}C_{min}/C_{max}) = function(NTU, c)$$

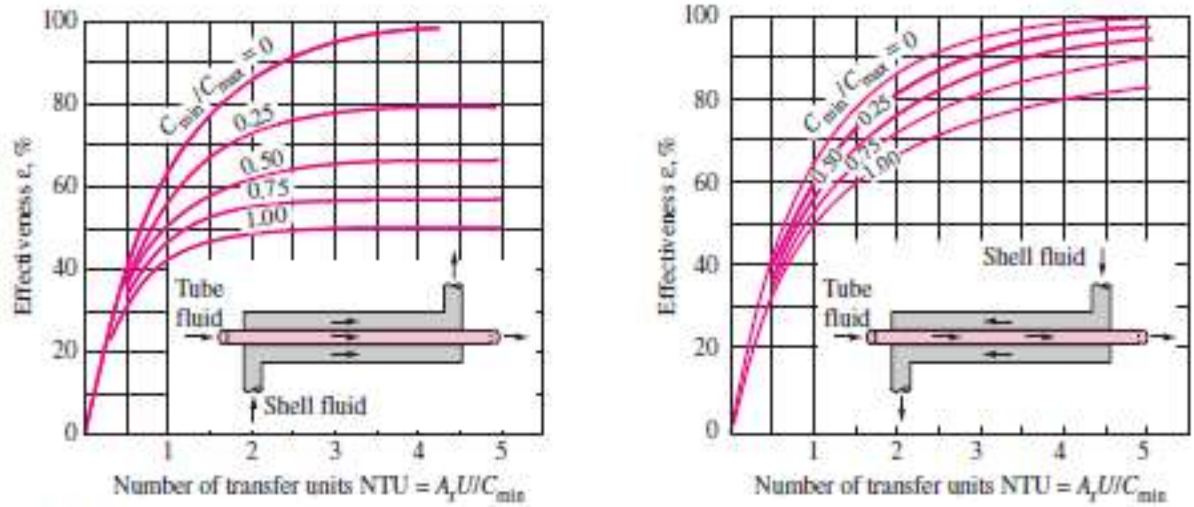
Effectiveness relations have been developed for a large number of heat exchangers, and the results are given in Table 5 .1. The effectivenesses of some common types of heat exchangers are also plotted in Figure 5.11

Table 5.1 Effectiveness relation for heat exchanger



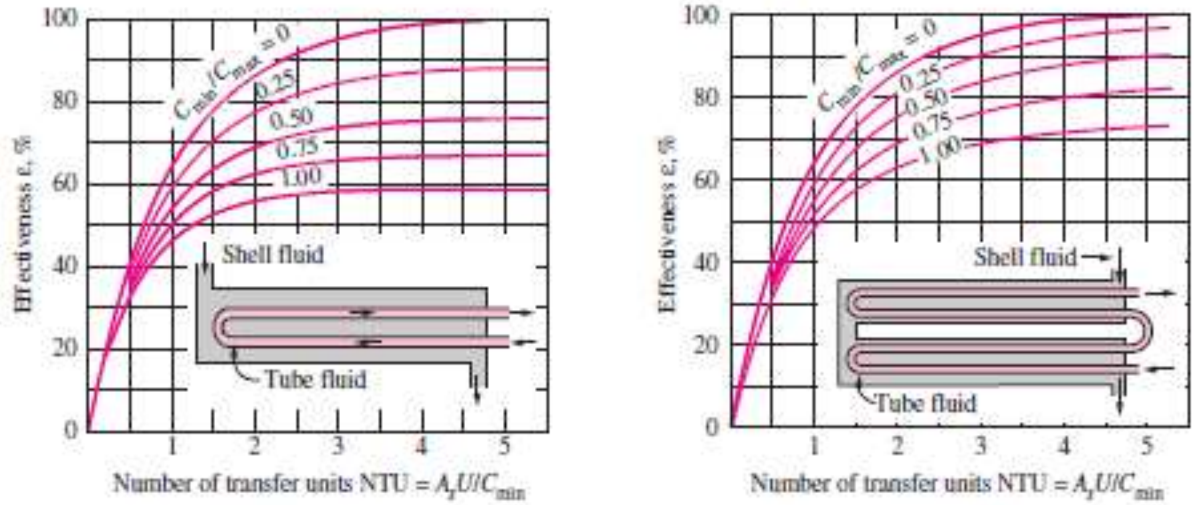
Effectiveness relations for heat exchangers: NTU = $UA_s/C_{\min}$ and $c = C_{\min}/C_{\max} = (\dot{m}C_p)_{\min}/(\dot{m}C_p)_{\max}$ (Kays and London, Ref. 5.)

