KEMENTERIAN PENDIDIKAN TINGGI
JABATAN PENDIDIKAN POLITEKNIK DAN KOLEJ KOMUNITI

POLITEKNIK MALAYSIA
TUN SYED NASIR

NUMERICAL METHOD

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NUMERICAL METHOD

Compatible with Polytechnic syllabus

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PREFACE

Purpose and structure of the book

This book provides a comprehensive, thorough, and up-to-date syllabus of Engineering Mathematics 3 and Electrical Engineering Mathematics as well as compatible with Malaysian polytechnic's syllabus. In particular, this book suitable for all diploma programmes at Malaysian polytechnic. It is intended to introduce students of engineering and those areas most related fields of applied mathematics that are relevant for solving practical problems.

Each of the subject matter is arranged into four subtopics as follows:

1. Gaussian Elimination Method
2. Crout Method
3. Doolittle Method
4. Fixed Point Iteration Method
5. False placement Method
6. Newton Raphson Method

As a whole, the theory is introduced in each subtopic by a simple definition and formula followed by a procedure for each example related to each topic. Every subtopic is followed by self practice at the end of each chapters that can be done as a drill.

NUMERICAL METHOD

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IN THE NAME OF ALLAH

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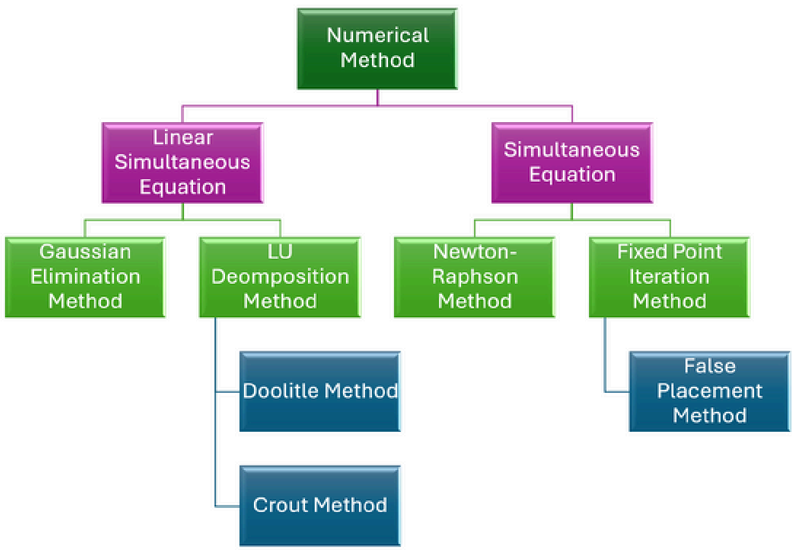
Table of Contents

INTRODUCTION

Numerical methods are techniques used to approximate solutions to mathematical problems that cannot be solved exactly. They are especially useful for solving equations, differential equations, and optimization problems where analytical solutions are difficult or impossible to obtain. Numerical methods are widely used in various fields such as engineering, physics, finance, and computer science.



3 METHODS TO SOLVE LINEAR SIMULTANEOUS EQUATIONS

1

GAUSSION ELIMINATION METHOD

Row elimination method is applied

2

Doolitle Method

A=LU

The value of 1 is on Lower Triangular Matrix

3

CROUT METHOD

A=LU

The value of 1 is on Upper Triangular Matrix

GAUSSIAN ELIMINATION METHOD

The Gaussian Method, often referred to as Gaussian Elimination, is a fundamental algorithm for solving systems of linear equations. It transforms a given system of linear equations into a simpler form that can be solved using back substitution. The method is named after the mathematician Carl Friedrich Gauss.

Steps of Gaussian Elimination

1

Form the Augmented Matrix:

Represent the system of equations in matrix form $[A|b]$, where A is the matrix of coefficients and b is the column matrix of constants.

2

Perform Row Operations:

Use elementary row operations to convert the matrix into upper triangular form. The row operations allowed are:

- **Swapping Rows:** Interchanging two rows.
- **Scaling Rows:** Multiplying a row by a non-zero scalar.
- **Row Addition/Subtraction:** Adding or subtracting a multiple of one row to another row.

The goal is to create zeros below the main diagonal (the diagonal from the top-left to the bottom-right of the matrix).

For example, to eliminate the entry in the i-th row and j-th column, you would perform:

$$R_i \leftarrow R_i - \frac{a_{ij}}{a_{jj}} R_j$$

where R_i and R_j are rows, and a_{ij} and a_{jj} are the matrix elements.

3

Repeat:

Continue the row operations to zero out elements below the diagonal for each column, moving from left to right and top to bottom.

Based on the following linear equations:

$$10x + 5y + 2z = 12$$
$$8x + 10y - 1z = 8$$
$$6x + 15y - 3z = 6$$

Calculate the value of x, y and z.

Answer

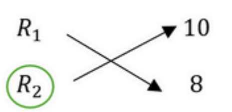
STEP 1: Transform the linear equations into matrices

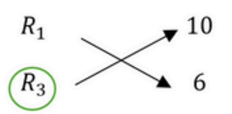
$$\begin{bmatrix} 10 & 5 & 2 \\ 8 & 10 & -1 \\ 6 & 15 & 1-3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 8 \\ 6 \end{bmatrix}$$

STEP 2: Form Augmented Matrix [A|B]

$$\left[\begin{array}{ccc|c} 10 & 5 & 2 & 12 \\ 8 & 10 & -1 & 8 \\ 6 & 15 & 1-3 & 6 \end{array}\right]$$

STEP 3: Express $a_{21} = 0$ & $a_{31} = 0$



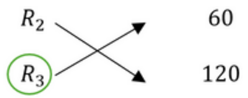


$R_{2(new)} = 10R_2 - 8R_1$	$R_{3(new)} = 10R_3 - 6R_1$
$10(8) - 8(110) = 0$	$10(6) - 6(10) = 0$
$10(10) - 8(5) = 60$	$10(15) - 6(5) = 120$
$10(-1) - 8(2) = -26$	$10(-3) - 6(2) = -42$
$10(8) - 8(12) = -16$	$10(6) - 6(12) = -12$

The new matrix is

$$\left[\begin{array}{ccc|c} 10 & 5 & 2 & 12 \\ 0 & 60 & -26 & -16 \\ 0 & 120 & -42 & -12 \end{array}\right]$$

STEP 4: Express $a_{32} = 0$



$$R_{3(new)} = 60R_3 - 120R_2$$

$$60(0) - 120(0) = 0$$

$$60(120) - 120(60) = 0$$

$$60(-42) - 120(-26) = 600$$

$$60(-12) - 120(-16) = 1200$$

The new matrix is

$$\left[\begin{array}{ccc|c} 10 & 5 & 2 & 12 \\ 0 & 60 & -26 & -16 \\ 0 & 0 & 600 & 1200 \end{array}\right]$$

START

STEP 5: Transform the matrices into linear equations

$$600z = 1200$$

$$z = 2$$

$$60y - 26z = -16$$

$$60y - 26(2) = -16$$

$$60y = 36$$

$$y = \frac{3}{5}$$

$$10x + 5y + 2z = 12$$

$$10x + 5\left(\frac{3}{5}\right) + 2(2) = 12$$

$$10x + 7 = 12$$

$$10x = 12$$

$$x = \frac{1}{2}$$

$$\therefore x = \frac{1}{2}, y = 5, z = 2$$

Based on the following linear equations:

$$2x + y + 3z = 16$$
$$7x - 2y - 3z = 7$$
$$x + 5y - 6z = 11$$

Calculate the value of x, y and z.

Answer

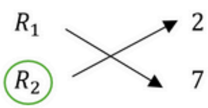
STEP 1: Transform the linear equations into matrices

$$\begin{bmatrix} 2 & 1 & 3 \\ 7 & -2 & -3 \\ 1 & 5 & -6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 16 \\ 7 \\ 1 \end{bmatrix}$$

STEP 2: Form Augmented Matrix [A|B]

$$\left[\begin{array}{ccc|c} 2 & 1 & 3 & 16 \\ 7 & -2 & -3 & 7 \\ 1 & 5 & -6 & 1 \end{array}\right]$$

STEP 3: Express $a_{21} = 0$ & $a_{31} = 0$



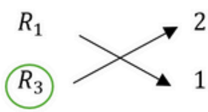
$R_{2(new)} = 2R_2 - 7R_1$

$2(7) - 7(2) = 0$

$2(-2) - 7(1) = -11$

$2(-3) - 7(3) = -27$

$2(7) - 7(16) = -98$



$R_{3(new)} = R_3 - 3R_1$

$2(1) - (2) = 0$

$2(5) - (1) = 9$

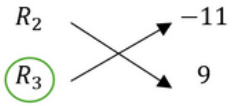
$2(-6) - (3) = -15$

$2(11) - (15) = 6$

The new matrix is

$$\left[\begin{array}{ccc|c} 2 & 1 & 3 & 16 \\ 0 & -11 & -29 & -98 \\ 0 & 9 & -15 & 6 \end{array}\right]$$

STEP 4: Express $a_{32} = 0$



$$R_{3(new)} = -11R_3 - 9R_2$$

$$-11(0) - 9(0) = 0$$

$$-11(0) - 9(-11) = 0$$

$$-11(0) - 9(-27) = 408$$

$$-11(0) - 9(-98) = 816$$

The new matrix is

$$\left[\begin{array}{ccc|c} 2 & 1 & 3 & 16 \\ 0 & -11 & -27 & -98 \\ 0 & 0 & 408 & 816 \end{array}\right]$$

STEP 5: Transform the matrices into linear equations

$$408z = 816$$

$$z = 2$$

$$-11y - 27z = -98$$

$$-11y - 27(2) = -98$$

$$-11y = -44$$

$$y = 24$$

$$2x + y + 3z = 16$$

$$2x + 4 + 3(2) = 162$$

$$2x + 10 = 16$$

$$2x = 6$$

$$x = 3$$

$$\therefore x = 3\ , y = 24\ , z = 2$$

Based on the following linear equations:

$$x - 3y = 8 - 2z$$

$$4x - y - z = 9$$

$$3x + 2y + z = 21$$

Calculate the value of x, y and z.

