KEMENTERIAN PENGAJIAN TINGGI

STRENGTH OF MATERIALS

PART 1
CHAPTER 1-3

Ts. MASNIZA YUSOF
Ts. NAZERA DAN
Ts. NADZRI CHE KAMIS

DEPARTMENT OF MECHANICAL ENGINEERING

STRENGTH OF MATERIALS

PART 1 (CHAPTER 1-3)

Ts.MASNIZA BINTI YUSOF
Ts.NAZERA BINTI DAN
Ts.NADZRI BIN CHE KAMIS
2021
JABATAN KEJURUTERAAN MEKANIKAL

Diterbitkan oleh:
Politeknik TuankuSyed Sirajudddin (PTSS)
Pauh Putra, 02600 Arau, Perlis
e ISBN 978-967-2258-46-9

Acknowledgement

We wish to express our sincere gratitude to The Almighty Allah swt for establishing us to complete this book – STRENGTH OF MATERIALS: PART 1 (CHAPTER 1-3)

We also wish to express our appreciation to the numerous parties who have enable the realization of this book. The parties include Mechanical Engineering Department, Politeknik Tuanku Syed Sirajuddin, Perlis and all parties who involve in the publication process directly or indirectly. We would like to thank our friends and colleagues for the assistance, guidance and constant support from them during the publication of this book. We would also like to thank our family members who provided encouragement, patience and support.

Finally, we would wish readers happy reading and apologize for any omissions and errors. We hope that this module will be beneficial to all.

Ts.MASNIZA BINTI YUSOF
Ts.NAZERA BINTI DAN
Ts.NADZRI BIN CHE KAMIS

Preface

This book is designed to provide a frame of reference for Polytechnic diploma courses in Mechanical Engineering Department or related courses. The book is alternatively helpful for third semester student in Diploma Mechanical Engineering.

The book has been structured into three chapters, which cover apart from the topics addressed in course syllabus adapted by the Malaysia Polytechnics. Students are guided to understand the concept of forces on materials, thermals and composite bars, shear forces and bending moments. Each chapter consist of notes, examples of solutions and exercises which is suitable for teaching and learning session.

Finally, may this book be beneficial to students and others who directly or indirectly used this book as a reference. Hope this book be beneficial in helping them achieve an excellent result during the final examination and in their life.

TABLE OF CONTENTS

TABLE OF CONTENTS

TABLE OF CONTENTS

TABLE OF CONTENTS

CHAPTER 1

1. FORCES ON MATERIALS

This topic analyses the effects of forces on materials, Hooke's law, shear stress and shear strain.

BASIC SYMBOL USING IN STRENGTH OF MATERIALS:-

- *A – area*
- *d – diameter*
- *Δd – change in diameter*
- *E – Young's modulus*
- *F or P – Force*
- *G – Modulus of rigidity*
- *L – length*
- *U – strain energy*
- *ΔL – change in length*
- *ε - strain*
- *ϕ - shear strain*
- *ρ - density*
- *σ - stress*
- *τ - shear stress*
- *υ - poisson's ratio*

1.1 TYPES OF LOAD:-

a. static

b. dynamic

c. impact

d. fatigue and alternating loads

a. Static load

- do not change
- example:- the building

b. Dynamic load

- constantly changing
- example:- the vehicle on the bridge

c. Impact load

- acting immediately
- example: the hammer knocks on the nail

d. Fatigue and alternating loads

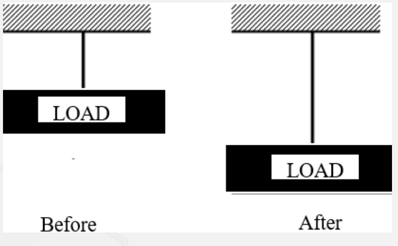
- - Charges in effect at certain times.
  - example:-

i. the shaft is mounted on the windmill.

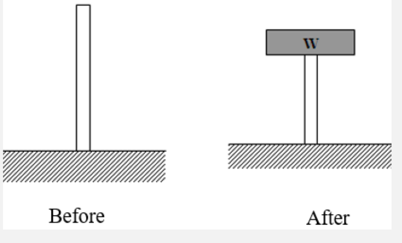
ii. when a load is suspended on a spring.

1.2 EFFECTS OF LOAD:-

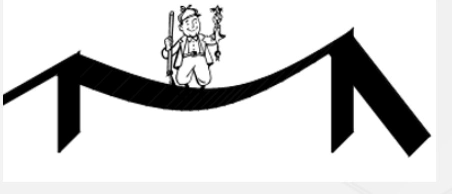
1. Causing Extension



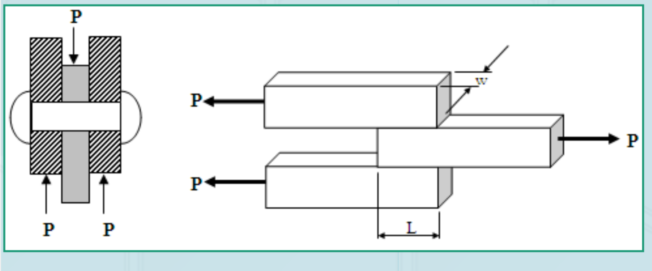
2. Causing Shortening



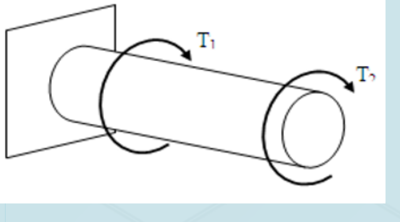
3. Causing Bending



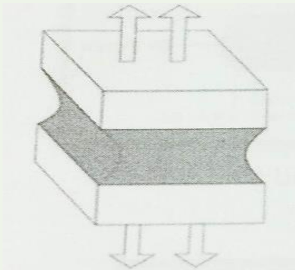
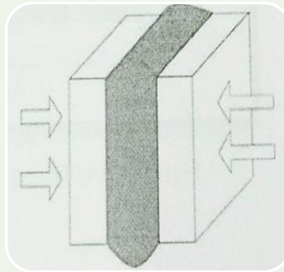
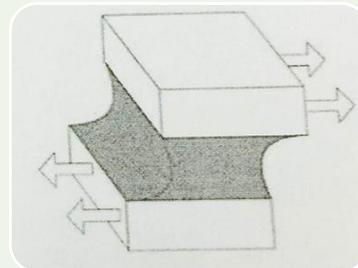
4. Causing Shear



5. Causing Twisting



1.3 TYPES OF FORCE:-

A. TENSILE FORCE	B. COMPRESSION FORCE	C. SHEAR FORCE
		

1.2 STRESS AND STRAIN

1.4.1 Type of stress

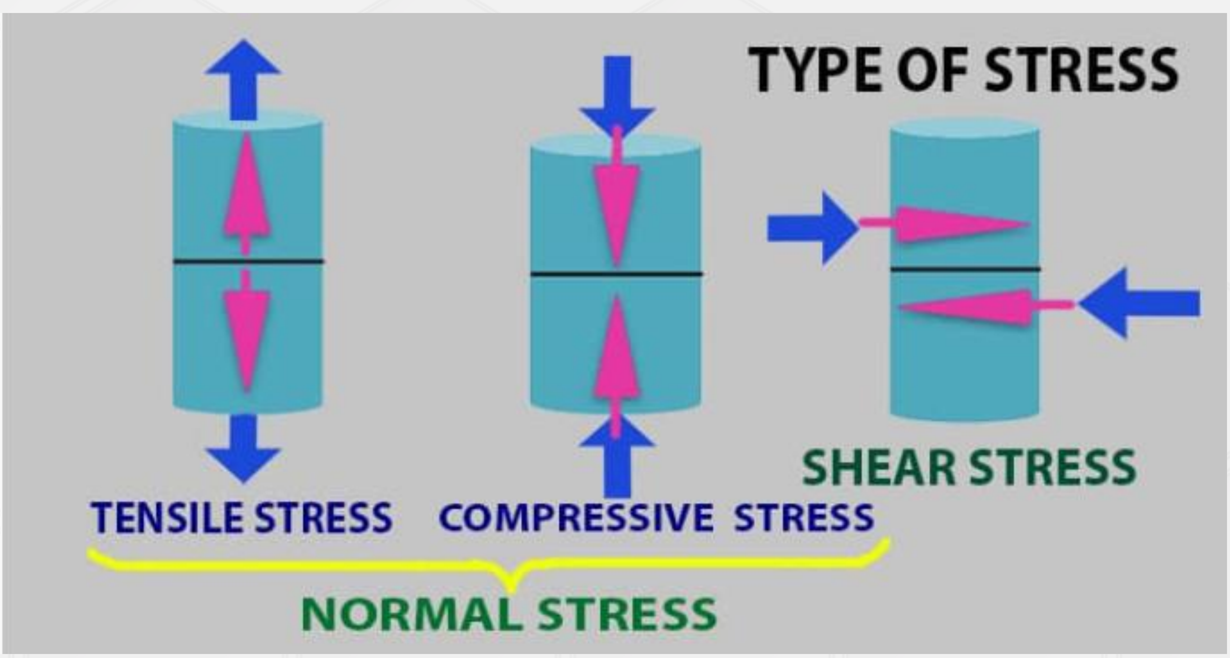
There are three types of stress;-

a. Tensile stress.

b. Compressive stress.

c. Shear stress.

- Stress depends on the **magnitude** and **direction** of force applied and the cross-sectional area of the **stress (σ)** is the ratio of **force (P)** with a cross-sectional **area (A)**.



STRESS

$$\sigma = \frac{F}{A}$$

- The average unit stress, here after call **stress (σ), is a measure of the intensity of the applied and resisting force.**
- The stress is often determined by dividing the total applied load (F) by the total area resisting deformation by the load (A);
- The **unit** of stress are Newtons per square meter **(N/m^2).**
- One Newton per square meter is equal to a Pascal (Pa).

STRAIN

$$\varepsilon = \frac{\Delta L}{L}$$

- The **average amount of distortion per unit length** is called the unit strain hereafter called **strain (ε)**.
- The strain is often determined by **dividing the total change in length of sample (ΔL) by the original length (L);**
- Strain has no net unit since it is defined as the meters of change per meter of length (m/m). **The units always cancel**.
- Consequently any units may be attached to the number representing strain.

1.5 YOUNG'S MODULUS

- In solid mechanics, the slope of the stress-strain curve at any point is called the tangent modulus. The tangent modulus of the initial, linear portion of a stress-strain curve is called **Young's modulus**, also known as the **tensile modulus.** It is defined as the ratio of the uniaxial stress over the uniaxial strain in the range of stress in which Hooke's Law holds. It is a measure of the stiffness of an elastic material and is a quantity used to characterize materials.
- It can be experimentally determined from the slope of a stress-strain curve as Figure 1.3 created during tensile tests conducted on a sample of the material. In anisotropic materials, Young's modulus may have different values depending on the direction of the applied force with respect to the material's structure.

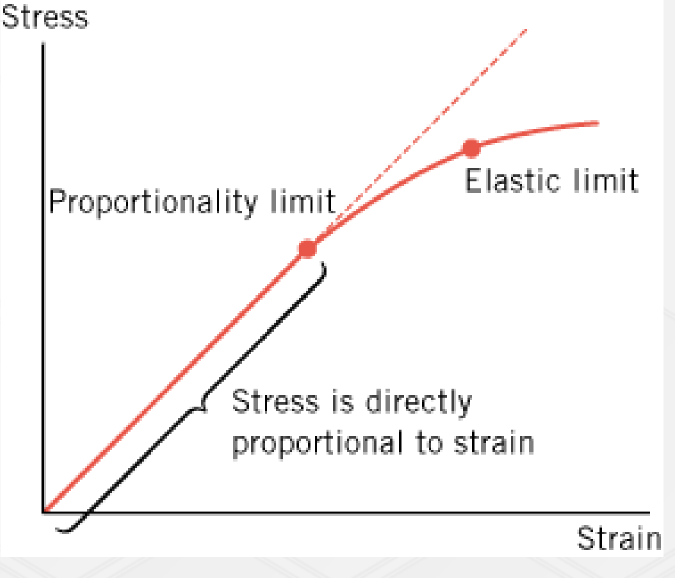
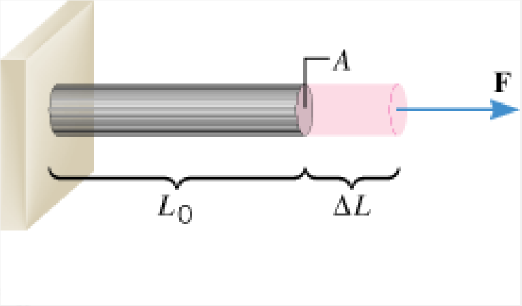


Figure 1.3. stress-strain curve

YOUNG'S MODULUS

$$E = \frac{\sigma}{\varepsilon}$$



- It is also commonly, but incorrectly, called the *elastic modulus* or *modulus of elasticity*, because Young's modulus is the most common elastic modulus used, but there are other elastic moduli measured, too, such as the bulk modulus and the shear modulus.
- It is defined as the ratio of the uniaxial stress over the uniaxial strain in the range of stress in which Hooke's Law holds.

Sample Question

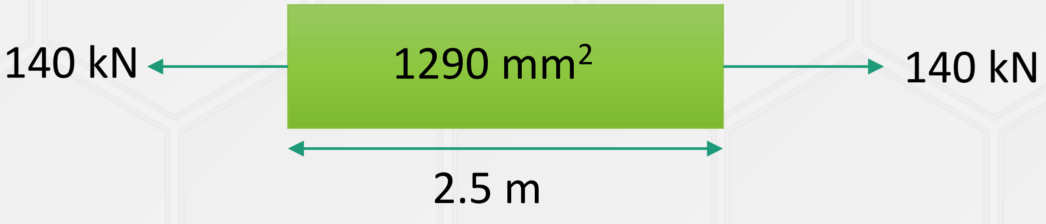
Exercise 1

A 2.5 m rod with cross sectional area of 1290 mm^2 extends by 1.5 mm when applied with a tensile force of 140 kN at both ends.

a. Draw a free body diagram for the above situation.
b. Calculate the tensile stress in the rod.
c. Determine the strain.
d. Determine the Young's Modulus of the rod..

Solution:-

a. Free body diagram



b. The tensile stress in the rod.

Given;

$L = 2.5m$
$A = 1290mm^2 = 1290 \times 10^{-6}\ m^2$
$\Delta L = 1.5\ mm$
$F = 140\ kN$

Tensile stress, σ

$$\sigma = \frac{F}{A}$$

$$= \frac{140\ x\ 10^3}{1290\ x\ 10^{-6}}$$

$$= 108.527 \times 10^6\ N/m^2$$

c. The strain.

$$\varepsilon = \frac{\Delta L}{L}$$

$$= \frac{1.5\ x\ 10^{-3}}{2.5}$$

$$= 6 \times 10^{-4}$$

d. The Young's Modulus of the rod.

$$E = \frac{\sigma}{\varepsilon}$$

$$= \frac{108.527\ x\ 10^6}{6\ x\ 10^{-4}}$$

$$= 180.527 \times 10^9\ N/m^2$$

1.6 UNDERSTAND HOOKE'S LAW

- Through verification by several experiments, it was Robert Hooke who stated the relationship of great importance.
- His statement "*ut tensio sic vis*" could be translated as "***as is the tension so is the extension***".
- A better way of making the statement will be that " ***extension is proportional to force***"

$P \;\alpha\; \delta L$ or $\delta L \;\alpha\; P$

Area of cross section = A
Length = l
Extension = δl

- According to above figure, dividing *P by A*, and *δl by l* where both *A* and *l* are constants in the given situation, it is identified *that P/A* is the stress and *δl/l* is the strain in the bar.
- Hence

stress α strain

or

$$\sigma \;\alpha\; \varepsilon$$

- The statement of proportionality is convert into an equation by introduction of a constant. Thus:

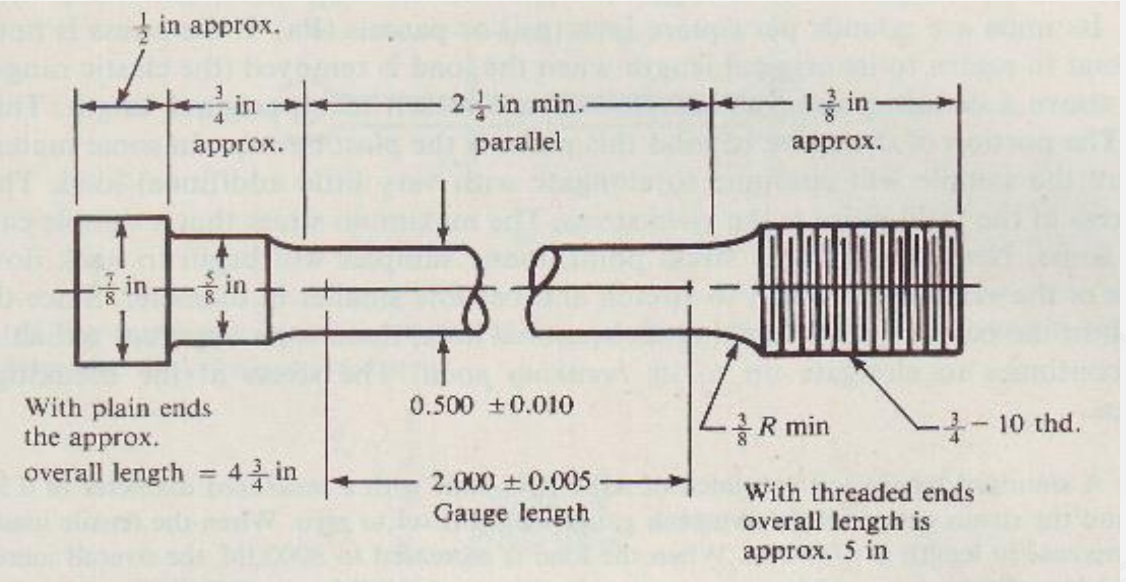
$$\sigma = E\,\varepsilon$$

where *E* is a constant.

- The constant of proportionality, *E* has assumed great importance in engineering practice.
- For different materials there will be different values of this constant. This constant of material is called ***modulus of elasticity*** of ***Young's modulus***.

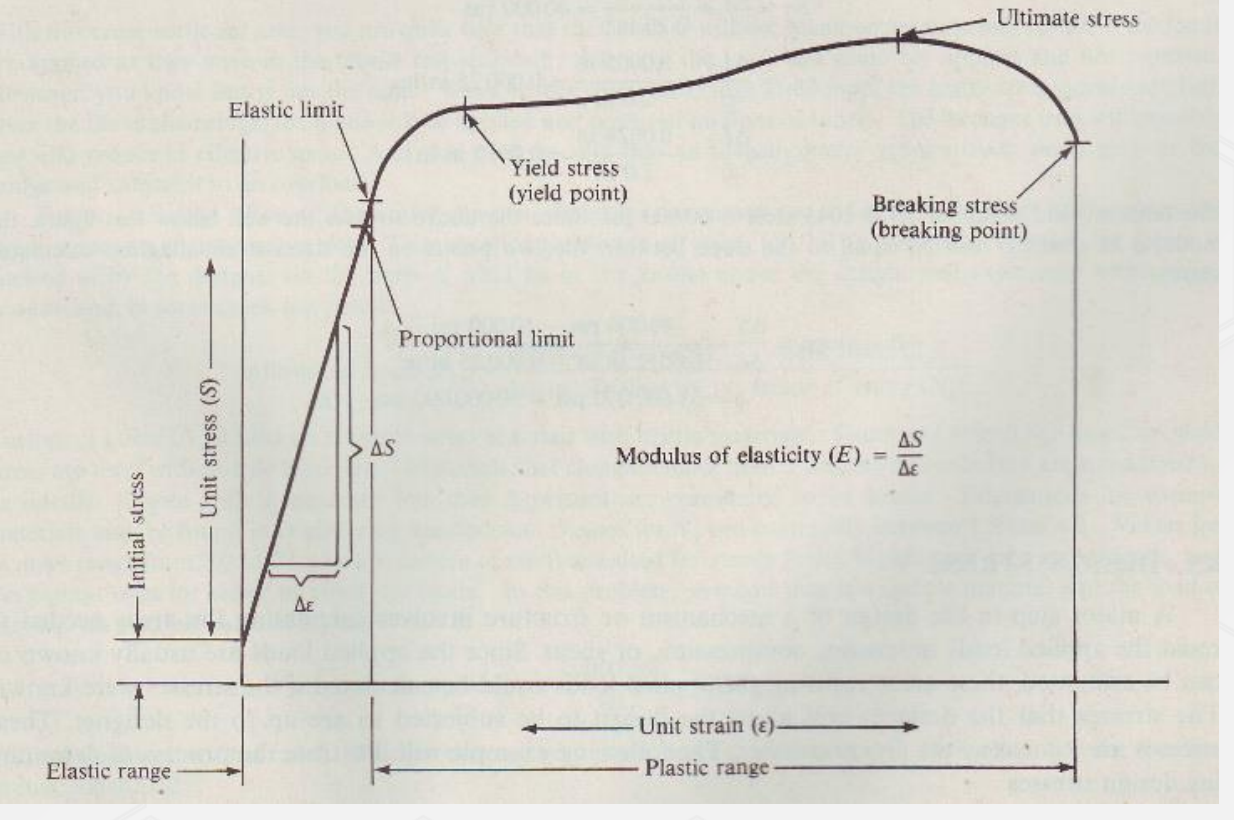
1.7 ELASTIC STRESS-STRAIN RELATIONSHIP- Tensile Test

- The most commonly used test to determine the strength of materials is Standard Tension Test.
- Standard tension test specimen devised by American Society for Testing and Materials (ASTM) is shown in below figure.