Heat exchanger type	Effectiveness relation
1 *Double pipe:*	
Parallel-flow	$\varepsilon = \dfrac{1-\exp[-\mathrm{NTU}(1+c)]}{1+c}$
Counter-flow	$\varepsilon = \dfrac{1-\exp[-\mathrm{NTU}(1-c)]}{1-c\exp[-\mathrm{NTU}(1-c)]}$
2 *Shell and tube:*	
One-shell pass 2, 4, . . . tube passes	$\varepsilon = 2\left\{1+c+\sqrt{1+c^2}\,\dfrac{1+\exp[-\mathrm{NTU}\sqrt{1+c^2}]}{1-\exp[-\mathrm{NTU}\sqrt{1+c^2}]}\right\}^{-1}$
3 *Cross-flow (single-pass)*	
Both fluids unmixed	$\varepsilon = 1-\exp\left\{\dfrac{\mathrm{NTU}^{0.22}}{c}[\exp(-c\ \mathrm{NTU}^{0.78})-1]\right\}$
$C_{\max}$ mixed, $C_{\min}$ unmixed	$\varepsilon = \dfrac{1}{c}(1-\exp\{1-c[1-\exp(-\mathrm{NTU})]\})$
$C_{\min}$ mixed, $C_{\max}$ unmixed	$\varepsilon = 1-\exp\left\{-\dfrac{1}{c}[1-\exp(-c\ \mathrm{NTU})]\right\}$
4 *All heat exchangers with $c = 0$*	$\varepsilon = 1-\exp(-\mathrm{NTU})$



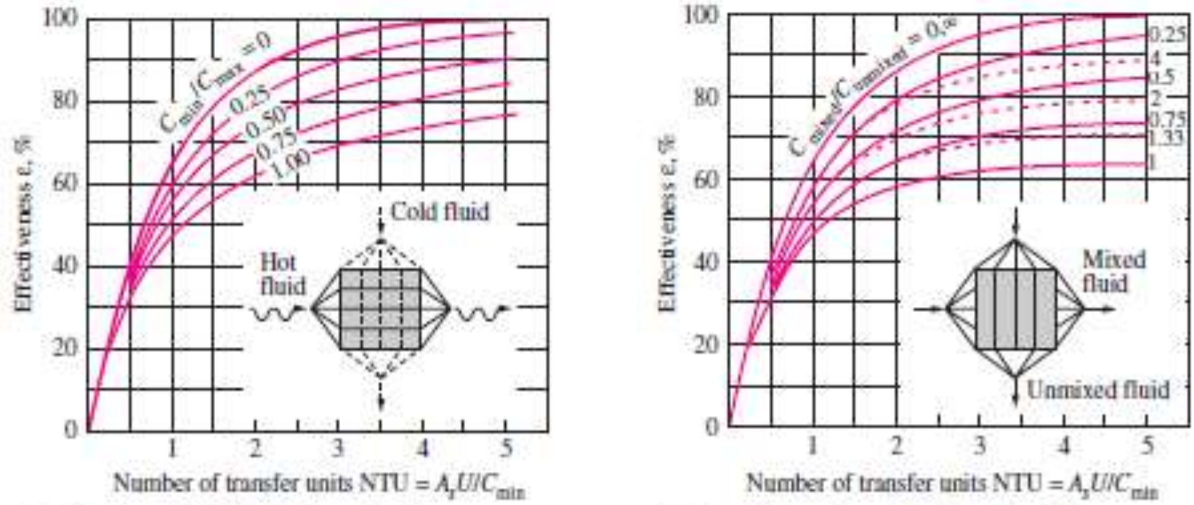
(*a*) Parallel-flow

(*b*) Counter-flow



(*c*) One-shell pass and 2, 4, 6, … tube passes

(*d*) Two-shell passes and 4, 8, 12, … tube passes



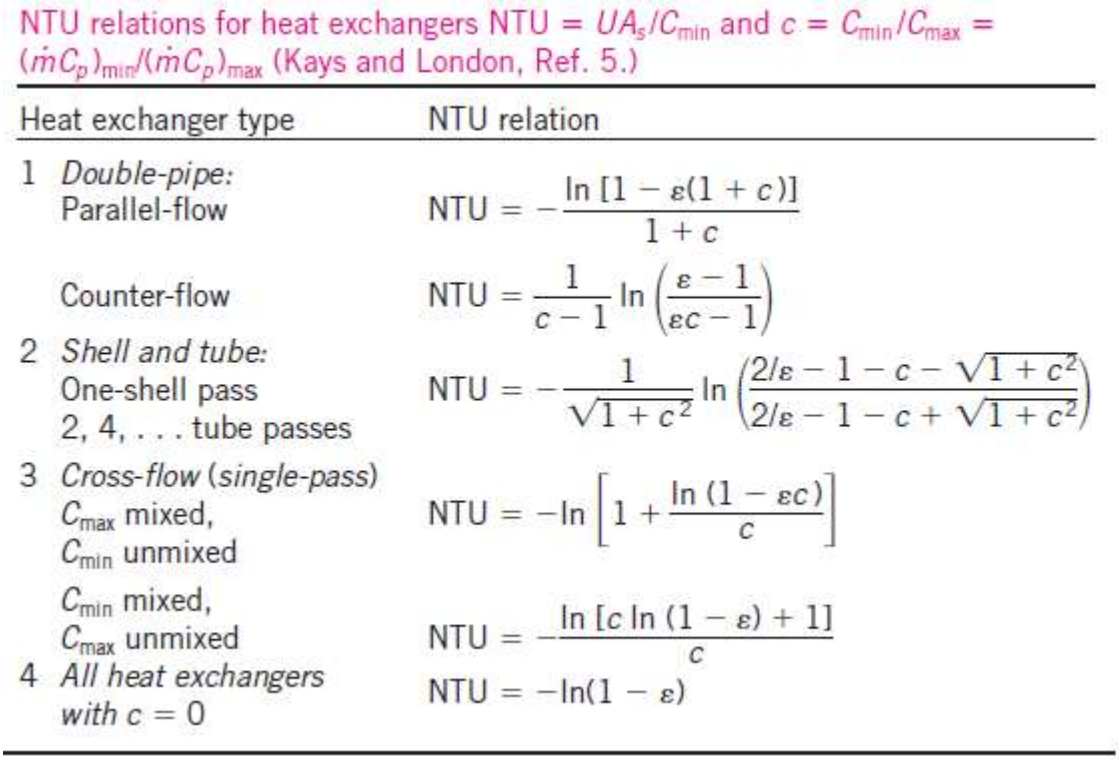
(*e*) Cross-flow with both fluids unmixed

(*f*) Cross-flow with one fluid mixed and the other unmixed

Figure 5.11 Effectiveness for heat exchangers (from Kays and London, Ref. 5).

