

KEMENTERIAN PENGAJIAN TINGGI



## BUSINESS MATHEMATICS

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Perdagangan

#### **BUSINESS MATHEMATICS**

@ Politeknik METrO Betong Sarawak, 2021

ISBN : 978-967-25866-0-9

https://www.pmbs.edu.my/v3/index.php/muat-turun/category/21e-book?download=64:business-mathematics

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#### PUBLISHED BY: POLITEKNIK METrO BETONG SARAWAK 95700 BETONG SARAWAK

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## PREFACE

Politeknik METrO Betong Sarawak: Business Mathematics is especially written for PMBS students who are pursuing diploma course in banking and finance, and tourism management. Students of other polytechnics and other institutions of higher learning who are pursuing professional courses in finance, business studies, accounting, business administration and commerce may also find this e-book useful.

This e-book covers six essential chapters: Equations, Breakeven-Point Analysis, Calculus, Interest, Project Evaluation, and Transportation Model in line with the latest syllabus prescribed in Malaysian polytechnics. The preparation of this e-book is to guide students for a better understanding in completing the course taken.

Each chapter begins with Learning Outcomes on the important concepts and theories related to mathematics in business. Well-designed exercises also include in every of the chapters to enhance students' analytical and thinking skills.

We would like to acknowledge the assistance and encouragement of our friends and colleagues as well our family who have contributed and support us to the successful publication of this e-book.



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## Chapter 1 EQUATIONS



### Learning Outcomes:

Students will be able to do the following at the end of this chapter:

- Use the principles and equations to solve problems.
- Equations are used to solve commercial mathematics difficulties.
- Determine the relationship between total cost, revenue, and profit, as well as the differences between fixed and variable costs.
- Using equations, calculate the variable cost, sales price, and profit.

## 1.0 INTRODUCTION

A mathematical statement featuring an equal sign between two algebraic expressions with the same value is known as an equation. One or more variables are used in the most basic and typical algebraic equations.

## 1.1 TYPES OF EQUATION

There are two types of the equation:

- a. Linear equation
- b. Quadratic equation

#### 1.1.1 Linear Equation

Linear equation also known as First Degree Equation which consist of two variables,  $\boldsymbol{\mathcal{X}}$  and  $\boldsymbol{\mathcal{Y}}$ , is

 $y = \mathcal{M}\mathcal{X} + \mathcal{C}$ , where *m* and *c* are constants.

For example, 4x + 3 = 12

**Example 1.1** Solve the following linear equation.

(a) 4x - 3 = 13 (b) 3(u + 6) = 8u + 8

Solution:Solution:4x - 3 = 133(u + 6) = 8u + 84x - 3 - 13 = 03u + 18 = 8u + 84x - 16 = 03u + 18 - 8u - 8 = 0

$$4x = 16 -5u + 10 = 0 -5u = -10$$
$$x = 4 \qquad u = -10 / (-5) = 9$$



Find the value of  $\chi$  in the following equations.

a. 
$$\frac{6}{x+4} = \frac{3}{x-2}$$

b. 
$$\frac{4x-3}{5} = \frac{6-2x}{2}$$

Solution:Solution:
$$6(x-2) = 3(x+4)$$
 $2(4x-3) = 5(6-2x)$  $6x-12 = 3x+12$  $8x-6 = 30-10x$  $6x-3x-12-12 = 0$  $8x+10x-6-30 = 0$  $3x-24 = 0$  $18x-36 = 0$  $3x = 24$  $18x = 36$  $x = 8$  $x = 2$ 

**?** Let's test your understanding. Self Check 1.1

#### Self Check 1.1 Solve the following linear equation.

Q1. 
$$8x - 3 = -19$$
  
Q3.  $\frac{3 - 5y}{4} = \frac{2 - 4y}{3}$   
Q2.  $4 = -2(x + 3)$ 

#### 1.1.2 Quadratic Equation

#### \_\_\_\_\_

Quadratic equation also known as second degree equations. This equation can be solved using:

a. Factoring method

b. Quadratic equation :  $ax^{2+}bx + c = 0$  where a, b, and c are constants.

#### **Factoring Methods**

A quadratic function  $ax^2 + bx + c = 0$  may be factorised into the form of  $(p_x + q)(rx + s)$ , where p, q and s are constant.

Solve the quadratic equations below.

a. 
$$x^2 - 4x + 4 = 0$$

Solution:

Example 1.3

Example 1.4 Solve the quadratic equations below.

b. 
$$x^2 - 13x = -36$$

Solution:

#### **Quadratic Equation**

There are some cases quadratic equation unable to be factorised into form  $(p_x + q)(r_x + s)$ . The solution is using the quadratic formula.

Given the quadratic equation  $ax^2 + bx + c = 0$ , the solution is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solve the quadratic equations below. Example 1.5

Substitute into the formula:

*x* = 2.22

-4(2)(-5)

x = -0.22

a. 
$$2x^{2} - 4x - 5 = 0$$
  
Solution:  
Where,  $a = 2$   
 $b = -4$   
 $c = -5$   
 $x = \frac{-(-4) \pm \sqrt{-4}^{2} - 4(2)(-5)}{2(2)}$   
 $= \frac{4 + \sqrt{24}}{4}$  or  $\frac{4 - \sqrt{24}}{4}$ 

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b. 6x(x+1) = 2(x+3)

**Solution**:

Rearrange the equation according to  $ax^2 + bx + c = 0$ 

$$6x (x + 1) = 2(x + 3)$$
  
 $6x^{2} + 6x = 2x + 6$   
 $6x^{2} + 6x - 2x - 6 = 0$   
 $6x^{2} + 4x - 6 = 0$   
Where,  $\alpha = 6$   
 $b = 4$   
 $c = -6$   
Substitute into the formula:

$$x = \frac{-4 \pm \sqrt{4^2 - 4(6)(-6)}}{2(6)}$$
$$= \frac{-4 \pm \sqrt{160}}{12} \quad \text{or} \quad \frac{-4 \pm \sqrt{160}}{12}$$
$$x = 0.72 \quad x = -1.39$$



Let's test your understanding. Self Check 1.2

Solve the following quadratic equations.

Q1.  $x^2 - 3x + 2 = 0$ Q3.  $x^2 + 4x = 10$ Q2.  $4x^2 = 11x - 6$ Q4.  $24x^2 = 40x - 10$ 

### **1.2** SIMULTANEOUS EQUATIONS

Simultaneous equations are made up of two or more equations that are part of a system of equations with few variables.

#### **1.2.1** The Solution of Two Equations

The solution for the following equations is:

5x + 4y = 643x + 2y = 36

#### **Solution**:

$$5x + 4y = 64 - 1$$

$$3x + 2y = 36 - 2$$
Use 1:  

$$5x + 4y = 64$$

$$4y = 64 - 5x$$

$$y = 64 - 5x$$

$$y = 16 - 5x$$

Substitute 3 into 2 :  

$$3x + 2\left(\frac{16}{4} - \frac{5x}{4}\right) = 36$$
  
 $3x + 32 - \frac{5x}{2} = 36$   
 $\frac{6x}{2} - \frac{5x}{2} = 36 - 32$   
 $\frac{x}{2} = 4$   
 $x = 8$ 

Substitute x = 8 into 1: 5(8) + 4y = 6440 + 4y = 64

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$$4y = 24$$
$$y = 6$$

Thus, x = 8 and y = 6.

Let's test your understanding. Self Check 1.3

Solve the following equations.

Q1. 8x + 9y = 3 x + y = 0 *Answer: x=-3, y=3* Q2. 2x + 13y = 36 13x + 2y = 69*Answer: x=5, y=2* 

## **1.2.1** The Solution of Three Equations

The solution for the following equations is:

$$4p + 5q + r = 400$$
  
 $3p + 5q + 3r = 500$   
 $7p + 3q + 2r = 600$ 

Solution:

$$4p + 5q + r = 400$$
 — 1  
 $3p + 5q + 3r = 500$  — 2  
 $7p + 3q + 2r = 600$  — 3

Use 1 :

$$4p + 5q + r = 400$$
  
 $r = 400 - 4p - 5q$ 

Substitute 4 into 2 & 3:

2 
$$3p + 5q + 3(400 - 4p - 5q) = 500$$
  
 $3p + 5q + 1200 - 12p - 15q = 500$   
 $-9p - 10q = -700$   
 $p = \frac{-700 + 10q}{-9}$  \_\_\_\_\_5

$$\begin{array}{rl} 3 & 7p + 3q + 2(400 - 4p - 5q) = 600 \\ & 7p + 3q + 800 - 8p - 10q = 600 \\ & -p - 7q = -200 \\ & p + 7q = 200 \\ & p = 200 - 7q \end{array}$$

Substitute 5 into 6:

 $700 \pm 10a$  000 7

$$\frac{-700 + 10q}{-9} = 200 - 7q$$
  
-700 + 10q = -1800 + 63q  
-53q = -1100  
q = 20.8

Substitute q = 20.8 into 6 : p = 200 - 7(20.8) = 54.4Substitute p = 54.4 and q = 20.8 into 1 : 4(54.4) + 5(20.8) + r = 400 321.6 + r = 400r = 78.4

Thus, *p* = 54.4, *q* = 20.8 and *r* = 78.4

Let's test your understanding. Self Check 1.4

Solve the following quadratic equations.

Q1. u - 2v + 3w = 7 2u + v + w = 4 -3u + 2v - 2w = -10Answer: u=2, v=-1, w=1Q2. 2x - 4y + 5z = -33 4x - y = -5 -2x + 2y - 3z = 19Answer:  $x = -\frac{1}{2}$ , y = 3, z = -4

### **1.3** APPLICATION OF EQUATIONS IN BUSINESS

The use of equations in business allows the firm's stakeholders to address difficulties by translating them into mathematical formulae.

The relevant equations can be used to calculate fixed costs (FC), variable costs (VC), total costs (TC), total revenue (TR), profit (p), and quantity (Q).



Total Cost = Fixed Cost  $\times$  Total Variable Cost TC = FC  $\times$  TVC



Dianne & Sis has released a new batch of cookies. The fixed cost per jar is RM50,000, and the variable cost of each jar is RM7. The price of each jar will be RM18. Calculate the number of jars that must be sold in order to make an RM60,000 profit.

#### Solution:

VCpu = RM7 FC = RM50,000 Sp = RM18 TP = RM60,000 Quantity, Q =  $\frac{FC + TP}{Sp - VC}$ =  $\frac{RM50,000 + RM60,000}{RM18 - RM7}$ = 10,000 jars Example 1.7

> Mr. Arabie intends to spend RM50,000 on two different projects: Project Catering and Project Restaurant. Both projects are expected to produce a total return of RM2,800. Due to market rivalry, Project Catering will pay back 7% per year and Project Restaurant will payback 5% per year. What amount of money should he put into each project?

Solution: Project Catering = *c* Project Restaurant = *r* 

The investment: c + r = 50,000 — 1 : 0.07c + 0.05r = 2,800 \_\_\_\_\_2 Return Use 1 : r = 50,000 - c 3 Substitute 3 into 2 : 0.07c + 0.05(50,000 - c) = 2,8000.07c + 2,500 - 0.05c = 2,8000.02c = 800c = 40,000

Substitute c = 40,000 into 1 : 40,000 + r = 50,000r = 10,000

Thus, Mr. Arabie should invest RM40,000 in Project Catering and RM10,000 in Project Restaurant.



Let's test your understanding. Self Check 1.5

Kuyon Foodie use three machines in its operation - Machine a, b, Q1. and c. These three machines produce material J, K, and L.

To produce material J, machine a, b, and c need 4, 7, and 1 hour(s) respectively. 400 units of material *J* are produced monthly.

To produce material K, machine a, b, and c need 4, 5, and 4 hours respectively. 540 units of material K are produced monthly.

To produce material L, machine a, b, and c need 8, 4, and 2 hours respectively. 600 units of material L are produced monthly.

Determine the operating hours for each machine every month to produce three different materials by using simultaneous equations.

Answer: =50, b=20, c=60

- Q2. Kay Nod Sea Ent. produce product with RM2.50 of variable cost per unit. The fixed cost per unit is RM50,000 and the selling price is RM8.50 per unit. Yearly sales is 10,000 units. You are require to calculate:
  - i. Total Revenue
  - ii. Total Cost
  - iii. Profit earn
  - iv. Number of quantity sold if the target profit is RM35,000

#### EXERCISE 1.0

1. Solve the following linear equation.

a. 
$$3x - 1 = 2(x - 5)$$
  
b.  $5 + 3(x - 1) = 5x - 6$   
c.  $\frac{4(x + 2)}{5} = 7 + \frac{5x}{13}$   
d.  $\frac{x + 5}{6} - \frac{x + 1}{9} = \frac{x + 3}{4}$ 

- 2. Solve the following quadratic equations.
  - a.  $5x^2 + 20x 25 = 0$ b.  $4x^2 = 11x - 6$ c.  $x^2 - 70 = -3x$ d.  $25x^2 = 40x - 16$
- 3. Solve the following simultaneous equations.
  - a. y 2x = 3 y - 5x = -3c.  $2x + \frac{1}{3}y = 1$  3x + 5y = 6b. 7u + 3r = -15 12r - 5u = 39d.  $3u + 2r = \frac{5}{2}$  $\frac{1}{3}u + 3r = -\frac{4}{3}$
- 4. Solve the following equations.
  - a. -2x + 3y 4z = 7b. 2p + 3q r = 15x y + 2z = 134p + q 3r = 113x + 2y z = 173p 2q + 5r = 21
- 5. Bahagia Berhad produces product named C-Syiok. The variable cost per unit is RM6 and the fixed cost is RM80,000. The price of each unit is RM10. Number of units sold monthly is 30,000 units. You are required to calculate:
  - i. Total Cost
  - ii. Total Revenue
  - iii. Total Profit
  - iv. Determine the number of units that need to be sold if the company wants to earn an income of RM60,000.

- 6. RM35,000 was invested in 3 businesses. Those businesses will earn 6%, 7%, and 8% incomes respectively. The income first year was RM2,800 and the capital was reinvested. In the second year, the amount which earned 8% income had increased to 10% and other incomes were maintained as in year one. The total income for the second year was RM3,000. Calculate the initial amount invested in business 1, 2, and 3 respectively.
- 7. The following data was obtained from Branch A.

Selling price per unit	:	RM35
Total Variable cost (500 units)	:	RM5,000
Fixed Cost	:	RM250,000

Based on the data given, determine:

- a. The function of total cost, TC(Q)
- b. The function of total revenue, TR(Q)
- c. The function of profit earned by the company, P(Q)
- d. The quantity that needs to be sold if the target profit is RM50,000.
- e. The new profit if the price increase by 20% for the units sold in (d).
- f. The new profit if the fixed costs reduces by 10% with the quantity sold in (d).

