KEMENTERIAN PENGAJIAN TINGGI

ENGINEERING MATHEMATICS 2

INDICES AND LOGARITHMS

VOLUME 1

AUTHOR
NITHYA PERIASAMY.

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VOLUME 1

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PREFACE

The book is about Engineering Mathematics 2 Volume 1 on chapter 1 Indices and Logarithms. This Topic introduces the basic laws of indices and logarithms. It also covers their applications to simplify expression and problem solving using laws of indices and laws of logarithms. This book contains the formulas, examples, exercises and quizzes for each sup topic.

CONTENTS

1.0 INDEX

1) When a number a is multiplied by itself any number of times, $a^n = a \times a \times ... \times a \times a$ (n times), the expression is simplified by using index notation. n is a positive integer, a is called the base and n is the index
2) For example,

$$3^4 = 3 \times 3 \times 3 \times 3$$
$$= 81$$

Where 3 is called the base and 4 is called the index or power. Therefore, 3^4 is read as '3 to the power of 4'.

1.1 INDICES

Integer Indices

$$a^n = a \times a \times ... \times a \times a$$
$$a^{-n} = \frac{1}{a^n}$$
$$a^0 = 1$$

Fractional Indices

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$
$$a^{\frac{m}{n}} = \sqrt[n]{a^m}$$

EXAMPLE 1

Evaluate each of the following.

a) 5^3 b) $(-2)^4$ c) $\left(\frac{2}{3}\right)^5$

d) 8^0 e) 2^{-4} f) 0.2^{-3}

Solution a) $5^3 = 5\times5\times5 = 125$

b) $(-2)^4 = -2\times-2\times-2\times-2 = 16$

c) $\left(\frac{2}{3}\right)^5 = \left(\frac{2}{3}\right)\times\left(\frac{2}{3}\right)\times\left(\frac{2}{3}\right)\times\left(\frac{2}{3}\right)\times\left(\frac{2}{3}\right) = \frac{32}{243}$

d) $8^0 = 1$

e) $2^{-4} = \frac{1}{2^4} = \frac{1}{2\times2\times2\times2} = \frac{1}{16}$

f) $0.2^{-3} = \frac{1}{0.2^3} = \frac{1}{0.2\times0.2\times0.2} = \frac{1}{0.008}$

EXAMPLE 2

Evaluate each of the following.

a) $8^{\frac{1}{3}}$ b) $27^{\frac{2}{3}}$ c) $\left(\frac{64}{125}\right)^{-\frac{1}{3}}$

Solution a) $8^{\frac{1}{3}} = \sqrt[3]{8} = 2$

b) $27^{\frac{2}{3}} = \sqrt[3]{27^2} = 9$

c) $\left(\frac{64}{125}\right)^{-\frac{1}{3}} = \frac{1}{\left(\frac{64}{125}\right)^{\frac{1}{3}}} = \frac{1}{\sqrt[3]{\frac{64}{125}}} = \frac{5}{4}$

EXERCISE 1

Find the value of the following:

a) 4^3

b) $(-3)^4$

c) $(-4)^{-3}$

d) $\left(\frac{2}{5}\right)^5$

e) $\left(\frac{2}{357}\right)^0$

f) $6561^{\frac{3}{4}}$

g) $262144^{\frac{5}{6}}$

h) $2048^{\frac{5}{11}}$

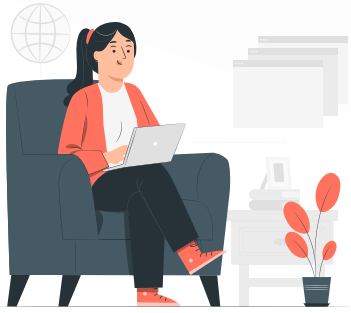
i) $0.00032^{\frac{2}{5}}$

j) $\left(\frac{32}{243}\right)^{-\frac{2}{5}}$

k) $1024^{-\frac{3}{5}}$

Answers

http://tiny.cc/C1_Exercise1



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1.2 LAWS OF INDICES

1) $a^m \times a^n = a^{m+n}$

2) $a^m \div a^n = a^{m-n}$

3) $(a^m)^n = a^{mn}$

4) $(ab)^m = a^m \times b^m$

5) $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

Tips!
Remember that, in general

$(a+b)^n \neq a^n + b^n$

$(a-b)^n \neq a^n - b^n$

EXAMPLE 3

Evaluate the value of the following:

a) $4^3 \times 4^5$ b) $5^6 \times 5^3 \div 5^7$ c) $(4^{-2})^3$ d) $\frac{9^{\frac{2}{3}}}{9^2 \times 9^{-\frac{5}{3}}}$

Solution

a) $4^3 \times 4^5$

$= 4^{3+5}$

$= 4^8 = 65536$

b) $5^6 \times 5^3 \div 5^7$

$= 5^{6+3-7}$

$= 5^2 = 25$

c) $\left(\frac{64}{125}\right)^{-\frac{1}{3}} = \frac{1}{\left(\frac{64}{125}\right)^{\frac{1}{3}}} = \frac{1}{\sqrt[3]{\frac{64}{125}}} = \frac{5}{4}$

EXAMPLE 3

d) $\dfrac{9^{\frac{2}{3}}}{9^2 \times 9^{-\frac{5}{3}}}$

$= \dfrac{9^{\frac{2}{3}}}{9^{2+\left(-\frac{5}{3}\right)}}$

$= \dfrac{9^{\frac{2}{3}}}{9^{\frac{1}{3}}}$

$= 9^{\frac{2}{3}-\frac{1}{3}}$

$= 9^{\frac{1}{3}}$

EXAMPLE 4

Simplify each of the following:

a) $5^{4n} \div 5^{3n}$

b) $\left(2^3\right)^{5n}$

c) $\left(p^6 q^{3n}\right)^{\frac{1}{3}}$

d) $a^n \times a^{2-n} \div a^{2n-1}$

e) $4^{n+1} \times 2^{2n} \div 16^{3n}$

f) $\dfrac{5^{2n+1}}{25^{n-1} \times 125^{n+1}}$

SOLUTION

a) $5^{4n} \div 5^{3n}$

$= 5^{4n-3n}$

$= 5^{n}$

b) $\left(2^{3}\right)^{5n}$

$= 2^{15n}$

c) $\left(p^{6}q^{3n}\right)^{\frac{1}{3}}$

$= \left(p^{6}\right)^{\frac{1}{3}}\left(q^{3n}\right)^{\frac{1}{3}}$

$= p^{2}q^{n}$

d) $a^{n} \times a^{2-n} \div a^{2n-1}$

$= a^{n+(2-n)-(2n-1)}$

$= a^{n+2-n-2n+1}$

$= a^{-2n+3}$

e) $4^{n+1} \times 2^{2n} \div 16^{3n}$

$= \left(2^{2}\right)^{n+1} \times 2^{2n} \div \left(2^{4}\right)^{3n}$

$= 2^{2n+2} \times 2^{2n} \div 2^{12n}$

$= 2^{2n+2+2n-12n}$

$= 2^{2-8n}$

f) $\dfrac{5^{2n+1}}{25^{n-1} \times 125^{n+1}}$

$= \dfrac{5^{2n+1}}{\left(5^{2}\right)^{n-1} \times \left(5^{3}\right)^{n+1}}$

$= \dfrac{5^{2n+1}}{5^{2n-2} \times 5^{3n+3}}$

$= \dfrac{5^{2n+1}}{5^{5n+1}}$

$= 5^{(2n+1)-(5n+1)}$

$= 5^{-3n}$

EXERCISE 2

Evaluate each of the following:

a) $8^{3} \times 16^{\frac{1}{2}} \times 256^{\frac{1}{4}}$

b) $9^{\frac{1}{6}} \times 27^{\frac{1}{3}} \times 81^{\frac{1}{6}}$

c) $25^{\frac{1}{4}} \times 125^{-\frac{1}{5}} \div 5^{\frac{3}{4}}$

d) $\dfrac{6^{-3} \times 216^{2}}{36}$

Answers

http://tiny.cc/C1_Exercise2

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EXERCISE 3

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Simplify each of the following:

a) $b^{n} \times b^{n-1} \div b^{2n}$

b) $2a^{3}c^{3} \times 3a^{2}c$

c) $16^{2n} \times 4^{n-1} \div 64^{n-2}$

d) $9^{2-n} \times 3^{2n+1} \div 81^{n-1}$

e) $\left(\dfrac{1}{6}\right)^{2} \times 36^{3n+1} \div 216^{n+1}$

f) $\dfrac{10m^{5}n^{4} \times n^{2}}{2m^{2}n}$

g) $\dfrac{4^{3n-1}}{16^{n-2} \times 8^{n+2}}$

h) $\dfrac{\left(s^{-2}r^{3}\right)^{\frac{1}{6}}}{s^{4}r^{\frac{1}{3}}}$

i) $\dfrac{5^{2n-4}}{25^{3n} \times 625^{n-1}}$

Answers

http://tiny.cc/C1_Exercise3

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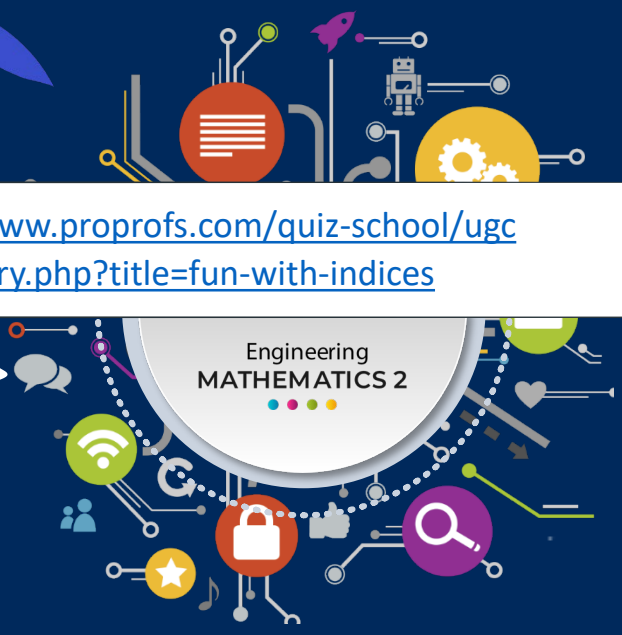

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1.3 SOLVING EQUATION INVOLVING INDICES

When solving an equation involving indices, we need to simplify the algebraic expression on both sides of the equation before expression in terms of the same base or of the same index and then equating the indices or base accordingly. For instance,

If $a^m = a^n$, then $m = n$

If $a^m = b^m$, then $a = b$

EXAMPLE 5

Solve the following equations:

a) $4^x = 16$

b) $9^x \bullet 3^{x-1} = 81$

c) $3^{x+1} \bullet 27^x = \frac{1}{9}$

Solution

a) $4^x = 16$

$4^x = 4^2$

$\therefore \ x = 2$

b) $9^x \bullet 3^{x-1} = 81$

$(3^2)^x \bullet 3^{x-1} = 3^4$

$3^{2x+x-1} = 3^4$

$3^{3x-1} = 3^4$

$\therefore 3x - 1 = 4$

$x = \frac{5}{3}$

c) $3^{x+1} \bullet 27^{x} = \frac{1}{9}$

$$3^{x+1} \bullet \left(3^{3}\right)^{x} = \frac{1}{3^{2}}$$

$$3^{x+1+3x} = 3^{-2}$$

$$3^{4x+1} = 3^{-2}$$

$$\therefore 4x+1 = -2$$

$$x = -\frac{3}{4}$$

EXERCISE 4

Solve the following equations.

