KEMENTERIAN PENDIDIKAN TINGGI

POLITEKNIK MALAYSIA PORT DICKSON

CIVIL ENGINEERING

DCC 2063

MECHANICS OF CIVIL ENGINEERING STRUCTURES

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DCC 2063

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PRAKATA

Buku Modul Mechanics Of Civil Engineering Structures ini ditulis untuk membantu pelajar Politeknik Port Dickson untuk memperolehi pemahaman yang lebih baik tentang kokurikulum Mechanics Of Civil Engineering Structures. Buku ini juga dapat membantu para pensyarah untuk menjalankan proses pengajaran dan pembelajaran kursus Mechanics Of Civil Engineering Structures dengan lebih terancang dan berkesan.

Di dalam setiap topik, contoh – contoh soalan pengukuhan diberikan supaya pelajar lebih memahami konsep dan dapat menguasai topik tersebut. Pada akhir setiap topik, soalan – soalan berkaitan topik diberikan bagi menguji dan menilai pemahaman setiap pelajar mengenai topik yang telah dipelajari.

Semoga dengan adanya buku ini dapat membantu para pelajar serta pensyarah di dalam proses pembelajaran dan pengajaran Mechanics Of Civil Engineering Structures.

Yusra Binti Saion

Siti Rahayu binti Ab Razak

SYLLABUS

UNIT 1	:	INTRODUCTION TO MECHANICS OF STRUCTURES
UNIT 2	:	EQUILIBRIUM FORCES, SHEAR FORCES AND BENDING MOMENT
UNIT 3	:	DIRECT STRESS
UNIT 4	:	BENDING STRESS IN BEAM
UNIT 5	:	SHEAR STRESS
UNIT 6	:	SLOPE AND DEFLECTION OF BEAM DUE TO SYMMETRICAL BENDING

COURSE OUTLINE

Credit(s): 3
LECTURE : 60 hours

SYNOPSIS:

MECHANICS OF STRUCTURES covers knowledge of facts and basic principles of types of forces, strength of materials and behavior of loaded structures. This course provides exposure to loaded structures on direct and shear stresses, slope and deflection. The domination of cognitive domain that brings intellectual force and student thinking from the lower to the higher level which include knowledge level and understanding level to application level in solving problems that involve calculations. In this approach all concepts learned can be correlated with structural behavior in everyday life

PROGRAMME LEARNING OUTCOMES (PLO)

Upon completion of this course, students should be able to:

1. Apply knowledge of mathematics, science and engineering fundamentals to well defined civil engineering theories and procedures.
2. Use necessary skills and technique to conduct experiment and civil engineering practices.
3. Communicate effectively both in written and spoken form with other colleague and community.
4. Provide effective solution to civil engineering problems.
5. Develop social responsibilities and humanistic values.
6. Recognise the need and to engage in, lifelong learning and professional development.
7. Apply entrepreneurship skill for career development.
8. Adhere to professional codes of ethics to adapt in working environment.
9. Demonstrate effective leadership's skills and team work responsibilities.

LEARNING OUTCOMES

Upon completion of this course, students should be able to:-

No.	Course Learning Outcome (CLO)	PLO	GSA, LD
1.	Understand generally the fundamental knowledge and principles of structure mechanics in determinate structure. (C2, PLO1)	PLO1	C2, LD1
2.	Solve problem systematically in determinate structure using analysis correlated with structure behavior. (C3, PLO2)	PLO2	C3, LD4
3.	Demonstrate efficiently continuous learning and information management skill while engaging in independent acquisition of new knowledge. (A3, PLO8)	PLO8	A3, LD6

Learning Domain (LD)

LD1 knowledge
LD2 Practical Skills
LD3 Communication Skills
LD4 Critical Thinking and Problem Solving Skills
LD5 Social Skills and Responsibilities
LD6 Continuous Learning and Information Management Skills
LD7 Management and Entrepreneurial Skills
LD8 Professionalism, Ethics and Moral
LD9 Leadership and Teamwork Skills

General Student Attribute (GSA)

Cognitive Domain
C1 Knowledge
C2 Comprehensive
C3 Application
C4 Analysis
C5 Synthesis
C6 Evaluation

Psychomotor Domain
P1 Imitate
P2 Manipulate
P3 Precision
P4 Articulation
P5 Naturalization

Affective Domain
A1 Receiving Phenomena
A2 Responding to phenomena
A3 Valuing
A4 Organizing Values
A5 Internalizing values

STUDENT LEARNING TIME (SLT)

No.	Learning and Teaching Activity	SLT
FACE TO FACE		
1.0	**Delivery Method**	
1.1	Lecture	45
1.2	Practical	
1.3	Tutorial	15
2.0	**Coursework Assessment (CA)**	
2.1	Lecture-hour-assessment -Test [2] -Quiz [2]	4
2.2	Practical-hour-assessment -None	

2.3	Tutorial-hour-assessment -None	
INDEPENDENT LEARNING		
3.0	**Coursework Assessment (CA)** - Assignment [2]	4
4.0 4.1 4.2 4.3 4.4	**Preparation and Review** - *Lecture* *- Preparation before theory class eg: download lesson notes.* *- Review after theory class eg: additional references, group discussion* *Practical* -None *Tutorial* *- None* Assessment *- Preparation for test.* [2] *- Preparation for final examination.*	45 2 3
FINAL EXAMINATION		2
Total		120
Credit=SLT/40		3

Remarks:
1. 1. Suggested time for
2. Quiz : 10 - 15 minutes
3. Test (Theory) : 30 - 60 minutes
4. 2. 40 hours is equivalent to 1 credit
5. Learning and Teaching Activity
6. Preparation before theory class eg: download lesson notes.
7. Review after theory class eg: additional references, group discussion

TEACHING METHODOLOGY

Lecture
Question & Answer
Demonstration
Self-directed Learning
Presentation
Tutorial
Assignment

GRADING (CONTINOUS ASSESSMENT)

No.	Assessment	Number (minimum)	% total
1	QUIZ - INTRODUCTION TO MECHANICS OF STRUCTURES	2	20

	- DIRECT STRESS		
2	TEST - EQUILIBRIUM FORCES, SHEAR FORCES AND BENDING MOMENT - SHEAR STRESS	2	30
3	ASSIGNMENT - BENDING STRESS IN BEAM - SLOPE AND DEFLECTION OF BEAM DUE TO SYMMETRICAL BENDING	2	50
Overall Total		**6**	**100**

<u>REFERENCES</u>

Main :

- R. C. Hibbeler. 2011. *Mechanics of Materials*. 8th Edition: Pearson Education Inc.

Additional:

- Ferdinand P. Beer,E Russell Johnston Jr. 2004. *Mechanics of Materials*. 3nd Edition: McGraw-Hill Book Company.
- Ferdinand P. Beer, E Russell Johnston Jr., John T. De Wolf . 2002. *Mechanics of Materials.* 3rd Editon: McGraw-Hill International Edition.
- James M. Gere. 2000. *Mechanics of Materials*. 5th Edition: Brooks Cole Thomson Learning.
- James M. Gere, S. P. Timoshenko.. 1997. *Mechanics of Materials*. 4th Edition. New York: PWS Engineering Warsdworth International

SYMBOLS

Abjad Greek

Α	α	Alpha	Ν	ν	Nu
Β	β	Beta	Ξ	ξ	Xi
Γ	γ	Gamma	Ο	ο	Omicron
Δ	δ	Delta	Π	π	Pi
Ε	ε	Epsilon	Ρ	ρ	Rho
Ζ	ζ	Zeta	Σ	σ	Sigma
Η	η	Eta	Τ	τ	Tau
Θ	θ	Theta	Υ	υ	Upsilon
Ι	ι	Iota	Φ	φ	Phi
Κ	κ	Kappa	Χ	χ	Chi
Λ	λ	Lamda	Ψ	ψ	Psi
Μ	μ	Mu	Ω	ω	Omega

List of Symbols

A	luas, pemalar
a,b,c	dimensi, jarak, pemalar
C	sentroid, pemalar
d	Garispusat, dimensi , jarak
E	modulus keanjalan
F	daya
g	pecutan graviti
h	ketinggian, dimensi
I	momen sifat tekun (momen luas kedua)
L	panjang, jarak panjang rentang
M	momen lentur, jisim
m	momen per unit panjang, jisim per unit panjang
n	faktor keselamatan, nombor
P	daya, daya paksi, kuasa
R	tindakbalas, jejari, daya
r	jejari, jarak
s	Jarak, panjang di sepanjang lengkok garisan
t	tebal, masa
v	daya ricih, isipadu
W	berat, kerja
α	sudut, pemalar
β	sudut
γ	terikan ricih, berat tentu
δ	Pesongan, anjakkan, pemanjangan
ε	terikan normal
θ	sudut
μ	pesongan satu rasuk, halaju
υ	nisbah Poisson
τ	tegasan ricih
φ	sudut
σ	Tegasan normal

INTRODUCTION TO MECHANICS OF STRUCTURES

UNIT 1

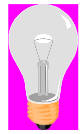
GENERAL OBJECTIVE :

Learn and understand the types of loads or forces and types of supports and their reactions.

SPECIFIC OBJECTIVES :

At end this unit the students should be able to:-

- Define Mechanic of Structure.
- Define types of external and internal forces.
- Deferentiate force of gravity, pressure and reactions.
- Define structure in Civil Engineering.
- Deferentiate types of supports, their reactions and directions.
- Identify number of unknowns at joints and supports.



INPUT 1

1.0 INTRODUCTION

Human body is made up of several number of members and limbs and each member and limb has its own functions *(peranan)*. Each and every member is joined and connected together through the bones. Can you emagine hows a human body without bones? Weak? Soft? It may not be able to stand straight. Refer figure 1.0 (a) & 1.0(b).

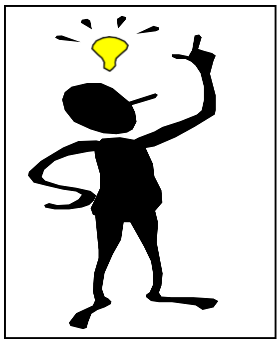


Figure 1.0 (a) :
Human body supported by bones

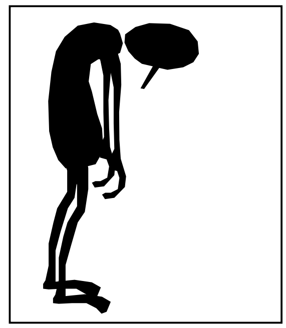


Figure 1.0 (b):
Boneless human body...?

Likewise, it is related to construction in Civil Engineering (e/g: buildings and bridges). A building will not be able to stand rigidly without the support of the structural members (e/g: beams, columns and slabs, etc). Even a construction needs to have a strong structure to support loads and/or forces applied to it. Therefore, analysis should be carried out to determine its size, shape and the most optimum structural material to be used that encompasses economic and safety.

1.1.1 MECHANIC OF STRUCTURE

Definition of 'mechanic' - study on movement of an object (force that causes movement).
Definition of 'structure' - something being formed from various parts and members into a specific shape.

'Mechanic of structure' is related to study or analysis on the structures' characteristic and behaviour when subjected to loads/forces. Rationally, when a mass or an object is loaded, it will experience a change of shape that depand on the magnitude and direction of the load/force. (Refer ffigure 1.1)

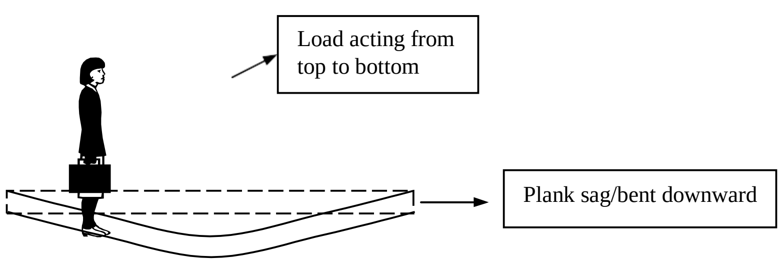


Figure 1.1 : Change of shape of floor structure

Thus, before a structural member can be used, it should be escertained that it is rigid and safe so that it ables to support the applied load throught its live span.

Structural mechanic is very synonym with the load/force. We will see in more detail about this load/force.

1.1.2 STRUCTURE

Definition : Structure is a system of joints of structural members designed to support loads/forces stably and to remains in shape.
Example : Building – combination of several and various structural members such as column, beams, slabs, wall and frameworks (refer ffigure 1.5)

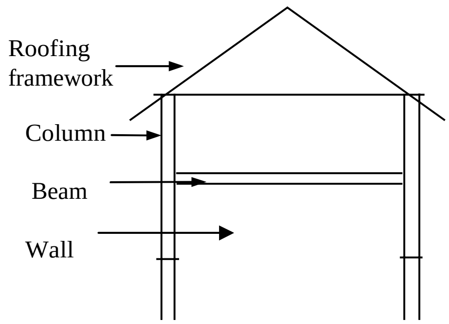


Figure 1.5 : Building Structure

1.1.3 EXTERNAL AND INTERNAL FORCES

> What is a force/load?
> How is a force/load
> acts?

Force is an important factor in science and engineering fields. It is defined as an agent that will cause or inclined to cause, eliminate or inclined to eliminate movement.

a. External Forces

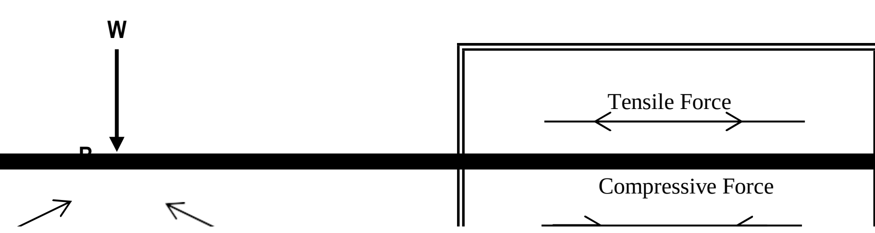
External force is related to load applied to structures. It includes the structure's own weight and the applied external loads. The action of this external load causes a reaction to counter balance the structure. Refering to figure 1.2 (a), a rod AB is subjected to an external force with a magnitude P Newton (as if it is being pulled outwards at both ends).

Figure 1.2 (a): External Force

b. Internal Forces

An internal force is a force that exists in the structural body that counter support the applied load. Internal forces act against the external forces. Refering to figure 1.2(b), a 3-member frame structure AB, BC and AC is supporting an external load of magnitude W Newton at point B. Structural member AB, BC and AC will regenerate internal force that against this external load.

Internal forces act in opposite direction of the external force. Therefore, externals and internals forces are couple of forces reactions and each of these couples can exists as either a **compressive or tensile force**. (Refer figure 1.2(c))



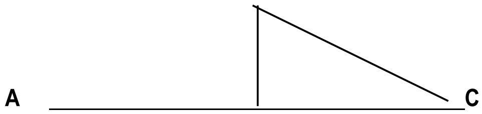


Figure 1.2 (b) : Internal Force **Figure 1.2 (c) : Direction of Reaction Force**

c. Types of External and Internal Forces

Generally, external and internal forces are devided into 4 types, that is :-

(i) Axial force/load
(ii) Shear force
(iii) Bending force
(iv) Torsion/twist

(i) Axial Force/load

Force that acts perpendicular to the cross sectional area of a body.

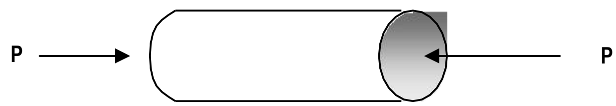


Figure 1.3 (a) : Axial Force / Load

This force acts either toward (compressive) or outward (tensile) of the cross section and respectively will cause deformation or ;

- change of chape or shortening (e/g: column that support load)
- change of shape or expansion (e/g: load supported by spring coil)

(ii) Shear Force

Force that acts parallel to the cross sectional area of a body.

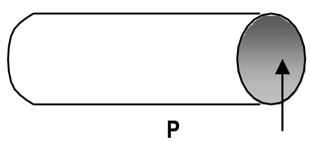


Figure 1.3 (b) : Shear Force

It causes change of shape in shear *(e/g: bolt joint and rivet that bears loading*)

(iii) Bending Force

It is caused by two moments of the same magnitude but acting in opposite directions at ends of a body.

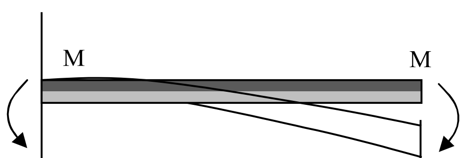
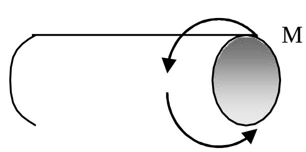


Figure 1.3 (c) : Bending Force

It causes change in shape in bending *(e/g:- beam that carries loads)*

(iv) Torsion/twist *(Daya Putiran/Kilasan)*

Two moments of the same magnitude but of opposite directions at both ends of a body.



It causes rotation (e/g: a rod rotating about its longitudinal axis)

> Key words:
>
> - Action of a pair of external and internal forces causes equilibrium so that the structure is stable.
> - If the internal force is unable to sustain the external force, the structure will experience deformation or change of shape and eventually fails.
> - External forces are real but internal forces are imaginary..

1.1.4 TYPES OF SUPPORTS, REACTIONS AND THEIR DIRECTIONS

There are three (3) types of supports (refer diagram 1.6), they are :

a. Roller
- It has reaction force that acts perpendicular to the plane of the support. They are usually vertical in action. $\Sigma F_y = 0$

b. Pin, also known as joint or hinge
- There are two (2) reaction forces that act horizontally and vertically. $\Sigma F_x = 0$, and $\Sigma F_y = 0$

c. Fixed end or built-in
- It is also known as internal fixture.
- There are 3 reaction forces ie the vertical force, horizontal force and moment. $\Sigma F_x = 0$, $\Sigma F_y = 0$ and $\Sigma M = 0$

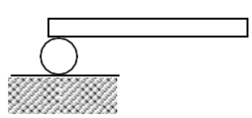
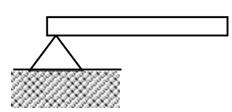
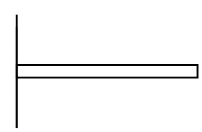
		
Roller	Pin / joint / hinge	Fixed / Built-in

Figure 1.6 : Types of Supports

1.1.4.1 NUMBER OF UNKNOWNS AT JOINTS

Total unknowns is referred to the number of values that are needed to be calculated or determined. Figure 1.7 shows reaction forces and their directions of action as explained in section 1.5 above.

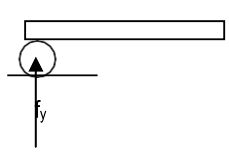
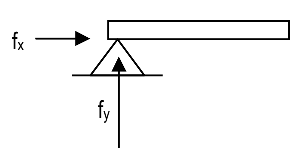
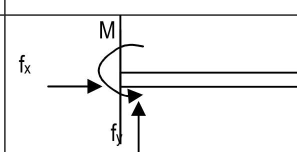
Roller	Pin / joint / hinge	Fixed / Built-in
		
One Uknown	Two Unknowns	Three Unknowns

Figure 1.7: Reaction Forces & Directions of Action

1.1.5 FORCE OF GRAVITY, PRESSURE AND REACTIONS

Force of gravity, pressure and recations are forms of force reaction. They are inter connected in causing equilibrium due to the action of forces.

a. Force of gravity

Figure 1.4 shows a box placed on a horizontal plane. If the box of a mass *m*, and the pull of gravity, *g*; weight of the box is the product of its mass and the pull of gravity.

Force (F) = Mass (m) x Pull of gravity (g)

Or simply

$F = m \times g$

Unit force is $kgms^{-2}$ @ Newton, N

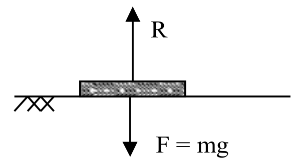


Figure 1.4: Force and Reaction

Pull of gravity = 9.81 ms^2

b. Pressure

If the surface area of contact between the box and the plane (refer figure 1.4) is *A*, hence pressure, *p* experienced by the surface in contact is the ratio of the foce to the area or its the force per unit area;

Pressure (p) = Force (F) / Area (A)

ie, $p = F / A$

Unit for Pressure is Nm^{-2}

c. Reactions

When a mass is exerting a force of *mg* on to the surface in contact, there exists reaction force, *R* of the same magnitude acting on to the mass. Action of this reaction force is in opposite direction to that of the applied force (refer figure 1.4).

Weight or Force = Mass x Gravity
Pressure = Force / Area

$1\ N/m^2 = 1$ Pascal (Pa)
$1\ kN/m^2 = 1$ kPa

These formulae and their units ***should be memorised.***

1.1.6 TYPES OF SUPPORTS

Supports are objects to support or withheld structural members. There are three types of supports that is roller or wheel, pin or hinge and fixed end.

a. Roller

Roller acts like wheels of a vehicle. If it is applied with a force parallel to plane it will move. In order to picture this type of support, imagine two cylindrical steel rods placed horizontally on a flat (refer diagram 2.9 (a)). A beam is placed on top of both rods. When the beam is loaded with bricks, the beam remains in its original position. This shows that the beam is in the state of equilibrium. This state of equilibrium is achieved because the supports exerted reactions against the applied load. (refer figure 2.9(b)).

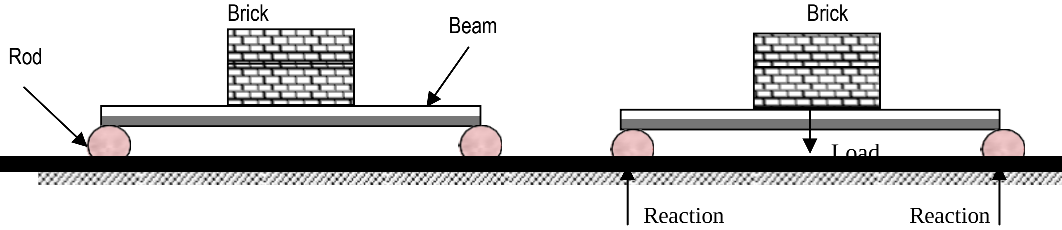
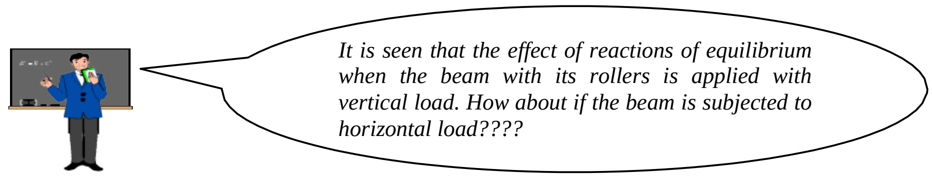
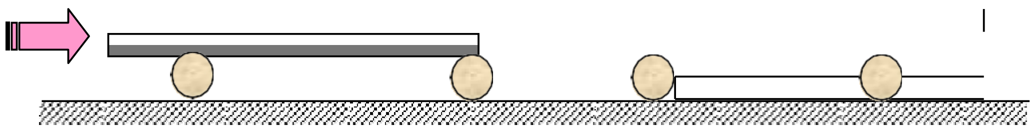


Figure 2.9(a): Beam with Load **Figure 2.9(b): Equilibrium and Reactions**



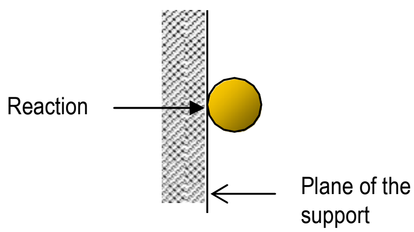
If the beam is applied with horizontal load such as pushing it to the left (refer figure 2.10a). Imagine what will happen. The beam will move to the right and eventually slipped out of the left rod (Figure 2.10b). This shows that there is no equilibrium reaction. Thus, roller supports are able to react in



vertical direction only.

Figure 2.10(a): Beam subjected to horizontal force **Figure 2.10(b) : Beam slipped from the roller**

How about if plane of the supports is not horizontal? In this case, reactions at the supports will always be perpendicular to the plane of the supports.(Diagram 2.11a & 2.11b)



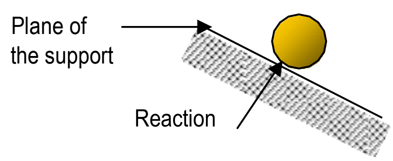


Figure 2.11(a) : Vertical Support Plane **Figure 2.11(b): Inclined Support Plane**

Roller is presented in two symbols as shown in figure 2.12.

Figure 2.12 : Roller

b. Pin (or Hinge)

Pin support is also known as joint or hinge. Imagine a door with the hinge. The door can be open or closed. (Figure 2.13) This shows that it is unable to support moment load as there is no obstruction or reaction of the hinge.