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## Chapter 2 BREAKEVEN ANALYSIS

#### Learning Outcomes:

You will be able to do the following at the end of this chapter:

- Calculate the breakeven point.
- Using mathematics, calculate the sales levels (ringgit/unit), sales price, and profit.
  Using a graph, determine the link between cost, revenue, and profit, as well as the concepts of fixed and variable costs.
- Using a graph, draw the sales levels (value/unit) and profit.

The break-even point (BEP) is a point or stage where total costs (expenses) and total sales (revenue) are equal and there is no profit or loss.

The number of units or dollars of revenue required to cover all costs is determined using a break-even point analysis.

## 2.1 Break-even Point Analysis

The following are two popular methods for estimating the breakeven point:

i. Plan and control costs in relation to sales.

ii. Track and measure sales results.

The following are some of the limitations in determining the breakeven point:

- i. All fixed costs are the same.
- ii. The variable per-unit cost is fixed.
- iii. Total cost = Fixed + Variable
- iv. There are no changes in the operation's efficiency or productivity.
- v. The costs are solely determined by the volume of production or sales.

## 2.2 Analyzing BEP Using Algebra

The gap between the selling price and variable costs is known as the **contribution margin (CM)**.

Contribution Margin = Selling Price - Variable Costs CM = Sp - VC

The contribution margin ratio (CMR) is the percentage difference between the selling price and variable costs, indicating the percentage of profit generated on that given sale.

Contribution Margin Ratio =  $\frac{\text{(Selling Price - Variable Costs)}}{\text{Selling Price}} \times 100\%$  $CMR = \frac{\text{(Sp - Vc)}}{\text{Sp}} \times 100\%$  A break-even point can be calculated in terms of both quantity (in units) and value (in dollars) (in RM).









The following information was provided by Senah Sdn Bhd for the production of *Kek Lapis Serawak* for the year ended 31 December 2020.

Sales volume (Packet)	=	30,000
Selling Price	=	RM12.00
Variable Cost per packet	=	RM7.00
Annual fixed cost	=	RM50,000

Calculate:

(a) Profit for the year

(b) BEP in units and RM

(c) Units to be sold to get a profit of RM120,000

Solution:

(a) Profit for the year

I otal Revenue	$=$ Sp $\times$ Q
	= RM12.00 × 30,000
	= RM360,000
Total Costs	= Fixed Cost + Total Variable Cost
	= RM50,000 + RM7.00(30,000)
	= RM260,000
Profit	= Total Revenue - Total Cost
	= RM360,000 - RM260,000
	= RM100,000

(b) BEP in units and RM

BEP (Units)	=	<u>Fixed Costs</u> Sp - VC
	=	<u>RM50,000</u> RM12.00 - RM7.00
	=	10,000 packets
BEP (RM)	=	Fixed Cost
		(Sp - Vc) Sp
	=	RM50,000
		(RM12.00 - RM7.00) RM12.00
	=	RM120,000
		OR
BEP (RM)	=	$\mathrm{Sp}  imes \mathrm{BEP}$ (units)
	=	RM12.00 $ imes$ 10,000
	=	RM120,000

(c) Units to be sold to get a profit of RM120,000

Quantity to be sold = <u>Fixed Costs + Target Profit</u> Sp - VC

#### = <u>RM50,000 + RM120,000</u> RM12.00 - RM7.00

= 34,000 packets



In 2018, the sales of KasiMurni Ent. was RM122,000. Goods were sold at a price of RM8.00 per unit. The fixed cost in that year was RM20,650 and the total cost was RM42,000. In 2019, the company gained a profit of RM90,000.

Calculate:

- (a) BEP in units and value (RM) for year 2018
- (b) Profit earned in 2018
- (c) Required sales volume to achieve the profit as in year 2019



Metroyce Sdn Bhd is considering two new products, Vanilla Cheese Cupcake and Chocolate Mint Cupcake. Data for both products are in the following table.

	Vanilla Cheese	<b>Chocolate Mint</b>
Annual Fixed Cost (RM)	21,500	14,000
Variable Cost per Unit (RM)	1.20	3.50
Selling Price (RM)	6.00	6.00

Find:

- (a) BEP in units and value (RM) for both products
- (b) Total sales (in RM) for both products if the total sales volume (in units) is 10% above BEP.
- (c) Based on the sales volume in (b), calculate the profit for each product and decide which product will bring higher profit and should be chosen by the company.

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### 2.3 Analyzing BEP Analysis Using Graph

The graph can be used to determine the break-even point.

- (a) BEP is the point where the graphs for total revenue and total cost cross.
- (b) The x-axis is used to read the quantity of BEP.
- (c) The y-axis is used to read the BEP value (RM).
- (d) The profit area is the region above the break-even point.
- (e) The loss area is the area below the BEP.

#### Example 2.2

The following data is obtained from a factory in Klang.

Selling price	:	RM24
Variable cost of 1000 units	:	RM15,000
Fixed cost	:	RM200,000

Based on the data provided above, state:

- (a) Total cost function, *C(x)*
- (b) Total revenue function, *TR(x)*
- (c) Profit function, *P(x)*. Construct the table for graphs of the break-even point.
- (d) Draw the straight line graphs and estimate the BEP (quantity & value)
- (e) Shade the profit & loss areas on the graph.
- (f) Profit (Loss) if:
  - i. 20,000 units sold
  - ii. 5,000 units sold

Solution:

(a) Total cost function, C(x)

C(x) = FC + VC(Q)= 200,000 + 15x

(b) Total revenue function, *TR(x)* 

 $TR(x) = Sp \times Q$ = 24x

(c) Profit function, *P*(*x*)

$$P(x) = TR(x) - C(x)$$
  
= 24x - (200,000 + 15x)  
= 24x - 20,000 - 15x  
= 9x - 20,000

Quantity, x	10,000	20,000	30,000	40,000	50,000
Fixed Cost (RM)	200,000	200,000	200,000	200,000	200,000
Variable Cost (RM)	150,000	300,000	445,000	600,000	750,000
Total Cost (RM)	350,000	500,000	645,000	800,000	975,000
Total Revenue (RM)	240,000	480,000	720,000	960,000	1,200,000

(d) Draw the straight line graphs and estimate the BEP (quantity & value)



(f) Profit (Loss) if:
i. 20,000 units sold
P(x) = 9(20,000) - 200,00
= -RM20,000 (Loss)

ii. 5,000 units sold

P(x) = 9(5,000) - 200,00= -RM155,000 (Loss)



Banisah & Co. produces disposable plate and cup for the market in Sri Aman. Below is the information given by the company:

Selling price	:	RM22.00
Variable cost per unit	:	RM7.00
Fixed cost	:	RM85,000

Based on the data provided above, state:

- (a) Total cost function, *C(x)*, total revenue function, *TR(x)*, profit function, *P(x)*
- (b) BEP (units & value) using formulae
- (c) Construct the graph of the fixed cost, total cost & total revenue.
- (d) From the graph, show the BEP (units & value)
- (e) Shade the profit & loss areas on the graph.

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# Chapter **3** CALCULUS

### Learning Outcomes:

You will be able to do the following at the end of this chapter:

- Apply the differentiation idea and guidelines.
- Fill in the blanks for the first and second derivate.
- Determine the variable, maximum and minimum points, and point of inflection.
- Solve corporate mathematics problems using the principles and procedures of differentiation from calculus.

The science of calculus was simultaneously developed by Isaac Newton (1642 - 1727) and Gottfried Wilheim von Leibniz (1646 - 1716).

Differential and integral calculus make up calculus, which is a dynamic idea. Finding the rate of change of one quantity with respect to another is the subject of differential calculus. If we know the rate at which one quantity changes in relation to another, we can build a relationship between the two.

## 2.1 Differentiation

Let y = f(x) be a given function. The derivative of y with respect to x, denoted by f'(x), is defined by

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

The method of finding the derivative using the aforementioned definition is the first principle of derivative. Differentiation is also known as the differential coefficient, and it is the process of calculating a function's derivative. If a function's derivative exists at a given position, we say that f(x) is differentiable at that location. y's derivative

with respect to x also can be done through f'(x),  $\frac{dy}{dx}$  or y'.

The derivative have several meanings depending on its use; gradient & rate of change.

## 3.1.1 Rules of Differentiation Rule 1 (Constant Rule): If y = y(x) = c, then y'(x) = 0





Rule 4  
(Sum Rule): If 
$$y(x) = ax^n + bx^m$$
, then  $y'(x) = \frac{dy}{dx} = nax^{n-1} + mbx^{m-1}$ 



Find y'(x) if (a)  $y(x) = 8x^2 + 4x$ 

Solution:

$$y(x) = 8x^{2} + 4x$$

$$\frac{dy}{dx} = 2(8)x^{2-1} + 4x$$

$$= 16x + 4$$

(b)  $y(x) = -49x^3 + 7x^2$ 

$$y(x) = -49x^{3} + 7x^{2}$$

$$\frac{dy}{dx} = 3(-49)x^{3-1} + 2(7x)^{2-1}$$

$$= -147x^{2} + 14x$$

If 
$$y(x) = uv$$
,  $u$  and  $v$  are functions of  $x$   
Rule 5  
(Product Rule):  $\frac{dy}{dx} = u(x)\frac{dv}{dx} + v(x)\frac{du}{dx}$   
Find  $y'(x)$  if  
(a)  $y(x) = 8x^2 (4x + 2)$   
Solution:  
 $u(x) = 8x^2$   $v(x) = 4x + 2$   
 $\frac{du}{dx} = 2(8)x^{2-1}$   $\frac{dv}{dx} = 4$   
 $= 16x$   
Substitute into formulae:  
 $\frac{dy}{dx} = u(x)\frac{dv}{dx} + v(x)\frac{du}{dx}$   
 $\frac{dy}{dx} = 8x^2(4) + [(4x + 2)(16x)]$ 

$$\frac{3}{dx} = 8x^{2}(4) + [(4x + 2)(16x)]$$
$$= 32x^{2} + 64x^{2} + 32x$$
$$= 96x^{2} + 32x$$

(b) 
$$y(x) = (2x + 7)(x - 9)$$
  
Solution:

$$u(x) = 2x + 7$$

$$\frac{du}{dx} = 2$$

$$v(x) = x - 9$$

$$\frac{dv}{dx} = 1$$

Substitute into formulae:

$$\frac{dy}{dx} = u(x)\frac{dv}{dx} + v(x)\frac{du}{dx}$$
$$\frac{dy}{dx} = [(2x+7)(1)] + [(x-9)(2)]$$
$$= 2x + 7 + 2x - 18$$
$$= 4x - 11$$

Rule 6  
(Quotient Rule):  

$$\frac{dy}{dx} = \frac{v(x) \frac{du}{dx} - u(x) \frac{dv}{dx}}{[v(x)]^2}$$
Example 3.6  
Find y'(x) if  
(a)  $y(x) = \frac{8x^2}{4x + 2}$   
Solution:  
 $u(x) = 8x^2$   $v(x) = 4x + 2$   
 $\frac{du}{dx} = 16x$   $\frac{dv}{dx} = 4$   
Substitute into formulae:  
 $\frac{dy}{dx} = \frac{v(x) \frac{du}{dx} - u(x) \frac{dv}{dx}}{[v(x)]^2}$   
 $= \frac{[(4x + 2)(16x)] - [(8x^2)(4)]}{[4x + 2]^2}$   
 $= \frac{64x^2 + 32x - 32x^2}{[4x + 2][4x + 2]}$   
 $= \frac{32x^2 + 32x}{[16x^2 + 8x + 8x + 4]}$ 

$$32x^2 + 32x$$

 $16x^2 + 16x + 4$ 

b) 
$$y(x) = \frac{5x^2 + 6}{x^3 + 6}$$
  
Solution:  
$$u(x) = 5x^2 + 6$$
$$\frac{du}{dx} = 10x$$
$$\frac{dv}{dx} = 3x^2$$

Substitute into formulae:

$$\frac{dy}{dx} = \frac{v(x) \frac{du}{dx} - u(x) \frac{dv}{dx}}{[v(x)]^2}$$

$$= \frac{[(x^3 + 6)(10x)] - [(5x^2 + 6)(3x^2)]}{[x^3 + 6]^2}$$

$$= \frac{10x^4 + 60x - 15x^4 - 18x^2}{[x^3 + 6][x^3 + 6]}$$

$$= \frac{-5x^4 - 18x^2 + 60x}{x^6 + 6x^3 + 6x^3 + 36}$$

$$= \frac{-5x^4 - 18x^2 + 60x}{x^6 + 12x^3 + 36}$$
If  $y(x) = [u(x)]^2$ ,  $\frac{dy}{dx} = n$ .

$$y(x) = [u(x)]^{2}, \ \frac{dy}{dx} = n \cdot [u(x)]^{n-1}, \text{ then}$$
$$y'(x) = \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

Example 3.7

Find y'(x) if  
(a) 
$$y(x) = (7x^2 + 1)^3$$

Solution:  $u(r) = 7r^2 + 1$   $v(r) = u^3$ 

Substitute into formulae:

$$y'(x) = \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$
$$= (147x^4 + 42x^2 + 3)(14x)$$
$$= 2,058x^5 + 588x^3 + 42x$$

(b) 
$$y(x) = (3x - 5)^5$$
  
Solution:  
 $u(x) = 3x - 5$   
 $\frac{du}{dx} = 3$   
 $y(x) = u^5$   
 $\frac{dy}{du} = 5u^4$   
 $= 5(3x - 5)^4$ 

Substitute into formulae:

$$y'(x) = \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$
$$= 5(3x - 5)^4 \cdot 3$$
$$= 15(3x - 5)^4$$



Self-check 3.1 - Try to do this.

Differentiate the following with respect to *x*:

(a) 
$$y(x) = 2 - 7x$$
  
(b)  $y(x) = x + 2x^2 + 3x^3$   
(c)  $y(x) = (2x + 3x^2)(2 - 8x)$   
(d)  $y(x) = \frac{5x - 3}{x^2 + 1}$   
(e)  $y(x) = \sqrt[5]{3x^2 - x} + 7$ 

### **3.2** Application of Derivatives



The Marginal Functions of total revenue, profit, cost, and so on are created using the first derivative. The marginal function can be used to find the function's Critical Quantity or Volume, which is the quantity at which the function's value changes.



The profit function of a product is given by  $P(x) = 20x - 0.02x^2$ - 500; x represents the quantity. Find the critical quantity for the functions.