Standard test specimen showing alternative end configurations

- The sample is subjected to increasing tensile load and the changes in length are noted. Average unit stresses and strains are calculated and a stress-strain curve is commonly plotted.
- The stress-strain diagram in below figure has a shape typical of steel. Significant points on the curve are indicated.



- According to graph stress-strain in the tensile test, the results are obtained by:-

➢ *original length* - *Lo*

➢ *original diameter* - *do*

➢ *original cross-sectional area* - *Ao*

➢ *length of the end* - *Lf*

➢ *the diameter of the end* - *df*

➢ *final cross-sectional area* - *Af*

➢ *load at yield point* - *Py*

➢ *maximum load* - *Pm*

- From the tensile results , we calculate:-

Ultimate stress	$\sigma_m = \frac{P_m}{A_o}$
Yield stress	$\sigma_y = \frac{P_y}{A_o}$
Percentage of elongation	$L = \frac{L_f - L_o}{L_o}$ x 100 %
Percentage reduction in area	$A = \frac{A_f - A_o}{A_o}$ x 100 %

1.8 FACTOR OF SAFETY

- **Factor of safety** is defined as the ratio of maximum stress with work stress or the maximum load with the workload.

F.S = $\frac{maximum\ stress}{work\ stress}$ or F.S = $\frac{maximum\ load}{work\ load}$

- If the **yield load** used, the factor of safety is defined as the ratio of yield stress with work stress or yield load with the workload.

F.S = $\frac{yield\ stress}{work\ stress}$ or F.S = $\frac{yield\ load}{work\ load}$

1.9 PROOF STRESS

❖Work stress is defined as the ratio of proof stress with a factor of safety.

❖So, to determine the proof stress:-

Proof Stress = Work stress x. F.S

1.10 STRAIN ENERGY

❖The **work that is done is absorbed by the component as stored** is known as strain energy.

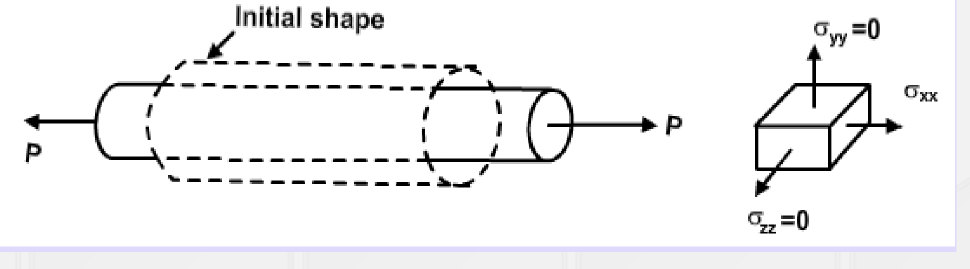
❖Strain energy, U defined as;

❖$U = ½ (P\Delta L)$ knowing that, $\Delta L = PL/AE$

❖So,

$$U = \frac{P^2L}{2AE}$$

1.11 POISON'S RATIO



❖ Consider a rod under an axial tensile load P as shown in above figure such that the material is within the elastic limit.

❖ The normal stress on x plane is $\sigma xx = P/A$ and the associated longitudinal strain in the x direction can be found out from $\varepsilon xx = \sigma xx /E$

❖Hence, despite the absence of normal stresses in *y* and *z* directions, strains do exist in those directions and they are called lateral strains, εy **(change in diameter).**

❖The ratio between the lateral strain and the axial/longitudinal, εx **(change in length)** strain for a given material is always a constant within the elastic limit and this constant is referred to as *Poisson's ratio*. It is denoted by υ.

$$\upsilon = \frac{\textit{Lateral strain}}{\textit{Longitudinal strain}}$$

$$\upsilon = \frac{\varepsilon_y}{\varepsilon_x}$$

Sample Question

A copper wire measuring 4 m long carrying a load of 100 kN. If the stress applied is 60 MN/m^2 and given E_{copper} = 112 GN/m^2, calculate:

i. The strain in the copper

ii. The elongation of copper.

iii. The diameter of copper.

Solution:

The strain;

$$\varepsilon = \frac{\sigma}{E}$$

$$\varepsilon = \frac{60x10^6}{112x10^9}$$

$$= 5.34\text{x}10^{-4}$$

ii. **The elongation:**

$$\Delta L = \varepsilon \, \text{L}$$

$$= 5.34\text{x}10^{-4}(4)$$

$$= 2.143\text{x}10^{-3}$$

iii. **The diameter:**

$$A = \frac{F}{\sigma}$$

$$A = \frac{100x10^3\, N}{60x10^6\, N/m^2}$$

$$A = \frac{\pi d^2}{4}$$

$$d = \sqrt{\frac{4A}{\pi}}$$

$$d = \sqrt{\frac{4(1.667\ x\ 10^{-3})}{\pi}}$$

$$d = 0.046\text{ m}$$

TUTORIAL TIME!

1. A rod with a diameter of 35mm is subjected to compressive load of 150kN. This load causes a reduction in length of 0.17×10^{-3}m. The original length of the rod is 200mm. Determine the Modulus of Elasticity of this material. (Answer: 183.419 GN/m^2)

2. A steel bar has length of 420mm and diameter of 45mm was loaded with tensile load 65kN and the elongation of the bar is 0.032mm and the diameter reduction of 0.01mm. From the test, determine;

a. Stress in the bar. (Answer: 40.869 MN/m^2)

b. Strain relatives to x-axis. (Answer: 7.619×10^{-5})

c. Strain relatives to y-axis. (Answer: 2.222×10^{-4})

d. Poisson ratio. (Answer: 2.917)

e. Safety factor if the ultimate stress was 200MN/m^2. (Answer: 4.894)

3. A circular sample from 1030 carbon steel with a diameter of 0.500 inches pulled to failure using a tensile testing machine. Diameter sample fracture surfaces are 0.343 inches. Determine the percentage of reduction area for the sample. (Answer: 53 %)

4. A 30mm diameter and 80mm long bar us subjected to a tensile force of 20kN. The elongation of bar is 0.0585mm and the diameter of the bar changed to 29.994mm. if the maximum stress of the bar is 230MN/m^2, determine;

a. The stress. (Answer: 28.30 MN/m^2)

b. Tensile strain. (Answer: 731.25×10^{-6})

c. Modulus Young. (Answer: 38.70 GN/m^2)

d. Lateral strain. (Answer: 200×10^{-6})

e. Poisson ratio. (Answer: 0.27)

f. Percentage of elongation. (Answer: 0.073 %)

g. Percentage of reduction area. (Answer: 0.04 %)

h. Strain energy. (Answer: 0.585J)

i. Safety factor. (Answer: 8.127)

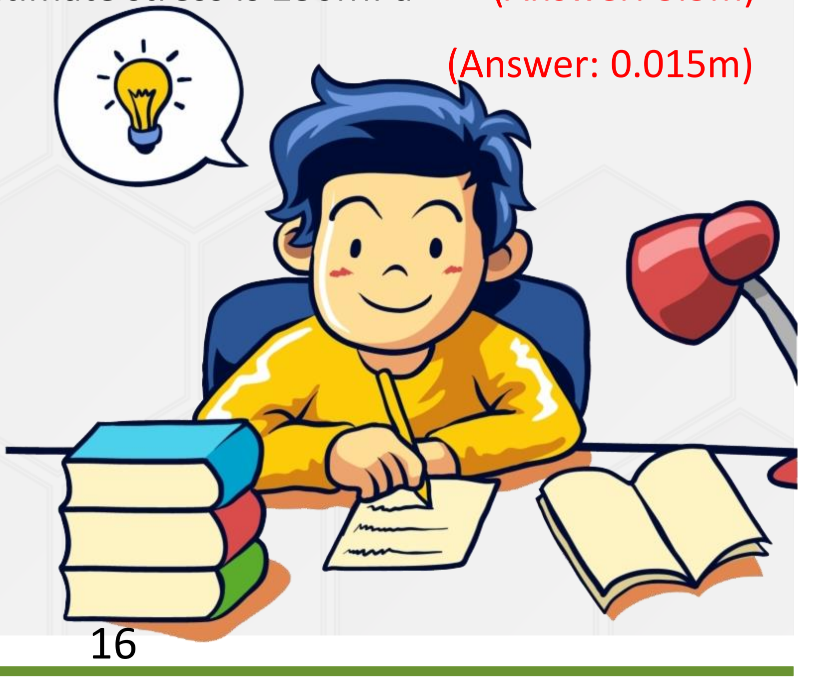
5. A copper wire of 4 meters in length is applied with a force of 10kN. If the stress in the wire is 60MPa. Given E_{copper} = 112GN/m^2. Calculate;

a. The strain in the wire. (Answer: 5.36×10^{-4})

b. The elongation of the wire. (Answer: 2.14×10^{-3}m)

c. The factor of safety, if the ultimate stress is 230MPa (Answer: 3.5m)

d. The diameter of the wire. (Answer: 0.015m)



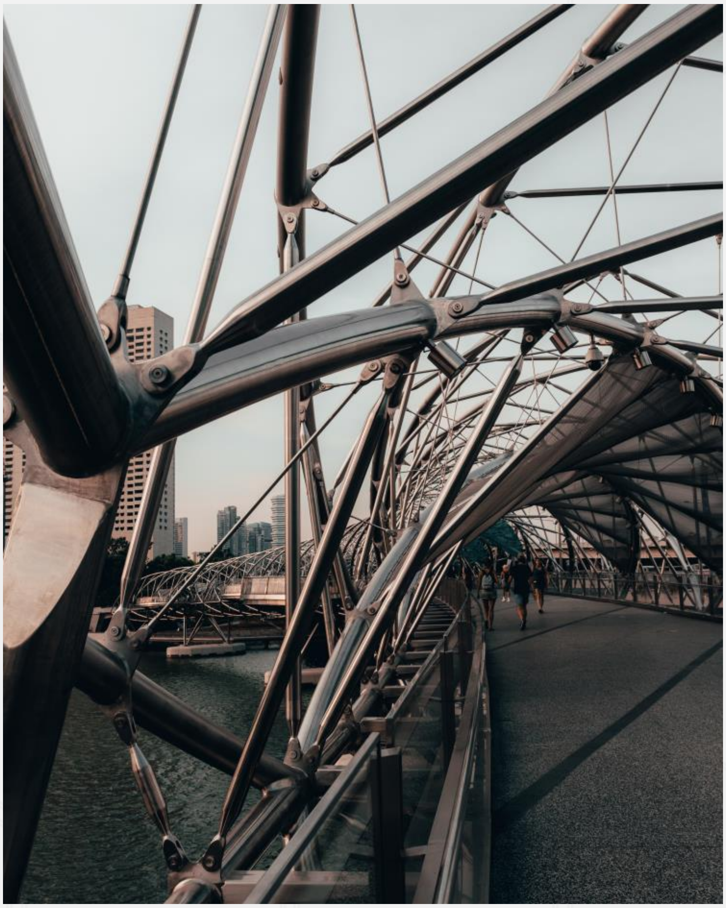
TOPIC 2

2. THERMAL STRESSES ON COMPOSITE BARS

2.1 Interpret thermal stresses on composite bars
2.2 Distinguish the following composite bars

2.1 . Introduction to Composite Bars

- **Compound/composite** bar is a bar **containing two or more rods, pipes or tubes connected by a rigid support either in parallel or series**.
- Bar rods may be of **different material types or different sizes**.
- It is permanently or temporarily connected to all the bars together expands or contracts when experiencing the action of external forces or the impact of temperature action.



2.2. Coefficient of Linear Expansion (Linear)

- Is a material change in length per unit length when the temperature changes of 1 degree (1^o). Unit expansion coefficient is the per ^oC (C^{-1})or per ^oK (K^{-1})
- Let say:-

ΔL = amount of expansion

Δt = temperatures changes (increase or reduces)

L = the original length of the rod

- So that;

$$\Delta L = \alpha L \Delta t$$

- For steel, $\alpha = 11.5 \times 10^{-6}$ per ^oC. That means for a 1 m long steel bars it will experienced an expansion of 5.11 x 10-6 m for a temperature rise 1 °C.

LIST OF THERMAL EXPANSION COEFFICIENTS (CTE) FOR COMMON MATERIALS

Materials	Linear Temperature Expansion Coefficient α (10^{-6} m/(m °C))
Aluminum	21 – 24
Brass	18 – 19
Bronze	17.5 – 18
Concrete	13 – 14
Copper	16 – 16.7
Glass, hard	5.9
Glass, plate	9
Glass, Pyrex	4
Gold	14.2
Iron, cast	10.4 – 11
Magnesium	25 – 26.9
Nickel	13

Materials	Linear Temperature Expansion Coefficient α $(10^{-6}\ m/(m\ ^{\circ}C))$
Plastics	40 – 120
Platinum	9
Polypropylene (PP), unfilled	72 – 90
Polystyrene (PS)	70
Rubber, hard	80
Silver	19 – 19.7
Steel	10.8 – 12.5
Titanium	8.5 – 9
Tungsten	4.5
Wood, pine	5
Zinc	30 – 35

2.3. Temperature Stresses in Compound Bars

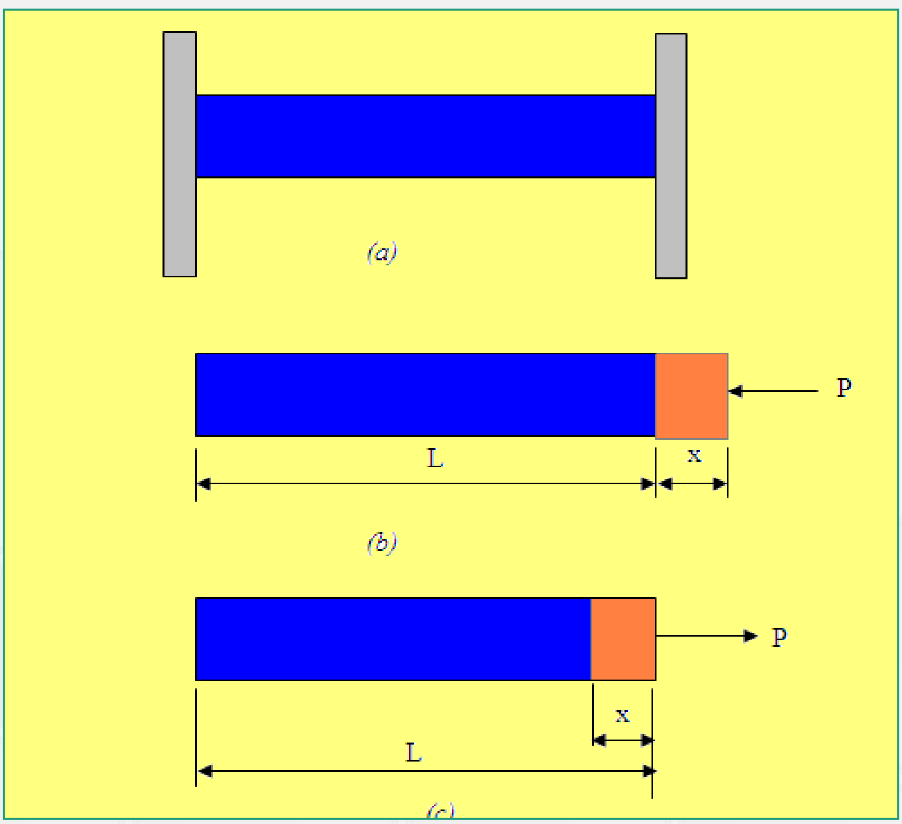


Figure 2.1.Temperature Stress

- In Figure 2.1 (a) a rod with length L, rigidly mounted on two walls. When the temperature is raised, the rod will expand as in Figure 2.1 (b) and Figure 21 (c), but the expansion did not occur due to blocked by the wall.
- Therefore, it is obvious that the bar is experiencing compressive stress. To determine the value of the stress, assuming that the bar was free and allowed to expand.
- The expansion occurs is:-

$$x = \alpha L \Delta t$$

And new length, $L' = L + \alpha L \Delta t$

$$= L (1 + \alpha \Delta t)$$

- We could assume that the expansion of $\alpha L \Delta t$ is the result of compressive stresses and it need for return to the original length bar.
- So in effect the compression strain is:

$$\varepsilon = x / L' = \frac{\alpha L \Delta t}{L(1 + \alpha \Delta t)}$$

- As often Δt is small and α for most metals is very small, so that the product of $\alpha L \Delta t$ is very small compared with L until $L + \alpha L \Delta t \approx L$.
- So that means, the **temperature strain**, $\varepsilon = \alpha \Delta t$.

- However, the stress = young's modulus x strain.
- So that, Temperatures stresses= $E \alpha \Delta t$
- Conversely, if the temperature is lowered the tensile stress was obtained.

Expansion

$$\Delta L = \alpha L \Delta t$$

Temperatures strain

$$\varepsilon = \alpha \Delta t$$

Temperatures stress

$$\sigma = E \alpha \Delta t$$

Sample Question

1. The temperature of a 1.4 meter long rod was raised from 30^0C to 70^0C. Calculate the expansion of this rod.

Given: E= 200GPa and $\alpha = 12 \times 10^{-6}/^0C$

Solution:-

the expansion , $\Delta L = \alpha L \Delta t$, $\Delta t = 70^0C - 30^0C = 40$

$\Delta L = (12 \times 10^{-6})(1.4)(40)$

$= 6.72 \times 10^{-4}$m

= 0.672mm

2. Determine the coefficient of linear expansion of 1.2 meter long steel when the temperature raised at 30^0C and expand to 1.2888 meter.

Given: E= 200GPa

Solution:-

the expansion , $\Delta L = \alpha L \Delta t$, $\Delta t = 30^0C$, $\Delta L = 1.2088 - 1.2 = 8.8 \times 10^{-3}$

$\Delta L = \alpha L \Delta t$

$$\alpha = \frac{\Delta L}{L \Delta t}$$

$$= \frac{8.8 X 10^{-3}}{1.2 (30)}$$

$$= 2.444 X 10^{-4} /^0C$$

3. A steel bar is subjected to temperature rise at 80 ^{0}C. Determine the stress in the steel bar.

Given: E= 200GPa and $\alpha = 11.7 X 10^{-6} /^0C$

Solution:-

the stress , $\sigma = \alpha L \Delta t$, $\Delta t = 80^0C$

$\sigma = E\alpha \Delta t$

$$= (200 \times 10^9)(11.7 X 10^{-6})(80)$$

$$= 187.2 \times 10^6 N/m^2$$

$$= 187.2 MPa$$

2.4. Series Composite Bars (Subjected To External Load)

- Figure 2.2 below shows a bar connected in series. Bars of this type has a different size of cross section.
- The forces acting on each bar is the same but the extension or shorten of each bar is different.

