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KOLEJ KOMUNITI MALAYSIA

FLUID MECHANICS

VOL.1

Malaysian Polytechnics Version

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PREFACE

This e-book is intended to provide Malaysian Polytechnics' students with a clear yet simple explanations of the basic theory and applications of fluid mechanics. The FLUID MECHANICS VOL.1 e-book covers the first three (3) topics in the Polytechnics' Fluid Mechanics syllabus namely; introduction to fluid mechanics, physical properties of fluid and fluid statics.

The explanations in this book are mostly aided by diagrams for better understanding. Also, every topic in this e-book contained solved examples and tutorial questions for the students to enhance their knowledge of the topic. With these features, it is hoped that the book will be useful and tremendous help for the students in the basic of Fluid Mechanics at diploma level.

A few notable reference books on Fluid Mechanics has been referred while writing this e-book, in which are mentioned at the last section of the book. Students are also encourages to search and refer the books mentioned for further reading.

CONTENTS

Chapter 1:

INTRODUCTION TO FLUID MECHANICS

1.1 INTRODUCTION

Mechanics is a physical science that deals with both stationary and moving bodies with under the effect of forces on the bodies. **Statics** is the study of bodies at rest, and **dynamics** is the study of bodies in motion under the action of forces.

Fluid Mechanics is a branch of applied mechanics which studies the statics and dynamics behaviour of liquid and gas. In other words, Fluid Mechanics is the study of behaviour of fluid (gas or liquid) that is either at rest or in motion. The subcategories of this subject includes:

a) Fluid Statics - study of behaviour of fluid at rest
b) Fluid Dynamics - study of behaviour of fluid in motion
c) Hydrodynamics - study of motion of incompressible fluids (e.g; liquids and gasses at low speed)
d) Gas Dynamics - study of flow of fluids that undergo significant density changes (e.g; flow of gases through nozzles at high speed)
e) Aerodynamics - study of flow of gases (especially air) over bodies such as aircraft, rockets and automobiles at high or low speeds.

Fluid Mechanics become important because the use of fluids are so common in many engineering disciplines. The applications of this subject can be seen in the following area (Figure 1.1);

a) Aeronautical and aerospace engineers – to study flight and to design propulsion systems.

b) Civil engineers – to design channels, water networks, sewer system and water resisting structures such as dam and levees.

c) Mechanical engineers – to design pumps, compressors, control systems, heating and air conditioning equipment, wind turbines and solar heating devices.

d) Chemical and petroleum engineers – to design equipment used for filtering, pumping and mixing fluids.

e) Electronics and computer industry – to design switches, screen displays and data storage equipment.

f) Also used in the field of biomechanics – to study circulatory, digestive and respiratory systems

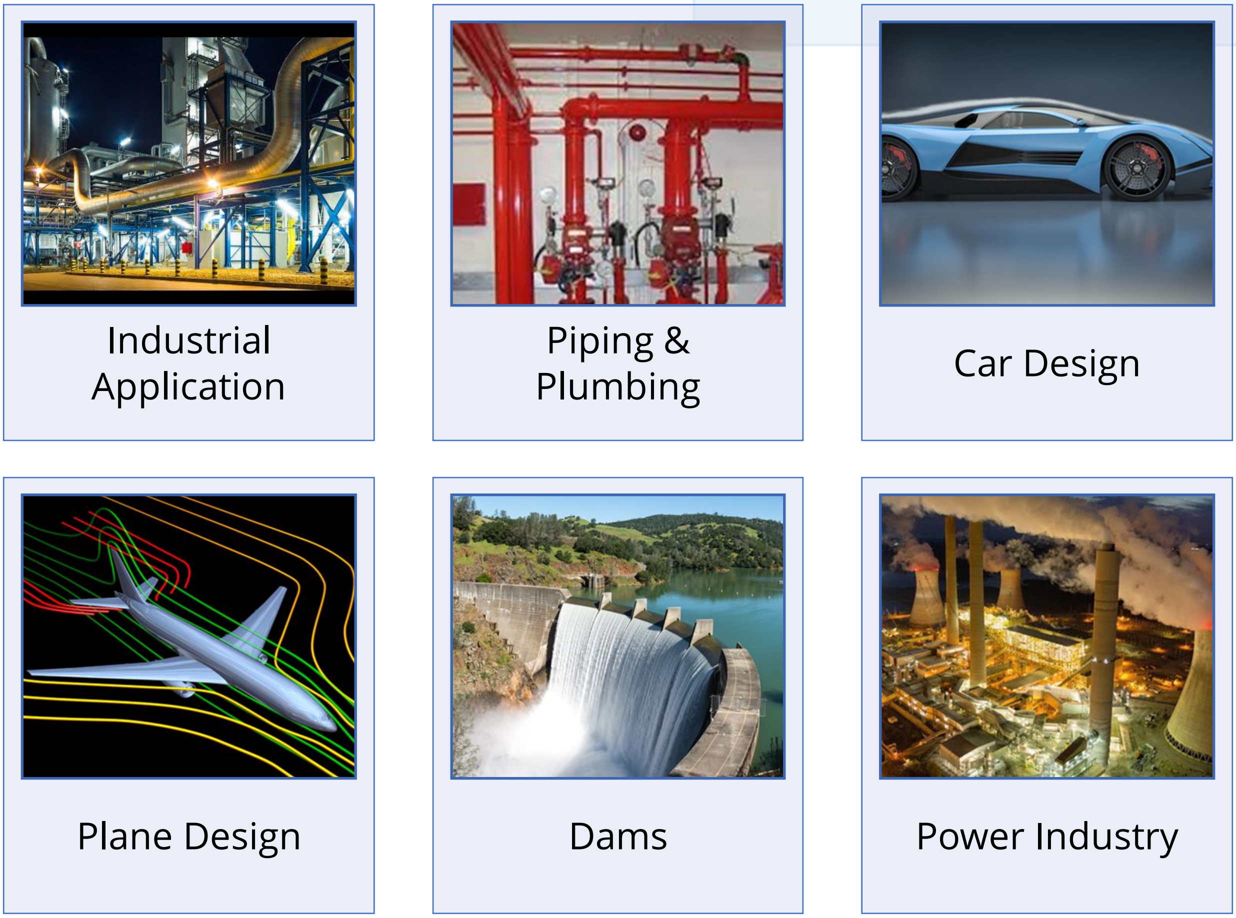


Figure 1.1: Some applications of Fluid Mechanics in engineering.

1.2 FLUID CHARACTERISTICS

Generally, matter exists in 3 states;

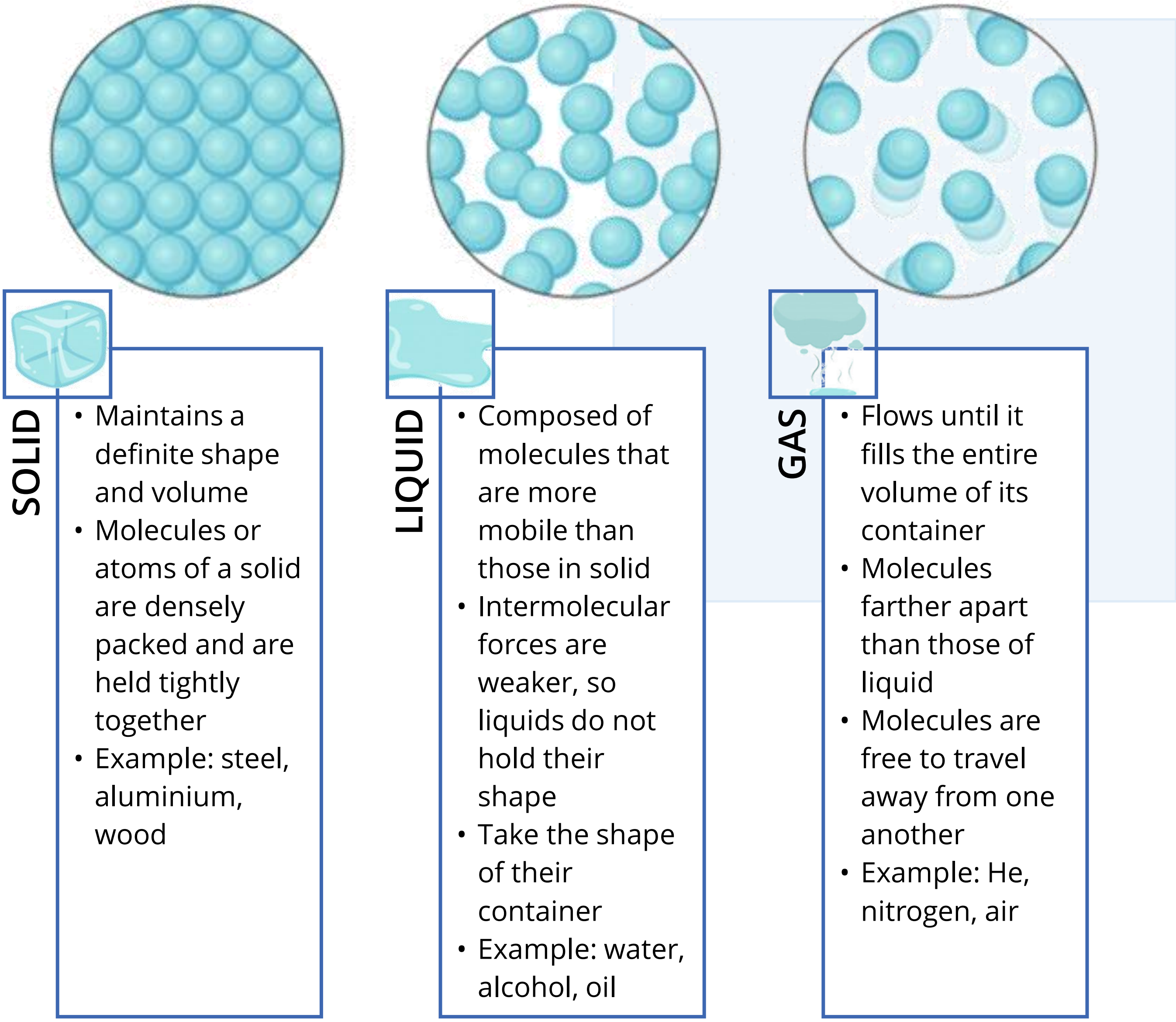


Figure 1.2: Characteristics of matter; solid, liquid and gas.

Although all states of matter is different in many aspects, liquid and gas have a common characteristics that differs them from solid state (Figure 1.2). Liquid and gas; both are **fluid** in nature.

Fluid refers to substance that can flow and by definition, fluid consist of substance in **liquid** or **gas** phase. Examples of fluid: water, oil, mercury, air, helium, oxygen.

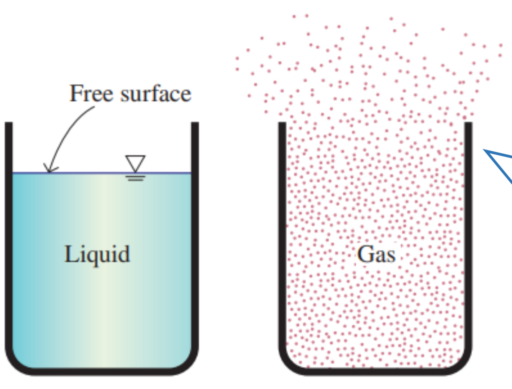
The general characteristics of fluid are;

a) Does not have a definite shape
b) Fluid will takes in the shape of the container it is in, although gas will expands until it encounters the walls of the container and fills the entire available space.
c) Fluid flows continuously under the influence of shear stress, no matter how small the shear stress is applied.
d) Fluid when at rest is at the state of zero shear stress.
e) Fluid contains molecules that are not closed bonded.

The comparison of fluid in liquid and gas phase is shown in Figure 1.3.

Liquid	Gas
Does not have a definite shape, but volume remain unchanged.	Have neither definite shape nor definite volume
Molecules move relative to each other.	Molecules are widely spaced, moves about randomly.
Cohesive forces between molecules stronger than gas.	Cohesive forces between molecules are very small.
Able to form free surface (Fig. 1.4) in open container.	Unable to form free surface (Fig. 1.4) .

Figure 1.3: Comparisons of liquid and gas phase.



Gas unable to form free surface because of the molecules are widely spaced and cohesive forces between them are too small.

Figure 1.4: Free surface of liquid vs gas.
(Source: Cengel & Cimbala)

1.3 DIMENSIONS AND UNITS

Any physical quantity can be characterized by **dimensions**. Magnitudes assigned to the dimensions are called **units**. A fluid and its flow characteristics can be described by combinations of units based on **primary** or **fundamental dimensions**; length, mass, time and temperature. Other dimensions such as velocity, pressure, volume and energy are expressed in terms of primary dimensions are called **secondary** or **derived dimensions**.

Unit systems that is widely used for scientific and engineering work in most industrialized nations is the metric **SI system**, also known as the **International System**. The SI unit system is a simple system based on a decimal relationship between various units. The primary dimensions and the common prefixes used to express multiples of various units in SI system is shown in Table 1.1.

Table 1.1: SI system primary dimensions and common prefixes.

Primary Dimension	Unit
Length, L	meter (m)
Mass, m	kilogram (kg)
Time, t	second (s)
Temperature, T	Kelvin (K)
Electric current, I	Ampere (A)
Luminous intensity, I_V	candela (cd)
Amount of matter, n	mole (mol)

		Exponential Form	Prefix
Multiple	1000 000 000 1000 000 1000 100	10^9 10^6 10^3 10^2	Giga (G) Mega (M) kilo (k) hecto (h)
Submultiple	0.01 0.001 0.000 001 0.000 000 001	10^{-2} 10^{-3} 10^{-6} 10^{-9}	centi (c) mili (m) micro (μ) nano (n)

1.4 TYPES OF PRESSURES

Pressure refers to physical force exerted on an object. In physics term, pressure is a force applied perpendicular to the surface of an object per unit area,

In Fluid Mechanics, pressure is defined by a normal force exerted by a fluid per unit area. It has units of **N/m^2**, which normally referred as **pascal (Pa)**. That is,

$$1\ Pa\ =\ 1\ N/m^2$$

The pressure unit pascal, is relatively too small for most pressures in engineering practice, therefore, kilopascal (1 kPa = 10^3 Pa) and megapascal (1 MPa = 10^6 Pa) is often used. Also, other pressure units commonly used in practice are; **bar** and **standard atmosphere**:

$$1\ bar\ =\ 10^5 Pa = 0.1\ MPa = 100\ kPa$$
$$1\ atm = 101325\ Pa = 101.325\ kPa = 1.01325\ bar$$

In perfect vacuum, which is completely empty space, the pressure is called **zero absolute pressure** (p = 0 Pa). **Absolute pressure, P_{abs}** refers to actual pressure measured above the zero absolute pressure (absolute vacuum). On that account, the standard **atmospheric pressure, P_{atm}** refers to the absolute pressure measured at sea level and at a temperature of 15°C. The value of atmospheric pressure is,

$$p_{atm} = 1.013 \times 10^5 N/m^2$$
$$= 101.3\ kPa = 1\ atm$$

Gauge pressure, P_g refers to any pressure measured above atmospheric pressure. It is often used to measure pressure relative to the atmospheric pressure.

Pressure measuring devices (gauges) are mostly calibrated to read zero in atmospheric surroundings. Thus, the gauge pressure readings indicate the difference between absolute pressure and atmospheric pressure. This pressure can be positive of negative, as pressures below atmospheric pressure are sometimes referred as **vacuum pressure, P_{vac}**. Absolute, gauge and vacuum pressures are related by,

$$p_g = p_{abs} - p_{atm} \quad \text{——— Eq. 1-1}$$

$$p_{vac} = p_{atm} - p_{abs} \quad \text{——— Eq. 1-2}$$

The relation between the pressures is illustrated in Figure 1.5

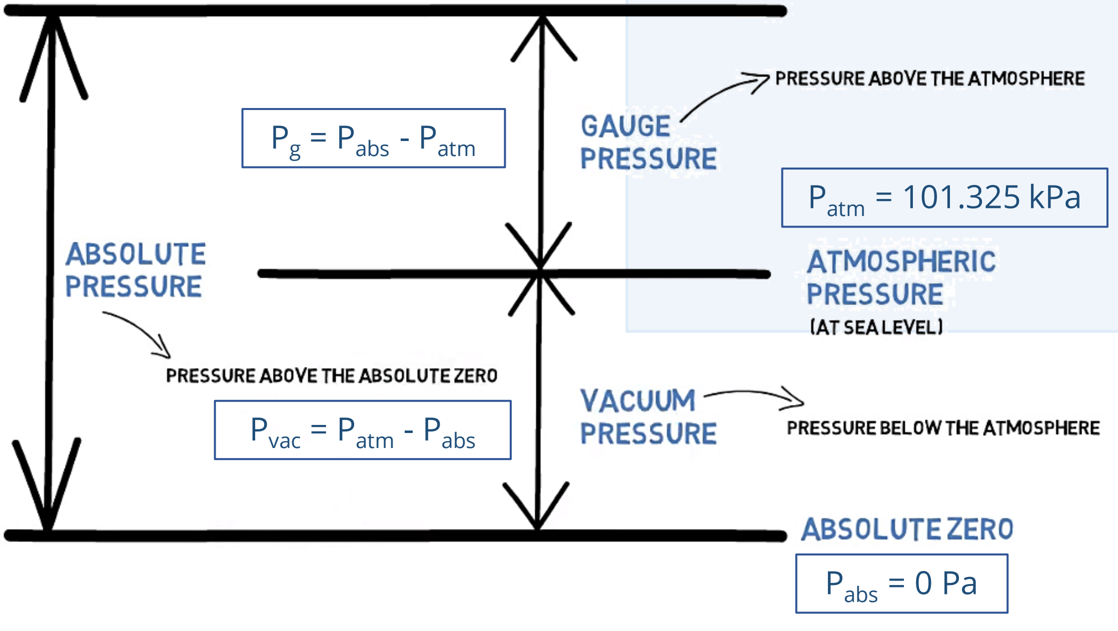
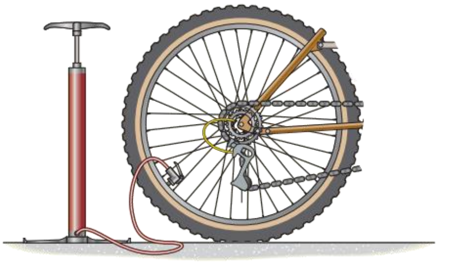


Figure 1.5: Relation between types of pressures.

Example 1.1:

The air pressure within the bicycle tyre is determined from a gauge to be 70 kPa in the figure shown. If the local atmospheric pressure is 104 kPa, determine the absolute pressure in the tire.



Given: $P_g = 70$ kPa; $P_{atm} = 104$ kPa

Solution:

Absolute pressure in the tyre,

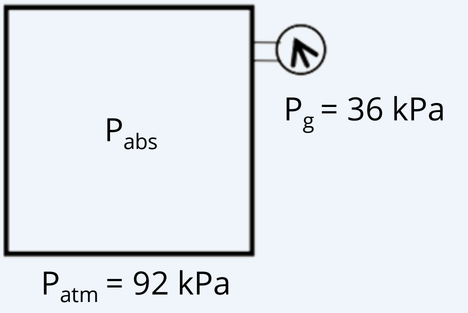
$$p_{abs} = p_{atm} + p_{gauge}$$

$$p_{abs} = 104 + 70 = 174\ kPa\ (Ans)$$

Example 1.2:

A vacuum gauge connected to a chamber reads 36 kPa at a location where the atmospheric pressure is 92 kPa. Determine the absolute pressure in the chamber.

Given: P_{vac} = 36 kPa; P_{atm} = 92 kPa



Solution:

Absolute pressure in the chamber,

$$p_{abs} = p_{atm} - p_{vac}$$
$$p_{abs} = 92 - 36 = 56\ kPa\ (Ans)$$

Example 1.3:

Calculate the pressure gauge of air in a cylinder when the measured absolute pressure is 350 kN/m^2 and the local atmospheric pressure is 101.3 kN/m^2.