We mentioned earlier that when all the inlet and outlet temperatures are specified, the size of the heat exchanger can easily be determined using the LMTD method. Alternatively, it can also be determined from the effectiveness–NTU method by first evaluating the effectiveness ε from its definition and then the NTU from the appropriate NTU relation in Table 5.2.

Table 5.2 NTU relations for heat exchanger



NTU relations for heat exchangers $\text{NTU} = UA_s/C_{\min}$ and $c = C_{\min}/C_{\max} = (\dot{m}C_p)_{\min}/(\dot{m}C_p)_{\max}$ (Kays and London, Ref. 5.)

Heat exchanger type	NTU relation
1 *Double-pipe:* Parallel-flow	$\text{NTU} = -\dfrac{\ln[1-\varepsilon(1+c)]}{1+c}$
Counter-flow	$\text{NTU} = \dfrac{1}{c-1}\ln\left(\dfrac{\varepsilon-1}{\varepsilon c-1}\right)$
2 *Shell and tube:* One-shell pass 2, 4, . . . tube passes	$\text{NTU} = -\dfrac{1}{\sqrt{1+c^2}}\ln\left(\dfrac{2/\varepsilon-1-c-\sqrt{1+c^2}}{2/\varepsilon-1-c+\sqrt{1+c^2}}\right)$
3 *Cross-flow* (*single-pass*) $C_{\max}$ mixed, $C_{\min}$ unmixed	$\text{NTU} = -\ln\left[1+\dfrac{\ln(1-\varepsilon c)}{c}\right]$
$C_{\min}$ mixed, $C_{\max}$ unmixed	$\text{NTU} = -\dfrac{\ln[c\ln(1-\varepsilon)+1]}{c}$
4 *All heat exchangers* *with* $c = 0$	$\text{NTU} = -\ln(1-\varepsilon)$

Note that the relations in Table 5.2 are equivalent to those in Table 5.1. Both sets of relations are given for convenience. The relations in Table 5.1 give the effectiveness directly when NTU is known, and the relations in Table 5.2 give the NTU directly when the effectiveness ε is known.

EXAMPLE 5.3 Upper Limit for Heat Transfer in a Heat Exchanger

Cold water enters a counter-flow heat exchanger at 10°C at a rate of 8 kg/s, where it is heated by a hot water stream that enters the heat exchanger at 70°C at a rate of 2 kg/s. Assuming the specific heat of water to remain constant at C_p 4.18 kJ/kg · °C, determine the maximum heat transfer rate and the outlet temperatures of the cold and the hot water streams for this limiting case.

SOLUTION

Cold and hot water streams enter a heat exchanger at specified temperatures and flow rates. The maximum rate of heat transfer in the heat exchanger is to be determined.

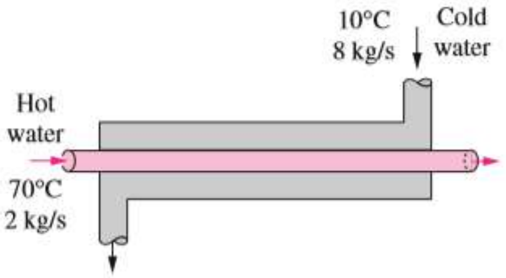
Assumptions

1. Steady operating conditions exist.
2. The heat exchanger is well insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to heat transfer to the cold fluid.

3. Changes in the kinetic and potential energies of fluid streams are negligible. 4 Heat transfer coefficients and fouling factors are constant and uniform. 5 The thermal resistance of the inner tube is negligible since the tube is thin-walled and highly conductive.

Properties
The specific heat of water is given to be C_p =4.18 kJ/kg · °C.

Analysis
A schematic of the heat exchanger is given in Figure.



The heat capacity rates of the hot and cold fluids are determined from

$$C_h = \dot{m}_h C_{ph} = (2\text{ kg/s})(4.18\text{ kJ/kg}\cdot{}^\circ\text{C}) = 8.36\text{ kW/}^\circ\text{C}$$

And

$$C_c = \dot{m}_c C_{pc} = (8\text{ kg/s})(4.18\text{ kJ/kg}\cdot{}^\circ\text{C}) = 33.4\text{ kW/}^\circ\text{C}$$

Therefore

$$C_{\min} = C_h = 8.36\text{ kW/}^\circ\text{C}$$

which is the smaller of the two heat capacity rates. Then the maximum heat transfer rate is

$$\begin{aligned}\dot{Q}_{\max} &= C_{\min}(T_{h,\text{ in}} - T_{c,\text{ in}})\\ &= (8.36\text{ kW/}^\circ\text{C})(70 - 10)^\circ\text{C}\\ &= 502\text{ kW}\end{aligned}$$

That is, the maximum possible heat transfer rate in this heat exchanger is 502 kW. This value would be approached in a counter-flow heat exchanger with a *very large* heat transfer surface area.
The maximum temperature difference in this heat exchanger is
$\Delta T_{\max} = T_{h,\text{ in}} - T_{c,\text{ in}} = (70 - 10)^\circ\text{C} = 60^\circ\text{C}$.