Figure 2.2. Series Composite Bars

- Let say that, the forces on the bar 1 is P1 and the forces on the bar 2 is P2, so that: -

Force;

$$P_1 = P_2$$

$$\sigma_1 A_1 = \sigma_2 A_2$$

$$\sigma_1 = \sigma_2 \frac{A_1}{A_2}$$

Deformation;

$$\Delta L = \Delta L_1 + \Delta L_2$$

$$\Delta L = \left[\frac{P_1 L_1}{A_1 E_1}\right] + \left[\frac{P_2 L_2}{A_2 E_2}\right]$$

For equal length; $\Delta L = \left[\frac{\sigma_1}{E_1}\right] + \left[\frac{\sigma_2}{E_2}\right]$

For unequal length; $\Delta L = \left[\frac{\sigma_1 L_1}{E_1}\right] + \left[\frac{\sigma_2 L_2}{E_2}\right]$

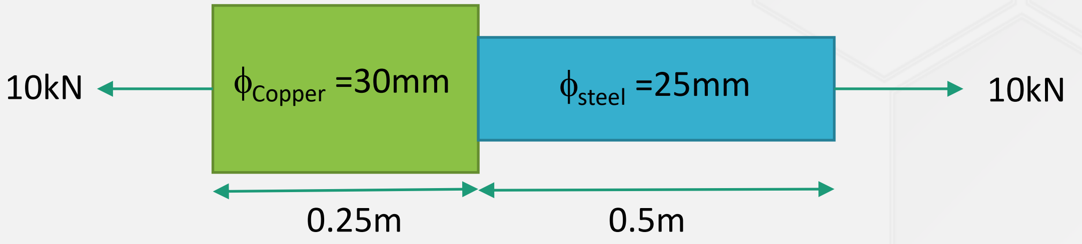
Sample Question

A composite series bar is made by copper and steel with diameter of 30mm and length of 0.5m respectively as shown in Figure 2.3 below. If end of both bar are subjected to axial tensile load of 10kN, determine;

i. Elongation for each bar

ii. Stress occur in both copper and steel bar

Given : E_{steel} = 200GN/m^2 and E_{copper} = 100GN/m^2



Since tensile load is similar for both copper and steel bar, so it shared the same load of 10kN. It is known the series composite bar has;

Solution:-

$$P_{copper} = P_{steel} \quad \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_(1)$$

$$\Delta L = \Delta L_{copper} + \Delta L_{steel} \quad \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_(2)$$

$$A_{copper} = \frac{\pi D^2}{4}$$

$$= \frac{\pi(30\ X\ 10^{-3})^2}{4}$$

$$= 7.069\ X\ 10^{-4} m^2$$

$$A_{steel} = \frac{\pi D^2}{4}$$

$$= \frac{\pi(25\ X\ 10^{-3})^2}{4}$$

$$= 4.909\ X\ 10^{-4} m^2$$

Elongation in copper bar

$$\Delta L = \left[\frac{P_c L_c}{A_c E_c}\right]$$

$$= \frac{(10\ X\ 10^3)(0.25)}{(7.069\ X\ 10^{-4})(100\ X\ 10^9)}$$

$$= 3.537\ X\ 10^{-5} m$$

Elongation in steel bar

$$\Delta L = \left[\frac{P_s L_s}{A_s E_s}\right]$$

$$= \frac{(10\ X\ 10^3)(0.5)}{(4.909\ X\ 10^{-4})(200\ X\ 10^9)}$$

$$= 5.092\ X\ 10^{-5} m$$

From Equation 1;

$$P_{copper} = P_{steel}$$

$$(\sigma A)_c = (\sigma A)_s$$

$$(\sigma_c)(7.069\ X\ 10^{-4}) = (\sigma_s)(4.909\ X\ 10^{-4})$$

$$\sigma_c = \frac{4.909\ X\ 10^{-4}}{7.069\ X\ 10^{-4}}$$

$$\sigma_c = 0.694\ \sigma_s \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_}(3)$$

From Equation 2;

$$\Delta L = \Delta L_{copper} + \Delta L_{steel}$$

$$= 3.537\, X\, 10^{-5} + 5.092 \text{ X } 10^{-5}$$

$$= 8.629 \text{ X } 10^{-5}\, m$$

$$\Delta L = \left[\frac{P_c L_c}{A_c E_c}\right] + \left[\frac{P_s L_s}{A_s E_s}\right]$$

$$= \left[\frac{\sigma L_c}{E_c}\right] + \left[\frac{\sigma_s L_s}{E_s}\right] \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_}(4)$$

Then, substitute Equation (3) into (4)

$$8.629 \text{ X } 10^{-5} = \left(\frac{(0.694\, \sigma_s)(0.25)}{100 \text{ X } 10^9}\right) + \left(\frac{(\sigma_s)(0.5)}{200 \text{ X } 10^9}\right)$$

$$8.629 \text{ X } 10^{-5} = 4.235 \text{ X } 10^{-12}\, \sigma_s$$

$$\sigma_s = 20.375 \text{ X } 10^6 \text{N/m}^2$$

$$= 20.375 \text{ MN/m}^2$$

Substitute $\sigma_s = 20.375 \text{ X } 10^6 \text{N/m}^2$ into equation (3)

$$\sigma_c = 0.694\, \sigma_s$$

$$= 0.694\, (20.375 \text{ X } 10^6)$$

$$= 14.140 \text{ X } 10^6 \text{N/m}^2$$

$$= 14.140 \text{ MN/m}^2$$

2.5. Parallel Composite Bars (Subjected To External Load)

- Figure 2.3 below shows the bars in parallel connection. Extension or shorten of the both bars is the same.
- The total load carried by each bars is same with the external load acting, even if forces acting on each bars is different each other.

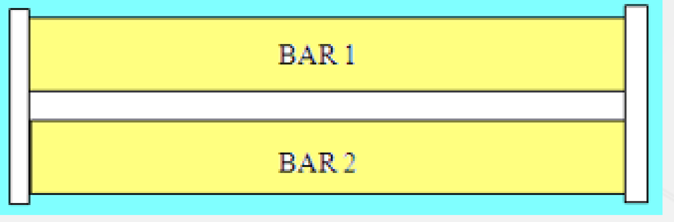


Figure 2.3. Parallel Composite Bars

- Let say that, the forces on the bar 1 is P1 and the forces on the bar 2 is P2, so that: -

Force;

$$P = P_1 + P_2$$

$$P = \sigma_1 A_1 + \sigma_2 A_2$$

Deformation; $\Delta L = \Delta L_1 = \Delta L_2$

$$\Delta L = \left[\frac{\sigma_1}{E_1}\right] = \left[\frac{\sigma_2}{E_2}\right]$$

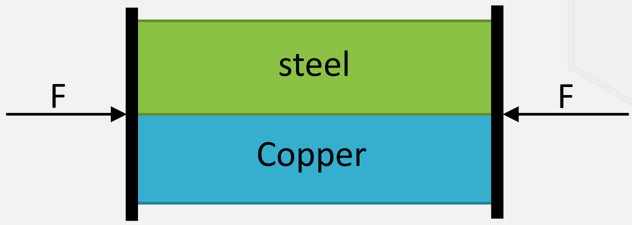
$$\left[\frac{\sigma_1}{E_1}\right] = \left[\frac{\sigma_2}{E_2}\right]$$

$$\sigma_1 = \left[\frac{\sigma_2}{E_2}\right] E_1$$

Sample Question

A composite parallel bar is made by copper and steel with cross sectional area is 600 mm2 for steel and 1000 mm2 for the copper. Both ends are rigidly mounted. Calculate the stress in each bar if 50kN load is applied.

Given : $E_{steel} = 200GN/m^2$ and $E_{copper} = 100GN/m^2$



Solution:-

$F_{copper} + F_{steel} = 50kN$

$\sigma_c A_c + \sigma_{st} A_{st} = 50$ ______________________ (1)

Knowing that,

$\Delta L = \Delta L_c = \Delta L_{st}$

$Thus, \quad \varepsilon_c = \varepsilon_{st}$

$$\frac{\sigma_c}{E_c} = \frac{\sigma_{st}}{E_{st}}$$

$\sigma_c = \dfrac{E_c \sigma_{st}}{E_{st}}$ ______________________________ (2)

Substitute equation (2) into (1);

$$\left(\frac{E_c \sigma_{st}}{E_{st}}\right) A_c + \sigma_{st} A_{st} = 50$$

$$\sigma_{st}[(\frac{E_c}{E_{st}}A_c) + (A_{st})\,] = 50(10^3)$$

$$\sigma_{st}[(\frac{100(\,10^9\,)}{200(\,10^9\,)}\ 1000(10^{-6})) + (600(10^{-6}))] = 50(10^3)$$

$\sigma_{st} = 79.923 \text{ X } 10^6 \text{ N/m}^2$

$= 79.923 \text{ MN/m}^2$

Then, Stress for copper;

$$\sigma_c = \frac{E_c\sigma_{st}}{E_{st}}$$

$$= \frac{100(\,10^9\,)}{200(\,10^9\,)}(79.923 \text{ X } 10^6\,)$$

$= 39.961 \text{ X } 10^6 \text{ N/m}^2$

$= 39.961 \text{ MN/m}^2$

2.6.Series Composite Bars (Subjected To Temperature Changes)

Δ L (F) = Δ L (T)

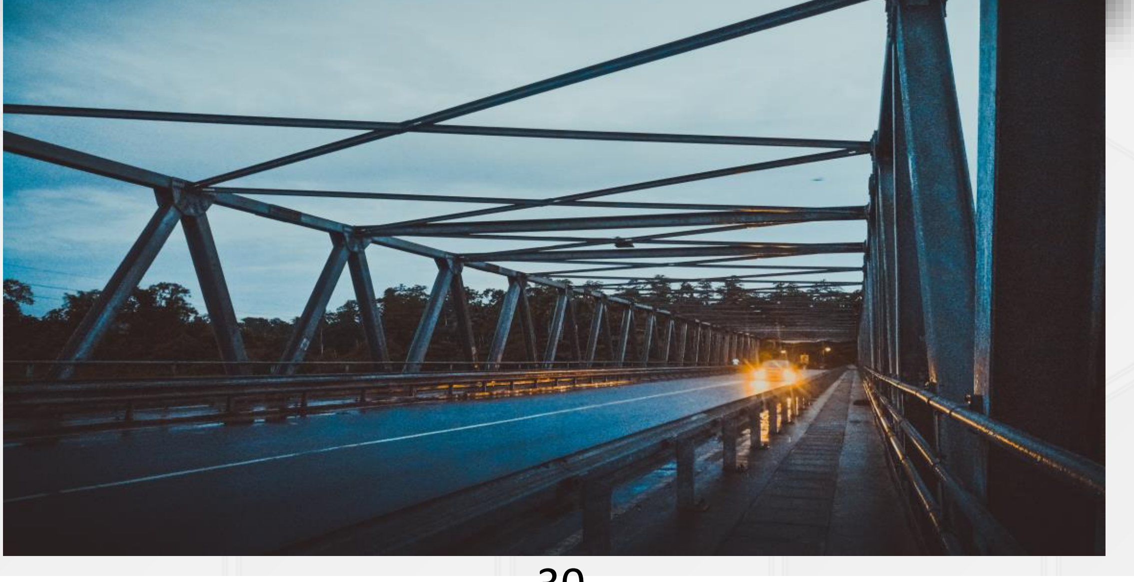
$$\left(\frac{FL}{AE}\right)_A + \left(\frac{FL}{AE}\right)_B = (\alpha\, L\, \Delta T\,)_A + (\alpha\, L\, \Delta T\,)_B$$

Stress in material 1:-

$$\sigma_1 = \frac{\Delta t(\alpha_1 L_1 + \alpha_2 L_2)}{A_1\left(\frac{L_1}{A_1E_1} + \frac{L_2}{A_2E_2}\right)}$$

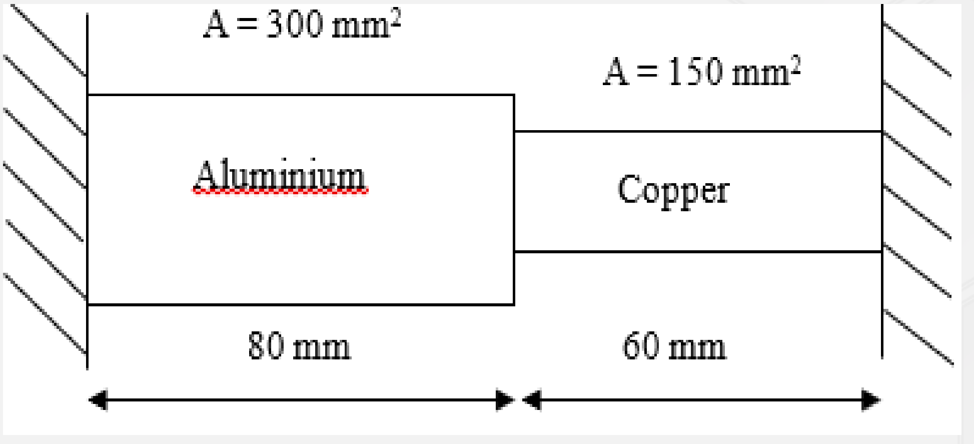
Stress in materials 2:-

$$\sigma_2 = \frac{\Delta t(\alpha_1 L_1 + \alpha_2 L_2)}{A_2\left(\frac{L_1}{A_1E_1} + \frac{L_2}{A_2E_2}\right)}$$



Sample Question

A series of bar consist of copper and aluminium bar which is fixed in between two rigid walls as Figure below. Determine the thermal stress induced in each bar if the temperature is increased by 80°C. Given that $E_{aluminium}$ = 69 GN/m^2 and E_{copper} = 112GN/m^2. $\alpha_{aluminium}$= 23 x 10^{-6} /°C, α_{copper} = 17 x 10^{-6} /°C.



Solution:-

The series composite bar:-

$$P = P_{Al} = P_c$$

$$\sigma_{Al} A_{Al} = \sigma_c A_c$$

$$\sigma_{Al} = \frac{\sigma_c A_c}{A_{Al}}$$

$$\sigma_{Al} = \frac{\sigma_c (150\ x\ 10^{-6})}{300\ x\ 10^{-6}}$$

$$\sigma_{Al} = 0.5\ \sigma_c\ \ldots\ldots\ldots\ldots\ldots (1)$$

Using equation of deformation due to compressive load equation:

$$\left(\frac{PL}{AE}\right)_{Al} + \left(\frac{PL}{AE}\right)_{c} = \Delta T\ [(\alpha L)_{Al} + (\alpha L)_{c}\ ]$$

$$\left(\frac{\sigma L}{E}\right)_{Al} + \left(\frac{\sigma L}{E}\right)_{c} = 80\ [(23 \text{ x } 10^{-6}\ x\ 0.08)_{Al} + (17 \text{ x } 10^{-6}\ x\ 0.06)_{c}]$$

………(2)

Substitute equation (1) into equation (2):

$$\left(\frac{0.5\sigma_c 0.08}{69\ x\ 10^9}\right) + \left(\frac{\sigma_c 0.06}{112\ x\ 10^9}\right) = 80\ [(1.84 \text{ x } 10^{-6} + (1.02 \text{ x } 10^{-6}\ )]$$

$$(5.797\ x\ 10^{-13}\ \sigma_c) + (5.36\ x\ 10^{-13}\ \sigma_c) = 2.288\ x\ 10^{-4}$$

$$1.116\ x\ 10^{12}\sigma_c = 2.288\ x\ 10^{-4}$$

$$\sigma_c = \frac{2.288\ x\ 10^{-4}}{1.116\ x\ 10^{-12}}$$

$$\sigma_c = 205.018\ x\ 10^6\ N/m^2$$

Substitute $\sigma_c = 205.018\ x\ 10^6\ N/m^2$ into equation (1):

$$\sigma_{Al} = 0.5\ \sigma_c$$

$$\sigma_{Al} = 0.5\ (205.018 x\ 10^6)$$

$$\sigma_{Al} = 102.509\ x\ 10^6\ N/m^2$$

2.7. Parallel Composite Bars (Subjected To Temperature Changes)

$$\Delta L\,(F) = \Delta L\,(T)$$

$$\frac{\sigma_A}{E_A} + \frac{\sigma_B}{E_B} = \Delta T\,(\alpha_B - \alpha_A) \quad where \quad \alpha_B > \alpha_A$$

Stress in material 1:-

$$\sigma_1 = \frac{\Delta t(\alpha_2 - \alpha_1)}{A_1\left(\frac{1}{A_1E_1} + \frac{1}{A_2E_2}\right)}$$

Stress in materials 2:-

$$\sigma_2 = \frac{\Delta t(\alpha_2 - \alpha_1)}{A_2\left(\frac{1}{A_1E_1} + \frac{1}{A_2E_2}\right)}$$

Sample Question

A compound bar installed in parallel is made from two metals, which are from steel and copper. The cross-sectional area for the steel is 600 mm^2 and the copper is 1000 mm^2. The bar is attached rigidly to its support. The bar temperature is increased through 100°C when the load is applied. Calculate the stress for both types of bar.

Given : $E_{STEEL} = 200$ GN/m^2 $\qquad \alpha_{STEEL} = 12 \times 10^{-6}$/°C

$E_{COPPER} = 107$ GN/m^2 $\qquad \alpha_{COPPER} = 16.5 \times 10^{-6}$/°C

Solution:-

$\sigma_1 = \sigma_{steel}$ and $\sigma_2 = \sigma_{copper}$

$$\sigma_1 = \frac{P}{A_1} = \left[\frac{\Delta t \left(\alpha_2 - \alpha_1 \right)}{A_1 \left(\frac{L_1}{A_1 E_1} + \frac{L_2}{A_2 E_2} \right)} \right]$$

$$= \frac{100 \ [(16.5 \times 10^{-6}) - (12 \times 10^{-6})]}{600 \times 10^{-6} \left\{ \frac{1}{(600 \times 10^{-6} \times 200 \times 10^{9})} + \frac{1}{(600 \times 10\text{-}6 \times 200 \times 109)} \right\}}$$

$$= \frac{4.5 \times 10^{-4}}{600 \times 10^{-6} \ (1.7679 \times 10^{-8})}$$

$\sigma_{STEEL} = 42.423 \times 10^6$ N/m^2

$$\sigma_2 = \frac{P}{A_2} = \left[\frac{\Delta t\left(\alpha_2 - \alpha_1\right)}{A_1\left(\frac{L_1}{A_1E_1} + \frac{L_2}{A_2E_2}\right)} \right]$$

$$= \frac{100 \;\; [(16.5 \times 10^{-6}) - (12 \times 10^{-6})]}{1000 \times 10^{-6} \left\{ \frac{1}{(600 \times 10^{-6} \times 200 \times 10^{9})} + \frac{1}{(600 \times 10\text{-}6 \times 200 \times 10^{9})} \right\}}$$

$$= \frac{4.5 \times 10^{-4}}{1000 \times 10^{-6} (1.7679 \times 10^{-8})}$$

$$\sigma_{COPPER} = 25.45 \times 10^{6} \text{ N/m}^2$$

TUTORIAL TIME!

1. A 1.2m length copper bar with diameter of 14mm is cooled from 800C to 200C. Determine the force needed to hold the elongation.

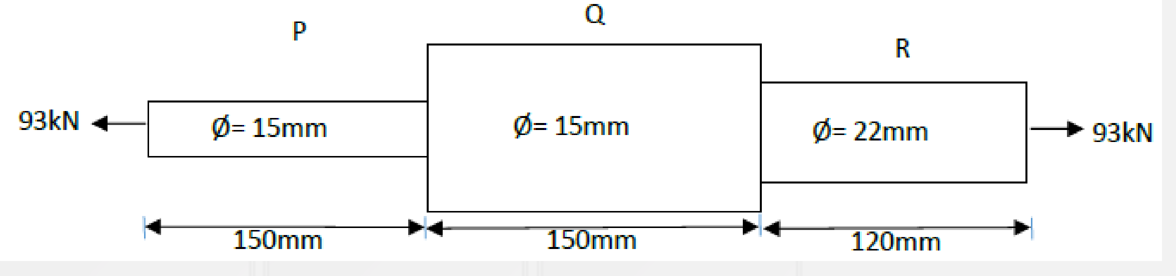
Given: E = 107 GN/m2, α= 18 x 10 -6/0C

(answer: σ = 115.56MN/m2, P=17.78kN)

2. A copper rod has 250mm length, thick 25mm and width 35mm is cooled from 700C to 230C. Determine the force needed to hold the elongation. Given: E = 103 GN/m^2, α= 17.7 x 10 -6/^{0}C

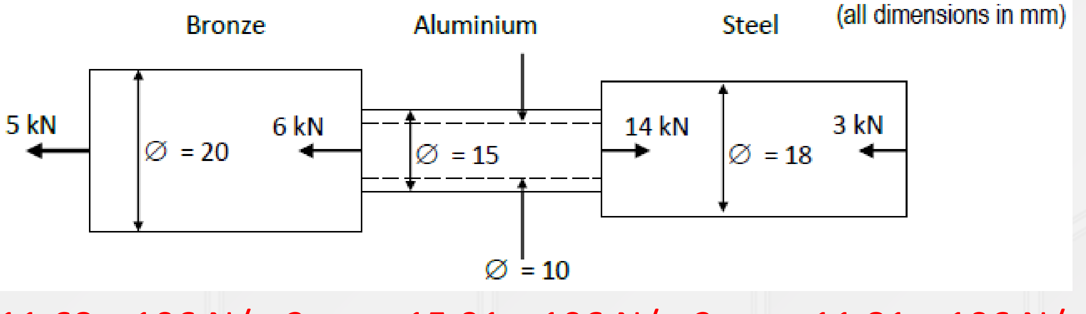
(answer: σ = 85.69MN/m2, P=74.98kN)

3. P and Q is made from steel and R is made from aluminium. Determine the total elongation of the bar. Assume Esteel = 205 GN/m^2 and Ealuminium = 71 GN/m^2



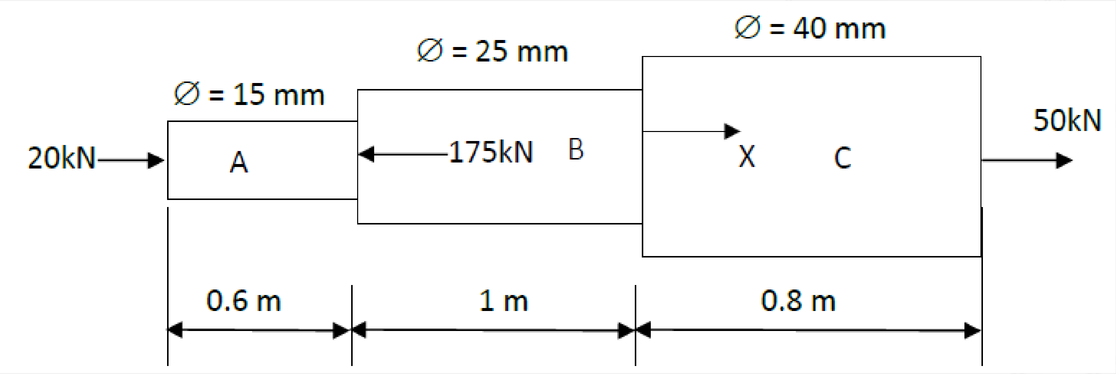
(answer: ΔL = 1.183 x 10-3 m)

4. Determine the stress in each bar below.



(σ_A = 111.68 x 106 N/m2, σ_B = 15.91 x 106 N/m2, σ_S = 11.81 x 106 N/m2)

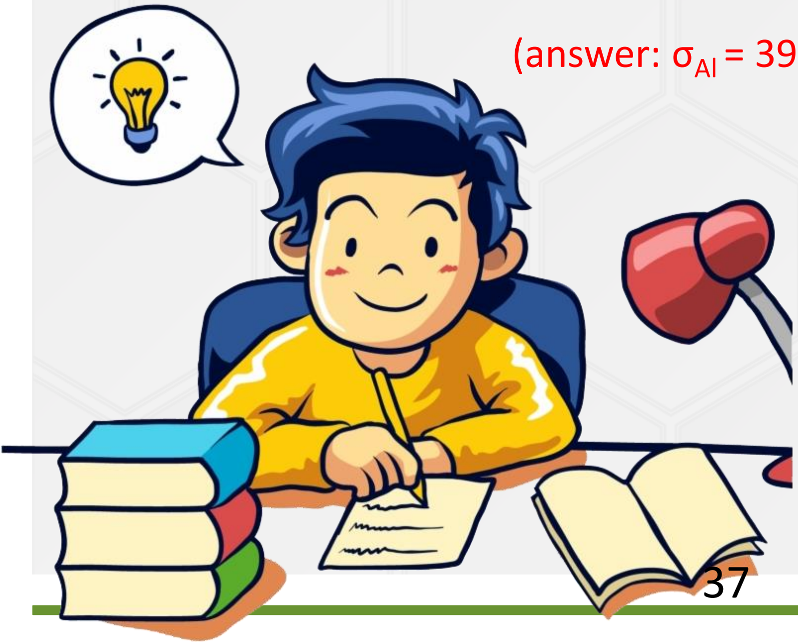
5. An ABC bar of 2.4m length is composed as series and load has been as shown in figure below. Determine the X load and total elongation of the bar. Notice that A and C have the same material of aluminum and B is made of steel. Given: E_{steel} = 200 GN/m^2 and $E_{aluminium}$ = 69 GN/m^2.