Given: P_{abs} = 350 kN/m^2; P_{atm} = 101.3 kN/m^2

Solution:

Gauge pressure in the cylinder,

$$p_g = p_{abs} - p_{atm}$$
$$p_g = 350 - 101.3 = 248.7\ kN/m^2\ (Ans)$$

TUTORIAL 1.1

Q1-1

A pressure gauge connected to a tank reads 500 kPa at a location where the atmospheric pressure is 94 kPa. Determine the absolute pressure in the tank. [Ans: 594 kPa]

Q1-2

The vacuum pressure of a condenser is given to be 80 kPa. If the atmospheric pressure is 98 kPa, what is the absolute pressure?
[Ans: 18 kPa]

1.4 PRESSURE AND DEPTH

Pressure within a hydrostatic fluid varies due to the weight of the fluid. Consider a closed container filled with fluid (Figure 1.6), let P be the hydrostatic pressure at the depth of h from the top surface of fluid as follows,

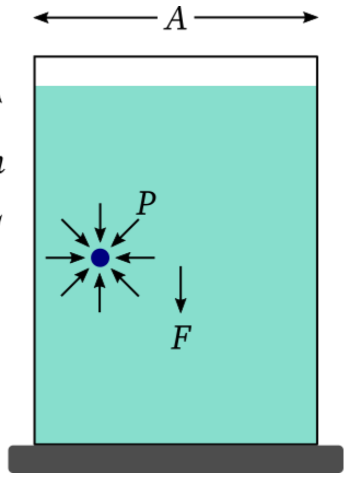


Figure 1.6: Fluid in closed container.

The pressure at the depth h is due to the pressure exerted by the fluid above it, that can be proven from pressure equation,

$$P = \frac{F}{A} = \frac{mg}{A}$$

Known, that, $\rho = \frac{m}{V}$, where $V = hA$, the mass of fluid becomes,

$$\rho = \frac{m}{V} = \frac{m}{hA} \rightarrow m = \rho hA$$

Eq. 1-3

Thus, substituting mass into pressure equation gives,

$$\therefore P = \frac{mg}{A} = \frac{\rho hAg}{A} = \rho gh$$

Eq. 1-4

From this equation, it can be concluded that, the pressure of hydrostatic fluid will increase linearly with depth. Pressure of fluid however does not change in horizontal direction. This is caused by more fluid rests on deeper layers, which is balanced by an increased in pressure. A simple experiment in Figure 1.7, demonstrated the variation of pressure in hydrostatic liquid with depth.

Nozzles at different depth **Nozzles at the same depth**

Figure 1.7: Variation of pressure with depth.

When an object submerged in water, with the top part touching the atmosphere as shown in Figure 1.8, the forces acting on the object are; weight of the object (W), force of atmosphere, (F_{atm}) pressing down and force of water (F_w) pushing up.

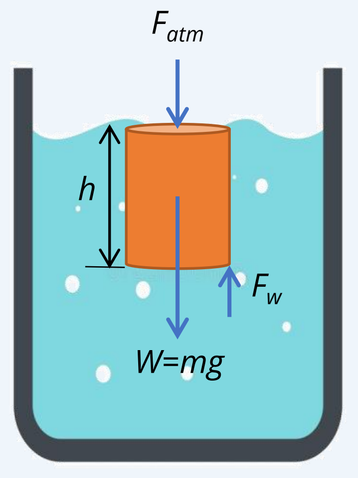


Figure 1.8: Forces acting on submerged object.

From the pressure equation,

$$F_{water} = PA, \quad F_{atm} = P_o A$$

Total forces in vertical direction,

$$F_w = F_{atm} + W$$

$$PA = P_o A + mg$$

Substituting $m = \rho h A$ from equation 1-4,

$$PA = P_o A + \rho A h g$$

Thus, pressure exerted on the object is given by,

$$P = P_o + \rho g h \quad \text{——— Eq. 1-5}$$

*where in this case, $P_0 = P_{atm}$

Variation of pressure in liquid with depth often applied in municipal water distribution systems, where water tanks or reservoir usually located at high elevation (Figure 1.9) to provide sufficient pressure flow for consumers located at lower levels. It also helps in dams construction, in which wall is constructed much thicker at the bottom to sustain high pressure at the bottom.

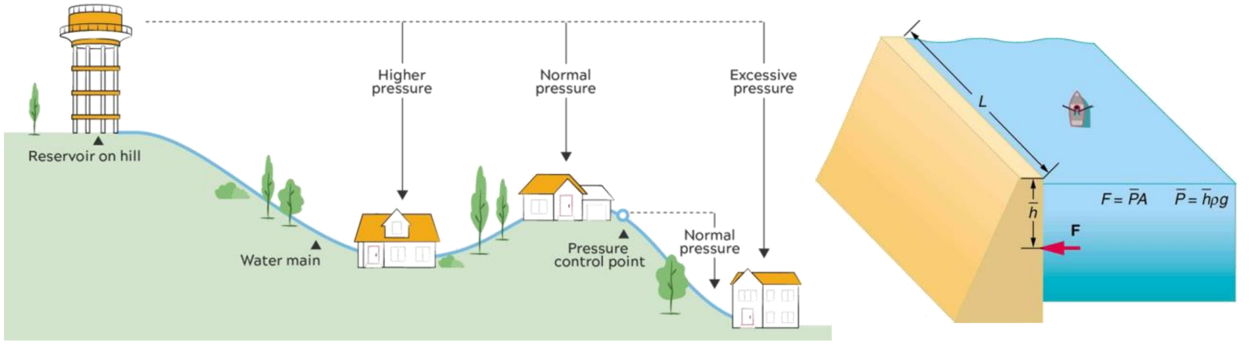
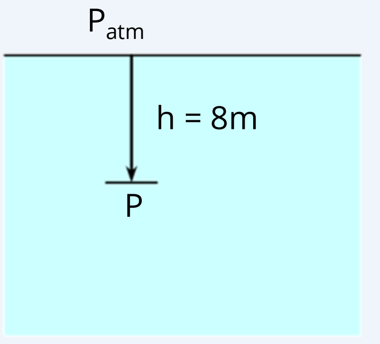


Figure 1.9: Applications of variation of pressure in liquid.

Example 1.4:

The absolute pressure in water at a depth of 8 m is measured to be 175 kPa. Determine,

a) the local atmospheric pressure, and
b) the absolute pressure at a depth of 8m in a liquid with density of 780 kg/m³.



Given: P_{abs} = 175 kPa; h = 8 m in water

Solution:

a) Local atmospheric pressure,

$$p_{atm} = p_g - \rho g h$$
$$p_{atm} = 175000 - 1000(9.81)(8)$$
$$= 96.52\ kPa\ (Ans)$$

b) Absolute pressure at h = 8 for ρ_f = 780 kg/m³,

$$p_{abs} = p_{atm} + \rho_f g h$$
$$p_{abs} = 96520 + 780(9.81)(8)$$
$$= 157.734\ kPa\ (Ans)$$

Example 1.5:

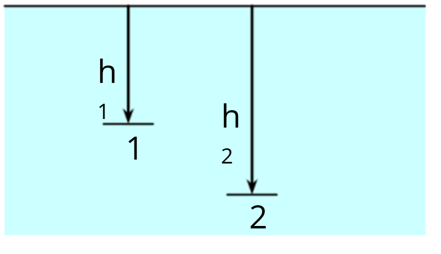
The gauge pressure in a liquid at a depth of 3 m is measured 28 kPa. Determine the gauge pressure in the same liquid at a depth of 12 m.

Given: P_g = 28 kPa at h_1 = 3m; at h_2 = 12 m, P_g = ?

Solution:

The pressures at two different depths of a liquid can be expressed as, $p_1 = \rho g h_1$ and $p_2 = \rho g h_2$, taking their ratio;

$$\frac{p_2}{p_1} = \frac{\rho g h_2}{\rho g h_1}$$



Solving for p_2,

$$p_2 = \frac{h_2}{h_1}(p_1) = \frac{12}{3}(28) = 112\ kPa\ (Ans)$$

Example 1.6:

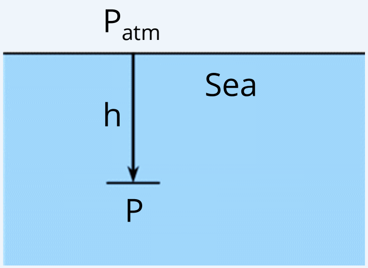
Determine the pressure exerted on a diver at 20 m below the free surface of the sea. Assume a barometric pressure of 101 kPa and density of seawater is 1030 kg/m^3.

Given: $h = 20$ m; $P_{atm} = 101$ kPa; $\rho_{sw} = 1030$ kg/m^3

Solution:

The pressure exerted on the diver at 20 m below surface of the sea,

$$p = p_{atm} + \rho_{sw} g h$$
$$= 101000 + 1030(9.81)(20)$$
$$= 303.086\ kPa\ (Ans)$$



TUTORIAL 1.2

Q1-3

Oxygen in a tank has an absolute pressure of 130 kPa. Determine the pressure head in mm of mercury. The atmospheric pressure is 102 kPa. Take the density of mercury, ρ = 13550 kg/m^3. [Ans: 286 mm]

Q1-4

The height of a head for a gas taken from gauge pressure reads 68 mm water and the height of mercury caused by atmospheric pressure is 750 mm mercury. Find the absolute pressure in kN/m^2.
[Ans: 100.7 kN/m^2]

Q1-5

Determine the pressure exerted on the surface of a submarine cruising 68.6 m below the free surface of the sea. Assume that the barometric pressure is 101.3 kPa and the density of the sea water is 1030 kg/m^3. [Ans: 794.5 kPa]

Q1-6

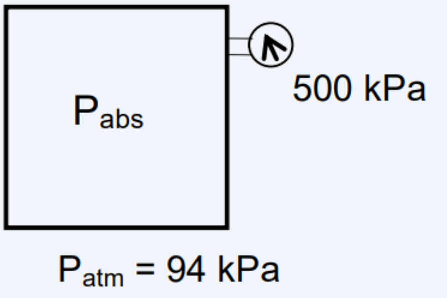
Consider a 1.73 m tall man standing vertically in water and completely submerged in a pool. Determine the difference between the pressures acting at the head and the toes of this man in kPa.
[Ans: 16.97 kPa]

TUTORIAL 1 WORKED SOLUTIONS

Q1-1

A pressure gauge connected to a tank reads 500 kPa at a location where the atmospheric pressure is 94 kPa. Determine the absolute pressure in the tank. [Ans: 594 kPa]

Given: P_{abs} = 500 kPa; P_{atm} = 94 kPa



Solution:

Absolute pressure in the tank,

$$p_{abs} = p_{atm} + p_{gauge}$$
$$p_{abs} = 94 + 500 = 594\ kPa\ (Ans)$$

Q1-3

Oxygen in a tank has an absolute pressure of 130 kPa. Determine the pressure head (in mm) of mercury. The atmospheric pressure is 102 kPa. Take the density of mercury, ρ = 13550 kg/m^3. [Ans: 286 mm]

Given: P_{abs} = 130 kPa; P_{atm} = 94 kPa

Solution:

Gauge pressure of the oxygen,

$$p_{abs} = p_{atm} + p_{gauge}$$
$$140 = 102 + p_{gauge}$$
$$p_{gauge} = 38\ kPa$$

Pressure head of mercury,

$$p_{gauge} = \rho_{Hg} g h_{Hg}$$
$$38000 = 13550(9.81)\ h_{Hg}$$
$$h_{Hg} = 0.2859\ m = 286\ mm\ (Ans)$$

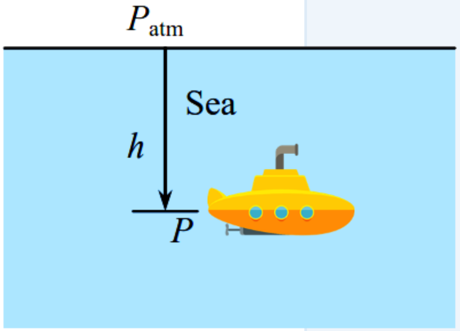
TUTORIAL 1 WORKED SOLUTIONS

Q1-5

Determine the pressure exerted on the surface of a submarine cruising 68.6 m below the free surface of the sea. Assume that the barometric pressure is 101.3 kPa and the density of the sea water is 1030 kg/m^3. [Ans: 794.5 kPa]

Given: h = 68.6 m; P_{atm} = 94 kPa

Solution:



The pressure exerted on the sub marine at 68.6 m below the free surface of the sea,

$$p = p_{atm} + \rho_{sw} g h$$
$$= 101300 + 1030(9.81)(68.6)$$
$$= 794.454\ kPa\ (Ans)$$

Chapter 2:

PHYSICAL PROPERTIES OF FLUID

2.1 BASIC PHYSICAL PROPERTIES OF FLUID

Physical property refers to any characteristics of a system that is measurable, in which the value describes the state of the physical system. A fluid has several important physical properties used to describe its behaviour. In this chapter, we will define its density, specific gravity (also known as relative density), specific weight, specific volume, fluid compressibility and viscosity .

Density. The **density**, **ρ** (rho) of a fluid is mass per unit volume of the substance. Density is measured in **kg/m^3** and can be determined from,

$$\rho = \frac{m}{V} \qquad \text{Eq. 2-1}$$

Density of a substance in general, depends on temperature and pressure. In most gases, density is proportional to pressure and inversely proportional to the temperature. For incompressible liquids, the variation of their density with pressure is negligible, however it varies heavily on temperature.

Specific gravity. In certain cases, density of a substance is given relative to the density of a well known substance. This is called specific gravity or ***relative density***. **Specific gravity, *SG*** is the ratio of the density of a substance to the density of standard substance at a specified temperature. Water at atmospheric pressure of 101.3 kPa and a temperature of 4°C is often taken as the standard. Thus,

$$SG = \frac{\rho}{\rho_w} = \frac{\gamma}{\gamma_w} \qquad \text{—— Eq. 2-2}$$

The density of water commonly used in this expression is 1000 kg/m³. This property is often used for liquids and is a dimensionless quantity. Note that, substances with SG value more than 1 is heavier than water, while the SG less than 1 are lighter, and would float on water.

Specific weight. The **specific weight**, **γ** (gamma) of a fluid is its weight per unit volume. It can be determine by,

$$\gamma = \frac{W}{V} \qquad \text{—— Eq. 2-3}$$

Also, since the weight of fluid is given by $W = mg$, and the density is $\rho = m/V$, the specific weight is related to density by,

$$\gamma = \frac{W}{V} = \frac{\rho m g}{m}$$

$$\therefore \gamma = \rho g \qquad \text{—— Eq. 2-4}$$

Specific weight is measured in **N/m³** and sometimes written as, ***w***.

Specific volume. The inverse of density of a substance is called the specific volume. **Specific volume**, ***v*** is defined as volume per unit mass. It is measured in **m³/kg** unit and expressed as,

$$\upsilon = \frac{V}{m} = \frac{1}{\rho} \qquad \text{—— Eq. 2-5}$$

Like density, specific volume also varies with temperature and pressure.

Example 2.1:

Determine the specific weight, γ of a fluid, which weighs 10 N and having 5 litre of volume.

Given: W = 10 N; V = 5 litre = 5 x 10^{-3} m^3

Solution:

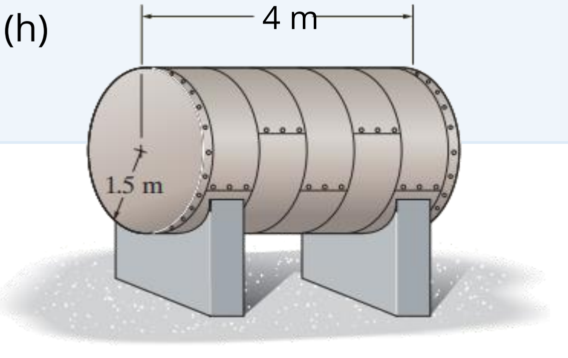
Specific weight,

$$\gamma = \frac{W}{V} = \frac{10}{5 \times 10^{-3}} = 2000\ N/m^3 (\text{Ans})$$

Example 2.2:

Determine the mass of the air contained in the tank below. Take, density of air; 0.628 kg/m^3.

Given: ρ_{air} = 0.628 kg/m^3 ; V = πr^2 (h)



Solution:

The mass of air within the tank is,

$$\rho = \frac{m}{V}$$

$$m = \rho V = 0.628(\pi \times 1.5^2 \times 4)$$

$$= 17.76\ kg\ (Ans)$$

Example 2.3:

A fluid that occupies a volume of 24 L weighs 225 N at a location where the gravitational acceleration is 9.80 m/s^2. Determine the mass of this fluid and its density.

Given: V = 24 L = 24 x 10^{-3} m^3 ; W = 225 N;

Solution:

The mass of fluid,

$$W = mg$$

$$m = \frac{W}{g} = \frac{225}{9.8} = 22.96\ kg\ (Ans)$$

Density of fluid,

$$\rho = \frac{m}{V} = \frac{22.96}{24 \times 10^{-3}}$$

$$= 956.63\ kg/m^3 (\text{Ans})$$

Example 2.4:

Determine the density, specific volume and specific weight of a fluid with a mass of 450 g and volume of 900 cm^3.

Given: m = 450 g = 0.45 kg; V = 900 cm^3 = 900 x 10^{-6} m^3

Solution:

Density of fluid,

$$\rho = \frac{m}{V} = \frac{0.45}{900 \times 10^{-6}}$$
$$= 500\ kg/m^3 (\text{Ans})$$

The specific volume,

$$\upsilon = \frac{V}{m} = \frac{1}{\rho} = \frac{1}{500}$$
$$= 0.002 m^3/kg (Ans)$$

The specific weight,

$$\gamma = \frac{W}{V} = \frac{mg}{V}$$
$$= \frac{0.45(9.81)}{900 \times 10^{-6}}$$
$$= 4905\ N/m^3 (\text{Ans})$$

Example 2.5:

Calculate the density, specific volume and specific weight of a fluid with specific gravity, SG of 0.89.

Given: SG = 0.89 ;

Solution:

Density of fluid,

$$SG = \frac{\rho}{\rho_w}$$
$$\rho = SG \rho_w = 0.89(1000)$$
$$= 890\ kg/m^3 (Ans)$$

The specific volume,

$$\upsilon = \frac{1}{\rho} = \frac{1}{890}$$
$$= 0.00112\ m^3/kg (Ans)$$

The specific weight,

$$\gamma = \rho g = 890\ (9.81)$$
$$= 8730.9\ N/m^3 (\text{Ans})$$

Example 2.6:

The fuel of a jet engine has a density of 680.3 kg/m^3 . If the total volume of the fuel tank A is 1420 litre, determine the weight of the fuel when the tanks are completely full.

Given: ρ_f = 680.3 kg/m^3 ; V = 1420 L = 1.42 m^3

Solution:

Specific weight of jet engine fuel,

$$\gamma_f = \rho_f g = 680.3 \times 9.81$$
$$= 6673.743\ N/m^3$$

Thus, total weight of the fuel when the tank is full,

$$W = \gamma_f(V) = 6673.743(1.42)$$
$$= 9476.7\ N = 9.48\ kN(\text{Ans})$$

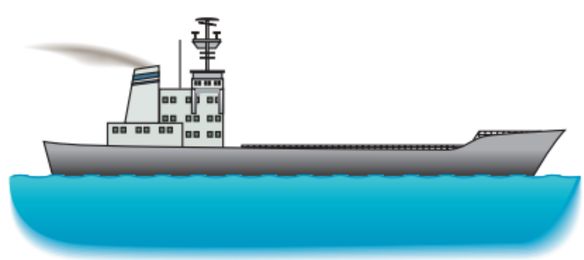
Example 2.7:

The tanker carries 1.5 x 10^6 barrels of crude oil in its hold. Determine the total weight of the oil if its specific gravity is 0.940. Each barrel contains 159 litres.

