POLITEKNIK
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BASIC ALGEBRA

Engineering Mathematics

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ACKNOWLEDGEMENT

PREFACE

Grateful to Allah because with His permission, the eBook Basic Algebra (Engineering Mathematics) was published. This eBook is written by lecturers who have been teaching in Engineering Mathematics for more than 15 years . This eBook can be used by all institutions of higher learning such as Polytechnics and Colleges as well as private and public universities. The purpose for this eBook was written is to make it easier for students to gain knowledge and review the topic of calculus in a simpler and more concise way.

Many examples in various forms of questions are included in this eBook with detailed steps of solution to make it easier for students to quickly understand the method of its solution. In addition, students will also be able to improve and strengthen their understanding through the included practice questions. The authors hope that this eBook can benefit all students as well as educators around the world in the field of Engineering Mathematics.

Thank You So Much.

SUBTOPICS

Click on Page Number

Introduction

What is Algebra?

- ∞ Algebra is a discipline of mathematics in which letters are used to represent unknown quantities. We've been utilizing algebra from our early schooling, in which we studied formulas such as the area of a rectangle, where width w and height h, therefore the formula for the area of a rectangle is $A = w \times h$.

- ∞ Once we knew the width and height, we could substitute them into the formula and find our area.

- ∞ Another example is the **area of a circle**, with its radius r. Thus, the formula for the area of a circle is $A = \pi r^2$.

Terms (factor) and Coefficient

- ∞ An algebraic expression with an expression involving letters and/or numbers (called **factors**) which are multiplied together is known as **term**.

- ∞ While a numerical or constant quantity placed before and multiplying the variable in an algebraic expression is known as **coefficient**.

- ∞ Therefore;

Single term	
$5x$	x is variable 5 is coefficient
More than one term	
$5x + 7y$	First term: $5x$, has factors 5 and x Second term: $7y$, has factors 7 and y

Basic Algebra Rule

Basic Mathematics Operation

$$12 + 8 = 20 \qquad 20 - 8 = 12 \qquad 56 \div 7 = 80 \qquad 15 \times 5 = 75$$

$$\frac{3}{5} = 0.6 \qquad \frac{1}{4} = 0.25 \qquad 0.4 = \frac{2}{5} \qquad 0.125 = \frac{1}{8}$$

Order of Operation

P	Parentheses	
E	Exponents	
M	Multiplication	
D	Division	
A	Addition	
S	Subtraction	

PEMDAS

$$8 + \boxed{3 \times 5}$$
$$= \boxed{8 + 15}$$
$$= 23 \quad ✔$$

$$8 + 3 \times 5$$
$$= 11 \times 5$$
$$= 55 \quad ✘$$

Negative Numbers

$$-20 + 4 = -16 \qquad -20 \times 4 = -80 \qquad -20 \div 4 = -5$$

$$-20 - 4 = -24 \qquad -20 \times -4 = 80 \qquad -20 \div -4 = 5$$

Long Algebra Problem

- Need to follow the order of the operation (PEMDAS)

$$12 \div 4 + 9 - 2 \times 5$$
$$= (12 \div 4) + 9 - (2 \times 5)$$
$$= 3 + 9 - 10$$
$$= 2$$

Understanding Variable

Variable

- In mathematics, it is common to have an unknown value for a given number.

- Variable is a **symbol**, usually a letter that represents an unknown quantity.

- Example of variable:

Letters	Symbols
A, c, m, S, t, W, z etc	$\pi, \varphi, \gamma, \beta, \rho$ etc

Single variable	More than one variable
$2x + 5 = 15$ $2x = 15 - 5$ $2x = 10$ $x = 5$	$7n + 5m - 6m = 30$ $7n - m = 30$

EXAMPLE

Find the value of x.

$$2x + 5 = 25$$
$$2x = 25 - 5$$
$$2x = 20$$
$$x = \frac{20}{2}$$
$$x = 10$$

Find the value of s.

$$7s + 6s - 3s = 40$$
$$10s = 40$$
$$s = \frac{40}{10}$$
$$s = 4$$

Solve Equation by "Cancelling"

- Need to get the variable by itself in algebra equation
- We need **only the variable** on the left side of the equation
- Example $\Rightarrow x + 5 = 3$

  Therefore, we need to get rid "**5**" from both side of the equation

  $$x + 5 = 3$$
  $$x + 5 - \mathbf{5} = 3 - \mathbf{5}$$
  $$x = -2$$

- Cancel addition with subtraction (vice versa)

  $$p + 8 = 10$$
  $$p + 8 - \mathbf{8} = 10 - \mathbf{8}$$
  $$p = 10 - 8$$
  $$p = 2$$

  $$q - 7 = 5$$
  $$q - 7 + \mathbf{7} = 5 + \mathbf{7}$$
  $$q = 5 + 7$$
  $$q = 12$$

- Cancel multiplication with division (vice versa)

  $$8w = 40$$
  $$w = \frac{40}{\mathbf{8}}$$
  $$p = 5$$

  $$\frac{v}{5} = 5$$
  $$v = 5 \times \mathbf{5}$$
  $$v = 25$$

- Cancel exponent by taking the root (vice versa)

  $$z^2 = 49$$
  $$\sqrt{z^2} = \sqrt{49}$$
  $$z = 7$$

  $$\sqrt[3]{x} = 2$$
  $$\sqrt{x^3} = 2^{\mathbf{3}}$$
  $$x = 8$$

Understanding Algebraic

∞ The *difference* between an **algebraic equation** and **algebraic expression** is:

Algebraic Expression	Algebraic Equation
A mathematical statement consisting of constants, variables, and algebraic operators is called an algebraic equation.	A mathematical statement that equates two algebraic expressions is called an algebraic equation.
An algebraic expression is a type of rule, and any value can be found using it.	An algebraic equation is always equal to some value
For all possible values of the variables, an algebraic expression holds true.	Algebraic equations are valid for certain specific variable values.
We find zeros of an algebraic expression	We find the root or solution of an algebraic equation
For example: $3x^2 + 5$, $4x - 3y$, etc	For example: $x^2 - 5 = 3$, $4x - 3 = y + 2$, etc

Algebraic Expression

What is Algebraic Expression?

- ∞ In mathematics, an **algebraic expression** is an expression which consist of algebraic operations, variables, and constants. (addition, subtraction, etc.)

- ∞ Expressions are made up of terms.

- ∞ Example of algebraic expression:

$$2x + 3y$$

$$x^2 - 5 + 6y$$

- ∞ An algebraic expression **has no sides** or**" equal to" sign**.

- ∞ Algebraic Expression/Variable Expression

term

$$8x + 5$$

coefficient   variable   constant

- ∞ Algebraic operations include:
    - § addition,
    - § subtraction,
    - § multiplication,
    - § division

Algebraic operations

Addition and Subtraction

EXAMPLE 1

Simplify the expressions below

$$7 + 5x + 12 - 3x$$

$$= 19 + 2x$$

$$3m - 2m - 7 + 3n + 6$$

$$= m + 3n - 1$$

$$5d^2 + 2d - 4d^2 + 10 + 8d - 3$$

$$= d^2 + 10d + 7$$

WATCH NOW

EXAMPLE 2

Simplify the expressions below

$$5t(1 - 2t) + 3t + 6t^2 \quad \text{expand}$$

Solution:

$$= 5t - 10t^2 + 3t + 6t^2$$

$$= 8t - 4t^2$$

$$= 4t(2 - t) \quad \text{factorize}$$

$$2(s + 5) - 3(3 - s) \quad \text{expand}$$

Solution:

$$= 2s + 10 - 9 + 3s$$

$$= 5s + 1$$

Addition and Subtraction

Rules for Algebraic Fraction

Same denominator
$\bullet \dfrac{a}{c} + \dfrac{b}{c} = \dfrac{a+b}{c}$
$\bullet \dfrac{a}{c} - \dfrac{b}{c} = \dfrac{a-b}{c}$

$$\dfrac{5}{m} + \dfrac{3}{m} = \dfrac{8}{m}$$

Denominator can be equal using common factor
$\bullet \dfrac{a}{b} + \dfrac{c}{d} = \dfrac{ad+bc}{bd}$
$\bullet \dfrac{a}{b} - \dfrac{c}{d} = \dfrac{ad-bc}{bd}$

$$\dfrac{5}{m} + \dfrac{3}{n} = \dfrac{5n + 3m}{mn}$$

EXAMPLE 3

Simplify the following algebraic fraction expression

$$\dfrac{2s}{t} - \dfrac{2 + 3s}{t}$$

Solution:

$$= \dfrac{2s - 2 - 3s}{t}$$

$$= \dfrac{-s - 2}{t}$$

$$\dfrac{3x}{4y} + \dfrac{2 - 5x}{8y}$$

Solution:

$$= \dfrac{3x}{4y} \dfrac{\times 2}{\times 2} + \dfrac{2 - 5x}{8y}$$

$$= \dfrac{6x + 2 - 5x}{8y}$$

$$= \dfrac{x + 2}{8y}$$

Addition and Subtraction

Steps to add and subtract fraction

1 The denominators (the bottom numbers) **MUST BE** the same.

2 Add/subtract the numerators (the top numbers).

3 Reduce/simplify the resultant fraction to its lowest terms.
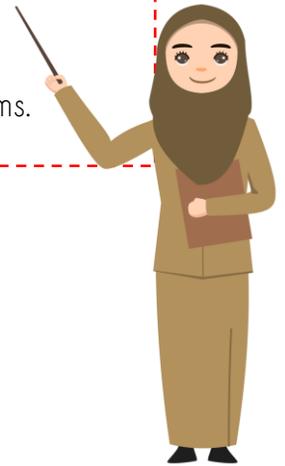
EXAMPLE 4

Simplify the following algebraic fraction expression

$$\frac{3 - 2p}{2} + \frac{5p}{3}$$

Solution:

$$= \frac{3 - 2p}{2} \frac{\times 3}{\times 3} + \frac{5p}{3} \frac{\times 2}{\times 2}$$

$$= \frac{9 - 6p}{6} + \frac{10p}{6}$$

$$= \frac{9 - 6p + 10p}{6}$$

$$= \frac{9 + 4p}{6}$$

$$\frac{2}{v + 1} - \frac{2}{3}$$

Solution:

$$= \frac{2}{v + 1} \frac{\times 3}{\times 3} - \frac{2}{3} \frac{\times (v + 1)}{\times (v + 1)}$$

$$= \frac{6 - 2(v + 1)}{3(v + 1)} \quad \textit{expand}$$

$$= \frac{6 - 2v - 2}{3(v + 1)}$$

$$= \frac{4 - 2v}{3(v + 1)}$$

Addition and Subtraction

EXAMPLE 5

Simplify the following algebraic fraction:

$$\frac{2}{n+1} - \frac{4}{n-2}$$

Solution:

$$= \frac{2}{n+1} \frac{\times (n-2)}{\times (n-2)} - \frac{4}{n-2} \frac{\times (n+1)}{\times (n+1)}$$