Answer

New Equation

$$x - 3y + 2z = 8$$

$$4x - y - z = 9$$

$$3x + 2y + z = 21$$

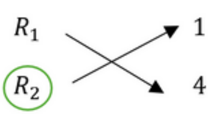
STEP 1: Transform the linear equations into matrices

$$\begin{bmatrix} 1 & -3 & 2 \\ 4 & -1 & -1 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 9 \\ 21 \end{bmatrix}$$

STEP 2: Form Augmented Matrix [A|B]

$$\left[\begin{array}{ccc|c} 1 & -3 & 2 & 8 \\ 4 & -1 & -1 & 9 \\ 3 & 2 & 1 & 1 \end{array}\right]$$

STEP 3: Express $a_{21} = 0$ & $a_{31} = 0$



$$R_{2(new)} = R_2 - 4R_1$$

$$(4) - 4(1) = 0$$

$$(-1) - 4(-3) = 11$$

$$(-1) - 4(2) = -9$$

$$(9) - 4(8) = -23$$

$$R_{3(new)} = R_3 - 3R_1$$

$$(3) - 3(1) = 0$$

$$(2) - 3(-3) = 11$$

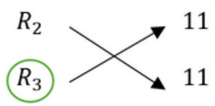
$$(1) - 3(2) = -5$$

$$(21) - 3(8) = -3$$

The new matrix is

$$\left[\begin{array}{ccc|c} 1 & -3 & 2 & 8 \\ 0 & 11 & -9 & -23 \\ 0 & 11 & -5 & -3 \end{array}\right]$$

STEP 4: Express $a_{32} = 0$



$$R_{3(new)} = 11R_3 - 11R_2$$

$$11(0) - 11(0) = 0$$

$$11(11) - 11(11) = 0$$

$$11(-5) - 11(-9) = 44$$

$$11(-3) - 11(-23) = 220$$

The new matrix is

START

$$\left[\begin{array}{ccc|c} 1 & -3 & 2 & 8 \\ 0 & 11 & -9 & -23 \\ 0 & 0 & 44 & 220 \end{array}\right]$$

STEP 5: Transform the matrices into linear equations

$$44z = 220$$

$$z = 5$$

$$11y - 9z = -23$$

$$11y - 9(5) = -23$$

$$11y = 22$$

$$y = 2$$

$$x - 3y + 2z = 8$$

$$x - 3(2) + 2(5) = 8$$

$$x = 14 - 10$$

$$x = 4$$

$$\therefore x = 4, y = 2, z = 5$$

Based on the following linear equations:

$$2x + 3z = 16$$
$$3x - 2y + 4z = 20$$
$$5x + 4y - 2z = 10$$

Calculate the value of x, y and z.

Answer

STEP 1: Transform the linear equations into matrices

$$\begin{bmatrix} 2 & 0 & 3 \\ 3 & -2 & 4 \\ 5 & 4 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 16 \\ 20 \\ 10 \end{bmatrix}$$

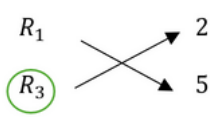
STEP 2: Form Augmented Matrix [A|B]

$$\left[\begin{array}{ccc|c} 2 & 0 & 3 & 16 \\ 3 & -2 & 4 & 20 \\ 5 & 4 & -1 & 10 \end{array}\right]$$

STEP 3: Express $a_{21} = 0$ & $a_{31} = 0$

$$R_{2(new)} = 2R_2 - 3R_1$$

$$2(3) - 3(2) = 0$$
$$2(-2) - 3(0) = -4$$
$$2(4) - 3(3) = -1$$
$$2(20) - 3(16) = -8$$



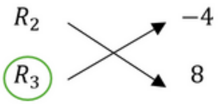
$$R_{3(new)} = 2R_3 - 5R_1$$

$$2(5) - 5(2) = 0$$
$$2(4) - 5(0) = 8$$
$$2(-1) - 5(3) = -17$$
$$2(10) - 5(16) = -60$$

The new matrix is

$$\left[\begin{array}{ccc|c} 2 & 0 & 3 & 16 \\ 0 & -4 & -1 & -8 \\ 0 & 8 & -17 & -60 \end{array}\right]$$

STEP 4: Express $a_{32} = 0$



$$R_{3(new)} = -4R_3 - 8R_2$$

$$-4(0) - 8(0) = 0$$

$$-4(8) - 8(-4) = 0$$

$$-4(-17) - 8(-1) = 76$$

$$-4(-60) - 8(-8) = 304$$

The new matrix is

START

$$\left[\begin{array}{ccc|c} 2 & 0 & 3 & 16 \\ 0 & -4 & -1 & -8 \\ 0 & 0 & 76 & 304 \end{array}\right]$$

STEP 5: Transform the matrices into linear equations

$$76z = 304$$

$$z = 4$$

$$-4y - z = -8$$

$$-4y - 4 = -8$$

$$-4y = -4$$

$$y = 1$$

$$2x + 3z = 16$$

$$2x + 3(4) = 16$$

$$2x = 4$$

$$x = 2$$

$$\therefore x = 2\,, y = 1\,, z = 4$$

CROUT METHOD

The Crout Method is a variant of LU Decomposition used for solving systems of linear equations. It decomposes a given square matrix A into the product of a lower triangular matrix L and an upper triangular matrix U, but unlike standard LU Decomposition, Crout's method computes L such that L is lower triangular and U is upper triangular with 1s on its diagonal.

1

In the Crout Method, a square matrix A is decomposed into:

$A = L \cdot U$

where:

- L is a lower triangular matrix with possibly non-unit diagonal elements.
- U is an upper triangular matrix with 1s on the diagonal.

Crout's Decomposition Process

2

Initialize Matrices:

- Let A be the matrix to be decomposed.
- Initialize L as a lower triangular matrix with entries l_{ij} and U as an upper triangular matrix with entries u_{ij}, where $u_{ii} = 1$ (for the diagonal elements of U).

3

Decomposition:

Compute L and U such that $A = L \cdot U$ using the following steps:

- For $i = 1$ to n:

$$l_{ii} = a_{ii} - \sum_{k=1}^{i-1} l_{ik} u_{ki}$$

$$u_{ij} = \frac{1}{l_{ii}} \left(a_{ij} - \sum_{k=1}^{i-1} l_{ik} u_{kj} \right) \text{ for } j > i$$

CROUT METHOD

$$l_{ji} = a_{ji} - \sum_{k=1}^{i-1} l_{jk} u_{ki} \text{ for } j > i$$

This process ensures that the matrix A is decomposed into L and U.

4

Solve the System:

Once you have L and U, solve the system of equations in two steps:

- **Solve $L \cdot y = b$ for y (forward substitution):**

$$L \cdot y = b$$

- **Solve $U \cdot x = y$ for x (back substitution):**

$$U \cdot x = y$$

Based on the following linear equations:

$$10x + 5y + 2z = 12$$

$$8x + 10y - 1z = 8$$

$$6x + 15y - 3z = 6$$

i. Calculate matrix L and U by using Crout Method.
ii. Then, calculate the value of x, y and z.

ANSWERS :

i. Calculate matrix L and U by using Crout Method.

STEP 1: Convert the simultaneous equation to the matrix form $Ax = B$

$$\begin{bmatrix} 10 & 5 & 2 \\ 8 & 10 & -1 \\ 6 & 15 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 8 \\ 6 \end{bmatrix}$$

STEP 2: Form matrix A = L x U

$$\begin{bmatrix} 10 & 5 & 2 \\ 8 & 10 & -1 \\ 6 & 15 & -3 \end{bmatrix} = \begin{bmatrix} a & 0 & 0 \\ b & d & 0 \\ c & e & f \end{bmatrix} \text{x} \begin{bmatrix} 1 & g & h \\ 0 & 1 & i \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 10 & 5 & 2 \\ 8 & 10 & -1 \\ 6 & 15 & -3 \end{bmatrix} = \begin{bmatrix} a & ag & ah \\ b & bg + d & bh + di \\ c & cg + e & ch + ei + f \end{bmatrix}$$

STEP 3: Find value of a, b, c, d, e, f, g, h and i

$\boldsymbol{a = 10}$	$ag = 5$ $(10)g = 5$ $g = \frac{5}{10}$ $\boldsymbol{g = \frac{1}{2}}$	$ah = 2$ $(10)h = 2$ $h = \frac{2}{10}$ $\boldsymbol{h = \frac{1}{5}}$
$\boldsymbol{b = 8}$	$bg + d = 10$ $(8)(\frac{1}{2}) + d = 10$ $\boldsymbol{d = 10 - 4}$ $\boldsymbol{d = 6}$	$bh + di = -1$ $(8)\left(\frac{1}{5}\right) + (6)i = -1$ $6i = -1 - \frac{8}{5}$ $\boldsymbol{i = -\frac{13}{30} = -0.43}$
$\boldsymbol{c = 6}$	$cg + e = 15$ $(6)(\frac{1}{2}) + e = 15$ $e = 15 - 3$ $\boldsymbol{e = 12}$	$ch + ei + f = -3$ $(6)\left(\frac{1}{5}\right) + (12)\left(-\frac{13}{30}\right) + f = -3$ $f = -6 - \frac{6}{5} + \frac{26}{5}$ $\boldsymbol{f = 1.00}$

STEP 4: Substitute the value of a, b, c, d, e, f, g, h and i into Matrix L and U

$$L = \begin{bmatrix} a & 0 & 0 \\ b & d & 0 \\ c & e & f \end{bmatrix} = \begin{bmatrix} 10 & 0 & 0 \\ 8 & 6 & 0 \\ 6 & 12 & 1 \end{bmatrix}$$

$$U = \begin{bmatrix} 1 & g & h \\ 0 & 1 & i \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{5} \\ 0 & 1 & -\frac{13}{30} \\ 0 & 0 & 1 \end{bmatrix}$$

STEP 5: Calculate value of $y1, y2, y3$ using $Ly = b$

$$Ly = b$$

$$\begin{bmatrix} 10 & 0 & 0 \\ 8 & 6 & 0 \\ 6 & 12 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 8 \\ 6 \end{bmatrix} \text{-------------} ①$$

$$(10)y_1 + 0(y_2) + 0(y_3) = 12$$

$$y_1 = \frac{6}{5}$$

$$(8)y_1 + (6)(y_2) + 0(y_3) = 8$$

$$(8)\left(\frac{6}{5}\right) - 6y_2 = 8$$

$$\frac{48}{5} - 6y_2 = 8$$

$$-6\, y_2 = 8 - \frac{48}{5}$$

$$-6y_2 = -\frac{8}{5}$$

$$y_2 = -\frac{4}{15} = -0.27$$

$$(6)y_1 + 12(y_2) + 1(y_3) = 6$$

$$(6)\left(\frac{6}{5}\right) + 12(-\frac{4}{15}) + y_3 = 6$$

$$\frac{36}{5} - \frac{48}{15} + y_3 = 6$$

$$y_3 = 6 - 4$$

$$y_3 = 2$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} \frac{6}{5} \\ -\frac{4}{15} \\ 2 \end{bmatrix}$$

STEP 6: Calculate value of x, y and z using $Ux = y$

$$Ux = y$$

$$\begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{5} \\ 0 & 1 & -\frac{13}{30} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{6}{5} \\ -\frac{4}{15} \\ 2 \end{bmatrix} \quad \text{----------------} (2)$$

$$(0)x + 0(y) + 1(z) = 2$$

$$\mathbf{z = 2}$$

$$(0)x + 1(y) + (-\frac{13}{30})(z) = -\frac{4}{15}$$

$$y - \frac{13}{30}(2) = -\frac{4}{15}$$

$$y = -\frac{4}{15} + \frac{13}{15}$$

$$\boldsymbol{y = \frac{3}{5}}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ \frac{3}{5} \\ \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0.6 \\ 2.0 \end{bmatrix}$$

Based on the following linear equations:

$$2x + y + 3z = 16$$
$$7x - 2y - 3z = 7$$
$$x + 5y - 6z = 11$$

i. Calculate matrix L and U by using Crout Method.

ii. Then, calculate the value of x, y and z.