Given: SG= 0.940; No of barrel = 1.5 x 10^6;
V per barrel = 159 L = 159 x 10^{-3} m^3

Solution:

Specific weight of jet engine fuel,



$$SG = \frac{\gamma_o}{\gamma_w}$$
$$\gamma_o = SG(\gamma_w) = 0.94\ (9810)$$
$$= 9221.4\ N/m^3$$

Thus, total weight of crude oil on the tanker,

$$W = \gamma_f(V) = 6673.743(1.42)$$
$$= 9476.7\ N = 9.48\ kN(\text{Ans})$$

TUTORIAL 2.1

Q2-1

Mercury has a specific weight of 133 kN/m^3 when the temperature is 20°C. Determine its density and specific gravity at this temperature. [Ans: 13557.6 kg/m^3, 13.6]

Q2-2

Determine the mass and the weight of the air contained in a room with dimensions are 6m x 6m x 8m. Assume the density of the air is 1.16 kg/m^3. [Ans: 334 kg, 3.27 kN]

Q2-3

The cargo ship carries 85800 barrels of crude oil in its hold. Determine the weight of the oil if its specific gravity is 0.940. Each barrel contains 159 litres. [Ans: 1.26 GN]

Q2-4

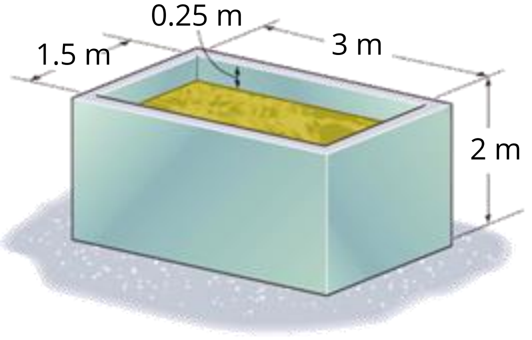
A 1m diameter cylindrical container is filled with water at a depth of 2.5 m. If the container has a mass of 30 kg, determine he combined weight of the container and the water. [Ans: 19.6 kN]

Q2-5

A cylindrical tank of methanol has a mass of 40 kg and a volume of 51 L. Determine the methanol's weight, density and specific gravity. Take the gravitational acceleration to be 9.81 m/s^2.
[Ans: 392.4 N, 784.3 kg/m^3, 0.784]

Q2-6

The tank contains a liquid having a density of 1.3 Mg/m^3. Determine the weight of the liquid when it is at the level shown. [Ans: 100.43 kN]



2.2 COMPRESSIBILITY

Compressibility. **Compressibility, 1/κ** (kappa) is a measure of relative volume change in response to pressure or temperature change. Compressibility is most commonly referred as the reciprocal of the bulk modulus of elasticity (also called **coefficient of compressibility** or **bulk modulus of compressibility**). **Bulk modulus of elasticity, κ** is define as ratio of compressive stress to volumetric strain and written as,

$$\kappa = \frac{increase\ of\ pressure}{volumetric\ strain} = -V\left(\frac{dp}{dV}\right) = \rho\left(\frac{dP}{d\rho}\right) \quad \text{——— Eq. 2-6}$$

$$\therefore Compressibility = \frac{1}{\kappa} \quad \text{——— Eq. 2-7}$$

Bulk modulus and compressibility both is measured in the dimension of pressure, **Pa** or **N/m²**. Fluid usually expands when heated (depressurized) and contracts when cooled (pressurized). The amount of volume change is different for different fluid. The compressibility in liquid and gas is illustrated in Figure 2.1 below;

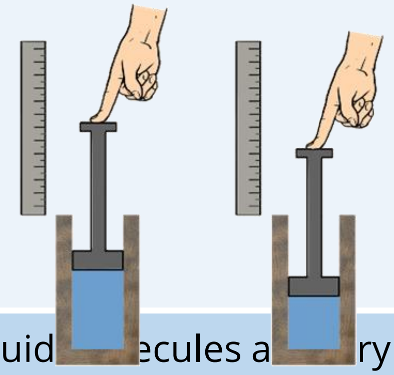
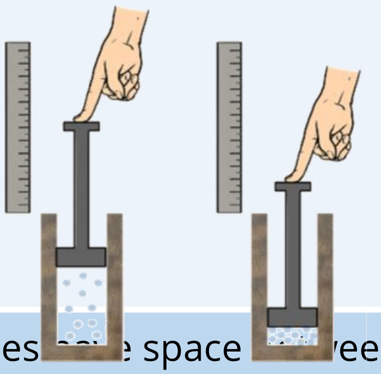
LIQUID	GAS
	
• Liquid ecules a ry close together, thus it do <u>NOT</u> compress easily. • Has **high** bulk modulus (κ). • Compressibility (1/κ) is **low**.	• Gases e space een molecules, thus it <u>CAN</u> be compressed easily. • Has **low** bulk modulus (κ). • Compressibility (1/κ) is **high**.

Figure 2.1: Compressibility of liquid and gas.

Example 2.8:

Determine the bulk modulus of elasticity of a liquid, if the pressure of the liquid is increased from 70 kPa to 130 kPa. The volume of the liquid decreases by 0.15 %.

Given: P_1 = 70 kPa ; P_2 = 130 kPa; decrease of V = 0.15 %

Solution:

Increase in pressure,

$$dp = P_2 - P_1 = 130 - 70 = 60\ kPa$$

Decrease in volume,

$$-\frac{dV}{V} = \frac{0.15}{100} = 0.0015$$

Bulk modulus of elasticity,

$$\kappa = -V\left(\frac{dp}{dV}\right) = -\frac{V}{dV}dp = \frac{60}{0.0015}$$
$$= 40000\ kPa = 40\ MPa\ (Ans)$$

TUTORIAL 2.2

Q2-7

Determine the bulk modulus of elasticity of a liquid which is compressed in a cylinder from a volume of 0.0125 m^3 at 80kPa to a volume of 0.0124 m^3 at 150 kPa pressure. [Ans: 8.75 MPa]

Q2-8

An amount of glycerin has a volume of 1 m^3 when the pressure is 120 kPa. If the pressure is increased to 400 kPa, determine the change in volume of the glycerin. The bulk modulus for glycerin is κ = 4.52 GPa. [Ans: - 61.9 x 10^{-6} m^3]

2.3 VISCOSITY

Viscosity. **Viscosity, μ** (mu) is a property of fluid that measure the internal resistance of fluid to motion or flow. The nature of fluid is that they continuously deform (or flow) when subjected to shear force. When a small force, F is applied to a thin plate, the adjacent thin layer of fluid will be dragged along, resulting deformation. This deformation can be depicted in the Figure 2.2 below,

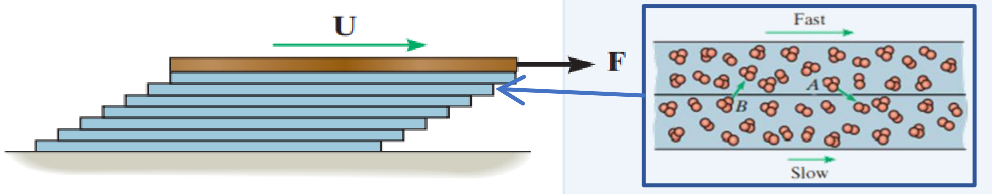


Figure 2.2: Deformation of fluid when subjected to shear force. (source: Hibbeler)

The top layer causes shear stress on the adjacent lower layer, while the lower layer causes shear stress on the adjacent top layer, causing resistance. This shear stress is proportional to the rate of change of velocity with respect to y (vertical distance from the bottom). Thus, **shear stress, τ** is expressed as,

$$\tau = \mu \frac{du}{dy} \rightarrow \mu = \frac{\tau}{du/dy} = \tau \frac{dy}{dx} \quad \text{——— Eq. 2-8}$$

Where, **μ** is the viscosity in **Ns/m²** or **Poise** for CGS (centimetre-gram-second) system and **τ** is measured in pressure unit, **Pa** or **N/m²**.

Viscosity is an important property of fluid because it causes internal friction within the fluid, which results in energy loss that must be taken into account when designing vehicles or conduits such as pipes and channels. Higher viscosity of fluid means higher resistance of fluid to motion. Also, note that, it is easier to move through air (low viscosity) compared to water (high viscosity) as shown in Figure 2.3.

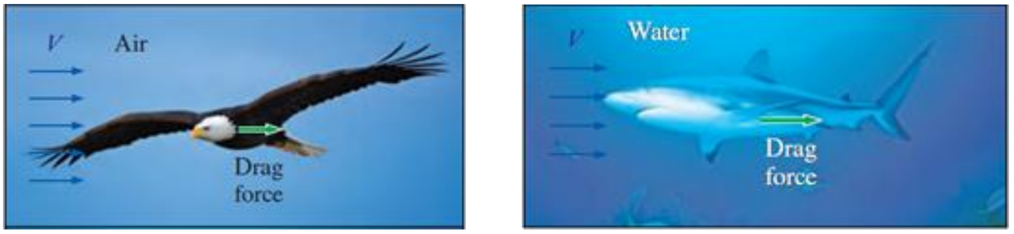
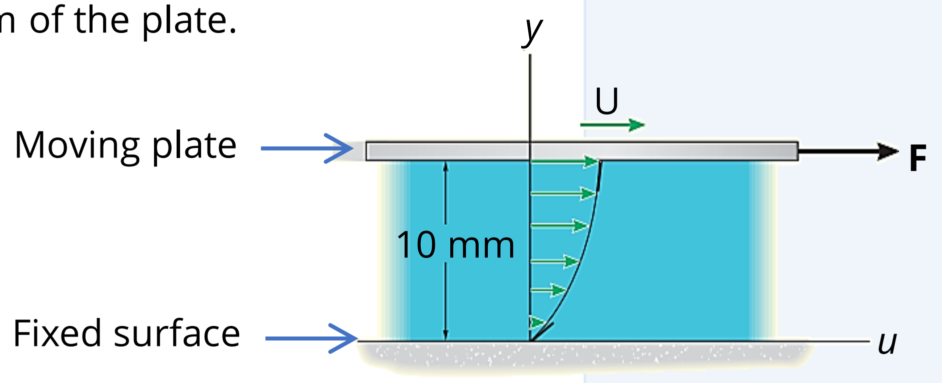


Figure 2.3: Resistance of movement in air and water. (source: Cengel and Cimbala)

Example 2.9:

A plate rests on top of the thin film of water. If a small force F is applied to the plate, the velocity profile across the thickness of the water is given by u = (40y – 800 y^2) m/s, at a distance y metre above the fixed plate. Take, dynamic viscosity of water as 0.897 x 10^{-3} Ns/m^2. Determine the shear stress acting on the fixed surface and on the bottom of the plate.



Given: u = (40y – 800 y^2) m/s;
μ = 0.897 x 10^{-3} Ns/m^2

Solution:

Velocity gradient,

$$\frac{du}{dy} = \frac{d}{dy} 40y - 800y^2 = 40 - 1600y$$

Thus, shear stress on the fixed surface, y = 0,

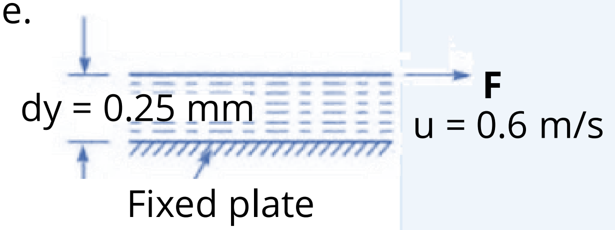
$$\tau = \mu \left.\frac{du}{dy}\right|_{y=0} = 0.897 \times 10^{-3}(40 - 0)$$
$$= 0.0359\ N/m^2 (Ans)$$

Shear stress on the bottom of the moving plate, y = 0.01m,

$$\tau = \mu \left.\frac{du}{dy}\right|_{y=0.01} = 0.897 \times 10^{-3}(40 - 1600(0.01))$$
$$= 0.0215\ N/m^2 (Ans)$$

Example 2.10:

A plate is placed on a thin film of fluid at a distant of 0.25 mm from a fixed plate. The plate moves at 0.6 m/s and requires a shear force of 2 N/m^2 to maintain this speed. Determine the fluid viscosity between the plates in Poise.



Given: dy = 0.25 mm = 0.25 x 10^{-3} m; u = 0.6 m/s; τ = 2 N/m^2

Solution:

Change of velocity,

$$du = u - 0 = 0.60\ m/s$$

Thus, the viscosity of fluid between the plates,

$$\mu = \tau \frac{dy}{du} = (2) \frac{0.00025 \times 10^{-3}}{0.6}$$

$$= (0.833\ \times 10^{-3}) Ns/m^2 \times \frac{10\ Poise}{1\ Ns/m^2}$$

$$= 0.00833\ \boldsymbol{Poise}\ (Ans)$$

TUTORIAL 2.3

Q2-9

If the velocity distribution over a plate is given by u = 2/3y – y^2 in which u is the velocity in m/s at a distance y metre above the plate, determine the shear stress at y = 0 and y = 0.15 m. Take dynamic viscosity of fluid as 0.863 Ns/m^2. [Ans: 0.5753 N/m^2, 0.3164 N/m^2]

Q2-10

The space between two square plates is filled with oil. The thickness of oil film is 12.5 mm. The upper plate moves at 2.5 m/s and requires a shear force of 272.5 N/m2 to maintain this velocity. Determine the dynamic viscosity of the oil in Poise. [Ans: 13.63 Poise]

TUTORIAL 2 WORKED SOLUTIONS

Q2-2

Determine the mass and the weight of the air contained in a room with dimensions are 6m x 6m x 8m. Assume the density of the air is 1.16 kg/m^3. [Ans: 334 kg, 3.27 kN]

Given: Volume = 6 m x 6 m x 8 m = 288 m^3; ρ_{air} = 1.16 kg/m^3

Solution:

Mass of the air in the room,

$$\rho_{air} = \frac{m}{V}$$

$$m = \rho V = 1.16(288) = 334.08\ kg\ (Ans)$$

Hence, the weight of air in the room,

$$W = mg = 334.08(9.81)$$

$$= 3277.325\ N = 3.27\ kN\ (Ans)$$

Q2-4

A 1m diameter cylindrical container is filled with water at a depth of 2.5 m. If the container has a mass of 30 kg, determine he combined weight of the container and the water. [Ans: 19.6 kN]

Given: Volume of water = 2.5(π/4) = 1.9635 m^3; m_{cont} = 30 kg

Solution:

Mass of water in container,

$$\rho_w = \frac{m_w}{V}$$

$$m_w = \rho_w V = 1000(1.9635) = 1963.5\ kg\ (Ans)$$

Total mass,

$$m = m_w + m_{container}$$

$$= 1963.5 + 30 = 1993.5\ kg$$

Thus, total weight,

$$W = mg = 1993.5(9.81) = 19556.19\ N = 19.56\ kN\ (Ans)$$

TUTORIAL 2 WORKED SOLUTIONS

Q2-8

An amount of glycerin has a volume of 1 m^3 when the pressure is 120 kPa. If the pressure is increased to 400 kPa, determine the change in volume of the glycerin. The bulk modulus for glycerin is κ = 4.52 GPa. [Ans: - 61.9 x 10^{-6} m^3]

Given: V = 1 m^3 ; κ = 4.52 GPa = 4.52 x 10^{-9} kPa;
P_1 = 120 kPa = 120 kN/m^2; P_2 = 400 kPa = 400 kN/m^2;

Solution:

The increase in pressure,

$$dp = P_2 - P_1 = 400 - 120 = 280\ k\text{N}/m^2$$

Thus, the change in volume,

$$\kappa = -V\left(\frac{dp}{dV}\right) \rightarrow \frac{dV}{V} = -V\frac{dp}{\kappa} = -(1)\frac{280 \times 10^3 k\text{N}/m^2}{4.52 \times 10^{-9} k\text{N}/m^2}$$

$$\frac{dV}{V} = -61.9 \times 10^{-6}\ m^3 (Ans)$$

Q2-10

The space between two square plates is filled with oil. The thickness of oil film is 12.5 mm. The upper plate moves at 2.5 m/s and requires a shear force of 272.5 N/m2 to maintain this velocity. Determine the dynamic viscosity of the oil in Poise. [Ans: 13.63 Poise]

Given: dy =12.5 mm = 0.0125 m; u = 2.5 m/s; τ = 272.5 N/m^2

Solution:

Change of velocity,

$$du = u - 0 = 2.5\ m/s$$

The viscosity of fluid between the plates,

$$\mu = \tau\frac{dy}{du} = (272.5)\frac{0.0125}{2.5} = 1.3625\ Ns/m^2 \times \frac{10\ Poise}{1\ Ns/m^2}$$
$$= 13.625\ \boldsymbol{Poise}\ (Ans)$$

Chapter 3:

FLUID STATICS

3.1 FLUID STATICS

Fluid statics is the study of incompressible fluids at rest or flow with constant velocity. In fluid statics, there is no relative motion between adjacent fluid layers, thus, no shear stress in the fluid trying to deform it. The only stress in fluid statics is the normal stress due to pressure. The variation of pressure is mainly depends on the weight of fluid.

Fluid statics often provides physical explanations for many daily life phenomena such as the change of pressure with altitude, the floating and submerging of matters in liquid and why the surface of water is always level. The engineering application of fluid statics include; the measurement of pressure using hydrostatics, floating or submerged bodies, water dams and gates, liquid storage tanks, etc.

The physical characteristics of static or stationary fluids and the laws that govern their behaviour, discussed in this chapter are:

- Pascal's Law
- Measurement of static pressure using Piezometer, Barometer, Manometer and Bourdon Gauge
- Buoyancy

3.2 PASCAL'S LAW

Pressure in static fluid is independent of the shape or cross section of the container. The pressure changes with altitude (vertical distance), but remains constant in horizontal directions. When liquid is poured into a set of connected tubes of different shapes, it rises up until the levels ate the same in all tubes, as demonstrated in Figure 3.1 below. Note that the pressure at point **A**, **B**, **C**, **D** and **E** are the same since they are at the same depth.

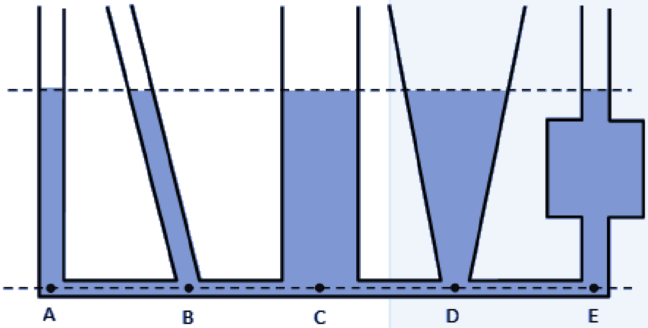


Figure 3.1: Pressure remains constant in horizontal directions.

A consequence of the pressure in a fluid remaining constant in horizontal direction is that the pressure applied to a confined fluid increases the pressure throughout by the same amount. This is known as **Pascal's Law** after Blaise Pascal (1623 – 1662), a French Mathematician.

Pascal's Law stated that the intensity of the pressure acting at a point the a fluid is the same in all direction. Hence, in an enclosed fluid at rest, a change in the pressure applied to a fluid is transmitted undiminished to every point of the fluid and to the walls of the container (Figure 3.2).

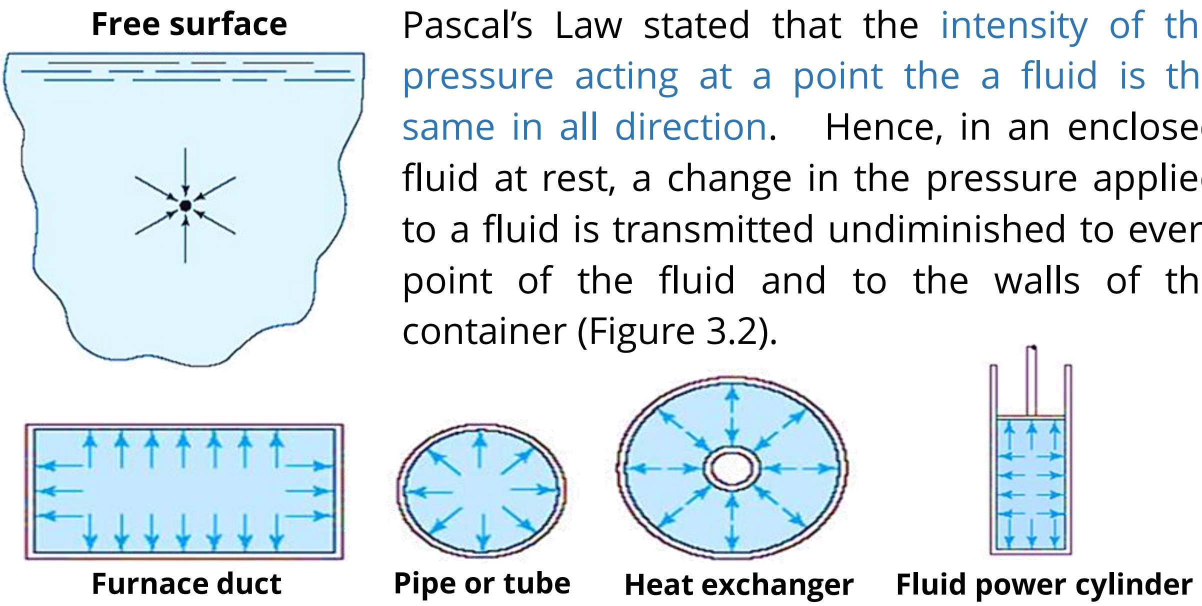


Figure 3.2: Pressure applied to a fluid transmitted perpendicular to the wall of container.