$$= \frac{2(n-2) - 4(n+1)}{(n+1)(n-2)} \quad \textit{expand}$$

$$= \frac{2n - 4 - 4n - 4}{(n+1)(n-2)}$$

$$= \frac{-2n - 8}{(n+1)(n-2)}$$

$$(2-m) + \frac{3}{(2+m)}$$

Solution:

$$= \frac{(2-m)}{1} \frac{\times (2+m)}{\times (2+m)} + \frac{3}{(2+m)}$$

$$\textit{expand}$$

$$= \frac{(2-m)(2+m) + 3}{(2+m)}$$

$$= \frac{4 + 2m - 2m - m^2 + 3}{(2+m)}$$

$$= \frac{7 - m^2}{(2+m)}$$

Addition and Subtraction

EXAMPLE 6

Simplify the following algebraic fraction:

$$\frac{2}{3w} + \frac{1}{w} - \frac{5}{6w}$$

Solution:

$$= \frac{2}{3w} \frac{\times 2}{\times 2} + \frac{1}{w} \frac{\times 6}{\times 6} - \frac{5}{6w}$$

$$= \frac{4 + 6 - 5}{6w}$$

$$= \frac{5}{6w}$$

$$\frac{1}{x+1} + \frac{x}{x-1} - \frac{x}{x^2-1}$$

Solution:

$$= \frac{1}{x+1} \frac{\times (x-1)}{\times (x-1)} + \frac{x}{x-1} \frac{\times (x+1)}{\times (x+1)} - \frac{x}{x^2-1}$$

$$= \frac{x - 1 + x^2 + x - x}{(x+1)(x-1)}$$

$$= \frac{x - 1 + x^2}{(x+1)(x-1)}$$

Addition and Subtraction

EXAMPLE 7

Simplify the following algebraic fraction:

$$\frac{2x}{(x-1)^2} - \frac{2}{x-1} + \frac{3}{x}$$

Solution:

$$= \frac{2x}{(x-1)^2} \frac{\times x}{\times x} - \frac{2}{x-1} \frac{\times x(x-1)}{\times x(x-1)} + \frac{3}{x} \frac{\times (x-1)^2}{\times (x-1)^2}$$

expand   *expand*

$$= \frac{2x^2 - 2(x^2 - x) + 3(x^2 - 2x + 1)}{x(x-1)^2}$$

$$= \frac{2x^2 - 2x^2 + 2x + 3x^2 - 6x + 3}{x(x-1)^2}$$

$$= \frac{3x^2 - 4x + 3}{x(x-1)^2}$$

$$\frac{2m}{m^2 + m} + \frac{m-1}{m^2}$$

Solution:

$$= \frac{2m}{m(m+1)} \frac{\times m}{\times m} + \frac{m-1}{m^2} \frac{\times (m+1)}{\times (m+1)}$$

$$= \frac{2m^2 + m^2 + m - m - 1}{m^2(m+1)}$$

$$= \frac{3m^2 - 1}{m^2(m+1)}$$

Simplify the following algebraic expression:

a) $(4m - 5) + 3 - 2m - 4$

b) $7(n - 4) + 3(3 - 2n) + 9$

c) $2(-5 - 3y) + 2 - 5y + 8$

d) $-4(t - 15) + 5t - 10 + 3t$

e) $7 + (-2q + 15) - 5q - 9$

f) $(5w - 12) + 3w - (-2w - 6)$

g) $10 - (4p - 3) - p + 5$

h) $(11s - 5) + (4 - 3s)(-2) + 5s$

i) $(-5r - 4) - (-10r - 2) + 5$

j) $6(4 - 4b) - 3(b + 3b) - 12$

ANSWER

a) $2m - 6$ b) $n - 10$ c) $-11y$ d) $4t + 50$ e) $13 - 7q$

f) $10w - 6$ g) $18 - 5p$ h) $22s - 13$ i) $5r + 3$ j) $12 - 36b$

Simplify the following algebraic fraction:

a) $\dfrac{3x}{2} + \dfrac{4x}{8}$

b) $\dfrac{x}{3} + \dfrac{3x}{4}$

c) $\dfrac{5}{m} + \dfrac{2}{3m^2}$

d) $\dfrac{3}{4xy} + \dfrac{5}{8y}$

e) $\dfrac{3}{2s} - \dfrac{4}{5t}$

f) $\dfrac{1-2w}{3} - \dfrac{4w}{5}$

g) $\dfrac{3d}{5c} - \dfrac{1-2c}{10d}$

h) $(1-2x) + \dfrac{2}{1+2x}$

i) $\dfrac{5}{3+a} - \dfrac{3}{2} + 4a$

j) $\dfrac{3y}{4x} + \dfrac{2x}{3y} - \dfrac{4}{xy}$

ANSWER

a) $2x$

b) $\dfrac{13x}{12}$

c) $\dfrac{15m+2}{3m^2}$

d) $\dfrac{6+5x}{8xy}$

e) $\dfrac{15t-8s}{10st}$

f) $\dfrac{5-22w}{15}$

g) $\dfrac{6d^2-c+2c^2}{10cd}$

h) $\dfrac{3-4x^2}{1+2x}$

i) $\dfrac{8a^2+21a+1}{6+2a}$

j) $\dfrac{9y^2+8x^2-48}{12xy}$

Algebraic operations

Multiplication

Steps to multiply fraction

1 Multiply the numerators together.

2 Multiply the denominators together.

3 Reduce/simplify the resultant fraction to its lowest terms.
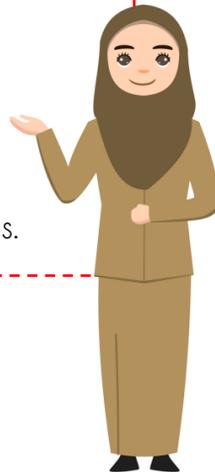
EXAMPLE 1

Simplify the following algebraic fraction:

$$\frac{5}{4m} \times \frac{1}{m}$$

Solution:

$$= \frac{5 \times 1}{4m \times m}$$

$$= \frac{5}{4m^2}$$

$$\frac{s}{t+s} \times \frac{t}{t-s}$$

Solution:

$$= \frac{s \times t}{(t+s) \times (t-s)}$$

expand

$$= \frac{st}{t^2 - ts + st - s^2}$$

$$= \frac{st}{t^2 - s^2}$$

Multiplication

EXAMPLE 2

Simplify the following algebraic fraction expressions:

$$\frac{2a - 6}{9a}\left(\frac{3a^2}{a - 3}\right)$$

Solution:

factorize

$$= \frac{2(a - 3) \times 3a^2}{9a \times (a - 3)}$$

$$= \frac{2a}{3}$$

Cancel common factor

$$\frac{2p + 6q}{16p}(4p^2)$$

Solution:

factorize

$$= \frac{2(p + 3q) \times 4p^2}{16p \times 1}$$

$$= \frac{p(p + 3q)}{2}$$

Cancel common factor

$$\frac{x - 5}{4y}\left(\frac{6xy}{x^2 - 10x + 25}\right)$$

Solution:

$$= \frac{x - 5}{4y}\left(\frac{6xy}{(x - 5)(x - 5)}\right)$$

factorize

$$= \frac{3x}{2(x - 5)}$$

Cancel common factor

$$\frac{m - n}{x + y} \times \frac{x^2 - y^2}{(m - n)^2}$$

Solution:

factorize

$$= \frac{m - n \times (x - y)(x + y)}{x + y \times (m - n)(m - n)}$$

factorize

$$= \frac{x - y}{m - n}$$

Cancel common factor

Multiplication

EXAMPLE 3

Simplify the following algebraic fraction:

$$\frac{2v^2 + 8v^3}{5w} \times \frac{(10w - 15)}{1 + 4v}$$

Solution:

$$= \frac{\overset{factorize}{2v^2(1 + 4v)} \times \overset{factorize}{5(2w - 3)}}{5w \times (1 + 4v)}$$

Cancel common factor

$$= \frac{2v^2(2w - 3)}{w}$$

$$\frac{x - 2}{x^2 - 5x + 6} \times \frac{6x^3 - 18x^2}{12x^3}$$

Solution:

$$= \frac{(x - 2) \times \overset{factorize}{6x^2(x - 3)}}{\underset{factorize}{(x - 3)(x - 2)} \times 3x^3}$$

Cancel common factor

$$= \frac{2}{x}$$

$$\left(\frac{x + 3}{x^2 - 5x + 6}\right)\left(\frac{x^2 - 4}{x^2 + 7x + 12}\right)$$

Solution:

$$= \frac{(x + 3) \times \overset{factorize}{(x - 2)(x + 2)}}{\underset{factorize}{(x - 3)(x - 2)} \times (x + 3)(x + 4)}$$

Cancel common factor

$$= \frac{x + 2}{(x - 3)(x + 4)}$$
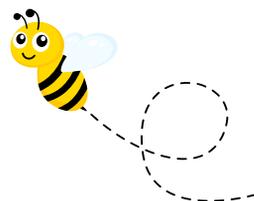
Algebraic operations

Division

Steps to divide fraction

1 Turn the second fraction upside down.

2 Multiply the first fraction by the second fraction.

3 Reduce/simplify the resultant fraction to its lowest terms.

EXAMPLE 1

Solve the following algebraic fraction:

$$\frac{3xy^2}{4s} \div \frac{x^3y^3}{2st}$$

Solution:

$$= \frac{3xy^2}{4s} \times \frac{2st}{x^3y^3}$$

$$= \frac{3t}{2sx^2y}$$

Cancel common factor

$$4x^2 \div \frac{2x^3}{5}$$

Solution:

$$= \frac{4x^2}{1} \times \frac{5}{2x^3}$$

$$= \frac{10}{x}$$

Cancel common factor

Division

EXAMPLE 2

Solve the following algebraic fraction:

$$\frac{a+5}{p^2-q^2} \div \frac{2a+10}{p-q}$$

Solution:

$$= \frac{a+5}{(p-q)(p+q)} \times \frac{p-q}{2(a+5)}$$

factorize   *factorize*   Cancel common factor

$$= \frac{1}{2(p+q)}$$

$$\frac{\frac{25}{8m}}{\frac{5}{4m-16}}$$

Solution:

$$= \frac{25}{8m} \div \frac{5}{4m-16}$$

$$= \frac{25}{8m} \times \frac{4(m-4)}{5}$$

factorize   Cancel common factor

$$= \frac{5(m-4)}{2m}$$

$$\frac{x-2}{x^2-6x+8} \div \frac{x^2-2x-8}{4x+2x^2}$$

Solution:

$$= \frac{x-2}{x^2-6x+8} \times \frac{4x+2x^2}{x^2-2x-8}$$

factorize

$$= \frac{x-2}{(x-2)(x-4)} \times \frac{2x(2+x)}{(x+2)(x-4)}$$

factorize   *factorize*   Cancel common factor

$$= \frac{2x}{(x-4)(x-4)}$$

$$= \frac{2x}{(x-4)^2}$$

Division

EXAMPLE 3

Solve the following algebraic fraction:

$$\frac{x^3 + x^2 y}{x - y} \div \frac{(x + y)^2}{x^2 - y^2}$$

Solution:

$$= \frac{x^3 + x^2 y}{x - y} \times \frac{x^2 - y^2}{(x + y)^2}$$

$$= \frac{\overset{factorize}{x^2(x + y)}}{x - y} \times \frac{\overset{factorize}{(x - y)(x + y)}}{\underset{factorize}{(x + y)(x + y)}}$$