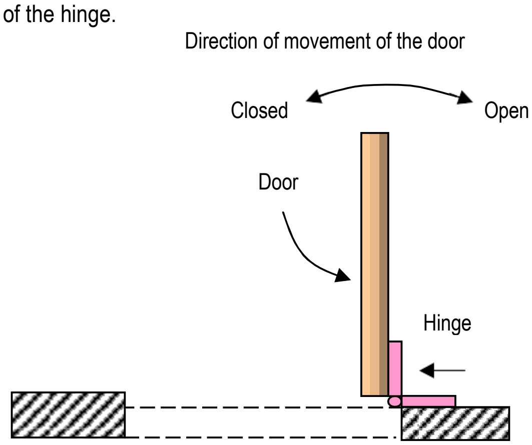


Figure 2.13: Plan View of a Hinged Door

In a beam situation, assumed a beam AB supported by a pin support at end A and a roller at support B (refer figure 2.14). When a vertical load is applied the beam system is still in stable situation because there are reactions in the vertical direction at the supports. Suppose the beam is subjected to horizontal force, the beam still remains stable. This shows that there exists reaction in horizontal direction. Concluding, this shows that pin support provides reactions in two directions that is vertical and horizontal.

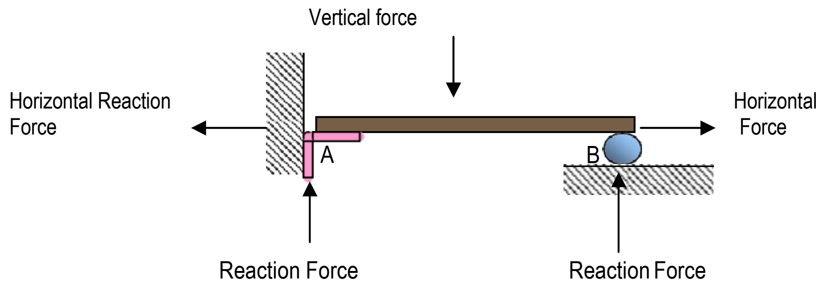


Diagram 2.14: Reactions on Beam with Pin Support

Pin supports may be represented in two forms (Figure 2.15)

Figure 2.15: Pin Support

c. **Fixed End / Built-In**

This type of support may be assumed as a beam projecting outward from a wall construction as in figure 2.16(a). When this beam is applied with a vertical load at point A and a horizontal load at point B, the beam will remain fixed at the supported end. This shows that fixed end support provides in both horizontal and vertical directions.

The vertical load can also cause a moment to the system and will be experienced at the fixed end. This effect is similar to that moment applied at end B. The fixed end does not permit the beam to rotate thus providing moment reaction to the beam. Thus shows such support provides reaction to moment. This concludes that a fixed end support provides three directions of reaction; vertical, horizontal and moment as shown in figure 2.16 (b).

Figure 2.16(a): Fixed End Beam

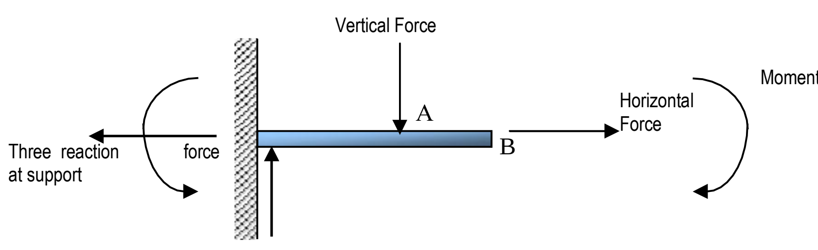


Figure 2.16(b): Fixed End Beam with Reactions

EQUILIBRIUM FORCES, SHEAR FORCES AND BENDING MOMENT

UNIT 2

GENERAL OBJECTIVE :

to understand the relation between forces and reactions that act on various type of beams and supports.

SPECIFIC OBJECTIVES :

At end of this unit, students should be able to :

- Know the principles and concepts of equilibrium forces
- Describe and identify beams
- Understand shear force and bending moment
- Apply the equilibrium concept in shear force and bending moment

2.0 INTRODUCTION

In Unit 1, it has been introduced about forces, supports and reactions. Unit 2 is the continuation of Unit 1. In this unit it will expose further about forces and reactions. The relationship between forces and reaction, supports of beam structure and equilibrium will be explained in detail. Refer figure 2.1(a & b) and try to think its connection with equilibrium

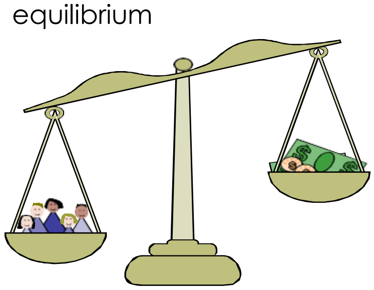


Figure 2.1 (a): Balance

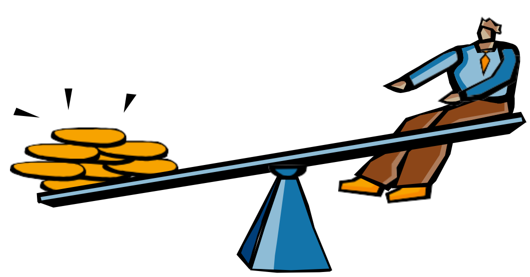
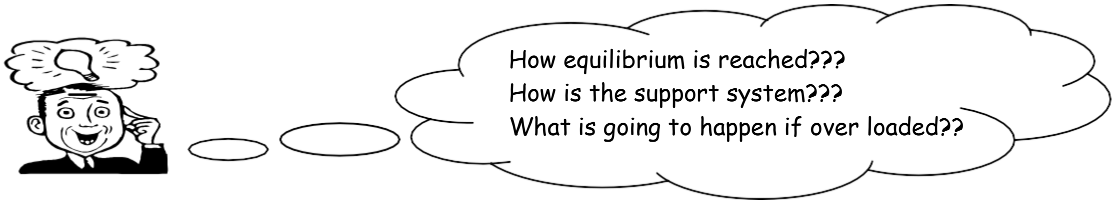


Figure 2.1 (b): See Saw



2.1.1 Reactions and Principle of Equilibrium of Forces

Law of basic static states that if a structural construction is in the state of equilibrium, any member in the said structure is also in equilibrium.

Conditions of static equilibrium.

a. Algebraic sum of force vectors = 0

$\Sigma f_x = 0$ and $\Sigma f_y = 0$

b. Algebraic sum of external and internal moments in the construction = 0

$\Sigma M_A = 0$

> **Principle of static equilibrium**
> - **Sum of forces to the right is equal to sum of forces to the left.**
> - **Sum of forces vertically up is equal to sum of forces vertically down**
> - **Sum of clockwise moments is equal to sum of anticlockwise moments**

2.1.2 TYPES OF LOAD

Loads are classified according to the method of its being distributed on the structure. There are three main types of load distribution. However, normally loads carried by the structure are the combination of various types of these loads.The three types of loads are ;

a. Concentrated or point load
b. Uniformly distributed load
c. Moment

a. Point Load

Point load is also known as concentrated load. It acts on a very small area and can be assumed as acting on a point. It usually carries a symbol of an arrow head with unit in N, kN etc (figure 2.23).

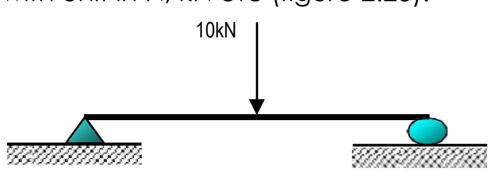


Figure 2.1.2 (a): Point Load

b. Uniformly Distributed Load (UDL)

This load may be assumed as acting and distributed evenly over or part of the beam (figure 2.24) carrying Unit in N/m, kN/m etc.

To simplify the reaction analysis it is usual to consider total load carried and assumed it acting at mid-span of the laod. Example (refer figure 2.24), if the magnitude of the UDL is 15 kN/m and is applied through the entire span of the beam of 4m, total load carried by the beam is 15 x 4 = 60 kN and acting at 2m from the supports.

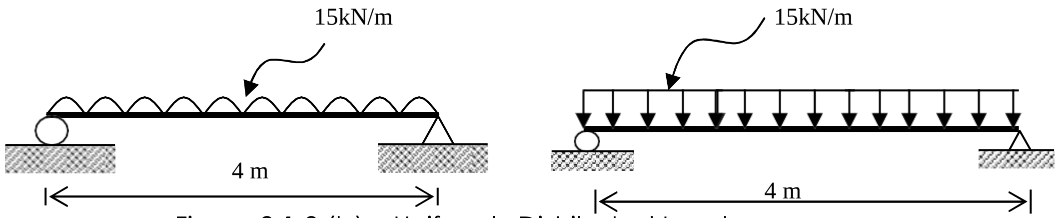


Figure 2.1.2 (b) : Uniformly Distributed Load

c. **Moment**

Moment is generated from a pair of forces. These forces acting from a certain point and it causes turning or twist at that particular point. It acts in a clockwise or anticlockwise direction. If it follows in the clockwise direction it is assumed as positive and conversely an anticlockwise direction will noted as negative. Its unit is in Nm, kNm etc. Moment is rpresented in two forms as shown in figure 2.25 (a & b).

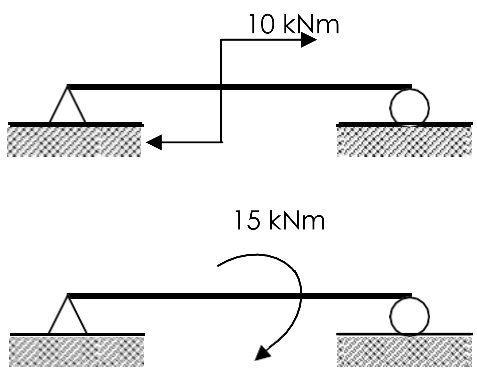


Figure 2..1.2(c.i): Clockwise Moment

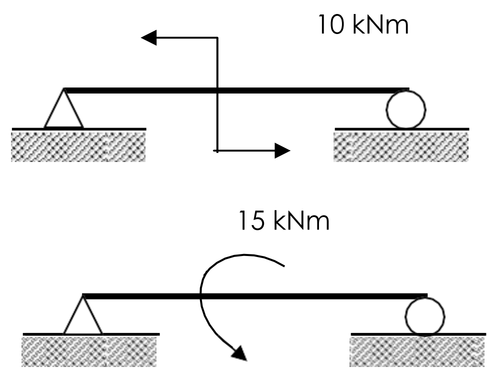


Figure 2.1.2(c.ii) : Anticlockwise Moment

2.1.3 DEFINITION OF BEAM

Beam may be defined as a long and thin member that support loads perpendicular to its longitudinal axis. A beam is usually supported horizontally (refer figure 2.4).

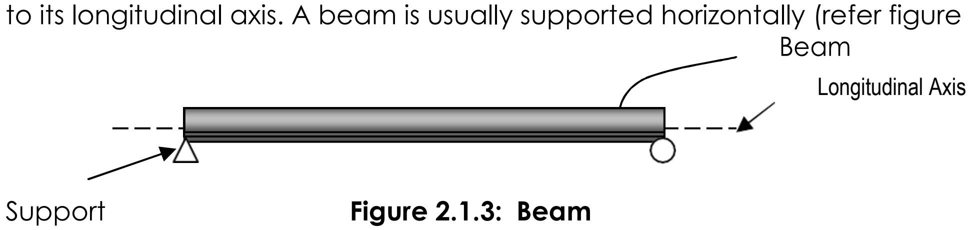


Figure 2.1.3: Beam

2.1.3.1 Types of Beams

A beam is placed on supports and it is categorized based on its position at the supports. Beams may be categorized in 4 categories:

a. Simply supported beams
b. Overhanging beams
c. Cantilever beams
d. Continuous beams

a. Simply supported beam

- It is also known as a simply placed beam

- This beam is simply supported at its both ends either by rollers or pins.(refer figure 2.5)

Figure 2.1.3.1(a): Simply Supported Beam

b. Overhanging Beam

- It is also known as over projected beam.
- One or both ends may be over hang.
- Supports used are rollers or pins or combination of both.
- Three ways of beam may be over hanged (refer figure 2.6).

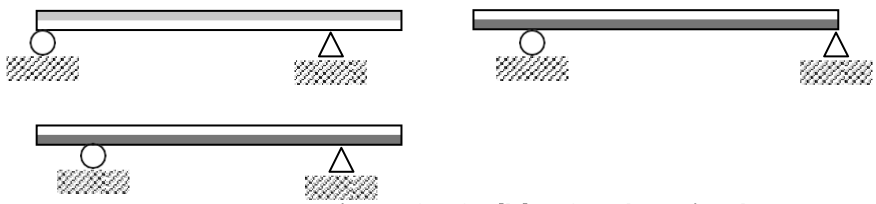


Figure 2.1.3.1(b): Overhanging beam

c. Cantilever Beam

- This beam is supported at one end only
- It used the fixed end as a support with the other end freely unsupported. (refer figure 2.7)

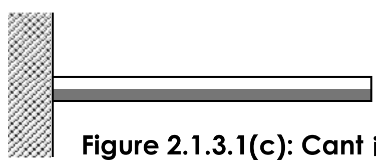


Figure 2.1.3.1(c): Cant ilever Beam

d. Continuous Beam

- It is similar to that of simply supported and over hanging beams but is supported with more than two supports.

- The number of reactions at supports is greater than the number of basic static equations. As such this type of beam is said as in statically indeterminate condition (refer figure 2.8)

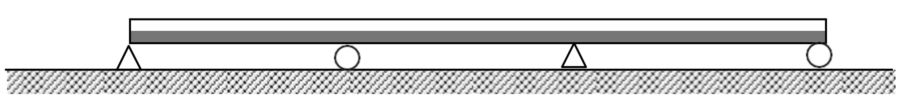


Figure 2.1.3.1(d): Continuous Beam

2.2 SHEAR FORCE AND BENDING MOMENT

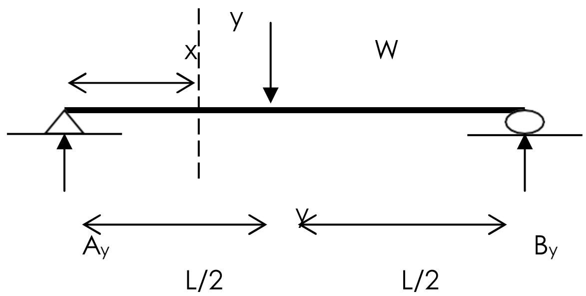


Figure 2.2

Figure 4.1 shows a SSB subjected to a point load, *W* kN at mid-span spanding *L* m. Load W is distributed to support A dan B with a value of *W*/2. If the baem is sectioned at y-y, load *V* and moment, M_X will appear at both section ends. This is because before the beam is sectioned it is already under state of equilibrium.

Both force and moment exists in pairs with the same magnitude but of opposite directions as shown in Figure 4.2. Due to this shear force the baem will be sliced in vertical section and the moment will bend the beam along its longitudinal section.

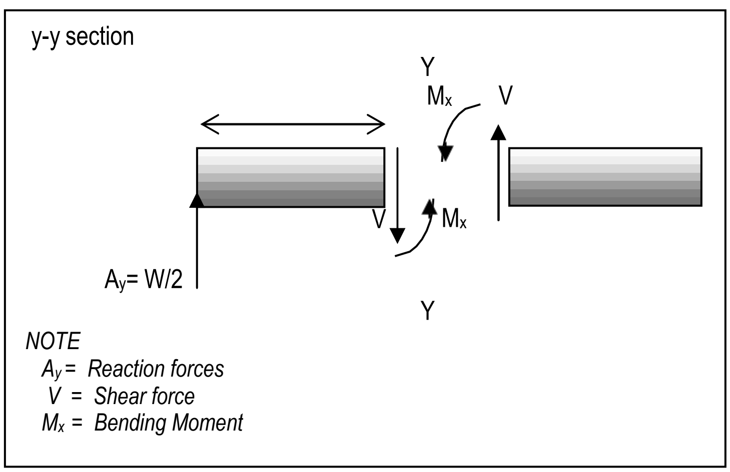


Figure 2.2 (a)

Shear Force
Shear force at any beam section is the algebraic sum (forces pependicular to its longitudinal axis) of all vertical forces acting on the left and right of the beam. This shear force acts vertically (direction of y axis).

Bending Moment
Bending moment at every beam section is defined as the algeraic sum of moments on the left and right of the section. Bending moment is caused by bending.

SIGN CONVENTION

Sign convention means assumption that is being used during analysing a structure.

Shear force

- Magnitude of a compunded shear force on the left at any section is equal to the magnitude of a compounded shear force on the right with opposite direction.
- Positive shear force is taken when the left section slides upward or the section on the right slides downward and vise versa for negative shear forces as shown in Figure 4.3.

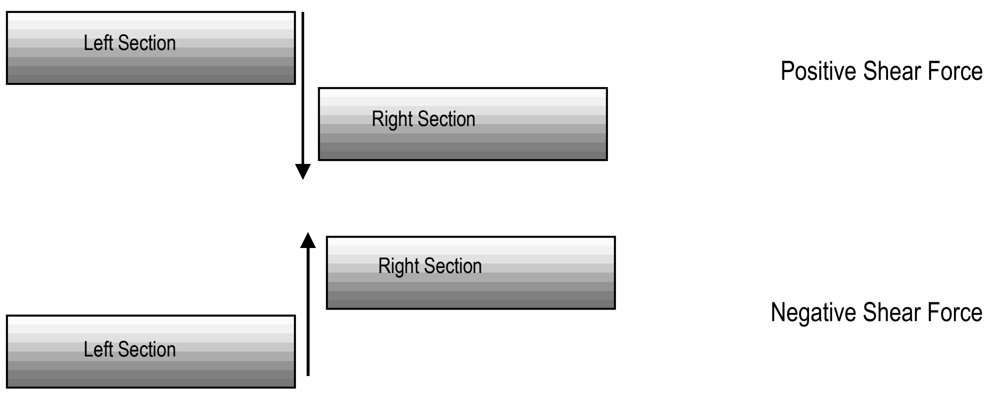


Figure 2.2 (b)

Bending Moment

- Forces acting on the left or right side of the beams section yields a clockwise moment.
- Bending moment taken as positive when the compounded moment on the left side is in clockwise direction and likewise for the negative moment.
- Positive bending moment sags the beam and negative bending moment hogs the beam as shown in Figure 4.4

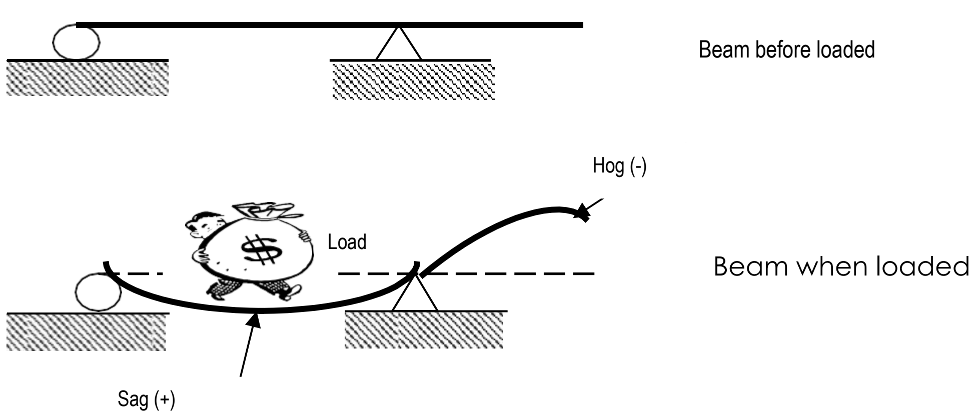


Figure 2.2 (c)

2.2.1 RELATION BETWEEN FORCE AND REACTION

Every object has its mass and the magnitude is different according to its material characteristic. Each object has a specific mass. Unit for mass is kilogram (kg).

Force is the product of mass and pull of gravity. An object with a specific mass is said to have weight if it is influenced by the gravitational pull. An object is said to have no

weight if it is not under the influence of the gravitational pull such as the astronauts on the moon that is always floating in space (in vacuum). Refer figure 2.2.

F = m x g Where *F = the generated force*
m = mass of object
g = gravitational acceleration

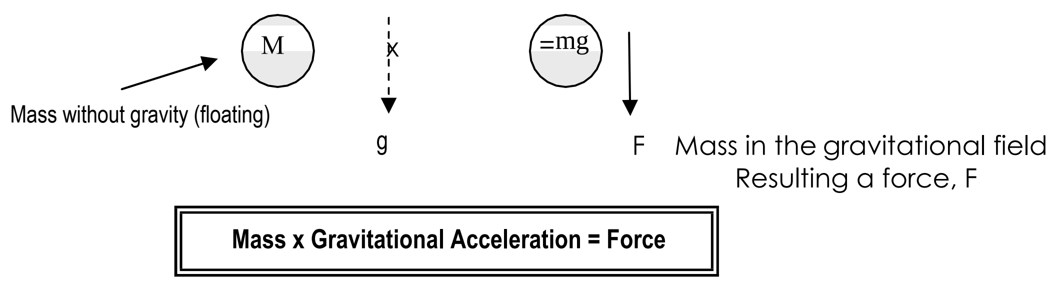


Figure 2.2.1 (a) : Object and Gravitational Influence

Figure 2.3 shows an object placed on a flat surface of a material. The force, ***F*** is caused by the object acting on the surface. This means that the object acts as a load on to the surface.

When this load is applied, there exists an internal force in the material acting against the load with the same magnitude. This force is known as reaction force, ***R***.

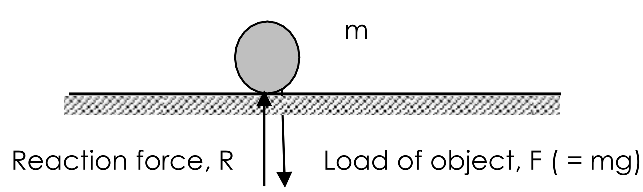


Figure 2.2.1 (b) : Force and Reaction

2.2.2 STATICALLY DETERMINATE AND STATICALLY INDETERMINATE BEAM

These two categories of beams are differentiated in the form of analysis. Briefly, a statically determinate beam has reaction forces not exeeding 3. A statically indeterminate beam has more than three recation forces.

a) Statically Determinate Beam

A structural beam is said to be in the state of static condition when all the unknown reaction forces can be determined using the Basic Static Equations. (Table 2.21)

i.	Algebraic sum of vertical forces = 0,	$\Sigma f_x = 0$
ii.	Algebraic sm of horizontal forces = 0,	$\Sigma f_y = 0$
iii.	Algebraic sum of moments = 0	$\Sigma M = 0$

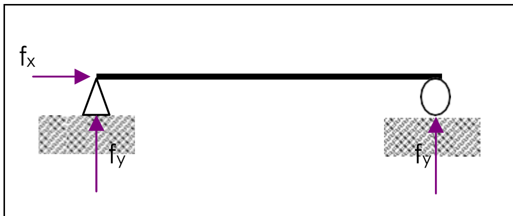
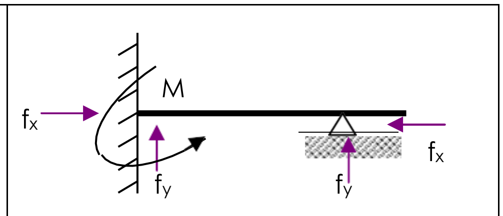
Table 2.2.2 (a) : Basic Static Equilibrium

The determination of the reaction forces may be carried out using the three basic static equations. Figure 2.22(a) shows the shape of a structural beam with the reactions that can be solved from the said equations.

b) Statically Indeterminate Beam

If the total unknown reaction forces and moments are greater than the number of unknowns (3 unknowns) then the beam is known as Statically

Indeterminate Beam. The magnitude of these forces cannot be determined with only using the basic static equations (figure 2.22(b)).

	
Three Unknowns	Five Unknowns

Figureh 2.2.2 (b.i): Statically Determinate Beam

Figure 2.2.2(b.ii): Statically Indeterminate Beam

2.3.1 FREE BODY DIAGRAMS

Before going further on the free body diagram, lets go back to Unit 1, regarding on external and internal forces. It should be clearly understood now of the consept of internal forces in order to be able to understand `free body diagram'.

Consider a loaded beam AB as in figure 3.1(a). If the beam is cut at XX plane and saperated apart as AX and XB, then in order to be under state of equilibruim, there exists internal forces; a force V and a moment M that are acting as shown in the free body digram (figure 3.1(b))

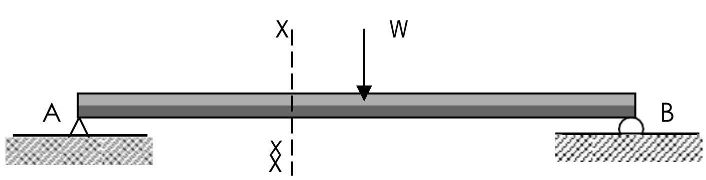


Figure 2.3.1(a) : Beam sectioned at plane XX

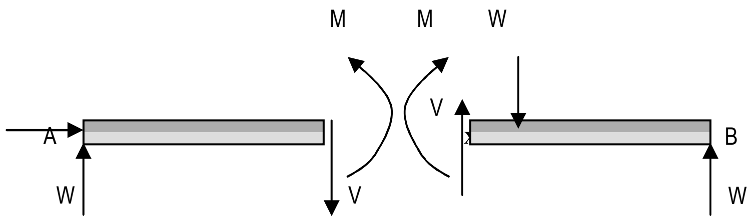


Figure 2.3.1(b) : Free Body Diagram

This vertical force *V* is known as shear force, acting at point *X*. Due to this shear force couple the beam may be chopped at *XX* plane. The moment M is known as the bending moment and also acting at point *X*. This moment causes beam to bend in vertical plane that contains the longitudinal axis of the beam.