Pascal also founded that the force applied by a fluid is proportional to the surface area. Therefore, when two hydraulic cylinders of different surface areas could be connected and the larger could be used to exert greater force proportional to the force applied to the smaller. This is called the **Pascal's Machine**. This enables the lifting of heavy masses by only one arm, as shown in the Figure 3.3 below.

Pressure P_1 is equal to P_2 because of the pistons are at the same level. Ratio of output force to input force is determined by,

$$P_1 = P_2$$

$$\frac{F_1}{A_1} = \frac{F_2}{A_2} \rightarrow \frac{F_2}{F_1} = \frac{A_2}{A_1} \quad \text{— Eq. 3-1}$$

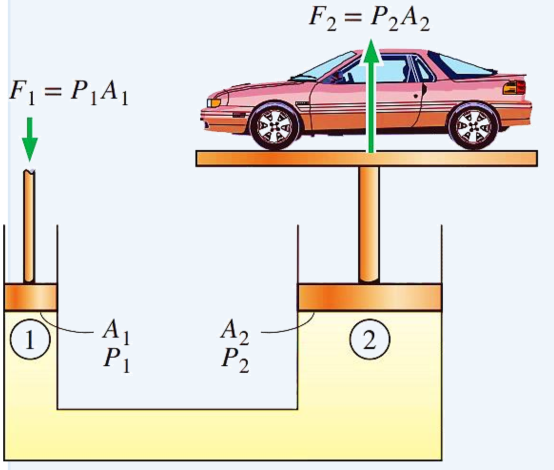


Figure 3.3: Hydraulic jack.
(source: Cengel & Cimbala)

The Pascal's law application are widely used in hydraulic lifts, hydraulics or air brakes, hydraulic jacks and forklifts. Also, Pascal's law application can also be seen in the following (Figure 3.4);

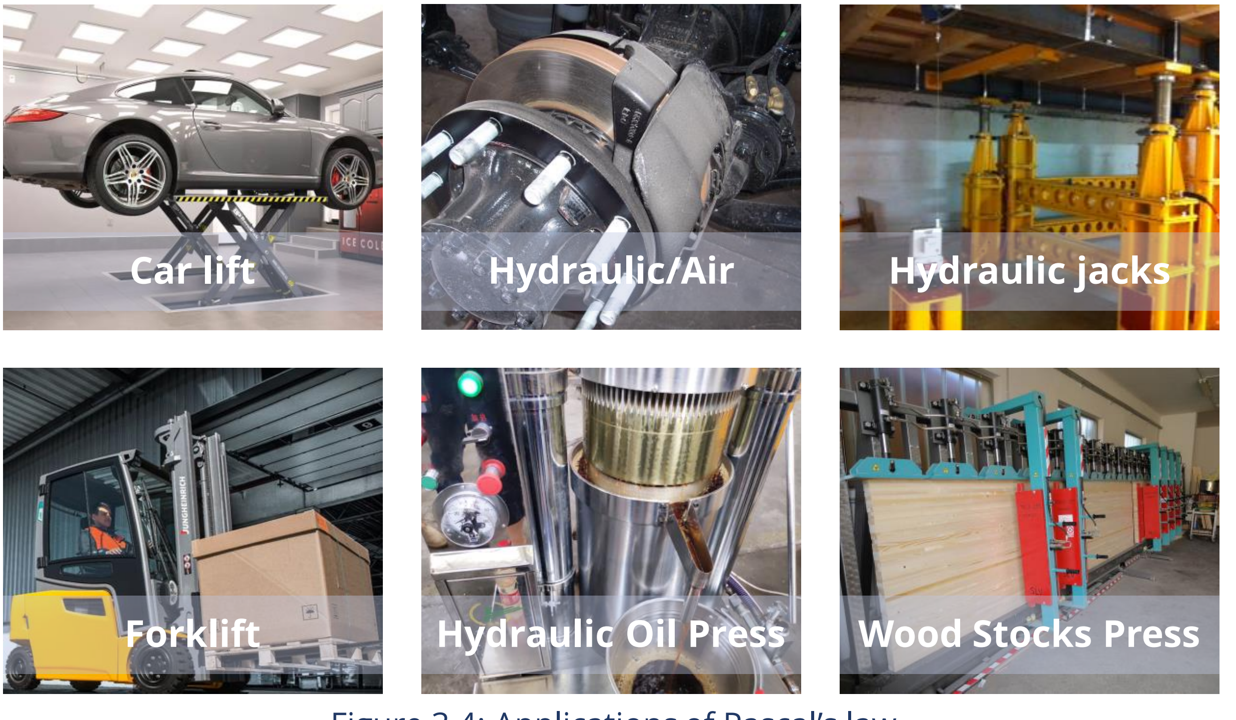


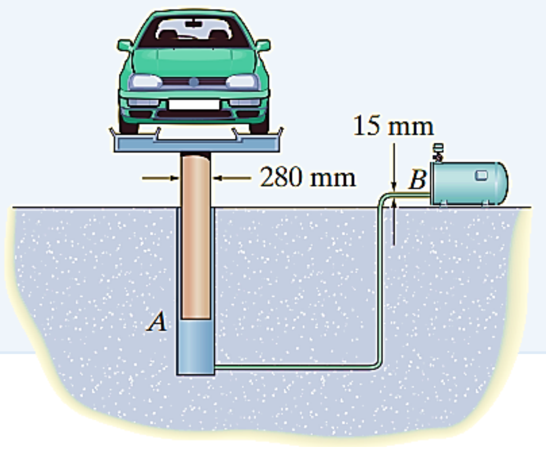
Figure 3.4: Applications of Pascal's law.

Example 3.1:

A pneumatic jack is used in a service station as shown in figure. If the car and lift weigh 25 kN, determine the force that must be developed by the air compressor at B to raise the lift at a constant velocity. Air is in the line from B to A. The air line at B has an inner diameter of 15 mm, and the post at A has a diameter of 280 mm.

Given: F_A = 25 kN = 25000 N; d_A = 280 mm = 0.28 m, r_A = 0.14 m; d_B = 15 mm = 0.015 m, r_B = 0.0075 m

Solution:



Due to equilibrium, the force created by air pressure at A is equal and opposite to weight of car and lift. Therefore,

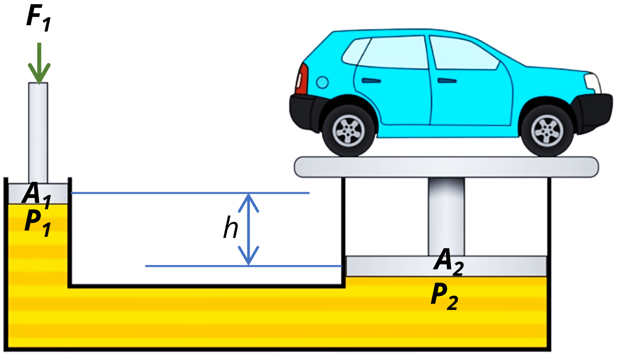
$$P_A = P_B \rightarrow \frac{F_A}{A_A} = \frac{F_B}{A_B}$$

$$\frac{25000}{\pi(0.14^2)} = \frac{F_B}{\pi(0.0075^2)}$$

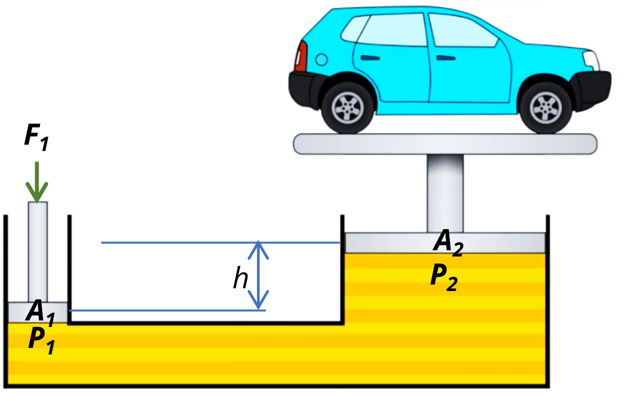
$$F_B = 71.747\ N\ (Ans)$$

This small force of 71.7 N is sufficient to lift the 25 kN load.

In the application of the hydraulic jack, the lifting and lowering jacks can also be considered as follows;



$$P_2 = P_1 + \rho g h = \frac{F_1}{A_1} + \rho g h$$



$$P_2 = P_1 - \rho g h = \frac{F_1}{A_1} - \rho g h$$

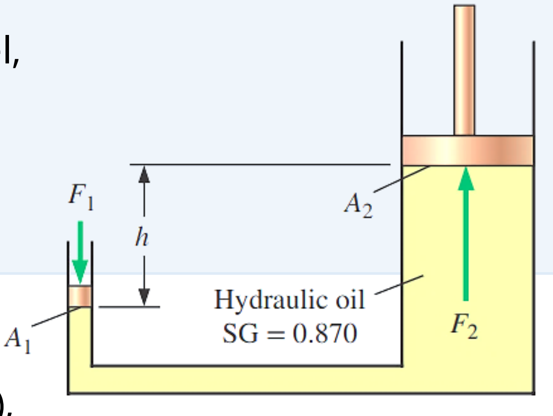
Example 3.2:

A hydraulic jack being used in a car repair shop, as in figure. The pistons have an area of A_1 = 0.8 cm² and A_2 = 0.04m². Hydraulic oil with a specific gravity of 0.87 is pumped in as the small piston on the left side is pushed up and down, slowly rising the larger piston on the right side. A car that weighs 13 000 N is to be jacked up. Calculate,

a) The force F_1 required to hold the weight of car at the beginning.
b) The force F_1 after the car has been lifted 2 m.

Given: $A_1 = 0.8\ cm^2 = 0.8 \times 10^{-4}\ m^2$; $A_2 = 0.04 m^2$;
SG = 0.870; F_2 = 13000 N

Solution:



a) When both piston at the same level,

$$P_1 = P_2 \rightarrow \frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$$F_1 = \frac{13000}{0.04}(0.8 \times 10^{-4})$$

$$= 26\ N\ (Ans)$$

b) When car being lifted 2 m (h = 2 m),

Density of hydraulic oil,

$$SG = \frac{\rho}{\rho_{H_2O}} \rightarrow \rho = SG(\rho_{H_2O})$$

$$\rho = 0.870(1000) = 870\ kg/m^3$$

Hence,

$$P_1 = P_2 + \rho g h$$

$$\frac{F_1}{A_1} = \frac{F_2}{A_2} + \rho g h$$

$$F_1 = \left(\frac{13000}{0.04} + (870)(9.81)(2)\right)(0.8 \times 10^{-4})$$

$$= 27.366\ N\ (Ans)$$

Example 3.3:

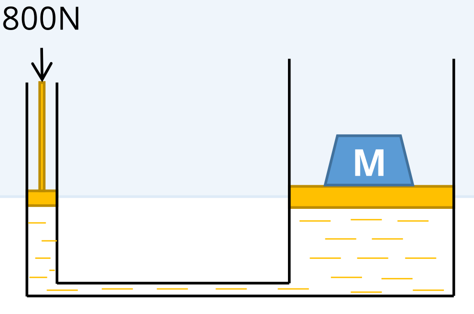
A force, F = 800 N is applied to the smaller cylinder of a hydraulic jack. The area of the small piston is 20 cm^2 while the area of the larger piston is 250 cm^2. Determine the mass that can be lifted on the larger piston if; (Take, ρ = 870 kg/m^3)

a) The pistons are at the same level.
b) The large piston is 0.7 m below the smaller piston.
c) The small piston is 0.55 m below the larger piston.

Given: F_1 = 800 N; F_2 = Mg; ρ = 870 kg/m^3;
A_1 = 20 cm^2 = 0.002 m^2; A_2 = 250 cm^2 = 0.025 m^2

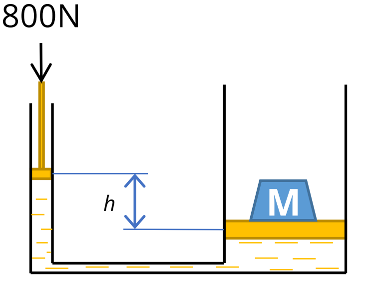
Solution:

a) When pistons at the same level (h = 0),



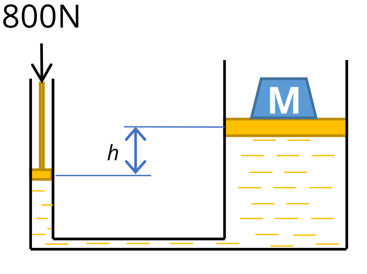
$$P_1 = P_2 \rightarrow \frac{F_1}{A_1} = \frac{F_2}{A_2}$$
$$F_2 = \frac{800}{0.002}(0.025)$$
$$M = \frac{10000}{9.81} = 1019.37\ kg\ (Ans)$$

b) When larger piston is 0.7 m below small piston (h = 0.7 m)



$$P_2 = P_1 + \rho gh \rightarrow \frac{F_2}{A_2} = \frac{F_1}{A_1} + \rho gh$$
$$F_2 = \left(\frac{800}{0.002} + 870(9.81)(0.7)\right)(0.025)$$
$$M = \frac{11493.57}{9.81} = 1071.62\ kg\ (Ans)$$

c) When smaller piston is 0.55 m below large piston (h = 0.55 m)

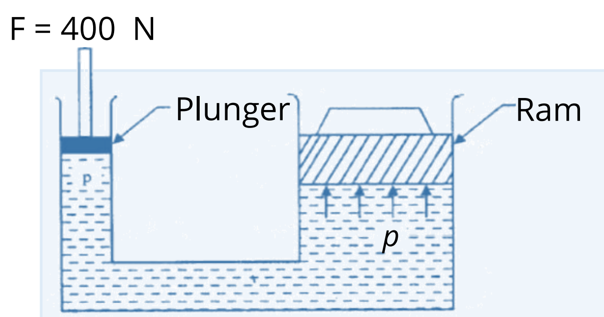


$$P_2 = P_1 + \rho gh \rightarrow \frac{F_2}{A_2} = \frac{F_1}{A_1} + \rho gh$$
$$F_2 = \left(\frac{800}{0.002} - 870(9.81)(0.55)\right)(0.025)$$
$$M = \frac{9982.65}{9.81} = 1007.41\ kg\ (Ans)$$

TUTORIAL 3.1

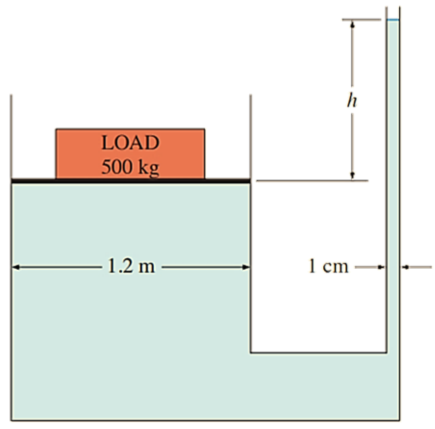
Q3-1

The diameters of ram and plunger of a hydraulic press are 200 mm and 30 mm respectively. Find the weight lifted by the hydraulic press when the force applied at the plunger is 400 N. [Ans: 17.77 kN]



Hydraulic press

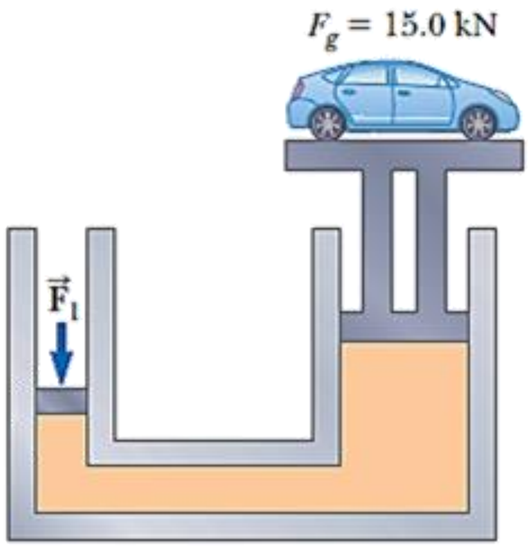
Q3-2



The 500 kg load on the hydraulic lift shown in the figure is to be raised by pouring oil (ρ = 780 kg/m^3) into a thin tube. Determine how high h should be in order to begin to raise the weight. [Ans: 0.57 m]

Q3-3

The small piston of a hydraulic lift in the figure has a cross-sectional area of 3 cm^2, and its large piston has cross-sectional area of 200 cm^2. What downward force of magnitude, F_1 must be applied to the small piston for the lift to raise a load whose weight Fg = 15 kN? Neglect the effects due to the difference in vertical positions of the two pistons. Assume that piston's is weightless. [Ans: 225 N]



3.3 MEASUREMENT OF STATIC FLUID PRESSURE

There are several ways to measure the atmospheric and gauge pressure at points within a fluid. Among all the common ones are; barometer, piezometer, manometer and bourdon gauge.

Barometer. **Barometer** is a simple device to measure atmospheric pressure, invented by Evangelista Toricelli (1608-1647). Barometer use mercury as preferred manometric fluid since it has high density and very small vapor pressure.

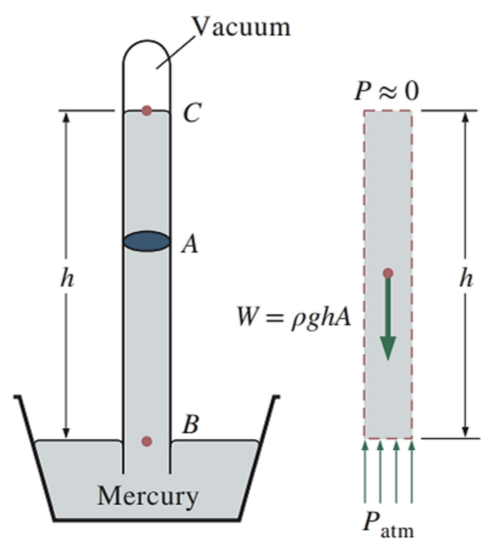


Figure 3.5: Mercury barometer. (source: Cengel & Cimbala)

Barometer consists of inverted mercury-filled tube in a mercury dish that is open to the atmosphere (Figure 3.5). The pressure at point B is equal to the atmospheric pressure. At point C, the pressure is taken as zero (0) since there is only mercury vapour above point C, in which the pressure is very low relative to P_{atm}. Atmospheric pressure is written as,

$$P_{atm} = \rho g h \qquad \textbf{Eq. 3-2}$$

Where, ρ is the density of mercury, often taken as ρ_{Hg} = 13,600 kg/m^3.

The length and the cross sectional area of tubes have no effect on the height of the fluid column of a barometer (Figure 3.6), provided the tube diameter is large enough to avoid surface tension effect. Standard atmospheric pressure is defined as the pressure produced by a column of mercury at 760 mm in height at 0°C under g = 9.807 m/s^2. For barometer reading, standard atmospheric pressure is 760 mmHg and the unit **mmHg** is also called **torr**. Hence, 1 atm = 760 torr and 1 torr = 133.3 Pa.

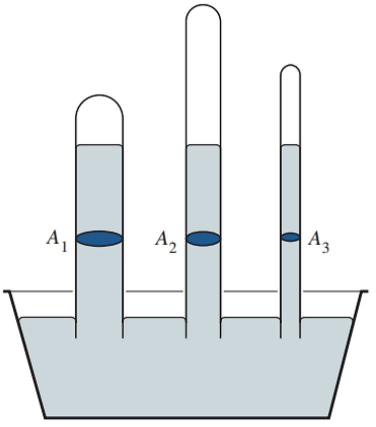


Figure 3.6: Different length and cross section area of barometer tube. (source: Cengel & Cimbala)

Example 3.4:

Determine the atmospheric pressure at a location where the barometric reading is 740 mmHg and the gravitational acceleration is g = 9.805 m/s^2. Assume the temperature of mercury to be 10°C, at which its density is 13,570 kg/m^3.