a) $7^{3x-2} = 1$

b) $15(25^{x}) = 75$

c) $64^{4x+4} = 256^{x-3}$

d) $81^{2x} = 729^{x-3}$

e) $14(7^{x-4}) = 2(49^{3x})$

f) $4^{2x-1} - \frac{1}{8^{2x}} = 0$

g) $4^{2x}.8^{x-2} = 32$

h) $27^{m-3}.3^{m^{2}} = 3^{3m}$

i) $2^{x+4} - 2^{x+3} = 1$

j) $2^{3x+4} - 2^{3x+2} - 24 = 0$

Answers

http://tiny.cc/C1_Exercise4

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1.4 LOGARITHMS

In the equation $y = a^x$, the number x is called an index. This equation $y = a^x$ can be written as $\log_a y = x$ Then, x is the logarithms of y to the base a.

In general,

$$y = a^x \quad \leftrightarrow \quad \log_a y = x$$

EXAMPLE 6

Express the following equations in logarithm forms.

a) $625 = 5^4$

b) $\frac{1}{64} = 4^{-3}$

Solution a) $625 = 5^4$

$\log_5 625 = 4$

b) $\frac{1}{64} = 4^{-3}$

$\frac{1}{64} = \frac{1}{4^3}$

$64 = 4^3$

$\log_4 64 = 3$

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EXAMPLE 7

Express the following equations in index forms.

a) $\log_3 \frac{1}{81} = -4$

b) $\log_{10} 1000 = 3$

Solution

a) $\log_3 \frac{1}{81} = -4$

$$\frac{1}{81} = 3^{-4}$$

$$\frac{1}{81} = \frac{1}{3^4}$$

$$81 = 3^4$$

b) $\log_{10} 1000 = 3$

$$1000 = 10^3$$

EXAMPLE 8

Determine the value of x for the following equations.

a) $\log_7 x = 4$

b) $\log_x 16 = 2$

Solution

a) $\log_7 x = 4$

$$x = 7^4$$

$$x = 2401$$

b) $\log_x 16 = 2$

$$16 = x^2$$

$$4^2 = x^2$$

$$\therefore \quad x = 4$$

EXERCISE 5

Find the value of x of the following equations.

a) $\log_3 243 = x$

b) $\log_6 x = 3$

c) $\log_4 x = -3$

d) $\log_x 625 = 4$

Answers

http://tiny.cc/C1_Exercise5

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1.5 LAWS OF LOGARITHMS

1) $\log_a xy = \log_a x + \log_a y$

2) $\log_a \frac{x}{y} = \log_a x - \log_a y$

3) $\log_a M^c = c \log_a M$

4) $\log_a a = 1$

5) $\log_a a^0 = 0$

6) $\log_N M = \frac{\log_a M}{\log_a N}$

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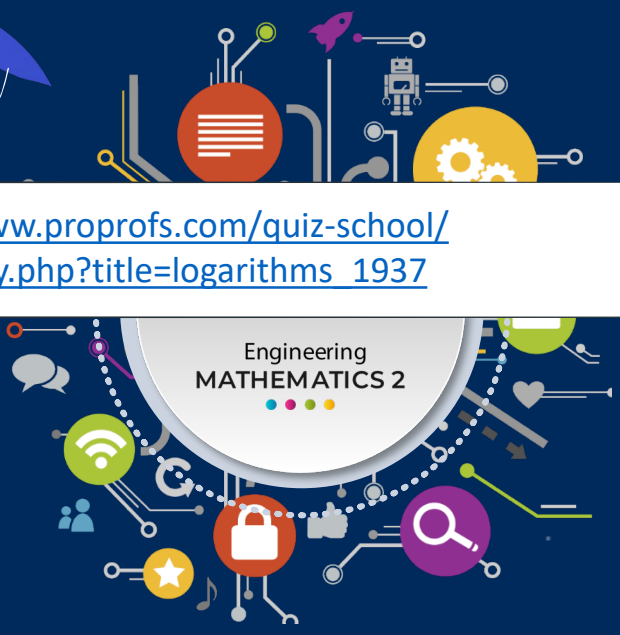

EXAMPLE 9

Evaluate each of the following by using the laws of logarithms.