2.3.2 Calculate Values and Directions of Vertical Reaction, Horizontal Reaction and Moment

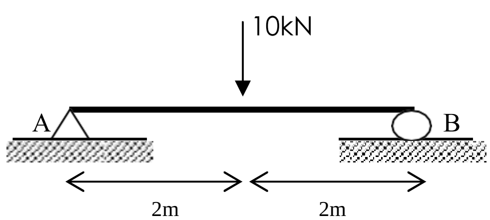
Below are the guide lines simplifying the calculations of the reaction forces :

(i) Determine number of reactions at every support.
(ii) Sketch the beam and assume the direction of the reactions.
(iii) Determine the magnitude of the reactions using the basic static eqquilibrium equations.

Follows are examples of calculation and analysis in determining reactions at supports of a loaded beam. Every example exhibits different types of beam with different load distributions.

a) Simply Supported Beam with A Point Load

Figure 3.2 shows a beam is simply pin-supported at end A and a roller support at end B. Determine the magnitude of the reaction at the supports.



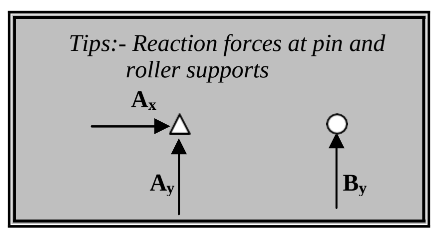


Figure 2.3.2 (a.i): Simply Supported Beam **Figure 2.3.2 (a.ii)**

Solution

Step 1:- Determine the number of reactions at each support

There are two reaction forces at support A and one reaction force at support B. Figure 3.3 shows the force system exists.

Step 2:- Sketch the load and reaction forces positions (refer figure 3.4)

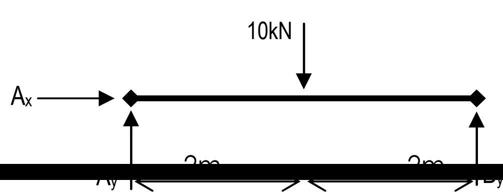


Figure 2.3.2 (a.iii)

Step 3: Determine reaction forces by using the basic static equilibrium equations.

Tips: Basic Static Equation

$\Sigma F_y = 0$
$\Sigma F_x = 0$
$\Sigma M = 0$

Tips : Standard Symbols
- Force to the right positive and force to the left negative
- Forces upward positive and forces downward negative
- Clockwise moment positive and anticlockwise moment negative

$\Sigma f_x = 0$
$\therefore A_x = 0$

$\Sigma f_y = 0$
$A_y + B_y - 10 = 0$
$A_y + B_y = 10$ ……...*(i)*

Tips:- Moment may be taken at point of the support A or B, if moment taken at point A, magnitude of B_y is determined and vice versa.

Take moment at point A:

$\Sigma M_A = 0$ +ve
$10 (2) - B_y (4) = 0$
$B_y = \frac{20}{4}$, $\therefore B_y = 5kN$

From equation (i):-
$A_y + B_y = 10$
$A_y = 10 - 5$ $\therefore A_y =$ 5kN

Tips:- As a check, take moment at point B and determine magnitude of A_y.

Take moment at point B:

$\Sigma M_B = 0$ +ve
$A_y (4) - 10 (2) = 0$
$A_y = 20/4$ $\therefore A_y = 5kN$ (ok!)

b) Simply Supported Beam with Inclined Point Load

Figure 3.5 shows a simply supported beam with a pin at end A and a roller at end B. Determine the magnitude of the reactions at the supports.

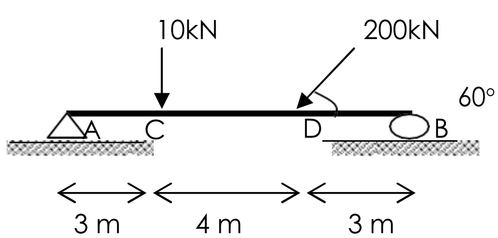


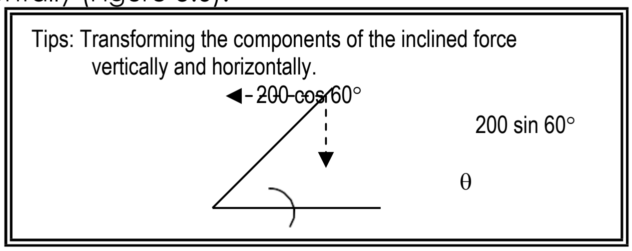
Figure 2.3.2 (b)

Solution

Step 1: Determine the number of reaction forces at each support.

There are two reaction forces at support A and one reaction force at support B. Inclined force should be transform into two components of forces that is vertically and horizontally (Figure 3.6).

Figure 2.3.2 (b.i)



Step 2: Sketch position of the applied forces and the reactions.

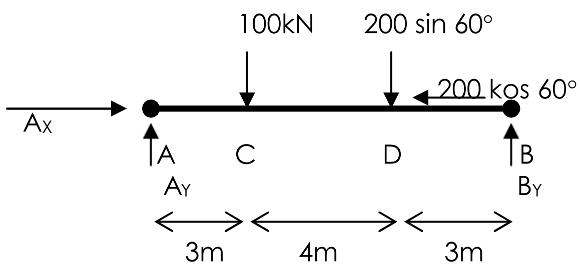


Figure 2.3.2 (b.ii)

Step 3: Determine the reaction forces using basic static equations

$\Sigma f_x = 0$

$A_x - 200 \text{ kos } 60° = 0$

$A_x = 100\text{kN}$

$\Sigma f_y = 0$

$A_y - 100 - 200 \sin 60° + B_y = 0$

$A_y + B_y = 17.32 + 100$

$A_y + B_y = 117.32$ ……...(i)

Moment may be determined from any support A or B. If moment is taken at point A then the magnitude of B_Y will be deduced and vice versa.

Take moment at point B:

$\Sigma M_B = 0$ +ve

$A_Y(10) - 100\,(7) - 200 \sin 60°\,(3) = 0$

$A_Y\,(10) = 873.21$

$A_y = \dfrac{873.21}{10}$, $\therefore$ $A_Y = \underline{87.32\text{kN}}$

From equation (i):

$A_y + B_y = 117.32$

$B_y = 117.32 - 87.32$ $\therefore$ $B_y = \underline{30\text{kN}}$

Now try take moment at point A to determine the magnitude of B_y as a check.

c) Simply Supported Beam with Uniformly Distributed Loading (UDL)

Figure 3.8 shows a simply supported subjected to a uniformly distributed load. Determine the reaction forces at the supports.

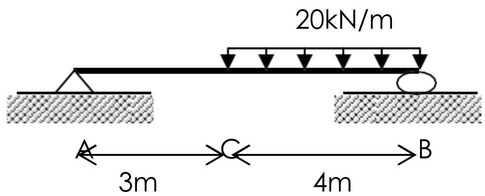


Figure 2.3.2 (c.i)

Solution

> *Tips:*
> - *UDL is given in the form of load per unit length. In order to get its total load, the load per unit length should be multiplied with the length represented. (eg: 20 kN/m x 4m = 80kN)*
> - *For analytical purposes the position of the UDL is assumed at the middle of its span.(eg: 4m ÷ 2 = 2m that is 5mfrom end A or 2mfrom end B)*
> - *Diagram 3.9shows this picture.*

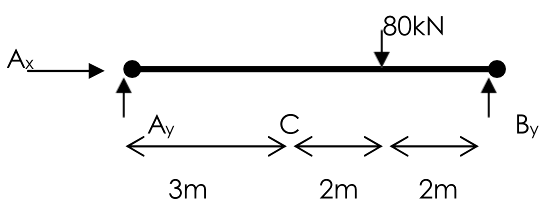


Figure 2.3.2 (c.ii)

$\Sigma f_x = 0$
$\therefore A_x = 0$

$\Sigma f_y = 0$
$A_y - 80 + B_y = 0$
$A_y + B_y = 80kN$ ....*(i)*

$\Sigma M_A = 0$ +ve

$80\left(\frac{4}{2} + 3\right) - B_y(7) = 0$

$B_y = \frac{400}{7}$ $\therefore B_y = \underline{57.14kN}$

From equation (i):

$A_y + B_y = 80$
$A_y = 80 - 57.14$ $\therefore$ $A_y = \underline{22.86kN}$

d) Simply Supported Beam with Uniformly Distributed Load (UDL) and A Moment

Figure 3.10 shows a simply supported beam subjected to a UDL and a moment. Determine the magnitude of their reaction forces.

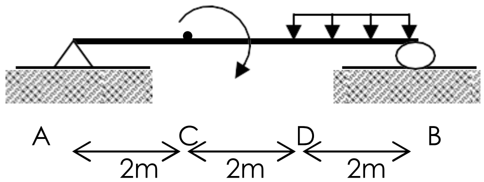


Figure 2.3.2 (d.i)

Solution

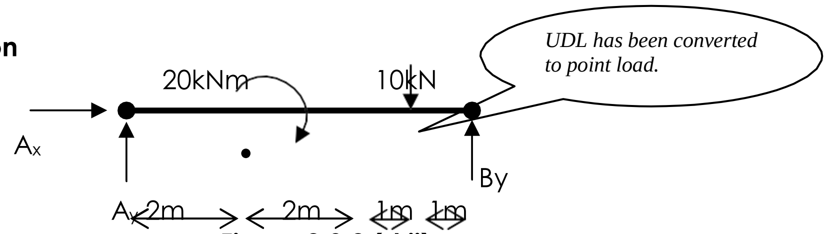


Figure 2.3.2 (d.ii)

$\Sigma f_x = 0 \qquad \therefore A_x = 0$

$\Sigma f_y = 0$

$A_y - 10 + B_y = 0$

$A_y + B_y = 10kN$ ....*(i)*

ΣM_A ↻ $= 0$ + ve

$20 + 5(2)\left(\frac{2}{2} + 4\right) - B_y(6) = 0$

$B_y = \frac{70}{6} = \underline{11.67kN}$

From equation (i):

$A_y + B_y = 80$

$A_y = 10 - 11.67 = \underline{-1.67kN}$ (↓)

Negative value for A_y shows that the actual direction of the reaction is downward.

e) Overhanging Beam with Uniformly Distributed and Point Loadings

Figure 3.16 shows an overhanging beam. Determine the reaction forces at the supports.

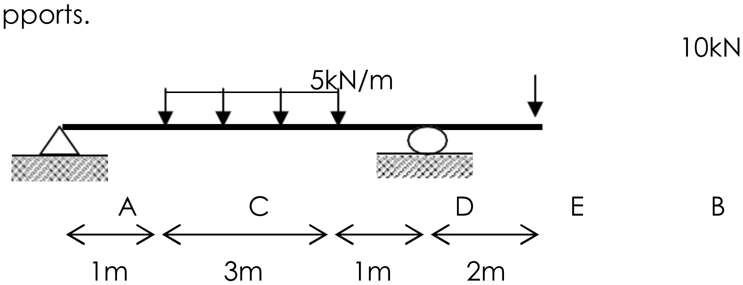


Figure 2.3.2 (e.i)

Solution

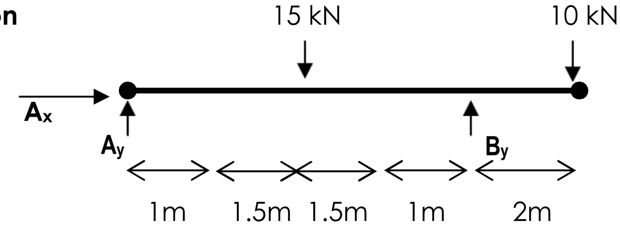


Figure 2.3.2 (e.ii)

> *Tips….*
> *Figure 3.17 is just helping to visualize the positions of the loading, there is no need to change the sketch diagram.*

$\Sigma f_x = 0$ $\quad\quad\quad\quad$ $\therefore A_x = 0$

$\Sigma f_y = 0$

$A_y - 15 + B_y - 10 = 0$

$A_y + B_y = 25kN$ .... *(i)*

Taking moment at B:

$\Sigma M_B = 0$ + ve

$A_y(5) - 15\left(\frac{3}{2} + 1\right) + 10(2) = 0$

$B_y = \frac{17.5}{5}$, $\quad\quad\quad\quad$ $B_y = \underline{3.5kN}$

$A_y + B_y = 25$

$A_y = 25 - 3.5$, $\quad\quad\quad\quad$ $A_y = \underline{21.5kN}$

f) Overhanging Beam with Uniformly Distributed and Point Loads

Figure 3.18 shows a loaded overhanging beam. Determine the reaction forces at the supports.

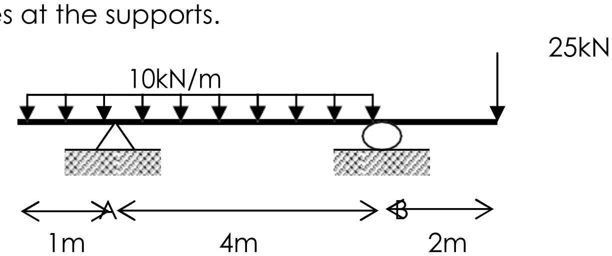
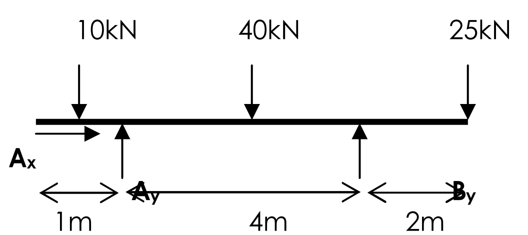


Figure 2.3.2 (f.i)

Solution



> *Tips:*
> *Break the UDL into two components of point loads that will simplifies in the calculation of the reactions.*
>
> - 10kN/m x 1m = 10kN
> - 10kN/m x 4m = 40kN

Figure 2.3.2 (f.ii)

$\Sigma f_x = 0$
$\therefore A_x = 0$

$\Sigma f_y = 0$
$-10 + A_y - 40 + B_y - 25 = 0$
$A_y + B_y = 75kN$ ....*(i)*

Taking moment at point A:

$\Sigma M_A = 0$ ↻ + ve
$-10(1)(0.5) + 40(2) - B_y(4) + 25(6) = 0$
$4B_y = 225$, $B_y = \underline{56.25kN}$

From equation *(i)*

$A_y + B_y = 75$
$A_y = 75 - 56.25 = \underline{18.75kN}$

g) Cantilever Beam with Uniformly Distributed And Inclined Point Load

Diagram 3.20 shows a cantilever beam. Determine the reaction forces at the support.

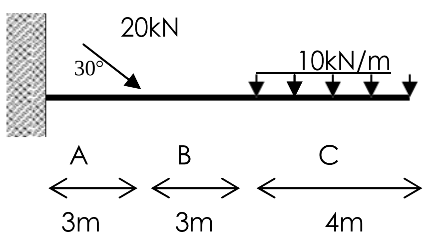
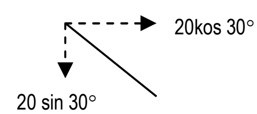


Figure 2.3.2 (g.i)

- Cantilever beam has three unknowns:
- Horizontal and vertical forces and a moment.

Tips: Converting inclined load into two components



Solution

$\Sigma f_x = 0$
$A_x + 20 \text{ kos } 30° = 0$
$A_x = -17.32kN$ (←)

$\Sigma f_y = 0.$
$A_y - 20 \sin 30° - 10(4) = 0$
$A_y = 50$ kN ....*(i)*

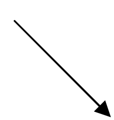


Figure 2.3.2 (g.ii)

Taking moment at point A:

$\Sigma M_A = 0$
$-M_A + 20 \text{ Sin } 30° (3) + 40 (8) = 0$
$M_A = 350kNm$

h) Cantilever Beam with Uniformly Distributed and A Point Load

Figure 3.21 shows a cantilever beam with uniformly distributed and a point loads. Determine the reaction forces at the support.

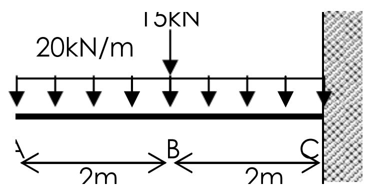


Figure 2.3.2 (h.i)

Solution:

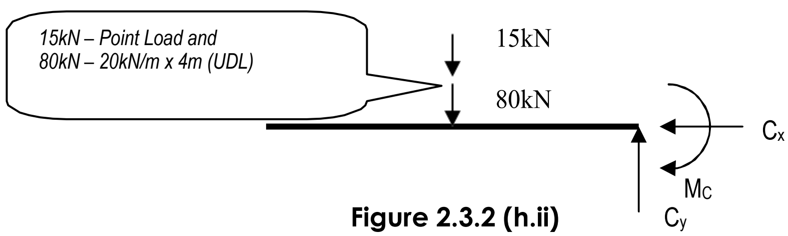


Figure 2.3.2 (h.ii)

$\Sigma f_x = 0$

$\therefore C_x = 0$

$\Sigma f_y = 0$

$-80 - 15 + C_y = 0$

$C_y = 95kN$

Taking moment at point C:

$\Sigma M_C = 0$ + ve

$-80(2) - 15(2) + M_C = 0$

$M_C = 190kNm$

2.3.3 Calculate Shear Force and Bending Moment in Beam

2.3.4 Draw Shear Force and Bending Moment Diagrams for Statically Determinate Bemas

Magnitude of shear and bending moment usually changes at any section of the beam. Changes of the shear and bending moment through the entire length of the beam may be explored much clearly through a Shear Force Diagram (SHD) and bending moment Diagram (BMD).

From SHD and BMD, maximum shear force and bending moment and their positions of action may be determined. These magnitudes and positions are important in structural designs.

i) Tips in Drawing Shear Force Diagram (SFD)

- The calculation of forces begins from the left to the right of a simply supported, centerliver and overhanging beam.
- Forces ating upward as positive and that downward as negative.
- Add or substract magnitude of the force according to vertically upward or downward directions.

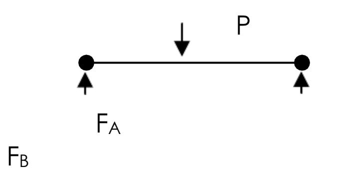
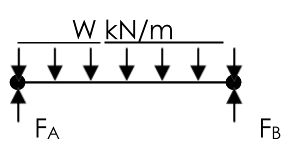
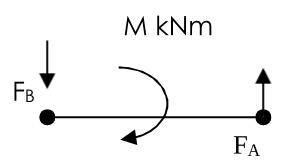
- When there exists UDL and point load at the same point of action, there exists two terms representing the loads at that point (eg F_B and $F_{B'}$) . First take the UDL and then followed by the point load.
- Sum of the shear forces at both ends of a SSB equals to zero.
- Every load acting on the beam (inclusive of their reactions) should be taken in the calculation.
- Draw SHD by connecting values obtained in the calculation.
- UDL produces an inclined straight line on the SHD.
- Point load produces vertical line on the SFD.
- Moment didnot make any changes to the SFD.

ii) Tips in Drawing Bending Moment Diagram (BMD)

- Calulation begins from the left for a SSB and overhanged beam.
- For a centilever beam, the calculation starts from the free end.
- Calculation of moment is taken from point to another saperately.

- When there exists moment at any spesific point then there exists two terms at that particular point (eg. M_B and $M_{B'}$). For M_B the calculation doesnot include magnitude of moment at that point. Please refer Figure 4.35.
- Sum of moment at both ends equals to zero.
- Point load produces inclined or sloping line.
- UDL produces a bending or parabolic lines.
- Moments result in vertical lines.
- Maximum moment may occur if there exists from SHD shear force line crossing the x-axis = 0, at that point happens maximum moment at BMD.

Table 2.3.4 (ii) : Shape of SFD and BMD of a SSB according to types of Loadding

Type of Load	Simply Supported Beam	SHD	BMD
Point		+ve -ve	+ve
UDL		+ve -ve	+ve
Moment		-ve	+ve -ve

iii) Maximum Bending Moment

For designing purposes, magnitude maximum of bending moment value is used. Bagi tujuan rekabentuk nilai momen lentur yang digunakan adalah nilai maksima. Magnitude of maximum bending moment is obtained firstly by determining its position from the SHD. Shear force line that cuts x-axis = 0 is the clue, then using it to calculate the moment at that point.

iv) Steps in drawing SFD and BMD

1. Calculate reaction forces at supports.
2. Calculate value of shear force at spesific points.
3. Sketch SHD
4. Calculate value of bending moment at spesific points.
5. Sketch BMD.

a) SSB with Point Load

Figure 4.6 shows a SSB subjected to a point load of 10kN. Determine value of shear force and bending moiment and hence sketch their SHD and BMD.

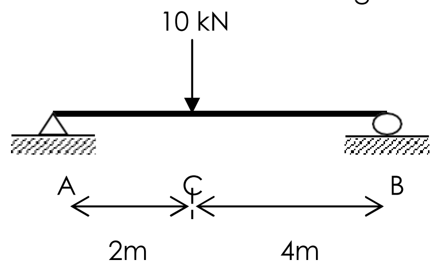


Figure 2.3.4 (a.i)

Step 1

Determine reactions at supports, refer Figure 2.3.4 (iv.a)

Sum of horizontal forces = 0
$\Sigma f_x = 0$
$A_x = 0$

Sum of vertical forces = 0
$\Sigma f_y = 0$
$A_y + B_y = 10$

Sum of moment = 0
$\Sigma M_A = 0$
$10 (2) – B_y (6) = 0$

$B_y = 20 / 6$

$B_y = 3.33$ kN

$A_y = 10 – 3.33$
$= 6.67$ kN

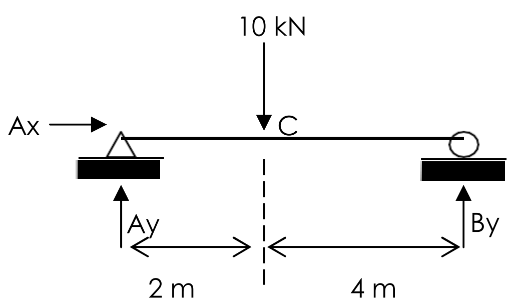


Figure 2.3.4 (a.ii)

Step 2

Determining value of shear force by observing every section point from left to right of the beam.

i. At point A – there is a force $A_y = 6.67$ kN acting upward, this value is positive.
Hence it is written as $F_A = 6.67$ kN

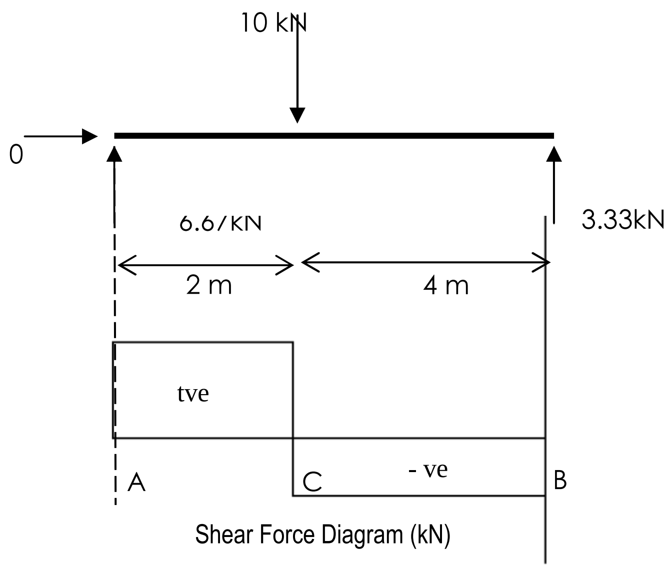
ii. At point C – there is a force 10 kN acting downward, hence this value is negative.
It is written as $F_C = 6.67 – 10$
$F_C = - 3.33$ kN

iii. At point B - there is a force $B_y = 3.33$ kN acting upward and it is positive.
It is written as $F_B = -3.33 + 3.33$
$F_B = 0$ kN

Step 3

Drawing/sketching shear force diagram. (Refer Figure 4.8)

✓ Mark navules of the shear force.
✓ Connect these marks/points.



Guide :
- Value of shear force vertically upward is positive and vertically downward negative

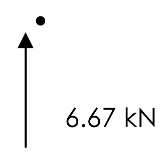
Figure 2.3.4 (a.iii)

Step 4

Determine value of bending moment by observing forces at left section to right section of beam.