Given: ρ = 13570 kg/m^3; g = 9.805 m/s^2; h = 740 mm = 0.74 m

Solution:

Atmospheric pressure,

$$
\begin{aligned}
P_{atm} &= \rho g h \\
&= 13570(9.805)(0.74) \\
&= 98459.849\ Pa = 98.5\ kPa\ (Ans)
\end{aligned}
$$

Example 3.5:

What is the atmospheric pressure in N/m^2 if the level of mercury in a barometer tube is 760 mm above the level of the mercury in the reservoir? Give the specific gravity of mercury is 13.6 and specific weight of water 9810 N/m^3.

Given: h = 760 mm = 0.76 m; SG_{Hg} = 13.6; γ_w = 9810 N/m^3

Solution:

Specific weight of mercury,

$$SG_{Hg} = \frac{\gamma_{Hg}}{\gamma_{H_2O}}$$

$$\gamma_{Hg} = SG_{Hg}\gamma_{H_2O} = (13.6)(9810) = 133416\ N/m^3$$

Atmospheric pressure,

$$
\begin{aligned}
P_{atm} &= \rho g h \\
&= 13570(9.805)(0.74) \\
&= 98459.849\ Pa = 98.5\ kPa\ (Ans)
\end{aligned}
$$

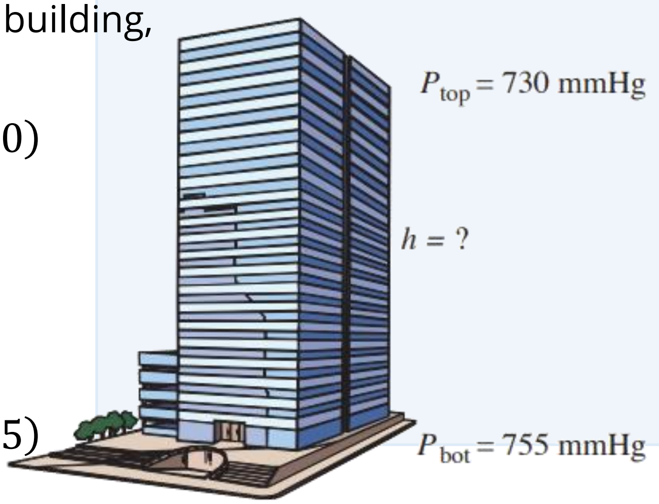
Example 3.6:

The basic barometer can be used to measure the height of a building. If the barometer reading at the top and at the bottom of a building are 730 mmHg mm and 755 mmHg, respectively, determine the height of the building. Assume an average air density of 1.18 kg/m^3.

Given: ρ = 1.18 kg/m^3; P_{top} = 730 mmHg; P_{bottom} = 755 mmHg

Solution:

Pressure on top and bottom of building,



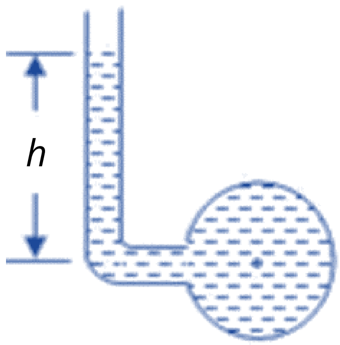
$$P_{top} = \rho g h_{top}$$
$$= 13600(9.81)(0.730)$$
$$= 97393.68\ Pa$$
$$= 97.39\ kPa$$

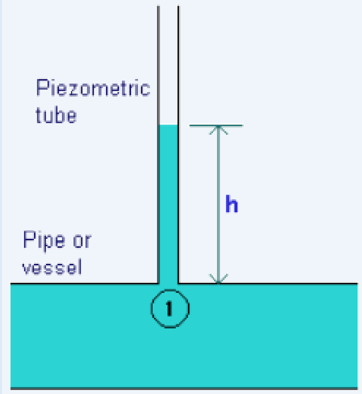
$$P_{top} = \rho g h_{top}$$
$$= 13600(9.81)(0.755)$$
$$= 100739.08\ Pa$$
$$= 100.74\ kPa$$

Therefore, taking an air column between the top and bottom of the building,

$$P_{air} = P_{bottom} - P_{top}$$
$$(\rho g h)_{air} = 100739.08 - 97393.68$$
$$1.18(9.81)(h) = 3345.4$$
$$\therefore h = 289\ m$$

Piezometer. **Piezometer** is the simplest form of instrument used to measure gauge pressure in static liquid. Piezometer consists of long transparent tube, open at one end to the atmosphere and other end of piezometer is connected to the point where pressure is to be measured (refer Figure below).





It usually used to measure pressure of liquids in vessels. Any pressure at the top of the vessel will push the liquid at a distance, h up the tube. The rise of liquid in the tube gives the pressure head at that point,

$$P = \rho g h \rightarrow P = \gamma h \quad \text{——— Eq. 3-3}$$

Limitations of piezometer;

1. Can not be employed when large pressures in the lighter liquids to be measured – this would require very long tubes.
2. Not effective at measuring high negative (suction) gauge pressures – since air may leak into the vessel.
3. Unable to measure gas pressures – since gases form no free surface.

Example 3.7:

A pressure tube is used to measure the pressure of oil (ρ = 715 kg/m^3) in a pipeline. If the oil rises to a height of 0.93 m above the centre of the pipe, calculate the gauge pressure in N/m^2 at that point.

Given: ρ = 715 kg/m^3; h = 0.93 m

Solution:

Gauge pressure of the oil,

$$P_g = \rho g h = 715(9.81)(0.93) = 6253.1595\ N/m^2\ (Ans)$$

Manometer. **Manometer** is a device used for measuring pressure at a point in a fluid by balancing the column of fluid with the same or another type of fluid. It is often used to measure small and moderate pressure in liquid. A manometer consists of a glass or plastic U-tube containing one or more manometric fluid (refer figure below). To keep the size of manometer to a manageable level, heavy fluids such as mercury are used if large pressure are to be measured.

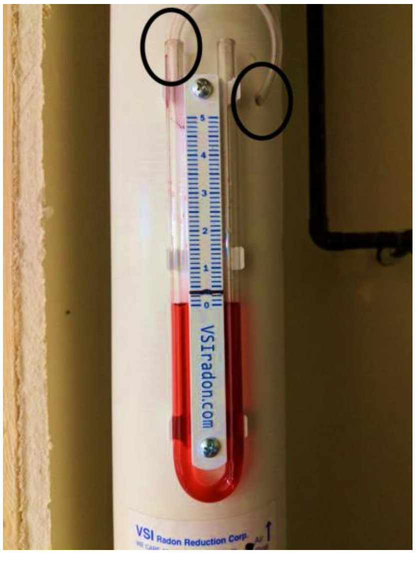


Figure 3.7: Simple U-tube manometer

Types of manometer;

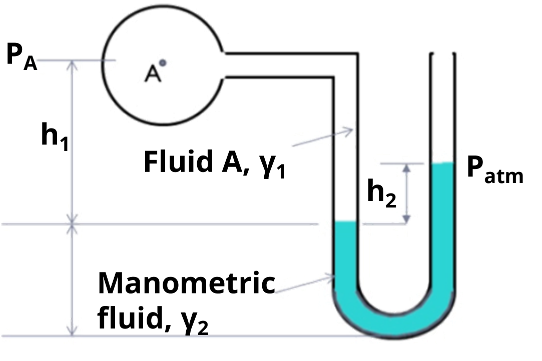
- Simple U-tube manometer (Figure 3.7)
- Differential U-tube manometer
- Inverted differential U-tube manometer
- Combined / multifluid manometer

Types of manometric fluids commonly used;

- Mercury
- Water
- Alcohol
- Oil

Simple U-tube manometer;

Have a U-shaped tube filled with manometric fluid. Due to pressure, level of manometric fluid rises on one side while it falls on other side. Difference in levels is measured to estimate the pressure.

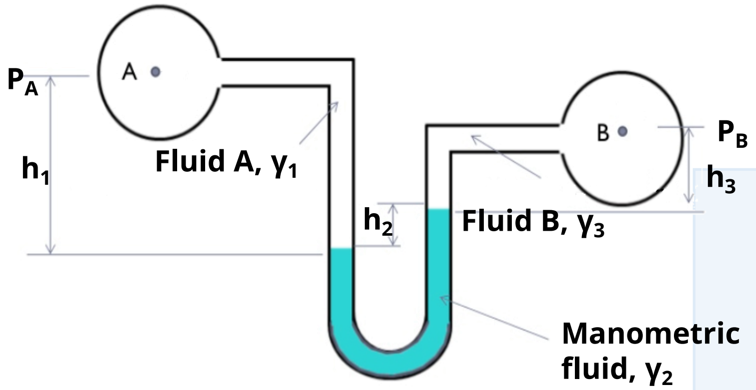


The differential fluid column of height h_2, is in static equilibrium, and it is open to the atmosphere. Then the pressure at point A is determined from,

$$p_A + \rho_1 g h_1 - \rho_2 g h_2 = p_{atm} \qquad or$$

$$p_A + \gamma_1 h_1 - \gamma_2 h_2 = p_{atm} \qquad \text{Eq. 3-4}$$

Differential U-tube manometer;



Differential manometer is used to measure difference of pressure between two points on a closed conduit where fluid is flowing through it.

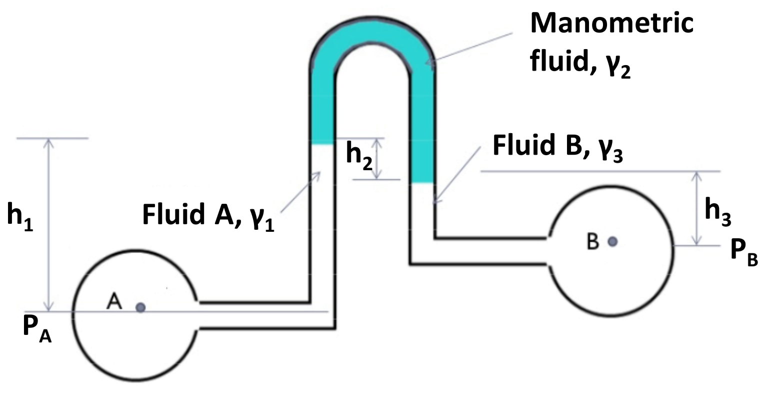
It consists of a U-tube containing heavy liquid (high density) with two ends connected to two points to be measured. Point A and B in Figure are at different level and also contains liquids of different specific gravities. Let pressure at A and B are p_A and p_B,

$$p_A + \rho_1 g h_1 - \rho_2 g h_2 - \rho_3 g h_3 = p_B \qquad or$$

$$p_A + \gamma_1 h_1 - \gamma_2 h_2 - \gamma_3 h_3 = p_B$$

$$p_A - p_B = \gamma_2 h_2 + \gamma_3 h_3 - \gamma_1 h_1 \qquad \text{——— Eq.3-5}$$

Inverted Differential U-tube manometer;



Inverted differential manometer is used to measure difference of low pressures, where accuracy is the prime consideration.

It consists of inverted U-tube containing light liquid (low density), and the two ends of the tube are connected to the points where the difference of pressure is to be measured. Let the pressure at A is more than the pressure at B as in Figure,

$$p_A - \rho_1 g h_1 + \rho_2 g h_2 + \rho_3 g h_3 = p_B \qquad or$$

$$p_A - \gamma_1 h_1 + \gamma_2 h_2 + \gamma_3 h_3 = p_B$$

$$p_A - p_B = \gamma_1 h_1 - \gamma_2 h_2 - \gamma_3 h_3 \qquad \text{——— Eq.3-6}$$

Multifluid / Combined Manometer;

Multifluid manometer (Figure 3.8) also known as combined manometer combines two manometric fluids to measure pressure. Fluid in the middle is used to prevent the toxicity of mercury affecting the water in the tank. To analyse this system, note that,

1) Pressure change across fluid is $\Delta P = \rho g h$
2) Pressure increases downward in a given fluid and decreases upward (i.e. P bottom > P top)
3) Two points at the same elevation in a continuous fluid at rest are at the same pressure.

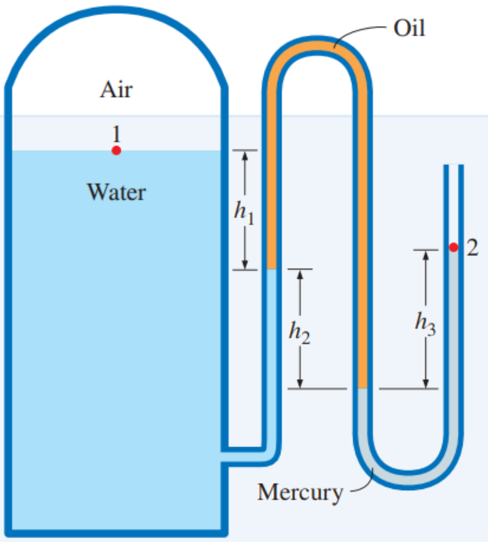


Figure 3.8: Multifluid manometer (source: Cengel & Cimbala)

In this type of manometer as in Figure, the system can be written as,

$$p_1 + \rho_w g h_1 + \rho_{oil} g h_2 - \rho_{Hg} g h_3 = p_2 = p_{atm}$$

$$p_1 + \gamma_w h_1 + \gamma_{oil} h_2 - \gamma_{Hg} h_3 = p_2 = p_{atm}$$

$$p_{air} = p_{atm} - \gamma_w h_1 - \gamma_{oil} h_2 + \gamma_{Hg} h_3 \quad \text{—— Eq.3-7}$$

Manometer Rule;

It is more effective to work manometer problems with general approach for each case. No formulas for particular manometer should be memorized. A general procedure to work all manometer problems:

1) Start at a point in the fluid where the pressure is to be determined.
2) Proceed to add to the pressure (algebraically) from one vertical fluid interface to the next.
   - Pressure term is positive (plus) if it is below (lower) the level of the next point – since it will cause an increase in pressure.
   - Pressure term is negative (minus) if it is above (higher) the level of the next point – since it will cause a decrease in pressure.
3) Continue until reaching liquid surface at the other end of the manometer.

Working Manometer Example;

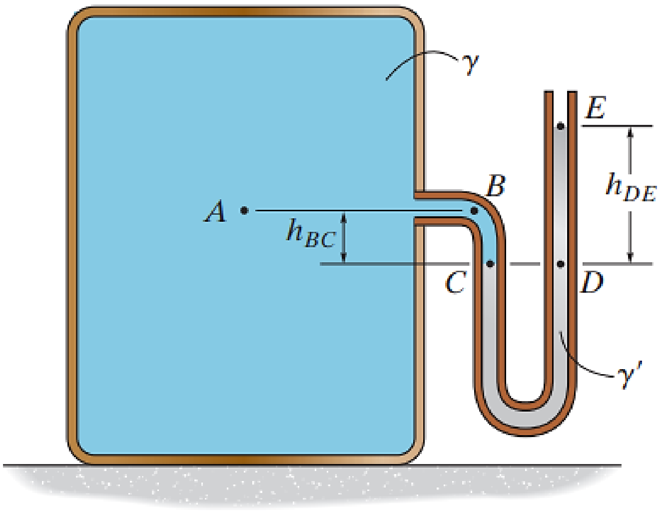


Figure 3.9: U-tube manometer analysis. (source: Hibbeler)

To work out the manometer in Figure 3.9,

1) Start with point A.
2) Pressure at point A & B and C & D are the same, since they are at the same level.
3) Add pressure at C (lower than point A – positive).
4) Subtract pressure at point E (higher than point C – negative).
5) Hence, pressure at point A,

$$p_A + \gamma h_{BC} - \gamma' h_{DE} = 0$$
$$p_A = \gamma' h_{DE} - \gamma h_{BC}$$

Example 3.8:

A manometer is used to measure the pressure of a gas in a tank. The fluid used has a specific gravity of 0.85, and the manometer column height is 55 cm , as shown in figure. If the local atmospheric pressure is 96 kPa, determine the absolute pressure within the tank.

Given; SG = 0.85, h = 55 cm = 0.55 m, p_{atm} = 96 kPa

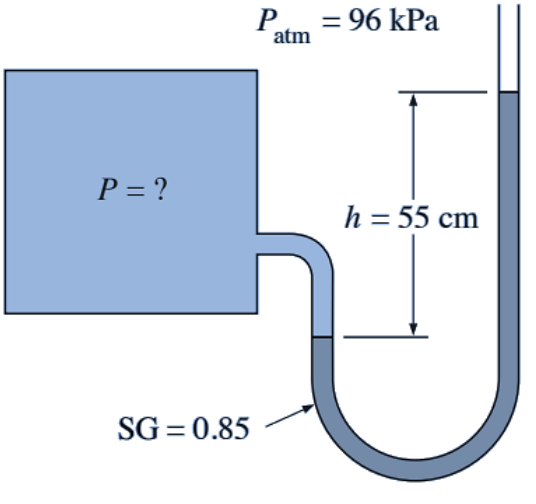
Solution:

Density of the fluid,

$$\rho = SG(\rho_w) = 0.85(1000)$$
$$= 850\ kg/m^3$$

Absolute pressure within the tank,

$$P - \rho g h = p_{atm}$$
$$P = p_{atm} + \rho g h$$
$$= 96 + (850)(9.81)(0.55)$$
$$= 100.6\ kPa\ (Ans)$$



Example 3.9:

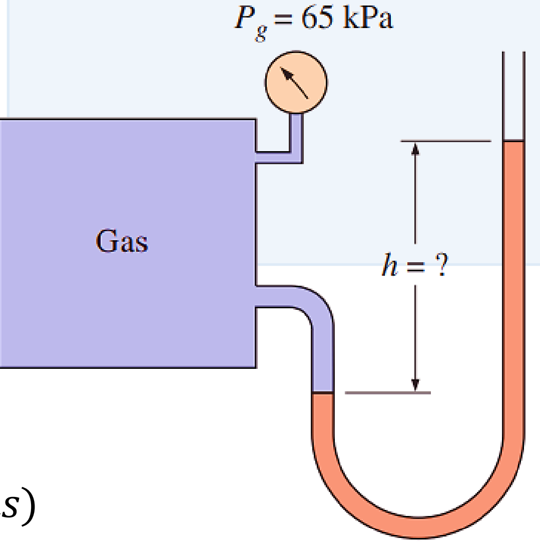
Both a gauge and a manometer are attached to a gas tank to measure its pressure. If the reading on the pressure gauge is 65 kPa, determine the distance between two fluid levels of the manometer of the fluid is;

a) Mercury (ρ = 13 600 kg/m³)
b) Water (ρ = 1000 kg/m³)

Given: P_{gauge} = 65 kPa = 65 000 Pa

Solution:

The gauge pressure is related to the vertical distance h between the two fluid levels by,



$$P_{gauge} = \rho g h$$

$$h = \frac{P_{gauge}}{\rho g}$$

a) For mercury,

$$h = \frac{p_{gauge}}{\rho_{Hg} g}$$

$$= \frac{65000}{13600(9.81)} = 0.49\ m\ (Ans)$$

b) For water,

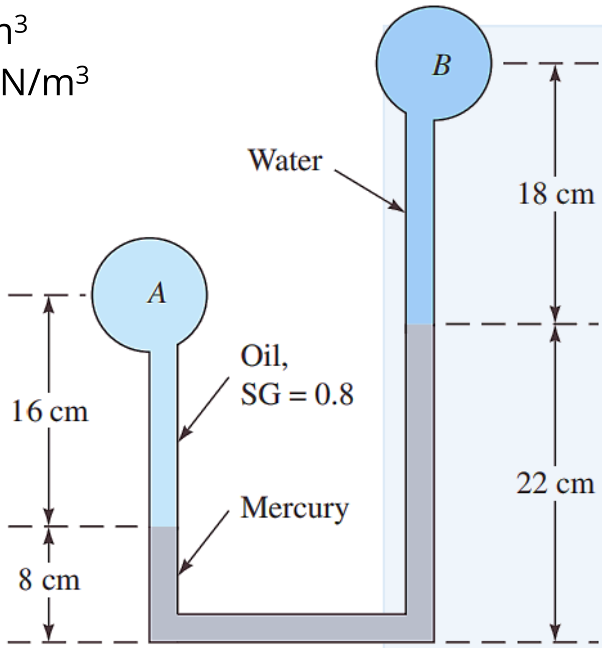
$$h = \frac{p_{gauge}}{\rho_{w} g}$$

$$= \frac{65000}{1000(9.81)} = 6.63\ m\ (Ans)$$

Example 3.10:

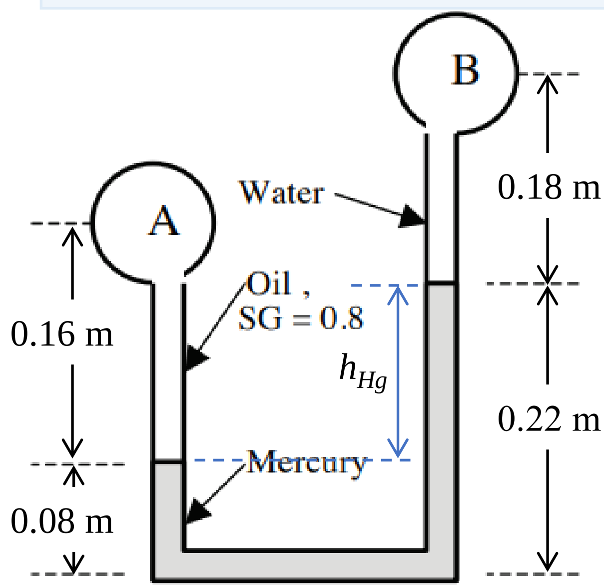
If the pressure in container A in the figure below is 200 kPa, calculate the pressure in container B. Take,

γ_w = 9790 N/m^3
γ_{Hg} = 133 100 N/m^3



Given: P_A = 200 kPa = 200 000 Pa

Solution:



Specific weights of oil,

$$\gamma_{oil} = SG\gamma_w$$
$$= (0.8)(9790)$$
$$= 7832\ N/m^3$$

Height difference of mercury,

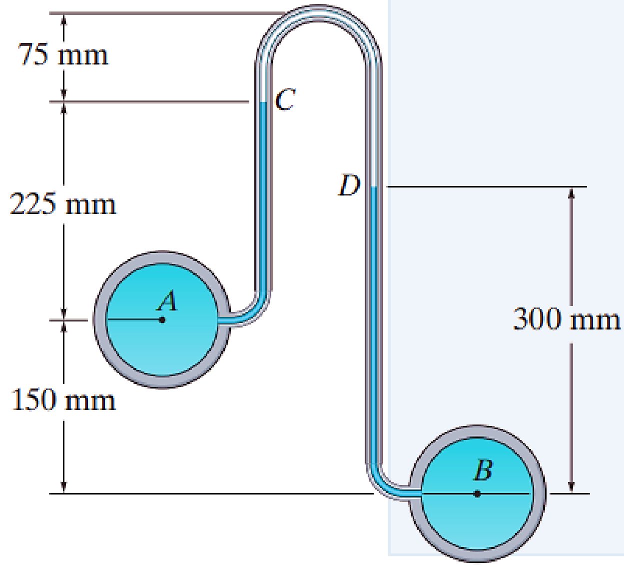
$$h_{Hg} = 0.22 - 0.08 = 0.14\ m$$

Using manometric rule from B to A,

$$p_B + \gamma_w h_w + \gamma_{Hg} h_{Hg} - \gamma_{oil} h_{oil} = p_A$$
$$p_B = p_A - \gamma_w h_w - \gamma_{Hg} h_{Hg} + \gamma_{oil} h_{oil}$$
$$p_B = 200000 - (9790)(0.18) - (133100)(0.14) + (7832)(0.16)$$
$$p_B = 180856.92\ Pa = 180.86\ kPa\ (Ans)$$

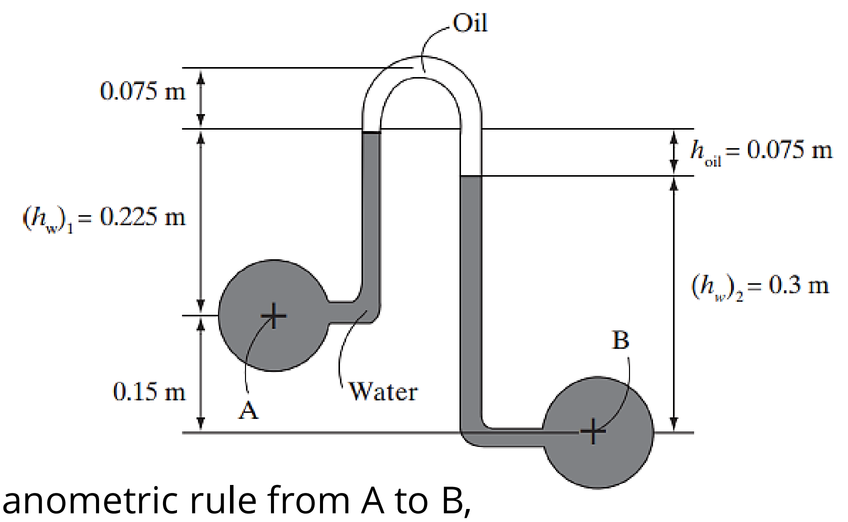
Example 3.11:

The inverted U-tube manometer is used to measure the difference in pressure between water flowing in the pipes at A and B. If the top segment is filled with an oil with ρ_{oil} = 800 kg/m^3, and the water levels in each segment are as indicated, determine this pressure difference between A and B.



Given: ρ_{oil} = 800 kg/m^3

Solution:

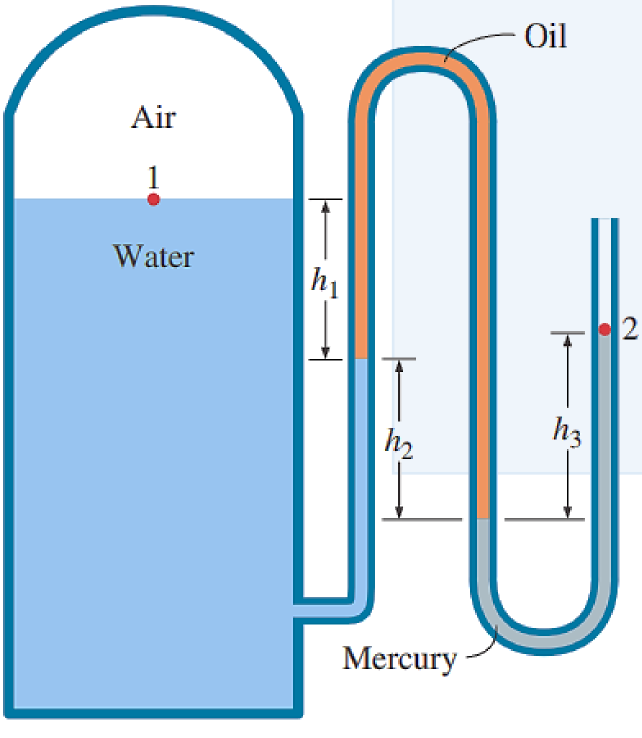


Using manometric rule from A to B,

$$p_A - \rho_w g h_{wA} + \rho_{oil} g h_{oil} + \rho_w g h_{wB} = p_B$$
$$p_B - p_A = \rho_w g(h_{wB} - h_{wA}) + \rho_{oil} g h_{oil}$$
$$p_B - p_A = (1000)(9.81)(0.3 - 0.225) + (800)(9.81)(0.075)$$
$$p_B - p_A = 1324.35\ Pa = 1.32\ kPa\ (Ans)$$

Example 3.12:

The water in a tank is pressurized by air, and the pressure is measured by a multifluid manometer as shown in the figure below. The tank is located on a mountain at an altitude of 1400 m where the atmospheric pressure is 85.6 kPa. Determine the air pressure in the tank if h_1 = 0.1 m, h_2 = 0.2 m, and h_3 = 0.35 m. Take the densities of water, oil, and mercury to be 1000 kg/m^3, 850 kg/m^3, and 13,600 kg/m^3, respectively.



Given: h_1 = 0.1 m; h_2 = 0.2 m; h_3 = 0.35 m

ρ_w = 1000 kg/m^3; ρ_{oil} = 850 kg/m^3; ρ_{Hg} = 13600 kg/m^3

Solution:

Using manometric rule starting from point 1,

$$p_1 + \rho_w g h_1 + \rho_{oil} g h_2 - \rho_{Hg} g h_3 = p_2 = p_{atm}$$

$$p_1 = p_{atm} + g(\rho_{Hg} h_3 - \rho_w h_1 - \rho_{oil} h_2)$$

$$p_1 = 85\,600 + (9.81)[(13600)(0.35) - (1000)(0.1) - (850)(0.2)]$$

$$= 130\ kPa\ (Ans)$$

Bourdon Gauge. **Bourdon gauge** is a type of mechanical pressure measurement device, typically used when the gauge pressure is too high for manometer to measure.

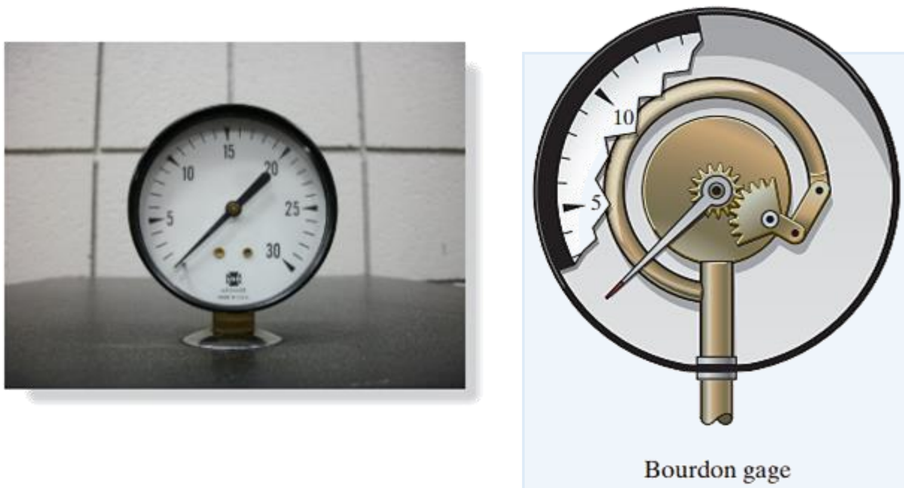


Figure 3.10: Bourdon gauge. (source: Hibbeler)

Bourdon gauge (Figure 3.10) basically applied the principle of elastic deformation of metal. It consists of a bent, coiled or twisted hollow metal tube which end is closed and connected to a dial indicator needle. The other end is connected to vessel where pressure is to be measured (Figure 3.11). The needle is calibrated to read zero (gauge pressure) when the tube is open to the atmosphere undeflected. When the fluid inside the tube is pressurized, the tube stretched and moves needle in proportion to the applied pressure.

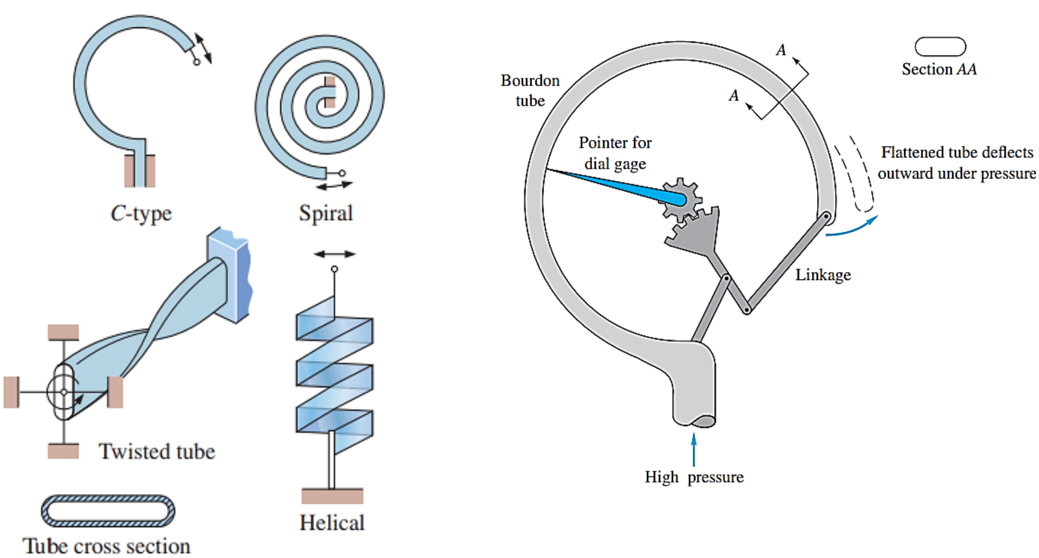
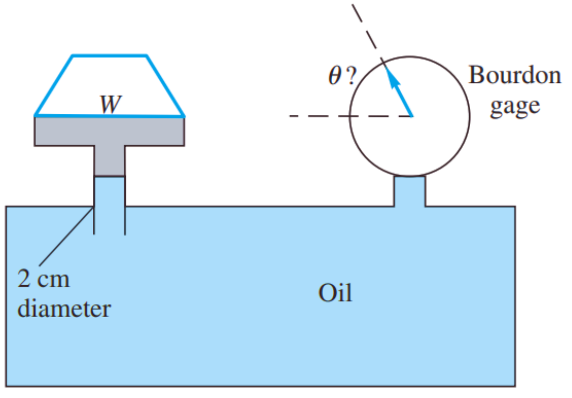


Figure 3.11: Hollow metal tube and needle assembly of Bourdon gauge. (source: Cengel & Cimbala)

Example 3.13:



The bourdon gauge in the figure, is calibrated with a deadweight piston. If the bourdon gauge is designed to rotate the pointer 10° for every 14 kPa of internal pressure, how many degrees does the pointer rotate if the piston and weight together total 44 newtons?

Given: pointer rotate 10° for every 14 kPa

Solution:

Pressure applied to the bourdon gauge,

$$p_{Bourdon} = \frac{F}{A_{piston}} = \frac{44}{\pi 0.01^2} = 140056\ Pa = 140.06\ kPa$$

Pointer rotates 10° for every 14 kPa, hence,

$$rotation\ of\ pointer = \frac{140.06}{14} = 10 \times 10° \approx 100°\ (Ans)$$

The pointer should move approximately 100°.

Example 3.14:

A Bourdon pressure gauge is attached to a boiler located at sea level shows a reading pressure of 10 bar. If the atmospheric pressure is 101.3 kPa, determine the absolute pressure of the boiler and the pressure head of water, h.

Given: $p_{bourdon}$ = 10 bar = 10 x 100 kN/m^2 , p_{atm} = 101.3 kPa

Solution:

The absolute pressure,

$$p_{abs} = p_{atm} + p_{bourdon}$$
$$= 101.3 + 1000$$
$$= 1101.3\ kN/m^2 (Ans)$$

The pressure head of water,

$$p_{bourdon} = \rho g h$$

$$h = \frac{p_{bourdon}}{\rho g} = \frac{1000000}{1000(9.81)}$$
$$= 101.937\ m\ (Ans)$$

TUTORIAL 3.2

Q3-4

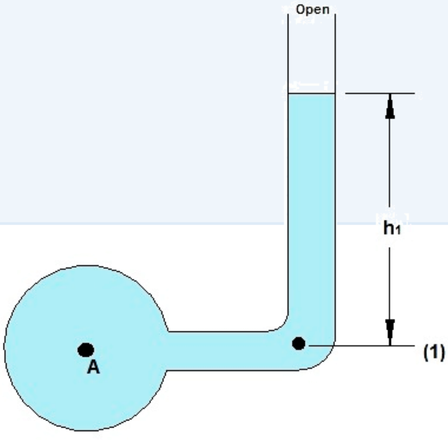
The atmospheric pressure in a location is measured by mercury barometer (ρ_{Hg} = 13,600 kg/m^3). If the height of the mercury column is 715 mm, calculate the atmospheric pressure at that location.
[Ans: 95.4 kPa]

Q3-5

Determine the pressure head, h in mm of mercury in a barometer if the atmospheric pressure is 102 kPa. Take, ρ_{Hg} = 13,600 kg/m^3.
[Ans: 764.5 mm]

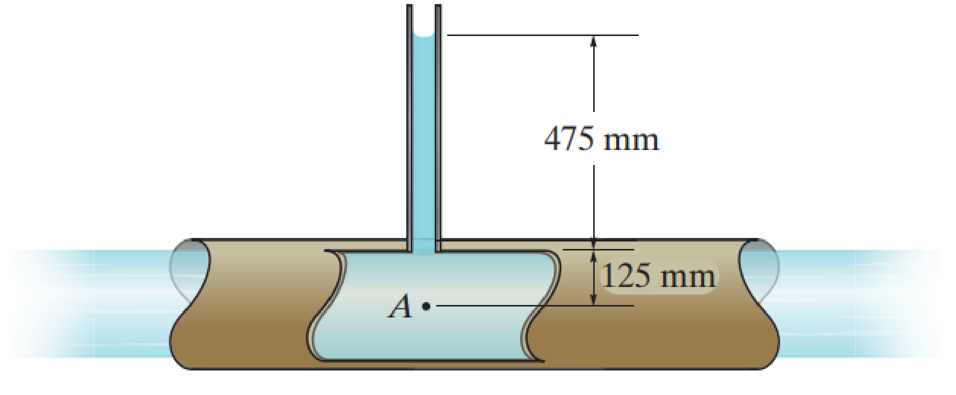
Q3-6

A pressure tube is used to measure the pressure of oil with mass density of 640 kg/m^3 in a pipeline. If the oil rises to a height of 1.2 m above the centre of the pipe, what is the gauge pressure in N/m^2 at that point?. [Ans: 7.53 kN/m^2]



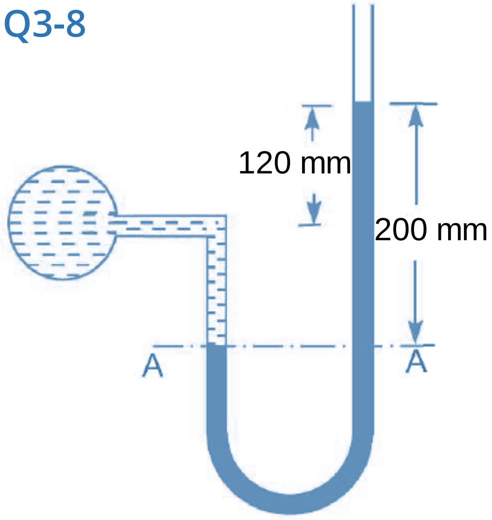
Q3-7

If the height of water in the piezometer is 475 mm, determine the absolute pressure at point A. Compare this pressure with that using kerosene. Take ρ_w = 1000 kg/m^3 and ρ_{ke} = 814 kg/m^3. [Ans: 105.96 kPa, 105.09 kPa]



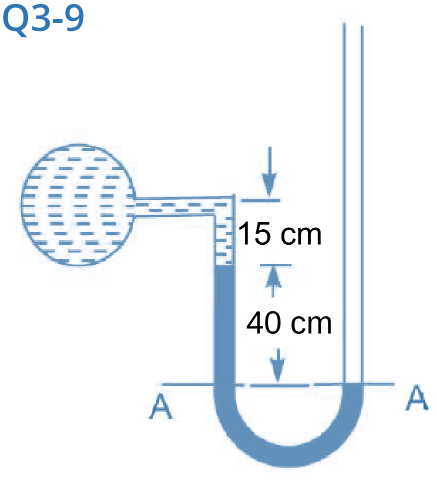
TUTORIAL 3.2

Q3-8



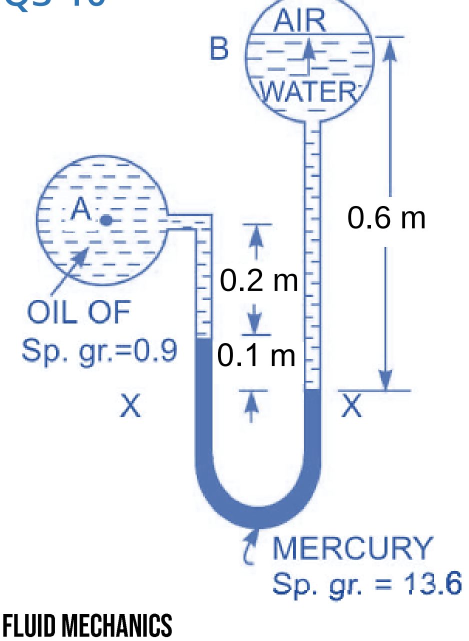
A simple U-tube manometer containing mercury with an open end was connected to a pipe containing a fluid with specific gravity of 0.9. The centre of the pipe is 120 mm below the level of mercury in the right limb. Find the pressure of fluid in the pipe if the difference of mercury level in the two limbs is 200 mm. [Ans: 25.98 kPa]