Solution:  $P(x) = 20x - 0.02x^{2} - 500$  $\frac{dP}{dx} = 20 - 0.04x$ 

Assume that 
$$\frac{dP}{dx} = 0$$
  
 $20 - 0.04x = 0$   
 $0.04x = 20$   
 $x = 500$  units

The revenue function of a product is given by  $R(x) = 18x - 0.04x^2$ ; x represents the quantity. Find the critical quantity for the functions.

Solution:  $R(x) = 18x - 0.04x^{2}$   $\frac{dR}{dx} = 18 - 0.08x$ Assume that  $\frac{dR}{dx} = 0$  18 - 0.08x = 0 0.08x = 18x = 225 units

The revenue function of a product is given by  $R(x) = 40x - 0.08x^2$ ; x represents the quantity. Find the marginal revenue function.

Solution:

$$R(x) = 40x - 0.08x^{2}$$
$$\frac{dR}{dx} = 40 - 0.16x$$

The marginal revenue function is R'(x) = 40 - 0.16x.

3.2.2 Second Derivative

The second derivative or second derived function is obtained by further differentiating the first derived function. The second derivatives can be used to determine whether a function produces a critical quantity's maximum, minimum, or inflection point.

Example 3.11

Find the first and second derivatives of  $P(x) = 20x - 0.02x^2 - 500$ 

Solution:  $P(x) = 20x - 0.02x^{2} - 500$   $\frac{dP}{dx} = 20 - 0.04x$ (First derivative)  $\frac{d^{2}P}{dx^{2}} = -0.04$ (Second derivative)



Find the first and second derivatives of  $R(x) = 40x - 0.08x^2 - 500$ 

Solution:

$$R(x) = 40x - 0.08x^{2}$$

$$\frac{dR}{dx} = 40 - 0.16x$$
(First derivative)
$$\frac{d^{2}R}{dx^{2}} = -0.16$$
(Second derivative)

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The critical point comprise the **local maximum point**, **local minimum point**, and **inflection point**. Illustration of some of the critical points are shown as follows:



The point x = c is called a **critical point** for a continuous function, *f* if: (a) f'(c) = 0 or f'(c) fails to exist, and (b) f(c) is well defined.

#### Second Derivative and First Derivative Tests

The second derivative test and the first derivative test are two tests that can be used to investigate the nature of critical points.

#### Second Derivative Test

3 steps in the second derivative test:

- 1. Find *f* ′(*x*) and *f* ′′(*x*).
- 2. Let f'(x) = 0 and solve x for critical values, c.
- 3. Substitute the critical values, c into f "(x). If f "(c) is
  a. Positive, the point is a local minimum point = U
  - b.**negative**, the point is a local **maximum** point = n c.**zero**, the **test fails** and the first derivative test has to be used.

#### **First Derivative Test**

- 1. Find *f* ′(*x*).
- 2. Let f'(x) = 0 and solve x for critical values, c.
- 3. Select points in the neighbourhood of c, one slightly less and the other slightly more than the critical value, c and substitute into f'(x). If f'(x)
  - a.changes sign from **negative to positive**, the point is a local **minimum point**. = **u**
  - b.changes sign from **positive to negative**, the point is a local **maximum point.** = **n**
  - c.does not change sign, the point is an inflection point.



Determine the critical point(s) for the curve  $y = x^2$ .

Solution:

 $\frac{dy}{dx} = 2x$  (First derivative)  $\frac{d^2y}{dx^2} = 2$  (Second derivative) Let  $\frac{dy}{dx} = 0$ Hence, 2x = 0 x = 0When  $x = 0, y = (0)^2 = 0$ . The point (0,0) is a critical point. When x = 0 $\frac{d^2y}{dx^2} = 2$  (Positive) = u

Hence, the point (0,0) is a local minimum point.



da

$$\frac{dy}{dx} = 3(2x^{(3-1)}) - 2(3x^{(2-1)}) - 12$$
  
=  $6x^2 - 6x - 12$   
$$\frac{d^2y}{dx^2} = 2(6x^{(2-1)}) - 6$$
  
=  $12x - 6$ 

At critical points,  $\frac{dy}{dx} = 0$ Thus,  $6x^2 - 6x - 12 = 0$ Factorise,  $\begin{array}{c|cccc} x & 1 & 6x & (x+1)(6x-12) = 0 \\ \hline 6x^2 & -12 & -12x & (x+1) = 0 & \text{or} & (6x-12) = 0 \\ \hline 6x^2 & -12 & -6x & x = -1 & x = 2 \end{array}$ Substitute x = -1 into  $y(x) = 2x^3 - 3x^2 - 12x + 2$   $y(x) = 2(-1)^3 - 3(-1)^2 - 12(-1) + 2 = 9$ Substitute x = 2 into  $y(x) = 2x^3 - 3x^2 - 12x + 2$  $y(x) = 2(2)^3 - 3(2)^2 - 12(2) + 2 = -18$ 

Thus, the critical points are (-1,9) and (2,-18). Substitute x = -1 into  $\frac{d^2y}{dx^2} = 12x - 6$   $\frac{d^2y}{dx^2} = 12(-1) - 6 = -18$  (local maximum point) = n Substitute x = 2 into  $\frac{d^2y}{dx^2} = 12x - 6$  $\frac{d^2y}{dx^2} = 12(2) - 6 = 18$  (local minimum point) = u





Find the coordinates of the turning point(s) of the following curves and determine their nature.

(a)  $y(x) = x^2 - x - 2$  (b)  $y(x) = x^3 - 3x^2 - 9x - 8$
# 3.3 Application of Calculus in Economics & Business

The goal of using calculus in business and economics is to maximize profits while minimizing costs. To maximize earnings, a company may need to identify the appropriate price for its products and services.

# 3.3.1 Total Cost, Average Cost and Marginal Cost

Total Cost = Fixed Costs + Variable Costs

If C(x) represents the total cost of producing x units of a product, then

Average Cost, 
$$C(x) = \frac{C(x)}{x}$$

Marginal Cost, MC(
$$x$$
) = C'( $x$ ) =  $\frac{dC}{dx}$ 

Marginal Cost is the rate of change in cost per unit change in production at an output level of x units.

# Example 3.15

A firm produces x units of a product per month and the total cost per month in RM is given by  $C(x) = 150 + 0.02x^2$ . Find:

(a) the total cost when 20 units are produced.

- (b) the average cost function.
- (c) the average cost when 20 units are produced.

(d) the marginal cost function.

(e) the marginal cost when 20 units are produced.

#### Solution:

(a)  $C(r) = 150 \pm 0.09r^2$ 

(d) MC(x) = C'(x) = 
$$\frac{dC}{dx}$$

(a) 
$$C(x) = 150 + 0.02x$$
  

$$= 150 + 0.02(20)^{2}$$

$$= RM158$$
(b)  $AC(x) = \frac{C(x)}{x}$ 

$$= \frac{150 + 0.02x^{2}}{x}$$

$$= \frac{150}{x} + \frac{0.02x^{2}}{x}$$

$$= \frac{150}{x} + 0.02x$$
(c)  $AC(x) = \frac{150}{20} + 0.02(20)$ 

$$C(x) = 150 + 0.02x^{2}$$
  

$$C'(x) = 2(0.02x)^{2-1}$$
  

$$= 0.04x$$

(e) MC(x) = C'(x) = 
$$\frac{dC}{dx}$$
  
= 0.04(20)

$$= RM0.80$$

(c) AC(x) = 
$$\frac{150}{20} + 0.02(20)$$
  
= RM7.90

# 3.3.2 Total Revenue, Average Revenue and Marginal Revenue

Revenue Function, R(x) = pxWhere  $p = f(x) \longrightarrow$  Demand function Thus,  $R(x) = f(x) \bullet x$ 

Marginal revenue is the rate of change in revenue per unit change in production at an output level of x units of the product. It is the additional revenue obtained when an additional unit of the product is produced and sold. Thus, marginal revenue is the derivative of the total revenue function.

Marginal Revenue, MR(
$$x$$
) =  $R'(x) = \frac{dR}{dx}$ 

Average Revenue, 
$$AR(x) = \frac{R(x)}{x}$$

Example 3.15

If the total revenue per month in RM, R(x) is given by  $R(x) = 20x - 0.03x^2$  where x is the number of units produced and sold per month. Find

(a) the total revenue when 30 units are produced and sold.

- (b) the average revenue function.
- (c) the average revenue when 30 units are produced and sold.
- (d) the marginal revenue function.

(e) the marginal revenue when 30 units are produced and sold.

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#### Solution:

(a) 
$$R(x) = 20x - 0.03x^{2}$$
  
 $= 20(30) - 0.03(30)^{2}$   
 $= RM573$ 
(b)  $AR(x) = \frac{R(x)}{x}$   
 $= \frac{20x - 0.03x^{2}}{x}$ 
(c)  $MR(x) = R'(x) = \frac{d}{dx}$   
 $= 20 - 0.06x$ 
(c)  $MR(x) = R'(x) = \frac{d}{dx}$   
 $= 20 - 0.06(30)$   
 $= RM18.20$ 

(c) 
$$AR(x) = 20 - 0.03(30)$$

= RM19.10

# 3.3.3 Profit, Breakeven Analysis and Optimisation

The total revenue function, R(x) and the total cost function, C(x) are known, then the total profit function, P(x) is given by

$$P(x) = R(x) - C(x)$$

The break-even level of production is given by R(x) = C(x) or P(x) = 0

Differentiating the total profit function, P(x) = R(x) - C(x), we get

$$P'(x) = R'(x) - C'(x)$$

or maximum profit, P'(x) = 0. Thus, R'(x) - C'(x) = 0 or R'(x) = C'(x)

Thus, for a firm to obtain maximum profit, marginal revenue must be equal to marginal cost.

# Example 3.16

The demand for an item produced by Weelux is given by p + 0.2x = 100 where p is the price per unit and x is the quantity demanded. The total cost, C(x) of producing x units of the item is given by C(x) = 800 + 30x where x is the level of output. Find (a) the total revenue function.

- (b) the profit function.
- (c) the total profit when 100 units are sold.
- (d) the marginal profit function.
- (e) the marginal profit when(i) 90 units are sold

#### (ii) 300 units are sold

Solution: From p + 0.2x = 100, we get p = 100 - 0.2x(a) R(x) = px = (100 - 0.2x)(x)  $= 100x - 0.2x^2$ (b) P(x) = R(x) - C(x)  $= (100x - 0.2x^2) - (800 + 30x)$   $= 100x - 0.2x^2 - 800 - 30x$  $= -0.2x^2 + 70x - 800$ 

(c) 
$$P(x) = -0.2(100)^2 + 70(100) - 800$$
  
= RM4,200

(d) 
$$P(x) = -0.2x^2 + 70x - 800$$
  
 $P'(x) = \frac{dP}{dx} = -0.4x + 70$ 

(e) (i) 
$$P'(x) = \frac{dP}{dx} = -0.4(90) + 70$$
  
= RM34

When 90 units are sold, the marginal profit (extra profit per extra item when production is increased by a small quantity) is RM34 per additional unit.

(ii) 
$$P'(x) = \frac{dP}{dx} = -0.4(300) + 70$$
  
= -RM50

When 300 units are sold, a small increase in production results in a loss (negative profit) of RM50 per extra unit.



# **Question 1**

The revenue function and cost function of a product are  $R(x) = 200x - x^2$  and  $C(x) = 0.1x^2 + 2x + 800$  respectively, find

- (a) the profit function.
- (b) the maximum revenue.
- (c) the price at maximum revenue.
- (d) the increase in cost if the number of product increases from 50 units to 100 units.

# **Question 2**

Given,

Revenue Function:  $R(x) = 150x - 0.2x^2$ Cost Function:  $C(x) = 0.1x^2 - 15x + 1,500$ Find

- (a) the marginal revenue function.
- (b) the marginal cost revenue.
- (c) the profit function.
- (d) the level of production which will maximise profit. Also, find the maximum profit and the price per unit when profit is maximised.





# INTEREST

Learning Outcomes:

At the end of this chapter, you will be able to:

- Calculate the simple interest, principal rate, time, maturity date, and maturity value
- Solve the concepts of promissory notes
- Apply of interest based on financial and business practices in Malaysia
- Calculate rebate and early payments methods for a personal loan and housing loan
- Apply the concept of compound interest to calculate the present value and future value
- Apply the concept of annuity to calculate the present value and future value

# 4.1 INTRODUCTION

According to Lau, T. K., Phang, Y. N., and Wee, K. K. (2012), the development of interest at the first place is due to the concept where lenders will face losses in lending money. It is important to understand on the economy of a country when interest rates fall, stock prices will increase, and when rate increase, stock prices will fall.

# 4.2 INTEREST

Interest can be defined in two ways:

First : Interest is money earned when money is invested. When you deposited RM5,000 in a bank for 5 years, you find that at the end of year 1, you have RM5,350 in your account. The additional RM350 is the interest you earned when you invested RM5,000.

Second : Interest is a charge incurred when a loan or credit is obtained.

When you borrowed RM3,000 from the same bank for 3 years and you are actually paying RM3,252 at the end of the third year. The additional RM252 is the the charge that you have paid to the bank when you are borrow RM3,000.

There are two types of interest: simple interest, and compound interest.



Simple interest is the interest calculated on the original principal for the entire period it is borrowed and invested. Simple interest can be calculate using the following formula:

Where, P = Principal r = Interest rate t = Terms or period

# 4.3.2 Simple Amount Formula

Simple amount is the sum of the original principal and the interest earned. The formula is as follow:

$$S = P + I$$

Where, *P* = Principal *I* = Simple interest

OR

$$S = P(1 + rt)$$

Where, P = Principal r = Interest rate t = Terms or period



RM10,000 is invested for 4 years 9 months in a bank earning a simple interest rate of 10% per annum. Find the simple interest and the total amount at the end of the investment period.

Solution:

P = RM10,000 r = 10% = 0.1t = 4 years 9 months = 4.75

Simple interest, *I* = *Prt* 

= RM10,000 (0.1) (4.75) = RM4,750

Total amount at the end of the period: S = P(1 + rt)

- = RM10,000 [1 + 0.1(4.75)]
- = RM14,750



Ayan invests RM12,000 in an investment fund for 5 years. At the end of the investment period, his investment will be worth RM15,000. Find the simple interest rate offered by the company.

Solution:

```
P = RM12,000

r = ?

t = 5 years

I = RM15,000 - RM12,000 = RM3,000

Using, I = Prt

RM3,000 = RM12,000 (?) (5)

= RM60,000

r = \frac{RM3,000}{RM60,000} \times 100\%

= 5\%
```



How long does it take to get RM5,200 if Mary start to invest RM2,500 today with a simple interest rate of 12%?