ANSWERS :

i. Calculate matrix L and U by using Crout Method.

STEP 1: Convert the simultaneous equation to the matrix form $Ax = B$

$$\begin{bmatrix} 2 & 1 & 3 \\ 7 & -2 & -3 \\ 1 & 5 & -6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 16 \\ 7 \\ 11 \end{bmatrix}$$

STEP 2: Form matrix A = L x U

$$\begin{bmatrix} 2 & 1 & 3 \\ 7 & -2 & -3 \\ 1 & 5 & -6 \end{bmatrix} = \begin{bmatrix} a & 0 & 0 \\ b & d & 0 \\ c & e & f \end{bmatrix} \times \begin{bmatrix} 1 & g & h \\ 0 & 1 & i \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 & 3 \\ 7 & -2 & -3 \\ 1 & 5 & -6 \end{bmatrix} = \begin{bmatrix} a & ag & ah \\ b & bg + d & bh + di \\ c & cg + e & ch + ei + f \end{bmatrix}$$

STEP 3: Find value of a, b, c, d, e, f, g, h and i

$\boldsymbol{a = 2}$	$ag = 1$ $(2)g = 1$ $\boldsymbol{g = \frac{1}{2}}$	$ah = 3$ $(2)h = 3$ $\boldsymbol{h = \frac{3}{2}}$
$\boldsymbol{b = 7}$	$bg + d = -2$ $(7)(\frac{1}{2}) + d = -2$ $\boldsymbol{d = -2 - \frac{7}{2}}$ $\boldsymbol{d = -\frac{11}{2} = -5.5}$	$bh + di = -3$ $(7)\left(\frac{3}{2}\right) + \left(-\frac{11}{2}\right)i = -3$ $-\frac{11}{2}i = -3 - \frac{21}{2}$ $\boldsymbol{i = \frac{27}{11} = 2.45}$
$\boldsymbol{c = 1}$	$cg + e = 5$ $(1)(\frac{1}{2}) + e = 5$ $e = 5 - \frac{1}{2}$ $\boldsymbol{e = \frac{9}{2} = 4.5}$	$ch + ei + f = -6$ $(1)\left(\frac{3}{2}\right) + \left(\frac{9}{2}\right)\left(\frac{27}{11}\right) + f = -6$ $f = -6 - \frac{3}{2} - \frac{243}{22}$ $\boldsymbol{f = -\frac{204}{11} = -18.55}$

STEP 4: Substitute the value of a, b, c, d, e, f, g, h and i into Matrix L and U

$$\begin{bmatrix} 2 & 0 & 3 \\ 3 & -2 & 4 \\ 5 & 4 & -1 \end{bmatrix} = \begin{bmatrix} a & 0 & 0 \\ b & d & 0 \\ c & e & f \end{bmatrix} \text{x} \begin{bmatrix} 1 & g & h \\ 0 & 1 & i \\ 0 & 0 & 1 \end{bmatrix}$$

$$L = \begin{bmatrix} a & 0 & 0 \\ b & d & 0 \\ c & e & f \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 7 & -\frac{11}{2} & 0 \\ 1 & \frac{9}{2} & -\frac{204}{11} \end{bmatrix}$$

$$U = \begin{bmatrix} 1 & g & h \\ 0 & 1 & i \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & \frac{1}{2} & \frac{3}{2} \\ 0 & 1 & \frac{27}{11} \\ 0 & 0 & 1 \end{bmatrix}$$

STEP 5: Calculate value of $y1, y2, y3$ using $Ly = b$

$$Ly = b$$

$$\begin{bmatrix} 2 & 0 & 0 \\ 7 & -\frac{11}{2} & 0 \\ 1 & \frac{9}{2} & -\frac{204}{11} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 16 \\ 7 \\ 11 \end{bmatrix} \text{------------} (1)$$

$$(2)y_1 + 0(y_2) + 0(y_3) = 16$$

$$y_1 = 8$$

$$(7)y_1 + (-\frac{11}{2})(y_2) + 0(y_3) = 7$$

$$(7)(8) - \frac{11}{2}y_2 = 7$$

$$56 - \frac{11}{2}y_2 = 7$$

$$y_2 = (7 - 56)(-\frac{2}{11})$$

$$y_2 = \frac{98}{11} = 8.91$$

$$(1)y_1 + \frac{9}{2}(y_2) - \frac{204}{11}(y_3) = 11$$

$$(1)(8) + \frac{9}{2}\left(\frac{98}{11}\right) - \frac{204}{11}y_3 = 11$$

$$8 + \frac{441}{11} - \frac{204}{11}y_3 = 11$$

$$-\frac{204}{11}y_3 = 11 - \frac{441}{11} - 8$$

$$y_3 = -\frac{408}{11}(-\frac{11}{204})$$

$$y_3 = 2$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 8 \\ 8.91 \\ 2 \end{bmatrix}$$

STEP 6: Calculate value of x, y and z using $Ux = y$

$$Ux = y$$

$$\begin{bmatrix} 1 & \frac{1}{2} & \frac{3}{2} \\ 0 & 1 & \frac{27}{11} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 8.91 \\ 2 \end{bmatrix} \quad \text{---------------} (2)$$

$$(0)x + 0(y) + 1(z) = 2$$

$$\mathbf{z = 2}$$

$$(0)x + 1(y) + (\frac{27}{11})(z) = 8.91$$

$$y + \frac{27}{11}(2) = 8.91$$

$$y = 8.91 - \frac{54}{11}$$

$$\boldsymbol{y = 4}$$

$$x + \frac{1}{2}(y) + \frac{3}{2}(z) = 8$$

$$x + \frac{1}{2}(4) + \frac{3}{2}(2) = 8$$

$$x = 8 - 2 - 3$$

$$\boldsymbol{x = 3}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 2 \end{bmatrix}$$

Based on the following linear equations:

$$x - 3y = 8 - 2z$$

$$4x - y - z = 9$$

$$3x + 2y + z = 21$$

i. Calculate matrix L and U by using Crout Method.

ii. Then, calculate the value of x, y and z.

ANSWERS :

i. Calculate matrix L and U by using Crout Method.

STEP 1: Convert the simultaneous equation to the matrix form $Ax = B$

$$\begin{bmatrix} 1 & -3 & 2 \\ 4 & -1 & -1 \\ 3 & 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 9 \\ 21 \end{bmatrix}$$

STEP 2: Form matrix A = L x U

A = L x U

$$\begin{bmatrix} 1 & -3 & 2 \\ 4 & -1 & -1 \\ 3 & 2 & 1 \end{bmatrix} = \begin{bmatrix} a & 0 & 0 \\ b & d & 0 \\ c & e & f \end{bmatrix} \text{x} \begin{bmatrix} 1 & g & h \\ 0 & 1 & i \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -3 & 2 \\ 4 & -1 & -1 \\ 3 & 2 & 1 \end{bmatrix} = \begin{bmatrix} a & ag & ah \\ b & bg + d & bh + di \\ c & cg + e & ch + ei + f \end{bmatrix}$$

STEP 3: Find the value of a, b, c, d, e, f, g, h and i

$\boldsymbol{a = 1}$	$ag = -3$ $(1)g = -3$ $\boldsymbol{g = -3}$	$ah = 2$ $(1)h = 2$ $\boldsymbol{h = 2}$
$\boldsymbol{b = 4}$	$bg + d = -1$ $(4)(-3) + d = -1$ $d = -1 + 12$ $\boldsymbol{d = 11}$	$bh + di = -1$ $(4)(2) + (11)i = -1$ $11i = -1 - 8$ $\boldsymbol{i = \frac{-9}{11}}$
$\boldsymbol{c = 3}$	$cg + e = 2$ $(3)(-3) + e = 2$ $e = 2 + 9$ $\boldsymbol{e = 11}$	$ch + ei + f = 1$ $(3)(2) + 11(\frac{-9}{11}) + f = 1$ $f = 1 - 6 + 9$ $\boldsymbol{f = 4}$

STEP 4: Substitute the value of a, b, c, d, e, f, g, h and i into Matrix L and U

A = L x U

$$\begin{bmatrix} 1 & -3 & 2 \\ 4 & -1 & -1 \\ 3 & 2 & 1 \end{bmatrix} = \begin{bmatrix} a & 0 & 0 \\ b & d & 0 \\ c & e & f \end{bmatrix} \text{ x } \begin{bmatrix} 1 & g & h \\ 0 & 1 & i \\ 0 & 0 & 1 \end{bmatrix}$$

$$L = \begin{bmatrix} a & 0 & 0 \\ b & d & 0 \\ c & e & f \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 11 & 0 \\ 3 & 11 & 4 \end{bmatrix}$$

$$U = \begin{bmatrix} 1 & g & h \\ 0 & 1 & i \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -3 & 2 \\ 0 & 1 & \frac{-9}{11} \\ 0 & 0 & 1 \end{bmatrix}$$