Note: *Clockwise moment positive and anticlockwise moment negative.*

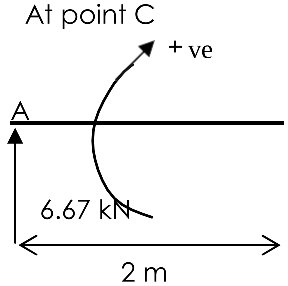
i. At point A



$M_A = 0$ kNm

Figure 2.3.4 (a.iv)

ii. At point C

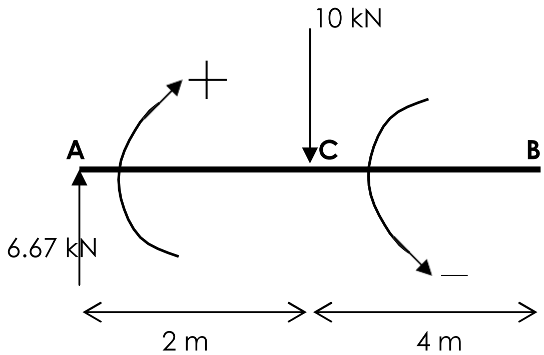


$M_C = 6.67 \times 2$
$M_C = 13.34$ kNm

Figure 2.3.4 (a.v)

M_C is positive because it is in clockwise direction

iii. At point B

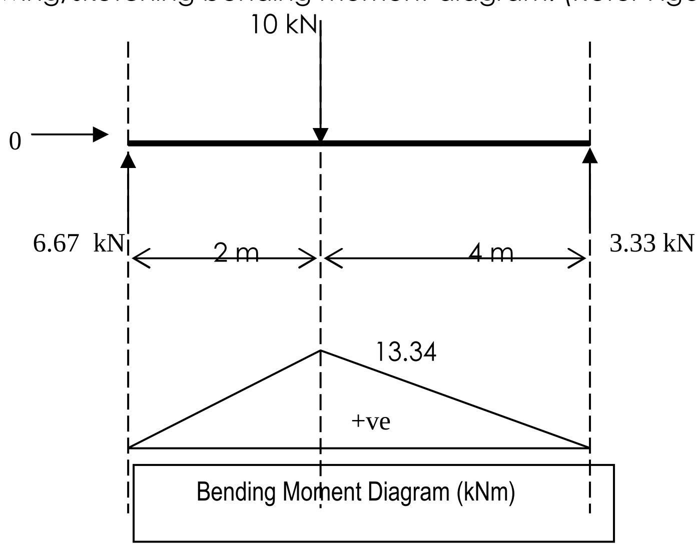


$$M_B = 6.67\,(6) - 10\,(4)$$
$$= 40 - 40$$
$$= 0$$

Figure 2.3.4 (a.vi)

Step 5

Drawing/sketching bending moment diagram. (Refer Figure 4.10)



Guide :

- *Positive moment is placed on top of x-axis and belaw it is the negative moment*

Figure 2.3.4 (a.vii)

b) Simply Supported Beam With Point Load and inclined Load

Sketch the shear force and bending moment diagrams for beam loaded as in Figure 4.12 below.

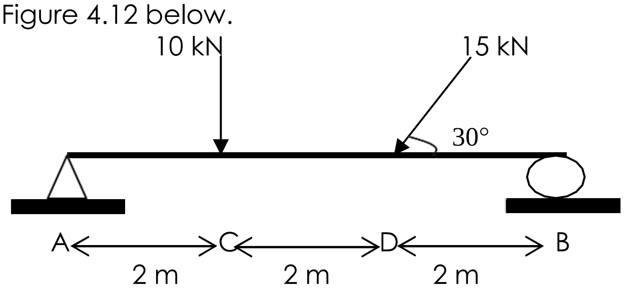


Figure 2.3.4 (b.i)

Solution

Analysis

The above question, the beam supports two point loads, first is 10kN vertically downward and the other is 15 kN inclined at an angle of 30^0. The components of the inclined load are acting in two directions ie vertical and horizontal. The value of these component forces should be determined first before calculating reaction forces at the supports, refer Figure 2.3.4 (b.ii)

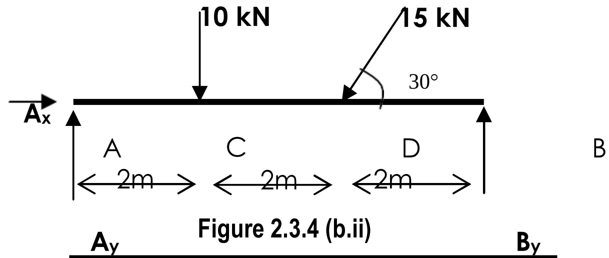


Figure 2.3.4 (b.ii)

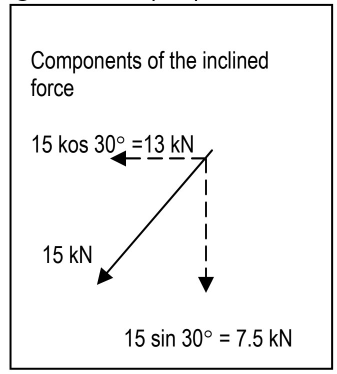


Figure 2.3.4 (b.iii)

Step 1

Reaction forces at supports.

Sum of horizontal forces = 0

$\Sigma f_x = 0$

$A_x - 13 = 0$

$A_x = 13$ kN

Sum of vertical forces = 0

$\Sigma f_y = 0$

$A_y + B_y = 10 + 7.5$

$A_y + B_y = 17.5$ kN

Sum of moment = 0

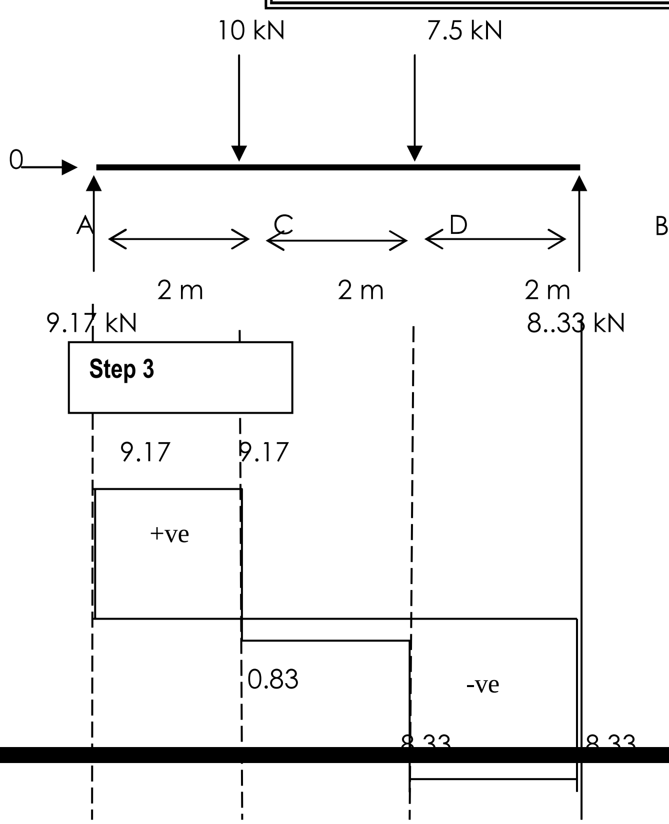
$\Sigma M_A = 0$

$10 (2) + 7.5 (4) - B_y (6) = 0$

$B_y = 50 / 6$

$B_y = 8.33$ kN

$A_y = 25 – 8.33$



Step 2

Magnitude of shear force (kN)

$F_A = 9.17$ kN

$F_C = 9.17 – 10$

$= -0.83$ kN

$F_D = -0.83 – 7.5$

$= -8.33$ kN

$F_B = -8.33 + 8.33$

$= 0$

Shear Force Diagram (kN)

Step 4

Magnitude of bending moment (kNm)

$M_A = 0$
$M_C = 9.17\ (2) = 18.34$

$M_D = 9.17\ (4) – 10\ (2)$
$= 16.68$

$M_B = 9.17\ (6) – 10\ (4) – 7.5\ (2)$
$= 0$

Step 5

18.34 16.68

+ve

Bending Moment Diagram (kNm)

Figure 2.3.4 (b.iv)

c) Simply Supported Beam With Uniformly Distributed Load (UDL)

Figure 4.27 shows a beam is simply supported is carrying a UDL through its span. Sketch the SHD and BMD of the beam.

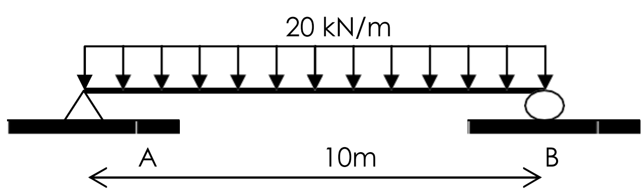
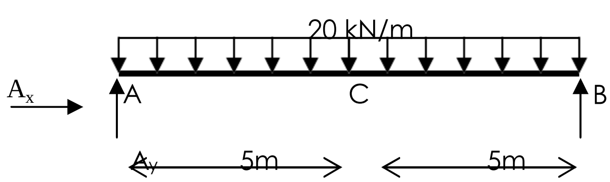


Figure 2.3.4 (c.i)

Solution

Step 1- determine their reaction forces.



$A_x = 0$ kN
$A_y = 100$ kN
$B_y = 100$ kN

Figure 2.3.4 (c.ii)

Step 2 – determine value of shear force at points.

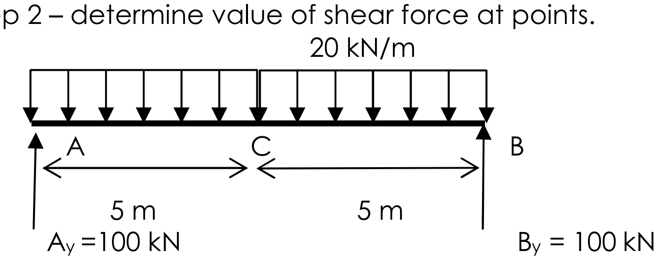


Figure 2.3.4 (c.iii)

F_A= 100 kN

F_C= 100 - 20(5) = 0

Sum of forces at point C is UDL multiplied by distance. It acts downward with negative value.

Value of shear force, F_A= 100 kN

F_B = – 20 (10) = - 100

UDL multiply distance C to B (10 m). It acts downward with negative value.

$F_{B'}$ = -100 + 100 = 0

Force B_Y acting upwards hence positive.

Magnitude of shear force (kN)
F_A = 100
F_C = 100 – 20 (5) = 0
F_B = 0 – 20 (10) = – 100
$F_{B'}$ = – 100 + 100 kN = 0

Step 3 – Sketch shear force diagram

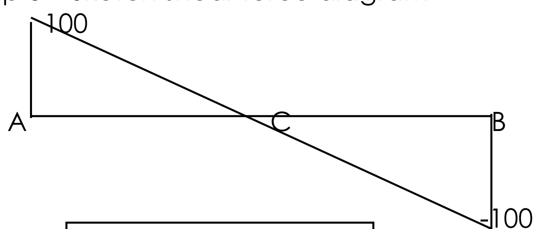
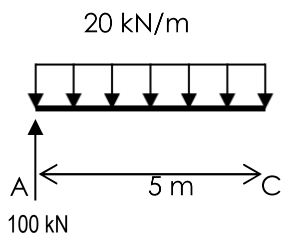


Figure 2.3.4 (c.iv)

Shear Force Diagram (kN)

Step 4 - determination of bending moment values



M_C = 100 (5) – 20 (5) ($\frac{5}{2}$) = 250 kNm

Moment = force x distance

UDL x distance

Negative – Moment in anticlockwise direction

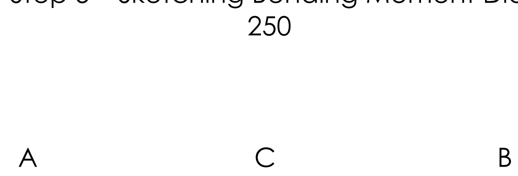
Figure 2.3.4 (c.v)

Note : UDL is changed to total load and acting at mid-span of beam.

Value of bending moment (kNm)
M_A = 0
M_C = 100 (5) – 20 (5) ($\frac{5}{2}$) = 250
M_B = 100 (10) – 20 (10)($\frac{10}{2}$) = 0

> Tips – Bending moment diagram
> *On the Shear Force Diagram the position of changing point that is when line cuts the x-axis shows that maximum bending moment occurs on the Bending Moment Diagram.*

Step 5 – Sketching Bending Moment Diagram



Bending Moment Diagram (kNm)

Figure 2.3.4 (c.vi)

> Tips
> **At the bending moment diagram, for a uniformly distributedload, bending moment diagram will produce a curving bending moment diagram . Therefore from point A to B is acurved line.**

d) Simply Supported Beam With Point Load and Moment

Sketch the shear force and bending moment diagrams for the following beam shown in Figure 4.15.

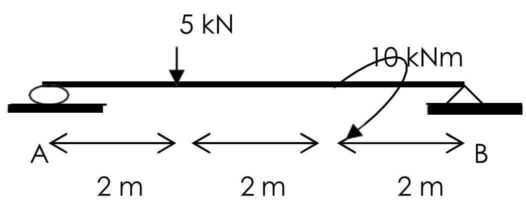
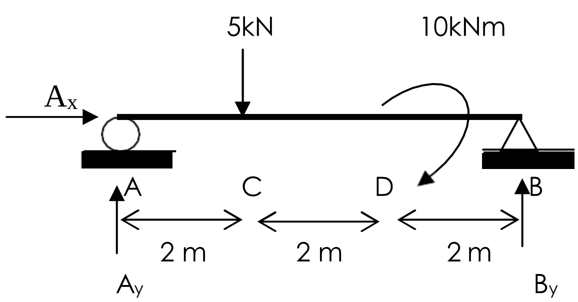


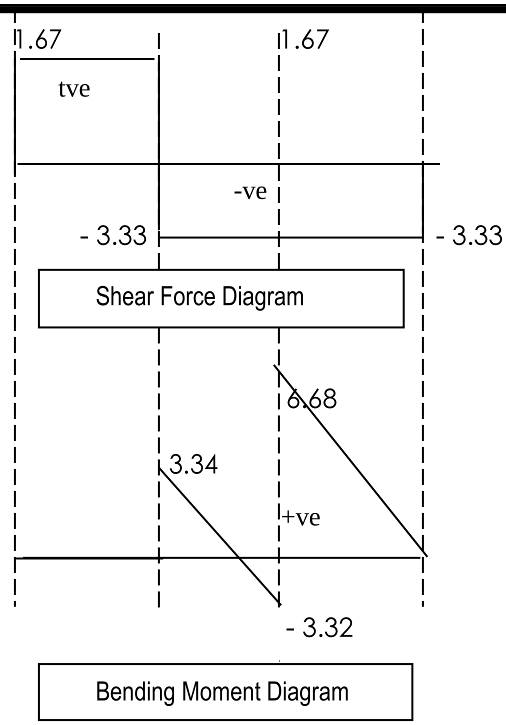
Figure 2.3.4 (d.i)

Solution
First step is neccesory to name all points on the beam where their distances are identified.

Then follow the steps that we have learned that is:
1. Calculate reactions at supports
2. Calculate value of shear forces
3. Sketch the shear force diagram
4. Calculate value of bending moments
5. Sketch bending moment diagram.



> Reaction forces (kN)
>
> $A_x = 0$
> $A_y = 1.67$
> $B_y = 3.33$



Magnitude of shear force (kN)

F_A = 1.67
F_C = 1.67 – 5 = - 3.33
F_D = - 3.33 – 0 = - 3.33
F_B = - 3.33 + 3.33 = 0

Magnitude of bending moment (kNm)

M_A = 0
M_C = 1.67(2) = 3.34
M_D = 1.67(4) – 5(2) = -3.32
$M_{D'}$ = -3.32 + 10 = 6.68
M_B = 1.67(6) – 5(4) + 10
= 0.02 = 0

Figure 2.3.4 (d.ii)

e) OVERHANGING BEAM WITH POINT LOAD

From figure 5.19 sketch the shear force and bending moment diagram,

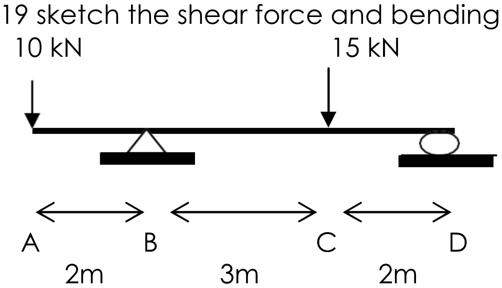
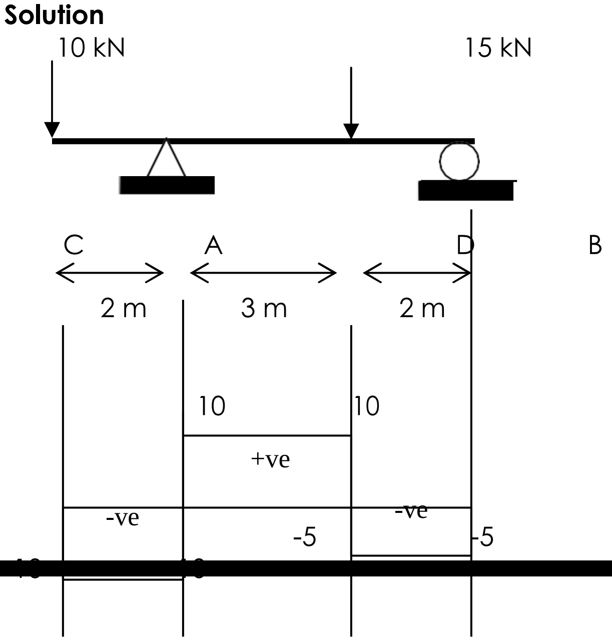


Figure 2.3.4 (e.i)

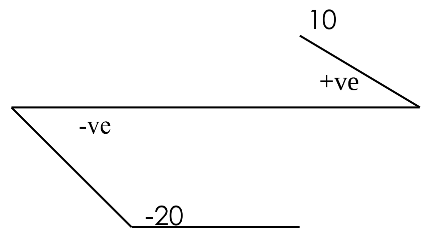
Solution



Reaction forces
A_X = 0 kN
A_Y = 20 kN
B_Y = 5 kN

Magnitude of shear force (kN)
F_C = -10
F_{CA} = -10
F_A = -10 + 20 = 10
F_D = 10 –15 = -5
F_{DB} = -5
F_B = -5 +5 = 0

Shear Force Diagram (kN)



Bending Moment Diagram (kNm)

Figure 2.3.4 (e.ii)

Magnitude of bending moment (kNm)

$M_C = 0$
$M_A = -10\ (2) = -20$
$M_D = -10\ (5) + 20\ (3) = 10$
$M_B = -10\ (7) + 20\ (5) - 15\ (2)$
$= 0$

f) Overhanging Beam with Uniformly Distributed and Point Loadings

Figure 3.16 shows an overhanging beam. Determine the reaction forces at the supports.

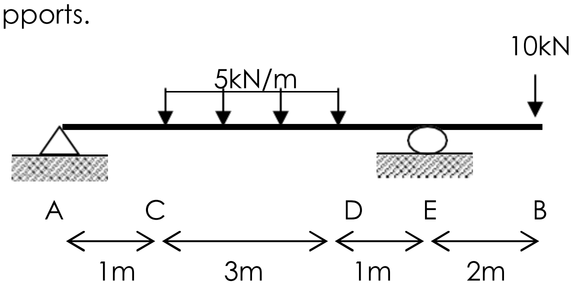


Figure 2.3.4 (f.i)

Solution

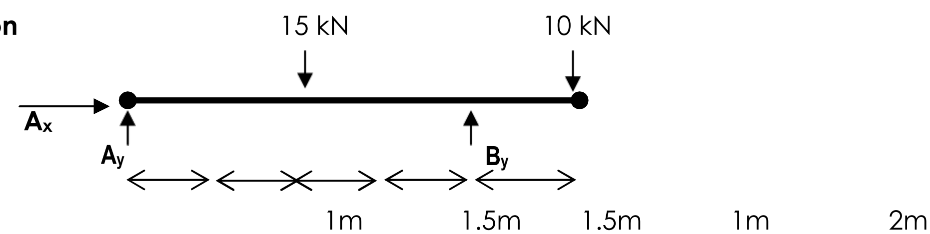


Figure 2.3.4 (f.ii)

> *Tips….*
> *Figure 3.17 is just helping to visualize the positions of the loading, there is no need to change the sketch diagram.*

$\Sigma f_x = 0 \qquad \therefore A_x = 0$

$\Sigma f_y = 0$
$A_y - 15 + B_y - 10 = 0$

$A_y + B_y = 25kN$ ………………..*(i)*

Taking moment at B:

$\Sigma M_B = 0$ + ve
$A_y(5) - 15(\frac{3}{2} + 1) + 10(2) = 0$

$B_y = \frac{17.5}{5}$ $\qquad B_y = 3.5kN$

$A_y + B_y = 25$
$A_y = 25 - 3.5,$ $A_y = \underline{21.5kN}$

g) Overhanging Beam with Uniformly Distributed and Point Loads

Figure 3.18 shows a loaded overhanging beam. Determine the reaction forces at the supports.

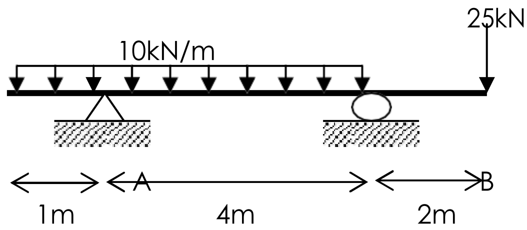


Figure 2.3.4 (g.i)

Solution

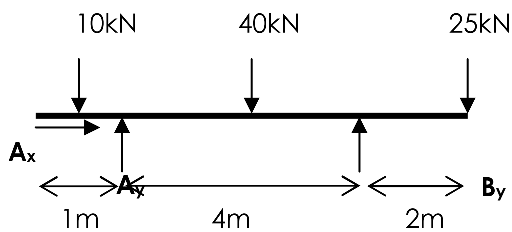


Figure 2.3.4 (g.ii)

> *Tips:*
> *Break the UDL into two components of point loads that will simplifies in the calculation of the reactions.*
>
> - 10kN/m x 1m = 10kN
> - 10kN/m x 4m = 40kN

$\Sigma f_x = 0$
$\therefore A_x = 0$

$\Sigma f_y = 0$
$-10 + A_y - 40 + B_y - 25 = 0$
$A_y + B_y = 75kN$ ....*(i)*

Taking moment at point A:

$\Sigma M_A = 0$ ↻ + ve
$-10(1)(0.5) + 40(2) - B_y(4) + 25(6) = 0$
$4B_y = 225,$ $B_y = \underline{56.25kN}$

From equation *(i)*

$A_y + B_y = 75$
$A_y = 75 - 56.25 = \underline{18.75kN}$

h) OVERHANGING BEAM WITH COMBINED LOADS

From figure 5.21 below, sketch the shear force and bending moment diagrams,

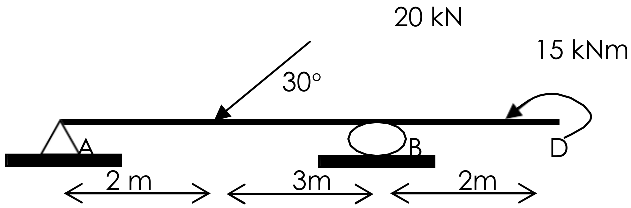
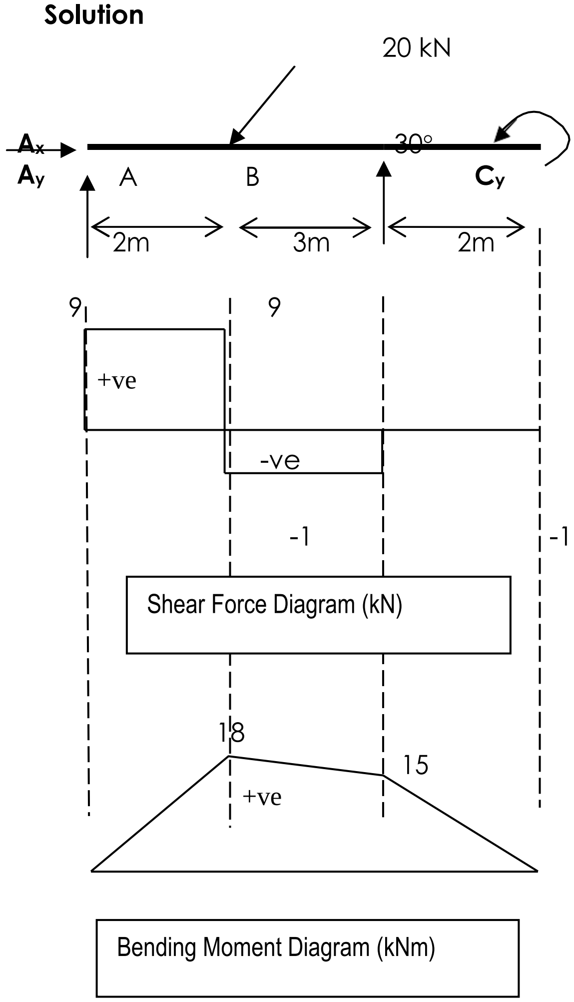


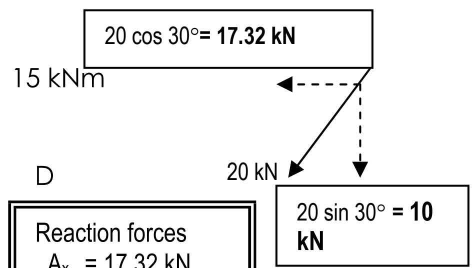
Figure 2.3.4 (h.i)

Analysis

Found out that the above question has a component horizontal and vertical force as there is an inclined force of 20 kN at 30° to the horizontal. Hence these component of forces should be determined first.