Solution:

```
P = RM2,500

r = 12\%

t = ?

I = RM5,200 - RM2,500 = RM2,700

Using, S = P[1 + 0.12(?)]
```

RM5,200 = RM2,500 [1 + (0.12) t] = RM2,500 + 300 t 300 t = RM2,700 $t = \underline{RM2,700}$ = 9 years



- 1. Maleficient deposited RM5,000 in a bank and obtained RM600 simple interest after 3 years. Find the simple interest rate offered.
- 2. Dayang borrows RM500 for 100 days at 3% per annum simple interest. How much is the interest charged? What is the total amount of her debt at the end of the 100 days?
- 3. Five years ago, Anoy invested RM6,600 in a bank at a simple interest rate of 7.2%. Find
  - a. The amount in the account today
  - b.The number of years of Anoy wanted the amount in the account to become RM9,500.

# 4.3.3 Four Basic Concept

The period of an investment may be calculated using the following concept:

1. Exact Time

It is the exact number of days between two given dates. For example, 1 - 31 January 2020 has 31 days

2. Approximate time

Assume that a month has 30 days.

3. Ordinary Simple Interest

In calculating ordinary simple interest, we use 360 per year.

4. Exact Simple Interest

This uses a 365 or 366-day per year in interest computation.



RM1,000 was invested on 2 January 2020. If the simple interest rate offered was 7.8% per annum, find the interest received on 24 May 2020 using

- a. Exact time and exact simple interest
- b. Exact time and ordinary simple interest (Banker's Rule)
- c. Approximate time and exact simple interest
- d. Approximate time and ordinary simple interest

#### Solution:

a. Exact time and exact simple interest

Date	Days	Interest Calculations
2 - 31 Jan	30	
1 - 29 Feb	29	I = Prt
1 - 31 Mar	31	$= RM1,000 (0.1) (\frac{144}{266})$
1 - 30 Apr	30	000 DN (200 24
1 - 24 May	24	= RM39.34
	144	

b. Exact time and ordinary st	nple interest (Banker's Rule)
-------------------------------	-------------------------------

Date	Days	<b>Interest Calculations</b>
2 - 31 Jan	30	T D
1 - 29 Feb	29	I = Prt
1 - 31 Mar	31	$= \text{RM1,000}(0.1)(\frac{144}{260})$
1 - 30 Apr	30	000 DM(40.00
1 - 24 May	24	= RM40.00
	144	

#### c. Approximate time and exact simple interest

Date	Days	<b>Interest Calculations</b>
2 - 30 Jan	29	
1 - 30 Feb	30	I = Prt
1 - 30 Mar	30	$= \text{RM1,000}(0.1)(\frac{143}{266})$
1 - 30 Apr	30	000 DM00.07
1 - 24 May	24	= RM39.07
	143	

# d. Approximate time and ordinary simple interest

Date	Days	Interest Calculations
2 - 30 Jan	29	
1 - 30 Feb	30	I = Prt
1 - 30 Mar	30	$= RM1,000 (0.1) (\frac{143}{260})$
1 - 30 Apr	30	500 DN 600 70
1 - 24 May	24	= RM39.72
	143	

Banker's Rule is mostly used by banks in the USA and in various international business transactions but not in Malaysia. In Malaysis, calculation of interest is governed by a banking rule which states that the **365-day** per year must be used.

The discussions throughout this chapter will using the exact time and 360-day year.

# 4.3.4 **Present Value**

Simple amount is the sum of the original principal and the interest earned. The formula is as follow:

From this formula, S = P(1 + rt)

We want to find the Present Value which is the value of P

$$P = \frac{S}{(1+rt)}$$

or

$$P = S (1 + rt)^{-1}$$



Find the present value at 10% simple interest of a debt RM2,500 due in 5 months.

Solution:

$$P = ? \qquad P = S (1 + rt)^{-1}$$
  

$$S = RM2,500 \qquad = RM2,500 \left[ 1 + 0.1 \left( \frac{5}{12} \right) \right]^{-1}$$
  

$$t = \frac{5}{12} \qquad = RM2,400$$



A debt of RM1,000 due in four months and another of RM1,500 due in nine months are to be settled by a single payment at the end of six months. Find the size of this payment using

- a. The present value as the focal date
- b. The date of settlement as the focal date



Assume the money is worth a simple interest of 6% per annum.

Amount of the RM1,000 debt at the focal date,

$$= \text{RM1,000} \left[ 1 + \left( 0.06 \times \frac{4}{12} \right) \right]^{-1}$$

= RM980.39

Amount of the RM1,500 debt at the focal date,

$$= \text{RM1,500} \left[ 1 + \left( 0.06 \times \frac{9}{12} \right) \right]^{-1}$$

= RM1,435.41

Amount of the single payment for the settlement

RM980.39 + RM1,435.41 = 
$$x \left[ 1 + \left( 0.06 \times \frac{6}{12} \right) \right]^{-1}$$
  
RM2,415.80 = 0.9709 $x$   
 $x = RM2,488.21$ 



Self-check 4.2 - Try to do this.

- 1. On 15 June 2019, Afifah put RM2,000 in a savings account that offers a simple interest rate of 2.8% per annum. Find the amount in her account on 25 November 2019 using Banker's Rule.
- 2.On 10 March 2020, Patrick deposited RM800 into a savings account which pays 4.8% simple interest per annum. Eight months later, he withdrew RM200 from the account. Find the amount that is left in the account after the withdrawal.
- 3.An obligation of RM8,400 is due in 6 months. What is the

present value of this obligation at a bank discount rate of 12%?4.Aniqa needs RM12,000 for 6 months. What should the loan size be if the bank charges her with a discount rate of 7%?



# **Promissory Notes**

Promissory notes are also known as debt instruments that can be used by anyone who wants to borrow money or lend money to another person or party. The function of this note is the promises state a sum of money with a future date signed by the borrower as the proof to make the payment to the lender.

An example of promissory note:

19 February 2021
p pay the order of <i>Khairul Lender</i> <i>Five Hundred Only</i> for value nnum until paid. Due: <i>19 April 2021</i>
Hanani

- Maker / Payer The person who signs the note.
- Payee • The person who receives the payment.
- Date of the note • The date on which the note is made
- Term of the note • The length of the period till the note is due for payment.
- Face value

The amount stated on the note.

• Maturity value

The total sum of money which the payee will receive on the maturity date. (Face value + Interest)

• Maturity Date

The date on which the maturity value is due.

In the promissory note :

- (a) Who is the maker of the note?
- (b) Who is the payee of the note?
- (c) What is the date of the note?
- (d) What is the term of the note?
- (e) What is the maturity date of the note?
- (f) How much is the face value of the note?
- (g) Calculate the maturity value of the note?

Solution:

- (a) The maker of the note is HANANI.
- (b) The payee of the note is KHAIRUL LENDER.
- (c) The date of the note is 19 FEBRUARY 2021.
- (d) The term of the note is SIXTY DAYS.
- (e) The maturity date of the note is 19 APRIL 2021.
- (f) The face value of the note is **RM2,500.00**.
- (g) Calculate the maturity value of the note? S = P(1 + rt)

$$= \text{RM2,500} \left[ 1 + \left( 0.065 \times \frac{60}{360} \right) \right] - \text{Banking rule}$$

= RM2,527.08

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Nally received 120 days promissory note for RM20,000 which mature on 2 August 2010 with a simple interest rate of 5% per annum.

Find:

- (a) The date of the note
- (b) Maturity value

Solution:

- (a) The date of the note
  - 1 2/8/2010 2 days 1 - 31/7/2010 31 days 1 - 30/6/2010 30 days 1 - 31/5/2010 31 days
  - 5 30/4/2010 26 days

**120** Days

Thus, the date of the note is 5 April 2010.

(b) Maturity value

$$S = P(1 + rt)$$

$$= \text{RM20,000} \left[ 1 + \left( 0.05 \times \frac{120}{360} \right) \right]$$
$$= \text{RM20,333.33}$$



# 4.3.6 Bank Discount

It is common for lenders, such as banks and financial institutions, to deduct the interest charge in advance for short-term loans. This charge is called a bank discount or interest in advance. The formula is:  $D = S \times d \times t$ 

Where, *D* = Bank discount

- *S* = Amount of maturity Value
- *d* = Discount rate per year
- *t* = term of discounts in years.

To calculate the proceeds, *R*, we can use the following formula: R = S - D

- = S Sdt
- = S(1 dt)

Where,

R = Proceeds (consider as the present value in time value of money)

S = Maturity Value

- *d* = Discount rate per year
- t = Term of discounts in years



(Already borrowed)

Marissa signs a note for a discounted loan and agrees to pay the lender RM5,000 in 10 maths at a 9% interest per annum compound annually. How much should Marissa receive?

Solution:

S = RM5,000 d = 9% = 0.09  $t = \frac{10}{12}$  D = Sdt  $= RM5,000 \times 0.09 \times \frac{10}{12}$  = RM375The proceed, R = S - D = RM5,000 - RM375= RM4,625

Example 4.9

(Not yet borrow)

Yakka Dee wishes to borrow RM10,000 for 6 months. if the person he is borrowing from offers a discounted loan at 8%, how much must he repay at the end of 6 months?

Solution:

S = ?d = 8% = 0.08

$$t = \frac{6}{12}$$
  

$$D = \text{RM10,000}$$
  

$$D = S(1 - dt)$$
  

$$\text{RM10,000} = S \left[ 1 - \left( 0.08 \times \frac{6}{12} \right) \right]$$
  

$$S = \text{RM10,416.67}$$

The repayment for the loan is RM10,416.67.

# 4.3.7 **Discounting promissory Notes**

Promissory notes can be sold to bank before its maturity date if the holder is in need of cash. Selling the note to the bank is called discounting the note. The amount received on the date of discounting is called the proceeds. The proceeds of a promissory note can be computed using the following procedure:

Step 1: Find the maturity value of the note.

Step 2: Find the Bank discount

Step 3: Compute the proceeds

Example 4.10

Geri receives a promissory note for RM1,500 with interest of 10% per annum that is due in 60 days. The note is dated 10 April 2021. The note is discounted on 15 April 2021 at a bank that charges 12% discount.

Determine:

- (a) The maturity date
- (b) The maturity value
- (c) The discount period
- (d) The proceeds

Solution:

(a) The maturity date

10 - 30/4/2021	21 days
1 - 31/5/2021	31 days
1 - 8/6/2021	8 days
	60 Days
Thus, the maturit	y date is 8 June 2021.

(b) The maturity value S = RM1,500[1 + (0.1 x 60/360)] = RM1,525

(c) The discount period 15/4/2021 - 30/4/2021 16 days 1/5/2021 - 31/5/2021 31 days 1/6/2021 - 8/6/2021 8 days 55 Days (d) The proceeds Bank Discount, D = Sdt= RM1,525×0.12×55/360 = RM27.96



Self-check 4.3 - Try to do this.

1. A promissory note dated 15 August 2020 reads "two months from date, I promise to pay RM5,000 with interest at 8% per annum. Find

a. The maturity date of the note

b. The maturity value of the note

- 2. The maturity value of a 60-days interest-bearing promissory note is RM2,020. If the interest is 6% per annum, what is the face value of the note?
- 3. Juliana borrows RM5,000 for six-months from a lender who charges a discount rate of 9%. Find

a. The discount

b. The proceeds

4. Acha receives a promissory note for RM8,000 with interest of 9% per annum. The note dated 12 April 2017 is due in 90 days. The note is discounted on 15 June 2017 at a bank that charges 6% discount. Find:

a.The maturity date

b.The maturity value

- c.The proceeds
- 5. A company had a note dated 26 March 2019 for RM7,500

with interest of 5.5% per annum. The team of the note was six months. The company discounts the note on 21 August 2019 at a bank that charges a discount rate of 4%. how much are the proceeds?

# 4.4 Loans

4.4.1

# Personal Loan

Personal loans are calculated using simple interest - usually involve small loan amounts which will be repaid within a short period.





Example 4.11

A computer has a cash price of RM4,500. If Shazam intends to pay through monthly repayment, he is required to pay a deposit of 10% of the cash payment. The balance must be paid by equal monthly payments with fixed interest rate of 8% for 2 years. Calculate:

- (a) Total interest charged
- (b) Instalment price
- (c) Monthly payment

Solution:

Loan Amount, P = RM4,500 - 10%(RM4,500) = RM4,050 (a) Total interest charged I = Prt = RM4,050 (8%) (2) = RM648 (b) Instalment price (a.k.a Total Loan, S) IP = Cash Price or Loan Amount + Interest

*IP* = Cash Price or Loan Amount + Interest = RM4,050 + RM648 = RM4,698

(c) Monthly payment  $MP = \underline{IP \text{ or } S}$  n  $= \underline{RM4,698}$  2 (12)= RM195.75

# Example 4.12

Ms Yasmin plans to buy a car at RM69,500. The purchase involves other costs such as air bag, CD player, etc. amounting to RM4,500. She afford to pay RM24,000 as the down payment and plans to borrow the balance from a bank the charges an interest rate of 5% per annum. Calculate the monthly repayment if she plans to repay the loan within 5 years.

Solution:

Loan Amount, *P* = RM69,500 + RM4,500 - RM24,000

= RM50,000

Total interest charged

*I* = *Prt* = RM50,000 (5%) (5) = RM12,500

Instalment price *IP* = Cash Price + Interest = RM50,000 + RM12,500 = RM62,500

Monthly payment  $MP = \underline{IP}$  n  $= \underline{RM62,500}$  5 (12)= RM1,041.67



Housing loans use the 'yearly-rest' method to determine the amount of interest charged to the borrower to reduce the debt burden that will be borne by the borrower over the payment period.

Assume that the principal remains same over the loan period, then the total interest charged is:

$$I = \frac{Pr + Yr}{2} \times t$$
  
Where,  $P$  = Loan Amount  
 $r$  = Interest Rate  
 $Y = \frac{P}{t}$   
 $t$  = Period  
Total Loan,  $S = P + I$ 

Monthly installment =  $\underline{S}$ =  $\underline{P+I}$ n



Shanty wants to buy a single storey house at RM390,000. The developers require 10% as a deposit and suggests that the balance can be borrowed from a finance company which offers interest at a rate of 2.8% per year for 30 years. Apart from that, Shanty was

charged some additional fees such as legal fees and insurance of RM2,500 and RM1,700 respectively. You are required to calculate: (a) Total interest charged (b) Total loan

(c) Monthly payment

#### Solution:

P = RM390,000 - 10%(RM390,000) + RM2,500 + RM1,700= RM355,200  $Y = \frac{RM355,200}{30} = RM11,840$ r = 2.8%t = 30 (a) Total interest charged

$$I = \frac{Pr + Yr}{2} \times t$$

$$I = \frac{[RM355,200(2.8\%)] + [RM11,840(2.8\%)]}{2} \times 30$$

$$= RM154,156.80$$

- (b) Total loan S = RM355,200 + RM154,156.80= RM509,356.80
- (c) Monthly payment MP = S n  $= \frac{RM509,356.80}{30(12)}$ = RM1,414.88



The initial payment or early settlement occurs when the debtors decide to pay all or settle the loans after paying some installments before the maturity date arrived. The borrower is eligible to receive a rebate when making an early settlement. The rebate is a reduction of the amount payable, for early redemption.