Cancel common factor

$$= x^2$$

$$\frac{x^2 - 2x - 15}{x^2 - 3x - 10} \div \frac{x^2 + 6x + 9}{x + 2}$$

Solution:

$$= \frac{x^2 - 2x - 15}{x^2 - 3x - 10} \times \frac{x + 2}{x^2 + 6x + 9}$$

$$= \frac{\overset{factorize}{(x + 3)(x - 5)}}{\underset{factorize}{(x - 5)(x + 2)}} \times \frac{x + 2}{\underset{factorize}{(x + 3)(x + 3)}}$$

Cancel common factor

$$= \frac{1}{(x + 3)}$$
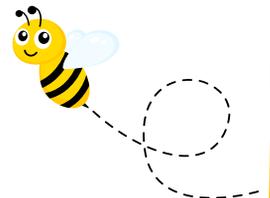
Simplify the following algebraic expression:

a) $\dfrac{3+x}{2} \times \dfrac{4-2x}{2}$

b) $\dfrac{x}{3} \cdot \dfrac{3x}{4}$

c) $\dfrac{2q}{12} \div \dfrac{3}{4q}$

d) $(2p+3) \div \dfrac{p}{3}$

e) $\dfrac{5m^3}{m} \times \dfrac{15}{3m^2}$

f) $\dfrac{6+2x}{3x} \times \dfrac{7+x}{x^2}$

g) $\dfrac{3}{4xy} \div \dfrac{5}{8y}$

h) $\dfrac{2m}{m^2-4} \div \dfrac{2}{m+2}$

i) $\dfrac{p^2-4}{q^2-9} \times \dfrac{q^2-6q+9}{p^2+4p+4}$

j) $\dfrac{12t-4t^2}{6t^2-7t+2} \div \dfrac{4t}{2t-1}$

ANSWER

a) $\dfrac{6-x-x^2}{2}$ b) $\dfrac{x^2}{4}$ c) $\dfrac{2q^2}{9}$ d) $\dfrac{6p+9}{p}$ e) 25

f) $\dfrac{2x^2+20x+42}{3x^3}$ g) $\dfrac{6}{5x}$ h) $\dfrac{m}{m-2}$ i) $\dfrac{(p-2)(q-3)}{(q+3)(p+2)}$ j) $\dfrac{3-t}{3t-2}$

Simplifying Algebraic Fraction Expression

EXAMPLE 1

Simplify each of the following into a simplest algebraic fraction:

$$\frac{15m^2 n}{25mn^3}$$

Solution:

$$= \frac{3m}{5n^2} \quad \text{Cancel common factor}$$

$$\frac{(2p)^3}{16pq^3}$$

Solution:

$$= \frac{8p^3}{16pq^3}$$

$$= \frac{p^2}{2q^3} \quad \text{Cancel common factor}$$

$$\frac{20(xy)^2 z^4}{(2xyz)^3}$$

Solution:

$$= \frac{20x^2 y^2 z^4}{8x^3 y^3 z^3}$$

$$= \frac{5z}{2xy} \quad \text{Cancel common factor}$$

$$\frac{6xy^2 - 18x^2 y}{y - 3x}$$

Solution:

$$= \frac{6xy(y - 3x)}{y - 3x} \quad \text{factorize} \quad \text{Cancel common factor}$$

$$= 6xy$$

Simplifying Algebraic Fraction Expression

EXAMPLE 2

Simplify each of the following into a simplest algebraic fraction:

$$\frac{2ab - 4b}{6a^2b + 2b}$$

Solution:

$$= \frac{\overset{factorize}{2b(a - 2)}}{\underset{factorize}{2b(3a^2 + 1)}}$$

Cancel common factor

$$= \frac{a - 2}{3a^2 + 1}$$

$$\frac{3s + 3t}{s^2 - t^2}$$

Solution:

$$= \frac{\overset{factorize}{3(s + t)}}{\underset{factorize}{(s - t)(s + t)}}$$

Cancel common factor

$$= \frac{3}{(s - t)}$$

$$\frac{x^2 - 2x - 15}{x^2 - 25}$$

Solution:

$$= \frac{\overset{factorize}{(x - 5)(x + 3)}}{\underset{factorize}{(x - 5)(x + 5)}}$$

Cancel common factor

$$= \frac{x + 3}{x + 5}$$

$$\frac{2m + 2n}{m + n}$$

Solution:

$$= \frac{\overset{factorize}{2(m + n)}}{\underset{factorize}{(m + n)}}$$

Cancel common factor

$$= 2$$

Simplifying Algebraic Fraction Expression

a) $\dfrac{18xy^2}{6x^2y}$

$= \dfrac{3y}{x}$

b) $\dfrac{p(p-5)^2}{(p+4)(p-5)}$

$= \dfrac{p(p-5)}{p+4}$

c) $\dfrac{3z+5z^3}{z}$

$= \dfrac{z(3+5z^2)}{z}$

$= 3+5z^2$

d) $\dfrac{16pq-4pr}{8p^2r^2+12pq^2}$

$= \dfrac{4p(4q-r)}{4p(2pr^2+3q^2)}$

$= \dfrac{4q-r}{2pr^2+3q^2}$

e) $\dfrac{q-5}{q^2-2x-15}$

$= \dfrac{q-5}{(q+3)(q-5)}$

$= \dfrac{1}{q+3}$

f) $\dfrac{3s^2+9s}{s^2-9}$

$= \dfrac{3s(s+3)}{(s-3)(s+3)}$

$= \dfrac{3s}{s-3}$

g) $\dfrac{x}{ac}-\dfrac{x}{bc}+\dfrac{x}{ab}$

$= \dfrac{xb-xa+xc}{abc}$

$= \dfrac{x(b-a+c)}{abc}$

h) $\dfrac{xy}{x-y}\left(\dfrac{x}{y}+\dfrac{y}{x}\right)$

$= \dfrac{xy}{x-y}\left(\dfrac{x^2+y^2}{xy}\right)$

$= \dfrac{x^2+y^2}{x-y}$

Simplifying Algebraic Fraction Expression

i)
$$\frac{2x+1}{y} - \frac{3x+4}{2y}$$
$$= \frac{2(2x+1) - (3x+4)}{2y}$$
$$= \frac{4x+2-3x-4}{2y}$$
$$= \frac{x-2}{2y}$$

j)
$$\frac{5+2g}{5} + \frac{g^2-1}{g}$$
$$= \frac{g(5+2g) + 5(g^2-1)}{5g}$$
$$= \frac{5g + 2g^2 + g^2 - 5}{5g}$$
$$= \frac{3g^2 + 5g - 5}{5g}$$

k)
$$(a-b) \div \left(\frac{1}{b} - \frac{1}{a}\right)$$
$$= \frac{a-b}{1} \div \frac{a-b}{ba}$$
$$= \frac{a-b}{1} \times \frac{ba}{a-b}$$
$$= ba$$

l)
$$\frac{5s-2t}{st} - \left(\frac{3}{s} + \frac{4s}{st}\right)$$
$$= \frac{5s-2t}{st} - \frac{(3t+4s)}{st}$$
$$= \frac{5s-2t-3t-4s}{st}$$
$$= \frac{s-5t}{st}$$

Simplify the linear algebraic expression below:

a) $\dfrac{5x^2y^9}{25y^6x^7}$

b) $\dfrac{27a^6b^2c^5}{57ab^6c^4}$

c) $\dfrac{9zy^2 + 6x^2y^3}{18y^2x - 12x^2z}$

d) $\dfrac{m^2 - 2mn + n^2}{(m-n)^2}$

e) $\dfrac{4a}{a-2} - 3$

f) $\dfrac{1}{2}(4p - 6q) - 5(2p + q)$

g) $\dfrac{x^2 + 2x + 1}{x^2 + 4x + 4} \div \dfrac{x^2 - 1}{x^2 - 4}$

h) $(8a^2b - 4a) \times \dfrac{ab}{4ab - 1}$

i) $c^2 + 3c \div \dfrac{c+3}{4c}$

j) $\dfrac{6v + 24}{3w^3} \times \dfrac{5w}{v^2 - 16}$

ANSWER

a) $\frac{y^2}{5x^5}$

b) $\frac{9a^5c}{19b^4}$

c) $\frac{y^2(3z+2x^2y^2)}{2x(3y^2-2xz)}$

d) 1

e) $\frac{a+6}{a-2}$

f) $-8p - 8q$

g) $\frac{(x+1)(x-2)}{(x-1)(x+2)}$

h) $4a^2b$

i) $4c^2$

j) $\frac{10}{w^2(v-4)}$

Transposition of the Formula

What is transposition of the formula?

- **Transposing** an equation is the process of rearranging the formula.
- A mathematical relationship involving two or more variable quantities is called a formula.
- Occasionally, it is required to modify the formula's subject for a standard formula.
- In any such transposition, the order of action needs to be carefully considered.
- For example:

  - ∞ $P = \dfrac{F}{A}$ is a formula for the pressure, P, where F is force and A is area.
  - ∞ In the formula, P is expressed in terms of F and A. Therefore, P is the subject of the formula.