Therefore, the hot water cannot be cooled by more than 60°C (to 10°C) in this heat exchanger, and the cold water cannot be heated by more than 60°C (to 70°C), no matter what we do. The outlet temperatures of the cold and the hot streams in this limiting case are determined to be

$$\dot{Q} = C_c(T_{c,\text{ out}} - T_{c,\text{ in}}) \longrightarrow T_{c,\text{ out}} = T_{c,\text{ in}} + \frac{\dot{Q}}{C_c} = 10^\circ\text{C} + \frac{502\text{ kW}}{33.4\text{ kW/}^\circ\text{C}} = 25^\circ\text{C}$$

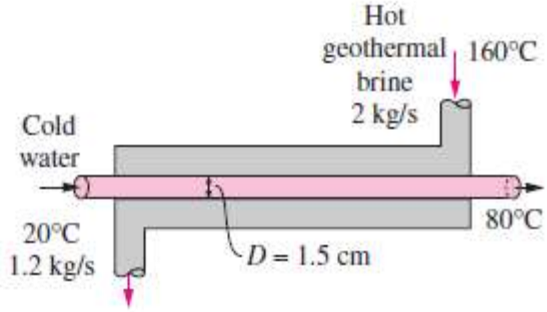
$$\dot{Q} = C_h(T_{h,\text{ in}} - T_{h,\text{ out}}) \longrightarrow T_{h,\text{ out}} = T_{h,\text{ in}} - \frac{\dot{Q}}{C_h} = 70^\circ\text{C} - \frac{502\text{ kW}}{8.38\text{ kW/}^\circ\text{C}} = 10^\circ\text{C}$$

EXAMPLE 3.4 Using the Effectiveness–NTU Method

Repeat Example 3.3, which was solved with the LMTD method, using the effectiveness–NTU method.

SOLUTION

The schematic of the heat exchanger is redrawn in Figure, and the same assumptions are utilized.



Analysis

In the effectiveness–NTU method, we first determine the heat capacity rates of the hot and cold fluids and identify the smaller one:

$$C_h = \dot{m}_h C_{ph} = (2\ \text{kg/s})(4.31\ \text{kJ/kg}\cdot{}^\circ\text{C}) = 8.62\ \text{kW/}^\circ\text{C}$$
$$C_c = \dot{m}_c C_{pc} = (1.2\ \text{kg/s})(4.18\ \text{kJ/kg}\cdot{}^\circ\text{C}) = 5.02\ \text{kW/}^\circ\text{C}$$

Therefore

$$C_{\min} = C_c = 5.02\ \text{kW/}^\circ\text{C}$$

And

$$c = C_{\min}/C_{\max} = 5.02/8.62 = 0.583$$

Then the maximum heat transfer rate is

$$\begin{aligned}\dot{Q}_{\max} &= C_{\min}(T_{h,\text{in}} - T_{c,\text{in}})\\ &= (5.02\ \text{kW/}^\circ\text{C})(160 - 20)^\circ\text{C}\\ &= 702.8\ \text{kW}\end{aligned}$$

That is, the maximum possible heat transfer rate in this heat exchanger is 702.8 kW. The actual rate of heat transfer in the heat exchanger is

$$\dot{Q} = [\dot{m}C_p(T_{\text{out}} - T_{\text{in}})]_{\text{water}} = (1.2\ \text{kg/s})(4.18\ \text{kJ/kg}\cdot{}^\circ\text{C})(80 - 20)^\circ\text{C} = 301.0\ \text{kW}$$

Thus, the effectiveness of the heat exchanger is

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\max}} = \frac{301.0\ \text{kW}}{702.8\ \text{kW}} = 0.428$$

Knowing the effectiveness, the NTU of this counter-flow heat exchanger can be determined from Figure 5.11b or the appropriate relation from Table 5.2. We choose the latter approach for greater accuracy:

$$\text{NTU} = \frac{1}{c-1}\ln\left(\frac{\varepsilon - 1}{\varepsilon c - 1}\right) = \frac{1}{0.583 - 1}\ln\left(\frac{0.428 - 1}{0.428 \times 0.583 - 1}\right) = 0.651$$

Then the heat transfer surface area becomes

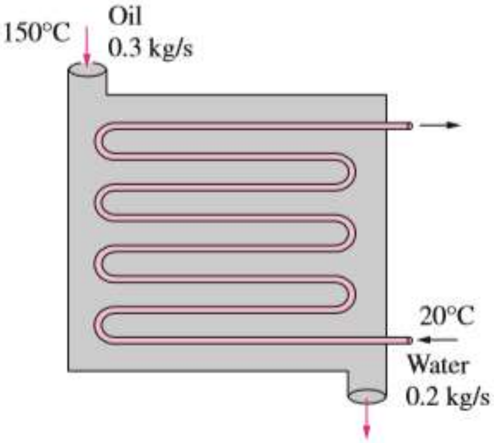
$$\text{NTU} = \frac{UA_s}{C_{\min}} \longrightarrow A_s = \frac{\text{NTU}\, C_{\min}}{U} = \frac{(0.651)(5020 \text{ W/°C})}{640 \text{ W/m}^2 \cdot \text{°C}} = 5.11 \text{ m}^2$$