Rebate, 
$$R = \frac{\sum n}{\sum N} \times I$$

Where,

*I* = Total interest

*n* = The remaining number of installments payable

*N* = The actual number of installment payments (total)

$$\Sigma n = \left| \frac{n+1}{2} \right| n$$
$$\Sigma N = \left| \frac{N+1}{2} \right| N$$

Amount payable = [No. of remaining installments × Monthly Payment] - Rebate



Mr. Brine borrowed RM62,000 from MBB to buy a new car at 2.4% per annum for 9 years. He paid RM700 per month and intends to settle his debts after his 50<sup>th</sup> installments. How much should he pay for an early settlement?

Solution:

Principal, P = RM62,000Interest rate, r = 2.4%Loan period, t = 9 years = 108 months Monthly payment = RM700 The remaining number of installments payable, n = 108 - 50 = 58The total number of installments payments, N = 108

To calculate rebate,  $r = \frac{\sum n}{\sum N} \times I$ 

$$I = Prt$$
  
= RM62,000 × 2.4% × 9  
= RM13,392

$$\Sigma n = \left| \frac{58+1}{2} \right| 58$$
$$= 1,711$$

$$\Sigma N = \left| \frac{108 + 1}{2} \right|$$
 108  
= 5,886

Rebate, 
$$R = \frac{1,711}{5,886} \times \text{RM13,392}$$

- - - -
  - = RM3,892.92

# Amount to be paid = Balance of Loans - Rebate = $[58 \times RM700] - RM3,892.92$ = RM36,707.08



Self-check 4.4 - Try to do this.

- 1. Ms Jessie plans to replace her old car with a new one that costs RM41,500. The downpayment that she can afford is RM13,500 and plans to borrow the rest either from Finance Companie HLA or HUO. Loans from HLA Company can be settled by the monthly installment of RM850 for 42 months. HUO Company charges on the other hand, an interest of 13% on all outstanding amounts due and payable in equal monthly installments over 42 months.
  - a. Calculate the monthly payment to be made by Ms Jessie if she intends to borrow from HUO Finance Company.
  - b.What is the interest rate charged by the HLA Finance Company?
  - c.Determine which financial company offers a more economical loan. How much are the savings?
- 2. Ron Weasly and his wife plan to build a bungalow residence priced at RM1,500,000 for the family. They decide to pay RM500,000 as a down payment. Other payments, including legal fees of RM3,000 and insurance RM2,300 will be loaned from a bank that charges an interest rate of 3% per annum. What is the monthly installment if they want to payback the loan within 25 years?
- 3. Mason borrowed RM50,000 to buy a car that is worth RM80,000. The interest rate charged is 3.7% per annum and the loan period is 7 years. If he wants to settle the loan after 20 payments, calculate the amount that must be paid.

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# 4.5 Compound Interest

Compound interest computation is based on the principal that changes from time to time. Interest earned is compounded or converted into principal and will earned interest thereafter. Thus, the principal increases from time to time.



RM500 is invested for 5 years. Find the interest received at the end of the fifth year if the investment earns 8% compounded annually.

Solution:

# Year

1	Principal = RM500 Interest for the 1st year = RM500 $\times$ 8% $\times$ 1 = RM40
2	Principal = RM500 + RM40 = RM540 Interest for the 2nd year = RM540 $\times$ 8% $\times$ 1 = RM43.20
3	Principal = RM540 + RM43.20 = RM583.20 Interest for the 3rd year = RM583.20 $\times$ 8% $\times$ 1 = RM46.66
4	Principal = RM583.20 + RM46.66 = RM629.86 Interest for the 4th year = RM629.86 $\times$ 8% $\times$ 1 = RM50.39
5	Principal = RM629.86 + RM50.39 = RM680.25 Interest for the 5th year = RM680.25 $\times$ 8% $\times$ 1 = RM54.42

Amount at the end of the 5th year :

RM680.25 + RM54.42 = RM734.67

Compound interest earned:

- = Amount at the end of the 5th year Original Principal
- = RM734.67 RM500
- = RM234.67

Based on the example, there are some common terms used concerning compound interest.

# Original Principal, P

- In this example the original principal is RM500

# Nominal Interest Rate, k

- The annual nominal rate is the interest rate for a year together with the frequency calculated in a year. In this case, is 8%.

# Interest period or conversion period, t

- Interest period is the length of time in which interest is calculated. Thus, the interest period in the example is 5 years.

# Frequency of Conversion, m

- The number of times interest is calculated in a year.
- Sometime the interest might be compounded:
  - > Daily (*m* = 360)
  - > Weekly (*m* = 48)
  - > Monthly (*m* = 12)
  - > Quarterly (m = 4)
  - > Semi-annually (m = 2)
  - > Annually (m = 1)

#### Periodic Interest Rate, i

- The interest rate for each interest period.

- To calculate 
$$i = \frac{k}{m} = \frac{8\%}{1} = 8\%$$
  
Number of Interest Periods in the Investment Period, n

- The number of times interest is calculated where n = mt in this case is n = 1, 5 = 5

**Compound Interest Formula** 

$$S = P \left( 1 + \frac{k}{m} \right)^{m}$$

Where,

S =Compound Amount

*P* = Original Principal

*k* = Annual Nominal Rate

*m* = Compounding Frequency in a year

t = Time or Term in years

# Example 4.16

Adeline decided to invest RM5,000 in a savings account at an annual interest rate of 7%. What is the amount after 3 years if the compounding takes place:

- (a) Annually
- (b) Semi-annually
- (c) Quarterly
- (d) Monthly

Solution:

(a) Annually

$$S = P \left(1 + \frac{k}{m}\right)^{mt}$$
$$= RM5,000 \left(1 + \frac{7\%}{1}\right)^{1 \times 3}$$
$$= RM6,125.22$$

(b) Semi-annually

$$S = P \left( 1 + \frac{k}{m} \right)^{mt}$$

$$= \text{RM5,000} \left(1 + \frac{7\%}{2}\right)^{2 \times 3}$$
$$= \text{RM6,146.28}$$

0.29

4.5.1

(c) Quarterly

$$S = P \left(1 + \frac{k}{m}\right)^{mt}$$
$$= RM5,000 \left(1 + \frac{7\%}{4}\right)^{4\times8}$$
$$= RM6,157.20$$

(d) Monthly

$$S = P \left(1 + \frac{k}{m}\right)^{mt}$$
$$= RM5,000 \left(1 + \frac{7\%}{12}\right)^{12\times3}$$
$$= RM6,164.63$$



Self-check 4.5 - Try to do this.

- 1.RM25,000 was invested for 4 years 9 months. If the investment was offered 12% compounded semi-annually, find the total amount of the investment and the compounded interest at the end of the term.
- 2.RM65,000 was invested for 6 years 9 months. If the investment was offered 5% compounded semi-annually for the first 2 years, and 8% compounded monthly for the rest of the period, find the total amount of the investment and the compounded interest at the end of the term.
- 3.Sam deposited RM1,700 into an account that pays 10% of interest compounded quarterly. Find the total amount of the investment and the compounded interest at the end of 3rd year.

# 4.5.2 Present Value $P = \frac{S}{\left(1 + \frac{k}{m}\right)^{mt}} @ P = S \left(1 + \frac{k}{m}\right)^{-mt}$ Example 4.17

A debt of RM5,000 will mature in 3 years' time. Assuming the money is worth 10% if compounded quarterly, find:

(a) The present value of this debt

(b) The value of this debt at the end of the first year

(c) The value of this debt at the end of four years

#### Solution:

(a) The present value of this debt Where, S = RM5,000 k = 10% m = 4 t = 3 years  $\text{PV} = \text{RM5,000} \left(1 + \frac{10\%}{4}\right)^{-(4 \times 3)}$   $= \text{RM5,000} \left(1 + \frac{10\%}{4}\right)^{-12}$ = RM3,717.78 (b) The value of this debt at the end of the first year

$$PV = RM5,000 \left(1 + \frac{10\%}{4}\right)^{-(4 \times 1)}$$
$$= RM5,000 \left(1 + \frac{10\%}{4}\right)^{-4}$$
$$= RM4,529.75$$

(c) The value of this debt at the end of four years





1.A debt of RM8,000 will mature in four years' time. Assuming the money is worth an interest of 3.4% compounded semi-annually, find:

a. The present value of this debtb. The value of this debt at the end of 2 yearsc. The value of this debt at the end of 7 years

- 2. Noel want to save some of his money into his saving accounts. If he intends to have RM10,000 in his account after five years, how much he need to save to have that amount of money? Assume that the interest rate is 5% compounded monthly.
- 3.A computer is estimated to cost RM4,500 in 2 years' time. If Mimi wishes to buy this computer in 2 years' time, how much she should save now which pays 9% of interest compounded quarterly?

# 4.5.3 Annuity

An annuity is a series of equal payments made at equal intervals of time. An annuity can be classified into annuity due, annuity certain, general annuity, perpetuity, and others. In this chapter, assume that money is invested at the end of every period which focuses more on ordinary annuity certain.

# **Future Value of Ordinary Annuity Certain**

Future values are also known as an accumulated value, S, is the sum of all the future values of the periodic payments.

$$S = R\left[\frac{\left(1+i\right)^n - 1}{i}\right]$$

Where, R = Periodic payments

- *i* = Interest rate per interest period
- *n* = Term of investment
- S = Future value of annuity at the end of *n* interest period

This is best explained with the following example.

RM100 is deposited every month for 5 years at an interest rate of 12% compounded monthly. What is the future value of this annuity at the end of the investment period? How much interest is earned?

Solution:



Where, 
$$R = RM100$$
  
 $i = \frac{12\%}{12} = 1\%$   
 $n = 5 \times 12 = 60$   
 $S = RM100 \left[ \frac{(1 + 1\%)^{60} - 1}{1\%} \right]$ 

= RM8,166.97

Interest earned,  $I = RM8,166.97 - (RM100 \times 60)$ = RM2,166.97



Minho invested RM50 every month for five years in an investment scheme. He was offered an interest rate of 7% compounded monthly for the first three years and 9% compounded monthly for the rest of the period. Find the accumulated amount at the end of the fifth year. Hence, determine the interest earned.

Solution:



Stream 1:

Amount after the 3rd year



= RM1,996.51

Amount after the 5th year =  $RM1,996.51 \left( 1 + \frac{9\%}{12} \right)^{24}$ = RM2,388.65

Stream 2:

$$\left[ \int \left( \begin{array}{c} \mathbf{q} \right)^{24} \right)^{24} \right]$$

Amount after the 5th year

$$= \text{RM50}\left[\frac{\left(1 + \frac{3\%}{12}\right) - 1}{\frac{9\%}{12}}\right]$$

= RM1,309.42

Total amount at the end of 5th year

= RM2,388.65 + RM1,309.42 = RM3,698.07

Interest earned, I

= S - nR= RM3,698.07 - (60 × RM50) = RM698.07



The following table shows the monthly deposits that were made into an investment account that pays an interest of 12% compounded monthly.

Year	Monthly Deposit
2019	RM150
2020	RM200
2021	RM250

Find the value of this investment at the end of 2021. Find also the interest earned.

Solution:



Stream 1:

Amount after the 1st year = RM150

RM150 
$$\left[\frac{\left(1 + \frac{12\%}{12}\right)^2 - 1}{\frac{12\%}{12}}\right]$$

= RM1,902.38

Amount after the 3rd year = RM1,902.38  $\left(1 + \frac{12\%}{12}\right)^{24}$ = RM2,415.52

Stream 2:

Amount after the 2nd year

$$= \text{RM200} \left[ \frac{\left(1 + \frac{12\%}{12}\right)^{12} - 1}{\frac{12\%}{12}} \right]$$

= RM2,536.50

Amount after the 3rd year

$$= \text{RM2,536.50} \left(1 + \frac{12\%}{12}\right)^{12}$$

= RM2,858.19

Stream 3:

Amount after the 3rd year

= RM250 
$$\boxed{\frac{\left(1 + \frac{12\%}{12}\right)^2 - 1}{\frac{12\%}{12}}}$$

Total amount at the end of 3rd year

= RM8,444.34

Interest earned, I

= S - nR

- = RM8,444.34 [(12 x RM150) + (12 x RM200) + (12 x RM250)]
- = RM1,244.34



- 1. Find the future value and the interest earned for each of the following annuities.
  - a.RM6,000 every year for 8 years at an interest rate of 12% compounded annually.
  - b.RM800 every month for 2 years 5 months at an interest rate of 5% compounded monthly.
  - c.RM950 every 3 months for 3 years 9 months at an interest rate of 6% compounded quarterly.
  - d.RM2,450 every 2 months for 1 year 8 months at an interest rate of 6% compounded every 2 months.
  - e.RM90 every day for 150 days at an interest rate 10% compounded daily.
- Eve invested RM300 every three months for 4 years. She was offered an interest of 5% compounded quarterly for the first 2 years, and interest of 8% compounded quarterly for the rest of the period. Find the accumulated amount at the end of the four years.
   The monthly deposits were made by Jason as shown in below table:

Year	Monthly Deposit
2015	RM500
2016	RM400
2017	RM700

Assume that the investment paid an interest of 8% compounded monthly. Find the accumulated amount in the account at the end of 2017.

# Present Value of Ordinary Annuity Certain

The present value, A, also known as discounted value of an ordinary annuity certain is the sum all present values of the periodic payments. The derivation of the formula of present value of ordinary annuity certain is illustrated as follow:

$$\mathcal{A} = R\left[\frac{1-(1+i)^{-n}}{i}\right]$$

Where, *R* = Periodic payments

- *i* = Interest rate per interest period
- *n* = Term of investment
- A = Present value of annuity at the end of n interest period

This is best explained with the following example.

Salina has to pay RM300 every month for 24 months to settle a loan at an interest rate of 6% compounded monthly.

- (a) What is the original value of the loan?
- (b) What is the total interest that he has to pay?

Solution:



(a) What is the original value of the loan?

$$R = RM300$$
  
 $i = \frac{6\%}{12}$   
 $n = 24$ 



= RM6,768.56

(b) What is the total interest that he has to pay?