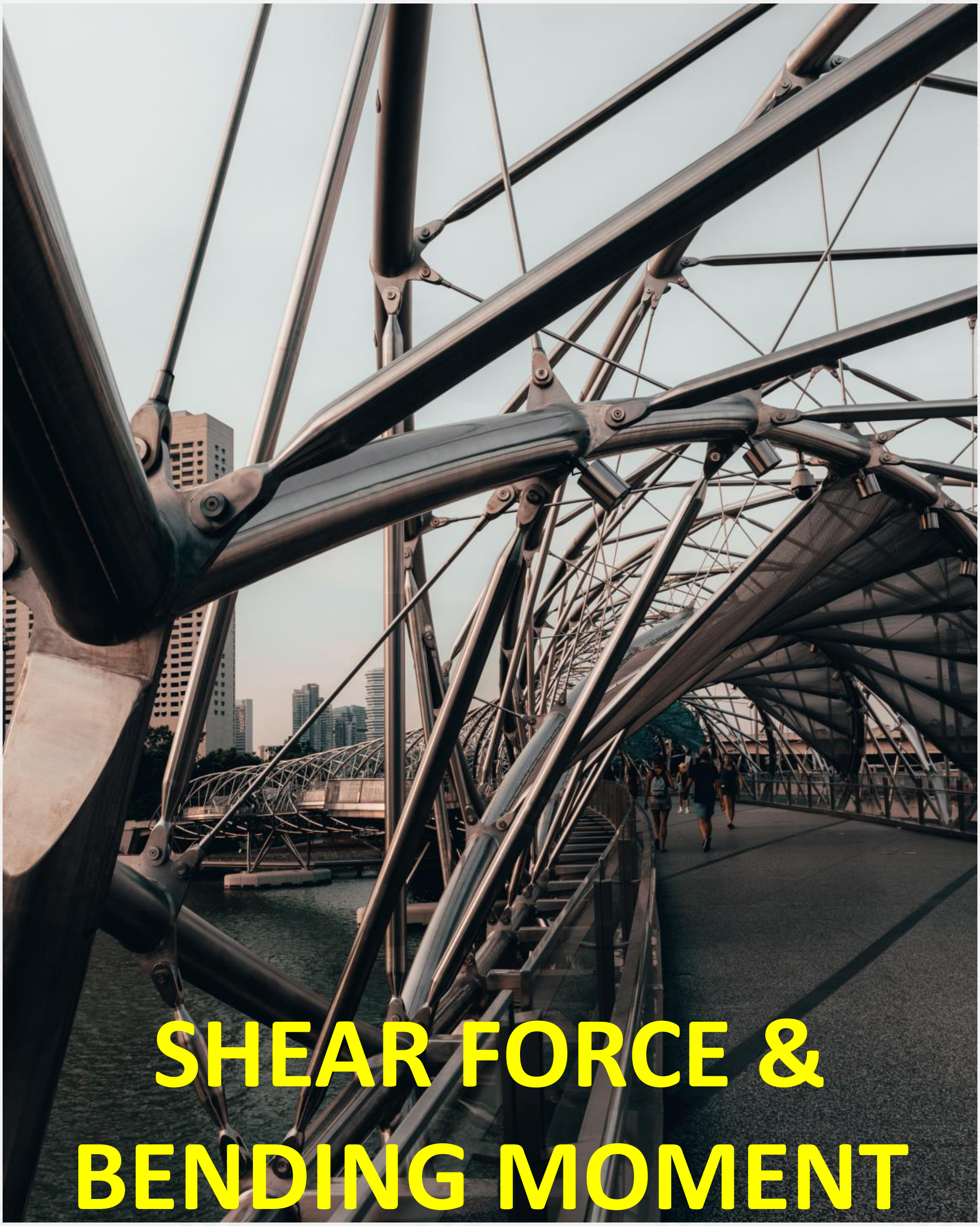
(Answer: ΔL = 1.057 x 10-3 m)

6. A composite aluminium and steel bar is fixed as series in between walls at temperature 95^{0}C. Aluminium diameter is 50mm and steel cross sectional area is 330mm^2. Steel length is 3 times longer than aluminium which is 753mm. If the temperature dropped to 43^{0}C, determine the stress in each bar. Given: E_{steel} = 220 GN/m^2, E_{Al} = 77 GN/m^2, α_{steel} = 12.5x 10^{-6}/^{0}C and α_{Al} = 23.6 x 10^{-6}/^{0}C.

(answer: σ_{Al} = 39.58MN/m^2, σ_{steel} = 235.61MN/m^2)



TOPIC 3



SHEAR FORCE & BENDING MOMENT

OBJECTIVES:

- In the end of this chapter, student should know to:
- Classify the types
- Classify the type of beam load
- Analyse the laws of equilibrium

3.1 DEFINATION OF BEAM:-

- A beam is a bar subject to forces or couples that lie in a plane containing the longitudinal section of the bar. According to determinacy, a beam may be determinate or indeterminate.
- Beams are generally horizontal members which transfer loads horizontally along their length to the supports where the loads are usually resolved into vertical forces.

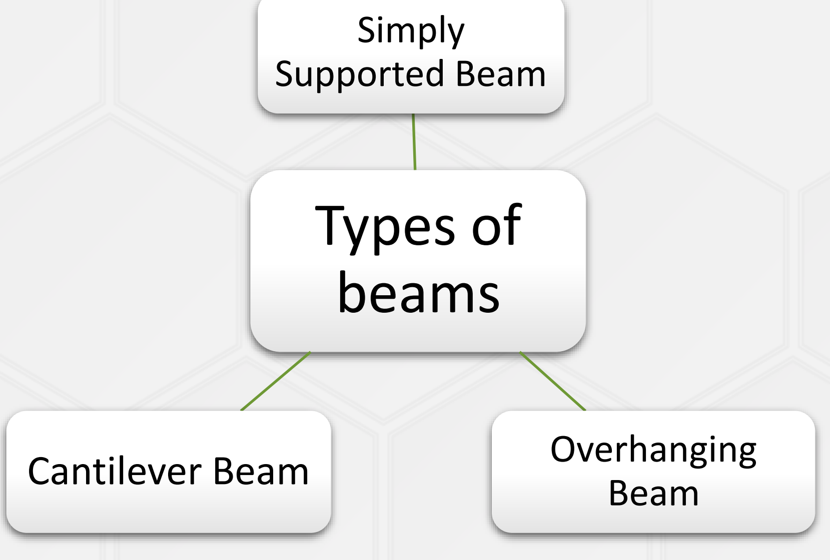
3.2 TYPES OF BEAM:-

- There are two types of beam:-

  a. simply supported beam

  b. cantilever beam

  c. Overhanging Beam

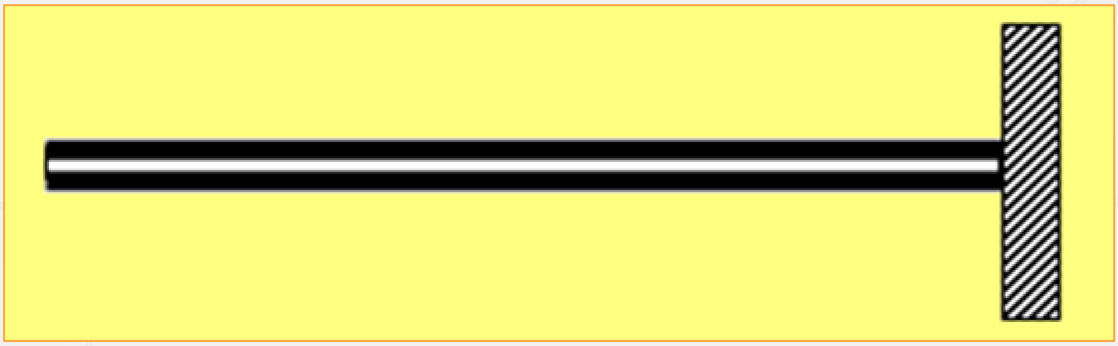


SIMPLY SUPPORTED BEAM:-

- It's pinned at one end and roller at the other

CANTILEVER OF BEAM:-

- It's fixed at one end and the other end free



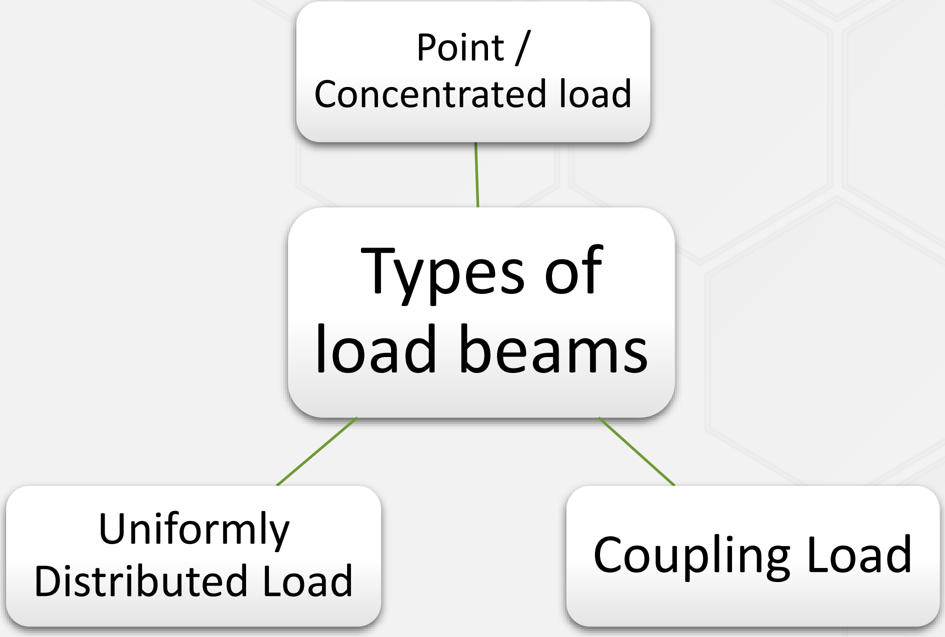
3.3 TYPE OF BEAM LOAD:-

- There are 3 types of beam load:-

  a. point or concentrated load

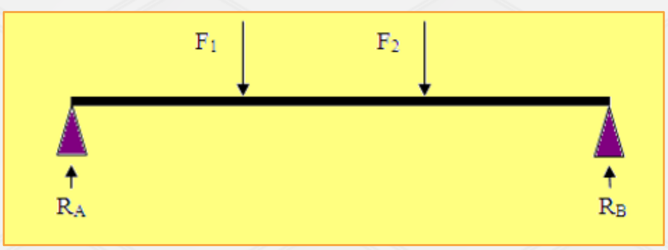
  b. uniformly distributed load

  c. couples



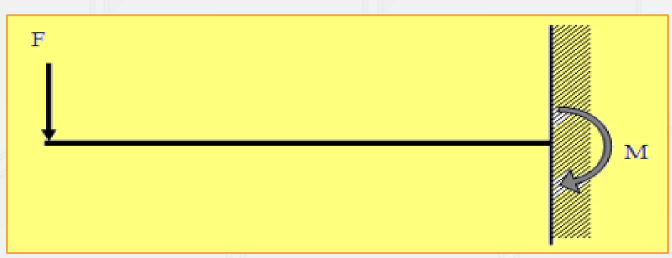
TYPE OF BEAM LOAD:-

- A concentrated load is the one which acts over so small length that it is assumed to act at a point. Practically, a point load can not be places as knife edge contact but for calculation purpose we consider that load is being transmitted at a point.



Concentrated load on simply supported beam

Concentrated load on cantilever beam



UNIFORMLY DISTRIBUTED LOAD:-

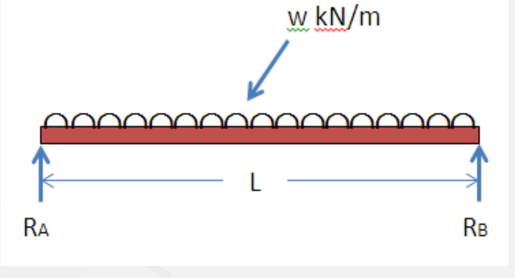
- To determine a single concentrated load from uniformly distributed load:-

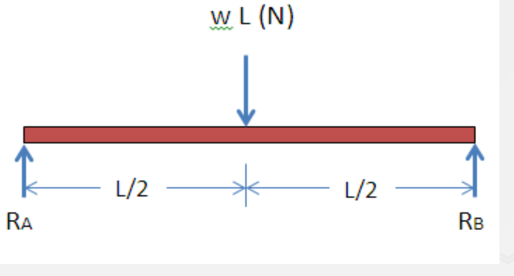
RA + RB = w L

where;

w.L is the single concentrated load in the middle of beam.

- A unit for u.d.l is N/m where in each meter length it carries a 50 kN of load.
- For example: - if w = 50 kN/m, that means for every meter length of the beam, it carries a load of 50 kN. If the length of the beam is 2 meters, the total load is 2 x 50 kN = 100 kN.

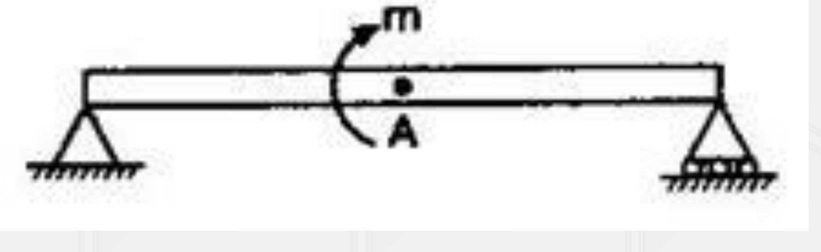




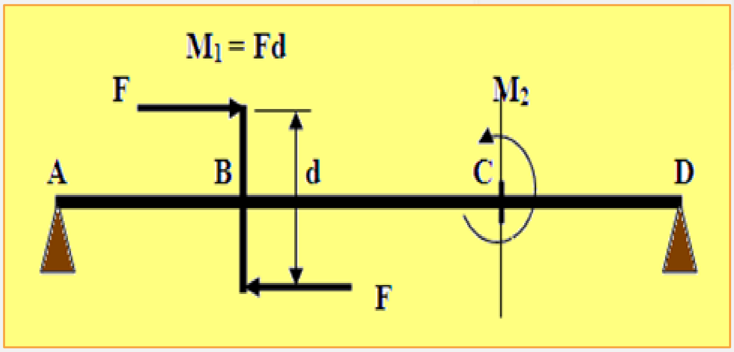
COUPLES

- A beam may also subjected to a couple ‘µ’ at any point. As shown in figure.

Note: In general, the load may be a combination of various types of loadings.



- The figure below shows the coupling beams by two forces F M1 doer in B and a coupling of M2 at C.



Here, the coupling of M1 is Fd Nm follow the clockwise direction and M2 is coupling follow counterclockwise direction

Moment = Force x perpendicular distance
M = F x d (N.m)

DETERMINATION OF REACTIONS AT SUPPORT

- Moment of force acting on a beam about any point must be zero.
- For taking moments, a uniformly distributed load may be treated as a single concentrated load acting at the center of gravity at the middle of the spread.

$$\sum M_X = \sum M_X \quad \text{or} \quad \sum M_X = \sum M_X$$

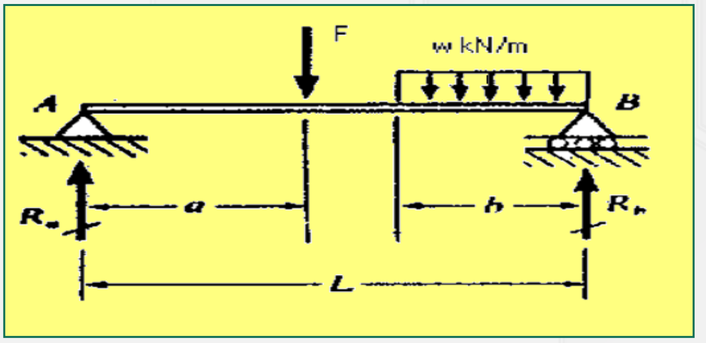
- Therefore:-

+ve $\sum M_X = 0$

or

+ve $\sum M_X = 0$

FORCE EQUILIBRIUM



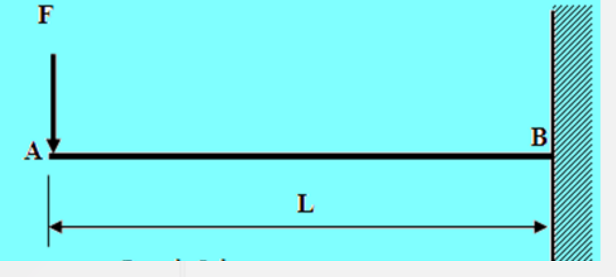
- To determine the total reaction force:-

$$\uparrow \Sigma F = \Sigma F \downarrow$$

$R_A + R_B = F + w(b)$ kN

REACTION AT BUILT IN BEAM

Reactions at B, R_B = F
&
M_B = - FL

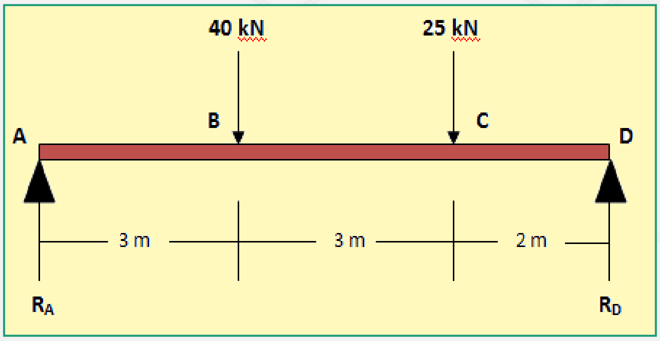


- The reaction at B (wall) is equal by F.
- Moment magnitude at B (wall) = - F.L (Nm)
- Moment unit is (Nm)

Sample Question

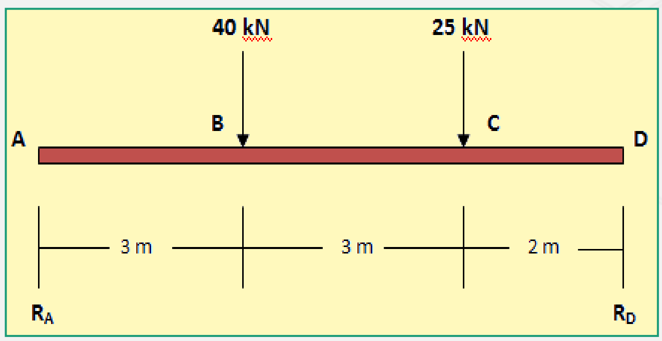
Exercise 1

- A simply supported beam of span 8 m is loaded by a single concentrated load at B and C as in Figure above. Calculate the reaction force at point A and D.