To provide this much heat transfer surface area, the length of the tube must be

$$A_s = \pi D L \longrightarrow L = \frac{A_s}{\pi D} = \frac{5.11 \text{ m}^2}{\pi(0.015 \text{ m})} = 108 \text{ m}$$

EXAMPLE 5.5 Cooling Hot Oil by Water in a Multipass Heat Exchanger

Hot oil is to be cooled by water in a 1-shell-pass and 8-tube-passes heat exchanger. The tubes are thin-walled and are made of copper with an internal diameter of 1.4 cm. The length of each tube pass in the heat exchanger is 5 m, and the overall heat transfer coefficient is 310 W/m^2 · °C. Water flows through the tubes at a rate of 0.2 kg/s, and the oil through the shell at a rate of 0.3 kg/s. The water and the oil enter at temperatures of 20°C and 150°C, respectively. Determine the rate of heat transfer in the heat exchanger and the outlet temperatures of the water and the oil.



SOLUTION

Hot oil is to be cooled by water in a heat exchanger. The mass flow rates and the inlet temperatures are given. The rate of heat transfer and the outlet temperatures are to be determined.

Assumptions

1 Steady operating conditions exist.
2 The heat exchanger is well insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid.
3 The thickness of the tube is negligible since it is thin-walled.
4 Changes in the kinetic and potential energies of fluid streams are negligible.
5 The overall heat transfer coefficient is constant and uniform.

Analysis The schematic of the heat exchanger is given in the figure. The outlet temperatures are not specified, and they cannot be determined from an energy balance. The use of the LMTD method in this case will involve tedious iterations, and thus the ε –NTU method is indicated. The first step in the ε –NTU method is to determine the heat capacity rates of the hot and cold fluids and identify the smaller one:

$$C_h = \dot{m}_h C_{ph} = (0.3 \text{ kg/s})(2.13 \text{ kJ/kg} \cdot \text{°C}) = 0.639 \text{ kW/°C}$$
$$C_c = \dot{m}_c C_{pc} = (0.2 \text{ kg/s})(4.18 \text{ kJ/kg} \cdot \text{°C}) = 0.836 \text{ kW/°C}$$

Therefore

$$C_{\min} = C_h = 0.639 \text{ kW/°C}$$

And

$$c = \frac{C_{\min}}{C_{\max}} = \frac{0.639}{0.836} = 0.764$$

Then the maximum heat transfer rate is

$$\begin{aligned}\dot{Q}_{\max} &= C_{\min}(T_{h,\text{ in}} - T_{c,\text{ in}}) \\ &= (0.639 \text{ kW/°C})(150 - 20)\text{°C} = 83.1 \text{ kW}\end{aligned}$$

That is, the maximum possible heat transfer rate in this heat exchanger is 83.1 kW. The heat transfer surface area is

$$A_s = n(\pi DL) = 8\pi(0.014 \text{ m})(5 \text{ m}) = 1.76 \text{ m}^2$$

Then the NTU of this heat exchanger becomes

$$\text{NTU} = \frac{UA_s}{C_{\min}} = \frac{(310 \text{ W/m}^2 \cdot \text{°C})(1.76 \text{ m}^2)}{639 \text{ W/°C}} = 0.853$$

The effectiveness of this heat exchanger corresponding to c = 0.764 and NTU = 0.853 is determined from Figure 5.11c to be

$$\varepsilon = 0.47$$

We could also determine the effectiveness from the third relation in Table 5.1 more accurately but with more labor. Then the actual rate of heat transfer becomes

$$\dot{Q} = \varepsilon \dot{Q}_{\max} = (0.47)(83.1 \text{ kW}) = 39.1 \text{ kW}$$

Finally, the outlet temperatures of the cold and the hot fluid streams are determined to be

$$\begin{aligned}\dot{Q} = C_c(T_{c,\text{ out}} - T_{c,\text{ in}}) \quad \longrightarrow \quad T_{c,\text{ out}} &= T_{c,\text{ in}} + \frac{\dot{Q}}{C_c} \\ &= 20\text{°C} + \frac{39.1 \text{ kW}}{0.836 \text{ kW/°C}} = 66.8\text{°C}\end{aligned}$$

$$\begin{aligned}\dot{Q} = C_h(T_{h,\text{ in}} - T_{h,\text{ out}}) \quad \longrightarrow \quad T_{h,\text{ out}} &= T_{h,\text{ in}} - \frac{\dot{Q}}{C_h} \\ &= 150\text{°C} - \frac{39.1 \text{ kW}}{0.639 \text{ kW/°C}} = 88.8\text{°C}\end{aligned}$$

Therefore, the temperature of the cooling water will rise from 20°C to 66.8°C as it cools the hot oil from 150°C to 88.8°C in this heat exchanger.

CHAPTER 6

MASS TRANSFER

At the end of this chapter, you should be able to:

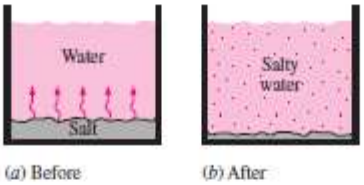
- Explain the principles of mass transfer and diffusion.
- Discuss the Fick’s Law for molecular diffusion.
- Explain the principle of molecular diffusion in gas

6.1 PRINCIPLES OF MASS TRANSFER AND DIFFUSION

Up to this point, we have focused solely on heat transfer problems without considering mass transfer. However, in practical applications, many critical heat transfer problems also involve mass transfer. For instance, approximately one-third of the heat loss from a resting person occurs due to evaporation. Interestingly, mass transfer shares many similarities with heat transfer, and the relationships between the two are closely related. In this chapter, we explore the mechanisms of mass transfer and establish equations to determine mass transfer rates for common practical scenarios.