 $I = (RM300 \times 24) - RM6,768.56$ 

= RM431.44

Example 4.20

Alan invests RM12,000 in an account that pays an interest of 6% compounded monthly. He intends to withdraw an equal amount every month for two years and when he makes his last withdrawal, his account will have zero balance. Find the size of these withdrawals.

Solution:



$$A = RM12,00$$
  
 $i = \frac{6\%}{12}$   
 $n = 24$ 

RM12,000 = 
$$R \left[ \frac{1 - \left(1 + \frac{6\%}{12}\right)^{-24}}{\frac{6\%}{12}} \right]$$

RM12,000 = 22.56R

*R* = RM531.91



Izat borrowed RM5,000 at an interest of 7% compounded monthly for one year.

(a) Find her monthly payment.

(b) If she had not paid her first 3 monthly payments, how much should she pay on her fourth payment to settle all outstanding arrears?

(c) If she wanted to settle all of the debt immediately after paying the fourth payments, how much additional payment does she have to make?

Solution:

(a) Find her monthly payment.

$$A = RM5,000$$
  
 $i = \frac{7\%}{12}$   
 $n = 12$ 

RM5,000 = 
$$R\left[\frac{1-\left(1+\frac{7\%}{12}\right)^{-12}}{\frac{7\%}{12}}\right]$$

RM5,000 = 11.56*R* 

R = RM432.53

(b) If she had not paid her first 3 monthly payments, how much should she pay on her fourth payment to settle all outstanding arrears?

R = RM432.53 $i = \frac{7\%}{12}$ n = 12

Outstanding balance,  $S = R \left[ \frac{(1+i)^n - 1}{i} \right]$ = RM432.53  $\left[ \frac{\left(1 + \frac{7\%}{12}\right)^4 - 1}{\frac{7\%}{12}} \right]$ = RM1,745.32

(c) If she wanted to settle all of the debt immediately after paying the fourth payments, how much additional payment does she have to make?

> R = RM432.53 $i = \frac{7\%}{19}$

$$n = 12$$
Settlement,  $S = R\left[\frac{(1+i)^n - 1}{i}\right]$ 

$$= RM432.53\left[\frac{\left(1 + \frac{7\%}{12}\right)^{12-4} - 1}{\frac{7\%}{12}}\right]$$

$$= RM9.591.79$$

= RM3,531.72
Example 4.22

Mulan borrowed RM70,000 at an interest rate of 3.45% compounded monthly. How many monthly payments of RM930 should she make? What would that concluding size of the final payment be?

Solution:

A = RM70,000 R = RM930 $i = \frac{3.45\%}{12}$ 

RM70,000 = RM930

$$\left[\frac{1 - \left(1 + \frac{3.45\%}{12}\right)^{-n}}{\frac{3.45\%}{12}}\right]$$

$$75.27 = \frac{1 - \left(1 + \frac{3.45\%}{12}\right)^{-n}}{\frac{3.45\%}{12}}$$

- $0.216398 = 1 \left(1 + \frac{3.45\%}{12}\right)^{-n}$
- $0.7836 = (1.002875)^{-n}$

lg 0.7836 = -n lg (1.002875)lg 0.7836

$$-n = \frac{\lg 0.7830}{\lg 1.002875}$$

8

$$= -84.94$$
  
 $n = 85$ 

Mulan can either pay 83 payments of RM930 each and one concluding payment that is more than RM930, or 84 payments and one concluding payment that is less than RM930.

If Mulan chooses to make 83 payments of RM930, the final payment will be more than RM930 on the 84th payment.



Let the concluding payment be *K* ringgit.

RM70,000 = RM930 
$$\left[\frac{1 - \left(1 + \frac{3.45\%}{12}\right)^{-83}}{\frac{3.45\%}{12}}\right] + K\left(1 + \frac{3.45\%}{12}\right)^{-84}$$

RM70,000 = RM68,583.88 + 0.7857K0.7857K = RM1,416.12*K* = RM1,802.37

If Mulan chooses to make 84 payments of RM930, the final payment will be less than RM930 on the 85th payment.



 $\frac{3.45\%}{12}$ 

┛

Let the concluding payment be *K* ringgit.

RM70,000 = RM930 
$$\left[\frac{1 - \left(1 + \frac{3.45\%}{12}\right)^{-84}}{\frac{3.45\%}{12}} + K\left(1 + \frac{3.45\%}{12}\right)^{-84}\right]$$

RM70,000 = RM69,314.60 + 0.7835K

0.7835K = RM685.40

*K* = RM874.79



- 1.Leha wants to send her daughter to further study in master degree level. She estimates that she need RM50,000 to pay for her daughter's tuition fee. How much shall she save in a bank every month for the next 10 years at an interest of 7% compounded monthly to accumulate the started amount?
- 2. To settle a loan of RM20,000 at an interest of 8% effective rate, Hector has to pay RM500 every month. Find the number of payments that must be made by Hoctor to settle the loan.
- 3. Ivon intends to accumulate RM10,000 by depositing RM650 every 6 months into an account that pays an interest of 5% compounded semi-annually.
  - a. How many full deposits of RM650 must he save?
  - b. Find the additional amount that must be saved together with the final full deposit so that he will have RM10,000.

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# EVALUATION

#### Learning Outcomes:

At the end of this chapter, you will be able to:

- Implement capital budgeting factors and environmental factors for investments decisions
- Apply the concept of cash flow and discounting to obtain present values
- Solve investment management problems which involve evaluation and selection of projects using the above methods

## 5.1 INTRODUCTION

Making decisions on where to invest the company's resources is an important move to be more competitive in the industry. Thus, try to get involved in the right projects to avoid wasting the company's time and resources to improve the chances of success. This chapter evaluates the project to determine whether a firm's investments are worth pursuing. Examples of projects that firms might invest in are producing new products, buying new machines, and investing in a new plant. Management needs to use capital budgeting techniques to determine which projects will yield the most return over an applicable period of time since its involves a major capital expenditure.

## 5.2 CAPITAL BUDGETING FACTORS

Factors that involved in capital budgeting are:

(a) Initial Cost

The initial investment or cash capital required to start a project.

(b) Cash Inflow

The estimated cash amount that flows into a business due to operating of the project or business.

(c) Investment Period

The duration of the project and when it is estimated to be completed.

(d) Discount Factor

The value of interest that will be received or charged during the period of the project's execution and it will affect the present value of cash in flows for different years.

(e) Time Value of Money

The idea that a ringgit now is worth more than a ringgit in the future, even after adjusting for inflation, because a ringgit now can earn interest or other appreciation until the time the ringgit in the future would be received. This theory has its base in the calculation for present value.

#### 5.3 FACTORS INFLUENCING INVESTMENT DECISION

A firm must make an investment decision to improve or increase the incomes of the company in order compete in the market. Investment environments include:

#### (a) **Product development / enhancement**

A company needs to invest some of the capital to make a new product or improve the quality of its existing products or to ensure its competitiveness in the market.

#### (b) Replacing equipment / machinery

The company will replace the equipment and machines that have been operating for too long in order to improve its productivity.

#### (c) Exploration of new fields or business

A company may need to explore new areas of business or conduct business in different locations. For example, in the field of oil exploration, a company need to find and develop new oil drilling locations.

## 5.4 **PROJECT EVALUATION METHODS**

Common methods used in evaluating projects, investments or alternatives are:

- (a) Payback period (PBP)
- (b) Accounting Rate of Return / Average Rate of Return (ARR)
- (c) Net Present Value (NPV)
- (d) Profitability Index (PI)
- (e) Internal Rate of Return (IRR)



The period of time required for the cumulative expected cash flows to equalize the initial investment or cash outflow. There are two-ways to calculate PBP:

#### (a) Equivalent / constant cash inflow

The payback period (PBP) may be calculated using the following formula.





Given the cash flow for hire new Plant Yoganess as follows:

Year	0	1	2	3	4	5
Cash Flow (RM)	(160,000)	55,000	55,000	55,000	55,000	55,000

Find the payback period.

Solution:

$$Payback \ Period \ (PBP) = \frac{Initial \ Investment}{Cash \ Inflow}$$
$$= \frac{RM160,000}{RM55,000}$$
$$= 2.9 \ years \ or \ 2 \ years \ 11 \ months$$

#### (b) Inconsistent cash inflow

The payback period (PBP) may be calculated using the following formula.



- Where *N* = Number of years for the accumulated cash flows that had **not exceeded** the capital or investment.
- Where *M* = Number of years for the accumulated cash flows that had **exceeded** the capital or investment.



# Given the cash flow for investment of new machine Maisie as follows:

Year	0	1	2	3	4	5
Cash Flow (RM)	(160,000)	45,000	50,000	60,000	65,000	70,000

#### Find the payback period.

Solution:

Year	0	1	2	3	4	5
Cash Flow (RM)	(160,000)	45,000	50,000	60,000	65,000	70,000
Accumulated Cash Inflow (RM)		45,000	95,000	155,000	220,000	290,000

As you can see, RM160,000 is between year 3 and 4. The N year is 3, and M is 4.

#### Accounting Rate of Return / Average Rate of Return 5.4.2

A financial ratio in capital budgeting which does not take into account the concept of time value of money. ARR calculates the return generated from net income of the proposed capital investment.

ARR is calculated by aggregating all expected cash flows and dividing by the number of years that the investment is expected to last. For example, an investment in real estate is expected to generate returns of RM22,000 in the first year, RM32,000 in the second year, and RM36,000 in the third year. The average of this amount is RM30,000. The initial investment was RM300,000, so the average rate of return is 10% (calculated as the RM30,000 average return divided by the RM300,000 investment).

The method used to calculate the ARR of an investment depends on the depreciation and cash inflow of the investment.



(a) Investment without scrap value				
Depreciation -	Total Investment			
Depreciation -	Useful Life			

(b) Investment with scrap value

Total Investment - Scrap Value Depreciation = Useful Life

(c) Inconsistent cash flow

Average Cash Flow =

Total Cash Flow Useful Life



Given the cash flow for hire new Plant Yoganess as follows:

Year	0	1	2	3	4	5
Cash Flow (RM)	(160,000)	55,000	55,000	55,000	55,000	55,000

Find the Average rate of return. Assume that the plant does not have scrap value.

Solution:

Depreciat	on = <u>Total Investment</u> Useful Life	
	$= \frac{\text{RM160,000}}{5 \text{ years}}$	
	= RM32,000	
ARR =	Average Cash Flow - Depreciation X 100%	
=	RM55,000 - RM32,000 × 100% RM160,000	
=	14.38%	

Example 5.4

Given the cash flow for investment of new machine Maisie as follows:

Year	0	1	2	3	4	5
Cash Flow (RM)	(160,000)	45,000	50,000	60,000	65,000	70,000

Find the Average rate of return. Assume that the machine have

scrap value RM6,000 at the end of year 5.

Solution:

Total Investment - Scrap value Depreciation = Useful Life RM160,000 - RM6,000 = 5 years RM30,800 = Average Cash Flow - Depreciation ARR =100% Х Initial investment RM58,000 - RM30,800 100% X = RM160,000 17% =



	Cash Inflow (RM)			
Year	Plant Aye	Plant Bee		
1	62,000	65,000		
2	65,000	65,000		
3	72,000	65,000		
4	75,000	65,000		
5	80,000	65,000		

1. Identify the best project. Use the payback period method.

Assume that the initial investment for Plant Aye is RM230,000 and Plant Bee is RM200,000.

2. Choose the best machine to invest in using average return of return method for the investment alternatives below:

	Cash Inflow (RM)			
Year	Machine Ace	Machine Bean		
1	80,000	60,000		
2	80,000	55,000		
3	80,000	85,000		
4	80,000	125,000		
5	80,000	160,000		

Assume that the initial investment for Machine Ace is RM250,000 and Machine Bean is RM255,000.

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#### 5.4.3 Net Present Value

Net present value (NPV) is used in capital budgeting and investment planning to analyse the profitability of a projected investment or project. It is an indicator of how much value an investment could contribute to the firm. The calculation of NPV takes into account the concept of the time value of money.

NPV is the difference between a project's net cash flows and the project's initial cash flow. The criteria below should be first considered before accepting or rejecting a project or an investment:

NPV	Indication	Decision
NPV > 0	The investment would add value to the firm	The project should be <b>ACCEPTED</b>
NPV < 0	The investment would subtract value from the firm, that is, the project reduces shareholder wealth	The project should be <b>REJECTED</b>
NPV = 0	The investment would neither bring gain nor lose value to the firm	We should be indifferent in the decision whether to accept or reject the project. This project adds no monetary value. Decision should be made based on other criteria.

NPV = Total Present Value - Initial Investment



Find the net present value for the cash flow generated by hiring new Plant Yoganess as follows:

Year	0	1	2	3	4	5
Cash Flow (RM)	(160,000)	55,000	55,000	55,000	55,000	55,000

Given that the discount rate is 14%.

#### Solution:

Year, n	Cash Flow, <i>CF</i> <sub>n</sub> (RM)	$\frac{1}{(1+r)^n}$	Present Value (RM) $CF_n  imes rac{1}{(1+r)^n}$
0	(160,000)		
1	55,000	$\frac{1}{(1+0.14)^1} = 0.8772$	48,246
2	55,000	$\frac{1}{(1+0.14)^2} = 0.7695$	42,322.50
3	55,000	$\frac{1}{(1+0.14)^3} = 0.6750$	37,125
4	55,000	$\frac{1}{(1+0.14)^4} = 0.5921$	32,565.50
5	55,000	$\frac{1}{(1+0.14)^5}$ = 0.5194	28,567
	Total Present Value		188,826
	(160,000)		
	Net Present Value, 1	NPV	28,826

Since the NPV is positive by RM28,826, the new hiring plant Yoganess is profitable to the firm. Thus, the firm should hire the new plant as it give value added to the company.

#### Example 5.6

Given the cash flow for investment of new machine Maisie as follows:

Year	0	1	2	3	4	5
Cash Flow (RM)	(160,000)	45,000	50,000	60,000	65,000	70,000

Find the Net Present Value (NPV) using PVIF Table for the cash flow generated by the machine. Assume that the discount rate is 24%.