Solution:-

a. Free body diagram



- **Determine reactions:-**

$\uparrow \Sigma F = \downarrow \Sigma F$

$R_A + R_D = 40 + 25$ kN

$= 65$ kN

- Taking moment about left hand support at A,

$\circlearrowright \Sigma M_A = 0$

$- R_D(8) + 40(3) + 25(6) = 0$

$-8R_D + 120 + 150 = 0$

$-8R_D + 270 = 0$

$-8R_D = -270$

$R_D = \dfrac{-270}{-8}$

$= 33.75$ kN

- Therefore, $R_A + R_D = 40 + 25$ kN

$R_A + 33.75 = 65$ kN

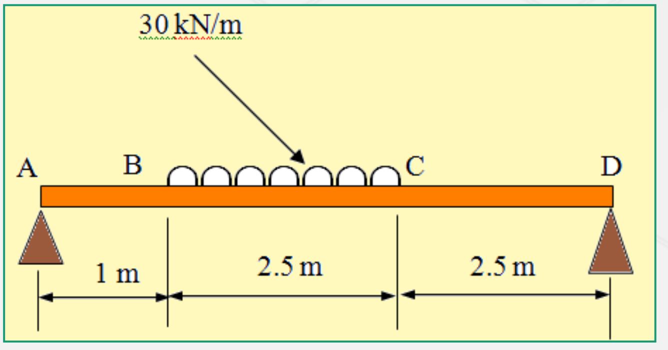
$R_A = 65 - 33.75$ kN

$= 31.25$ kN

Sample Question

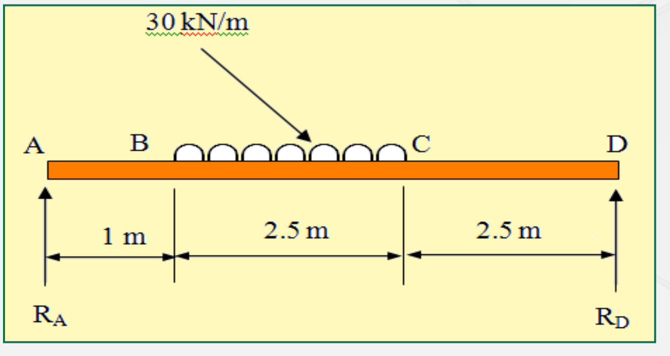
Exercise 2

A simply supported beam of span 6 m is loaded by a uniformly distributed load at B until C as in Figure above. Calculate the reaction force at point A and D.



Solution:-

a. Free body diagram



- Determine reaction:-

$\uparrow \Sigma F = \downarrow \Sigma F$

$R_A + R_D = 30$ kN/m (2.5 m)

$= 75$ kN

- Taking moment about right hand support at A,

$\circlearrowright \Sigma M_A = 0$

$- R_D(6) + 30 (2.5) (2.5/2 + 1) = 0$

$-6R_D + 75 (2.25) = 0$

$-6R_D + 168.75 = 0$

$-6R_D = - 168.75$

$R_D = \dfrac{-168.75}{-6}$

$= 28.125$ kN

- Therefore, $R_A + R_D = 75$ kN

$R_A + 28.125 = 75$ kN

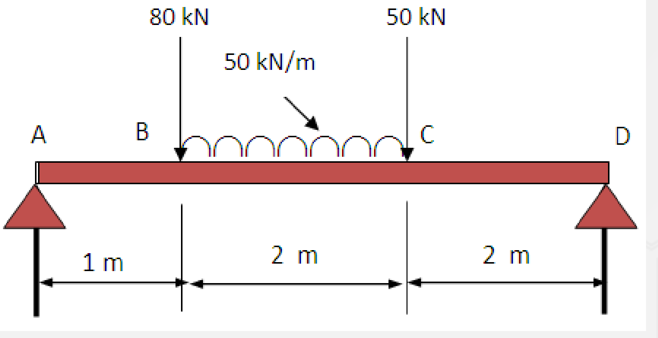
$R_A = 75 – 28.125$ kN

$= 46.875$ kN

Sample Question

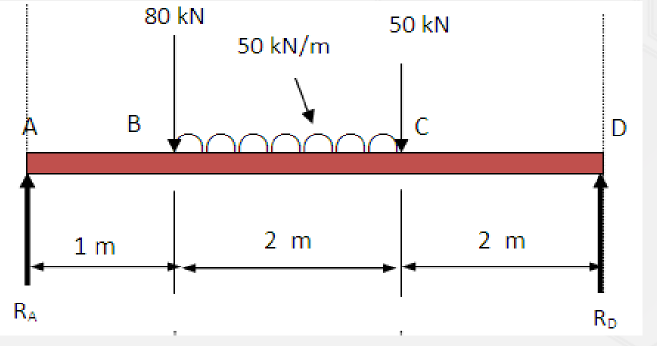
Exercise 2

A simply supported beam of span 5 m is loaded by a single concentrated and uniformly distributed load as in Figure above. Calculate the reaction force at point A and D.



Solution:-

a. Free body diagram



- Determine reaction:-

$\uparrow \Sigma F = \downarrow \Sigma F$

$R_A + R_D = 80 + 50(2) + 50$

$= 230$ kN

- Taking moment about right hand support at A,

$\circlearrowleft \Sigma M_A = 0$

$- R_D(5) + 50(3) + 50(2)\,(2/2 + 1) + 80(1) = 0$

$-5R_D + 150 + 200 + 80 = 0$

$-5R_D + 430 = 0$

$-5R_D = -430$

$R_D = \dfrac{-430}{-5}$

$= 86$ kN

- Therefore, $R_A + R_D = 230$ kN

$R_A + 86 = 230$ kN

$R_A = 230 - 86$ kN

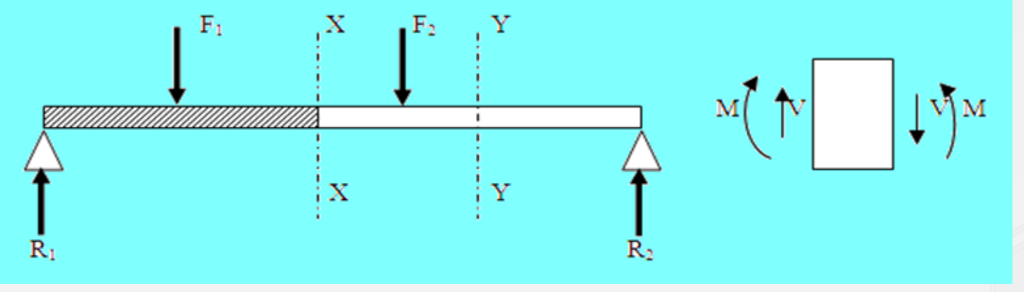
$= 144$ kN

3.5 SHEAR FORCE

- ***Shear force*** (V) is the force in the beam acting perpendicular to its longitudinal (x) axis. For design purposes, the beam's ability to resist shear force is more important than its ability to resist an axial force.

- The shear force at a section is **positive** when the resultant(reactions) force to the left of the section **acts upwards** in a horizontal beam.

3.5.1 SYMBOL & SIGN CONVENTION FOR SHEAR FORCE

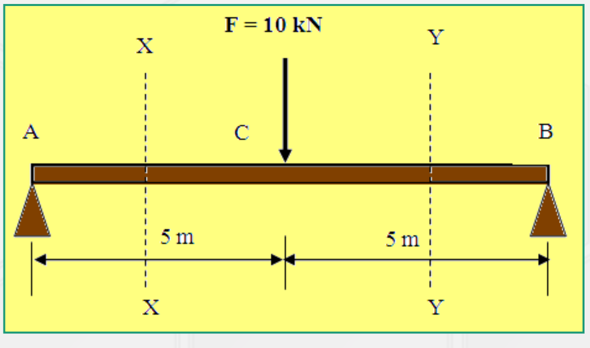


- Referring to the diagram above: -

Shear force at section x-x = R1 - F1

Shear force at section y-y = R1 - F1 – F2

3.5.2 SHEAR FORCE MAGNITUDE



Cut of beam at x distance from A. The section shown as below figure.

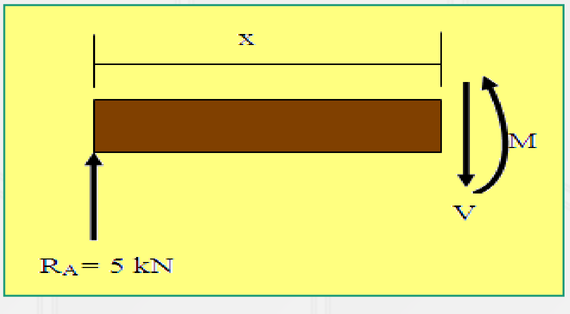


Figure 1: Left hand section

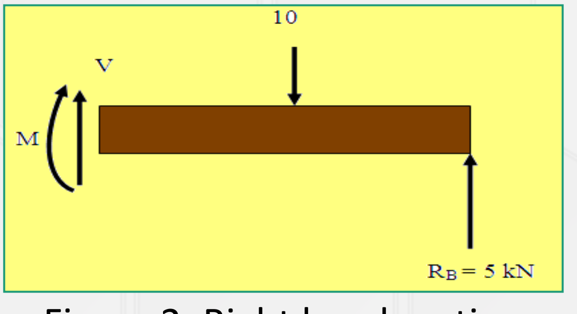
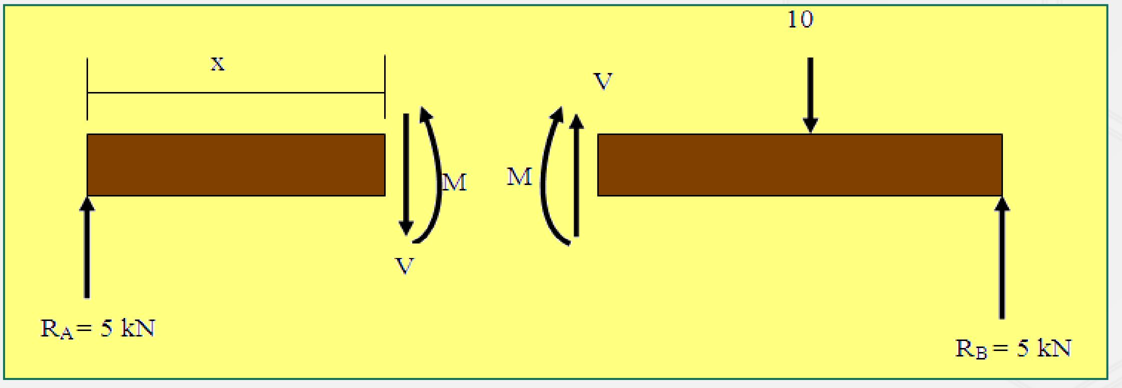
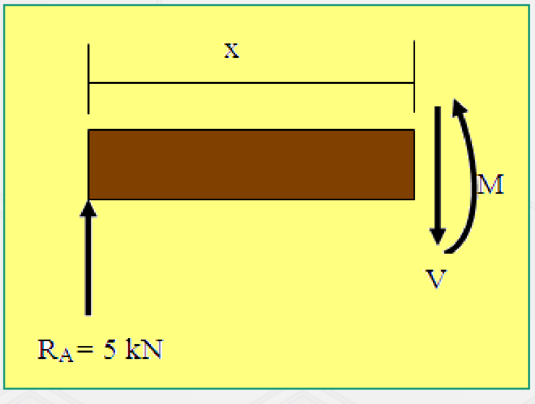


Figure 2: Right hand section

To determine the shear force, V we can consider at left hand section or right hand section.



Normally, the consideration at left hand section.



LEFT HAND SECTION

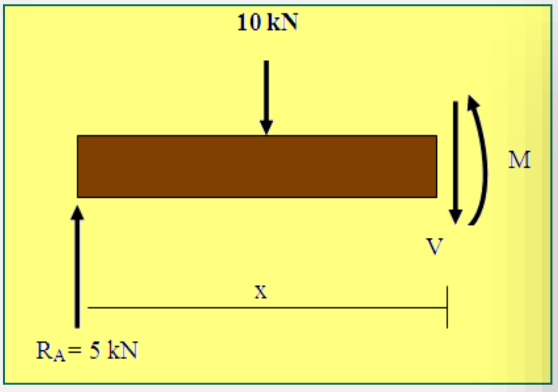
❖If consider the vertical force, we find that V = RA = 5 kN.

❖For a distance x = 0 to x = 5 m, the shear force is 5 kN.

❖According to the convention sign, we find the shear force is in positive range.

❖ Consider the left-hand section.

❖For 5 to 10 m distance, cut of beam at sections y-y and let say it has distance x meter from A.



LEFT HAND SECTION

Referring to left side figure, with totally the vertical force, we find that:-

$$V + 10 - 5 = 0$$
$$V + 5 = 0$$
$$V = -5 \text{ kN}$$

Therefore, for distance x = 5 m until x =10 m, the value of the shear force is -5 kN

SHEAR FORCE DIAGRAM (SFD)

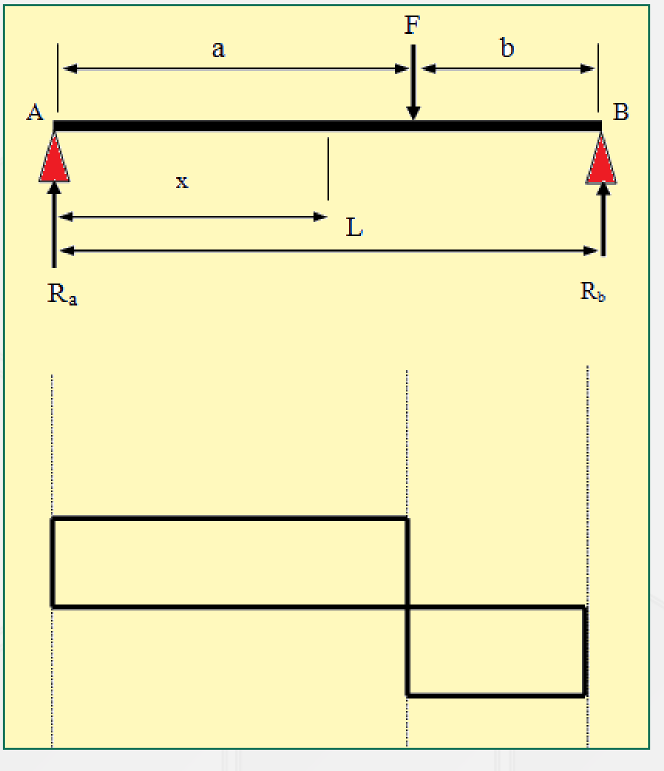
Shear Force Diagram

$$R_a = \frac{Fb}{L} \qquad R_b = \frac{Fa}{L}$$

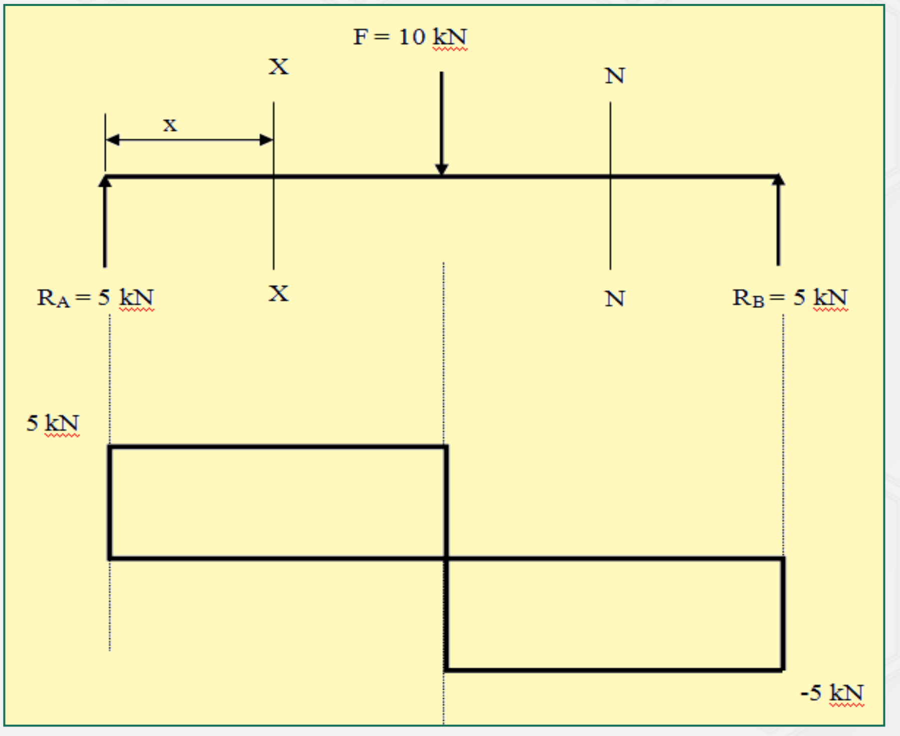
$$V = R_a = \frac{Fb}{L}$$

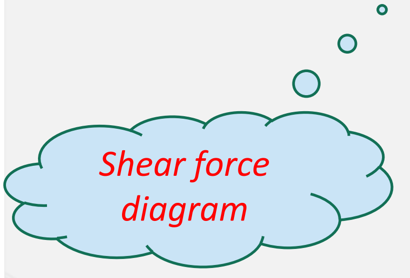
$$V = \frac{Fb}{L} - F = \frac{-Fa}{L}$$

$$V = -R_b = -\frac{Fa}{L}$$

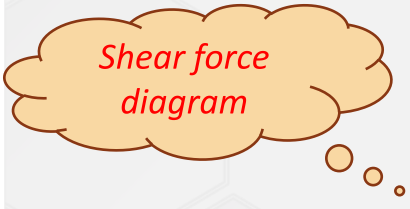


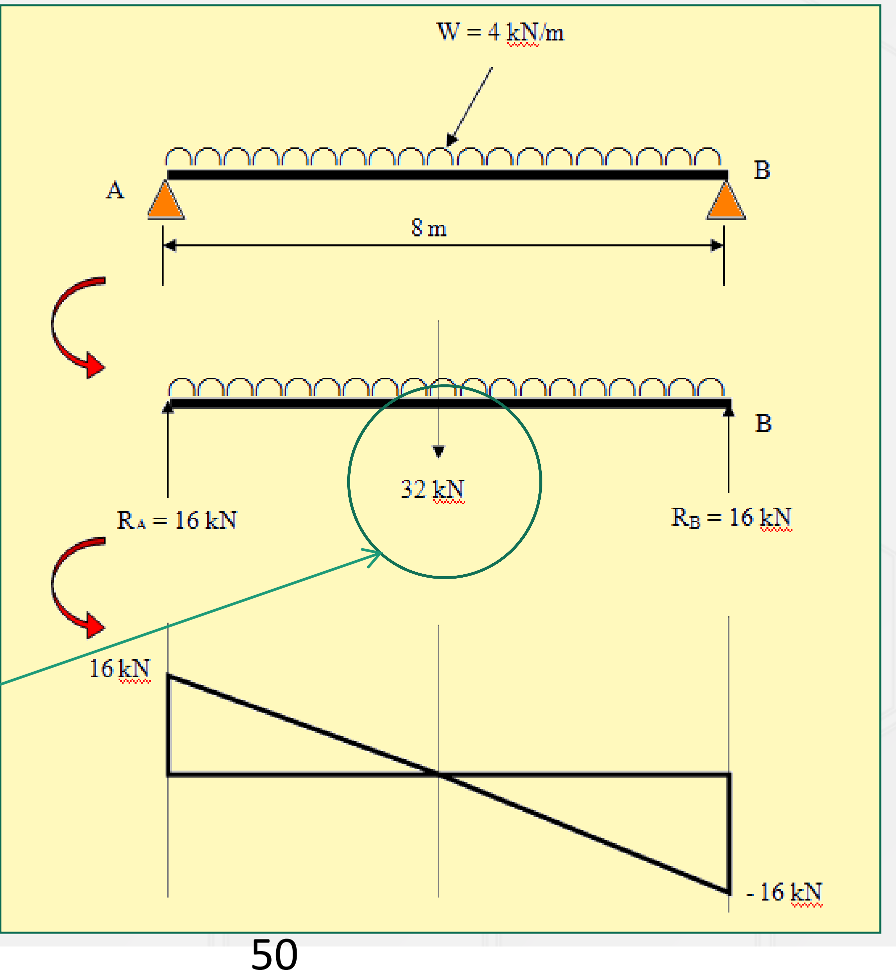
SHEAR FORCE DIAGRAM (SFD)-with concentrated load





SHEAR FORCE DIAGRAM (SFD)-with uniformly distributed load





F = w x d

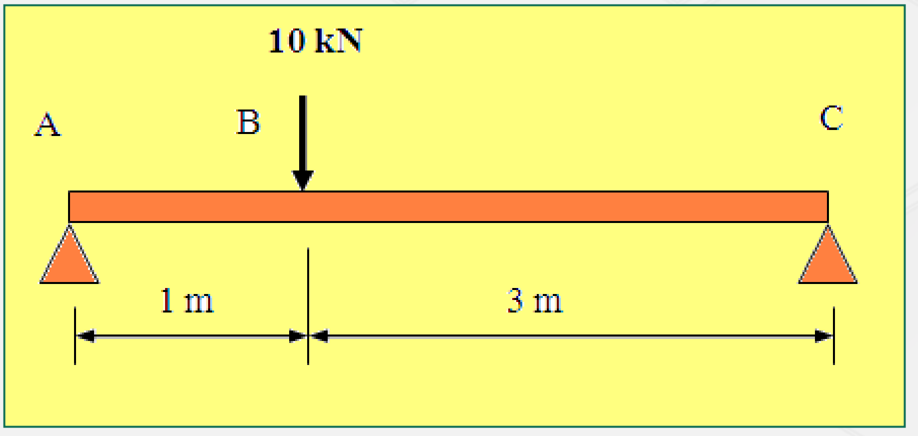
= 4 kN/m (8m)

= 32 kN

Sample Question

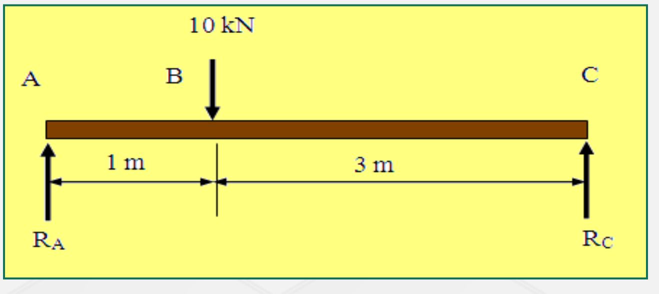
Exercise 1

A simply supported beam is support at A and C as shown in figure below. Determine the value of the shear force on the beam.