Solution





Reaction forces

A_x = 17.32 kN

A_y = 9 kN

C_y = 1 kN

Magnitude of shear forces (kN)

F_A = 9

F_{AB} = 9

F_B = 9 – 10 = - 1

F_{BC} = - 1

F_C = - 1 +1 = 0

F_{CD} = 0

F_D = 0

Magnitude of bending moment (kNm)

M_A = 0

M_B = 9 (2) = 18

M_C = 9 (5) – 10 (3) = 15

M_D = 9 (7) – 10 (5) + 1 (2)
= 15

$M_{D'}$ = 15- 15 = 0

Figure 2.3.4 (h.ii)

i) Cantilever Beam ith Point Load

Sketch shear force and bending moment diagrams for a beam as Figure 5.1 below:

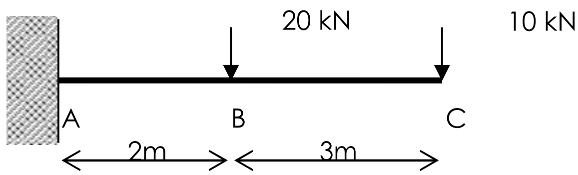


Figure 2.3.4 (i.i)

Tips in sketching SHD and BMD for a cantilever beam
- *Agnitude of the reaction and moment at support shoul firstly determined.*
- *Magnitude of the shear force should be determined by observing from left ro right of beam.*
- *Magnitude of bending moment is determined from the free end.*

Solution

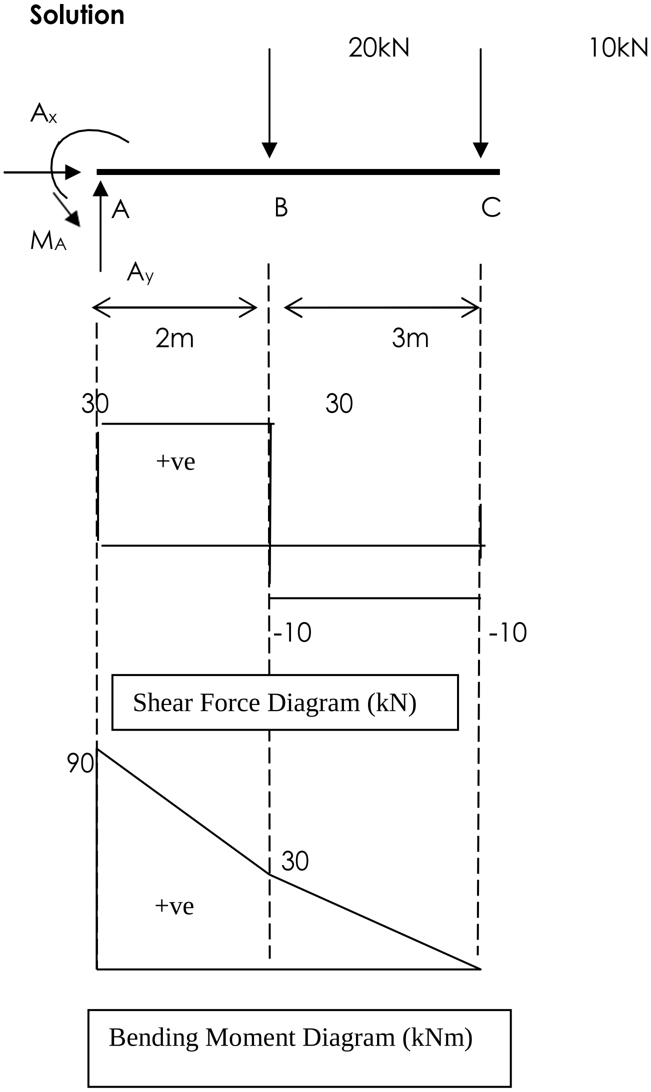


Figure 2.3.4 (i.ii)

1. Reaction at support

$A_x = 0$
$M_A = 90$ kNm
$A_y = 30$ kN

2. Magnitude of shear force (kN)

$F_A = 30$
$F_B = 30 – 20 = 10$
$F_C = 10 – 10 = 0$

3. Magnitude of bending moment (kNm)

$M_C = 0$
$M_B = 10 (3) = 30$
$M_A = 10 (5) + 20 (2) = 90$

$M_{A'} = 90 - 90 = 0$

Moment at A (M_A) with negative magnitude as it against clockwise direction

j) Cantilever Beam ith Uniformly Distributed Load

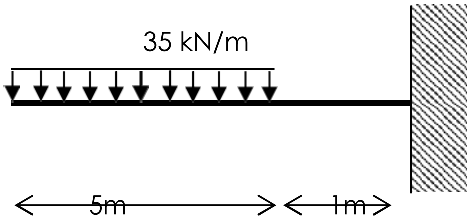
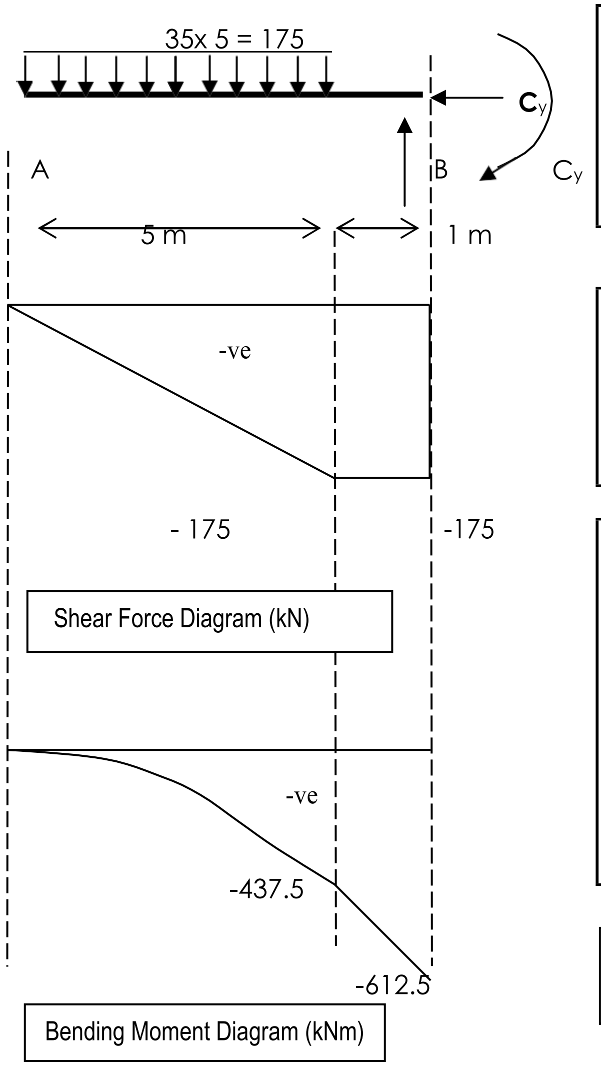


Figure 2.3.4 (j.i)

Based on Figure 5.3 determine :

a. Reaction forces at fixed end support.
b. Magnitude of shear force and sketch shear force diagram.
c. Magnitude of beding moment and sketch bending moment diagram
d. Magnitude of its maximum moment.

Solution



Reactions at support

$C_x = 0$
$C_y = 35 \times 5 = 175$ kN
$M_c = 612.5$ kNm

Magnitude of shear force (kN)

$F_A = 0$
$F_B = -175$
$F_C = -175 + 175 = 0$

Megnitude of bending moment (kNm)

$M_A = 0$
$M_B = -175\left(\frac{5}{2}\right) = -437.5$
$M_C = -175\left(\frac{5}{2} + 1\right) = -612.5$
$M_{C'} = -612.5 + 612.5 = 0$

Maximum Moment of 612.5 kNm occurs at the fixed end of the beam.

Figure 2.3.4 (j.ii)

k) Cantilever Beam With Compounded Loads

Figure 5.5 shows a cantilever beam loaded a UDL and a moment, Detremine:

a. Reaction forces at the fixed end support..
b. Magnitudes of shear force and bending moment .
c. Sketch the SH and BM Diagrams.

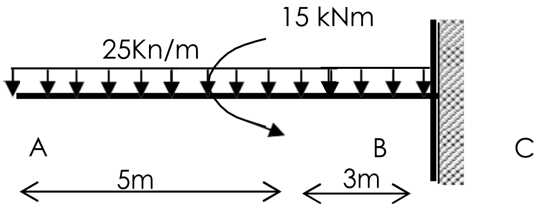
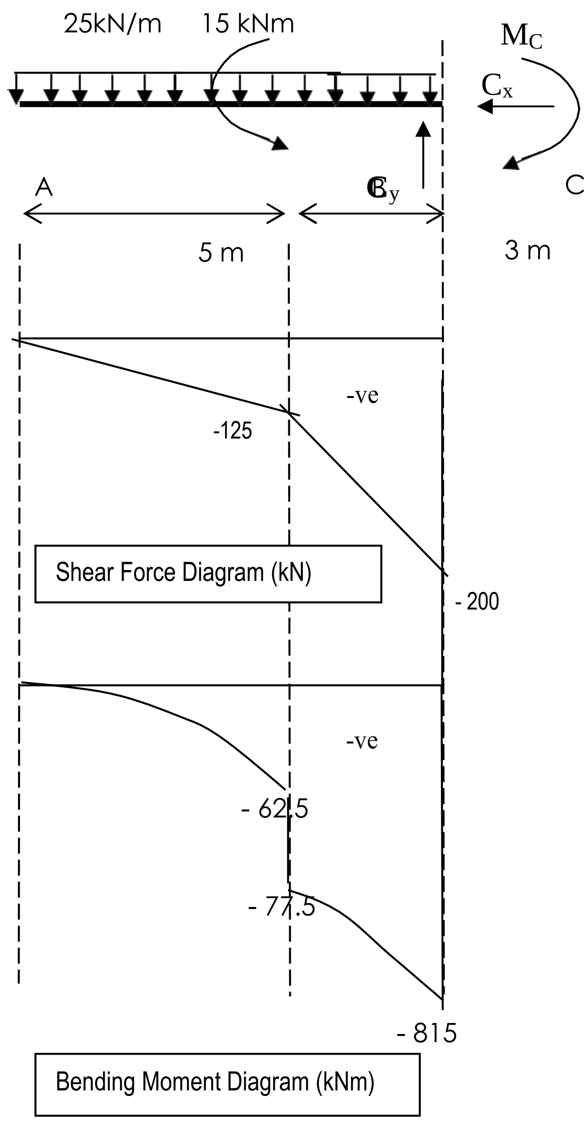


Figure 2.3.4 (k.i)

Solution



Reactions at support

$C_x = 0$

$C_y = 25 \times 8 = 200$ kN

$M_A = 15 + 25(8)(4)$

$= 815$ kNm

Magnitude of shear force (kN)

$F_A = 0$

$F_B = -25(5) = -125$

$F_C = -125 - 25(3) = -200$

$F_{C'} = -200 + 200 = 0$

Magnitude of bending moment (kNm)

$M_A = 0$

$M_B = -25(5)(\frac{5}{2}) = -62.5$

$M_{B'} = -25(5)(\frac{5}{2}) - 15 = -77.5$

$M_A = -25(8)(\frac{8}{2}) - 15 = -815$

$M_{A'} = -815 + 815 = 0$

Figure 2.3.4 (k.ii)

I) Cantilever Beam with Uniformly Distributed And Inclined Point Load

Diagram 3.20 shows a cantilever beam. Determine the reaction forces at the support.

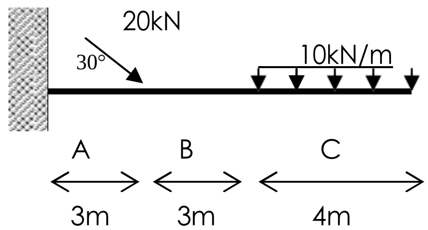
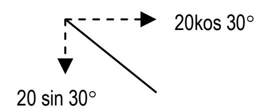


Figure 2.3.4 (I.i)

- Cantilever beam has three unknowns:
- Horizontal and vertical forces and a moment.

Tips: Converting inclined load into two components



Solution

$\Sigma f_x = 0$
$A_x + 20 \text{ kos } 30° = 0$
$A_x = - 17.32kN$ (⟵)

$\Sigma f_y = 0.$
$A_y – 20 \sin 30° - 10(4) = 0$
$A_y = 50$ kN ....*(i)*

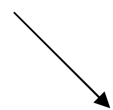


Figure 2.3.4 (I.ii)

Taking moment at point A:

$\Sigma M_A = 0$
$-M_A + 20 \text{ Sin } 30° (3) + 40 (8) = 0$
$M_A = 350kNm$

m) Cantilever Beam with Uniformly Distributed and A Point Load

Figure 3.21 shows a cantilever beam with uniformly distributed and a point loads. Determine the reaction forces at the support.

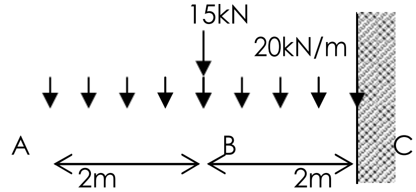


Figure 2.3.4 (m.i)

Solution:

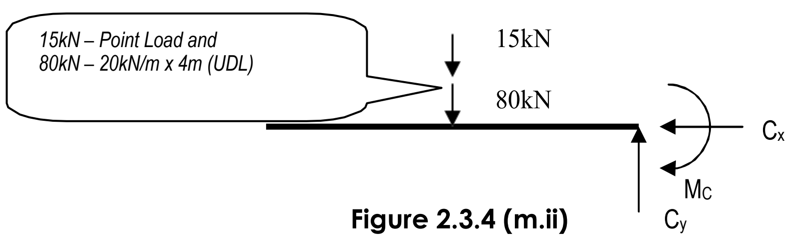


Figure 2.3.4 (m.ii)

$\Sigma f_x = 0$

$\therefore C_x = 0$

$\Sigma f_y = 0$

$-80 - 15 + C_y = 0$

$C_y = 95kN$

Taking moment at point C:

$\Sigma M_C = 0$ + ve

$-80(2) - 15(2) + M_C = 0$

$M_C = 190kNm$

2.3.5 Interpret to get the Maximum shear force and Bending Moment Value
a) Simply supported beam with combined loads

Based on figure 5.33, answer the following question :

a. Determine the reactions at supports
b. Sketch the shear force diagam inserting its principle values
c. Determine the maximum shear force
d. Sketch the bending moment diagam inserting its principle values
e. State the value of maximum bending moment and its position.

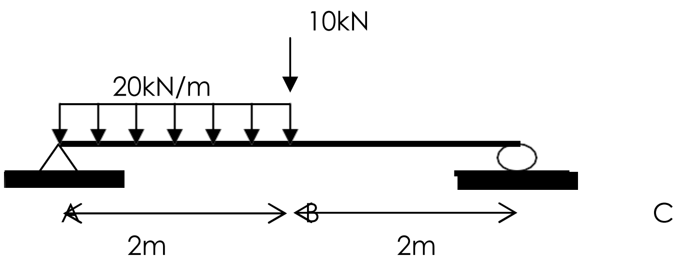
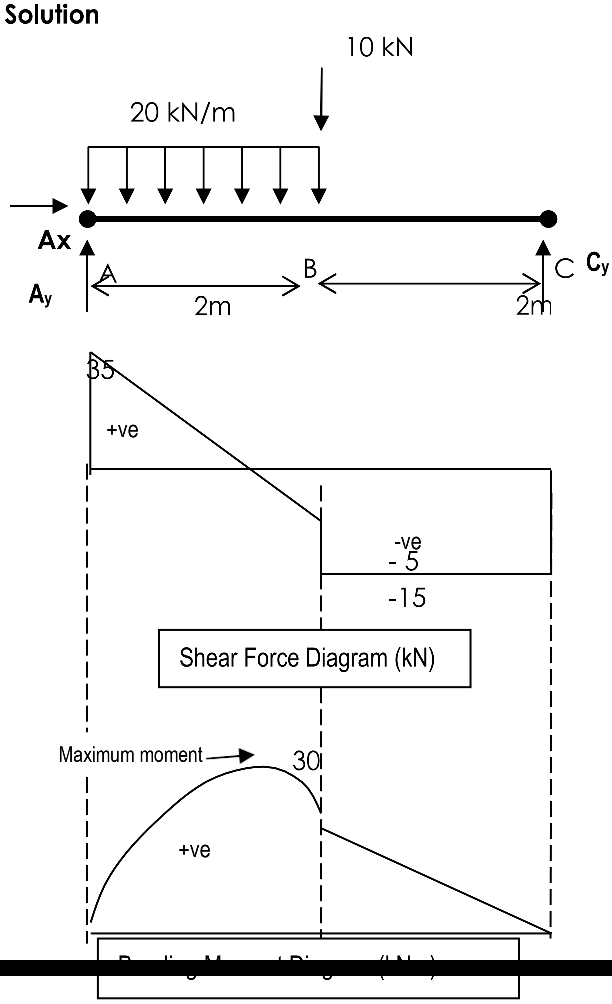


Figure 2.3.5 (a.i)

Analysis

The above question requires students to determine the magnitude of maximum bending moment and its position. Maximum bending moment exists at point where line of shear force intersect with the x-axis = 0. Hence, students should first find the position of this intersection before dealing with the maximum bending moment.

Solution



Reaction forces

$A_X = 0$
$A_Y = 35$ kN
$C_Y = 15$ kN

Magnitude of shear force (kN)

$F_A = 35$
$F_B = 35 – 20 (2) = - 5$
$F_{B'} = - 5 – 10 = - 15$
$F_C = – 15 + 15 = 0$

Magnitude of bending moment (kNm)

$M_A = 0$
$M_B = 35 (2) – 20 (2)(2/2) = 30$
$M_C = 35 (4) – 20 (2)(2/2 + 2) – 10(2)$
$= 0$

Figure 2.3.4 (a.ii)

> ***Tips.....***
> *In the above example shape of the curve from point A to B is determined by examinig the shear force diagram. Maximum moment occurs when shear force line intersect the origin x-axis = 0 of the shear force diagram. Hence maximum moment doesnot happen at point B (30 kNm) but at a point that is reffered at the shear force diagram.*

Determining maximum shear force

Observe the shear force diagram, then the maximum shear force value is 35 kN acting at point A

Determining maximum beneding moment

To determine the magnitude of maximum bending moment its position should be determined and calculated first. Look at the shear force diagram at point A and B (refer figure 5.35)

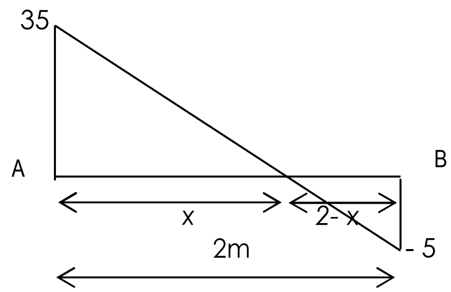


Figure 5.35

Using similar triangles determine value of x by forming the equation as follows. Students should still remember this!

$$x / 32 = (2 – x) / 5$$

Ignore the negative sign of the shear force, than cross multiply the equation,

$$5x = 35 (2-x)$$
$$5x = 70 – 35x$$
$$40x = 70 , \qquad x = 1.75 \text{ m}$$

∴ Maximum moment acts at 1.75m from point A.

Determining maximum bending moment

Look again figure 5.33 of the beam and makse section at 1.75 m from point A, the position where maximum moment occurs. Draw section's diagram of beam and name this section point, as an example say P as shown in figure 5.36

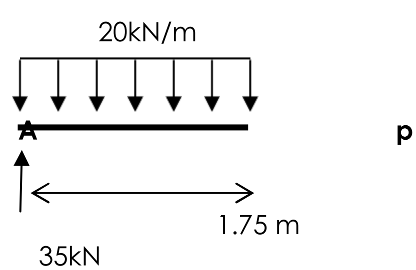


Figure 2.3.5 (a.iii)

$$M_P = 35 (1.75) – 20 (1.75) (1.75 / 2)$$
$$M_P = 30.63 \text{ kNm}$$

Magnitude of maximum moment 30.63 kNmm at a position 1.75m from support A.

b) Simply Supported Beam With Uniformly Distributed Load (UDL)

Based on the SSB in Figure 4.32 below:

i. Determine reaction forces at supports
ii. Sketch SH and BM Diagrams.

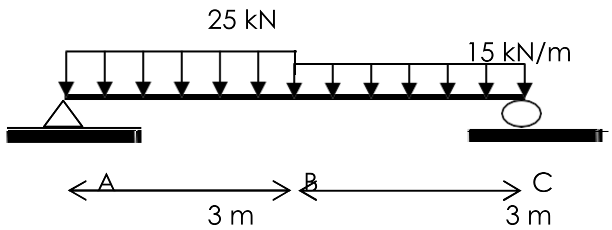
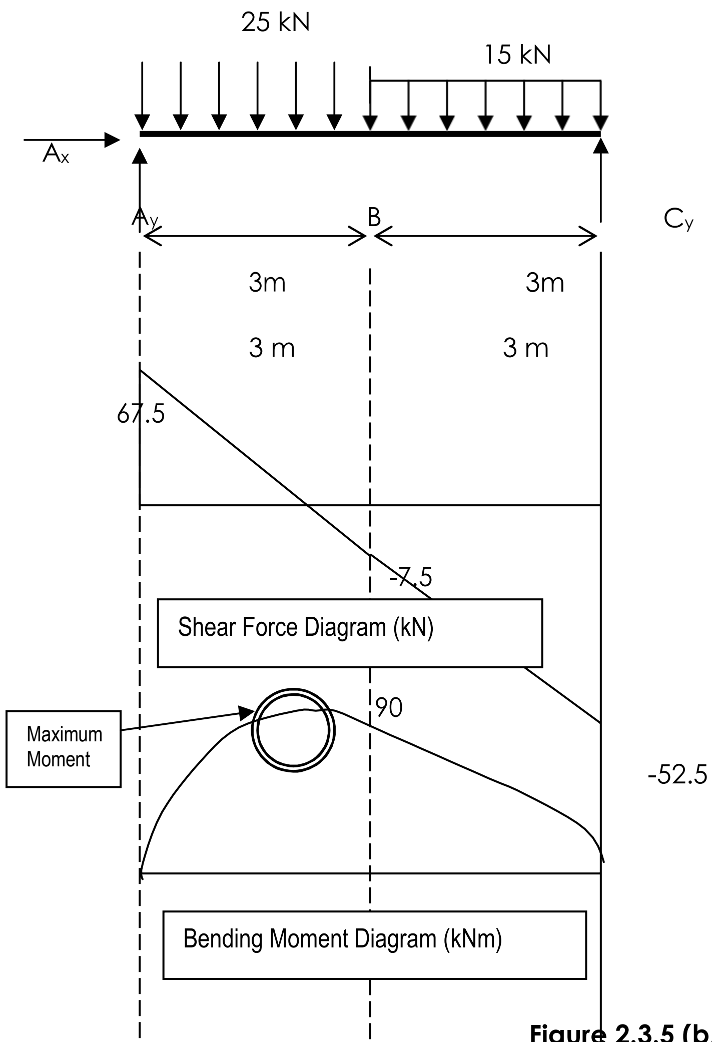


Figure 2.3.5 (b.i)

Solution



Reaction forces (kN)

$A_x = 0$

$A_y = 67.5$ kN

$C_y = 52.5$ kN

Value of shear force (kN)

$F_A = 67.5$

$F_B = 67.5 - 25\ (3) = -7.5$

$F_C = -7.5 - 15(3) = -52.5$

$F_{C'} = -52.5 + 52.5 = 0$

Value of Bending moment (kNm)

$M_A = 0$

$M_B = 67.5\ (3) - 25\ (3)\ (\frac{3}{2}) = 90$

$M_C = 67.5\ (6) - 25\ (3)\ (\frac{3}{2} + 3) - 15\ (3)\ (\frac{3}{2}) = 0$

Figure 2.3.5 (b.ii)

Note : Value of maximum moment will be detail in next sub unit

c) Simply Supported Beam With Several Types of Loadings

a. Baseed on the following Figure 4.35 of a loaded beam :
   i. Determine reaction forces at supports
   ii. Sketch the SF and BM Diagrams.