- To transpose the formula for A, or solve for A, then we must make A the subject of the formula. Therefore, when transposing a formula whatever is done to one side is also done to the other.
- You can perform the following operation to transpose the formula:
  - ∞ add the same quantity to both sides of the formula;
  - ∞ subtract the same quantity from both sides of the formula;
  - ∞ multiply both sides of the formula by the same quantity;
  - ∞ divide both sides of the formula by the same quantity.
  - ∞ take a "function" of both sides of the formula, such as find the reciprocal of both sides (i.e., invert);

Transposition of the Formula

EXAMPLE 1

Make the variable stated in the bracket as the subject of the formula:

Given $u = v - at$, (a)

$$u = v - at$$

Step 1	minus both side with v	$u - v = -at - v$
Step 2	divide both side by $-t$	$\dfrac{v - u}{-t} = \dfrac{-at}{-t}$
Step 3	a is the subject of the formula	$a = \dfrac{v - u}{-t}$

EXAMPLE 2

Make the variable stated in the bracket as the subject of the formula:

Given $y = a + \dfrac{1}{x}$, (x)

$$y = a + \dfrac{1}{x}$$

Step 1	minus both side with a	$y - a = \dfrac{1}{x} - a$
Step 2	multiply both side with x	$y - a(x) = \dfrac{1}{x}(x)$
Step 3	divide both side by $(y - a)$	$\dfrac{(y - a)(x)}{(y - a)} = \dfrac{1}{(y - a)}$
Step 4	x as the subject of the formula	$x = \dfrac{1}{(y - a)}$

Transposition of the Formula

EXAMPLE 3

Make the variable stated in the bracket as the subject of the formula:

Given $t = 2\pi\sqrt{\dfrac{h+k}{g}}$, (h)

$$t = 2\pi\sqrt{\dfrac{h+k}{g}}$$

Step 1	square both side	$t^2 = (2\pi)^2\left(\dfrac{h+k}{g}\right)$
Step 2	divide both side by $(2\pi)^2 = 4\pi^2$	$\dfrac{t^2}{4\pi^2} = \dfrac{h+k}{g}$
Step 3	multiply both side with g	$\dfrac{gt^2}{4\pi^2} = h+k$
Step 4	minus both side with k	$\dfrac{gt^2}{4\pi^2} - k = h$
Step 5	h is the subject of the formula	$h = \dfrac{gt^2}{4\pi^2} - k$

Simplify the following algebraic expression by making the **variable stated in the bracket** as the **subject of the formula**:

a) Given $p = 5t - 17$. (**t**)

b) Given $\sqrt{2q} = p$. (**q**)

c) Given $v = \sqrt{t^2 + w}$. (**w**)

d) Given $x = \dfrac{2}{3y}$. (**y**)

e) Given $\dfrac{2}{R} = \dfrac{3}{x+y}$. (**R**)

f) Given $\dfrac{2}{S} = \dfrac{2}{S_1} - \dfrac{3}{S_2}$. (**S**)

g) Given $v = \sqrt{x + 2y}$. (**y**)

h) Given $T = \dfrac{2v}{g}\left(\dfrac{1}{1-e}\right)$. (**e**)

i) Given $I = \dfrac{E-V}{R+r}$. (**R**)

j) Given $\dfrac{Q}{t} = P\left(\dfrac{T_2 - T_1}{x}\right)$. (**T₂**)

ANSWER

a) $t = \dfrac{p+17}{5}$ b) $q = \dfrac{p^2}{2}$ c) $w = v^2 + t^2$ d) $y = \dfrac{2}{3x}$ e) $R = \dfrac{2(x+y)}{3}$

f) $S = \dfrac{2S_1 S_2}{2S_2 - 3S_1}$ g) $y = \dfrac{v^2 - x}{2}$ h) $e = 1 - \dfrac{2v}{Tg}$ i) $R = \dfrac{E-v}{I} - r$ j) $T_2 = \dfrac{Qx}{Pt} + T_1$

Algebraic Equation

What is Algebraic Equation?

∞ An algebraic equation is a mathematical statement where two expressions are equated to each other.

∞ Algebraic equation contains a variable, coefficients, and constants.

∞ The variable must be multiplied by a coefficient that is not zero.

∞ An algebraic equation **has sides** or" **equal to" sign** and the left-hand side is equal to the right-hand side.

∞ Example of algebraic equation:

$$2x + 3y = 15$$

$$x^2 - 5 = 2 + 6y$$

∞ Algebraic Equation:

equation

expression

$$8x + 5 = 29$$

coefficient variable constant   constant

∞ Two types of algebraic equation that will be covered are:

   a) Linear Equation

   b) Quadratic Equation

Linear Equation

∞ A **linear equation** consists of one or two variables and involves setting two expressions which is equal to each other.

∞ A **linear equation** is an equation that can be written in the form of $ax + b = 0$, where a and b are real numbers and x is a variable.

∞ This form is sometimes called the **standard form of a linear equation**.

∞ An algebraic equation, such as $y = 2x + 5$ or $2x - 4y + 2z = 15$, in where the highest degree of any term involving the variable(s) is one.

Steps to solve linear equation

1. Simplify each side of the equation as much as possible.

2. Collect all the variable terms on one side of the equation.

3. Collect all the constant terms on the other side of the equation.

4. Make the coefficient of the variable term equal to 1.

5. Simplify the equation (if necessary).

Linear Equation

EXAMPLE 1

Solve the linear equation below:

$$5m - 2 = 5 + 4m$$

Solution:

$$5m - 4m = 5 + 2$$

$$m = 7$$

$$2(5 + 3z) = 5 - (1 - 5z)$$
expand *expand*

Solution:

$$10 + 6z = 5 - 1 + 5z$$

$$6z - 5z = 5 - 1 - 10$$

$$z = -6$$

$$\frac{u}{5} = \frac{2 - u}{4}$$

Solution:

expand
$$4u = 5(2 - u)$$

$$4u = 10 - 5u$$

$$4u + 5u = 10$$

$$9u = 10$$

$$u = \frac{10}{9}$$

$$\frac{y - 3}{y - 1} = 4$$

Solution:

expand
$$y - 3 = 4(y - 1)$$

$$y - 3 = 4y - 4$$

$$y - 4y = -4 + 3$$

$$-3y = -1$$

$$y = \frac{1}{3}$$

Linear Equation

EXAMPLE 2

Solve the linear equation below:

$$\frac{3w + 4}{2} = \frac{5w - 1}{3}$$

Solution:

expand   *expand*

$$3(3w + 4) = 2(5w - 1)$$

$$9w + 12 = 10w - 2$$

$$9w - 10w = -2 - 12$$

$$-w = -14$$

$$w = 14$$

$$\frac{p - 5}{2} + 3 = p - 2$$

Solution:

make denominator the same $\dfrac{p - 5 + 6}{2} = p - 2$

                     expand

$$p - 5 + 6 = 2(p - 2)$$

$$p + 1 = 2p - 4$$

$$-p = -5$$

$$p = 5$$

$$2 - \frac{3p - 1}{3} = \frac{4p + 5}{2}$$

Solution:

$$\frac{2 \times 3}{1 \times 3} - \frac{3p - 1}{3} = \frac{4p + 5}{2}$$

$$\frac{6 - (3p - 1)}{3} = \frac{4p + 5}{2}$$

$$\frac{6 - 3p + 1}{3} = \frac{4p + 5}{2}$$

$$\frac{7 - 3p}{3} \diagdown \frac{4p + 5}{2}$$

expand   *expand*

$$2(7 - 3p) = 3(4p + 5)$$

$$14 - 6p = 12p + 15$$

$$-12p - 6p = 15 - 14$$

$$-18p = 1$$

$$p = \frac{1}{-18}$$

Solve the following algebraic equation:

a) $8x - 7(2 - x) = 5x + 4$

b) $4(s + 3) - 5 = 2(6 - 7s) + 3s$

c) $2(n - 3) = 10 - 5(2 - n)$

d) $\dfrac{3}{2n + 1} = \dfrac{5}{n + 1}$

e) $\dfrac{b - 4}{3} + 3 = \dfrac{2b + 5}{4}$

f) $\dfrac{5}{m + 2} + \dfrac{1}{3} = \dfrac{3}{m + 2}$

g) $\dfrac{s - 2}{s + 2} = 4$

h) $15 - p = \dfrac{5p - 3}{3}$

i) $\dfrac{6h + 3}{5} = \dfrac{2h - 3}{2}$

j) $\dfrac{3q}{4} + \dfrac{2q}{8} - \dfrac{4q}{2} = 1$

ANSWER

a) $x = \frac{9}{5}$ b) $s = \frac{1}{3}$ c) $n = -2$ d) $n = -\frac{2}{7}$ e) $b = \frac{5}{2}$

f) $m = -8$ g) $s = -\frac{10}{3}$ h) $p = 6$ i) $h = -\frac{21}{2}$ j) $q = -1$

Quadratic Equation

- A **quadratic equation** is defined as an equation that can be rearranged into standard form $ax^2 + bx + c = 0$.

- Where x represents an unknown and a, b, and c are the coefficients of the equation which represent known numbers, where $a \neq 0$.

- In general, a quadratic equation must contain an x^2 term (**MUST NOT** contain terms with degrees higher than x^2 eg x^3 or x^4 etc)

- A quadratic equation can have a maximum of two solutions, which consist of two roots.

- A quadratic equation can be factored into an equivalent equation.

- There are 3 (three) ways to solve quadratic equation:

    a) Factorization

    b) Quadratic Formula

    c) Completing the Square

Quadratic Equation

Factorization

§ By using the "*Zero-Product Property*" method is how we solve quadratics by factoring.

Steps to solve quadratic equation by factoring

1. Obtain zero on one side and then factor.
2. Factor the non-zero side.
3. Set each factor equal to zero.
4. Solve each of the resulting equations.

Tips You can always check your answer using the calculator to see if you solved it correctly.

EXAMPLE 1

Solve the quadratic equation below:

Given $x^2 - 2x = 15$

Step 1	Bring all terms to the left and simplify, leaving zero on the right side.	$x^2 - 2x - 15 = 0$
Step 2	Factorize the quadratic expression.	$(x + 3)(x - 5) = 0$
Step 3	Set each factor equal to zero.	$x + 3 = 0$ and $x - 5 = 0$
Step 4	Solve the resulting linear equations	$x + 3 = 0$ $x - 5 = 0$ $x = -3$ $x = 5$

Note: we need to set the equation equal to zero with the terms written in descending order

EXAMPLE 2

Solve the quadratic equation below:

Given $x^2 - 7x + 12 = 0$

Step 1	The equation is already equal to zero.	$x^2 - 7x + 12 = 0$
Step 2	Factorize the quadratic expression.	$(x - 3)(x - 4) = 0$
Step 3	Set each factor equal to zero.	$x - 3 = 0$ and $x - 4 = 0$
Step 4	Solve the resulting linear equations	$x - 3 = 0$ $x = 0$ $\quad$ $x - 4 = 0$ $x = 4$

EXAMPLE 3

Solve the quadratic equation below:

Given $3x^2 - 16x + 5 = 0$

Step 1	The equation is already equal to zero.	$3x^2 - 16x + 5 = 0$
Step 2	Factorize the quadratic expression.	$(x - 5)(3x - 1) = 0$
Step 3	Set each factor equal to zero.	$x - 5 = 0$ and $3x - 1 = 0$
Step 4	Solve the resulting linear equations	$x - 5 = 0$ $x = 5$ $\quad$ $3x - 1 = 0$ $x = \dfrac{1}{3}$

Solve the following algebraic equation by using Factorization:

a) $x^2 + x - 72 = 0$

b) $n^2 - 8n + 12 = 0$

c) $b^2 = 4b + 21$

d) $x^2 - 9x + 14 = 0$

e) $6x^2 - 11x - 10 = 0$

f) $x^2 - 12x - 45 = 0$

g) $5x^2 - 7x - 6 = 0$

h) $2x^2 - 5x - 3 = 0$

i) $4x - x^2 = 0$

j) $(3x + 1)(x - 1) = 15$

ANSWER

a) $x_1 = -9, x_2 = 8$ b) $n_1 = 2, n_2 = 6$ c) $b_1 = -3, b_2 = 7$ d) $x_1 = 2, x_2 = 7$ e) $x_1 = -\frac{2}{3}, x_2 = \frac{5}{2}$

f) $x_1 = -3, x_2 = 5$ g) $x_1 = -\frac{3}{5}, x_2 = 2$ h) $x_1 = -\frac{1}{2}, x_2 = 3$ i) $x_1 = 0, x_2 = 4$ j) $x_1 = -2, x_2 = \frac{8}{3}$

Quadratic Equation

§ A quadratic equation with real or complex coefficients has two solutions, called **roots**. These two solutions may or may not be distinct, and they may or may not be real.