a) $\log_p rs^3$ b) $\log_p \dfrac{x^3}{\sqrt{y}}$ c) $\log_a \sqrt{\dfrac{ax^3}{y}}$

Solution

a) $\log_p rs^3$

$= \log_p r + \log_p s^3$

$= \log_p r + 3\log_p s$

b) $\log_p \dfrac{x^3}{\sqrt{y}}$

$= \log_p x^3 - \log_p \sqrt{y}$

$= \log_p x^3 - \log_p y^{\frac{1}{2}}$

$= \log_p x^3 - \dfrac{1}{2}\log_p y$

c) $\log_a \sqrt{\dfrac{ax^3}{y}}$

$= \log_a \left(\dfrac{ax^3}{y}\right)^{\frac{1}{2}}$

$= \dfrac{1}{2}\log_a \left(\dfrac{ax^3}{y}\right)$

$= \dfrac{1}{2}\left[\log_a a + \log_a x^3 - \log_a y\right]$

$= \dfrac{1}{2}\left[1 + 3\log_a x - \log_a y\right]$

$= \dfrac{1}{2} + \dfrac{3}{2}\log_a x - \dfrac{1}{2}\log_a y$

EXAMPLE 10

Simplify the following logarithm equations to the simplest form.

a) $3\log_c 2 + 2\log_c 5 - \log_c 20$

b) $2\log x - 3\log y + 2\log xy$

c) $\log\sqrt{a} - \log\frac{1}{b} + \frac{1}{3}\log c$

d) $\log_4 x + \log_8 x$

e) $\log_3 a + \log_9 a^2 - \log_{27} a$

Solution

a) $3\log_c 2 + 2\log_c 5 - \log_c 20$

$= \log_c 2^3 + \log_c 5^2 - \log_c 20$

$= \log_c \left(\frac{2^3 \times 5^2}{20}\right)$

$= \log_c 10$

b) $2\log x - 3\log y + 2\log xy$

$= \log x^2 - \log y^3 + \log(xy)^2$

$= \log x^2 - \log y^3 + \log x^2 y^2$

$= \log\left(\frac{(x^2)(x^2 y^2)}{y^3}\right)$

$= \log \frac{x^4}{y}$

c) $\log\sqrt{a} - \log\frac{1}{b} + \frac{1}{3}\log c$

$= \log\sqrt{a} - \log\frac{1}{b} + \log c^{\frac{1}{3}}$

$= \log\left(\frac{\sqrt{a} \times c^{\frac{1}{3}}}{\frac{1}{b}}\right)$

$= \log\sqrt{a}\ b\ c^{\frac{1}{3}}$

d) $\log_4 x + \log_8 x$

$= \frac{\log_2 x}{\log_2 2^2} + \frac{\log_2 x}{\log_2 2^3}$

$= \frac{\log_2 x}{2} + \frac{\log_2 x}{3}$

$= \frac{5}{6}\log_2 x$

e) $\log_3 a + \log_9 a^2 - \log_{27} a$

$= \log_3 a + \frac{\log_3 a^2}{\log_3 3^2} - \frac{\log_3 a}{\log_3 3^3}$

$= \frac{5}{3}\log_3 a$

Answers

http://tiny.cc/C1_Exercise6

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EXERCISE 6

Simplify each of the following by using the laws of logarithms.

a) $4\log_b x + 2\log_b y$

b) $4\log_2 x + 3\log_2 y - \log_2 z$

c) $3\log_x y + \log_x 2 - 5\log_x a$

d) $\frac{1}{2}\log_4 p - 2\log_4 q + 1$

EXAMPLE 11

Find the value of the following logarithms without using calculator.