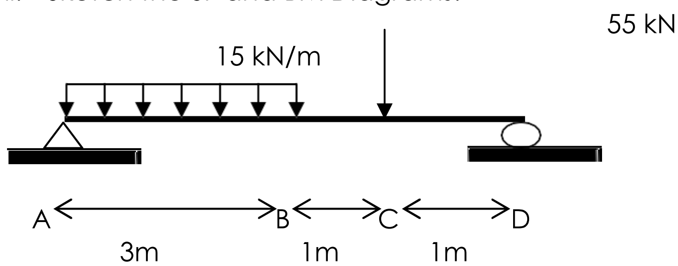


Figure 2.3.5 (c.i)

b. Based on a SSB in Figure 4.36
   i. Determine reaction forces at supports
   ii. Sketch the SF and BM Diagrams.

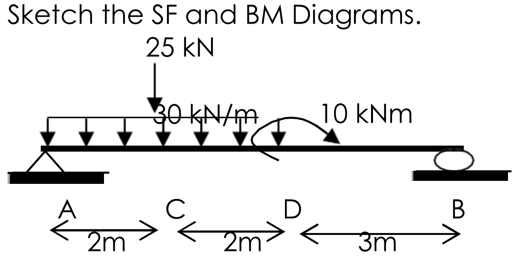
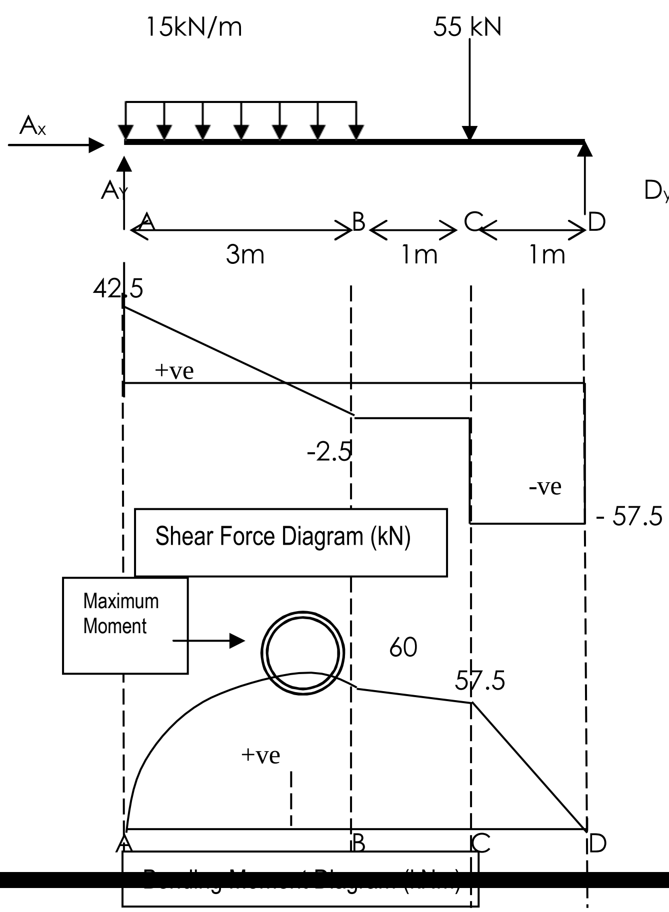


Figure 2.3.5 (c.ii)

Solution a.



Reaction Forces (kN)
$A_X = 0$
$A_Y = 42.5$ kN
$D_Y = 57.5$ kN

Magnitude of shear force (kN)
$F_A = 42.5$
$F_B = 42.5 - 15(3) = -2.5$
$F_C = -2.5 - 55 = -57.5$
$F_D = -57.5 + 57.5 = 0$

Bending moment (kNm)
$M_A = 0$

$M_B = 42.5\ (3) - 15(3)(\frac{3}{2}) = 60$

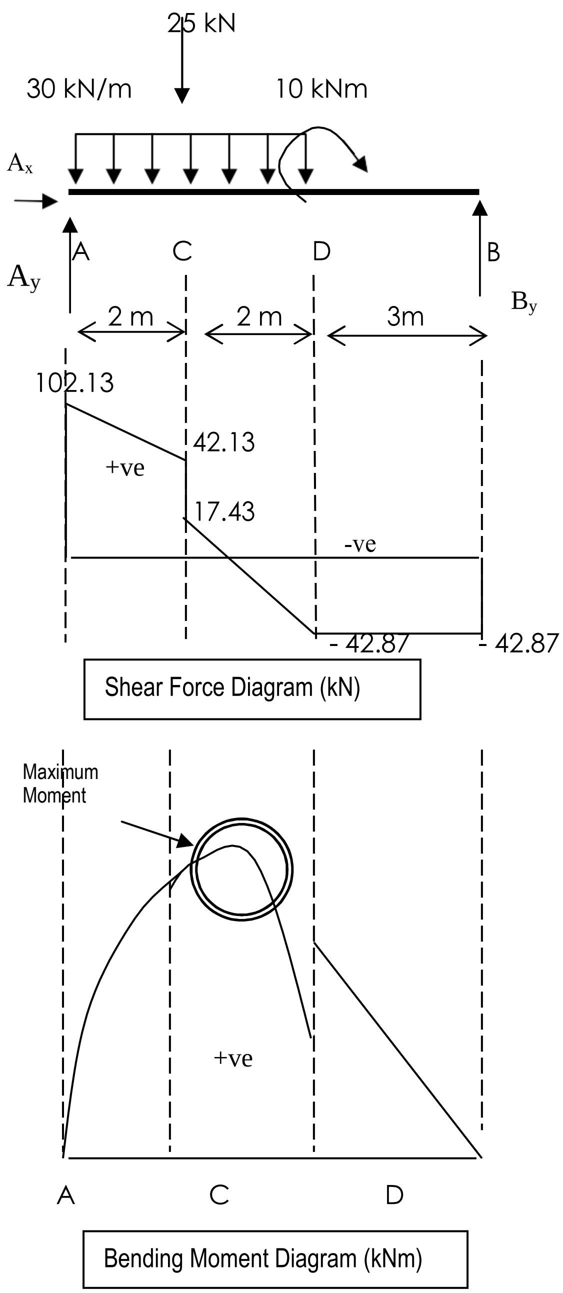
$M_C = 42.5(4) - 15(3)(\frac{3}{2} + 1) = 57.5$

$M_D = 42.5(5) - 15(3)(\frac{3}{2} + 2) - 55(1)$

$= 0$

Figure 2.3.5 (c.iii)

Solution b.



Reaction forces (kN)
$A_x = 0$
$A_y = 102.13$ kN
$B_y = 42.87$ kN

Magnitude of Shear Force (kN)
$F_A = 102.13$
$F_C = 102.13 – 30(2) = 42.13$
$F_{C'} = 42.13 – 25 = 17.43$
$F_D = 17.43 – 30(2) = - 42.87$
$F_B = -42.87 + 42.87 = 0$

Magnitude of bending moment (kNm)
$M_A = 0$
$M_C = 102.13(2) - 30(2)(1) = 144.26$
$M_D = 102.13(4) - 30(4)(1) - 25(2)$
$= 118.52$
$M_{D'} = 118.52 + 10 = 128.52$
$M_B = 102.13(7) - 30(4)(2+3) - 25(5) + 10$
$= 0$

Figure 2.3.5 (c.iv)

DIRECT STRESS

UNIT 3

GENERAL OBJECTIVE :

- To study and understand the concept of direct stress and strain..

SPECIFIC OBJECTIVE :

At end of this unit the students should be able to :

- Understand the relationship between direct stress and strain
- Apply the material mechanical characteristic in direct stress and strain

INPUT 3.0 **STRESS AND STRAIN**

3.0 INTRODUCTION

Have you sen a tug-of-war competition?

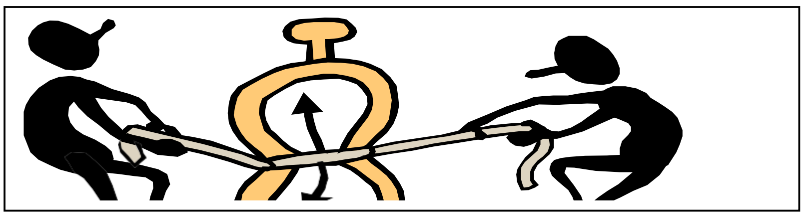


Figure 6.1 : Tug-of- war Competition

In this competition two sides pull the rope in opposite direction. Focus your mind on the rope, the question is,

Why the rope doesnot snap?
Is there any posibility that the rope will snap?
Is the rope expends while it is being streched?
Is there any change in cross-sectional dimension of the rope when it is stretched?
Is the rope return back to its original length after end of competition?

In short, stress is correlated with the force exists over the sectional area of the rope to resist tensile force applied in both directions. Strain on the other hand correlates with changes in size or shape that occurs to the rope resulting from tensile force applied to it.

A structure consists of a number of members combined or joined together with loads or forces being transfered form one member to another. Stress and strain concept is an analysis of the structural behaviour that bearing those loads or forces which is the basic theory to structure. If there exists any member that fail to bear the load, then it may caused failure or danger the the structure as a whole.

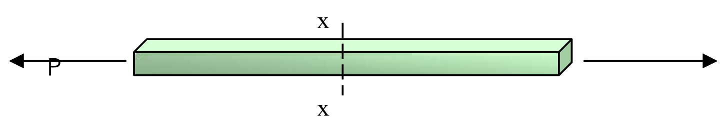
3.1.1.1 DIRECT STRESS

> ***Definition: Stress is defined as the intensity of internal load per unit area [****Tegasan ditakrifkan sebagai keamatan beban terhadap sesuatu luas keratan* ***].*** *The unit of stress depends on the unit for load/force and area of section, example N/mm²; kN/mm²; N/m²; kN/m² etc*

When a body is subjected to an external load, it tends to experience a change in shape (size or dimension). During the process of change of shape there is an internal ressistence in the body that against the change. If this internal resistant force ables to resist the applied load, the body is said to be in the stable state.

In brief, stress is an internal force in the form of reaction of material to an applied force/load. This internal force is against and in opposite direction to the applied force of the body. Unit for stress are stated as N/mm²; kN/mm²; N/m²; kN/m² (dependant to the unit of force and area).

Consider a prismatic bar subjected to a tensile load, P. Load P caused the bar to elongate. Assumed the bar is cut at x-x plane (figure 6.3a).



P

Figure 6.3(a) : Sectional Prismatic Bar

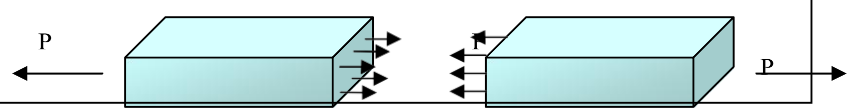


Figure 3.1.1.1 : Stress at sectioned plane

To be in state of equilibrium, the sectioned rod produces a force of the same magnitude P but acting in opposite direction to that external force, P (Figure 6.3b).

This internal force is said as stress and it is the reaction of the mateial of the bar to that of the applied external force. It is assumed uniformly distributed, acting over the entire bar section.

Thus,

Stress , $\sigma = \dfrac{Force}{Area}$ ⇨ $\boxed{\sigma = \dfrac{P}{A}}$

3.1.1.2 DIRECT STRAIN

> ***Definition : Strain is defined as the change of shape or length of a body per unit length.***
> ***Strain does not have unit.***

When a structural member is subjected to an external load, the member will experience a change of shape even though if it is very small. This eventually will cause the member to change in dimension. When a tensile force, P is applied, the member will extend longer in the direction of the force and followed with a reduction in sectional size (Figure 6.4a). On the other hand, when a compressive force P is applied, the member will become shorter in the direction of the force and followeed with an expension in section size of the member (Figure 6.4b).

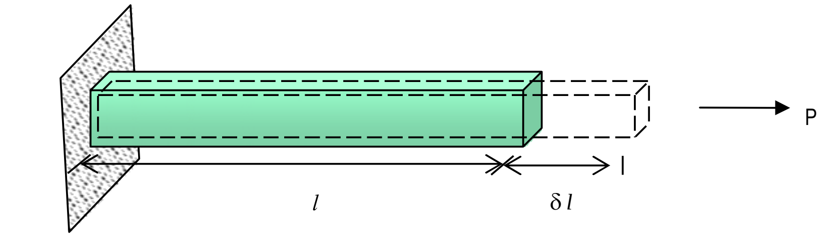


Figure 3.1.1.2 (a) : Elongation from *l* to (l + δ *l*)

δl = change of length

l = Original length

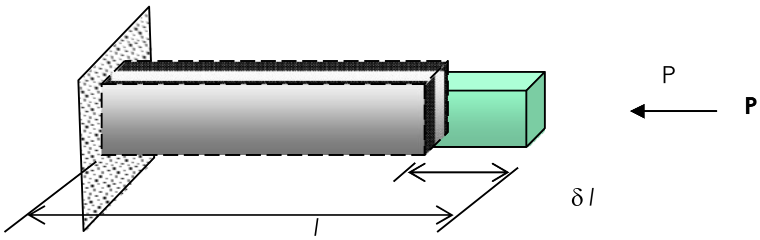


Figure 3.1.1.2 (b0 : Deformation (shortening) from *l* to *(l* - δ *l)*

Thus,

$$\text{Strain, } \varepsilon = \frac{\text{Change in dimension}}{\text{Original dimension}}$$

$$\varepsilon = \frac{\delta l}{l}$$

Note: Strain is a ratio of dimensions, hence strain has no unit.

3.1.2 EFFECT OF AXIAL LOAD TO DIRECT STRESS AND STRAIN

Axial load has been discuss in Unit 1. A force applied to a rod LS is an axial force as it acts parellel and in line with the longitudinal axis of the rod (figure 6.2).

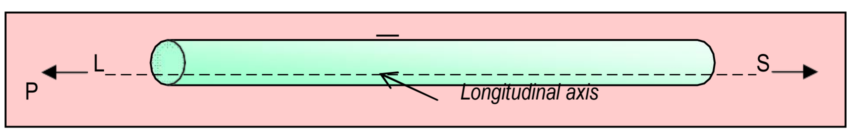


Figure 3.1.2 : Axial Force/Load

Magnitude and direction of the axial reaction forces will give different stress and strain effect. It may caused the rod a change from its original size and eventually fail (break). Rod failure may depends on its cross-sectional size of the rod. If the cross-sectional size is large, the magnitide of force, P would be much bigger to break the material compared to the same material which has a smaller cross-sectoional size.

Generally axial force is categorized into two effects:-
a. Direct stress
b. Direct strain

Difinition of Hooke's Law:

- Hooke states that a body will recover its original length or size after the applied load is revoved/ released, provided that the body is within its elastic limit. *(Hukum Hooke menyatakan bahawa sesuatu jasad akan kembali kepada panjang asal setelah beban yang dikenakan, dialihkan daripadanya selagi ia berada dalam had anjal).*

- Hooke states that when a body is loaded not exceeding the elastic limit, then the change of shape or size is directly proportional to load. *(Hukum Hooke menyatakan jika bahan yang dikenakan tidak melebihi had anjal, maka beban berkadar terus dengan ubahbentuk).*

Load (P) α Elongation (δl)

As load is directly proportional to stress and elongation is proportional to strain, then stress is directly proportional to strain.

Stress (σ) α Strain (ε)

<u>*Hooke's Law is observed with the following conditions;*</u>

- Loads applied are assumed axial loadings.
- Cross-sectional area of the body is constatnt and uniform
- Mass of body is homogeneous (characteristic of body is uniform throughout the length).

3.1.3 MECHANICAL CHARACTERISTIC OF MATERIALS

When a material is subjected to a tensile load several characteristics were identified, they are:

Ductile (Mulur)
It refers to a material that allows itself to be drawn to a smaller section by tensile force during which shows a certain degree of elasticity together with a considerable amount of plasticity ie it has a high rate of deformation.

Elastic (Anjal)
It refers to a situation when a material undergoes deformation under the action of axternal force, such deformation completely disappears on the removal of the force.

Elasto-plastic (Anjal-plastik)
It refers to material which undergoes deformation under the action of axternal force such deformation does not completely disappear after the removal of the force hence permanent defromation axists.

Plastic (Plastik)
It refers to a material which undergoes deformation under the action of axternal force such deformation does not disappear at all after the removal of the force.

Brittle (Rapuh)
A material is said to be brittle when it does not show any deformation and fails by rapture under the action of force.

Elastic Limit (Had Anjal)
When an external force acts on a material it under goes some deformation. The removal of this force causes the material to spring back to its original position. It found out that

within a certain limit the defrormation is completely disappears on removal of the force. This limiting force is known as the ***elastic limit*** of the material.

Stress (Tegasan)(σ)
Every material is elastic in nature thus it deforms under the action of external load. Under such deformation, the material sets up some resistance to deformation. This resistance to deformation per unit area is term as ***stress***, or may be defined as the force per unit area, carries a symbol σ.

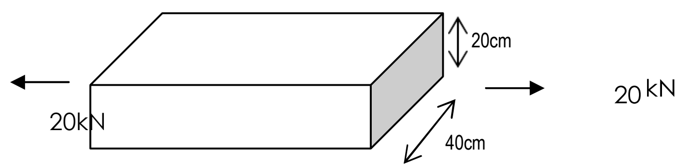
Strain (Terikan)(ε)
When a material undergoes the action of force, the material deforms. This deformation per unit length is called ***strain***, denoted by the symbol ε.

3.1.4 STRESS AND STRAIN OF A PRISMATIC BAR

SOLUTION OF PROBLEM 3a

To determine Stress

A bar with a sectional dimension of 20cm and 40cm is streched by pulling it at both ends with a 20kN force.
Determine the stress experienced by the bar.



Determination of cross-sectional area

A = 20 x 40 = 800cm^2

Solution
Tensile stress, $\sigma = \frac{P}{A} = \frac{20}{800} = 0.025\ kN/cm^2$ #

SOLUTION OF PROBLEM 3b

To determine Stress

A Steel rod having cross-sectional area of 175mm^2 experiences a compressive force of 370N.
Calculate stress in the rod.

Solution
Stress, $\sigma = \frac{P}{A} = \frac{370}{175} = 2.114\ N/mm^2$ #

SOLUTION OF PROBLEM 3c

To determine Strain

The elongation occurs to a bar of 3m long is 0.5mm. Calculate the strain.

Solution
Strain, $\varepsilon = \frac{\delta l}{l} = \frac{0.5}{3000} = 1.67 \times 10^{-4}$ #

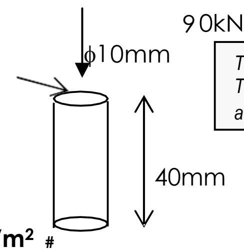
Tips….
Strain is a ratio of dimensions, thus stain does not have unit.

SOLUTION OF PROBLEM 3d

To determine stress and strain

A concrete cylinder as shown in figure below is suppoting a compressive load of 90kN. The cylinder shortened by 0.03mm. Find : a) Compressive stress b) Strain

Solution



Tips:
The concrete cylinder has a circular cross-section.

Determination of area

$A = \pi d^2 / 4$

$A = 3.14 \times 10^2 / 4$

$A = 78.54\ mm^2$

Stress, $\sigma = \frac{P}{A} = \frac{90}{78.54}$ = **$1.146\ kN/m^2$** #

Strain, $\varepsilon = \frac{\delta l}{L} = 0.03$ = **7.5×10^{-4}** #

SOLUTION OF PROBLEM 3e

Stress and strain of a hollow rod

A hollow rod is 600mm long having an external and internal diameter of 30mm and 20mm respectively. It is subjected to a load of 50kN and found that it extended by 0.2mm. Determine the direct stress and strain of the rod.

Solution

Given;

d_d = 20mm; l = 600mm; δl = 0.2 mm

d_l = 30mm; P = 50kN;

σ **?** **ε ?**

$\therefore$ Cross-sectional area, A = Area of the ring cross-section

$= (\pi d_l^2 / 4) - (\pi d_d^2 / 4) = (\pi 30^2 / 4) - (\pi 20^2 / 4)$

$= 706.86 - 314.16 = 392.7 mm^2$

Tips…
(i) Area of the hole is not accounted as it is not solid.
(ii) Solving may be done direct using the stress equation.

$\sigma = \frac{P}{A} = \frac{50}{392.7}$

$= 0.127\ kN/mm^2 = 127.32 N/mm^2$ #

$\varepsilon = \frac{\delta l}{l} = \frac{0.2}{600} = 3.33 \times 10^{-4}$ #

SOLUTION OF PROBLEM 3f

To determine the diameter of a column

A cast-steel column has an internal diameter of 200mm. What is the external dimeter of this column if it were to support
a load of 1.6MN with a stress not exceeding 90N/mm^2.

Solution

Given; $d_d = 200mm$; $P = 1.6$ MN @ 1.6×10^6 N
$d_o = ?$; $\sigma = 90$ N/mm^2

Tips: This is a hollow column. To determine its external diameter, consider area, A as the subject of the equation.

$$A = \frac{P}{\sigma} = \frac{1.6 \times 10^6}{90} = 17.78 \times 10^3 \text{ mm}^2$$

$$\left[\frac{\pi d_o^2}{4}\right] \quad \left[-\frac{\pi d_i^2}{4}\right]$$

$$\pi d_o^2 = [(17.78 \times 10^3)(4)] + \pi d_i^2$$

$$D_o^2 = \{[(17.78 \times 10^3)(4)] + \pi (200)^2\} / \pi$$

$$= (71120 + 125663.71) / \pi = 62638.2\text{mm}^2$$

$$\therefore \quad d_o = 250.28 \text{ mm } \#$$

3.2 MODULUS OF ELASTICITY (*Modulus Keanjalan*)

Definition : Modulus of Elasticity is the ratio of stress to strain.

As it has been elaborated earlier, when a body is subjected to a load, it will experience change in shape. When this load is released the body will come back to its original shape and size. This characteristic is known as **elasticity**. Theory about elasticity will be elaborated in detai in Unit 7.

Elasticity uses symbol E and its unit N/m^2 (or similar to stress)

$$E = \frac{\text{Stress}}{\text{Strain}} = \frac{\sigma}{\varepsilon}$$

Where $\sigma = \frac{P}{A}$ and $\varepsilon = \frac{\delta l}{l}$

$$\therefore \quad E = \frac{Pl}{A\,\delta l}$$

Table 7.1 shows the standard value of the Young's Modulus of the following materials;

Material	Young's Modulus (GN/m²)
Steel	200 – 220
Aluminium	60 - 80
Copper	90 – 110
Timber	10

Table 7.1: Value of Young's Modulus

The value of these moduli indicates the strength of the material; a high value shows a high stress-strain garph. Therefore a greater load is required to get the same deformation.

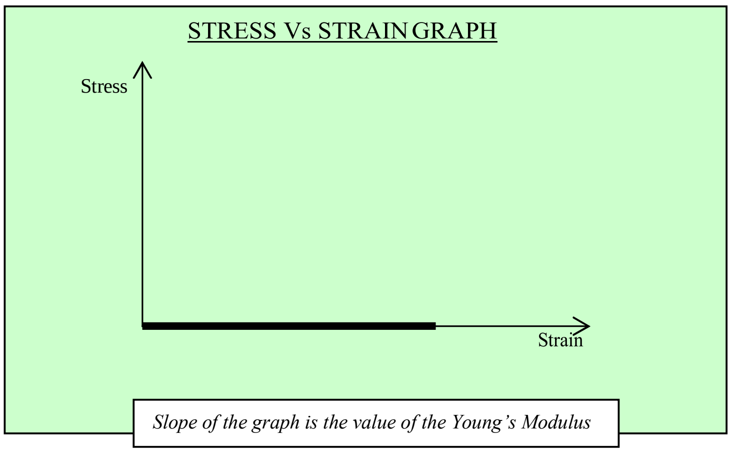


Diagram 7.2: Stress Vs Strain Graph

VALUE OF MODULUS OF ELASTICITY FROM GRAPH

The mechanical characteristic of a material is determined by performing a test on a sample of the said material. Diagram 7.3 shows a standard tensile test on cylindrical bar having a uniform cross-section fixed in a tensile testing machine. This machine ables to apply axial load on the test bar.

A test specimen is prepared according to the specified dimension and fixed in the tensile testing machine. The applied tensile load is gradually increased until the sample fails. The applied load and elongation of the sample is recorded. A graph of 'load vs elongation' is then plotted and the result obtained is as shown in diagram 7.4.