§ The roots are given by the quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

§ First, bring the equation to the form $ax^2 + bx + c = 0$ where a, b, and c are **coefficients**.

§ Then identify the values of these coefficients a, b, and c before using the formula. Then substitute the values of a, b, and c into the formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

§ Solving a quadratic equation, we shall follow these steps below:

Steps to solve quadratic equation by quadratic formula

1 Write the quadratic equation in standard form. (Identify the value of a, b and c.

2 Substitute the value of a, b and c into the quadratic formula.

3 Simplify the formula and solve for x.

4 Solve each of the resulting equations.

Tips You can always check your answer using the calculator to see if you solved it correctly.

Quadratic Formula

EXAMPLE 1

Solve the quadratic equation below:

Given $x^2 - 8x + 14 = 0$

Step 1	Identify the values of a, b, and c	$x^2 - 8x + 14 = 0$ $a = 1, b = -8, c = 14$
Step 2	Then substitute these values into the quadratic formula.	$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $= \dfrac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(14)}}{2(1)}$
Step 3	Use the order of operations to simplify the quadratic formula.	$x = \dfrac{8 \pm \sqrt{64 - 56}}{2}$ $= \dfrac{8 \pm \sqrt{8}}{2}$
Step 4	Simplify the radical, if you can and simplify.	$x = \dfrac{8 \pm 2\sqrt{2}}{2}$ $= 4 \pm \sqrt{2}$
Step 5	Solve the resulting linear equations.	$x_1 = 4 + \sqrt{2}$ $x_2 = 4 - \sqrt{2}$ $x_1 = 5.4142$ $x_2 = 2.5858$

Quadratic Formula

EXAMPLE 2

Solve the quadratic equation below:

Given $2x^2 = 7x + 6$

Step 1	Bring all terms to the left & simplify, leaving zero on the right side.	$2x^2 - 7x - 6 = 0$
Step 2	Identify the values of a, b, and c	$2x^2 - 7x - 6 = 0$ $a = 2, b = -7, c = -6$
Step 3	Then substitute these values into the quadratic formula.	$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $= \dfrac{-(-7) \pm \sqrt{(-7)^2 - 4(2)(-6)}}{2(2)}$
Step 4	Use the order of operations to simplify the quadratic formula.	$x = \dfrac{7 \pm \sqrt{49 + 48}}{4}$ $= \dfrac{7 \pm \sqrt{97}}{4}$
Step 5	Simplify the radical, if you can and simplify.	$x = \dfrac{7 \pm \sqrt{97}}{4}$
Step 6	Solve the resulting linear equations.	$x_1 = \dfrac{7 + \sqrt{97}}{4}$ $\qquad x_2 = \dfrac{7 - \sqrt{97}}{4}$ $x_1 = 4.2122 \qquad\qquad x_2 = -0.7122$

Note: we need to set the equation equal to zero with the terms written in descending order

Solve the following algebraic equation by using Formula Method:

a) $x^2 - 6x + 2 = 0$

b) $3t^2 - 4t = 7$

c) $2m^2 - 4m + 1 = 0$

d) $2x^2 + 3x = 5$

e) $2m(m - 2) = m - 2$

f) $3t^2 - 4t = 7$

g) $9q^2 - 9q = -2$

h) $3q^2 = 4 - 8q$

i) $18x^2 + 27x = 35$

j) $\dfrac{11x - 2}{x + 3} = 2x$

🐝 ANSWER

a) $x_1 = 0.3542, x_2 = 5.6458$ b) $t_1 = -1, t_2 = 2.3333$ c) $m_1 = 0.2929, m_2 = 1.7071$ d) $x_1 = -2.5, x_2 = 1$ e) $m_1 = 2, m_2 = \frac{1}{2}$

f) $t_1 = -1, t_2 = 2.3333$ g) $q_1 = -\frac{1}{3}, q_2 = \frac{2}{3}$ h) $x_1 = -\frac{1}{2}, x_2 = 3$ i) $x_1 = -\frac{7}{3}, x_2 = 0.8333$ j) $x_1 = 2, x_2 = \frac{1}{2}$

Quadratic Equation

§ The method of **Completing the Square** is used to solve quadratic equations that cannot be factorized. This method can solve all quadratic equations..

§ Completing the Square is a method used to solve a quadratic equation by transforming the equation so that the left side become a perfect square trinomial.

§ To solve a quadratic equation, follow these steps below:

Steps to solve quadratic equation by completing the square

1. Divide all terms by a (the coefficient of x^2). **MUST ensure** that $a = 1$

2. Take the coefficient of $x^2(b)$ and multiply by $\frac{1}{2}$.

3. Move the number term (c) to the right side of the equation.

4. Squared the answer and put it on each side of the equal sign.

5. Take the squared answers only & arrange the answer to be squared. Solve the right side of the equal sign.

6. Simplify the formula and solve for x.

7. Solve each of the resulting equations.

Tips You can always check your answer using the calculator to see if you solved it correctly.

Completing the Square

EXAMPLE 1

Solve the quadratic equation below:

Given $2x^2 + 12x + 10 = 0$

Step 1	Divide all terms by a (the coefficient of x^2). MUST ensure that $a = 1$	$2x^2 + 12x + 10 = 0$ $\frac{2}{2}x^2 + \frac{12}{2}x + \frac{10}{2} = 0$ $x^2 + 6x + 5 = 0$
Step 2	Take the coefficient of x^1 (b) and multiply by $\frac{1}{2}$.	$6 \times \frac{1}{2} = 3$
Step 3	Move the coefficient of x^0 (c) to the right side of the equal sign(=).	$x^2 + 6x = -5$
Step 4	Squared the answer and put it on each side of the equal sign.	$x^2 + 6x + 3^2 = -5 + 3^2$
Step 5	Take the squared answers only & arrange the answer to be squared. Solve the right side of the equal sign.	$(x + 3)^2 = 4$
Step 6	Solve the equation	$x + 3 = \sqrt{4}$ $x + 3 = \pm 2$
Step 7	Solve the resulting linear equations.	$x_1 = 2 - 3 \quad x_2 = -2 - 3$ $x_1 = -1 \quad\quad x_2 = -5$

Completing the Square

EXAMPLE 2

Solve the quadratic equation below:

Given $x^2 - x - 6 = 0$

Step 2	Take the coefficient of x^1 (b) and multiply by $\frac{1}{2}$.	$-1 \times \frac{1}{2} = -\frac{1}{2}$
Step 3	Move the coefficient of x^0 (c) to the right side of the equal sign(=).	$x^2 - x = 6$
Step 4	Squared the answer and put it on each side of the equal sign.	$x^2 - x + \left(-\frac{1}{2}\right)^2 = 6 + \left(-\frac{1}{2}\right)^2$
Step 5	Take the squared answers only & arrange the answer to be squared. Solve the right side of the equal sign.	$\left(x - \frac{1}{2}\right)^2 = \frac{25}{4}$
Step 6	Solve the equation	$x - \frac{1}{2} = \sqrt{\frac{25}{4}}$ $x - \frac{1}{2} = \pm\frac{5}{2}$
Step 7	Solve the resulting linear equations.	$x_1 = \frac{1}{2} - \frac{5}{2}$ $x_2 = -\frac{1}{2} + \frac{5}{2}$ $x_1 = -2$ $x_2 = 3$

Solve the following algebraic expression by using Completing the Square Method:

a) $x^2 + 5x + 6 = 0$

b) $p^2 + 8p = 2$

c) $f(f - 1) = 18$

d) $2q^2 + 8q = 5$

e) $4x^2 - 5x + 1 = 0$

f) $2x^2 + 4x - 3 = 0$

g) $2m^2 + 12m + 2 = 0$

h) $3x^2 = 5 + 8x$

i) $4x^2 + 8x + 3 = 0$

j) $x(3x + 10) = 77$

ANSWER

a) $x_1 = -3, x_2 = -2$

b) $p_1 = -8.2426, p_2 = 0.2426$

c) $t_1 = 4.7720, t_2 = -3.7720$

d) $q_1 = 4.5495, q_2 = 0.5495$

e) $x_1 = \frac{1}{4}, x_2 = 1$

f) $x_1 = -2.5811, x_2 = 0.5811$

g) $m_1 = -5.8284, m_2 = -0.1716$

h) $x_1 = -5.226, x_2 = 3.1893$

i) $x_1 = -\frac{3}{3}, x_2 = -\frac{1}{3}$

j) $x_1 = -7, x_2 = \frac{11}{3}$

Partial Fraction

∞ Given the algebraic fraction, $\frac{5x-4}{3x^2+2x-7}$, it can often be broken down into simpler parts called **Partial Fraction**.

∞ Partial fractions are commonly used to simplify functions that cannot be integrated into basic fractions.

∞ Partial fraction decomposition is a process which a fraction that the numerator and the denominator are both polynomials is broken down into simpler fraction.

∞ It involves starting with the simplified fraction and decomposing it into its initial polynomial fractions..

∞ If you divide an algebraic expression into multiple rational expressions, each part is referred to as a partial fraction. Therefore, essentially, it is the inverse operation of adding rational expressions.

∞ Just like fractions, a partial fraction consists of a numerator and denominator, with the denominator representing the broken-down portion of a rational function.

Type of Partial Fractions
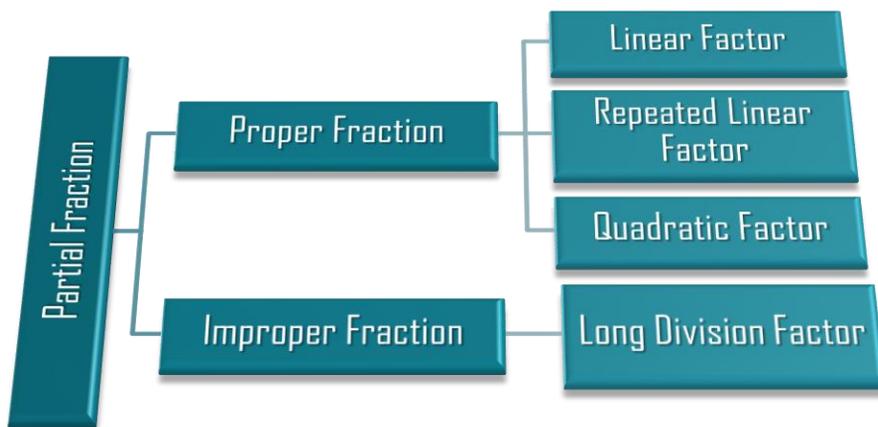
Decomposition of Partial Fractions

Here is the methodical steps to determine the partial fraction decomposition.