Solution:-

a. Sketching free body diagram



b. Calculate the reactions:-

$$\uparrow \Sigma F = \downarrow \Sigma F$$

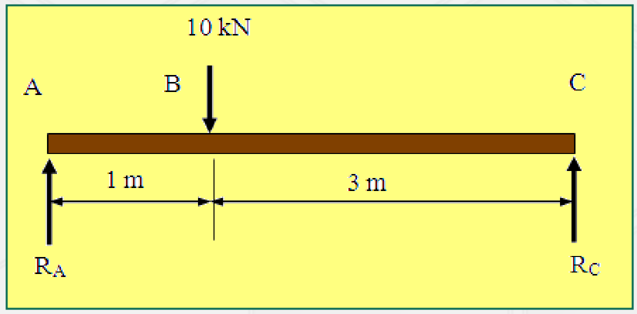
$$RA + RC = 10\ kN \longrightarrow (1)$$

c.

$$\sum M_A = \sum M_A$$

$$10(1) = R_C\,(4)$$

$$R_C = \frac{10}{4} = 2.5\ \text{kN}$$



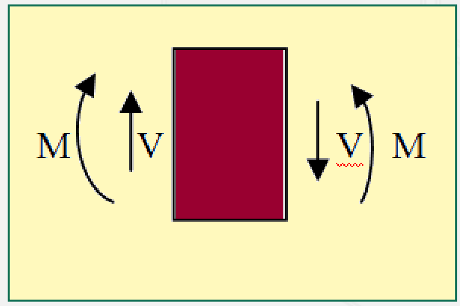
Substitute R_C = 2.5 kN in eq. (1)

$$RA + RC = 10\ \text{kN}$$

$$R_A + 2.5 = 10$$

$$R_A = 10 - 2.5$$

$$= 7.5\ \text{kN}$$



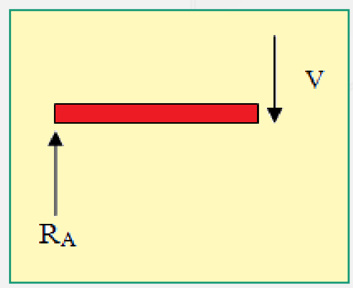
d. Determine shear force:-

Shear force at point A : $V - R_A = 0$ kN

$$V = R_A$$

$$V = 7.5\ \text{kN}$$

For left side, we are assume upwards force is negative, and downwards force is positive.



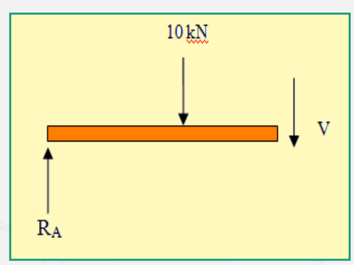
Shear force at point B : $V + 10 - R_A = 0$ kN

$$V + 10 - 7.5 = 0\ \text{kN}$$

$$V + 2.5 = 0\ \text{kN}$$

$$V = -2.5\ \text{kN}\ (\uparrow)$$

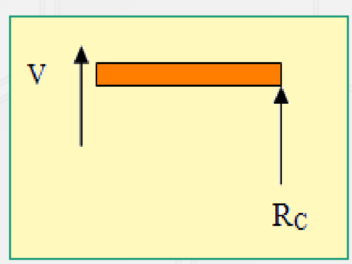
There fore, downwards force for shaer force is positive.



Shear force at point C : $V + R_C = -2.5 + 2.5$ kN

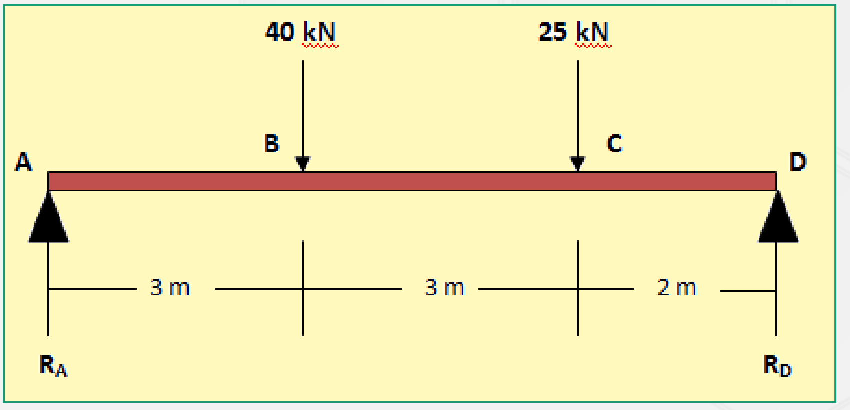
$$= 0\ \text{kN}$$

For right hand section, we describes that the sign convention of shear force is negative.



Sample Question

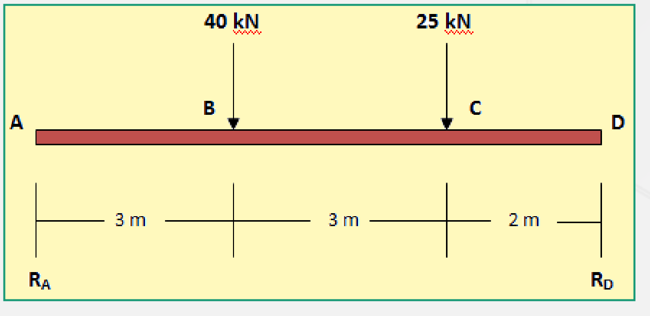
Exercise 2



A simply supported beam is support at A and D as shown in above figure. Calculate the shear force on the beam and draw the shear force diagram.

Solution:-

a. Sketching free body diagram



b. Determine the reactions:-

$\uparrow \Sigma F = \downarrow \Sigma F$

$R_A + R_D = 40 + 25$ kN

$= 65$ kN

c. Consider moment at point A,

$\Sigma M_A = 0$

$-R_D(8) + 40(3) + 25(6) = 0$

$-8R_D + 120 + 150 = 0$

$-8R_D + 270 = 0$

$-8R_D = -270$

$R_D = \frac{-270}{-8}$

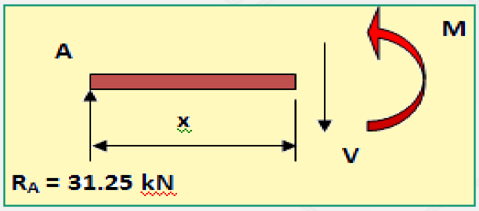
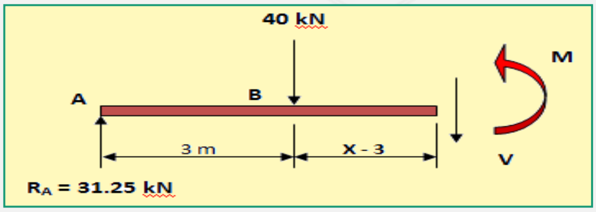
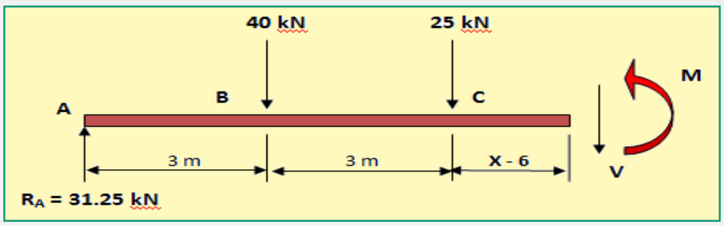
$= 33.75$ kN

- Therefore, $R_A + R_D = 60 + 25$ kN

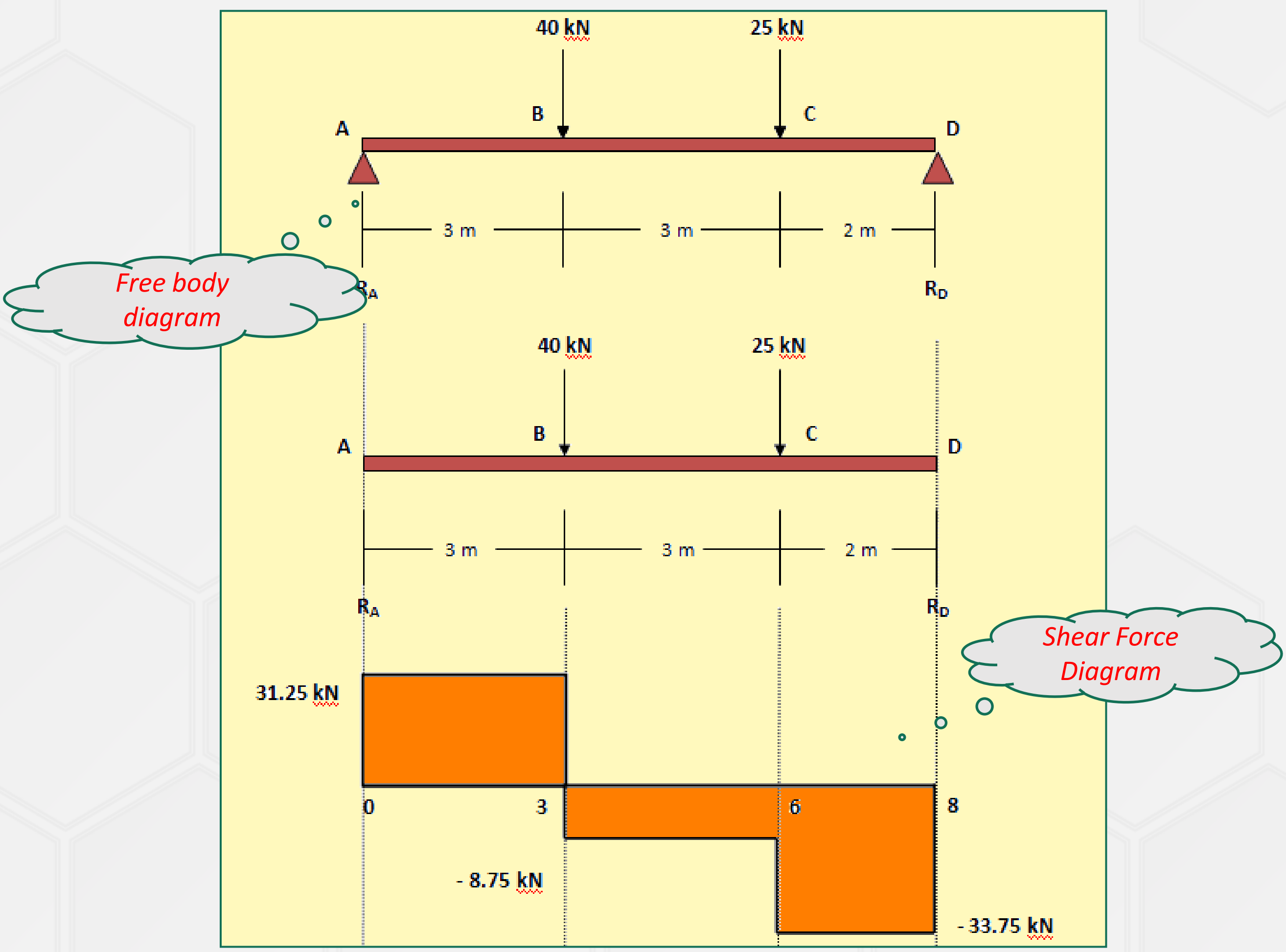
$R_A + 33.75 = 65$ kN

$R_A = 65 - 33.75$ kN

$= 31.25$ kN

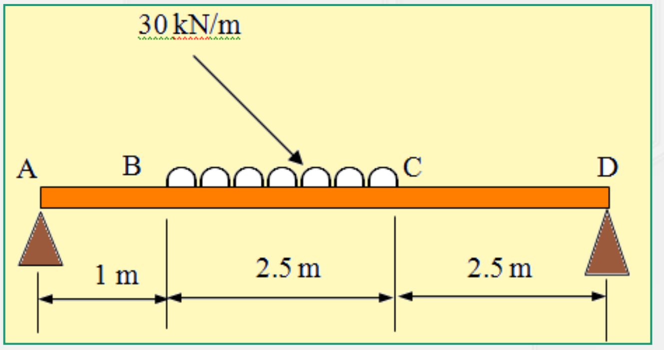
Range (m)	Free body diagram	Shear force(kN)
$0 \leq x < 3$		V = 31.25 kN x = 0, V = 31.25 kN x = 3, V = 31.25 kN
$3 \leq x < 6$		V = 31.25 – 40 x = 3, V = - 8.75 kN x = 6, V = - 8.75 kN
$6 \leq x < 8$		V = 31.25 - 40 - 25 x = 6, V = - 33.75 kN x = 8, V = - 33.75 kN

d. Shear Force Diagram



Sample Question

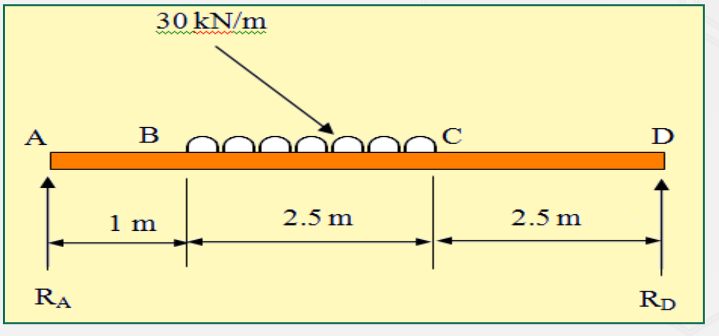
Exercise 3



A simply supported beam is support at A and D as shown in above figure. Calculate the shear force on the beam and draw the shear force diagram.

Solution:-

a. Sketching free body diagram



b. Determine the reactions:-

$\uparrow \Sigma F = \downarrow \Sigma F$

$R_A + R_D = 30\ \text{kN/m}\ (2.5\ \text{m})$

$= 75\ \text{kN}$

F = w x d

c. Consider moment at point A,

$\Sigma M_A = 0$

$- R_D(6) + 30\ (2.5)\ (2.5/2 + 1) = 0$

$-6R_D + 75\ (2.25) = 0$

$-6R_D + 168.75 = 0$

$-6R_D = - 168.75$

$R_D = \dfrac{-168.75}{-6}$

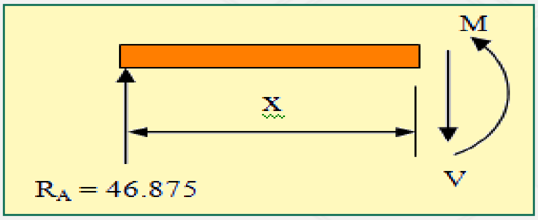
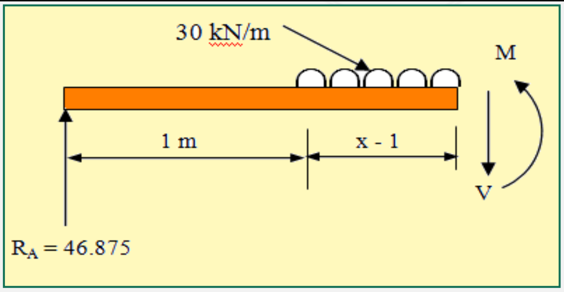
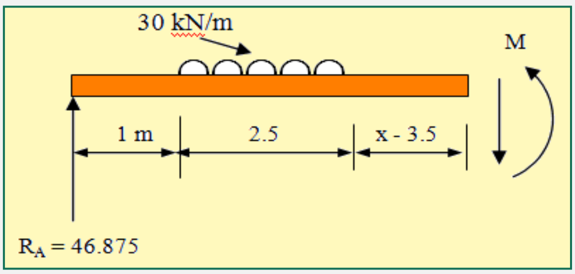
$= 28.125\ \text{kN}$

- Therefore, $R_A + R_D = 75\ \text{kN}$

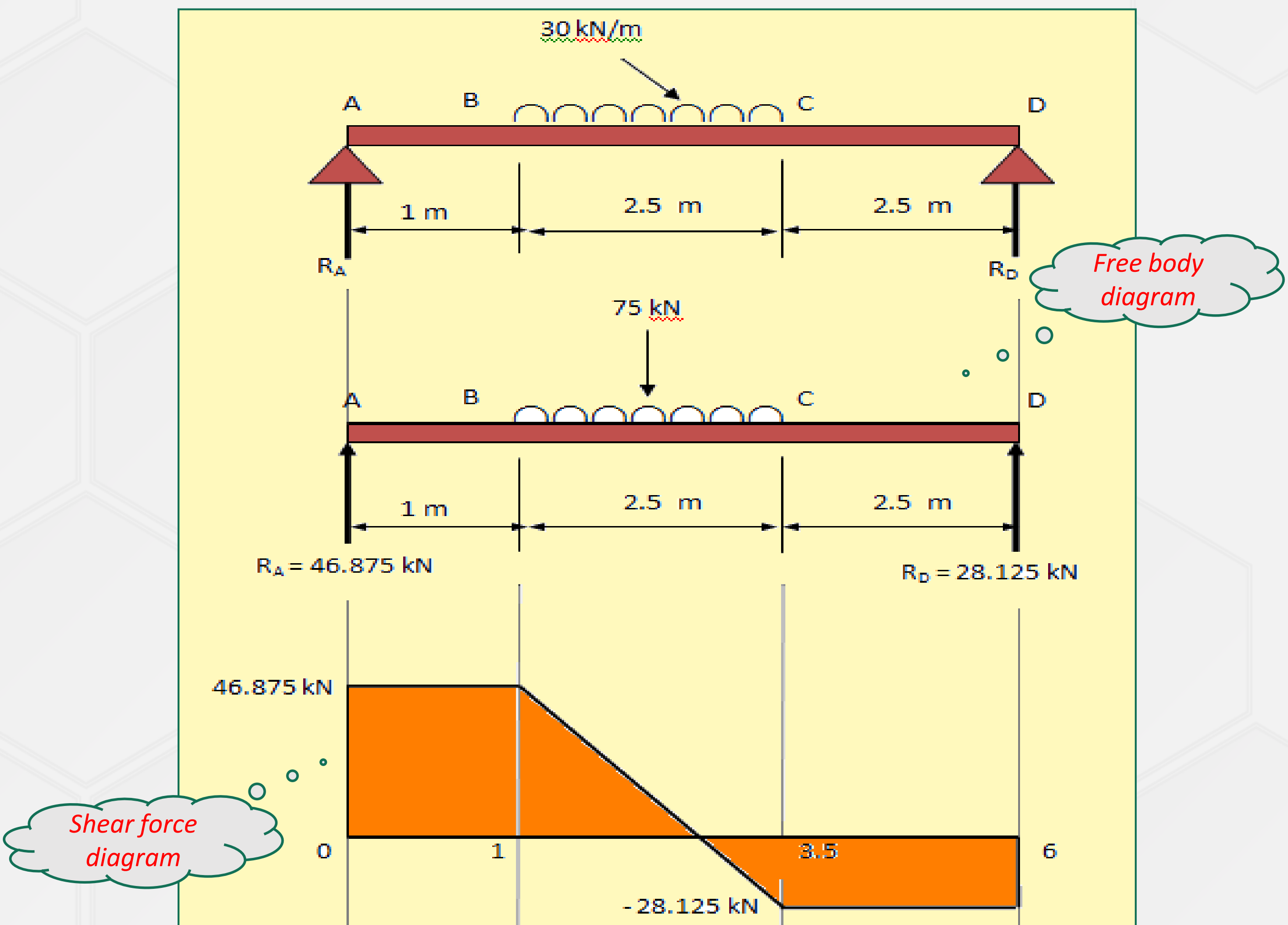
$R_A + 28.125 = 75\ \text{kN}$

$R_A = 75 – 28.125\ \text{kN}$

$= 46.875\ \text{kN}$

Range (m)	Free body diagram	Shear force(kN)
$0 \leq x < 1$		V = 46.875 kN x = 0, V = 46.875 kN x = 1, V = 46.875 kN
$1 \leq x < 3.5$		V = 46.875 – 30 (x-1) x = 1, V = 46.875 – 30 (0) kN = 46.875 kN x = 3.5, V = 46.875 – 30 (2.5) kN = - 28.125 kN
$3.5 \leq x < 6$		V = 46.875 – 30 (2.5) x = 3.5, V = - 28. 125 kN x = 6, V = - 28. 125 kN

d. Shear Force Diagram



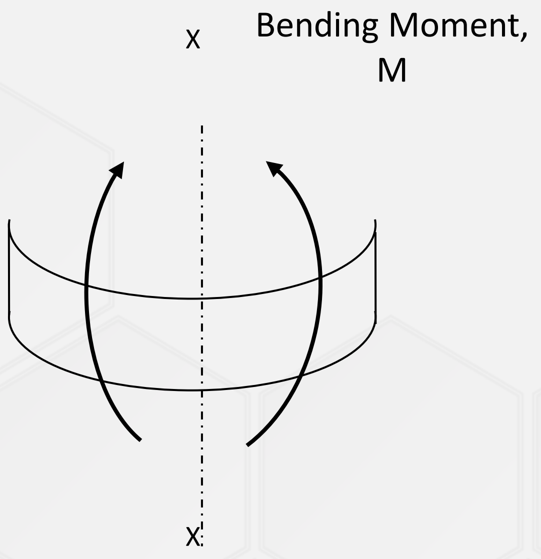
3.6 BENDING MOMENT

The bending moment is the algebraic sum of moments of the forces acting on the right or left section of the beam. It is usually marked by M.