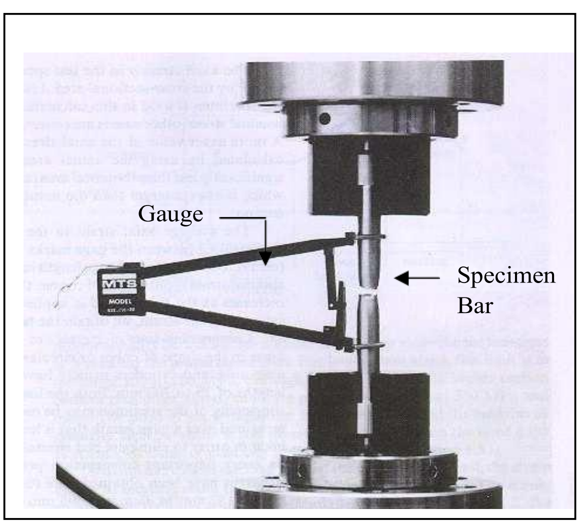


Figure 7.3: Tensile Test Apparatus

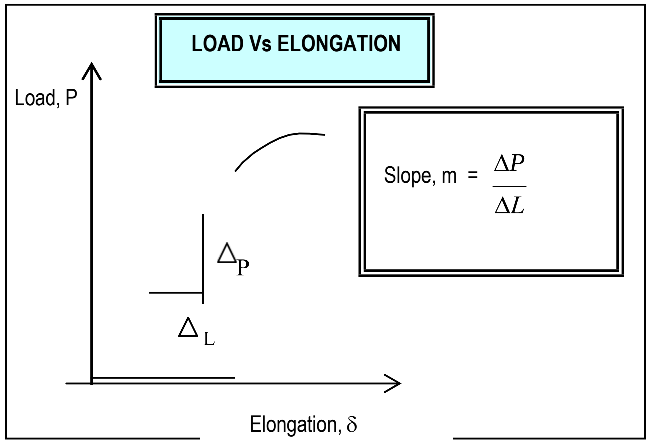


Figure 7.4: Graph of Load Vs Elongation

Young's Modulus, $E = \frac{PL}{\delta A}$

$$E = \frac{P}{\delta} \times \frac{L}{A}$$

$$E = m \times \frac{L}{A}$$

Where,

A = cross sectional area of sample
L = length of sample
$\frac{P}{\delta}$ = slope of graph, m

CHARACTERISTIC STATE OF MATERIAL FROM GRAPH

Diagram 7.5 shows tensile test result for a mild steel bar. The discription related to his graph is as follows.

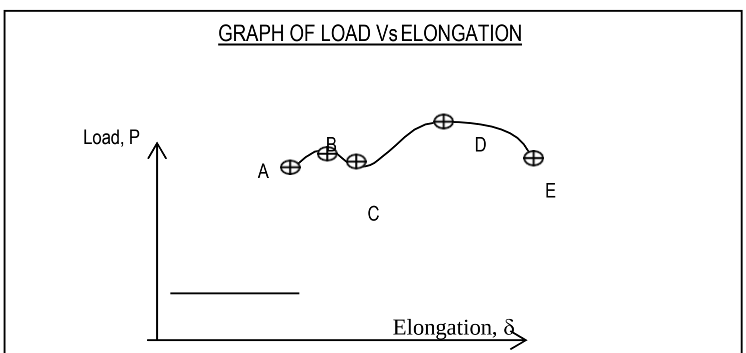


Figure 7.5: Stress-Strain Curve of a Mild Steel

A – Elastic limit, that is the end limit that the material obeys Hooke's Law

B – Plastic limit
The limit where the material begins showing plastic characteristic until fail/rapture hence it does no longer obeys Hooke's Law

C – Failure limit where elongation occurs without the increament of load.

D – Maximum load
A point where the load applied reaches the maximum and the material experience critical elongation till rapture.

E - Rapture
Material fails to sustain the load.

3.2.1

SOLUTION OF PROBLEM 3g

To determine strain and Modulus of Elasticity

A rod of 2.5m long with a cross-setional area of 1290mm^2 experiences an elongation of 1.5mm when subjected
to a tensile forcce of 142 kN. Calculate:-

(a) Tensile stress of rod
(b) Strain
(c) Modulus of Elasticity

Solution

Given; *l = 2.5m @ 2500mm, A = 1290mm^{2,} δl = 1.5mm, P = 142kN*

(a) $\sigma = \dfrac{P}{A} = \dfrac{142x10^3}{1290}$

$= 110.08 N/mm^2$ #

(b) $\varepsilon = \dfrac{\delta\ell}{\ell} = \dfrac{1.5}{2500}$

$= 6 \times 10^{-4}$ #

(c) $E = \dfrac{\sigma}{\varepsilon} = \dfrac{110.08}{6x10^{-4}} = 1.83 \times 10^5 N/mm^2$ #

Tips.....

- *You are encouraged to change to change informations given in words into numbers and symbols.*
- *To avoid mistakes, try to change all units in auniform manner at the early steps of the process..*

SOLUTION OF PROBLEM 3h

To determine area of section, stress and strain

A steel rod 3m long carries a load of 20kN. If the elongation is not permitted to exceed greater than 0.2mm, calculate;

(a) Minimum cross-sectional area of rod
(b) Tensile stress of rod
(c) Strain of rod. Given E = 206 kN/m^2

Solution

(a) $E = \dfrac{Pl}{A\delta l}$

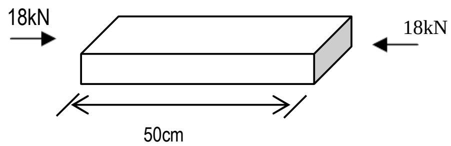
$\therefore \ A = \dfrac{Pl}{E\delta l} = \dfrac{(20 \times 10^3)\,3}{(206 \times 10^3)(0.2 \times 10^{-3})} = 1456.31 \ m^2$ #

(b) $\sigma = \dfrac{P}{A} = \dfrac{20 x10^3}{1456.31} = 13.73\ N/m^2$ #

(c) $\varepsilon = \dfrac{\delta \ell}{\ell} = \dfrac{0.2}{3000} = 6.67 \times 10^{-5}$ #

SOLUTION OF PROBLEM 3i To determine sectional area, stress and strain

A steel bar of 50cm x 7.5cm x 2.5cm is subjected to a load of 18 kN. Find the stress and strain in the bar when given that Young's Modulus is 215 GN/m².



All units be changed to m² so that it is the same as the unit of E.

Solution

(a) $\sigma = \dfrac{P}{A} = \dfrac{18 x10^3}{(0.075 \ x 0.025)} = 9.6 \times 10^6\ N/m^2$ #

(b) $E = \dfrac{\sigma}{\varepsilon}$, $\varepsilon = \dfrac{\sigma}{E} = \dfrac{9.6 x10^6}{215 \ x10^9}$ = 4.47 x 10 #

Note: 215 GN/m² = 215 x 10⁹ N/m²

Stress and Straian in Bar of Different Cross-Section

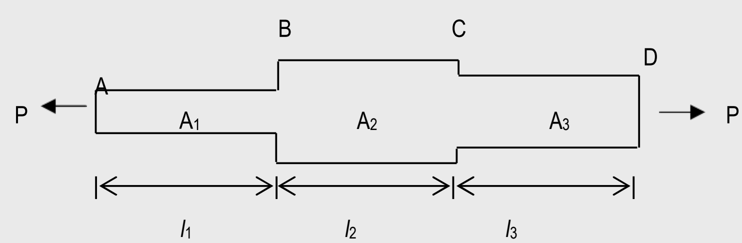


Figure 3.1.4 : Bar With Different Cross-Section

Note: Each section ecperiences the same load action

There exists a bar having different cross-sectional area and with various length (Figure 6.5). In this situation, stress, strain and deformation (change in length) of each section is dealt separately. The total change in length is sum of all changes in length of each individual section.

Consider figure 6.5:- P = Force
E = Modulus of Elasticity
L_1 = Length of section 1
A_1 = Cross-sectional area of section 1
l_2, A_2 = Sequence of values for section 2 and the rest

Thus; $\sigma_{AB} = \dfrac{P}{A_1}$; $\sigma_{BC} = \dfrac{P}{A_2}$; $\sigma_{CD} = \dfrac{P}{A_3}$

Hence; $\delta l_{AB} = \dfrac{Pl_1}{A_1 E}$; $\delta l_{BC} = \dfrac{Pl_2}{A_2 E}$; $\delta l_{CD} = \dfrac{Pl_3}{A_3 E}$

Therefore; $\Sigma \delta l = \delta l_{AB} + \delta l_{BC} + \delta l_{CD}$

$$= \frac{Pl_1}{A_1 E} + \frac{Pl_2}{A_2 E} + \frac{Pl_3}{A_3 E}$$

$$\Sigma \delta l = \frac{P}{E}\left[\frac{L_1}{A_1} + \frac{L_2}{A_2} + \frac{L_3}{A_3}\right]$$

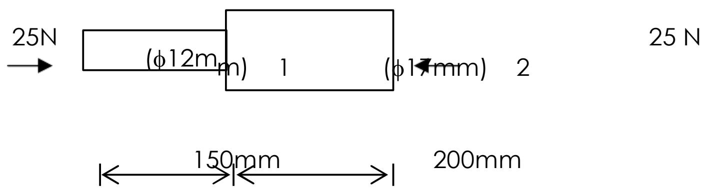
This Formula is applicable only when the material of the entire bar is the same i.e Modulus of Elasticity is equal

SOLUTION OF PROBLEM 3j To determine total elongation of a changing cross-section

A steel rod composed of two different cross-sectional parts subjected to a compressive load of 25N, Calculate :

(a) Stress for each section.

(b) Total deformation. Given E = 210 GN/m^2



Solution

Given; P = 25 N

E = 210 GN/m^2 @ 210 x 10^9 N/m^2

ϕ_1 = 12mm @ l_1 = 150mm

ϕ_2 = 17mm @ l_2 = 200mm

? σ_1; ? σ_2; ? $\Sigma \delta l$

(a) $\sigma_1 = \dfrac{P}{A_1} = \dfrac{25}{\pi (12)^2 \div 4} = 0.22 \text{ N/mm}^2$ #

$\sigma_2 = \dfrac{P}{A_2} = \dfrac{25}{\pi (17)^2 \div 4} = 0.11 \text{ N/mm}^2$ #

Tips…
Since diameter and length is i unit mm ,hence unit E should be changed the same:-

$$E = 210 \times 10^9 \frac{N}{m^2} \times \frac{(1)^2}{(1000)^2} = 210 \times 10^3 \text{ N/mm}^2$$

(b) $\Sigma \delta l = \delta l_1 + \delta l_2 = \dfrac{P}{E}\left[\dfrac{l_1}{A_1} + \dfrac{l_2}{A_2}\right]$

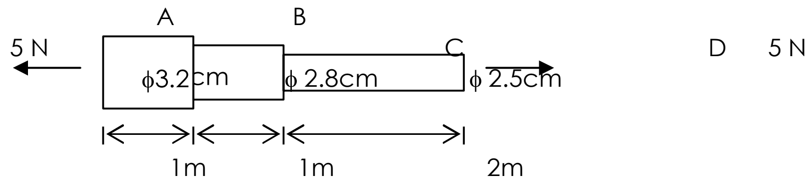
$$= \frac{25}{210 \times 10^3}\left[\frac{150}{\pi (12)^2 \div 4} + \frac{200}{\pi (17)^2 \div 4}\right]$$

$$= 1.19 \times 10^{-4}\left[\frac{150}{113.09} + \frac{200}{226.98}\right]$$

$$= 1.19 \times 10^{-4} (1.326 + 0.881) = 2.627 \times 10^{-4} \text{ N/mm}^2 \#$$

SOLUTION OF PROBLEM 3k To determine total

A steel bar of 4m in length is subjected to a tensile force of 5N. Calculate the elongation of the bar when given that
$E = 2.0 \times 10^6$ N/cm^2.



Solution

Given that; $L = 4$ m @ 400 cm
$P = 5$ N
$E = 2.0 \times 10^6$ N/cm^2
? $\Sigma \delta l$

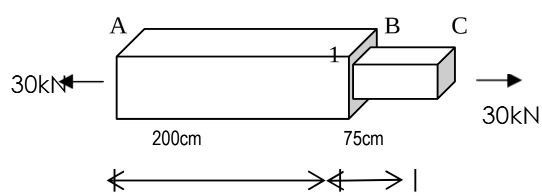
$$\Sigma \delta l = \delta l_{AB} + \delta l_{BC} + \delta l_{CD} = \frac{P}{E}\left[\frac{L_{AB}}{A_{AB}} + \frac{L_{BC}}{A_{BC}} + \frac{L_{CD}}{A_{CD}}\right]$$

$$= \frac{5}{2.0x10^6}\left[\frac{100}{\frac{\pi}{4}(3.2)^2} + \frac{100}{\frac{\pi}{4}(2.8)^2} + \frac{200}{\frac{\pi}{4}(2.5)^2}\right]$$

$$= 2.5 \times 10^{-6} (12.43 + 16.24 + 40.74) = 1.735 \times 10^{-4} \text{ cm } \#$$

SOLUTION OF PROBLEM 3l To determine stress and total elongation

A copper bar AC is subjected to a tensile force of 30 kN. Section AB and BC has a square cross-sectional area of
900cm^2 and 400cm^2 respectively. Calculate stress and elongation of the section. (E_{copper} = 110 GPa). Give the answer
in N and m units.



> *Tips…..*
> *Anda perlu seragamkan unit yang akan digunakan.*
>
> - E ✂ 110 GPa = 110 x 10^9 N/m^2
> - P ✂ 30kN = 30 x 10^3 N
> - *L* ✂ 1 cm = 1cm x $\frac{1m}{100cm}$
> - A ✂ 1 cm^2 = 1cm^2 x $\frac{1m^2}{100^2cm^2}$

Solution

(a) $\sigma_{ab} = \frac{P}{A_{ab}} = \frac{30 \, x10^{3}}{900 \, x10^{-3}} = 33.33 \times 10^3$ N/m^2 #

$$\delta l_{ab} = \frac{Pl_{ab}}{A_{ab}E} = \frac{(30 \times 10^{3})(2)}{(900 \times 10^{-3})(110 \times 10^{9})} = \frac{60000}{99 \times 10^{9}} = 6.06 \times 10^{-7}\ \text{m}\ \#$$

b) $$\sigma_{bc} = \frac{P}{A_{bc}} = \frac{30 \times 10^{3}}{400 \times 10^{-3}} = 75 \times 10^{3}\ \text{N/m}^2\ \#$$

$$\delta l_{bc} = \frac{Pl_{bc}}{A_{bc}E} = \frac{(30 \times 10^{3})(0.75)}{(400 \times 10^{-3})(110 \times 10^{9})} = \frac{22500}{44 \times 10^{9}} = 5.11 \times 10^{-7}\ \text{m}\ \#$$

3.2 STRESS AND STRAIN

STRESS AND STRAIN OF A COMPOSITE BAR

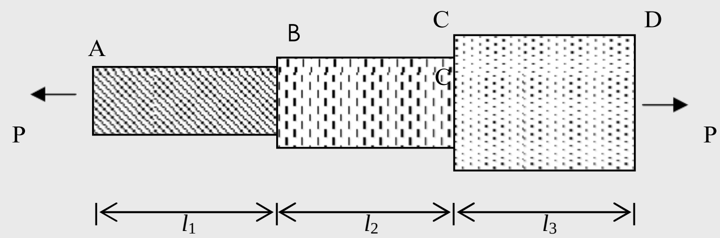


Figure 3.2 : Compound Bar Section
Note: Each section experiences the same force reaction

Definition : A composite bar section is a bar made of two or more materials with both ends are rigidly fixed so that the load applied to it is being shared together and hence experience the same elongation or deformation.

There are bars that have different characteristics in terms of their cross-sectional area and the materials that they area made of (Figure 6.6). As such, stress, strain and change in length for each section of bar will acts individually. Total change in length is the sum of every change in length of each section. As a composite bar consists of at least two different materials, then it involved with different value of Modulus of Elasticity.

Consider Figure 6.6:-

P = Force
E_1 = Modulus of Elasticity of section 1
l_1 = Length of section 1
A_1 = Cross-sectional area of sction 1
l_2, A_2, E_2 = Sequene of section 2 and that follows

Thus; $\sigma_{AB} = \dfrac{P}{A_1}$; $\sigma_{BC} = \dfrac{P}{A_2}$; $\sigma_{CD} = \dfrac{P}{A_3}$

hence; $\delta l_{AB} = \dfrac{Pl_1}{A_1 E_1}$; $\delta l_{BC} = \dfrac{Pl_2}{A_2 E_2}$; $\delta l_{CD} = \dfrac{Pl_3}{A_3 E_3}$

with that; $\Sigma \delta l = \delta l_{AB} + \delta l_{BC} + \delta l_{CD} = \dfrac{Pl_1}{A_1 E_1} + \dfrac{Pl_2}{A_2 E_2} + \dfrac{Pl_3}{A_3 E_3}$

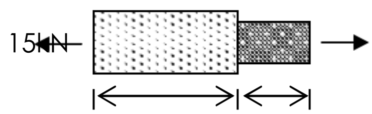
$$\Sigma \delta l = P\left[\frac{L_1}{A_1 E_1} + \frac{L_2}{A_2 E_2} + \frac{L_3}{A_3 E_3}\right]$$

This Formula is suitable used if material for each section of bar is different to each other i.e different Modulus of Elasticity

SOLUTION OF PROBLEM 3m

Elongation of a Compound Bar

A compound rod of 350mm in length consists of a copper rod 250mm long with a diameter of 15mm is rigidly connected to a bronze bar of 100mm long and 12mm in diameter. If a tensile load of 15kN is applied to this compound bar, determine its total elongation. [given E_b = 150 GN/m^2 & E_c = 110 GN/m^2]



15kN

250mm 100mm

Solution

Given that;

P = 15kN @ 15 x 10^3 N

E_1 = 110 GN/m^2 @ 110 x 10^9 N/m^2

E_2 = 150 GN/m^2 @ 150 x 10^9 N/m^2

l_1 = 250mm @ 0.25m ; ϕ_1 = 15mm @ 0.015m

l_2 = 100mm @ 0.1m ; ϕ_1 = 12mm @ 0.012m

$$\therefore A_1 = \frac{\pi d^2}{4} = \frac{\pi(0.015)^2}{4} = 1.767 \times 10^{-4} \text{ m}^2$$

$$\therefore A_2 = \frac{\pi d^2}{4} = \frac{\pi(0.012)^2}{4} = 1.131 \times 10^{-4} \text{ m}^2$$

> *Tips….*
>
> *You are free to choose the changes in units either from mm to m or vice versa*
>
> *You also may solve this problem by calculating elongation of each section individually and then summing them up.*

$$\Sigma \delta l = P\left[\frac{L_1}{A_1E_2} + \frac{L_2}{A_2E_2}\right]$$

$$= 15 \times 10^3 \left[\frac{0.25}{(1.767x10^{-4})(110x10^9)} + \frac{0.1}{(1.131x10^{-4})(150x10^9)}\right]$$

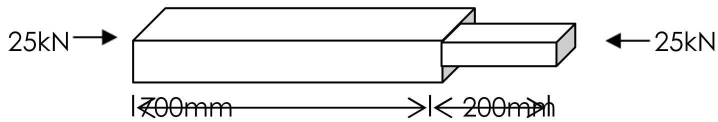
$$= 15 \times 10^3 \left[\frac{0.25}{19.44x10^6} + \frac{0.1}{16.97x10^6}\right]$$

= 15 x 10^3 (1.875 x 10^{-8}) = 2.813 x 10^{-4} m @ 0.281 mm #

SOLUTION OF PROBLEM 3n

To determine total deformation of a Compound Bar

A steel bar is rigidly fixed to an aluminium bar with their lenght of 700mm and 200mm respectively. Cross-sectional area of steel is 230mm^2 and that of aluminium is 150mm^2. Determine the total shortening of the compound bar when subjected to a compressive load of 25kN. [E_s = 207 GPa dan E_a = 70GPa]



Solution

Given; $L_k = 700mm$; $A_k = 230mm^2$
$L_a = 200mm$; $A_a = 150mm^2$

$$\Sigma \delta l = P\left[\frac{L_1}{A_1E_2} + \frac{L_2}{A_2E_2}\right]$$

$$= 25 \times 10^3 \left[\frac{700}{(230)(2.07 x10^5)} + \frac{200}{(150)(7 x10^4)}\right]$$

$$= 25 \times 10^3 \ (1.47 \times 10^{-5} + 1.90 \times 10^{-5}) = 0.844 \text{ m}$$

Conversion of unit N/m² $\Rightarrow$ N/mm²

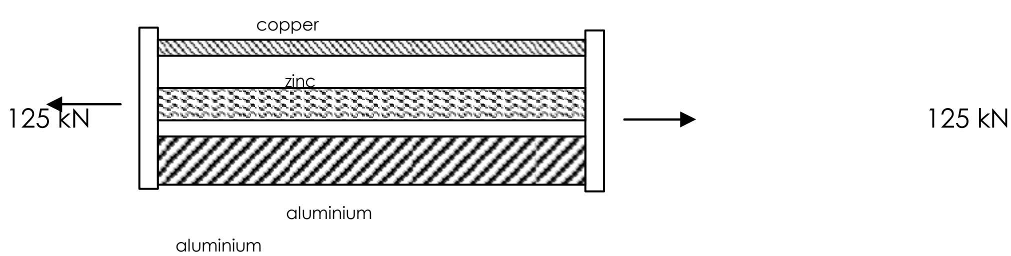
E_s = 207 Gpa
= 207×10^9 N/m²
= $207 \times 10^9 \times \frac{1^2 m^2}{1000^2 mm^2}$
= 2.07×10^5 N/mm²
@

E_a = 70 Gpa
= $70 \times 10^9 \times 10^{-6}$
= 7×10^4 N/mm²

SOLUTION OF PROBLEM 3o

Determination of stress and load distribution in a composite bar

A composite bar consists of 3 bars of copper, zinc and aluminium of the same length are rigidly fixed at both ends. Cross-sectional area of each bar is 250mm², 375mm² and 500mm² respectively. If this composite bar is subjected to a tensile load of 125kN, determine the load distribution in each bar. [E_b= 130 GN/m² ; E_z = 100 GN/m² ; E_a = 80 GN/m²]



Solution:

Load of 125 kN is distributeed to bronze, zinc and aluminium, where :

$P_c + P_z + P_a = 125$ kN -----(i)

Since all bars are regidly fixed, thence shange of length is the same :

$$\therefore \ \delta l = \frac{P_c L}{A_c E_c} = \frac{P_z L}{A_z E_z} = \frac{P_a L}{A_a E_a}$$

Select equation of change of length of copper and zinc;

$$\Rightarrow \quad \frac{P_c L}{A_c E_c} = \frac{P_z L}{A_z E_z}$$

$$P_c = \frac{A_c E_c}{A_z E_z} . P_z$$

$$P_c = \left[\frac{250 x10^{-6} \, x \, 130 x10^{9}}{375 x10^{-6} \, x \, 100 x10^{9}} \right] P_z$$

$$= \frac{32500}{37500} P_z = \frac{15}{13} P_z \text{ --------- (ii)}$$

Select equation of change of length of copper and aluminium;

$$\frac{PcL}{A_c E_c} = \frac{P_a L}{A_a E_a}$$

$$P_c = \frac{A_c E_c}{A_a E_a} . P_a$$

$$P_c = \left[\frac{250 x10^{-6} \, x \, 130 x10^{9}}{500 x10^{-6} \, x \, 80 x10^{9}} \right] P_a$$

$$= \frac{32500}{40000} P_a = \frac{13}{16} P_a \text{ -------(iii)}$$

Load distribution in each bar:

Insert value of P_a and P_z in (i):-

$$P_c + \frac{15}{13} P_c + \frac{16}{13} P_c = 125 \text{ kN}$$

$$\frac{44 P_c}{13} = 125$$

$$P_c = \frac{125 x13}{44} = 36.93 \text{ kN } \#$$

Thence;

$$P_z = \frac{15}{13}(36.93) = 42.61 \text{ kN } \#$$

$$P_{al} = \frac{16}{13}(36.93) = 45.45 \text{ kN } \#$$

Stress in each bar :

$$\sigma_c = \frac{P_c}{A_c} = \frac{36.93}{250} = 0.148 \text{ kN/mm}^2 \#$$

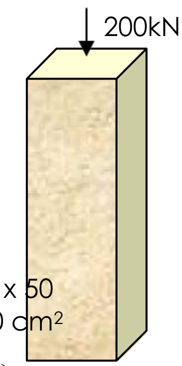
$$\sigma_z = \frac{P_z}{A_z} = \frac{42.61}{375} = 0.114 \text{ kN/mm}^2 \#$$

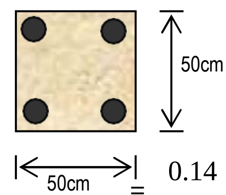
$$\sigma_{al} = \frac{P_{al}}{A_{al}} = \frac{45.45}{500} = 0.09 \text{ kN/mm}^2 \#$$

SOLUTION OF PROBLEM 3p

Determination of stress in composite bar

A concrete column 50cm x 50cm contains 4 reinforcement steel bar at each conner with diameter 2.5cm. This column carries an axial load of 200kN. Determine the stress in concrete and reinforcement. [$E_C = 0.14 \times 10^6$ kN/cm^2 ; $E_S = 2.1 \times 10^6$ kN/cm^2]





Solution :

Area of column, $A_T = 50 \times 50$
$= 2500$ cm^2

Area of steel, $A_s = 4\left(\dfrac{\pi . d^2}{4}\right)$
$= \pi (2.5)^2$
$= 19.63$cm^2

Area of concrete, $A_O = A_T - A_s$
$= 2500 - 19.63$
$= 2480.37$cm^2

> *Tips……*
> *Strain in concrete is the same as strain in steel i.e* $\sigma_c = \sigma_s$

Where ;

Strain, $\varepsilon = \dfrac{\sigma}{E}$

Hence strain of concrete is equal to strain in steel;
Strain of concrete, ε_C = Strain of steel, ε_s

$$\frac{\sigma_c}{E_c} = \frac{\sigma_s}{E_s}$$

$$\sigma_C = \frac{E_c}{E_s} . \sigma_s$$

$$= \frac{0.14 \times 10^6}{2.1 \times 10^6} . \sigma_s$$

$= 0.067\ \sigma_S$ ------- (i)

Knew that ;

$P = \sigma A$

$\therefore\ 200 = \sigma_C A_C + \sigma_S A_S$ ---------- (ii)

Insert *(i) in (ii);*

$200 = (0.067\ \sigma_S)\ A_S + \sigma_S A_S$

$= (0.067\ \sigma_S)\ 2480.37 + \sigma_S\ (19.63)$

$= 166.18\ \sigma_S + 19.63\sigma_S$
$= 185.81\ \sigma_S$

$$\therefore\ \sigma_S = \frac{200}{185.81}$$

$= 1.076$ kN/cm^2
$= 1076$ N/cm^2 #

From equation (i):

$\sigma_C = 0.067\ \sigma_S$
$= 0.067\ (1076)$
$= 72.12$ N/cm^2

3.2.2 CALCULATION OF MODULUS OF ELASTICITY FROM TENSILE TEST DATA

SOLUTION OF PROBLEM 3q

Calculate stress and percentage change of length from a test data

The following result was obtained from a tensile test.