STEP 5: Calculate value of $y1, y2, y3$ using $Ly = b$

$$Ly = b$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 4 & 11 & 0 \\ 3 & 11 & 4 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 8 \\ 9 \\ 21 \end{bmatrix} \quad \text{-----------} (1)$$

$$(1)y_1 + 0(y_2) + 0(y_3) = 8$$

$$y_1 = 8$$

$$(4)y_1 + 11(y_2) + 0(y_3) = 9$$

$$(4)(8) + 11y_2 = 9$$

$$32 + 11y_2 = 9$$

$$y_2 = \frac{9-32}{11}$$

$$y_2 = -\frac{23}{11}$$

$$(3)y_1 + 11(y_2) + 4(y_3) = 21$$

$$(3)(8) + 11\left(-\frac{23}{11}\right) + 4y_3 = 21$$

$$24 - 23 + 4y_3 = 21$$

$$4y_3 = 21 + 23 - 24$$

$$y_3 = \frac{20}{4}$$

$$y_3 = 5$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 8 \\ -\frac{23}{11} \\ 5 \end{bmatrix}$$

STEP 6: Find the value of x, y and z by using $Ux = y$

$$Ux = y$$

$$\begin{bmatrix} 1 & -3 & 2 \\ 0 & 1 & \frac{-9}{11} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ -\frac{23}{11} \\ 5 \end{bmatrix} \text{ ---------------- } (2)$$

$$(0)x + 0(y) + 1(z) = 5$$

$$\mathbf{z = 5}$$

$$(0)x + 1(y) + (\frac{-9}{11})(z) = \frac{-23}{11}$$

$$y - \frac{9}{11}(5) = \frac{-23}{11}$$

$$y = \frac{-23}{11} + \frac{45}{11}$$

$$\boldsymbol{y = 2}$$

$$x - 3(y) + 2(z) = 8$$

$$x - 3(2) + 2(5) = 8$$

$$x = 8 + 6 - 10$$

$$\boldsymbol{x = 4}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 5 \end{bmatrix}$$

Based on the following linear equations:

$$2x + 3z = 16$$

$$3x - 2y + 4z = 20$$

$$5x + 4y - 1z = 10$$

i. Calculate matrix L and U by using Crout Method.

ii. Then, calculate the value of x, y and z.

ANSWERS :

i. Calculate matrix L and U by using Crout Method.

STEP 1: Convert the simultaneous equation to the matrix form $Ax = B$

$$\begin{bmatrix} 2 & 0 & 3 \\ 3 & -2 & 4 \\ 5 & 4 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 16 \\ 20 \\ 10 \end{bmatrix}$$

STEP 2: Form matrix A = L x U

$$\begin{bmatrix} 2 & 0 & 3 \\ 3 & -2 & 4 \\ 5 & 4 & -1 \end{bmatrix} = \begin{bmatrix} a & 0 & 0 \\ b & d & 0 \\ c & e & f \end{bmatrix} \times \begin{bmatrix} 1 & g & h \\ 0 & 1 & i \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & 3 \\ 3 & -2 & 4 \\ 5 & 4 & -1 \end{bmatrix} = \begin{bmatrix} a & ag & ah \\ b & bg + d & bh + di \\ c & cg + e & ch + ei + f \end{bmatrix}$$

STEP 3: Find the value of a, b, c, d, e, f, g, h and i

$\mathbf{a = 2}$	$ag = 0$ $(2)g = 0$ $\mathbf{g = 0}$	$ah = 3$ $(2)h = 3$ $\mathbf{h = \frac{3}{2}}$
$\mathbf{b = 3}$	$bg + d = -2$ $(3)(0) + d = -2$ $\mathbf{d = -2}$	$bh + di = 4$ $(3)(\frac{3}{2}) + (-2)i = 4$ $-2i = 4 - \frac{9}{2}$ $\mathbf{i = \frac{1}{4}}$
$\mathbf{c = 5}$	$cg + e = 4$ $(5)(0) + e = 4$ $\mathbf{e = 4}$	$ch + ei + f = -1$ $(5)\left(\frac{3}{2}\right) + 4\left(\frac{1}{4}\right) + f = -1$ $f = -1 - 1 - \frac{15}{2}$ $\mathbf{f = -\frac{19}{2}}$

STEP 4: Substitute the value of a, b, c, d, e, f, g, h and i into Matrix L and U

A = L x U

$$\begin{bmatrix} 2 & 0 & 3 \\ 3 & -2 & 4 \\ 5 & 4 & -1 \end{bmatrix} = \begin{bmatrix} a & 0 & 0 \\ b & d & 0 \\ c & e & f \end{bmatrix} \times \begin{bmatrix} 1 & g & h \\ 0 & 1 & i \\ 0 & 0 & 1 \end{bmatrix}$$

$$L = \begin{bmatrix} a & 0 & 0 \\ b & d & 0 \\ c & e & f \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 3 & -2 & 0 \\ 5 & 4 & -\frac{19}{2} \end{bmatrix}$$

$$U = \begin{bmatrix} 1 & g & h \\ 0 & 1 & i \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & \frac{3}{2} \\ 0 & 1 & \frac{1}{4} \\ 0 & 0 & 1 \end{bmatrix}$$

STEP 5: Find the value of $y1, y2, y3$ by using $Ly = b$

$$Ly = b$$

$$\begin{bmatrix} 2 & 0 & 0 \\ 3 & -2 & 0 \\ 5 & 4 & -\frac{19}{2} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 16 \\ 20 \\ 10 \end{bmatrix} \text{---------} ①$$

$$(2)y_1 + 0(y_2) + 0(y_3) = 16$$

$$y_1 = 8$$

$$(3)y_1 + (-2)(y_2) + 0(y_3) = 20$$

$$(3)(8) - 2y_2 = 20$$

$$32 - 2y_2 = 20$$

$$y_2 = \frac{20-24}{-2}$$

$$y_2 = 2$$

$$(5)y_1 + 4(y_2) - \frac{19}{2}(y_3) = 10$$

$$(5)(8) + 4(2) - \frac{19}{2}y_3 = 10$$

$$40 + 8 - \frac{19}{2}y_3 = 10$$

$$-\frac{19}{2}y_3 = 10 - 48$$

$$y_3 = -38\left(\frac{2}{-19}\right)$$

$$y_3 = 4$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 8 \\ 2 \\ 4 \end{bmatrix}$$

STEP 6: Find the value of x, y and z by using $Ux = y$

$$Ux = y$$

$$\begin{bmatrix} 1 & 0 & \frac{3}{2} \\ 0 & 1 & \frac{1}{4} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 2 \\ 4 \end{bmatrix} \quad \text{---------------} \quad ②$$

$$(0)x + 0(y) + 1(z) = 4$$

$$\mathbf{z = 4}$$

$$(0)x + 1(y) + (\frac{1}{4})(z) = 2$$

$$y + \frac{1}{4}(4) = 2$$

$$y = 2 - 1$$

$$\boldsymbol{y = 1}$$

$$x - 0(y) + \frac{3}{2}(z) = 8$$

$$x - 0(2) + \frac{3}{2}(4) = 8$$

$$x = 8 + 0 - \frac{12}{2}$$

$$\boldsymbol{x = 2}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

EXERCISE 1

Based on the following linear equations:

$$3x + 2y + z = 2$$

$$4x + 2y + 2z = 4$$

$$2x + 4y - 3z = 3$$

i. Calculate matrix L and U by using Crout Method.

ii. Then, calculate the value of x, y and z.

Answer

i. Calculate matrix L and U by using Crout Method.

$$L = \begin{bmatrix} a & 0 & 0 \\ b & d & 0 \\ c & e & f \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 4 & -0.67 & 0 \\ 2 & 2.67 & -1 \end{bmatrix}$$

$$U = \begin{bmatrix} 1 & g & h \\ 0 & 1 & i \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & \mathbf{0.67} & \mathbf{0.33} \\ 0 & 1 & \mathbf{-1} \\ 0 & 0 & 1 \end{bmatrix}$$

ii. Then, calculate the value of x, y and z.

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ -9 \\ -7 \end{bmatrix}$$

EXERCISE 2

Based on the following linear equations:

$$x + 3y - 2z = 3$$
$$2x + y + 2z = 2$$
$$x + 2y - z = 1$$

i. Calculate matrix L and U by using Crout Method.

ii. Then, calculate the value of x, y and z.

Answer

i. Calculate matrix L and U by using Crout Method.

$$L = \begin{bmatrix} a & 0 & 0 \\ b & d & 0 \\ c & e & f \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & -5 & 0 \\ 1 & -1 & -0.2 \end{bmatrix}$$

$$U = \begin{bmatrix} 1 & g & h \\ 0 & 1 & i \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 & -2 \\ 0 & 1 & -1.2 \\ 0 & 0 & 1 \end{bmatrix}$$

ii. Then, calculate the value of x, y and z.

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -9 \\ 8 \\ 6 \end{bmatrix}$$

DOOLITLE METHOD

The Doolittle method is a numerical technique used for solving systems of linear equations. It is a form of LU decomposition, where the coefficient matrix A of a system $Ax = b$ is decomposed into the product of a lower triangular matrix L and an upper triangular matrix U. This method simplifies the process of solving linear systems by breaking down the problem into simpler steps.

Steps of the Doolittle Method:

1

Matrix Decomposition: Decompose the matrix A into L and U such that:

$$A = LU$$

where L is a lower triangular matrix with ones on the diagonal, and U is an upper triangular matrix.

2

Solve $Ly = b$: Once A is decomposed into L and U, solve the system:

$$Ly = b$$

for y using forward substitution. Since L is lower triangular, this step involves solving a sequence of equations from the top row down.

3

Solve $Ux = y$: After finding y, solve:

$$Ux = y$$

for x using backward substitution. Since U is upper triangular, this step involves solving a sequence of equations from the bottom row up.

Based on the following linear equations:

$$10x + 5y + 2z = 12$$
$$8x + 10y - 1z = 8$$
$$6x + 15y - 3z = 6$$

i. Calculate matrix L and U by using Doolittle Method.
ii. Then, calculate the value of x, y and z.

ANSWERS :

i. Calculate matrix L and U by using Doolittle Method.