a) $\log_6 1296$

b) $\log_5 \frac{1}{125}$

c) $\log_{27} 9$

d) $\log_2 14 + \log_2 12 - \log_2 21$

e) $\frac{1}{4}\log_9 625 - \log_9 45 + 4\log_9 3$

Solution

a) $\log_6 1296$

$= \log_6 6^4$

$= 4\log_6 6$

$= 4(1)$

$= 4$

b) $\log_5 \frac{1}{125}$

$= \log_5 \frac{1}{5^3}$

$= \log_5 5^{-3}$

$= -3\log_5 5$

$= -3(1)$

$= -3$

c) $\log_{27} 9$

$= \frac{\log_3 9}{\log_3 27}$

$= \frac{\log_3 3^2}{\log_3 3^3}$

$= \frac{2\log_3 3}{3\log_3 3}$

$= \frac{2}{3}$

d) $\log_2 14 + \log_2 12 - \log_2 21$

$= \log_2 \left(\frac{14 \times 12}{21} \right)$

$= \log_2 8$

$= \log_2 2^3$

$= 3 \log_2 2$

$= 3$

e) $\frac{1}{4} \log_9 625 - \log_9 45 + 4 \log_9 3$

$= \log_9 (625)^{\frac{1}{4}} - \log_9 45 + \log_9 3^4$

$= \log_9 5 - \log_9 45 + \log_9 81$

$= \log_9 \left(\frac{5 \times 81}{45} \right)$

$= \log_9 9$

$= 1$

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EXERCISE 7

Evaluate each of the following without using a calculator.

a) $\log_3 243$

b) $\log_2 \frac{1}{32}$

c) $\log_{\frac{1}{2}} 128$

d) $\log_{25} 625$

e) $\log_{\sqrt{2}} 256$

f) $\log_{10} 0.1$

g) $\log_3 81\sqrt{3}$

h) $3\log_5 2 + 2\log_5 10 - \log_5 32$

i) $2\log_2 4 + \log_2 8 - \frac{1}{2}\log_2 16$

Answers

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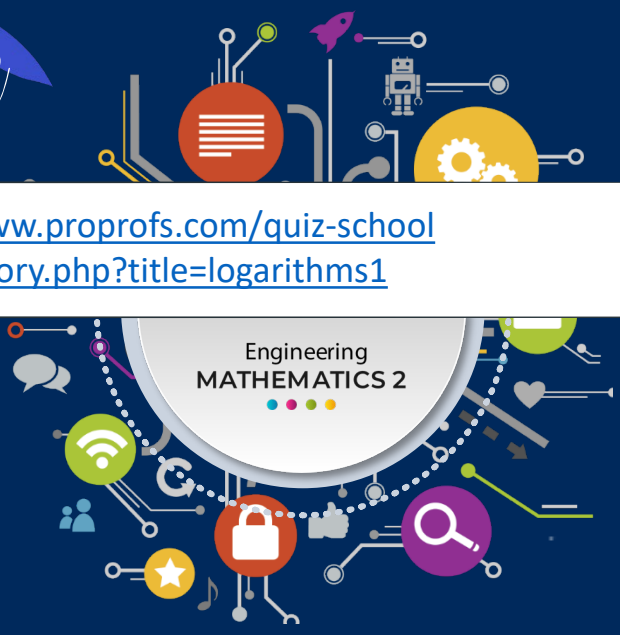

EXAMPLE 12

Solve the following logarithms.

a) $\log(4x+5)=1+\log(x-1)$

b) $\log_3 x-\log_3(1-x)=2$

c) $\log_5(5x-4)=2\log_5 3+\log_5 4$

Solution

a) $\log(4x+5)=1+\log(x-1)$

$$\log(4x+5)-\log=1$$

$$\log\frac{(4x+5)}{(x-1)}=1$$

$$\frac{(4x+5)}{(x-1)}=10^1$$

$$4x+5=10x-10$$

$$6x=15$$

$$x=\frac{15}{6}$$

b) $\log_3 x-\log_3(1-x)=2$

$$\log_3\frac{x}{(1-x)}=2$$

$$\frac{x}{(1-x)}=3^2$$

$$x=9-9x$$

$$10x=9$$

$$x=\frac{9}{10}$$

c) $\log_5 (5x-4) = 2\log_5 3 + \log_5 4$

$$\log_5 (5x-4) - 2\log_5 3 - \log_5 4 = 0$$

$$\log_5 (5x-4) - \log_5 3^2 - \log_5 4 = 0$$

$$\log_5 \frac{(5x-4)}{(3^2)(4)} = 0$$

$$\frac{(5x-4)}{(3^2)(4)} = 5^0$$

$$\frac{(5x-4)}{36} = 1$$

$$5x-4 = 36$$

$$5x = 40$$

$$x = 8$$

EXERCISE 8

Solve each of the following equations.