6.1.1 Introduction to Mass Transfer

Mass transfer is the movement of mass from one location to another, driven by differences in concentration. This phenomenon is central to many engineering and natural systems, such as distillation, evaporation, and biological processes. The driving force for mass transfer is the **concentration gradient**, which causes molecules to move from regions of higher concentration to lower concentration.



(a) Before (b) After

Figure 6.1: Whenever there is concentration difference of a physical quantity in a medium, nature tends to equalize things by forcing a flow from the high to the low concentration region.

6.1.2 Mechanisms of Mass Transfer

To better understand the diffusion process, imagine a tank divided into two equal sections by a partition. Initially, the left section contains pure nitrogen gas (N_2), while the right section contains air (approximately 21% oxygen (O_2) and 79% nitrogen (N_2)) at the same temperature and pressure. In this setup, O_2 and N_2 molecules are represented by dark and light circles, respectively.

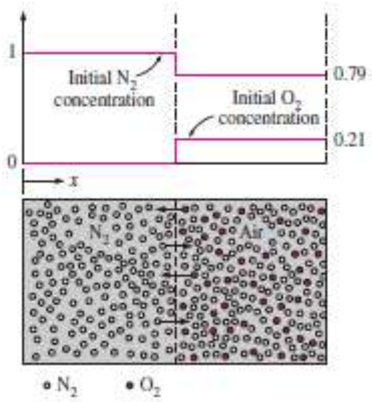
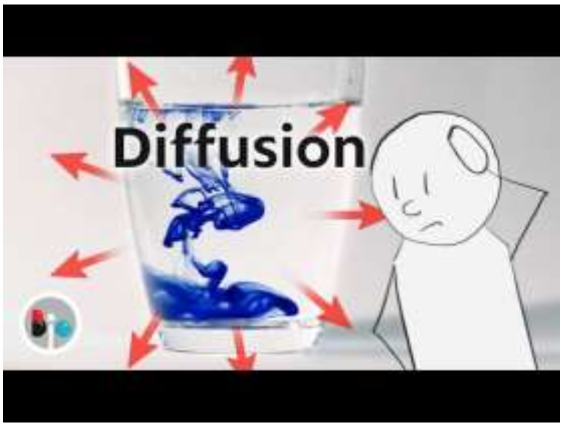


Figure 6.2: A tank that contains N_2 and air in its two compartments, and the diffusion of N_2 into the air when the partition is removed.

When the partition is removed, N_2 molecules start diffusing into the air, while O_2 molecules diffuse into the nitrogen, as illustrated in Figure 6.2. Over time, the gases mix uniformly, resulting in a homogeneous distribution of N_2 and O_2 throughout the tank.

This diffusion process can be analysed by considering an imaginary dashed line across the tank. Gas molecules move randomly, so the likelihood of a molecule crossing to either side of the line is equal. At any given moment, half of the molecules on one side of the line will cross to the other. However, since the concentration of N_2 is initially higher on the left side than on the right, more N_2 molecules move toward the right, creating a net flow of N_2 in that direction. Similarly, a net flow of O_2 occurs from the right to the left.

This process continues until the concentrations of N_2 and O_2 are uniform throughout the tank, reaching a state where the number of molecules moving in each direction is equal, resulting in zero net transfer of N_2 or O_2 across an imaginary line. Watch the video below to further understand how the particles move in a container.



6.2 Similarity of mass, heat and momentum transfer process

Mass, heat, and momentum transfer processes share fundamental similarities in the way they occur, despite involving different physical quantities. All three are driven by gradients that create imbalances in a system, prompting nature to restore equilibrium by facilitating the transfer of mass, heat, or momentum. This shared behaviour allows us to draw analogies between the governing principles and mathematical formulations of these processes.

6.2.1 Driving forces

The transfer of mass, heat, or momentum is driven by a gradient in the respective quantity:

1. **Mass Transfer** occurs due to a concentration gradient. Molecules naturally move from regions of higher concentration to lower concentration, as seen when salt dissolves in water or when perfume spreads through a room.
2. **Heat Transfer** is driven by a temperature gradient. Heat flows from areas of high temperature to areas of low temperature, such as when a metal rod conducts heat or a hot drink cools down.
3. **Momentum Transfer** happens due to a velocity gradient. Momentum is transferred between layers of fluid that move at different velocities, such as the frictional forces in a pipe's fluid flow or wind interacting with the surface of the Earth.

Despite the differences in what is being transferred (mass, heat, or momentum), all three processes follow the same principle: the quantity moves from a region of high intensity to a region of low intensity to restore balance.

6.2.2 Governing Laws

The mathematical laws governing each process are structurally similar:

1. **Mass Transfer** is described by Fick's Law, where the mass flux is proportional to the concentration gradient.

$$J*_{Az} = -A_B \frac{dC_A}{dz}$$

2. **Heat Transfer** follows Fourier's Law for conduction, where the heat flux is proportional to the temperature gradient.

$$\frac{q_z}{A} = -\alpha \frac{d(\rho C_\rho T)}{dz}$$

3. **Momentum Transfer** is defined by Newton's Law of for momentum transfer for constant density.

$$\tau_{zx} = -\frac{\mu}{\rho}\frac{d(v_x\rho)}{dz}$$

These laws all involve a proportional relationship between the transfer flux and the gradient of the driving force, highlighting a consistent mathematical framework across these processes.