STEP 1: Convert the simultaneous equation to the matrix form $Ax = B$

$$\begin{bmatrix} 10 & 5 & 2 \\ 8 & 10 & -1 \\ 6 & 15 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 8 \\ 6 \end{bmatrix}$$

STEP 2: Form matrix A = L x U

A = L x U

$$\begin{bmatrix} 10 & 5 & 2 \\ 8 & 10 & -1 \\ 6 & 15 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & c & 1 \end{bmatrix} \text{ x } \begin{bmatrix} d & e & f \\ 0 & g & h \\ 0 & 0 & i \end{bmatrix}$$

STEP 3: Find value of a, b, c, d, e, f, g, h and i

$$\begin{bmatrix} 10 & 5 & 2 \\ 8 & 10 & -1 \\ 6 & 15 & -3 \end{bmatrix} = \begin{bmatrix} d & e & f \\ ad & ae+g & af+h \\ bd & be+cg & bf+ch+i \end{bmatrix}$$

$\boldsymbol{d = 10}$	$\boldsymbol{e = 5}$	$\boldsymbol{f = 2}$
$ad = 8$ $a(10) = 7$ $\boldsymbol{a = 0.80}$	$ae + g = 10$ $(0.80)(1) + g = 10$ $\boldsymbol{g = 6}$	$af + h = -1$ $(0.80)(2) + h = -3$ $h = -3 - 1.60$ $\boldsymbol{h = -2.60}$
$bd = 6$ $b(10) = 6$ $\boldsymbol{b = 0.6}$	$be + cg = 15$ $(0.6)(5) + c(6) = 15$ $-\frac{11}{2}c = 5 - \frac{1}{2}$ $c = \frac{9}{2} \text{ x } (-\frac{2}{11})$ $c = \frac{9}{2} \text{ x } (-\frac{2}{11})$ $\boldsymbol{c = 2}$	$bf + ch + i = -6$ $\left(\frac{1}{2}\right)(3) + \left(-\frac{9}{11}\right)\left(\frac{27}{2}\right) + i = -6$ $i = -6 - \frac{3}{2} - \frac{243}{22}$ $\boldsymbol{i = 1}$

STEP 4: Substitute the value of a, b, c, d, e, f, g, h and i into Matrix L and U

$$L = \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & c & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \mathbf{0.8} & 1 & 0 \\ \mathbf{0.6} & \mathbf{2} & 1 \end{bmatrix}$$

$$U = \begin{bmatrix} d & e & f \\ 0 & g & h \\ 0 & 0 & i \end{bmatrix} = \begin{bmatrix} 10 & 5 & 2 \\ 0 & \mathbf{6} & \mathbf{-2.60} \\ 0 & 0 & \mathbf{1} \end{bmatrix}$$

STEP 5: Calculate value of $y1, y2, y3$ using $Ly = b$

$$Ly = b$$

$$\begin{bmatrix} 1 & 0 & 0 \\ \mathbf{0.8} & 1 & 0 \\ \mathbf{0.6} & \mathbf{2} & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 8 \\ 6 \end{bmatrix} \text{-------------- } ①$$

$$(1)y_1 + 0(y_2) + 0(y_3) = 12$$

$$\boldsymbol{y_1 = 12}$$

$$(0.8)y_1 + 1(y_2) + 0(y_3) = 8$$

$$(0.8)(12) + y_2 = 8$$

$$9.6 + y_2 = 8$$

$$y_2 = 8 - 9.6$$

$$\boldsymbol{y_2 = -1.6}$$

$$(0.6)y_1 + 2(y_2) + 1(y_3) = 6$$

$$(0.6)(12) + 2(-1.6) + y_3 = 6$$

$$7.2 - 3.2 + y_3 = 6$$

$$y_3 = 6 - 4$$

$$\boldsymbol{y_3 = 2}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 12 \\ -1.6 \\ 2 \end{bmatrix}$$

STEP 6: Calculate value of x, y and z using $Ux = y$

$$Ux = y$$

$$\begin{bmatrix} 10 & 5 & 2 \\ 0 & 6 & -2.60 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ -1.6 \\ 2 \end{bmatrix} \quad \text{----------} \; (2)$$

$$(0)x + 0(y) + (1)(z) = 2$$

$$\mathbf{z = 2}$$

$$(0)x + 6(y) + (-2.60)(z) = -1.6$$

$$6y - 2.60(2) = -1.6$$

$$6y = -1.6 + 5.2$$

$$\boldsymbol{y = 0.6}$$

$$10x + 5(y) + 2(z) = 12$$

$$10x + 5(0.6) + 2(2) = 12$$

$$10x = 12$$

$$\boldsymbol{x = 0.5}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0.6 \\ 2 \end{bmatrix}$$

Based on the following linear equations:

$$2x + y + 3z = 16$$
$$7x - 2y - 3z = 7$$
$$x + 5y - 6z = 11$$

i. Calculate matrix L and U by using Doolittle Method.
ii. Then, calculate the value of x, y and z.

ANSWERS :

i. Calculate matrix L and U by using Doolittle Method.

STEP 1: Convert the simultaneous equation to the matrix form $Ax = B$

$$\begin{bmatrix} 2 & 1 & 3 \\ 7 & -2 & -3 \\ 1 & 5 & -6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 16 \\ 7 \\ 11 \end{bmatrix}$$

STEP 2: Form matrix A = L x U

A = L x U

$$\begin{bmatrix} 2 & 1 & 3 \\ 7 & -2 & -3 \\ 1 & 5 & -6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & c & 1 \end{bmatrix} \times \begin{bmatrix} d & e & f \\ 0 & g & h \\ 0 & 0 & i \end{bmatrix}$$

STEP 3: Find value of a, b, c, d, e, f, g, h and i

$$\begin{bmatrix} 2 & 1 & 3 \\ 7 & -2 & -3 \\ 1 & 5 & -6 \end{bmatrix} = \begin{bmatrix} d & e & f \\ ad & ae + g & af + h \\ bd & be + cg & bf + ch + i \end{bmatrix}$$

$d = 2$	$e = 1$	$f = 3$
$ad = 7$ $a(2) = 7$ $a = \frac{7}{2}$	$ae + g = -2$ $(\frac{7}{2})(1) + g = -2$ $g = -\frac{11}{2}$	$af + h = -3$ $\left(\frac{7}{2}\right)(3) + h = -3$ $h = -3 - \frac{21}{2}$ $h = \frac{27}{2}$
$bd = 1$ $b(2) = 1$ $b = \frac{1}{2}$	$be + cg = 5$ $\left(\frac{1}{2}\right)(1) + c(-5.5) = 5$ $-\frac{11}{2}c = 5 - \frac{1}{2}$ $c = \frac{9}{2} \times (-\frac{2}{11})$ $c = \frac{9}{2} \times (-\frac{2}{11})$ $c = -\frac{9}{11}$	$bf + ch + i = -6$ $\left(\frac{1}{2}\right)(3) + \left(-\frac{9}{11}\right)\left(\frac{27}{2}\right) + i = -6$ $i = -6 - \frac{3}{2} - \frac{243}{22}$ $i = -\frac{204}{11}$

STEP 4: Substitute the value of a, b, c, d, e, f, g, h and i into Matrix L and U

$$L = \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & c & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{7}{2} & 1 & 0 \\ \frac{1}{2} & -\frac{9}{11} & 1 \end{bmatrix}$$

$$U = \begin{bmatrix} d & e & f \\ 0 & g & h \\ 0 & 0 & i \end{bmatrix} = \begin{bmatrix} 2 & 1 & 3 \\ 0 & -\frac{11}{2} & -\frac{27}{2} \\ 0 & 0 & -\frac{204}{11} \end{bmatrix}$$

STEP 5: Calculate value of $y1, y2, y3$ using $Ly = b$

$$\mathrm{Ly} = \mathrm{b}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ \frac{7}{2} & 1 & 0 \\ \frac{1}{2} & -\frac{9}{11} & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 16 \\ 7 \\ 11 \end{bmatrix} \text{--------------} \quad (1)$$

$$(1)y_1 + 0(y_2) + 0(y_3) = 16$$

$$y_1 = 16$$

$$\left(\frac{7}{2}\right) y_1 + 1(y_2) + 0(y_3) = 7$$

$$\left(\frac{7}{2}\right)(16) + \; y_2 = 7$$

$$56 + \; y_2 = 7$$

$$y_2 = 7 - 56$$

$$y_2 = -49$$

$$\left(\frac{1}{2}\right)y_1 - \frac{9}{11}(y_2) + 1(y_3) = 11$$

$$\left(\frac{1}{2}\right)(16) - \frac{9}{11}(-49) + y_3 = 11$$

$$8 + \frac{441}{11} + y_3 = 11$$

$$y_3 = 11 - \frac{441}{11} - 8$$

$$y_3 = -\frac{408}{11} = -37.09$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 16 \\ -49 \\ -\frac{408}{11} \end{bmatrix}$$

STEP 6: Calculate value of x, y and z using $Ux = y$

$$Ux = y$$

$$\begin{bmatrix} 2 & 1 & 3 \\ 0 & -\frac{11}{2} & -\frac{27}{2} \\ 0 & 0 & -\frac{204}{11} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 16 \\ -49 \\ -\frac{408}{11} \end{bmatrix} \text{ ---------- (2)}$$

$$(0)x + 0(y) + \left(-\frac{204}{11}\right)(z) = -\frac{408}{11}$$

$$z = -\frac{408}{11}\left(-\frac{11}{204}\right)$$

$$z = 2$$

$$(0)x + \left(-\frac{11}{2}\right)(y) + \left(-\frac{27}{2}\right)(z) = -49$$

$$-\frac{11}{2}y - \frac{27}{2}(2) = -49$$

$$-\frac{11}{2}y = -49 + 27$$

$$y = \frac{-22}{-11}(2)$$

$$y = 4$$

$$2x + 1(y) + 3(z) = 16$$

$$2x + 1(4) + 3(2) = 16$$

$$x = \frac{16-6-4}{2}$$

$$x = 3$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 2 \end{bmatrix}$$

Based on the following linear equations:

$$x - 3y = 8 - 2z$$
$$4x - y - z = 9$$
$$3x + 2y + z = 21$$

i. Calculate matrix L and U by using Doolittle Method.
ii. Then, calculate the value of x, y and z.

ANSWERS :

i. Calculate matrix L and U by using Doolittle Method.