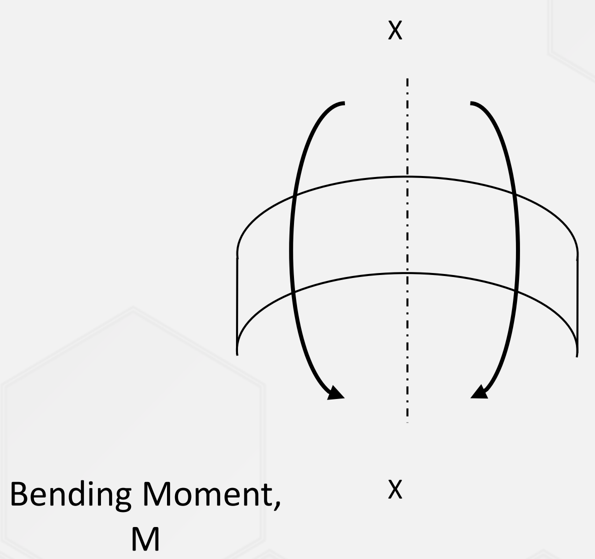
SIGN CONVENTION FOR BENDING MOMENT

The forces acting on the left @ right in the beam section will produces a moment clockwise or counterclockwise.

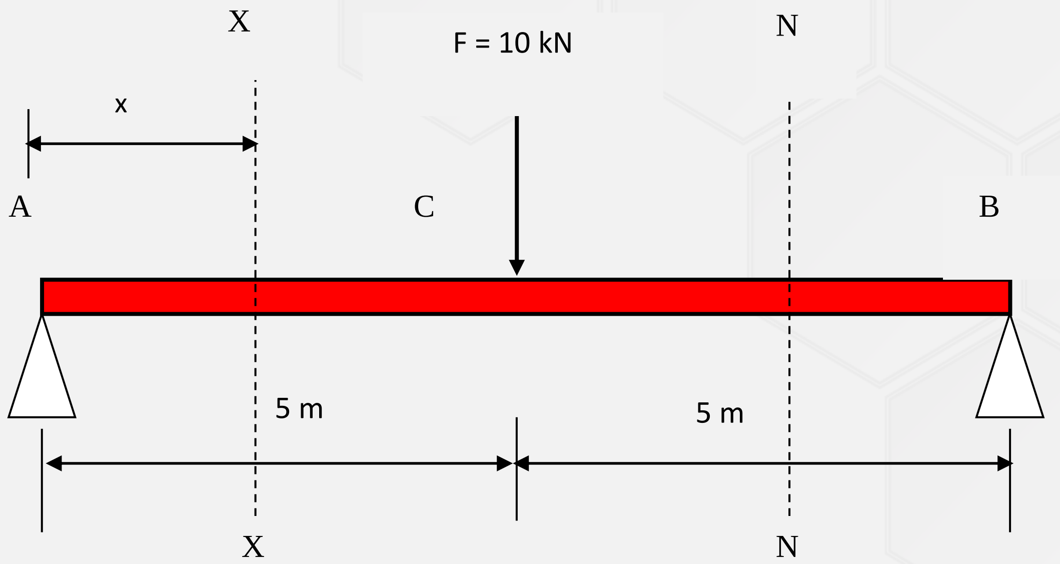
Also as shear forces, a sign convention for bending moment must be set to ensure that the same sign is used for the left @ left of the section under consideration.



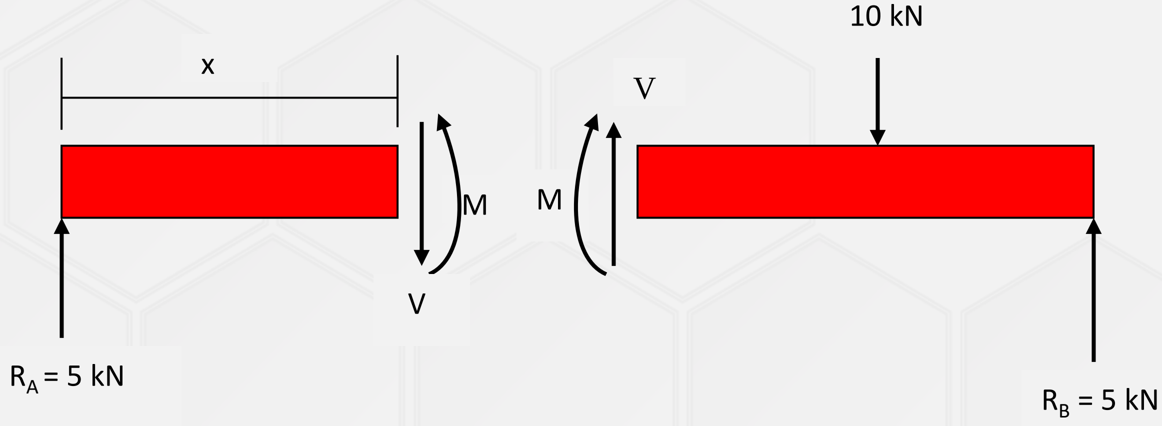
(a) Positive Bending Moment



(b) Negative Bending Moment



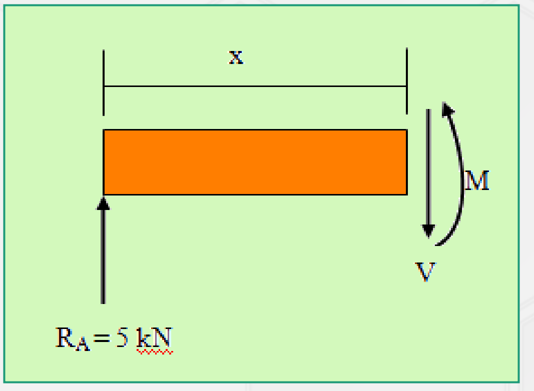
Cut beam at a distance x from A. The free-body diagram for the left and right side of this section is shown as below figure.



We need to determine the value of the bending moments at several sections of the beam in range of 0 <x <5. We cut of beam at the X-X axis as shown in figure. Taking a moment at this section provides: -

$\sum M_{NN} = 0$

$$M - 5x = 0$$
$$M = 5x$$



At $x = 0$, $M = 0$ Nm

$x = 5$, $M = 25$ Nm

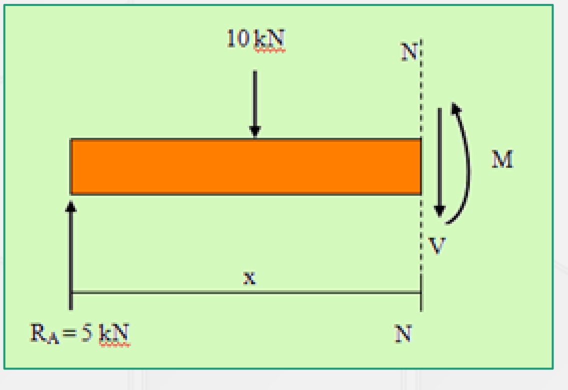
This equation gives the relationship of bending moments (M) with a range from A (x). Indicate that the above BMD, we need to draw a straight line with a slope of +5 between point A to point C.

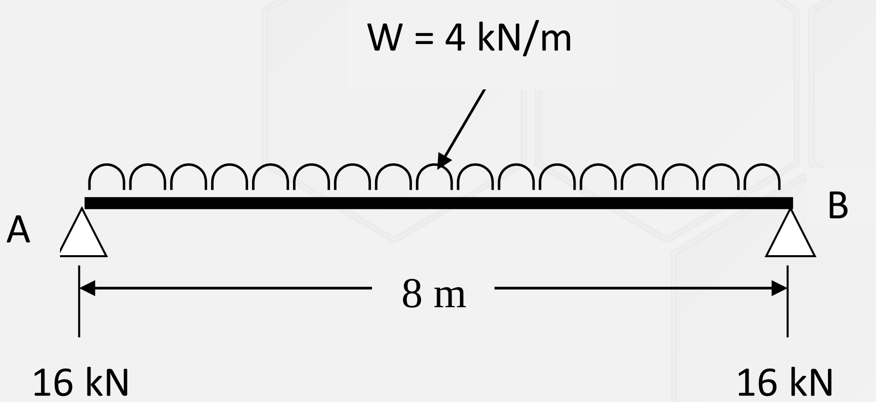
Between 5 m to 10 m, cut beam at N-N axis. Refer to below figure. Taking a moment at this section provides: -

$$\sum M_{NN} = 0$$
$$M + 10(x-5) - 5x = 0$$
$$M = 50 - 5x$$

At , $x = 5$, $M = 25$ Nm

$x = 10$, $M = 0$ Nm



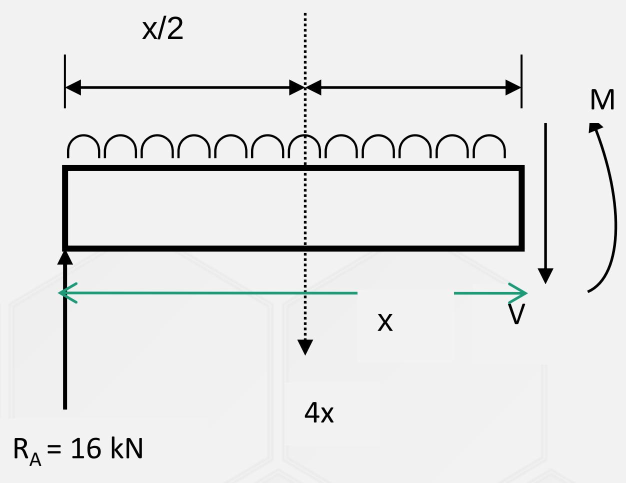


Based on the right figure, taking moment at x-x part, we find that:-

$$\sum M_{XX} = 0$$

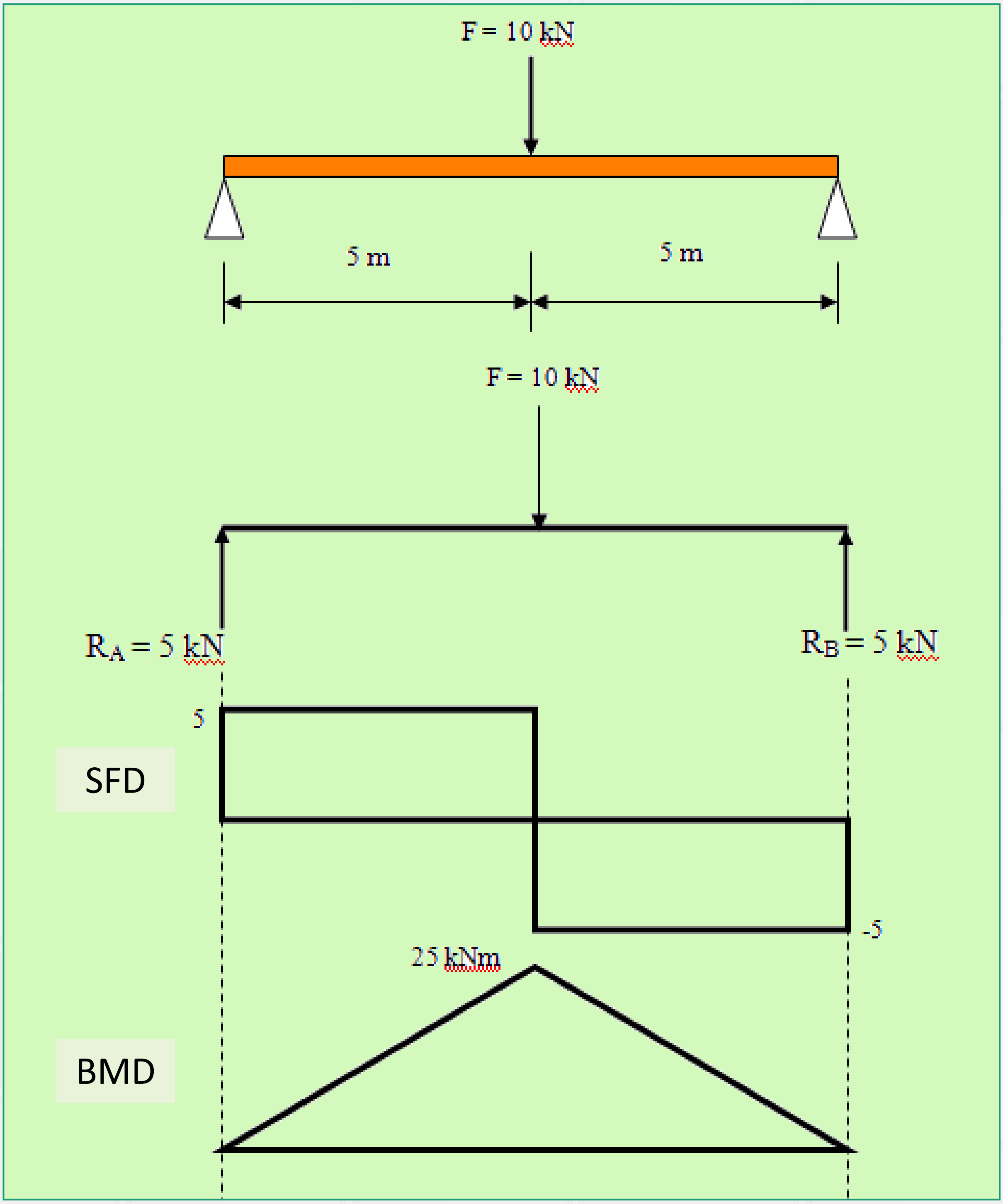
and $$M + 4x\,(x/2) - 16\,(x) = 0$$

$$M = 16x - 2x^2$$



To determine shear force and bending moment, firstly we must to determine the reactions force.

The equilibrium of force $\sum F = 0$ and the equilibrium of moment $\sum M = 0$ are used to determine the reaction.

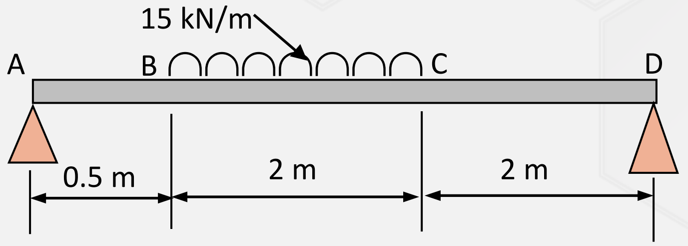


Shear Force Diagram (SFD) & Bending Moment Diagram (BMD)

Sample Question

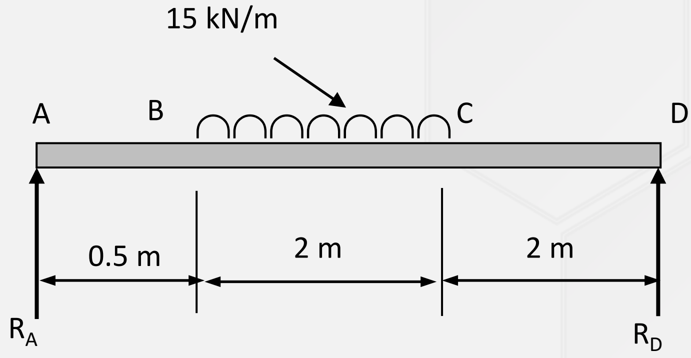
Exercise 1

A beam is simply supported at A and D and applied load as shown in below figure. Determine the shearing force and the bending moment.



Solution:-

a. Sketching free body diagram

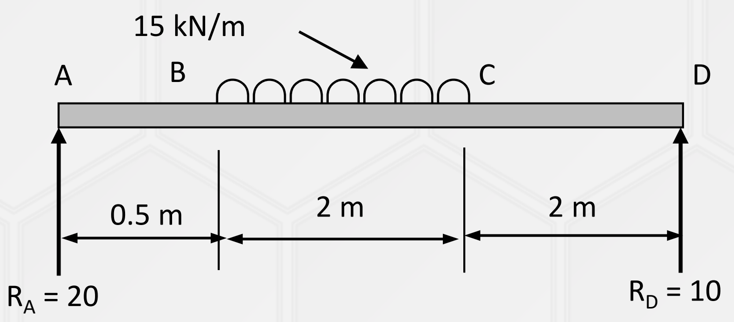


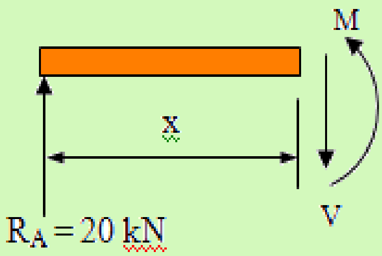
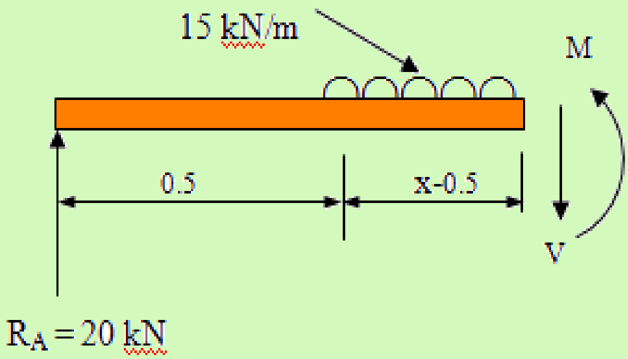
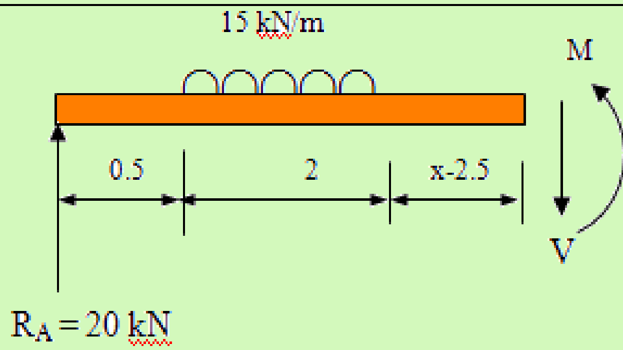
$$\circlearrowright \sum M_A = \circlearrowleft \sum M_A \qquad \uparrow \sum F = \downarrow \sum F$$

$$R_D(4.5) - 15(2)\left(\frac{2}{2} + 0.5\right) = 0 \qquad R_A + R_D - F = 0$$

$$4.5R_D - 30(1.5) = 0 \qquad R_A + (10) - (15 \times 2) = 0$$

$$R_D = \frac{45}{4.5} = 10 \text{ kN} \qquad R_A = 20 \text{ kN}$$



Range	Free Body Diagram	SF & BM
$0 \leq x \leq 0.5$		V = 20 x = 0, V = 20 x = 0.5, V = 20 M = 20x x=0, M=0 x=0.5, M=10
$0.5 \leq x \leq 2.5$		V = 20 – 15(x-0.5) X = 0.5, V = 20 X = 2.5, V = -10 $M=20x - \frac{15(x-0.5)(x-0.5)}{2}$ x=0.5, M=10 x=2.5, M=20
$2.5 \leq x \leq 4.5$		V = 20-30 = -10 x = 2.5, V = -10 x = 4.5, V = -10 M=20x-15(2)(x-1.5) M=20x-30(x-1.5) x=2.5, M=20 x=4.5, M=0

3.6.2 MAXIMUM BENDING MOMENT

❖ In some of cases, where a beam is loaded with easy loads, the position and the maximum bending moment can be found by inspection to the bending moment diagram.