Step 1: When breaking down the rational expression into partial fractions, start by considering the proper rational expression.

Step 2: Next, factor the bottom part of the rational expression into linear or irreducible quadratic factors (*Note: Do not factor the denominator using complex numbers*).

Step 3: Record the partial fraction for each factor obtained, using variables A and B in the numerators.

Step 4: It involves multiplying the entire equation by the denominator to determine the values of variables A and B.

Step 5: Find the values of the unknowns by replacing the factor variable with zero.

Step 6: Ultimately, replace the values of A and B in the partial fractions.

Decomposition of Partial Fractions

§ We will now discuss how to break down different type of partial fraction decomposition based on the factor of the denominator into its respective term in partial fraction decomposition.

§ Diagram below shows how to categorize the question given.

Linear Factor	• $\dfrac{2x+5}{x(x-2)} \rightarrow x$ has **NO POWER** • therefore $\dfrac{2x+5}{x(x-2)} = \dfrac{A}{x} + \dfrac{B}{x-2}$
Repeated Linear Factor	• $\dfrac{2x+5}{x(x-2)^2} \rightarrow x$ has **POWER OUTSIDE BRACKET** • therefore $\dfrac{2x+5}{x(x-2)^2} = \dfrac{A}{x} + \dfrac{B}{x-2} + \dfrac{C}{(x-2)^2}$
Quadratic Factor	• $\dfrac{2x+5}{x(x^2-2)} \rightarrow x$ has **POWER INSIDE BRACKET** • therefore $\dfrac{2x+5}{x(x^2-2)} = \dfrac{A}{x} + \dfrac{Bx+C}{x^2-2}$
Improper Fraction	• $\dfrac{x^3-15}{2x^2-5x+10} \rightarrow$ the **POWER** of numerator is **BIGGER OR EQUAL** to the **POWER** of denominator • therefore solve using division method

Partial Fraction

Linear Factor

EXAMPLE 1

Find partial fraction expansion for each of the followings:

Given $\dfrac{x-2}{x(x-3)}$

| Step 1 | Write one partial fraction for each of those factors. | $\dfrac{x-2}{x(x-3)} = \dfrac{A}{x} + \dfrac{B}{(x-3)}$ |

| Step 2 | Multiply through by the denominator so it have no fractions. | $x - 2 = A(x-3) + Bx$ |

Step 3

Now find the constants A & B.

Substituting the roots, or "zeros", of $x(x-3)$ can help.

$x - 3 = 0$
assume $x = 3$
$(3) - 2 = A((3) - 3) + B(3)$
$1 = 0 + B(3)$
$B = \dfrac{1}{3}$

---

assume $x = 0$
$(0) - 2 = A((0) - 3) + B(0)$
$-2 = A(-3) + 0$
$A = \dfrac{2}{3}$

Step 4

Substitute the value of A & B into the algebraic fraction equation.

$\dfrac{x-2}{x(x-3)} = \dfrac{\frac{2}{3}}{x} + \dfrac{\frac{1}{3}}{(x-3)}$

$= \dfrac{2}{3x} + \dfrac{1}{3(x-3)}$

Can also be written as:
$\dfrac{1}{3}\left(\dfrac{2}{x} + \dfrac{1}{(x-3)}\right)$

Partial Fraction

EXAMPLE 2

Find partial fraction expansion for each of the followings:

Given $\dfrac{3x}{(x+3)(x+2)}$

Step 1	Write one partial fraction for each of those factors.	$\dfrac{3x}{(x+3)(x+2)} = \dfrac{A}{(x+3)} + \dfrac{B}{(x+2)}$
Step 2	Multiply through by the denominator so it have no fractions.	$3x = A(x+2) + B(x+3)$
Step 3	Now find the constants A & B. Substituting the roots, or "zeros", of $(x+3)(x+2)$ can help.	$x + 2 = 0$ assume $x = -2$ $3(-2) = A\big((-2)+2\big) + B\big((-2)+3\big)$ $-6 = 0 + B(1)$ $B = -6$ $x + 3 = 0$ assume $x = -3$ $3(-3) = A\big((-3)+2\big) + B\big((-3)+3\big)$ $-9 = A(-1) + 0$ $A = 9$
Step 4	Substitute the value of A & B into the algebraic fraction equation.	$\dfrac{3x}{(x+3)(x+2)} = \dfrac{9}{(x+3)} + \dfrac{-6}{(x+2)}$ $\qquad = \dfrac{9}{(x+3)} - \dfrac{6}{(x+2)}$

Partial Fraction

EXAMPLE 3

Find partial fraction expansion for each of the followings:

Given $\dfrac{5x-4}{x^2-x-2}$

Step 1	Factorize the denominator.	$\dfrac{5x-4}{x^2-x-2} = \dfrac{5x-4}{(x-2)(x+1)}$
Step 2	Write one partial fraction for each of those factors.	$\dfrac{5x-4}{(x-2)(x+1)} = \dfrac{A}{(x-2)} + \dfrac{B}{(x+1)}$
Step 3	Multiply through by the denominator so it have no fractions.	$5x-4 = A(x+1) + B(x-2)$
Step 4	Now find the constants A & B. Substituting the roots, or "zeros", of $(x-2)(x+1)$ can help.	$x+1 = 0$ assume $x = -1$ $5(-1) - 4 = A\big((-1)+1\big) + B\big((-1)-2\big)$ $-9 = 0 + B(-3)$ $B = 3$ <hr>$x-2 = 0$ assume $x = 2$ $5(2) - 4 = A\big((2)+1\big) + B\big((2)-2\big)$ $6 = A(3) + 0$ $A = 2$
Step 5	Substitute the value of A & B into the algebraic fraction equation.	$\dfrac{5x-4}{(x-2)(x+1)} = \dfrac{2}{(x-2)} + \dfrac{3}{(x+1)}$
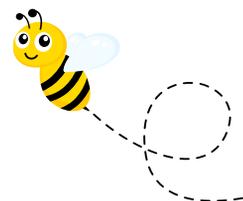
Partial Fraction

MORE EXAMPLE

a) Given $\dfrac{x+4}{x^2-1}$

$$\frac{x+4}{x^2-1} = \frac{x+4}{(x-1)(x+1)}$$

$$\frac{x+4}{(x-1)(x+1)} = \frac{A}{(x-1)} + \frac{B}{(x+1)}$$

$$x+4 = A(x+1) + B(x-1)$$

$x+1=0$

assume $x=-1$

$(-1)+4 = A((-1)+1) + B((-1)-1)$

$3 = 0 + B(-2)$

$B = -\dfrac{3}{2}$

$x-1=0$

assume $x=1$

$(1)+4 = A((1)+1) + B((1)-1)$

$5 = A(2) + 0$

$A = \dfrac{5}{2}$

$$\frac{x+4}{(x-1)(x+1)} = \frac{\frac{5}{2}}{(x-1)} + \frac{-\frac{3}{2}}{(x+1)}$$

$$= \frac{5}{2(x-1)} - \frac{3}{2(x+1)}$$

Can also be written as:
$$\frac{1}{2}\left(\frac{5}{(x-1)} - \frac{3}{(x+1)}\right)$$

b) Given $\dfrac{12x}{(x+1)(2x+3)(x-3)}$

$$\frac{12x}{(x+1)(2x+3)(x-3)} = \frac{A}{(x+1)} + \frac{B}{(2x+3)} + \frac{C}{(x-3)}$$

$$12x = A(2x+3)(x-3) + B(x+1)(x-3) + C(x+1)(2x+3)$$

$x+1 = 0$

assume $x = -1$

$$12(-1) = A(2(-1)+3)((-1)-3) + B((-1)+1)((-1)-3) + C((-1)+1)(2(-1)+3)$$
$$-12 = A(1)(-4) + 0 + 0$$
$$A = 3$$

$x-3 = 0$

assume $x = 3$

$$12(3) = A(2(3)+3)((3)-3) + B((3)+1)((3)-3) + C((3)+1)(2(3)+3)$$
$$36 = 0 + 0 + C(4)(9)$$
$$C = 1$$

$2x+3 = 0$

assume $x = -\dfrac{3}{2}$

$$12\left(-\frac{3}{2}\right) = A\left(2\left(-\frac{3}{2}\right)+3\right)\left(\left(-\frac{3}{2}\right)-3\right) + B\left(\left(-\frac{3}{2}\right)+1\right)\left(\left(-\frac{3}{2}\right)-3\right)$$
$$+ C\left(\left(-\frac{3}{2}\right)+1\right)\left(2\left(-\frac{3}{2}\right)+3\right)$$
$$-18 = 0 + B\left(\frac{1}{3}\right)\left(-\frac{11}{3}\right)$$
$$B = -8$$

$$\frac{12x}{(x+1)(2x+3)(x-3)} = \frac{3}{(x+1)} + \frac{-8}{(2x+3)} + \frac{1}{(x-3)}$$

$$= \frac{3}{(x+1)} - \frac{8}{(2x+3)} + \frac{1}{(x-3)}$$

Simplify the following partial fraction:

a) $\dfrac{x-2}{x(x-3)}$

b) $\dfrac{3x}{(x+2)(x-1)}$

c) $\dfrac{x}{(x+1)(x+2)}$

d) $\dfrac{7x-53}{x^2-2x-15}$

e) $\dfrac{5x}{(2x-1)(x-2)}$

f) $\dfrac{x+4}{x^2-1}$

g) $\dfrac{5x+1}{(x+1)(x-1)}$

h) $\dfrac{125+4x-9x^2}{(x-1)(x+3)(x+4)}$

i) $\dfrac{5x+13}{x^2+4x-5}$

j) $\dfrac{4-3x}{(3-x)(2-3x)}$

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Online Calculator

ANSWER

a) $=\dfrac{\frac{2}{3}}{x}+\dfrac{\frac{1}{3}}{x-3}$

b) $=\dfrac{2}{x+2}+\dfrac{1}{x-1}$

c) $=\dfrac{-1}{x+1}+\dfrac{2}{x+2}$

d) $=\dfrac{-\frac{9}{4}}{x-5}+\dfrac{\frac{37}{4}}{x+3}$

e) $=\dfrac{-\frac{5}{3}}{2x-1}+\dfrac{\frac{10}{3}}{x-2}$

f) $=\dfrac{\frac{5}{3}}{x-1}+\dfrac{-\frac{3}{2}}{x+1}$

g) $=\dfrac{3}{x-1}+\dfrac{2}{x+1}$

h) $=\dfrac{6}{x-1}-\dfrac{8}{x+3}-\dfrac{7}{x+4}$

i) $=\dfrac{3}{x-1}+\dfrac{2}{x+5}$

j) $=\dfrac{\frac{5}{7}}{3-x}+\dfrac{\frac{6}{7}}{2-3x}$

Partial Fraction

Repeated Linear Factor

EXAMPLE 1

Find partial fraction expansion for each of the followings:

Given $\dfrac{3x}{(x+3)^2}$

Step 1	Write one partial fraction for each of those factors.	$\dfrac{3x}{(x+3)^2} = \dfrac{A}{(x+3)} + \dfrac{B}{(x+3)^2}$
Step 2	Multiply through by the denominator so it have no fractions.	$3x = A(x+3) + B$
Step 3	Now find the constants A & B. Substituting the roots, or "zeros", of $(x+3)^2$ can help.	$x + 3 = 0$ assume $x = -3$ $3(-3) = A((-3)+3) + B$ $-9 = 0 + B$ $B = -9$ assume $x = 0$ $3(0) = A((0)+3) + (-9)$ $0 = A(3) - 9$ $A = 3$
Step 4	Substitute the value of A & B into the algebraic fraction equation.	$\dfrac{3x}{(x+3)^2} = \dfrac{3}{(x+3)} + \dfrac{-9}{(x+3)^2}$ $= \dfrac{3}{(x+3)} - \dfrac{9}{(x+3)^2}$

Partial Fraction

EXAMPLE 2

Find partial fraction expansion for each of the followings:

Given $\dfrac{4+2x-x^2}{x(x-2)^2}$

Step 1	Write one partial fraction for each of those factors.	$\dfrac{4+2x-x^2}{x(x-2)^2} = \dfrac{A}{x} + \dfrac{B}{x-2} + \dfrac{C}{(x-2)^2}$
Step 2	Multiply through by the denominator so it have no fractions.	$4+2x-x^2 = A(x-2)^2 + Bx(x-2) + Cx$
Step 3	Now find the constants A, B & C. Substituting the roots, or "zeros", of $x(x-3)^2$ can help.	$x-2=0$ assume $x=2$ $4+2(2)-(2)^2 = A(2-2)^2 + B(2)(2-2) + C(2)$ $4 = 0+0+2C$ $C=2$ assume $x=0$ $4+2(0)-(0)^2 = A(0-2)^2 + B(0)(0-2) + 3(0)$ $4 = A(4)$ $A=1$ assume $x=1$ $4+2(1)-(1)^2 = 1(1-2)^2 + B(1)(1-2) + 2(1)$ $5 = B(-1)+3$ $B=-2$
Step 4	Substitute the value of A & B into the algebraic fraction equation.	$\dfrac{4+2x-x^2}{x(x-2)^2} = \dfrac{1}{x} + \dfrac{-2}{x-2} + \dfrac{2}{(x-2)^2}$ $= \dfrac{1}{x} - \dfrac{2}{x-2} + \dfrac{2}{(x-2)^2}$

BASIC ALGEBRA

EXAMPLE 3

Find partial fraction expansion for each of the followings:

Given $\dfrac{x^2+1}{(x+1)(x-1)^2}$

Step 1 — Write one partial fraction for each of those factors.

$$\frac{x^2+1}{(x+1)(x-1)^2} = \frac{A}{x+1} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$$

Step 2 — Multiply through by the denominator so it have no fractions.

$$x^2+1 = A(x-1)^2 + B(x+1)(x-1) + C(x+1)$$

Step 3 — Now find the constants A, B & C.

Substituting the roots, or "zeros", of $(x+1)(x-1)^2$ can help.

$x-1=0$
assume $x=1$
$(1)^2+1 = A\big((1)-1\big)^2 + B\big((1)+1\big)\big((1)-1\big)$
$\qquad\qquad + C\big((1)+1\big)$
$2 = 0 + 0 + C(2)$
$C = 1$

$x+1=0$
assume $x=-1$
$(-1)^2+1 = A\big((-1)-1\big)^2 + B\big((-1)+1\big)\big((-1)-1\big)$
$\qquad\qquad + C\big((-1)+1\big)$
$2 = A(4) + 0 + 0$
$A = \dfrac{1}{2}$

assume $x=0$
$(0)^2+1 = A\big((0)-1\big)^2 + B\big((0)+1\big)\big((0)-1\big)$
$\qquad\qquad + C\big((0)+1\big)$
$1 = \dfrac{1}{2}(1) + B(1)(-1) + 1(1)$
$-\dfrac{1}{2} = -B$
$B = \dfrac{1}{2}$

Step 4 — Substitute the value of A, B & C into the algebraic fraction equation.

$$\frac{x^2+1}{(x+1)(x-1)^2} = \frac{\frac{1}{2}}{x+1} + \frac{\frac{1}{2}}{x-1} + \frac{1}{(x-1)^2}$$

$$= \frac{1}{2(x+1)} + \frac{1}{2(x-1)} + \frac{1}{(x-1)^2}$$
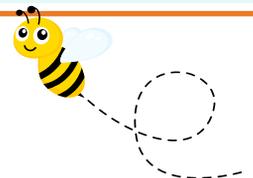
Simplify the following partial fraction:

a) $\dfrac{6x - 11}{(x - 1)^2}$

b) $\dfrac{x^2 - 2}{x^2(2x + 1)}$

c) $\dfrac{4 + 2x - x^2}{x(x - 2)^2}$

d) $\dfrac{x + 1}{x(x + 3)^2}$

e) $\dfrac{x^2 + 6x + 9}{(x - 3)^2(x + 5)}$

f) $\dfrac{9x^2 + 5x - 3}{(x - 2)(x + 1)^2}$

g) $\dfrac{3x + 13}{x^2 + 8x + 16}$

h) $\dfrac{17 + 21x + x^2}{(x - 5)(x + 2)^2}$

i) $\dfrac{x^2}{(x + 1)^2(x - 3)}$

j) $\dfrac{2x^2 + 1}{x^3 + 2x^2 + x}$

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Online Calculator

ANSWER

a) $= \dfrac{6}{x-1} + \dfrac{-5}{(x-1)^2}$

b) $= \dfrac{4}{x} + \dfrac{-2}{x^2} + \dfrac{-7}{2x+1}$

c) $= \dfrac{1}{x} + \dfrac{-2}{x-2} + \dfrac{2}{(x-2)^2}$

d) $= \dfrac{\frac{1}{9}}{x} + \dfrac{-\frac{1}{9}}{x-2} + \dfrac{\frac{2}{3}}{(x-2)^2}$

e) $= \dfrac{\frac{15}{16}}{x-3} + \dfrac{\frac{9}{2}}{(x-3)^2} + \dfrac{\frac{1}{16}}{x+5}$

f) $= \dfrac{\frac{43}{9}}{x} + \dfrac{\frac{38}{9}}{x-2} + \dfrac{-\frac{1}{3}}{(x-2)^2}$

g) $= \dfrac{3}{x+4} + \dfrac{1}{(x+4)^2}$

h) $= \dfrac{3}{x-5} + \dfrac{-2}{x+2} + \dfrac{3}{(x+2)^2}$

i) $= \dfrac{\frac{7}{16}}{x+1} + \dfrac{-\frac{1}{4}}{(x+1)^2} + \dfrac{\frac{9}{16}}{x-3}$

j) $= \dfrac{1}{x} + \dfrac{1}{x+1} + \dfrac{-3}{(x+1)^2}$

Partial Fraction

Quadratic Factor

EXAMPLE 1

Find partial fraction expansion for each of the followings:

Given $\dfrac{x+4}{x(x^2+2)}$

Step 1	Write one partial fraction for each of those factors.	$\dfrac{x+4}{x(x^2+2)} = \dfrac{A}{x} + \dfrac{Bx+C}{x^2+2}$
Step 2	Multiply through by the denominator so it have no fractions.	$x+4 = A(x^2+2) + (Bx+C)x$
Step 3	Distribute, rearrange, and group the coefficients of similar terms.	$x+4 = Ax^2 + 2A + Bx^2 + Cx$ $\qquad = (A+B)x^2 + Cx + 2A$
Step 4	Now find the constants A, B & C. Observe the correspondence of the coefficients on both sides of the equation.	coefficient x^1 $1 = C$ coefficient x^0 $4 = 2A$ $A = 2$ coefficient x^2 $0 = A+B$ $0 = 2+B$ $B = -2$
Step 5	Substitute the value of A, B & C into the algebraic fraction equation.	$\dfrac{x+4}{x(x^2+2)} = \dfrac{2}{x} + \dfrac{-2x+1}{x^2+2}$ $\qquad\qquad = \dfrac{2}{x} + \dfrac{1-2x}{x^2+2}$

Partial Fraction: Quadratic Factor

EXAMPLE 2

Find partial fraction expansion for each of the followings:

Given $\dfrac{5x^2-6x+7}{(x-1)(x^2+1)}$

Step 1	Write one partial fraction for each of those factors.	$\dfrac{5x^2-6x+7}{(x-1)(x^2+1)} = \dfrac{A}{(x-1)} + \dfrac{Bx+C}{x^2+1}$
Step 2	Multiply through by the denominator so it have no fractions.	$5x^2 - 6x + 7$ $= A(x^2+1) + (Bx+C)(x-1)$
Step 3	Distribute, rearrange, and group the coefficients of similar terms.	$5x^2 - 6x + 7$ $= Ax^2 + A + Bx^2 - Bx + Cx - C$ $= (A+B)x^2 + (C-B)x + (A-C)$
Step 4	Now find the constants A, B & C. Observe the correspondence of the coefficients on both sides of the equation.	$x - 1 = 0$ assume $x = 1$ $5(1)^2 - 6(1) + 7$ $= A\big((1)^2 + 1\big) + (B(1) + C)\big((1) - 1\big)$ $6 = A(2) + 0$ $A = 3$ coefficient x^0 $7 = A - C$ $7 = 3 - C$ $C = -4$ coefficient x^2 $5 = A + B$ $5 = 3 + B$ $B = 2$
Step 5	Substitute the value of A, B & C into the algebraic fraction equation.	$\dfrac{5x^2-6x+7}{(x-1)(x^2+1)} = \dfrac{3}{(x-1)} + \dfrac{2x-4}{x^2+1}$