a) $\log_{11} 9x = 2\log_{11} 3$

b) $\log_3 6x - \log_3 (3x-1) = 2$

c) $\log_{10}(x-1)+1 = \log_{10}(x-5)$

d) $\log_3 6 + \log_3 2x = 0$

e) $\log_5 (6x-3) = 2\log_5 3 + \log_5 4$

f) $\log_3 5x - \log_3 (2x-1) = 2$

g) $\log 4 + \log(3x+1) = \log 2 + \log(x+3)$

h) $\log_2 (5x-2) = 1 + \log_2 (2x+1)$

i) $\log_k 4 + \frac{1}{3}\log_k 8 = 3$

j) $\log_4 (x^2+8x-1) = 2 + \log_4 (x-1)$

k) $\log_{10}(m^2+3m+46) = 2$

l) $5\log_x 6 - \log_x 96 = 4$

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Answers

http://tiny.cc/C1_Exercise8

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EXAMPLE 13

Given that $\log_2 5 = 2.32$, find the value of each of the following without using a calculator

a) $\log_5 4$ b) $\log_5 2$ c) $\log_2 5\sqrt{5}$

Solution a) $\log_5 4$

$$= \frac{\log_2 4}{\log_2 5}$$

$$= \frac{\log_2 2^2}{\log_2 5}$$

$$= \frac{2}{2.32}$$

$$= 0.8621$$

b) $\log_5 2$

$$= \frac{\log_2 2}{\log_2 5}$$

$$= \frac{1}{2.32}$$

$$= 0.4310$$

c) $\log_2 5\sqrt{5}$

$$= \log_2 5 + \log_2 5^{\frac{1}{2}}$$

$$= \log_2 5 + \frac{1}{2}\log_2 5$$

$$== \frac{3}{2}\log_2 5$$

$$= \frac{3}{2}(2.32) = 3.48$$

EXAMPLE 14

Given $\log_2 3 = 1.59$ and $\log_2 5 = 2.32$, find the value of the following without using calculator.

a) $\log_2 45$ b) $\log_4 0.6$

Solution a) $\log_2 45$

$$= \log_2 (3^2 \times 5)$$

$$= \log_2 3^2 + \log_2 5$$

$$= 2(1.59) + 2.32$$

$$= 5.50$$

b) $\log_4 0.6$

$$= \frac{\log_2 0.6}{\log_2 4}$$

$$= \frac{\log_2 \left(\frac{3}{5}\right)}{\log_2 2^2}$$

$$= \frac{\log_2 3 - \log_2 5}{2 \log_2 2}$$

$$= \frac{1.59 - 2.32}{2} = -0.365$$

EXERCISE 9

a) Given that $\log_5 2 = 0.431$ and $\log_5 3 = 0.683$, without using a calculator, evaluate

i) $\log_5 6$

ii) $\log_5 1.5$

b) Given that $\log_3 2 = 0.63$ and $\log_3 5 = 1.46$, find the value of

i) $\log_3 \frac{9}{10}$

ii) $\log_3 \sqrt{20}$

c) Given that $\log_3 x = p$, find $\log_x 27x^6$ in terms of p.

d) Given that $\log_y 5 = p$ and $\log_y 2 = q$, express $\log_5 \frac{10}{y^4}$ in terms of p and q.

e) Given $\log_a 4 = m$, express in terms of a.

i) $\log_a 256$

ii) $\log_4 64\sqrt{a}$

f) Given that $\log_n 2 = 0.123$ and $\log_n 3 = 0.256$, calculate the value of $\log_n 36$.