Solution:

																			1	
Period	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%	13%	14%	15%	16%	20%	24%	25%	30%
1	0.9901	0.9804	0.9709	0.9615	0.9524	0.9434	0.9346	0.9259	0.9174	0.9091	0.9009	0.8929	0.8850	0.8772	0.8696	0.8621	0.833	0.8065	.8000	0.7692
2	0.9803	0.9612	0.9426	0.9246	0.9070	0.8900	0.8734	0.8573	0.8417	0.8264	0.8116	0.7972	0.7831	0.7695	0.7561	0.7432	0.694	0.6504	.6400	0.5917
3	0.9706	0.9423	0.9151	0.8890	0.8638	0.8396	0.8163	0.7938	0.7722	0.7513	0.7312	0.7118	0.6931	0.6750	0.6575	0.6407	0.578	0.5245	.5120	0.4552
4	0.9610	0.9238	0.8885	0.8548	0.8227	0.7921	0.7629	0.7350	0.7084	0.6830	0.6587	0.6355	0.6133	0.5921	0.5718	0.5523	0.482	0.4230	.4096	0.3501
5	0.9515	0.9057	0.8626	0.8219	0.7835	0.7473	0.7130	0.6806	0.6499	0.6209	0.5935	0.5674	0.5428	0.5194	0.4972	0.4761	0.401	0.3411	.3277	0.2693

Table A-3 Present Value Interest Factors for One Dollar Discounted at k Percent for n Periods:  $PVIF_{k,n} = 1 / (1 + k)^n$ 

Source: https://www.retailinvestor.org/pdf/futurevaluetables.pdf

Year, n	Cash Flow, <i>CF</i> <sub>n</sub> (RM)	<i>PVIF</i> ( <i>i</i> = <b>24</b> % )	Present Value (RM) $CF_n \times \text{PVIF}_{24\%}$								
0	(160,000)										
1	45,000	0.8065	36,292.50								
2	50,000	0.6504	32,520								
3	60,000	0.5245	31,470								
4	65,000	0.4230	27,495								
5	70,000	0.4311	30,177								
	Total Present Value		151,654.50								
	(-) Initial Investment										
	Net Present Value, 1	NPV	(8,345.50)								

Since the NPV is negative by RM8,345.50, the new investment of the new machine Maisie is not profitable to the firm. Thus, the firm should not invest in the new machine as it did not give value added to the company.



1. Use the net present value methods to identify which project is better. (Use Formula Method)

V	Cash Flow (RM)										
Year	Project A	Project B									
0	(220,000)	(220,000)									
1	50,000	55,000									
2	68,000	75,000									
3	82,000	95,000									
4	85,000	98,000									
5	88,000	100,000									

Discount rate for both projects is 16%.

2. Find the net present value for the planned plantation project, with the cash flow as follows: (Use PVIF Table)

Year	0	1	2	3	4	5
Cash Flow (RM)	(110,000)	42,000	52,000	62,000	72,000	85,000

The discount rate is 20%.

#### A Profitability Index

The ratio of the present value of the future net cash flows to the initial cash outflow.

A project or investment is **NOT PROFITABLE** if the profitability index is **LESS THAN 1.00**.



Example 5.7

Refer back on Example 5.5 and 5.6. Find the profitability index for both projects.

	Plant Yoganess	Machine Maisie
Initial Investment	(160,000)	(160,000)
Total Present Value	188,826	151,654.50

Solution:

Profitability Index	=	Total Present Value Total Investment
Plant Yoganess	=	RM188,826 RM160,000
	=	1.18 (Profitable, 1.18 > 1.00
Machine Maisie	=	RM151,654.50

5.4.4

#### RM160,000

#### = 0.95 (Not Profitable, 0.95 < 1.00)

#### 5.4.5 Internal Rate of Return (IRR)

The discount rate that generates a zero NPV for a series of future cash flows. It equates the PV of the future cash flows from an investment project with the initial cash outflow of the project.

IRR may be calculated by employing the trial and error method which requires us to seek a discount rate that will enable the discounted total cash flow to be equaled to the initial investment.

The calculation of IRR takes into account the concept of the **Time Value of Money**. Project or investment with IRR greater than the required rate of return is considered acceptable and profitable. By interpolation, the IRR can be determined as follows.



Solution:

Since NPV Plant Yoganess is positive for discounting rate of 14%, we need to find the negative NPV by selecting discounting rate higher than 14%. Assume that, we choose to use 30%. Try to calculate the NPV using PVIF table.

Year, n	Cash Flow, CF <sub>n</sub> (RM)	<i>PVIF</i> ( <i>i</i> = <b>30</b> % )	Present Value (RM) $CF_n \times \frac{1}{(1+r)^n}$
0	(160,000)		
1	55,000	0.7692	42,306
2	55,000	0.5917	32,543.50
3	55,000	0.4552	25,036
4	55,000	0.3501	19,255.50
5	55,000	0.2693	14,811.50
	Total Present Value		133,952.50
	(-) Initial Investmen	t	(160,000)
	Net Present Value, I	NPV	(26,047.50)

We already have a positive NPV at 14% and a negative NPV at 30% of the discount rate. So now try to substitute all the information into the formula.

Internal Rate of Return (IRR)

 $= \frac{\% \text{ for Positive NPV} + (\% \text{ for Negative NPV} - \% \text{ for Positive NPV})}{(\text{Positive NPV} - \text{Negative NPV})}$   $= \frac{14\% + (30\% - 14\%)}{(\text{RM28,826} - (-\text{RM26,047.50}))}$  = 22.41%

Since the IRR is 22.41% > 14% (required rate of return), we

should accept the project or investment.



Self-check 5.3 - Try to do this.

# Refer back on Example 5.6 (machine Maisie). Find the IRR for the project.



- 1. Mr. Lim plans to invest in a project. MEGA project requires an investment of RM120,000 while the PRIME project requires an investment of RM80,000. You are required to help Mr. Lim to make his decision using the following project evaluation methods:
  - (a) Payback period
  - (b) Net present value
  - (c) Profitability Index
  - (d) Average rate of return
  - (e) Internal rate of return

Veen	Cash Flo	ow (RM)
rear	PRIME	MEGA
1	25,000	15,000
2	30,000	20,000
3	30,000	20,000
4	35,000	25,000
5	30,000	20,000
5	30,000	35,000

Discount rate for both projects is 12%. Which project is profitable?

-								<u> </u>													
Per	iod 1%	%	2%	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%	13%	14%	15%	16%	20%	24%	25%	30%
1	0.99	901	0.9804	0.9709	0.9615	0.9524	0.9434	0.9346	0.9259	0.9174	0.9091	0.9009	0.8929	0.8850	0.8772	0.8696	0.8621	0.8333	0.8065	0.8000	0.7692
2	0.98	803	0.9612	0.9426	0.9246	0.9070	0.8900	0.8734	0.8573	0.8417	0.8264	0.8116	0.7972	0.7831	0.7695	0.7561	0.7432	0.6944	0.6504	0.6400	0.5917
3	0.97	706	0.9423	0.9151	0.8890	0.8638	0.8396	0.8163	0.7938	0.7722	0.7513	0.7312	0.7118	0.6931	0.6750	0.6575	0.6407	0.5787	0.5245	0.5120	0.4552
4	0.96	610	0.9238	0.8885	0.8548	0.8227	0.7921	0.7629	0.7350	0.7084	0.6830	0.6587	0.6355	0.6133	0.5921	0.5718	0.5523	0.4823	0.4230	0.4096	0.3501
5	0.95	515	0.9057	0.8626	0.8219	0.7835	0.7473	0.7130	0.6806	0.6499	0.6209	0.5935	0.5674	0.5428	0.5194	0.4972	0.4761	0.4019	0.3411	0.3277	0.2693
								J													
6	0.94	420	0.8880	0.8375	0.7903	0.7462	0.7050	0.6663	0.6302	0.5963	0.5645	0.5346	0.5066	0.4803	0.4556	0.4323	0.4104	0.3349	0.2751	0.2621	0.2072
	0.00		0.0700	0.0404		0 7407	0.0054	0.0007	0.5005	0.5470	0.5400	0.4047	0 4500	0.4054	0.0000	0.0750	0.0500	0.0704	0.0040	0.0007	0.4504

8	0.9235	0.8535	0.7894	0.7307	0.6768	0.6274	0.5820	0.5403	0.5019	0.4665	0.4339	0.4039	0.3762	0.3506	0.3269	0.3050	0.2326	0.1789	0.1678	0.1226
9	0.9143	0.8368	0.7664	0.7026	0.6446	0.5919	0.5439	0.5002	0.4604	0.4241	0.3909	0.3606	0.3329	0.3075	0.2843	0.2630	0.1938	0.1443	0.1342	0.0943
10	0.9053	0.8203	0.7441	0.6756	0.6139	0.5584	0.5083	0.4632	0.4224	0.3855	0.3522	0.3220	0.2946	0.2697	0.2472	0.2267	0.1615	0.1164	0.1074	0.0725
11	0.8963	0.8043	0.7224	0.6496	0.5847	0.5268	0.4751	0.4289	0.3875	0.3505	0.3173	0.2875	0.2607	0.2366	0.2149	0.1954	0.1346	0.0938	0.0859	0.0558
12	0.8874	0.7885	0.7014	0.6246	0.5568	0.4970	0.4440	0.3971	0.3555	0.3186	0.2858	0.2567	0.2307	0.2076	0.1869	0.1685	0.1122	0.0757	0.0687	0.0429
13	0.8787	0.7730	0.6810	0.6006	0.5303	0.4688	0.4150	0.3677	0.3262	0.2897	0.2575	0.2292	0.2042	0.1821	0.1625	0.1452	0.0935	0.0610	0.0550	0.0330
14	0.8700	0.7579	0.6611	0.5775	0.5051	0.4423	0.3878	0.3405	0.2992	0.2633	0.2320	0.2046	0.1807	0.1597	0.1413	0.1252	0.0779	0.0492	0.0440	0.0254
15	0.8613	0.7430	0.6419	0.5553	0.4810	0.4173	0.3624	0.3152	0.2745	0.2394	0.2090	0.1827	0.1599	0.1401	0.1229	0.1079	0.0649	0.0397	0.0352	0.0195
16	0.8528	0.7284	0.6232	0.5339	0.4581	0.3936	0.3387	0.2919	0.2519	0.2176	0.1883	0.1631	0.1415	0.1229	0.1069	0.0930	0.0541	0.0320	0.0281	0.0150
17	0.8444	0.7142	0.6050	0.5134	0.4363	0.3714	0.3166	0.2703	0.2311	0.1978	0.1696	0.1456	0.1252	0.1078	0.0929	0.0802	0.0451	0.0258	0.0225	0.0116
18	0.8360	0.7002	0.5874	0.4936	0.4155	0.3503	0.2959	0.2502	0.2120	0.1799	0.1528	0.1300	0.1108	0.0946	0.0808	0.0691	0.0376	0.0208	0.0180	0.0089
19	0.8277	0.6864	0.5703	0.4746	0.3957	0.3305	0.2765	0.2317	0.1945	0.1635	0.1377	0.1161	0.0981	0.0829	0.0703	0.0596	0.0313	0.0168	0.0144	0.0068
20	0.8195	0.6730	0.5537	0.4564	0.3769	0.3118	0.2584	0.2145	0.1784	0.1486	0.1240	0.1037	0.0868	0.0728	0.0611	0.0514	0.0261	0.0135	0.0115	0.0053
21	0.8114	0.6598	0.5375	0.4388	0.3589	0.2942	0.2415	0.1987	0.1637	0.1351	0.1117	0.0926	0.0768	0.0638	0.0531	0.0443	0.0217	0.0109	0.0092	0.0040
22	0.8034	0.6468	0.5219	0.4220	0.3418	0.2775	0.2257	0.1839	0.1502	0.1228	0.1007	0.0826	0.0680	0.0560	0.0462	0.0382	0.0181	0.0088	0.0074	0.0031
23	0.7954	0.6342	0.5067	0.4057	0.3256	0.2618	0.2109	0.1703	0.1378	0.1117	0.0907	0.0738	0.0601	0.0491	0.0402	0.0329	0.0151	0.0071	0.0059	0.0024
24	0.7876	0.6217	0.4919	0.3901	0.3101	0.2470	0.1971	0.1577	0.1264	0.1015	0.0817	0.0659	0.0532	0.0431	0.0349	0.0284	0.0126	0.0057	0.0047	0.0018
25	0.7798	0.6095	0.4776	0.3751	0.2953	0.2330	0.1842	0.1460	0.1160	0.0923	0.0736	0.0588	0.0471	0.0378	0.0304	0.0245	0.0105	0.0046	0.0038	0.0014
30	0.7419	0.5521	0.4120	0.3083	0.2314	0.1741	0.1314	0.0994	0.0754	0.0573	0.0437	0.0334	0.0256	0.0196	0.0151	0.0116	0.0042	0.0016	0.0012	•
35	0.7059	0.5000	0.3554	0.2534	0.1813	0.1301	0.0937	0.0676	0.0490	0.0356	0.0259	0.0189	0.0139	0.0102	0.0075	0.0055	0.0017	0.0005	•	•
36	0.6989	0.4902	0.3450	0.2437	0.1727	0.1227	0.0875	0.0626	0.0449	0.0323	0.0234	0.0169	0.0123	0.0089	0.0065	0.0048	0.0014	•	•	•
40	0.6717	0.4529	0.3066	0.2083	0.1420	0.0972	0.0668	0.0460	0.0318	0.0221	0.0154	0.0107	0.0075	0.0053	0.0037	0.0026	0.0007	•	•	•
50	0.6080	0.3715	0.2281	0.1407	0.0872	0.0543	0.0339	0.0213	0.0134	0.0085	0.0054	0.0035	0.0022	0.0014	0.0009	0.0006	•	•	•	•



# Chapter 6 **TRANSPORTATION** MODEL

Learning Outcomes:

At the end of this chapter, you will be able to:

- Identify transportation problems
- Determine the initial solution using Northwest Corner Rules
- Determine the initial solution using Least Cost Method
- Solve unbalanced transportation problems
- Solve maximum transportation using stepping stone method

## 6.1 INTRODUCTION

What is transportation model?

- A special model specifically used to solve transportation problems
- Requires less computation
- 3 main items:
  - Supply
  - Demand
  - Transportation cost or profit
- Objective:
  - To minimise the total transportation cost
  - To maximise profit or contribution in transporting items from various sources to destinations

## 6.2 TRANSPORTATION MODEL CONCEPT

To minimise the total transportation & production costs from several points of supply called sources to several points of demand called destinations.



Jimmy Company owns 3 farms in 3 areas -A, B, and C. The weekly productions from the 3 farms are given below.

Production Area	Weekly Production (Tons)
А	300
В	420
С	600

These 3 farms supply fish to the markets in 4 cities – W, X, Y

#### and Z with following demand.

Market Location	Weekly Demand (Tons)
W	280
X	200
Y	360
Z	480

The transportation cost (in RM) for each ton of fish is given below.