Simplify the following partial fraction:

a) $\dfrac{2x - 1}{x(x^2 + 4)}$

b) $\dfrac{5x^2 - 6x + 7}{(x^2 + 1)(x - 1)}$

c) $\dfrac{4x^2 - x - 8}{(x - 2)(x^2 + 4)}$

d) $\dfrac{x^2 - x + 2}{(x + 1)(x^2 + 3)}$

e) $\dfrac{5x^2 + 3x + 4}{(x + 1)(x^2 + 2)}$

f) $\dfrac{x^2 - 2x + 3}{(x + 1)(x^2 + 5)}$

g) $\dfrac{1}{x(x^2 + 4)}$

h) $\dfrac{2(1 + x)}{x(x^2 + 4)}$

i) $\dfrac{5x^2 - 2x + 1}{(x + 2)(x^2 + 5)}$

j) $\dfrac{3x^2 + 7x - 4}{(x^2 + 4)(x - 3)}$

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Online Calculator

ANSWER

a) $= \dfrac{-\frac{1}{4}}{x} + \dfrac{\frac{1}{4}x+2}{x^2+4}$

b) $= \dfrac{2x-4}{x^2+1} + \dfrac{3}{x-1}$

c) $= \dfrac{\frac{3}{4}}{x-2} + \dfrac{\frac{13}{4}x+\frac{11}{2}}{x^2+4}$

d) $= \dfrac{1}{x+1} + \dfrac{-1}{x^2+3}$

e) $= \dfrac{2}{x+1} + \dfrac{3x}{x^2+2}$

f) $= \dfrac{1}{x+1} + \dfrac{-2}{x^2+5}$

g) $= \dfrac{\frac{1}{4}}{x} + \dfrac{-\frac{1}{4}x}{x^2+4}$

h) $= \dfrac{\frac{1}{2}}{x} + \dfrac{-\frac{1}{2}x+2}{x^2+4}$

i) $= \dfrac{\frac{25}{9}}{x+2} + \dfrac{\frac{20}{9}x+\frac{58}{9}}{x^2+5}$

j) $= \dfrac{-\frac{5}{13}x+\frac{76}{13}}{x^2+4} + \dfrac{\frac{44}{13}}{x-3}$

Partial Fraction

EXAMPLE 1

Given $\dfrac{x^3+2x^2-3}{x^2-3x}$

Step 1 — Solve by using long division

$$\begin{array}{r} x+5 \\ x^2-3x\,\overline{\big)\,x^3+2x^2-3} \\ (-)\ \underline{x^3-3x^2} \\ 5x^2-3 \\ (-)\ \underline{5x^2-15x} \\ 12x-3 \end{array}$$

Step 2 — Write down the new equation

$$\frac{x^3+2x^2-3}{x^2-3x}=x+5+\frac{12x-3}{x^2-3x}$$

Step 3 — Take the fraction expression and write one partial fraction for each of those factors.

$$\frac{12x-3}{x^2-3x}=\frac{15x-3}{x(x-3)}=\frac{A}{x}+\frac{B}{x-3}$$

Step 4 — Multiply through by the bottom so we no longer have fractions.

$$12x-3=A(x-3)+Bx$$

Step 5 — Now find the constants A & B.

Substituting the roots, or "zeros", of $x(x-3)$ can help

$x-3=0$
assume $x=3$
$12(3)-3=A\big((3)-3\big)+B(3)$
$33=3B$
$B=11$

assume $x=0$
$12(0)-3=A\big((0)-3\big)+B(0)$
$-3=A(-3)$
$A=1$

Step 6 — Substitute the value of A & B into the algebraic fraction equation.

$$\therefore\frac{x^3+2x^2-3}{x^2-3x}=x+5+\frac{1}{x}+\frac{11}{x-3}$$

Partial Fraction

EXAMPLE 2

Find partial fraction expansion for each of the followings:

Given $\dfrac{x^2}{x^2+2x-15}$

Step 1	Solve by using long division	$\begin{array}{r} 1 \\ x^2+2x-15 \enclose{longdiv}{x^2 } \\ (-)\ \underline{x^2+2x-15} \\ -2x+15 \end{array}$
Step 2	Write down the new equation	$\dfrac{x^2}{x^2+2x-15} = 1 + \dfrac{15-2x}{x^2+2x-15}$
Step 3	Take the fraction expression and write one partial fraction for each of those factors.	$\dfrac{15-2x}{x^2+2x-15} = \dfrac{15-2x}{(x+5)(x-3)}$ $= \dfrac{A}{x+5} + \dfrac{B}{x-3}$
Step 4	Multiply through by the bottom so we no longer have fractions.	$15-2x = A(x-3) + B(x+5)$
Step 5	Now find the constants A & B. Substituting the roots, or "zeros", of $x(x-3)$ can help	$x-3=0$ assume $x=3$ $15-2(3) = A((3)-3) + B(3+5)$ $9 = B(8)$ $B = \dfrac{9}{8}$ $x+5=0$ assume $x=-5$ $15-2(-5) = A((-5)-3) + B(-5+5)$ $25 = A(-8)$ $A = -\dfrac{25}{8}$
Step 6	Substitute the value of A & B into the algebraic fraction equation.	$\therefore \dfrac{x^2}{x^2+2x-15} = 1 + \dfrac{-\dfrac{25}{8}}{(x+5)} + \dfrac{\dfrac{9}{8}}{x-3}$ $= 1 - \dfrac{25}{8(x+5)} + \dfrac{9}{8(x-3)}$

Simplify the following partial fraction:

a) $\dfrac{6x^3 + x^2 + 5x - 1}{x^3 + x}$

b) $\dfrac{x^2 + 2x + 1}{x^2 + x + 1}$

c) $\dfrac{2x^2 + 5x}{(x + 2)(x - 2)}$

d) $\dfrac{4x^3 + 10x + 4}{x(2x + 1)}$

e) $\dfrac{2x^3 + 3}{x^2 - 1}$

f) $\dfrac{x^3 + 4x^2 - 2x - 5}{x^2 - 4x + 4}$

g) $\dfrac{x^3 - 2x^2 + 3}{x^2 + 5x + 4}$

h) $\dfrac{2x^4 + 3x^2 + 1}{x^2 + 3x + 2}$

i) $\dfrac{2x^3 + 5x^2 - 4x}{x^2 + 2x}$

j) $\dfrac{2x^3 + 3x^2 + 1}{x(2x + 1)}$

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Online Calculator

ANSWER

a) $= 6 - \dfrac{1}{x} + \dfrac{2x-1}{x^2+1}$

b) $= 1 + \dfrac{x}{x^2+x+1}$

c) $= 2 + \dfrac{\frac{1}{2}}{x+2} + \dfrac{\frac{9}{2}}{x-2}$

d) $= 2x - 1 + \dfrac{4}{x} + \dfrac{3}{2x+1}$

e) $= 2x + \dfrac{-\frac{1}{2}}{x+1} + \dfrac{\frac{5}{2}}{x-1}$

f) $= x + 8 + \dfrac{26}{x-2} + \dfrac{15}{(x-2)^2}$

g) $= x - 7 + \dfrac{31}{x+4}$

h) $= 2x^2 - 6x + 17 + \dfrac{6}{x+1} + \dfrac{-45}{x+2}$

i) $= 2x + 1 + \dfrac{-6}{x+2}$

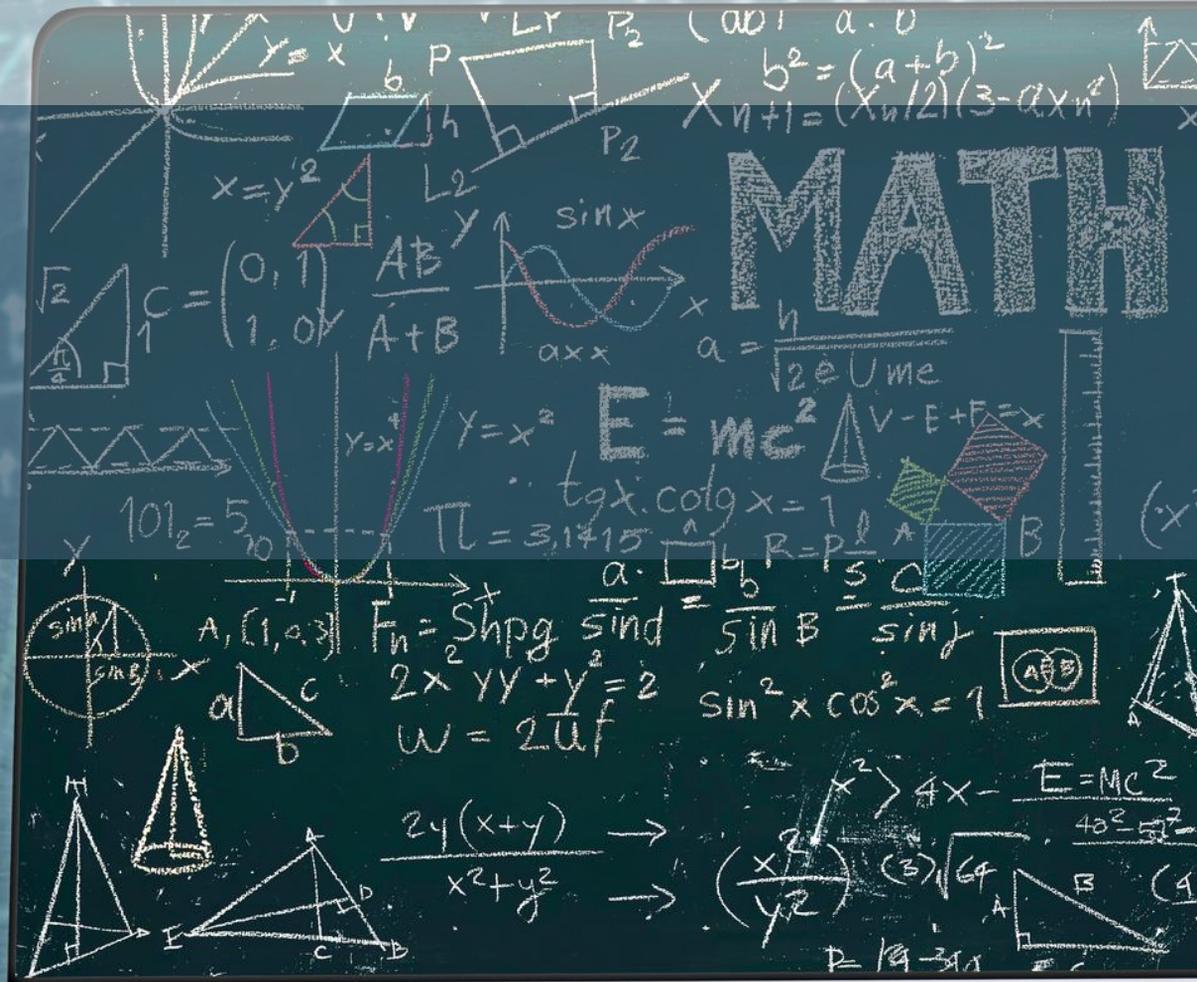j) $= x + 1 + \dfrac{1}{x} + \dfrac{-3}{2x+1}$

Reference

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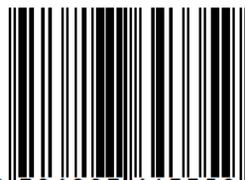Calculus 1 (article). (n.d.). Khan Academy. Retrieved 2021, from https://www.khanacademy.org/math/calculus-1

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