Rearrange the equation

$$x - 3y + 2z = 8$$
$$4x - y - z = 9$$
$$3x + 2y + z = 21$$

STEP 1: Convert the simultaneous equation to the matrix form $Ax = B$

$$\begin{bmatrix} 1 & -3 & 2 \\ 4 & -1 & -1 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 9 \\ 21 \end{bmatrix}$$

STEP 2: Form matrix A = L x U

A = L x U

$$\begin{bmatrix} 1 & -3 & 2 \\ 4 & -1 & -1 \\ 3 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & c & 1 \end{bmatrix} \times \begin{bmatrix} d & e & f \\ 0 & g & h \\ 0 & 0 & i \end{bmatrix}$$

STEP 3: Find value of a, b, c, d, e, f, g, h and i

$$\begin{bmatrix} 1 & -3 & 2 \\ 4 & -1 & -1 \\ 3 & 2 & 1 \end{bmatrix} = \begin{bmatrix} d & e & f \\ ad & ae + g & af + h \\ bd & be + cg & bf + ch + i \end{bmatrix}$$

$\boldsymbol{d = 1}$ $\quad\quad$ $\boldsymbol{e = -3}$ $\quad\quad$ $\boldsymbol{f = 2}$

$ad = 4$ $\quad\quad$ $ae + g = -1$ $\quad\quad$ $af + h = -1$

$a(1) = 4$ $\quad\quad$ $(4)(-3) + g = -1$ $\quad\quad$ $(4)(2) + h = -1$

$\boldsymbol{a = 4}$ $\quad\quad$ $\boldsymbol{g = 11}$ $\quad\quad$ $\boldsymbol{h = -9}$

$bd = 3$ $\quad\quad$ $be + cg = -1$ $\quad\quad$ $bf + ch + i = 1$

$b(1) = 3$ $\quad\quad$ $(3)(-3) + c(11) = -1$ $\quad\quad$ $(3)(2) + (1)(-9) + i = 1$

$\boldsymbol{b = 3}$ $\quad\quad$ $\boldsymbol{c = 1}$ $\quad\quad$ $\boldsymbol{i = 4}$

STEP 4: Substitute the value of a, b, c, d, e, f, g, h and i into Matrix L and U

$$L = \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & c & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 3 & 1 & 1 \end{bmatrix}$$

$$U = \begin{bmatrix} d & e & f \\ 0 & g & h \\ 0 & 0 & i \end{bmatrix} = \begin{bmatrix} 1 & -3 & 2 \\ 0 & 11 & -9 \\ 0 & 0 & 4 \end{bmatrix}$$

STEP 5: Calculate value of $y1, y2, y3$ using $Ly = b$

$$\mathrm{Ly} = \mathrm{b}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 3 & 1 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 8 \\ 9 \\ 21 \end{bmatrix} \quad \text{------------} (1)$$

$$(1)y_1 + 0(y_2) + 0(y_3) = 8$$

$$y_1 = 8$$

$$(4)y_1 + 1(y_2) + 0(y_3) = 9$$

$$(4)(8) + y_2 = 9$$

$$32 + y_2 = 9$$

$$y_2 = 9 - 32$$

$$y_2 = -23$$

$$(3)y_1 + 1(y_2) + 1(y_3) = 21$$

$$(3)(8) + (-23) + y_3 = 21$$

$$24 - 23 + y_3 = 21$$

$$y_3 = 21 + 23 - 24$$

$$y_3 = 20$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 8 \\ -23 \\ 20 \end{bmatrix}$$

STEP 6: Calculate value of x, y and z using $Ux = y$

$$\text{Ux} = \text{y}$$

$$\begin{bmatrix} 1 & -3 & 2 \\ 0 & 11 & -9 \\ 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ -23 \\ 20 \end{bmatrix} \quad \text{---------------} \; ②$$

$$(0)x + 0(\text{y}) + 4(\text{z}) = 20$$

$$\mathbf{z = 5}$$

$$(0)x + 11(y) + (-9)(\text{z}) = -23$$

$$11y - 9(5) = -23$$

$$11y = -23 + 45$$

$$y = \frac{22}{11}$$

$$\boldsymbol{y = 2}$$

$$x - 3(\text{y}) + 2(\text{z}) = 8$$

$$x - 3(2) + 2(5) = 8$$

$$x = 8 + 6 - 10$$

$$\boldsymbol{x = 4}$$

$$\begin{bmatrix} x \\ \text{y} \\ \text{z} \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 5 \end{bmatrix}$$

Based on the following linear equations:

$$
\begin{aligned}
2x + 3z &= 16 \\
3x - 2y + 4z &= 20 \\
5x + 4y - 2z &= 10
\end{aligned}
$$

i. Calculate matrix L and U by using Doolittle Method.
ii. Then, calculate the value of x, y and z.

ANSWERS :

i. Calculate matrix L and U by using Doolittle Method.

STEP 1: Convert the simultaneous equation to the matrix form $Ax = B$

$$\begin{bmatrix} 2 & 0 & 3 \\ 3 & -2 & 4 \\ 5 & 4 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 16 \\ 20 \\ 10 \end{bmatrix}$$

STEP 2: Form matrix A = L x U

A = L x U

$$\begin{bmatrix} 2 & 0 & 3 \\ 3 & -2 & 4 \\ 5 & 4 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & c & 1 \end{bmatrix} \times \begin{bmatrix} d & e & f \\ 0 & g & h \\ 0 & 0 & i \end{bmatrix}$$

STEP 3: Find value of a, b, c, d, e, f, g, h and i

$$\begin{bmatrix} 2 & 0 & 3 \\ 3 & -2 & 4 \\ 5 & 4 & -1 \end{bmatrix} = \begin{bmatrix} d & e & f \\ ad & ae + g & af + h \\ bd & be + cg & bf + ch + i \end{bmatrix}$$

$\boldsymbol{d = 2}$

$\boldsymbol{e = 0}$

$\boldsymbol{f = 3}$

$ad = 3$
$a(2) = 3$
$\boldsymbol{a = \frac{3}{2}}$

$ae + g = -2$
$(\frac{3}{2})(0) + g = -2$
$\boldsymbol{g = -2}$

$af + h = 4$
$(\frac{3}{2})(3) + h = 4$
$h = 4 - \frac{9}{2}$
$\boldsymbol{h = -\frac{1}{2}}$

$bd = 5$
$b(2) = 5$
$\boldsymbol{b = \frac{5}{2}}$

$be + cg = 4$
$\left(\frac{5}{2}\right)(0) + c(-2) = 4$
$\boldsymbol{c = -2}$

$bf + ch + i = -1$
$\left(\frac{5}{2}\right)(3) + (-2)\left(-\frac{1}{2}\right) + i = -1$
$\boldsymbol{i = -1 - \frac{15}{2} - 1 = -\frac{19}{2}}$

STEP 4: Substitute the value of a, b, c, d, e, f, g, h and i into Matrix L and U

$$L = \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & c & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{3}{2} & 1 & 0 \\ \frac{5}{2} & -2 & 1 \end{bmatrix}$$

$$U = \begin{bmatrix} d & e & f \\ 0 & g & h \\ 0 & 0 & i \end{bmatrix} = \begin{bmatrix} 2 & 0 & 3 \\ 0 & -2 & -\frac{1}{2} \\ 0 & 0 & -\frac{19}{2} \end{bmatrix}$$

STEP 5: Calculate value of $y1, y2, y3$ using $Ly = b$

$$Ly = b$$

$$\begin{bmatrix} 1 & 0 & 0 \\ \frac{3}{2} & 1 & 0 \\ \frac{5}{2} & -2 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 16 \\ 20 \\ 10 \end{bmatrix} \text{ -------------- } ①$$

$$(1)y_1 + 0(y_2) + 0(y_3) = 16$$

$$y_1 = 16$$

$$\left(\frac{3}{2}\right) y_1 + 1(y_2) + 0(y_3) = 20$$

$$\left(\frac{3}{2}\right)(16) + y_2 = 20$$

$$24 + y_2 = 20$$

$$y_2 = 20 - 24$$

$$y_2 = -4$$

$$\left(\frac{5}{2}\right) y_1 - 2(y_2) + 1(y_3) = 10$$

$$\left(\frac{5}{2}\right)(16) - 2(-4) + y_3 = 10$$

$$40 + 8 + y_3 = 10$$

$$y_3 = 10 - 40 - 8$$

$$y_3 = -38$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 16 \\ -4 \\ -38 \end{bmatrix}$$

STEP 6: Calculate value of x, y and z using $Ux = y$

$$Ux = y$$

$$\begin{bmatrix} 2 & 0 & 3 \\ 0 & -2 & -\frac{1}{2} \\ 0 & 0 & -\frac{19}{2} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 16 \\ -4 \\ -38 \end{bmatrix} \quad \text{------------} \; ②$$

$$(0)x + 0(y) + (-\frac{21}{2})(z) = 38$$

$$z = \frac{38}{19}(2)$$

$$\mathbf{z = 4}$$

$$(0)x + (-2)(y) + (-\frac{1}{2})(z) = -4$$

$$-2y - \frac{1}{2}(4) = -4$$

$$-2y = -4 + 2$$

$$y = \frac{-2}{-2}$$

$$\boldsymbol{y = 1}$$

$$2x + 0(y) + 3(z) = 16$$

$$2x + 0(1) + 3(4) = 16$$

$$x = \frac{16-12}{2}$$

$$\boldsymbol{x = 2}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}$$

Based on the following linear equations:

$$2x + y + 2z = 2$$
$$x + 2y - z = 4$$
$$2x + 2y + z = 2$$

i. Calculate matrix L and U by using Doolittle Method.

ii. Then, calculate the value of x, y and z.

ANSWERS :

i. Calculate matrix L and U by using Doolittle Method.

$$L = \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & c & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \mathbf{0.5} & 1 & 0 \\ \mathbf{1} & \mathbf{0.67} & 1 \end{bmatrix}$$

$$U = \begin{bmatrix} d & e & f \\ 0 & g & h \\ 0 & 0 & i \end{bmatrix} = \begin{bmatrix} \mathbf{2} & \mathbf{1} & \mathbf{2} \\ 0 & \mathbf{1.5} & \mathbf{-2} \\ 0 & 0 & \mathbf{0.33} \end{bmatrix}$$

ii. Then, calculate the value of x, y and z.

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ -6 \\ -6 \end{bmatrix}$$

Based on the following linear equations:

$$8x + 3y + 2z = 4$$

$$2x + 2y + 4z = 6$$

$$x + 6y + 2z = 2$$

i. Calculate matrix L and U by using Doolittle Method.

ii. Then, calculate the value of x, y and z.