6.2.3 Analogy between heat and mass transfer

The driving force for heat transfer is the temperature difference. In contrast, the driving force for mass transfer is the concentration difference. We can view temperature as a measure of "heat concentration," and thus a high temperature region as one that has a high heat concentration. Therefore, both heat and mass are transferred from the more concentrated regions to the less concentrated ones. If there is no temperature difference between two regions, then there is no heat transfer. Likewise, if there is no difference between the concentrations of a species at different parts of a medium, there will be no mass transfer.

Heat is transferred by conduction, convection, and radiation. Mass, however, is transferred by conduction (called diffusion) and convection only. The rate of heat conduction in a direction x is proportional to the temperature gradient dT/dx in that direction and is expressed by Fourier's law of heat conduction as

$$\dot{Q}_{cond} = -kA\frac{dT}{dx}$$

Likewise, the rate of mass diffusion m_{diff} or J^*_{AZ} of a chemical species A in a stationary medium in the direction x is proportional to the concentration gradient dC/dx in that direction and is expressed by Fick's law of diffusion

$$J^*_{Az} = -D_{AB}\frac{dc_A}{dz} \qquad \dot{m}_{diff} = -D_{AB}A\frac{dC_A}{dx}$$

6.2.4 Key analogies

Beyond their governing laws, mass, heat, and momentum transfer exhibit analogous behaviours:

- All involve the movement of a property through a medium, whether it is molecules diffusing, heat energy conducting, or momentum being exchanged between fluid layers.
- The transfer is always driven by an imbalance, which the system works to equalize.
- Each process depends on molecular motion at a microscopic level, showing a connection in their fundamental nature.

The shared principles and behaviours of mass, heat, and momentum transfer create a unified framework that simplifies problem-solving in engineering. Recognizing these similarities helps professionals design efficient systems and address challenges involving multiple transfer processes, enhancing our ability to innovate across various fields.

6.3 Example of mass transfer process

Mass Transfer in Nature

- Evaporation: When water in an open container evaporates into the air, it illustrates mass transfer driven by a concentration difference between the water vapor near the surface and the surrounding air.
- Diffusion in Liquids: A lump of sugar dissolving in a cup of coffee demonstrates diffusion as sugar molecules spread uniformly throughout the liquid.
- Perfume Diffusion: The scent of perfume spreads across a room due to the random movement of molecules from high to low concentration.

Industrial Applications

- Distillation: Separating alcohol from water through vaporization and condensation.
- Absorption: Removing sulphur dioxide from flue gases by dissolving it in a basic liquid solution.

Biological Significance

- In fermentation processes, oxygen and nutrients diffuse through a solution to reach microorganisms, ensuring their survival and activity.

6.4 Fick’s Law for molecular diffusion

We mentioned earlier that the rate of mass diffusion of a chemical species in a stagnant medium in a specified direction is proportional to the local concentration gradient in that direction. This linear relationship between the rate of diffusion and the concentration gradient proposed by Fick in 1855 is known as Fick’s law of diffusion and can be expressed as

Mass flux = Constant proportionality X Concentration gradient

Theoretically, Fick's Law for molecular diffusion can be understood with reference to Figure 6.3, which illustrates the random movement of a molecule A as it diffuses through molecules B from point (1) to point (2). If the concentration of A molecules is higher at point (1) than at point (2), the random motion of molecules in both directions will result in more A molecules moving from (1) to (2) than from (2) to (1). This imbalance leads to a net diffusion of A from the region of higher concentration to the region of lower concentration.

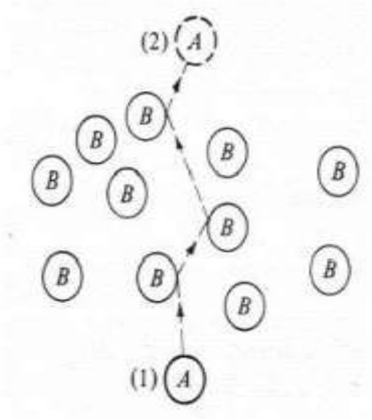


Figure 6.3: Schematic diagram of molecules diffusion process

The general Fick's Law equation:

$$J^*_{Az} = -D_{AB}\frac{dC_A}{dz}$$

Where:

- J_{Az}* : Molar flux of component A in the z-direction due to molecular diffusion (kg mol/A.s.m²).
- D_{AB}: Molecular diffusivity of molecules A in B (m²/s).
- C_A: Concentration of A (kg mol/m³).
- z: Distance of diffusion (m).

At steady state:

J_{Az}* and D_{AB} are constant. After rearranging and integrating the equation:

$$J^*_{Az}\int_{z_1}^{z_2} dz = -D_{AB}\int_{C_{A1}}^{C_{A2}} dC_A$$

$$\therefore J^*_{Az} = \frac{D_{AB}(C_{A1} - C_{A2})}{z_2 - z_1}$$

Example 6.1 Gas mixture

A mixture of He and N_2 gas is contained in a pipe at 298 K and 1 atm total pressure which is constant throughout. At one end of the pipe at point 1, the partial pressure p_{A1} of He is 0.6 atm and the other end 0.2 m (20 cm) p_{A2}= 0.2 atm. Calculate the flux of He at steady state if D_{AB} of the He-N_2 mixture is 0.687×10^{-4} m²/s (0.687 cm²/s).