Q3-9



A simple U-tube manometer containing mercury is connected to a pipe in which a fluid with WG 0.8 is flowing. The other end of the manometer is open to the atmosphere. Find the vacuum pressure in the pipe, if the difference of mercury level in the two limbs is 40 cm and the height of fluid in the left from the centre of pipe is 15 cm below. [Ans: 54.5 kPa]

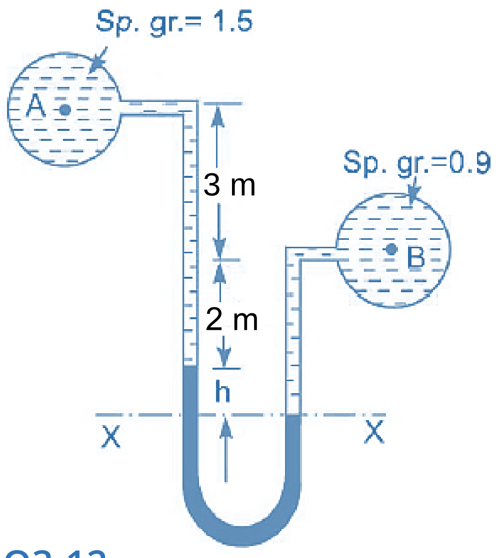
Q3-10



A differential manometer is connected at the two points A and B as shown in the figure. At B the absolute air pressure is 98.1 kN/m^2. find the absolute pressure at A. [Ans: 88.88 kN/m^2]

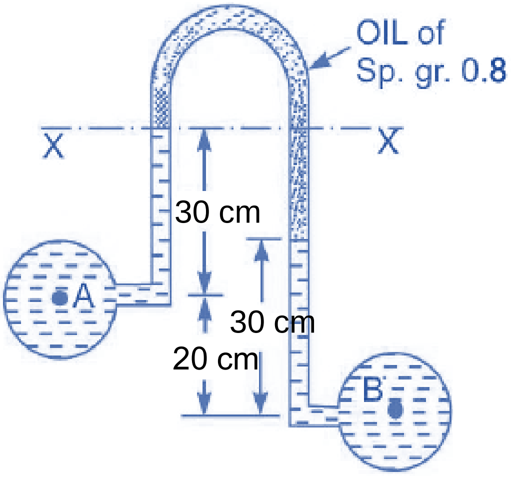
TUTORIAL 3.2

Q3-11



A differential manometer is connected at the two points A and B of two pipes as shown in the figure. The pipe A contains a liquid with SG of 1.5 while pipe B contains a liquid with SG of 0.9. The pressures at A and B are 98.1 kPa and 176.58 kPa respectively. Find the difference in mercury level, h in the differential manometer. [Ans: 0.1811 m]

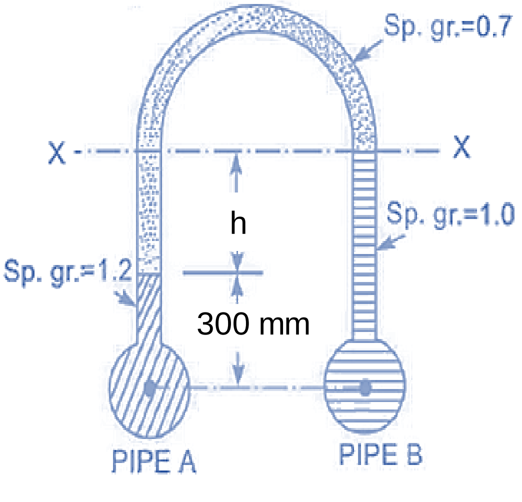
Q3-12



An inverted differential manometer is connected to two pipes A and B which convey water. The fluid in manometer is oil with SG 0.8. For the manometer reading shown in the figure, find the pressure difference between A and B.

[Ans: 1569.6 Pa]

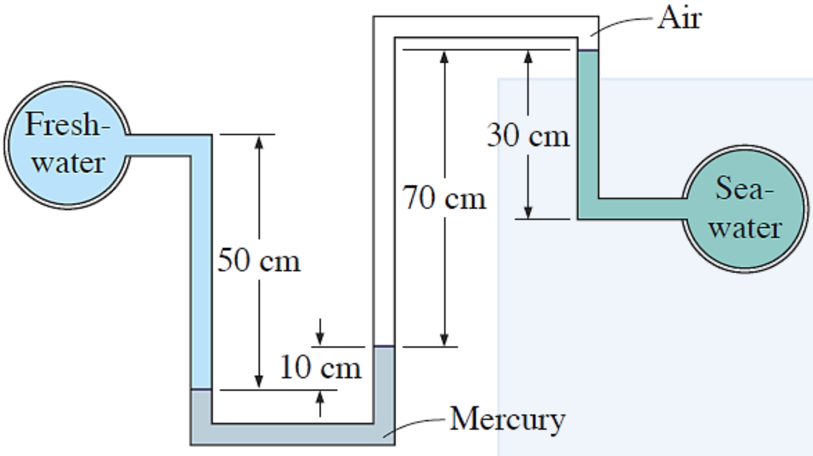
Q3-13



Find the differential reading, h of an inverted U-tube manometer containing oil of specific gravity of 0.7 as the manometric fluid when connected across pipes A and B as shown in the figure. Pipes A and B are located at the same level and assume the pressure A and B to be equal. [Ans: 200 mm]

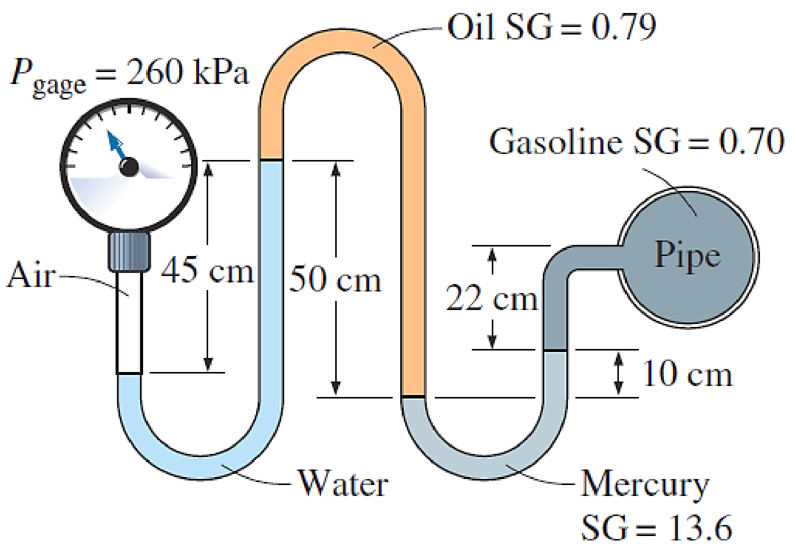
TUTORIAL 3.2

Q3-14



Freshwater and seawater flowing in parallel horizontal pipelines are connected to each other by a double U-tube manometer, as shown in the figure. Determine the pressure difference between the two pipelines. Take the density of seawater at that location to be ρ = 1035 kg/m^3 and oil density is 700 kg/m^3. [Ans: 10.2 kPa]

Q3-15



A gasoline line is connected to a pressure gauge through a double U-manometer, as shown in the figure. If the reading of the pressure gauge is 260 kPa. Determine the gauge pressure of the gasoline line. Take, ρ_{air} = 1.225 kg/m^3. [Ans: 245 kPa]

3.4 BUOYANCY

When a body is immersed in a fluid, an upward force is exerted by the fluid on the body. This upward force is equal to the weight of fluid displaced by the body and is called the **force of buoyancy**. This force is of great importance in the design of hot-air balloons, ships and submarines (Figure 3.12).

Figure 3.12: Applications of buoyancy in the design of ships, submarines and hot-air balloons.

Archimedes' Principle. **Archimedes' principle** states that when a body is placed in a static fluid, it is buoyed up by a force that is equal to the weight of the fluid displaced by the body**.** This principle is discovered by Archimedes, a Greek scientist (287 – 212 BC).

This can be demonstrated by weighing a heavy object in water by a spring scale as shown in Figure 3.13. From this observation, it can be concluded that a fluid exerts an upward force on a body immersed in it. The buoyant force is equal to the weight of liquid displaced by the object.

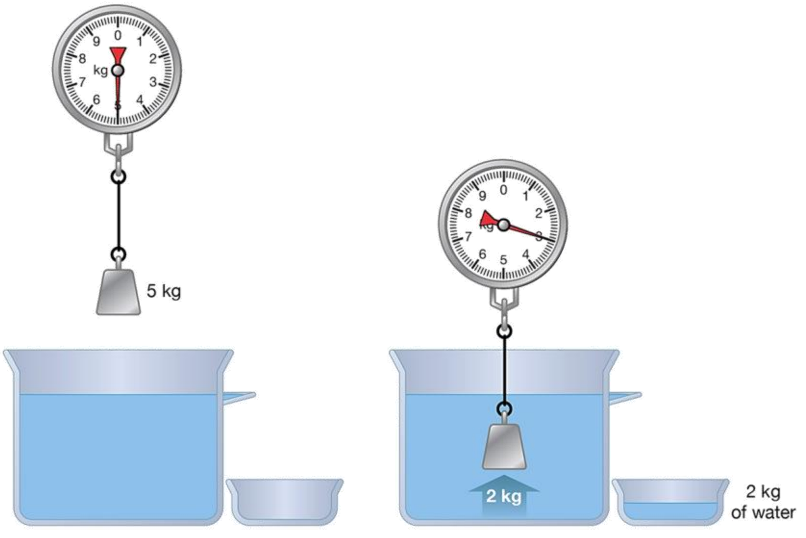


Figure 3.13: Simple demonstration of Archimedes' principle.

Buoyant Force. **Buoyant force, F_b** is the upward force on any object immersed in any fluid, caused by the increase of pressure with depth and acts through the centre of buoyancy, C_b. To demonstrate this, consider a floating body in Figure 3.13.

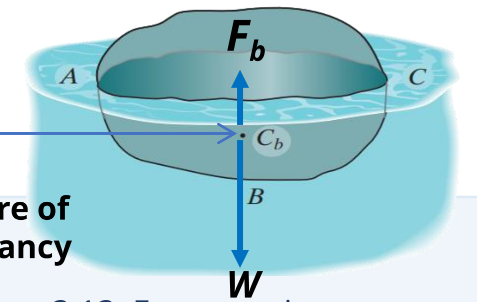


Figure 3.13: Forces acting on buoyant object. (source: Hibbeler)

A force acting downward on AC is equivalent to the weight of the whole body, W. The force acting upward on the body, ABC is equivalent to the weight of fluid displaced by region ABCA is called the buoyant force,

$$F_b = \rho_f ghA = \rho_f gV \quad \text{——— Eq. 3-8}$$

In the free-body diagram (FBD), the buoyant force acts upward at the centre of buoyancy, and the body's weight acts downward through its centre of gravity.

Float or Sink?

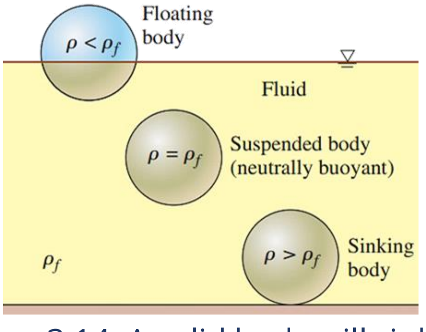


Figure 3.14: A solid body will sink or float depending on density. (source: Cengel and Cimbala)

The density of a body determines whether it will float or sink when placed in a liquid. For same volume objects placed in the same fluid, if it is less dense than the liquid, it will float, but it will sink if it is denser than the liquid (Figure 3.14). But, the buoyant force will be the same.

Archimedes' principle also explain why some objects float in fluids even though they are very heavy. It depends on how much fluid they displace. That is the reason why a boat floats and a block of same material having the same mass will sink (Figure 3.15). The boat due to its shape, displaces larger volume of water, so the the buoyant force will be larger.

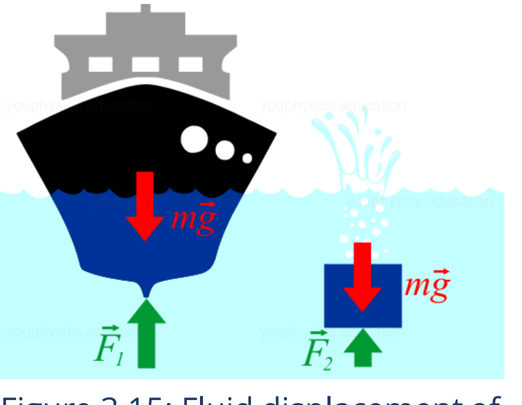
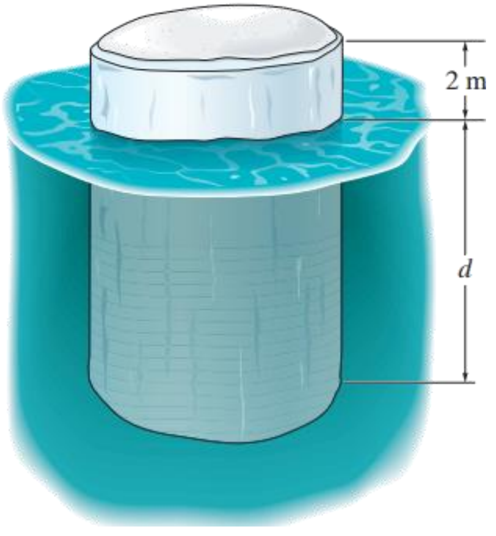


Figure 3.15: Fluid displacement of same material with same mass but different shape.

Example 3.15:



Consider an iceberg to be in the form of a cylinder of arbitrary and floating in the ocean as shown in the figure. If the cylinder extends 2 m above the ocean's surface, determine the depth of the cylinder below the surface. The density of the ocean water is ρ_{sw} = 1024 kg/m^3, and the density of the ice is ρ_i = 935 kg/m^3.

Given: ρ_{sw} = 1024 kg/m^3 ; ρ_{ice} = 935 kg/m^3

Solution:

The weight of the iceberg is,

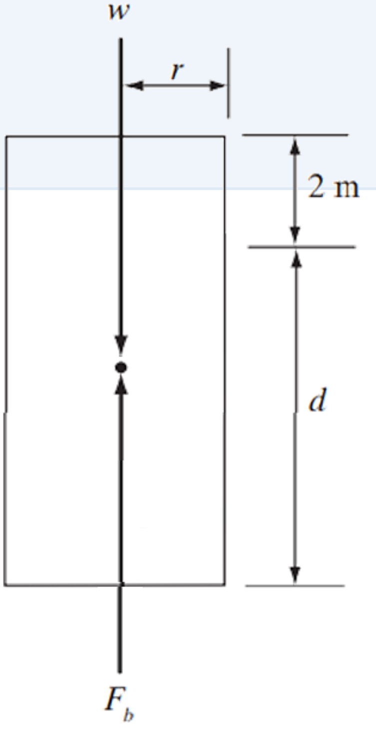
$$W = \rho_{ice} g V_{ice}$$
$$= (935)(9.81)[\pi r^2 (2 + d)]$$

The buoyant force,

$$F_B = \rho_{sw} g V_{sub}$$
$$= (1024)(9.81)(\pi r^2 d)$$

From the FBD,

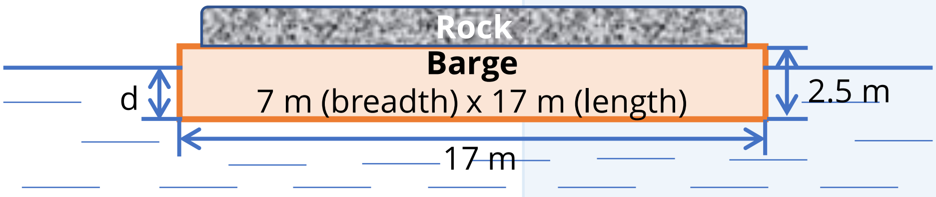
$$+\uparrow \Sigma F_y = 0;$$



$$F_B - W = 0$$
$$F_B = W$$
$$(1024)(9.81)(\pi r^2 d) = (935)(9.81)[\pi r^2 (2 + d)]$$
$$1024d = 935\ (2 + d)$$
$$d = 21.01\ m\ (Ans)$$

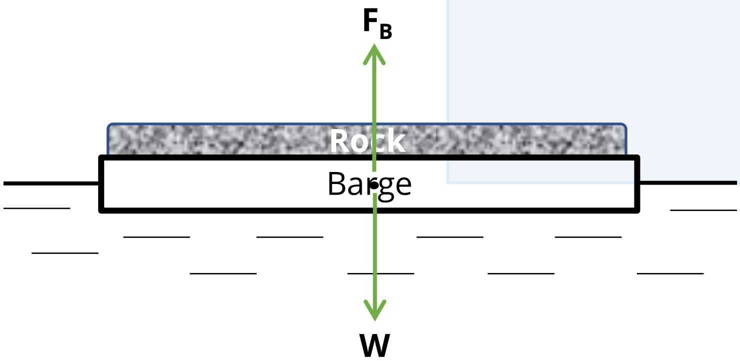
Example 3.16:

A barge with 7 m wide, 17 m long and 2.5 m in height, is filled with rock. If the barge and the rock weigh 2 MN, determine the depth of barge immersible in the water as shown in the figure below.



Given: γ_w = 9.81 kN/m^3 ; W = 2 MN = 2 x 10^6 N

Solution:



From Archimedes' Principle, the volume of displaced fluid (water) is equal to the weight of barge,

$$+\uparrow \Sigma F_y = 0;$$
$$F_B - W = 0$$
$$W = F_B$$
$$2\ MN = \rho_w g V$$
$$2 \times 10^6 = (9810)(7 \times 17 \times d)$$
$$d = 1.7132\ m\ (Ans)$$

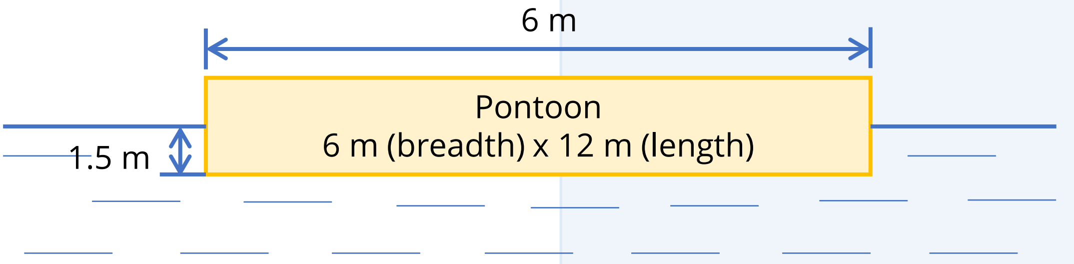
Example 3.17:

A rectangular pontoon has a breadth of 6 m, a length of 12 m, and draught of 1.5 m in fresh water (density = 1000 kg/m^3). Calculate:

a) Weight of the pontoon

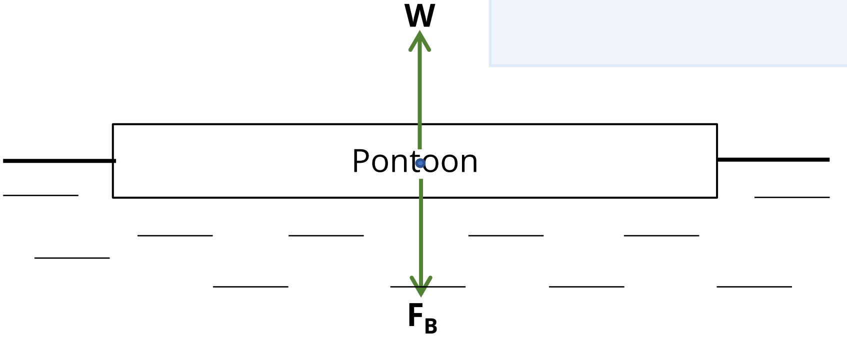
b) Depth of pontoon in sea water at density of 1025 kg/m^3

c) Load (in kg) that can be supported by the pontoon in fresh water if the maximum permissible depth is 2 m.