ANSWERS :

i. Calculate matrix L and U by using Doolittle Method.

$$L = \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & c & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \mathbf{0.25} & 1 & 0 \\ \mathbf{0.13} & \mathbf{4.5} & 1 \end{bmatrix}$$

$$U = \begin{bmatrix} d & e & f \\ 0 & g & h \\ 0 & 0 & i \end{bmatrix} = \begin{bmatrix} \mathbf{8} & \mathbf{3} & \mathbf{2} \\ 0 & \mathbf{1.25} & \mathbf{3.5} \\ 0 & 0 & \mathbf{-14} \end{bmatrix}$$

ii. Then, calculate the value of x, y and z.

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0.2 \\ -0.2 \\ 1.5 \end{bmatrix}$$

3 METHODS TO FIND SIMULTANEOUS EQUATIONS

1

FIXED POINT ITERATION METHOD

To find roots of simultaneous equation when x_0 value is given

2

FALSE PLACEMENT METHOD

To find roots of simultaneous equation when x_0 value is not given

3

NEWTON RAPHSON METHOD

To find roots of simultaneous equation when x_0 value is given

FIXED POINT ITERATION METHOD

The Fixed Point Iteration Method is a numerical technique used to find approximate solutions to equations of the form $x = g(x)$. The general idea is to start with an initial guess and then iteratively apply a function $g(x)$ to converge to a solution.

1

Rearrange the Equation:

Rewrite the equation $f(x) = 0$ into the form $x = g(x)$. This step involves isolating x on one side of the equation.

2

Choose an Initial Guess:

Select an initial guess x_0 for the solution. This guess should be reasonably close to the actual solution to ensure convergence.

3

Iterate:

Apply the iteration formula:

$$x_{n+1} = g(x_n)$$

where x_n is the current approximation, and x_{n+1} is the next approximation.

4

Check for Convergence:

Continue iterating until the difference between successive approximations is smaller than a predetermined tolerance ϵ, i.e.,

$$|x_{n+1} - x_n| < \epsilon$$

5

Terminate:

Once convergence is achieved (or the maximum number of iterations is reached), the current approximation x_{n+1} is taken as the approximate solution.

Use Fixed-Point Iteration Method to find the root of this equation which is near the given value. Give your answers correct to 3 decimal places.

$$x^5 - x - 7 = 0; x_0 = 2$$

$$f(x) = x^5 - x - 7$$

Answer

Step 1 : Re-arrange the equation to find the value of x:

<u>Equation 1</u>

$$x^5 = x + 7$$

$$x = (x + 7)^{\frac{1}{5}}$$

$$f'(x) = \left(\frac{1}{5}\right)(x + 7)^{\frac{-4}{5}}(1)$$

$$f'(2) = \left(\frac{1}{5}\right)(2 + 7)^{\frac{-4}{5}}(1)$$

$$= 0.034 < 1, \textit{the iteration will converge}$$

<u>Equation 2</u>

$$x = x^5 - 7$$

$$f'(x) = 5x$$

$$f'(2) = 5(2)$$

$$= 10 > 1, \textit{the iteration will not converge}$$

Therefore, choose equation 2 for the iteration process

Step 2 : Use the equation from step 1 to find the roots

x_n	$x_{n+1} = (x+7)^{\frac{1}{5}}$
$x_0 = 2$	$x_1 = 1.552$
$x_1 = 1.552$	$x_2 = 1.536$
$x_2 = 1.536$	$x_2 = 1.536$

Stop calculating when you get two repeated answers

Therefore, the root for equation $x^5 - x - 7 = 0$ is 1.536

Use Fixed-Point Iteration Method to find the root of this equation which is near the given value. Give your answers correct to 3 decimal places.

$$x^3 + 4x^2 + 7 = 0 \; ; x_0 = -4$$
$$f(x) = x^3 + 4x^2 + 7$$

Answer

Step 1 : Re-arrange the equation to find the value of x:

Equation 1

$$x^3 = -4x^2 - 7$$

$$x = (-4x^2 - 7)^{\frac{1}{3}}$$

$$f'(x) = \left(\frac{1}{3}\right)(-4x^2 - 7)^{\frac{-2}{3}}(-8x)$$

$$f'(-4) = \left(\frac{1}{3}\right)[-4(-4)^2 - 7]^{\frac{-2}{3}}[-8(-4)]$$

$$= 0.720 < 1, \textit{the iteration will converges}$$

Equation 2

$$4x^2 = -x^3 - 7$$

$$x = \left(\frac{-x^3 - 7}{4}\right)^{\frac{1}{2}}$$

$$f'(x) = \left(\frac{1}{2}\right)\left(\frac{1}{4}\right)(-3x^2)\left(\frac{-x^3 - 7}{4}\right)^{\frac{-1}{2}}$$

$$= \left(\frac{1}{2}\right)\left(\frac{1}{4}\right)(-3(-4)^2)\left(\frac{-(-4)^3 - 7}{4}\right)^{\frac{-1}{2}}$$

$$= 1.59 > 1, \textit{the iteration will not converge}$$

Therefore, choose equation 1 for the iteration process

Step 2 : Use the equation from step 1 to find the roots

x_n	$x_{n+1} = (-4x^2 - 7)^{\frac{1}{3}}$
$x_0 = -4$	$x_1 = -4.141$
$x_1 = -4.141$	$x_2 = -4.228$
$x_2 = -4.228$	$x_3 = -4.282$
$x_3 = -4.282$	$x_4 = -4.315$
$x_4 = -4.315$	$x_5 = -4.335$
$x_5 = -4.335$	$x_6 = -4.347$
$x_6 = -4.347$	$x_7 = -4.355$
$x_7 = -4.355$	$x_8 = -4.360$
$x_8 = -4.360$	$x_9 = -4.363$
$x_9 = -4.363$	$x_{10} = -4.365$
$x_{10} = -4.365$	$x_{11} = -4.366$
$x_{11} = -4.366$	$x_{12} = -4.366$

Stop calculating when you get two repeated answers

Therefore, the root for equation $x^3 + 4x^2 + 7 = 0$ is -4.366

Use Fixed-Point Iteration Method to find the root of this equation which is near the given value. Give your answers correct to 4 decimal places.

$$5x^2 - 4x^{\frac{3}{2}} - 6 = 0\ ;\ x_0 = 1.5$$

$$f(x) = 5x^2 - 4x^{\frac{3}{2}} - 6$$

Answer

Step 1 : Re-arrange the equation to find the value of x:

Equation 1

$$5x^2 = 4x^{\frac{3}{2}} + 6$$

$$x = \left(\frac{4x^{\frac{3}{2}}+6}{5}\right)^{\frac{1}{2}}$$

$$f'(x) = \frac{1}{2}\left(\frac{4x^{\frac{3}{2}}+6}{5}\right)^{\left(\frac{-1}{2}\right)} \times \left[\left(\frac{1}{5}\right)(6x^{\frac{1}{2}})\right]$$

$$f'(1.5) = 0.45$$

$$= 0.45 < 1, \textit{the iteration will converges}$$

Equation 2

$$-4x^{\frac{3}{2}} = 6 - 5x^2$$

$$x = \left(\frac{5x^2-6}{4}\right)^{\frac{2}{3}}$$

$$f'(x) = \frac{1}{4}(5x^2-6)^{\frac{-1}{3}}(10x)$$

$$f'(1.5) = \frac{1}{4}[5(1.5)^2 - 6]^{\frac{-1}{3}}[10(1.5)]$$

$$= 2.1576 > 1, \textit{the iteration will not converges}$$

Therefore, choose equation 1 for the iteration process

Step 2 : Use the equation from step 1 to find the roots

x_n	$x_{n+1} = \left(\frac{4x^{\frac{3}{2}} + 6}{5}\right)^{\frac{1}{2}}$
$x_0 = 1.5$	$x_1 = 1.6339$
$x_1 = 1.6339$	$x_2 = 1.6943$
$x_2 = 1.6943$	$x_3 = 1.7217$
$x_3 = 1.7217$	$x_4 = 1.7342$
$x_4 = 1.7342$	$x_5 = 1.7398$
$x_5 = 1.7398$	$x_6 = 1.7424$
$x_6 = 1.7424$	$x_7 = 1.7436$
$x_7 = 1.7436$	$x_8 = 1.7441$
$x_8 = 1.7441$	$x_9 = 1.7443$
$x_9 = 1.7443$	$x_{10} = 1.7444$
$x_{10} = 1.7444$	$x_{11} = 1.7444$

Stop calculating when you get two repeated answers

Therefore, the root for equation $5x^2 - 4x^{\frac{3}{2}} - 6 = 0$ is 1.7444

FALSE PLACEMENT METHOD

The "false placement method" is typically associated with problem-solving in mathematics and physics, particularly in the context of numerical analysis. It's a numerical technique used to find the roots of an equation. The method is closely related to the "bisection method" but offers a more refined approach.

How It Works:

1. **Define the Function**: Start with a function $f(x)$ for which you want to find the root (i.e., a value of x such that $f(x) = 0$).
2. **Initial Guesses**: Choose two initial points x_0 and x_1 such that $f(x_0)$ and $f(x_1)$ have opposite signs. This guarantees that there is at least one root between x_0 and x_1 by the Intermediate Value Theorem.
3. **False Position Formula**: Calculate the new approximation of the root using:

$$x_{\text{new}} = x_1 - f(x_1)\frac{x_1 - x_0}{f(x_1) - f(x_0)}$$

   This formula is derived from linear interpolation between the points $(x_0, f(x_0))$ and $(x_1, f(x_1))$.

4. **Update Points**: Evaluate $f(x_{\text{new}})$:
   - If $f(x_{\text{new}})$ is very close to zero or the desired accuracy is achieved, then x_{new} is the approximate root.
   - If $f(x_{\text{new}})$ has the same sign as $f(x_0)$, update x_0 to x_{new}; otherwise, update x_1 to x_{new}.
5. **Repeat**: Continue iterating steps 3 and 4 until the approximation is sufficiently accurate or a maximum number of iterations is reached.