❖For the combined load , it is difficult to obtain the maximum bending moment of the bending moment diagram.

❖For equation of bending moment, ***dM/dx = 0*** are used to determine the maximum bending moment positions.

3.6.3 CONTRAFLEXTURE POINT

❖From the deflection of the beam figure, AB is part of the sagging (positive bending moment), while the BC is hogging (negative bending moments).

❖Point K , is the curvature changes from the hogging to the sagging (bending moment) and it referred as the contraflexture point where the point is the sign of bending moment changes from positive to negative or otherwise.

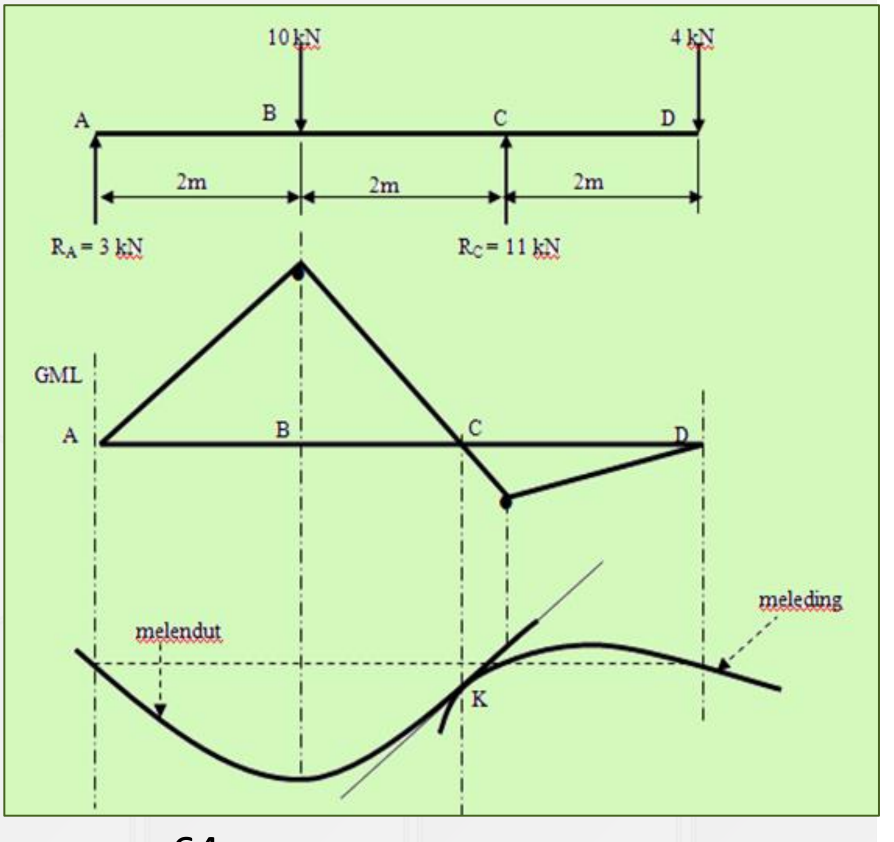
❖The position of this point can be obtained by solving the equation M = 0 in the range of position.

From the figure, we obtain the contraflexture point occurs in range $2 \leq x \leq 4$. So, the maximum moment:-

$$M = 3x - 10(x - 2)$$
$$= -7x + 20$$

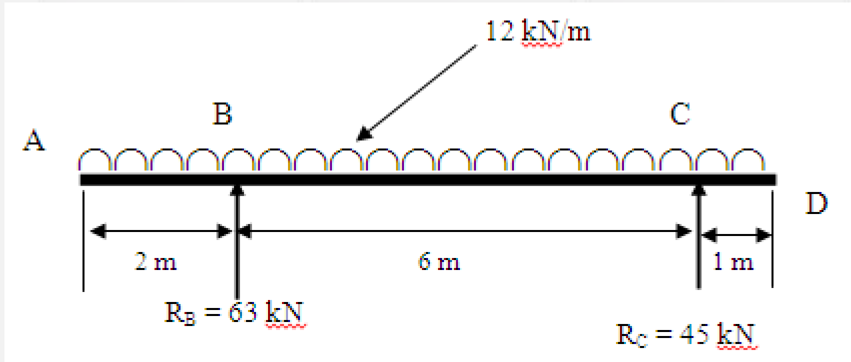
To determine the contraflexture point, we must used equation M = 0

$$M=0$$
$$-7x + 20 = 0$$
$$-7x = -20$$
$$x = -20 / -7$$
$$= 2.86 \text{ m}$$



Sample Question

Exercise 3

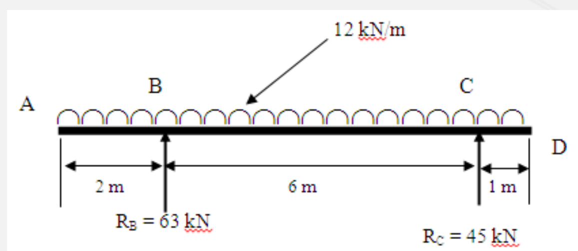


A beam ABCD with measuring length 9 m is simply supported at B and C. The distance of AB = 2m and **BC** = 6m. The beam is carrying a uniform load of 12 kN/m along the beam. Draw the SFD and BDM for the beam and determine:

a) the position and the maximum bending moment
b) the position of the contraflexture point.

Solution:-

a. Sketching free body diagram

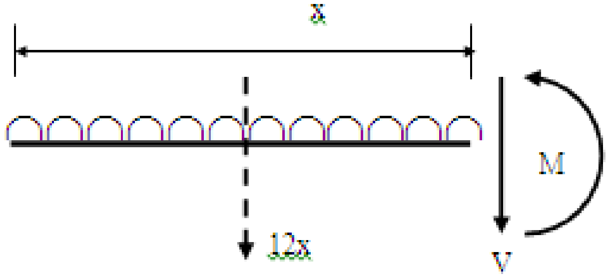
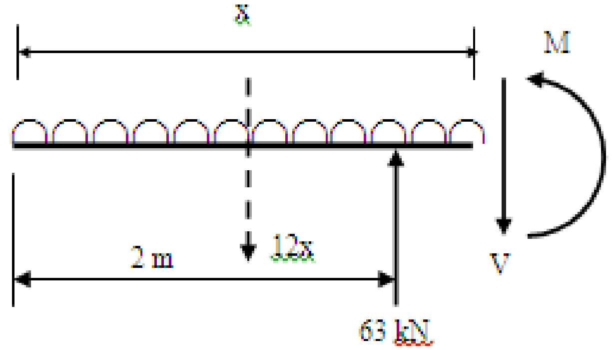
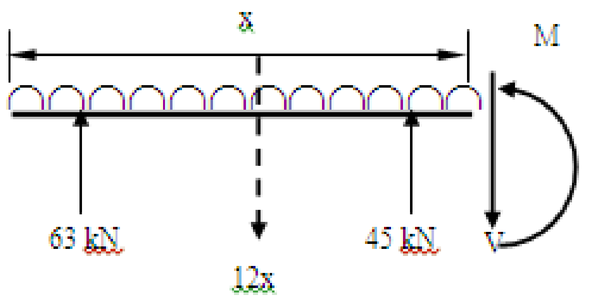


Taking moment at point B:-

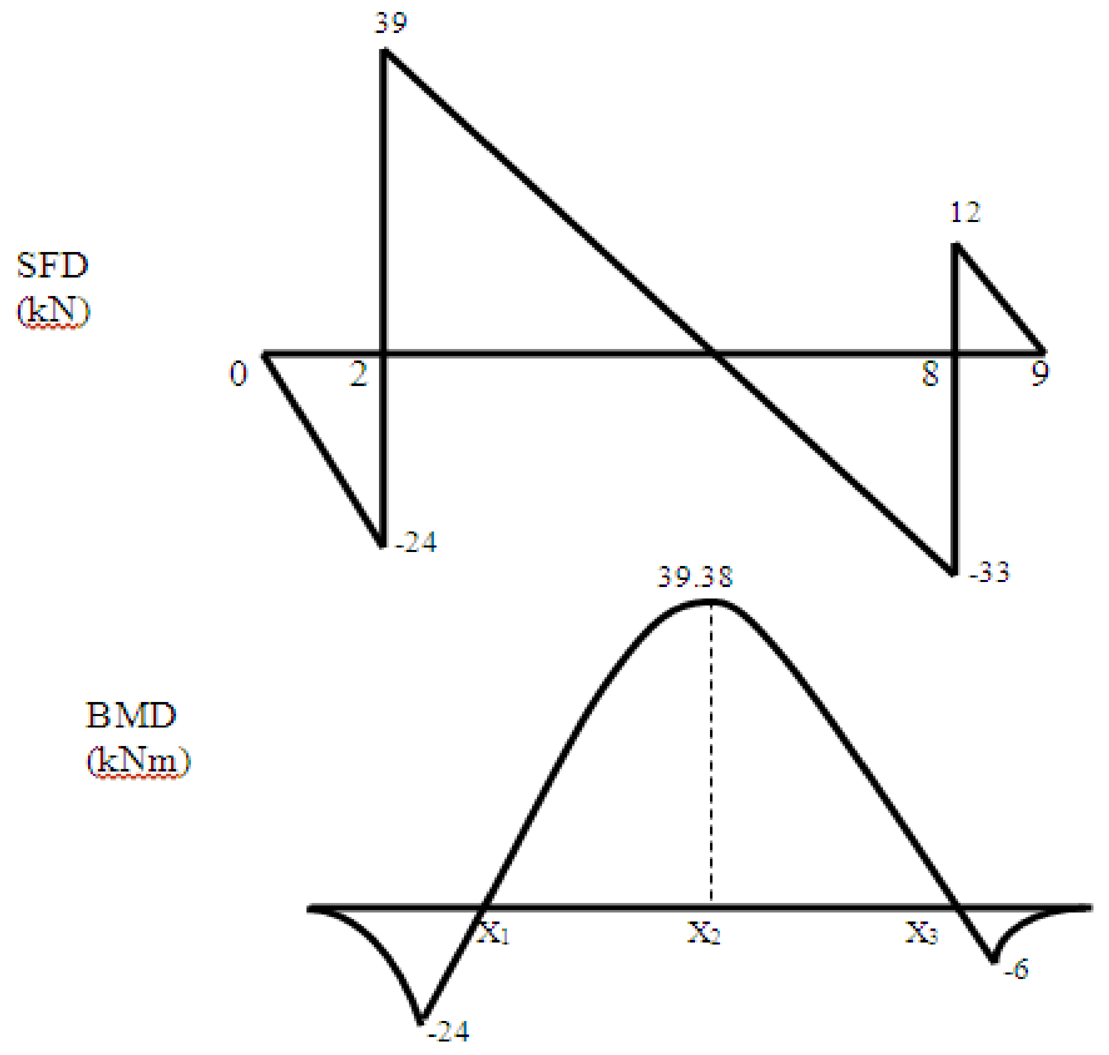
$$\sum M_B = 0$$
$$-6R_C + 12\,(9)\,(9/2 - 2) = 0$$
$$\therefore R_C = 45 \text{ kN}$$

Taking moment at point C:-

$$\sum M_C = 0$$
$$6R_B - 12\,(9)\,(9/2 - 1) = 0$$
$$\therefore R_B = 63 \text{ kN}$$

Range (x)	Free body diagram	Shear force (kN) & Bending moment (kNm)
$0 \le x \le 2$		$V = -12x$ $x = 0, V = 0$ $x = 2, V = -24$ $M = \frac{-12x^2}{2}$ $x = 0, M = 0$ $x = 2, M = -24$
$2 \le x \le 8$		$V = -12x + 63$ $x = 2, V = 39$ $x = 8, V = -33$ $M = \frac{-12x^2}{2} + 63\ (x-2)$ $x = 2, M = -24$ $x = 8, M = -6$
$8 \le x \le 9$		$V = -12x + 63 + 45$ $x = 8, V = 12$ $x = 9, V = 0$ $M = \frac{-12x^2}{2} + 63\ (x-2)$ $+ 45\ (x-8)$ $x = 8, M = -6$ $x = 9, M = 0$

Solution:



Shear Force Diagram (SFD) & Bending Moment Diagram (BMD)

The maximum bending moment, M_m is occurs in $2 \leq x \leq 8$ range.

Here, $$\mathbf{M} = \frac{-12x^2}{2} + \mathbf{63\ (x\text{-}2)}$$

$$\frac{dM}{dx} = \mathbf{-12x + 63 = 0}$$

$$\therefore \mathbf{x = 5.25}$$

Substitute x = 5.25 in moment equation in $2 \leq x \leq 8$ range.

The maximum bending moment $M_m = \dfrac{-12(5.25)^2}{2} + 63\ (5.25 - 2) = 39.38$ kNm

To determine the contraflexure point, used equation M = 0:-

$$\therefore M = \frac{-12x^2}{2} + 63\ (x\text{-}2) = 0$$

$$\therefore -6x^2 + 63x - 126 = 0$$

$$6x^2 - 63x + 126 = 0$$

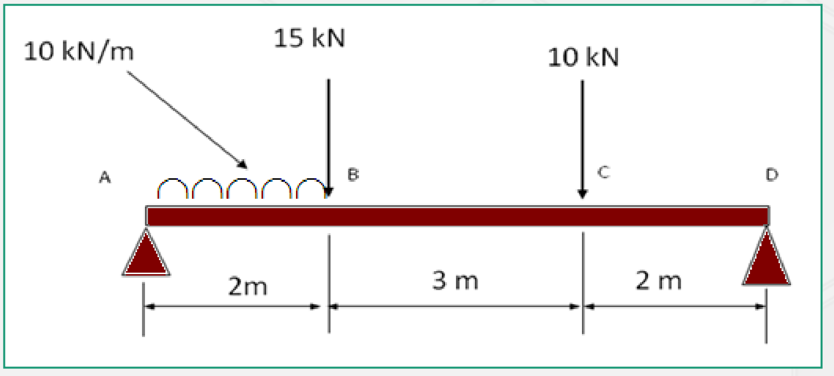
$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{63 \pm \sqrt{63^2 - 4 \times 6 \times 126}}{2 \times 6}$$

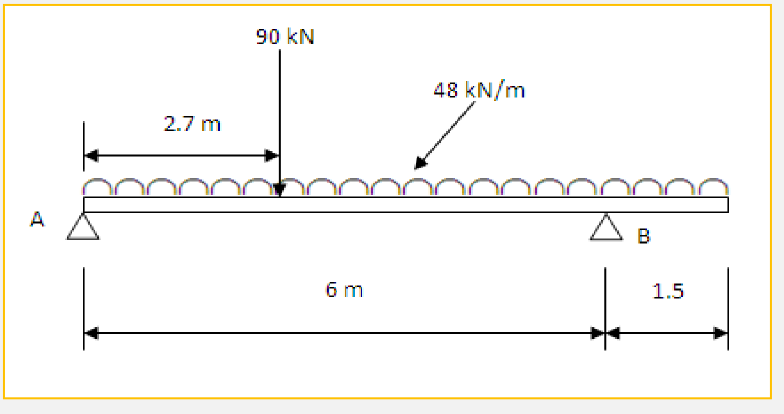
$$x = 7.81 \text{ m or } 2.69 \text{ m}$$

TUTORIAL TIME!

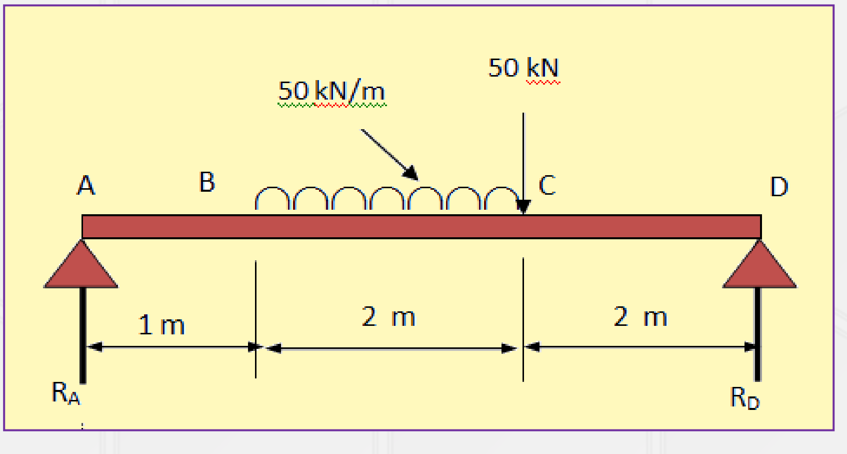
1. Determine the reaction force at the beam support as shown in below figure.



2. Determine the reaction force at the beam support as shown in below figure.

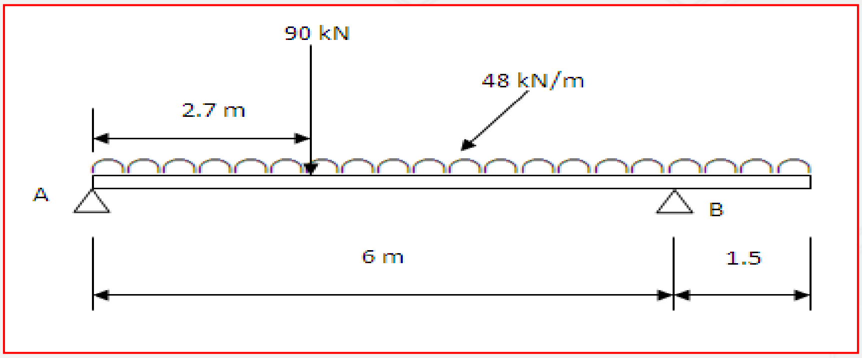


3. A simply supported beam is support at A and D as shown in above figure. Calculate the shear force on the beam and draw the shear force diagram.

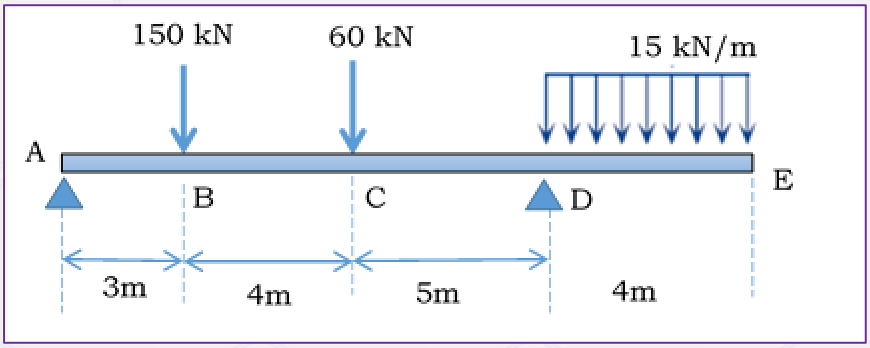


4. A simply supported beam is loaded as shown in below figure:-

a. Calculate the shear force, bending moment
b. Draw the shear force and bending moment diagram.
c. Determine the maximum bending moment and its position.



5. A simple supported beam is loaded as shown in figure below. Determine
a. The reaction force at support A and D
b. Draw the shear force and bending moment diagram
c. Determine the maximum bending moment and its position
d. Determine the position of contra flexure



REFERENCES

Abdul Mubeen.(2006), Mechanics Of Solids, Dorling Kindersley

Ahmad Zafri Bin Zainudin & Mohd Yazid Bin Yahya. (1998). Mekanik Pepejal 1, UTM.

Ahmad Zafri Bin Zainudin. (1998). Mekanik Bahan 1, UTM.

Beer & Johnston. (1992). Mechanics of Materials, Mc Graw Hill, Londan.

Beerand F. P., Johnston E. R. (1992),Mechanics of Materials, 2nd edition, McGraw Hill.

D.K Singh.(2007), Strength Of Materials, Ane books India.

Ferdinand P. Beer & E. Russell Johnston Jr. (1992). Mechanics of Materials, McGraw Hill.

Hibbler R.C . (2005), Mechanics Of Materials, Prentice Hall.

Hibbler. R. C. (1993),Statics and Mechanics of materials, Prentice-Hall.

R.S Khurmi.(2007), Strength Of Materials, S.Chand & Company Ltd.

Ros Saidatunnaziah Binti Md Yussof. (2015).Strength of Materials, First Edition in SI Units.

Terbitan

POLITEKNIK
MALAYSIA
Tuanku Syed Sirajuddin

e ISBN 978-967-2258-46-9

9 7 8 9 6 7 2 2 5 8 4 6 9