Given: ρ_w = 1000 kg/m^3; ρ_{sw} = 1025 kg/m^3

Solution:



a) In fresh water, the draught of pontoon is h = 1.5 m, thus, weight of pontoon immersible in water,

$$+\uparrow \Sigma F_y = 0;$$

$$F_B - W = 0$$

$$W = F_B$$

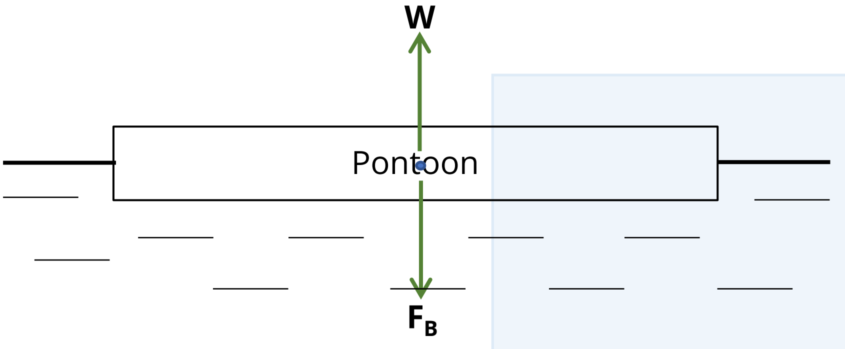
$$W = \rho_w g V$$

$$W = 1000(9.81)(6 \times 12 \times 1.5)$$

$$= 1059.48\ kN (Ans)$$

Example 3.17: (continued)

Solution:



b) Draught of pontoon in sea water,

$$+\uparrow \Sigma F_y = 0;$$
$$F_{BSW} - W = 0$$
$$F_{BSW} = W$$
$$\rho_{sw} g V = 1059.48$$
$$(1025)(9.81)(6 \times 12 \times h_{sw}) = 1059480$$
$$h_{sw} = 1.463\ m\ (Ans)$$

c) Maximum load supported in fresh water for the pontoon when permissible h = 2m,

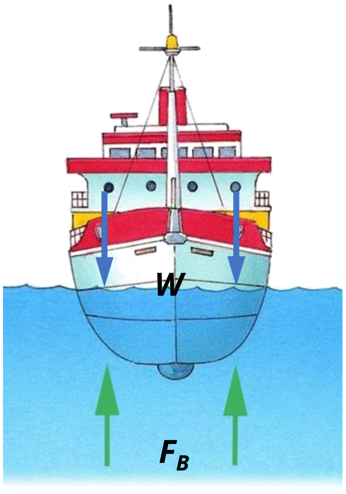
$$+\uparrow \Sigma F_y = 0;$$
$$F_B - W_{max} = 0$$
$$W_{max} = F_B$$
$$W_{max} = \rho_w g V$$
$$m_{max} = (1000)(6 \times 12 \times 2) = 144000\ kg$$

Max load that can be supported by the pontoon,

$$Max\ load = m_{max} - \frac{W}{g}$$
$$= 144000 - \frac{1059480}{9.81}$$
$$= 36000\ kg (Ans)$$

TUTORIAL 3.3

Q3-16



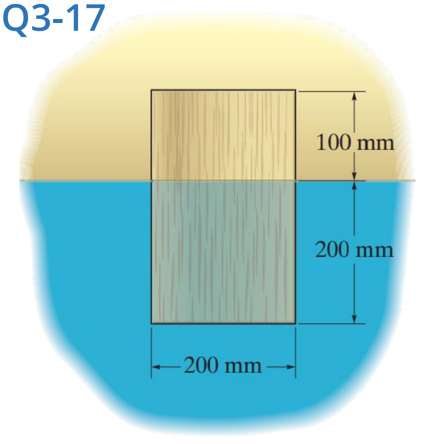
A ship floating in sea water displaces 115 m^3. Find,

a) The mass of the ship if the sea water has a density of 1025 kg/m^3

b) The volume of fresh water which the ship would displaced.

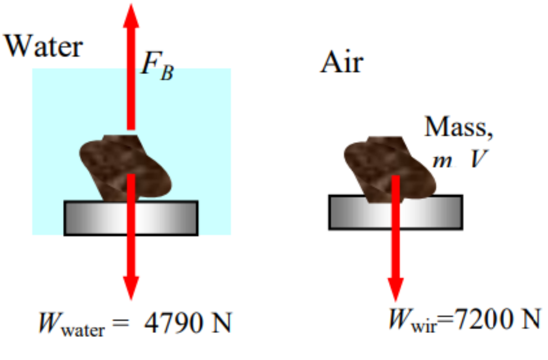
[Ans: 117875 kg, 118 m^3]

Q3-17



The cylinder floats in the water and oil to the level shown. Determine the weight of the cylinder. Take, density of the oil ρ_{oil} = 910 kg/m^3. [Ans: 89.7 N]

Q3-18



The volume and the average density of an irregularly shaped body are to be determined by using a spring scale. The body weighs 7200 N in air and 4790 N in water. Determine the volume and the density of the body. Assume the buoyancy force in air is negligible and the body is completely submerged in water.

[Ans: 0.25 m^3, 2988 kg/m^3]

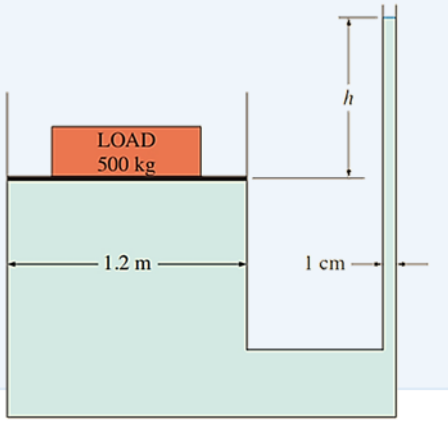
TUTORIAL 3 WORKED SOLUTIONS

Q3-2

The 500 kg load on the hydraulic lift shown in the figure is to be raised by pouring oil (ρ = 780 kg/m³) into a thin tube. Determine how high *h* should be in order to begin to raise the weight. [Ans: 0.57 m]

Given: m_1 = 500 kg; ρ = 780 kg/m³;
d_1 = 1.2 m, r_1 = 0.6 m;
d_2 = 1 cm = 0.01 m, r_2 = 0.005 m;

Solution:



Applying Pascal's Law,

$$P_1 = P_2$$

$$\frac{F_1}{A_1} = P_2$$

$$P_2 = \frac{500\ (9.81)}{\pi(0.6^2)}$$

$$\rho g h = 4336.972\ N$$

$$(780)(9.81)h = 4336.972$$

$$h = 0.5668\ m\ (Ans)$$

Thus, a 500 kg load can be raised by this hydraulic lift by raising the oil level in the tube by 56.7 cm.

Q3-4

The atmospheric pressure in a location is measured by mercury barometer (ρ_{Hg} = 13,600 kg/m³). If the height of the mercury column is 715 mm, calculate the atmospheric pressure at that location.
[Ans: 95.4 kPa]

Given: h = 0.715 m, ρ = 13600 kg/m³

Solution:

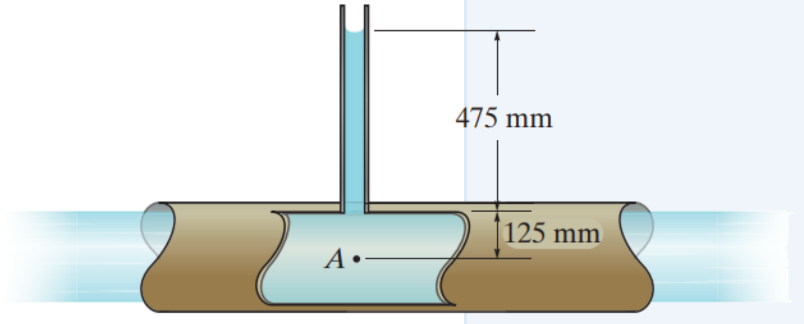
Atmospheric pressure,

$$P_{atm} = \rho g h = 3600(9.81)(0.715)$$
$$= 95392.44\ Pa = 95.4\ kPa\ (Ans)$$

TUTORIAL 3 WORKED SOLUTIONS

Q3-7

If the height of water in the piezometer is 475 mm, determine the absolute pressure at point A. Compare this pressure with that using kerosene. Take ρ_w = 1000 kg/m^3 and ρ_{ke} = 814 kg/m^3. [Ans: 105.96 kPa, 105.09 kPa]



Given: h_w = 475 mm = 0.475 m;

ρ_w = 1000 kg/m^3; ρ_{ke} = 814 kg/m^3;

Solution:

Absolute pressure at point A using water,

$$\begin{aligned} p_{abs} &= p_{atm} + \rho_w g h \\ &= 101300 + 1000(9.81)(0.475) \\ &= 105959.75\ Pa \\ &= 105.96\ kPa (Ans) \end{aligned}$$

Absolute pressure at point A using kerosene,

$$\begin{aligned} p_{abs} &= p_{atm} + \rho_{ke} g h \\ &= 101300 + 814(9.81)(0.475) \\ &= 105093.04\ Pa \\ &= 105.09\ kPa (Ans) \end{aligned}$$

Therefore, P absolute using water is higher than using kerosene; $p_w > p_{ke}$

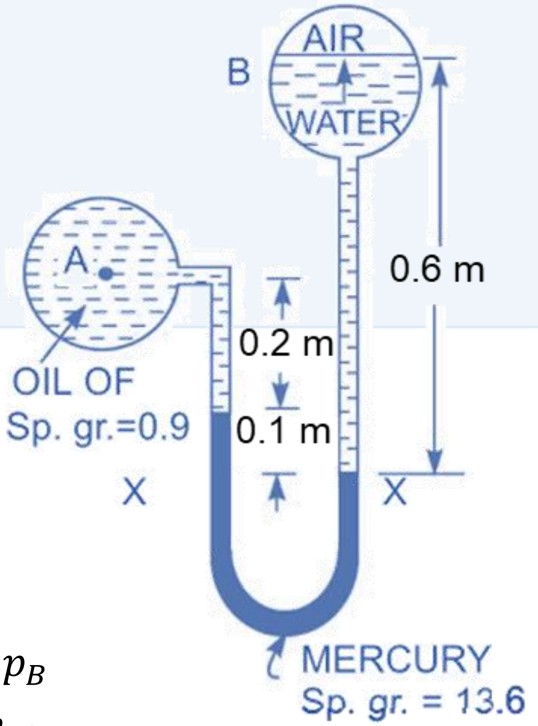
TUTORIAL 3 WORKED SOLUTIONS

Q3-10

A differential manometer is connected at the two points A and B as shown in the figure. At B the absolute air pressure is 98.1 kN/m². find the absolute pressure at A. [Ans: 88.88 kN/m²]

Given: P_B = 98.1 kN/m² = 98100 N/m² ;
SG_{oil} = 0.9; ρ_{oil} = 0.9 x 1000 = 900 kg/m³
SG_{Hg} = 13.6, ρ_{Hg} = 13.6 x 1000 = 13600 kg/m³

Solution:



Specific weight of oil,

$$\gamma_{oil} = \rho_{oil} g$$
$$= 900 \times 9.81$$
$$= 8829\ N/m^2$$

Specific weight of mercury,

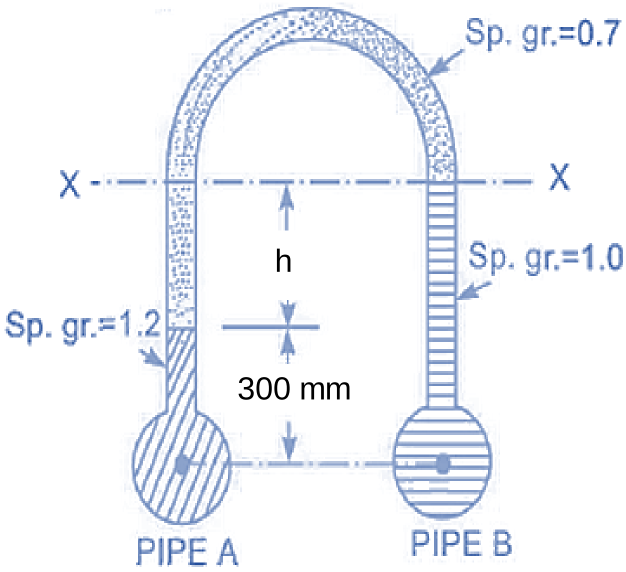
$$\gamma_{oil} = \rho_{Hg} g$$
$$= 13600 \times 9.81$$
$$= 13341.6\ N/m^2$$

Thus, the pressure at A,

$$p_A + \gamma_{oil} h_{oil} + \gamma_{Hg} h_{Hg} - \gamma_w h_w = p_B$$
$$p_A = p_B + \gamma_w h_w - \gamma_{Hg}\ h_{Hg} - \gamma_{oil} h_{oil}$$
$$= 98100 + (9810)(0.6) - (13341.6)(0.1) - (8829)(0.2)$$
$$= 88878.6 N/m^2 \ = 88.878\ kN/m^2 (Ans)$$

TUTORIAL 3 WORKED SOLUTIONS

Q3-13



Find the differential reading, h of an inverted U-tube manometer containing oil of specific gravity of 0.7 as the manometric fluid when connected across pipes A and B as shown in the figure. Pipes A and B are located at the same level and assume the pressure A and B to be equal. [Ans: 200 mm]

Given: $SG_A = 1.2$, $\rho_A = 1.2 \times 1000 = 1200\ kg/m^3$
$SG_B = 1.0$, $\rho_B = 1.0 \times 1000 = 1000\ kg/m^3$
$SG_m = 0.7$, $\rho_m = 0.7 \times 1000 = 700\ kg/m^3$

Solution:

Specific weight of fluid A,

$$\gamma_A = \rho_A g = 1200 \times 9.81 = 11772\ N/m^2$$

Specific weight of fluid B,

$$\gamma_B = \rho_B g = 1000 \times 9.81 = 9810\ N/m^2$$

Specific weight of manometric fluid,

$$\gamma_m = \rho_m g = 700 \times 9.81 = 6867\ N/m^2$$

Differential reading, h when pressure at A and B are equal,

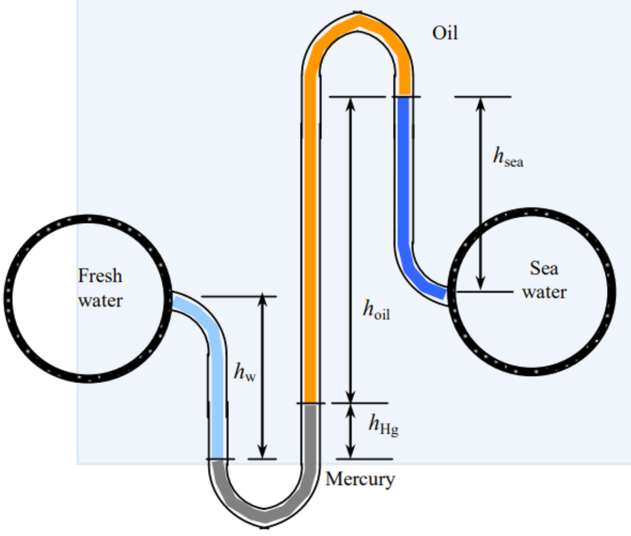
$$p_A - \gamma_A h_A - \gamma_m h_m + \gamma_B h_B = p_B$$
$$p_A - p_B = \gamma_A h_A + \gamma_m h_m - \gamma_B h_B$$
$$0 = (11772)(0.3) + (6867)h - (9810)(h + 0.3)$$
$$6867h - 9810h - 2943 = 3531.6$$
$$-2943h = -588.6$$
$$h = 0.2\ m = 200\ mm\ (Ans)$$

TUTORIAL 3 WORKED SOLUTIONS

Q3-14

Freshwater and seawater flowing in parallel horizontal pipelines are connected to each other by a double U-tube manometer, as shown in the figure. Determine the pressure difference between the two pipelines. Take the density of seawater at that location to be ρ = 1035 kg/m³ and oil density is 700 kg/m³. [Ans: 10.2 kPa]

Given: ρ_{sw} = 1035 kg/m³
ρ_{oil} = 700 kg/m³
ρ_{Hg} = 13600 kg/m³



Solution:

Specific weight of seawater,

$$\gamma_{sw} = \rho_{sw} g = 1035 \times 9.81$$
$$= 10153.35 \; N/m^2$$

Specific weight of oil,

$$\gamma_{oil} = \rho_{oil} g = 1700 \times 9.81$$
$$= 6867 \; N/m^2$$

Specific weight of mercury,

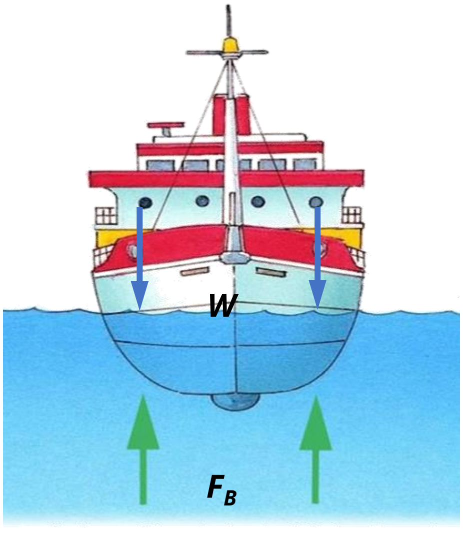
$$\gamma_{Hg} = \rho_{Hg} g = 13600 \times 9.81$$
$$= 133416 \; N/m^2$$

Pressure difference between the pipelines,

$$p_w + \gamma_w h_w - \gamma_{Hg} h_{hg} - \gamma_{oil} h_{oil} + \gamma_{sw} h_{sw} = p_{sw}$$
$$p_w - p_{sw} = \gamma_{Hg} h_{hg} + \gamma_{oil} h_{oil} - \gamma_w h_w - \gamma_{sw} h_{sw}$$
$$= (133416)(0.1) + (6867)(0.7)$$
$$-(9810)(0.5) - (10153.35)(0.3)$$
$$p_w - p_{sw} = 10197.495 \; PA = 10.2 \; kPa \; (Ans)$$

TUTORIAL 3 WORKED SOLUTIONS

Q3-16



A ship floating in sea water displaces 115 m³. Find,

a) The mass of the ship if the sea water has a density of 1025 kg/m³

b) The volume of fresh water which the ship would displaced.

[Ans: 117875 kg, 118 m³]

Given: Volume of water displaced = 115 m³; ρ_{sw} = 1025 kg/m³

Solution:

a) The mass of the ship in seawater,

$$+\uparrow \Sigma F_y = 0;$$
$$F_B - W = 0$$
$$W = F_B$$
$$mg = \rho_{sw} g V_{sw} = 1025(9.81)(115)$$
$$m = 117875\ kg (Ans)$$

b) The volume of fresh water in which the ship would displace,

$$+\uparrow \Sigma F_y = 0;$$
$$F_{Bsw} - W = 0$$
$$W = F_B$$
$$mg = \rho_w g V_w$$
$$117875 = 1000\ (V_w)$$
$$V_{sw} = 117.875\ m^3 (Ans)$$

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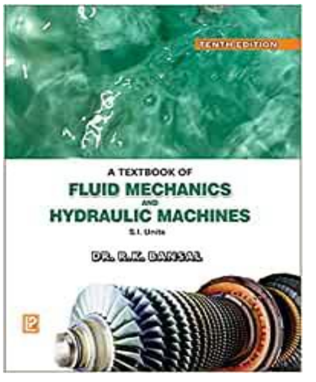

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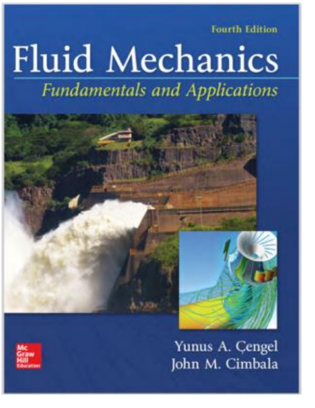

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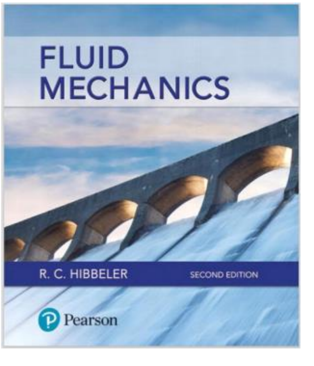

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