Destination Supply	W	X	Y	Z
Α	28	18	32	36
В	22	16	14	32
С	32	24	20	36

The network format for the transportation problem is shown in the following diagram.





#### **Northwest Corner Rule**

- Table 1st set up with rows as supplies and the columns as demands.
- Value in the small square in each cell indicates the cost of shipping
- Step:
  - Starting point by allocate the 1st cell with the maximum quantity of demand.
  - When the supply at each row is fully allocated, allocation is moved to the next row
  - Similarly, when the demand of each column is fully allocated, moved to the next column.
  - Ensure that all the supplies and demands are satisfied.

To From	N	W	>	K	Y	,	Z	2	Supply
А	280	28	20	18		32		36	300
В		22	180	16	240	14		32	420
С		32		24	120	20	480	36	600
Demand	28	30	20	00	3	60	4	80	1,320

Total transportation cost:

A to W	:	$RM28 \times 280$	=	RM7,840
A to $X$	:	$RM18 \times 20$	=	RM360
B to X	:	RM16 ×180	=	RM2,880
B to Y	:	$RM14 \times 240$	=	RM3,360
C to $Y$	:	$RM20 \times 120$	=	RM2,400
C to $Z$	:	$RM36 \times 480$	=	RM17,280

RM34,120

#### Least Cost Rule

- Initial allocations based on lowest cost
- Step:
  - Identify cell with the lowest cost & allocate maximum units to the cell
  - Cross out the row or column that had been satisfied
  - Identify the cell with second lowest cost and further allocate the max. unit to that cell such that it does not exceed the requirements for demand & supply
  - Repeat the process until the table is completed with all supplies allocated & demands fulfilled

To From	Ņ	W	>	<b>&lt;</b>	Y	1	Z	2	Supply
А	160	28	140	18		32		36	300
В		22	60	16	360	14		32	420
С	120	32		24		20	480	36	600
Demand	28	30	20	00	3	60	4	180	1,320

#### Total transportation cost:

A to $W$	•	$RM28 \times 160$	=	RM4,480
A to X	•	$RM18 \times 140$	=	RM2,520
B to $X$	:	$RM16 \times 60$	=	RM960

B to $Y$ :	RM14  imes 360	=	RM5,040
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- C to W : RM32  $\times$  120
- C to Z : RM36 × 480
- = RM3,840
  - = RM17,280





#### **Question 1**

Zaki company produces computers at plants located at 3 towns A, B & C. The computers are sent to warehouse at different locations U, V & W. The plants at A produces 400 units while B & C each produce 300 units of computers each week. Warehouses U, V & W require 200, 450, & 350 respectively. The shipping costs are given in the following table.

To	U	V	W
Α	5	4	6
В	3	5	4
с	6	3	3

The company would like to find the least cost way to meet the demands of the warehouses. Find the total transportation cost using:

- (a) Northwest corner rule
- (b) Least cost method

#### **Question 2**

Ayub company produces oil filters at plants located at towns A, B & C. The filters are sent to warehouse at different locations E, F & G. The plants at A, B & C produce 500, 400 & 300 filters each week. Warehouse E, F & G require 300, 550, and 350 respectively. The shipping costs vary & are as given in the following table.

To From	E	F	G
Α	5	4	6
В	3	5	4
С	6	3	3

The company would like to find the least cost way to meet the demands of the warehouses. Find the initial solution using:

- (a) Northwest corner rule
- (b) Least cost method

### 6.3 UNBALANCED TRANSPORTATION PROBLEMS

Unbalanced transportation happened when the supply does not equal demand. A dummy supply or dummy source is introduced to overcome this problem. If supply more than demand, a dummy destination or dummy demand is introduced. If supply is less than demand, a dummy supply is created.



Manchu Enterprise produces bottles in three plants A, B, and C. It has three major customers E, F, and G. The transportation cost per unit from various sources to destinations are given in the table below.

Destination Supply	E	F	G
Α	11	10	7
В	6	8	5
С	6	10	8

Factory	Capacity (Units)
А	130
В	100
С	120

Customer	Demand (Units)
E	110
_	



- (a) Determine initial solution using Northwest Corner Rule of bottles from the plants to the customers.
- (b) What is the total transportation cost?

Solution:

Step 1: Calculate the total amount of supply and demand. Total Supply = 350 unit Total Demand = 330 unit Thus, there will be extra column "To" act as dummy.

To From	E		F		G		Dummy		Supply
A	110	11	20	10		7		0	130
В		6	70	8	30	5		0	100
С		6		10	100	8	20	0	120
Demand	110		90		130		2	20	350

Step 2: Solve the initial solution.

Step 3: Calculate the transportation cost.

RM800
RM150
RM560
RM200
RM1,210

Example 6.3

The Bilbo Company faces the problem of supplying school meals from three caterers to four schools. The number of meals required at the four schools A, B, C, and D on each day is given below.

		_	-	_
Destination	A	В	С	D
Sulading				
		2	,	0
Х	6	2	6	3
Y	14	9	5	4
Z	8	4	3	5

Caterer	Number of Meals
Х	120
Y	80
Z	150

School	Number of Meals
А	130
В	95
С	75
D	100

#### (a) Determine initial solution using Northwest Corner Rule.

To From	A		A			В		с		D	Supply
x	120	6		2		6		3	120		
Y	10	14	70	9		5		4	80		
z		8	25	4	75	3	50	5	150		
Dummy		0		0	0	0	50	0	50		
Demand	130		95		;	75	1	00	400		

(b) What is the total transportation cost?

#### Total transportation Cost:

Z to D	:	RM5	×	50	=	RM250
Z to C	:	RM3	$\times$	75	=	RM225
Z to B	:	RM4	$\times$	25	=	RM100
Y to B	:	RM9	$\times$	70	=	RM630
Y to A	:	RM14	$\times$	10	=	RM140
X to A	:	RM6	$\times$	120	=	RM720

RM2,065

#### 6.4 OPTIMAL SOLUTION BY STEPPING STONE

Stepping stone is used to obtain the least cost solution for minimization problem. This method involves two steps.

: To determine if the current solution can be lst step improved by calculating the **improvement** indices.

2nd step : Improving the solution



#### **Testing for improvement**

For each unused cell, a closed path is traced by moving horizontally or vertically through the used cells. Assign a positive sign, '+' to the selected empty cell and then assign alternative '-' and '+' to the occupied cells following the closed path. Then, calculate improvement index by summing up the signed unit cost of the traced cells.

Repeat the process for the other empty cells.

Optimal solution is reached if all the improvement indices of all unused cells are positive or zero. A negative index indicates that the solution can be improved and the transportation cost can be reduced by the amount of improvement index for each unit moved.



Refer back to Jimmy Company in Example 6.1

To From	w		X		Y		Z		Supply
А	280	28	20	)	+	32 †		36	300
В		22	+	16	- 240	14		32	420
С		32		24	120	20	480	36	600
Demand	280		200		360		480		1,320

Solution:

1st step : Identify unused cells.

A to Y, A to Z, B to W, B to Z, C to W, and C to X.

2nd step : Improvement Indices

16  $I_{AY}$  : 32 - 18 + 16 - 14 =  $I_{AZ}$  : 36 - 18 + 16 - 14 + 20 - 36 4 =  $I_{BW}$  : 18 - 28 + 22 - 16 -4 =  $I_{BZ}$  : 32 - 14 + 20 - 36 2 =  $I_{CW}$  : 32 - 20 + 14 - 16 + 18 - 28 0 =  $I_{CX}$  : 24 - 20 + 14 - 16 12 =

Since the unused cells, BW has a negative improvement index, the solution is not optimal. For every unit moves along the path, the cost can be reduced by RM4 for BW.



The unused cell with negative index of largest magnitude is selected. This largest magnitude negative index shows the greatest decrease in cost. Consider the closed path with plus and negative signs. Select the smallest number S in the square containing minus sign. Subtract this number S from all negative cells and add S to all the plus cells on the closed path. By moving this S value, the cost is reduced by  $S \times I$ , where I is the improvement index.



Continue with Example 6.4

To improve the solution for Jimmy Company, we select the smallest value from the negative cells along the closed path. For cells BW the smallest value from the negative cells is 180.

To From	w	w x		Z	Supply
А	- 28 280	+ 18	32	36	300
В	+ 22	16	240 14	32	420
С	32	24	120 20	480 36	600
Demand	280	200	360	480	1,320

So, we need to minus 180 from negative cells BX and AW and add 180 to positive cells BW and AX.

To From	w		w x		Y		z		Supply
A	100	28	200	18		32		36	300
В	180	22		16	240	14		32	420
С		32		24	120	20	480	36	600
Demand	280		20	00	3	60	4	80	1,320

Improvement indices:

A to Y	:	32 - 28 + 22 - 14	=	12
A to $Z$	:	36 - 28 + 22 - 14 + 20 - 36	=	0
B to X	:	16 - 18 + 28 - 22	=	4
B to $Z$	•	32 - 14 + 20 - 36	=	2
C to W	:	32 - 20 + 14 - 22	=	4
C to $X$	:	24 - 20 + 14 - 22 + 28 - 18	=	6

Since all the improvement indices are non-negative, the solution is optimal.

Minimum total transportation cost:

				RM33,400
C to $Z$	:	$RM36 \times 480$	=	RM17,280
C to Y	:	$RM20 \times 120$	=	RM2,400
B to $Y$	:	$RM14 \times 240$	=	RM3,360
B to $W$	:	$RM22 \times 180$	=	RM3,960
A to $X$	:	$RM18 \times 200$	=	RM3,600
A to W	:	$RM28 \times 100$	=	RM2,800



Daisey Florist has three distribution centres at Sun Store, Rainbow Store, and Star Store. The dealers located at Station A, Station B, and Station C. The demand, supply and the cost (RM) per unit to transport the tiles from the distribution centres to the dealers are given in the following table.

To From	А	В	С	Supply
Sun Store	7	4	6	350
Rainbow Store	6	2	5	200
Star Store	9	5	8	150
Demand	300	160	210	

- (a) Find an initial solution and the total cost.
- (b) Determine an optimal solution and the minimum transportation cost.
- (c) Which distribution centres will have the suplus? How much is the surplus?

#### Solution:

#### (a) Initial solution using Northwest Corner Rule

To From	A		В		С		Dummy		Supply
Sun	300	7	50	4		6		0	350
Rainbow		6	110	2	90	5		0	200
Star		9		5	120	8	30	0	150
Demand	30	00	10	50	2	10	63	30	700

#### Total transportation cost:

Sun to A	:	RM7 $\times$	300	=	RM2,100
Sun to B	:	RM4 $\times$	50	=	RM200
Rainbow to <i>B</i>	:	RM2 $ imes$	110	=	RM220
Rainbow to C	•	RM5 $\times$	90	=	RM450
Star to C	•	RM8 $\times$	120	=	RM960



(b) Optimal solution and the minimum transportation cost. Improvement indices:

I <sub>Sun to C</sub>	•	6 - 4 + 2 - 5	=	-1
I <sub>Sun to Dummy</sub>	•	0 - 4 + 2 - 5 + 8 - 0	=	1
I Rainbow to A	•	6 - 2 + 4 - 7	=	1
I <sub>Rainbow to Dummy</sub>	:	0 - 5 + 8 - 0	=	3
I <sub>Star to A</sub>	:	9 - 8 + 5 - 2 + 4 - 7	=	1
I <sub>Star to B</sub>	:	5 - 8 + 5 - 2	=	0

Since the unused cells, Sun to C has a negative improvement index, the solution is not optimal. To improve the solution, we select the smallest value from the negative cells along the closed path. The smallest value from the negative cells is 50.

To From	А	В	С	Dummy	Supply
Sun	300 (	- 4 50	+ 6	0	350
Rainbow	6	110	- 5 90	0	200
Star	9	5	8 120	0 30	150
Demand	300	160	210	30	700

So, we need to minus 50 from negative cells *Sun to B* and *Rainbow to C* and add 50 to positive cells *Sun to C* and *Rainbow to B*.

To From		A	E	3	(	C	Dur	nmy	Supply
Sun	300	7	-	4	+ 50	6		0	350
Deinhow		6	+	2	-	5		0	200
Rainbow			160		40				200
Star		9		5		8		0	150
					120		30		
Demand	30	00	10	60	2	10	3	80	700

#### Improvement indices:

I <sub>Sun to B</sub>	•	4 - 2 + 5 - 6	=	1
I <sub>Sun to Dummy</sub>	:	0 - 6 + 8 - 0	=	2
I Rainbow to A	•	6 - 5 + 6 - 7	=	0
I <sub>Rainbow to Dummy</sub>	•	0 - 5 + 8 - 0	=	3
I <sub>Star to A</sub>	•	9 - 8 + 6 - 7	=	0
I <sub>Star to B</sub>	•	5 - 8 + 5 - 2	=	0

#### Minimum total transportation cost:

					RM3,880
Star to C	:	$RM8 \times$	120	=	RM960
Rainbow to C	:	RM5 $\times$	40	=	RM200
Rainbow to B	:	RM2 $ imes$	160	=	RM320
Sun to C	:	RM6 $\times$	50	=	RM300
Sun to A	:	RM7 $\times$	300	=	RM2,100

Thus, the minimum total transportation is RM3,880.

(c) Star Store distribution centres will have the suplus of 30 units.



#### **Question 1**

Megah Company has three plants, A, B, and C and it intends to distribute its new furniture to four outlets, K, L, M, and N. The plant capacity is given in the table below:

Plant	Capacity (Units per week)
А	250
В	175
С	325

#### The demands for the four outlets are as follows:

Outlet	Demand per Week
К	120
L	230
М	240
N	160

The estimated transportation cost (in RM) of a unit of furniture for the various routes are:

To From	K	L	м	N
А	15	20	16	21
В	25	13	5	11
С	15	15	7	17

- (a) Is the transportation problem balance?
- (b) Determine the optimal solution that will minimize total cost.
- (c) Is the optimal solution in (b) unique? Give a reason for your answer.

#### **Question 2**

M.Tech Enterprise produce cups and mugs at three plants, A, B, and C and transports them to three major stores P, Q, and R. The monthly capacity, demand and cost per unit are given in the tables below,

Plant	Capacity (Units)	
А	1,300	
В	1,000	
С	1,200	

Stores	Demand per Week	
Р	1,100	
Q	900	
R	1,300	

To From	Р	Q	R
А	15	20	16
В	25	13	5
С	15	15	7

- (a) Find the initial solution.
- (b) Determine the optimal solution that will minimize total cost.
- (c) Which stores will have the surplus? How much is the surplus?
- (d) Is the optimal solution in (b) unique? Give a reason for your answer.

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## Terbitan



e ISBN 978-967-25866-0-9