Given:

$$R = 8314.3\,\frac{m^3.\,Pa}{kg\ mol.K}$$

$$R = 82.057\ \times 10^{-3}\,\frac{m^3.\,atm}{g\ mol.K}$$

SOLUTION

$$PV = nRT$$

$$\frac{n}{V} = \frac{P}{RT} = C$$

$$J_{*AZ}\int_{z_1}^{z_2} dz = -D_{AB}\int_{C_{A1}}^{C_{A2}} dC_A$$

$$\int_{z_1}^{z_2} dz = -D_{AB}\int_{C_{A1}}^{C_{A2}} \frac{P}{RT}$$

$$J_{*AZ} = \frac{D_{AB}(P_{A1}-P_{A2})}{RT(z_2-z_1)}$$

$$J_{*AZ} = \frac{(0.687\times10^{-4}\,\frac{m^2}{s})(6.08\times10^{4}-2.027\times10^{4}\,Pa)}{(8314\,\frac{Pa.m^3}{K.kg\ mol})(298K)(0.20-0\ m)}$$

$$= 5.63\times10^{-6}\,\frac{kg\ mol}{s.m^2}$$

6.5 Equimolar counter diffusion in gases

Equimolar counter diffusion refers to a mass transfer process in which two different gas species diffuse through each other in opposite directions at equal molar rates. This means that the molar flux of one gas moving in one direction is exactly balanced by the molar flux of the other gas moving in the opposite direction. The total molar flux in the system is, therefore, zero, as the opposing fluxes cancel each other out.

This phenomenon can be explained by Fick's Law, which states that the molar flux of a species is proportional to the concentration gradient. For equimolar counter diffusion, the concentration gradients of the two gases are equal and opposite, ensuring that the molar flow rates are balanced. The driving force for diffusion in this scenario is the concentration gradient of each gas, which results in a steady-state system with no net movement of the total mixture. An example of equimolar counter diffusion is found in the diffusion of gas A and B gases as shown in Figure 6.4. As A diffuses toward B region, B simultaneously diffuses toward the A region at an equal rate.

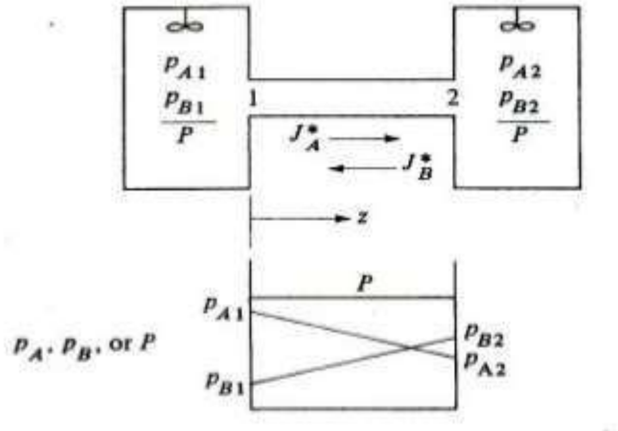


Figure 6.4 Equimolar counterdiffusion of gas A and B

This concept is particularly useful in simplifying the analysis of diffusion systems. By knowing that the molar fluxes are equal and opposite, engineers can focus on the individual behaviours of the gases without accounting for changes in the total mixture. Substitution of Fick's Law into the equation for equimolar counter diffusion;

$$J_{Az}^* = -J_{Bz}^*$$

$$-D_{AB}\frac{dc_A}{dz} = -(-D_{BA}\frac{dc_B}{dz})$$

Example 6.2: Equimolar Counter diffusion

Ammonia gas (A) is diffusing through a uniform tube 0.10 m long containing N_2 gas (B) at 1.0132×10^5 Pa pressure and 298K. The diagram is similar to Figure 6.4. At point 1, $P_{A1}= 1.013\times10^4$ Pa and at point 2, $P_{A2}= 0.507\times10^4$ Pa. The diffusivity $D_{AB} = 0.230\times10^{-4}$ m^2/s.

a) Calculate the flux J^*_A at steady state
b) Repeat for J^*_B

Solution

$$J^*_A = \frac{D_{AB}(P_{A1}-P_{A2})}{RT(z_2-z_1)}$$

$$= \frac{(0.23\times10^{-4}\ \frac{m^2}{s})(1.013\times10^4 - 0.507\times10^4\ Pa)}{(8314\ \frac{Pa.m^3}{K.kg\ mol})(298K)(0.10-0\ m)}$$

$$= 4.70\times10^{-7}\frac{kg\ mol\ A}{s.m^2}$$

$J^*_B = \frac{D_{AB}(P_{B1}-P_{B2})}{RT(z_2-z_1)}$ $\quad\quad$ $P_{B1} = P - P_{A1}$ $\quad\quad$ $P_{B2} = P - P_{A2}$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad = 9.119 \times 10^4$ $\quad\quad$ $= 9.625 \times 10^4\ Pa$

$$= \frac{(0.23 \times 10^{-4}\ \frac{m^2}{s})(9.119 \times 10^4 - 9.625 \times 10^4\ Pa)}{(8314\ \frac{Pa.m^3}{K.kg\ mol})(298K)(0.10 - 0\ m)}$$

$$= -4.70 \times 10^{-7} \frac{kg\ mol\ A}{s.m^2}$$

The negative value for J^*_B means the flux goes from point 2 to point 1

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