Answers

http://tiny.cc/C1_Exercise9

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REVIEW 1

1. Express the following in the simplest form.

a) $\dfrac{2^{n+4} \times 4(2^{n-1})}{2^n}$

b) $4 \log a + \dfrac{\log a}{2} - 2 \log a$

2. Calculate the value of x for the following equations.

a) $3^{x-2} = 9^{x+4}$

b) $\log_4 (5x-1) = 3$

c) $\log (x+3) - \log 5 = \log (1-3x)$

3. Given $\log_4 3 = 0.7925$ and $\log_4 6 = 1.2925$, calculate the value of $\log_3 6 + \log_4 \sqrt{6} - \log_4 9$

4. Write each of the following expressions in the simplest form:

a) $\left(\dfrac{81p^4}{16q^{-4}}\right)^{\frac{3}{4}}$

b) $2 + 4\log_8 p - \dfrac{1}{3}\log_8 q$

5. Calculate the value of x for the following equations:

a) $\log_3 (2x+7) = 3$

b) $6^{2x+1} = 17$

c) $\left(27^x\right)^x = 3^{x+2} \bullet 9^{2x}$

e) $\log_3 (5x-1) = 45^x$

6. Given $\log_6 4 = 0.7737$ and $\log_6 8 = 1.1606$. Determine the value of $\log_6 32 + \log_6 6^3$ without using the calculator.

7. State each of the following functions in the simplest form:

a) $\dfrac{3^n}{81^{n+1} \times 27}$

b) $5 \log m - \log \sqrt[3]{m} + \log 2m$

8. Determine the following equations:

a) $3^{4-x} = 9$

b) $\dfrac{25^{\frac{x}{2}} \times 125}{5} = 3125^x$

c) $\dfrac{1}{2}\log_3 x + \log_9 4x = 0$

d) $\log_4 (x-2) + 3\log_2 8 = 10$

9. Calculate the value of p for the following equation:

$$2\log_p 10 - 2\log_p \left(\frac{5}{2}\right) = 4 - \log_p p^2$$

10. Express each of the following in the simplest form.

a) $\dfrac{a^{4-3n}}{a^{2n} \times a^{3n+1}}$

b) $5^{n+2} \times 625^{n-1} \div 125^{2n-1}$

c) $3 \log a + \dfrac{\log a}{4} - 2 \log a$

REFERENCES

1. Bird, J.(2017). Higher Engineering Mathematics. (8th Edition). New York, NY: Routledge
2. Larson, R. and Edwards, B. (2018). Calculus. (11th Edition). Boston, MA. Cencage Learning. P.M.

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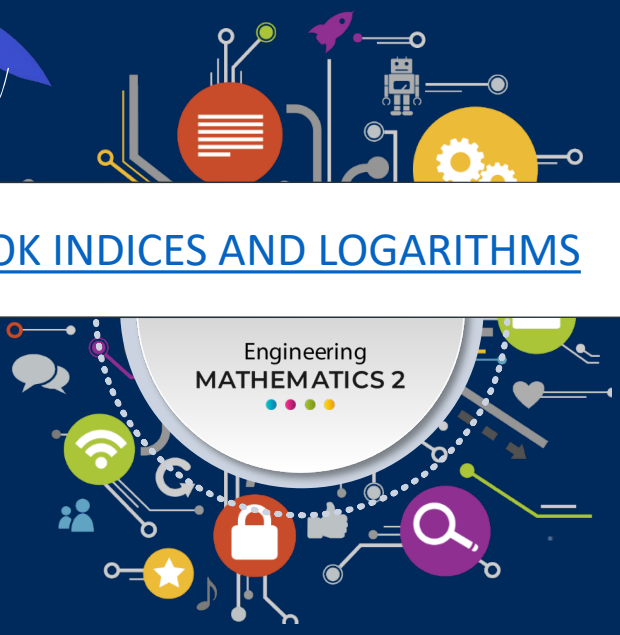

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