Use False Placement Method to find the root of this equation which is near the given value. Give your answers correct to 4 decimal places.

$$x^3 - 6x - 4 = 0$$

Answer

Step 1: Find the x_0 using false placement method

x	$-1(x_1)$	$0(x_2)$	1
$y = x^3 - 6x - 4$	$1(y_1)$	$-4(y_2)$	-44

(Calculate using calculator in mode radian)

The root lies between -1 to 0 (sign changes from positive (+) to negative (-)

$$x_0 = \frac{1}{y_2 - y_1}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix}$$

$$= \frac{1}{-4-1}\begin{vmatrix} -1 & 1 \\ 0 & -4 \end{vmatrix}$$

$$= \frac{1}{-5}(4 - 0)$$

$$= -0.8$$

Step 2: Re-arrange the equation to find the value of x

<u>**Equation 1**</u>

$$x = (6x + 4)^{\frac{1}{3}}$$

$$f'(x) = \left(\frac{1}{3}\right)(6x + 4)^{\frac{-2}{3}}(6)$$

$$f'(-0.8) = 2.32 > 1, \textit{the iteration will not converges}$$

Equation 2

$$x = \frac{x^3 - 4}{6}$$

$$f'(x) = \frac{3x^2}{6}$$

$$= \frac{x^2}{2}$$

$f'(-0.8) = -0.32 < 1$, *the iteration will converges*

Therefore, choose equation 2 for the iteration process

Step 3 : Use the equation from step 2 to find the roots

x_n	$x_{n+1} = \frac{x^3 - 4}{6}$
$x_0 = -0.8$	$x_1 = -0.7520$
$x_1 = -0.7520$	$x_2 = -0.7375$
$x_2 = -0.7375$	$x_3 = -0.7335$
$x_3 = -0.7335$	$x_4 = -0.7324$
$x_4 = -0.7324$	$x_5 = -0.7321$
$x_5 = -0.7321$	$x_6 = -0.7321$

Stop calculating when you get two repeated answers

Therefore, the root for equation $x^3 - 6x - 4 = 0$ is -0.7321

Use False Placement Method to find the root of this equation which is near the given value. Give your answers correct to 4 decimal places.

$$2x^3 - 5x = 1$$

Answer

Step 1: Find the x_0 using false placement method

x	$-1(x_1)$	$0(x_2)$	1
$y = 2x^3 - 5x - 1$	$2(y_1)$	$-1(y_2)$	-4

(Calculate using calculator in mode radian)

The root lies between -1 to 0 (sign changes from positive (+) to negative (-)

$$x_0 = \frac{1}{y_2 - y_1}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix}$$

$$= \frac{1}{-1-2}\begin{vmatrix} -1 & 2 \\ 0 & -1 \end{vmatrix}$$

$$= \frac{1}{-3}(1 - 0)$$

$$= -0.3$$

Step 2: Re-arrange the equation to find the value of x

Equation 1

$$x = \left(\frac{5x + 1}{2}\right)^{\frac{1}{3}}$$

$$f'(x) = \left(\frac{1}{3}\right)\left(\frac{5}{2}\right)\left(\frac{5x + 1}{2}\right)^{\frac{-2}{3}}$$

$$f'(-0.3) = 2.1 > 1, \textit{the iteration will not converges}$$

Equation 2

$$x = \frac{2x^3 - 1}{5}$$

$$f'(x) = \frac{2(3x^2)}{5}$$

$$f'(-0.3) = -0.108 < 1, \textit{the iteration will converges}$$

Therefore, choose equation 2 for the iteration process

Step 3 : Use the equation from step 2 to find the roots

x_n	$x_{n+1} = \frac{2x^3 - 1}{5}$
$x_0 = -0.3$	$x_1 = -0.2108$
$x_1 = -0.2108$	$x_2 = -0.2037$
$x_2 = -0.2037$	$x_3 = -0.2034$
$x_3 = -0.2034$	$x_4 = -0.2034$

Stop calculating when you get two repeated answers

Therefore, the root for equation $2x^3 - 5x = 1$ is -0.2034

Use False Placement Method to find the root of this equation which is near the given value. Give your answers correct to 4 decimal places.

$$x^3 - 7x + 2 = 0$$

Answer

Step 1: Find the x_0 using false placement method

Find the x_0 using false placement method

x	-1	$0(x_1)$	$1(x_2)$
$y = x^3 - 7x + 2$	8	$2(y_1)$	$-4(y_2)$

(Calculate using calculator in mode radian)

The root lies between 0 to 1 (sign changes from positive (+) to negative (-)

$$x_0 = \frac{1}{y_2 - y_1}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix}$$

$$= \frac{1}{-4-2}\begin{vmatrix} 0 & 2 \\ 1 & -4 \end{vmatrix}$$

$$= \frac{1}{-6}(0-2)$$

$$= 0.33$$

Step 2: Re-arrange the equation to find the value of x

<u>**Equation 1**</u>

$$x = (7x-2)^{\frac{1}{3}}$$

$$f'(x) = \frac{1}{3}(7x-2)^{\frac{-2}{3}}(7)$$

$$f'(0.33) = 5.094 < 1, \textit{the iteration will not converge}$$

Equation 2

$$x = \frac{x^3 + 2}{7}$$

$$f'(x) = \frac{1(3x^2)}{7}$$

$$f'(0.33) = 0.047 < 1, \textit{the iteration will converges}$$

Therefore, choose equation 2 for the iteration process

Step 3 : Use the equation from step 2 to find the roots

x_n	$x_{n+1} = \frac{x^3 + 2}{7}$
$x_0 = -0.33$	$x_1 = 0.2806$
$x_1 = 0.2806$	$x_2 = 0.2889$
$x_2 = 0.2889$	$x_3 = 0.2892$
$x_3 = 0.2892$	$x_4 = 0.2892$

Stop calculating when you get two repeated answers

Therefore, the root for equation of $x^3 - 7x + 2 = 0$ is 0.2892

NEWTON RAPHSON METHOD

The Newton-Raphson Method is a popular and powerful numerical technique for finding successively better approximations to the roots (or zeroes) of a real-valued function. It is particularly useful for solving nonlinear equations of the form $f(x) = 0$.

Iteration Formula

1

Given a function $f(x)$ and its derivative $f'(x)$, the iteration formula is:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

where:

- x_n is the current approximation.
- x_{n+1} is the next approximation.
- $f(x_n)$ is the function value at x_n.
- $f'(x_n)$ is the derivative of the function at x_n.

2

Choose an Initial Guess:

Select an initial guess x_0 for the root of the equation $f(x) = 0$. This guess should be close to the actual root for faster convergence.

3

Iterate:

Apply the iteration formula:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Update the approximation to x_{n+1} using the above formula.

4

Terminate:

Once convergence is achieved, the current approximation x_{n+1} is taken as the approximate root of the equation.

Use Newton Raphson Method to find the root of each of these equations which is near the given value. Give your answers correct to 3 decimal places.

$$x^4 - 2x^3 - x + 1 = 0 \ , \ x_0 = 1.5$$

Answer

Step 1 : Find the derivative of the equation

$$f(x) = x^4 - 2x^3 - x + 1$$

$$f'(x) = 4x^3 - 6x^2 - 1$$

Step 2 : Substitute the equations in the formula

$$x_{n+1} = x_n - \frac{x^4 - 2x^3 - x + 1}{4x^3 - 6x^2 - 1}$$

Step 3 : Use the formula to find the iteration

x_n	$x_{n+1} = x_n - \frac{x^4 - 2x^3 - x + 1}{4x^3 - 6x^2 - 1}$
$x_0 = 1.5$	$x_{1=} - 0.688$
$x_{1=} - 0.688$	$x_2 - 0.190$
$x_2 - 0.190$	$x_3 = 0.779$
$x_3 = 0.779$	$x_4 = 0.649$
$x_4 = 0.649$	$x_{5=}0.641$
$x_{5=}0.641$	$x_{6=}0.641$

STOP : repeat step 2 until you get the constant value of x

Step 4 : Conclusion

The root for equation $x^4 - 2x^3 - x + 1 = 0$ is 0.641

Use Newton Raphson Method to find the root of each of these equations which is near the given value. Give your answers correct to 3 decimal places.

$$x^3 + 4x^2 + 7 = 0\ ,\ x_0 = -4$$

Answer

Step 1 : Find the derivative of the equation

$$f(x) = x^3 + 4x^2 + 7$$

$$f'(x) = 3x^2 + 8x$$

Step 2 : Substitute the equations in the formula

$$x_{n+1} = x_n - \frac{x^4 - 2x^3 - x + 1}{4x^3 - 6x^2 - 1}$$

Step 3 : Use the formula to find the iteration

x_n	$x_{n+1} = x_n - \frac{x^4 - 2x^3 - x + 1}{4x^3 - 6x^2 - 1}$
$x_0 = -4$	$x_{1=} - 4.438$
$x_{1=} - 4.438$	$x_2 - 4.369$
$x_2 - 4.369$	$x_3 = -4.367$
$x_3 = -4.367$	$x_4 = -4.367$

STOP : repeat step 2 until you get the constant value of x

Step 4 : Conclusion

The root for equation $x^3 + 4x^2 + 7 = 0$ is -4.367

Use Newton Raphson Method to find the root of each of these equations which is near the given value. Give your answers correct to 3 decimal places.

$$5x^2 - 4x^{\frac{3}{2}} - 6 = 0 \, , x_0 = 1.5$$

Answer

Step 1 : Find the derivative of the equation

$$f(x) = 5x^2 - 4x^{\frac{3}{2}} - 6$$

$$f'(x) = 10x - 6x^{\frac{1}{2}}$$

Step 2 : Substitute the equations in the formula

$$x_{n+1} = x_n - \frac{x^4 - 2x^3 - x + 1}{4x^3 - 6x^2 - 1}$$

Step 3 : Use the formula to find the iteration

x_n	$x_{n+1} = x_n - \frac{x^4 - 2x^3 - x + 1}{4x^3 - 6x^2 - 1}$
$x_0 = 1.5$	$x_{1=}1.7743$
$x_{1=}1.7743$	$x_2 = 1.7449$
$x_2 = 1.7449$	$x_3 = 1.7445$
$x_3 = 1.7445$	$x_4 = 1.7445$

STOP : repeat step 2 until you get the constant value of x

Step 4 : Conclusion

The root for equation $5x^2 - 4x^{\frac{3}{2}} - 6 = 0$ is 1.7445

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NUMERICAL METHOD

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NUMERICAL METHOD is based on the latest syllabus prepared by Department of Mathematics, Science & Computer, PTSN. The content are comprehensive and concise to help student to learn this subject more effectively in order to achieve excellent results in examination.

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