Diameter = 10mm
Gauge length = 50mm
Maximum load = 40 kN
Final length = 58.88mm
Neck diameter = 7.7mm

Determine ; a) Ultimate stress
b) Percentage of elongation

Solution

a) Ultimate stress = $\dfrac{\text{Maximum Load}}{\text{Initial x-sectional Area}} = \dfrac{40}{\pi(10)^2/4}$

$= 0.509 \text{ kN/mm}^2 = \mathbf{509 \text{ N/mm}^2}$ #

b) %tage of elongation = $\dfrac{\text{Change of length}}{\text{Initial length}} = \dfrac{58.88 - 50.0}{50} \times 100 = \mathbf{17.76\%}$#

SOLUTION OF PROBLEM 3r

Calculate Young's Mudulus and change of shape from test data

During a tensile test on a sample, result as in Table 7.6 was obtained.

Load (kN)	5	10	15	20	25	30
Elongation x 10^{-3}mm	40	78	117	157	197	237

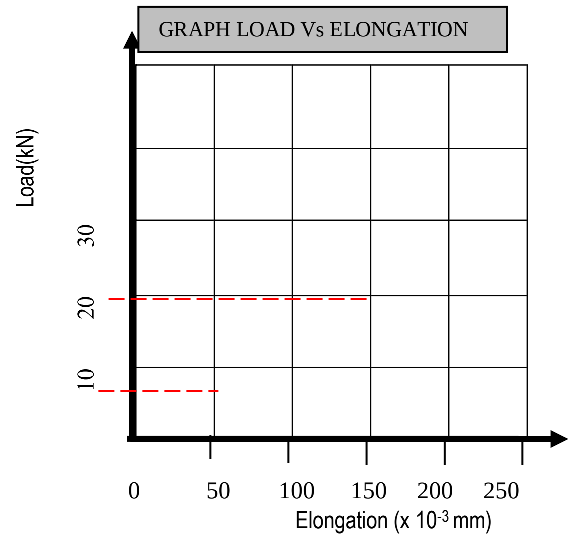
Table 7.6: Load-Elongation Data

The following shows data of the sample used:
Initial diameter = 12.5mm
Gauge length = 200mm
Final diameter = 8.0mm
Final length = 260mm

Determine: a) Young's Modulus
b) Percentage of elongation
c) Percentage reduction in area

Solution

Tips…
- *Graph drawn on graph paper.*
- *Choose a suitable scale for the graph.*
- *Points connection is a straight line that represent and joined most points.*



From graph:

$y_1 = 6.5$

$y_2 = 19$

$x_1 = 50$

$x_2 = 150$

(a) Slope of graph, m = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{19 - 6.5}{(150 - 50)x10^{-3}}$ = **125 kN/mm**

$\Rightarrow$ Young's Modulus, E = m x $\frac{L}{A}$ = 125 x $\frac{200}{\frac{\pi (12.5^2)}{4}}$ = **203.72 kN/mm²#**

(b) %tage of elongation = $\frac{\text{Change of length}}{\text{Initial length}}$ x 100 = $\frac{260 - 200}{200}$ x 100 = **30%**

(c) %tage reduction in area = Reduction in area x 100 = $\frac{\pi (12.2)^2/4 - \pi(8)^2/4}{\pi (12.2)^2/4}$ x 100

= $\frac{385.53 - 50.27}{385.53}$ x 100 = 86.96 %#

SOLUTION OF PROBLEM 3s Calculate Modulus of elasticitty from a test data

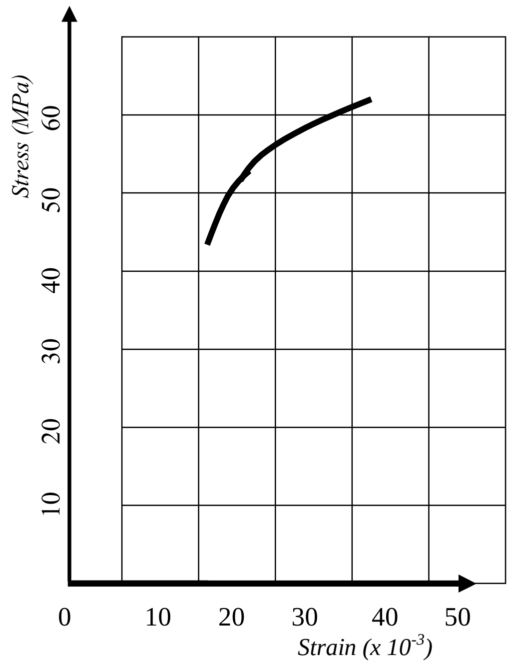
A sample of material is tested with tensile test dan resulted the stress-strain data as shown in Table 7.7. Plot the stress against strain graph and ddetermine the Modulus of Elasticity of the material. Do you classify this material as brittle or elastic?

Sterss (MPa)	8.0	17.5	25.6	31.1	39.8	44.0	48.2	53.9	58.1	62.0	62.5
Strain (x 10-3)	3.2	7.3	11.1	12.9	16.3	18.4	20.9	26.0	33.1	42.9	patah

Table : Stress-Strain Data

Solution

GRAPH OF STRESS Vs STRAIN



Tips….

- *Plot graph on graph paper.*
- *Joint connection is a linear line best representing and connecting most of the points.*
- *Slope is calculated at the straight portion of the line.*
- *Slope of graph is the value of the Modulus of Elasticity.*

(a) Modulus of Elasticity

$$= \text{slope of graph, } m$$

$$= \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{(24 - 5)10^{6}}{(10 - 2)10^{-3}}$$

$$= 2.4 \text{ GPa } \#$$

(b) Brittle (low elongation before rapture and without any warning).

SOLUTION OF PROBLEM 3t

Calculate Modulus of Elasticity and maximum stress from a test data

A tensile test on a speimen gives the the result as in Table 7.8;

Gauge length = 250mm
Initial diameter= 25mm
Final diameter = 18.6mm

Load (kN)	20	60	100	140	160	170	172	176	178
Elongation x 10^{-3} mm	50	160	260	360	410	440	470	550	720

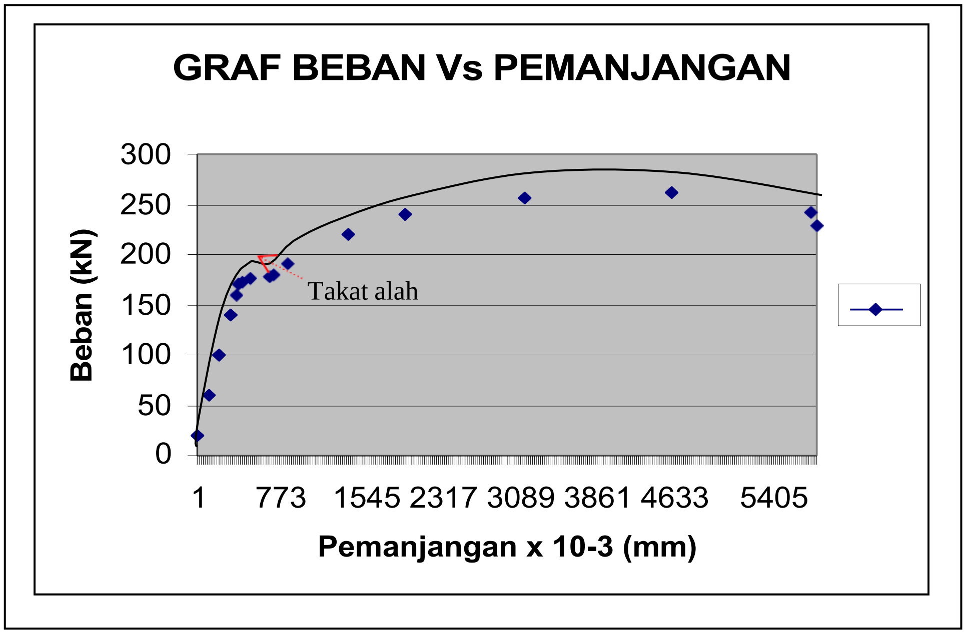
Load (kN)	180	190	220	240	257	261	242	229
Elongation x 10^{-3} mm	760	900	1460	1990	3120	4500	5800	5850

Table : Load - Elongation Data

By drawing load – elongation graph, determine:

a) Modulus of Elasticity
b) Maximum stress
c) Failure limit

Solution



From the plotted graph:-

(a) Slope of graph, $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{135 - 40}{(350 - 100)10^{-3}}$ = **380 kN/mm**

$\Rightarrow$ Modulus of elasticity, $E = m \times \dfrac{L}{A} = 380 \times \dfrac{250}{\pi(25)^2/4}$ = **193.53 kN/mm² @ 194 GN/m²**

(b) Maximum stress = $\dfrac{\text{Ultimate load}}{\text{Area}} = \dfrac{261}{\pi(25)^2/4}$ = **0.532 kN/mm²**

(c) Failure limit = refer to graph

Determining the slope of a graph may result in getting different answers; this depends to the accuracy of the plotted graph.

SPECIFIC OBJECTIVE:

At end this unit the students should be able to :

- Understand the basic knowledge of bending stress in beams.
- Apply the basic knowledge of bending stress in beams.

4.0 INTRODUCTION

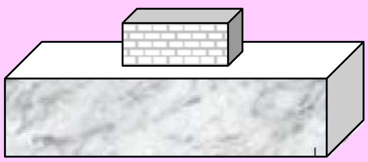


Figure 8.1(a) : Beam

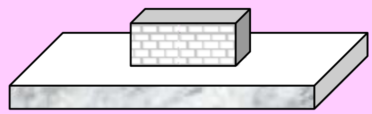


Figure 8.1(b): Beam

Observe the above two beams at figure 8.1. Both beams receive same loading.

- Which beam do you think is stronger and more stable?
- If uour answer is A, what is the rasionale?
- If uour answer is B, what is the rasionale?
- Is the shape mof the section would determine the strength and stableness of a beam?

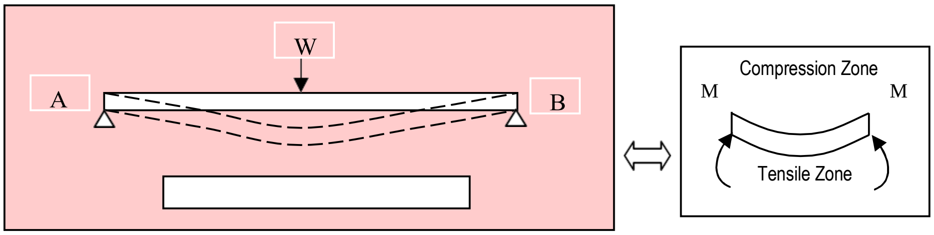
Generally, there exists structural members with different shape used in civil engineering. As shown in figure 8.1, beam A has a square section whereas beam B has a rectangular section. A part from the materials used, the shape of the structural member also detrmine the strength and stiffness of a structure.

The question of which beam is more strength and firm is determined by analyzing the second moment of area. The determnation of the second moment of area is closely related with centroid. Thus, this unit will discuss in detail about centroid and second moment of area for common sectional shape in engineering.

4.1 THE EFFECT OF SIDE LOADINGS

Consider two simple beams of different shape loaded as shown in figure 8.2(a) and 8.2 (b). Both beams experience deflection at different directions. The deflection is caused by the moment at side of the beams. The moment gives effect to bending moment and hence causing bending stress to occur in the beam.

Observe a filement at the bent beam (figure 8.2a), top surface of the beam experiences compression which reduses to zero at the centroidal plane whereas the lower part of the section experiencing tension to the maximum at the extreme outer lower surface. Imagine if the beam is made of hundreds of filement layers. Will there be filement that did not experience any change ie compression or tension? Yes there will be filement that will not experience any change ie compression or tension and it is situated at the neutral plane. Generally the neutral plane passes through the centre of gravity, thence for a sectional area it goes through the centroidal point. The following section will discuss centroid in more detail.



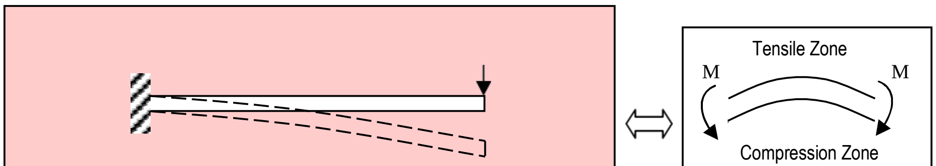


Figure 8.2(b) : Deflected beam

4.2 CENTROID

We knew that the world rotates at an axis through a centroid where the masses converge and centred, it is known as the centre of gravity. Centroid is also a term used to visualize the centre of an area plane (body that is considered as no mass). Both the centre of gravity and centroid are points of equilibrium. Generally centroid may be assumed as the centre of gravity for a shape of an area but of no mass. The determination of this centroid may be dermined based on the geometrical shape of the area.

4.2.1 Centroid of Basic Geometrical Shape

The position of the centroid is usually based on the reference axes that is the horizontal (x) and vertical (y) axes. The magnitude of the centroid is represented in the form of coordinate system (x, y). Figure 4.3 shows the position of the centroid, s of basic geometrical shapes.

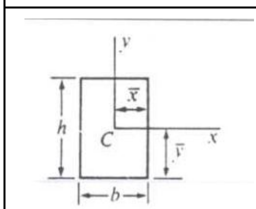
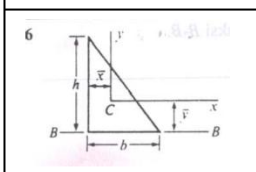
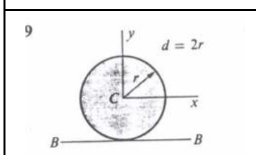
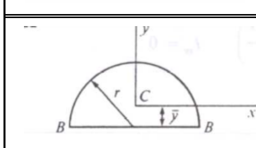
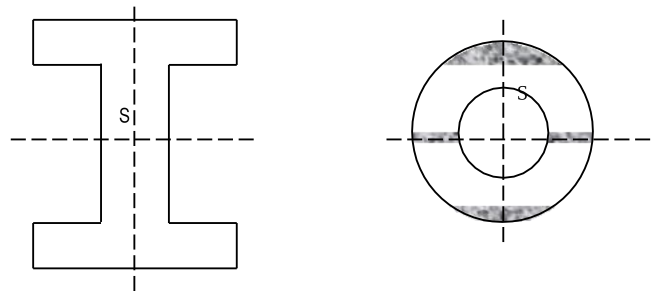
Shape	Area, A	x	y
	bh	$\frac{b}{2}$	$\frac{h}{2}$
	$\frac{bh}{2}$	$\frac{b}{3}$	$\frac{h}{3}$
	πrr^2	r	r
	$\frac{\pi rr^2}{2}$	r	$\frac{4r}{3\pi}$

Figure 4.3 : Centroid of Basic Geometrical Shapes

4.2.2 Centroid of Shapes Having Two Symetrical Axes

Generally there are shapes that are made from the combination of several and various basic geometrical shapes that have two symmetrical axes. The centroid for this particular shape is the point of intersections of the two axes at its centre of gravity. It is simple to determine, by just dividing the width to get the x-ordinate and dividing the height/depth to get the y-ordinate. (Refer Figure 4.4)



(a) 'I'-Shape (b) Hollow Circle

Figure 4.4 : Shape of Section With Two Symmetrical Axes

4.2.3 Centroid of A Composite Area

There are geometrical shapes that are not having any symmetrical axis. It may built-up from various basic geometrical shapes and areas joined or connected togather. The centroid of this compiste shape caan be determined by the moment of area method. Consider the shape as in Figure 8.5.

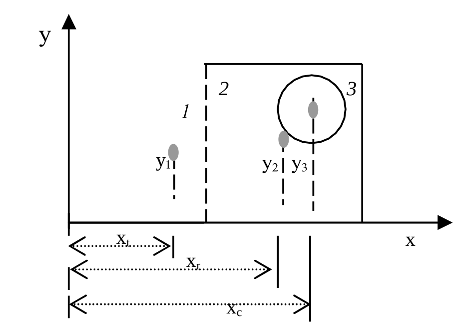
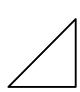
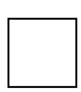


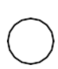
Figure 4.5 : Composite Setion

The composite shape shown contains a circular hole. To simplify the arithmetic, the shape is divided into three basic geometrical components that is triangle, rectangle and circle having an area of A_t, A_r and A_c respectively.

Say:-

 = Component 1 @ A_t

 = Component 2 @ A_r

 = Component 3 @ A_c

x_t, x_r, x_c = Distance of centroid from y-axis

y_t, y_r, y_c = Distance of centroid from x-axis

- Moment about the y-axis :

$$A\bar{x} = A_t x_t + A_r x_r - A_c x_c$$

$$\therefore \quad \bar{x} = \frac{A_t x_t + A_r x_r - A_c x_c}{A_t + A_r - A_c}$$

$$\bar{x} = \frac{\Sigma Ax}{\Sigma A}$$

- Moment about x-axis:

$$A\bar{y} = A_t y_t + A_r y_r - A_c y_c$$

$$\therefore \quad \bar{y} = \frac{A_t y_t + A_r y_r - A_c y_c}{A_t + A_r - A_c}$$

$$\bar{y} = \frac{\Sigma Ay}{\Sigma A}$$

SOLUTION OF PROBLEM 4

To determine centroid of a shape with two symmetrical axes

Determine the position of the centroid of cross-section the boxed culvert as in Figure 4.6. The section has a dimension of 1000mm x 800mm and 300mm thick.

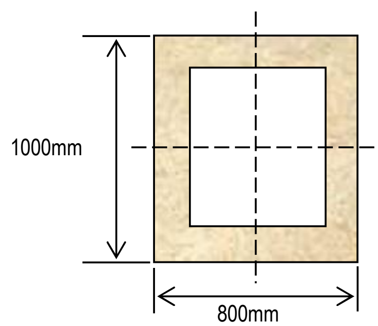
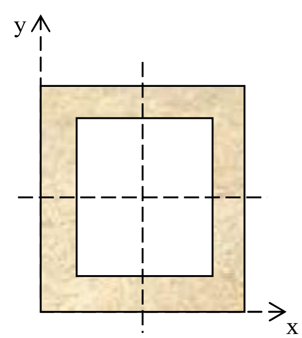


Figure 4.6 : Section of A Culvert

Solution

> *Tips…..*
> *The above section has two symmetrical axes, hence the centroid of the section is at the intersection of the axes.*



⇒ Position of centroid to the y-axis, $\bar{x}$ = b / 2

= 800 / 2 = **400mm**

⇒ Position of centroid to the x-axis, $\bar{y}$ = h / 2

= 1000 / 2 = **500mm**

SOLUTION OF PROBLEM 4 To determine centroid of shape with one symmetrical axial

Determine the position centroid of the T-section shown in Figure 4.7.

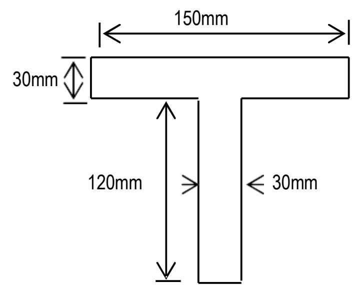
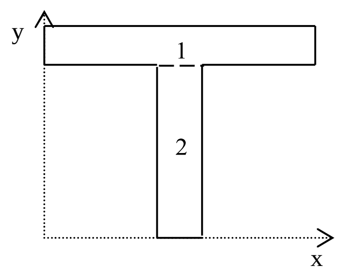


Figure 4.7: 'T' Section

Tips….
T shaped section has one symmetrical y-y axis. Hence the position of the centroid with reference to the y-axis, $\bar{x}$ can be determined by dividing the two width of the 'T' section i.e 75mm. Only the position of $\bar{y}$ has to be calculated.

Solution



Tips….
To simplify the arithmetic, divide the T section into two basic geometrical shapes i.e two perfect rectengles.

A_1 = 150 x 30 ; A_2 = 120 x 30 ; $\bar{y}_1$ = (30 4 2) + 120 ; $\bar{y}_2$ = 120 42

= 4500mm^2 = 3600mm^2 = 135mm = 60mm

$\Rightarrow$ $\bar{x}$ = b / 2 = 150 / 2 = **75mm#**

$$\therefore \bar{y} = \frac{A_1y_1 + A_2y_2}{A_1 + A_2} = \frac{(4500 \times 135) + (3600 \times 60)}{(4500 + 3600)} = 101.67\text{mm \#}$$

SOLUTION OF PROBLEM 4 Determine the centroid of a Composite Section

Determine the position of centroid of the section shown in Figure 4.8.

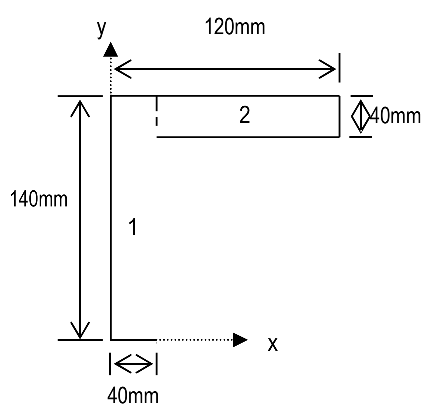


Figure 8.8: Non-symmetrical Section

Solution

$A_1 = 140 \times 40 = 5600mm^2$

$A_2 = 80 \times 40 = 3200mm^2$

> *Tips.....*
> *To simplify the arithmetic, divide the shape into two basic geometrical shapes ie perfect rectangles.*

Position of centroid of section 1 to the x-axis, $\bar{y}_1 = 140 / 2 = 70mm$

Position of centroid of section 2 to the x-axis, $\bar{y}_2 = 40 / 2 + 100 = 120mm$

Position of centroid of section 2 to the y-axis, $\bar{x}_1 = 40 / 2 = 20mm$

Position of centroid of section 2 to the y-axis, $\bar{x}_2 = 80 / 2 + 40 = 80mm$

Again, applying the formula for composite section;

$$\bar{x} = \frac{A_1x_1 + A_2x_2}{A_1 + A_2} = \frac{(55600 \times 20) + (3200 \times 80)}{(5600 + 3200)} = 41.82mm \#$$

$$\bar{y} = \frac{A_1y_1 + A_2y_2}{A_1 + A_2} = \frac{(5600 \times 70) + (3200 \times 120)}{(5600 + 3200)} = 88.18mm \#$$

SOLUTION OF PROBLEM 4 Determine the centroid of a composite area

Determine the position of centroid of the shaded area as shown in Figure 4.9.

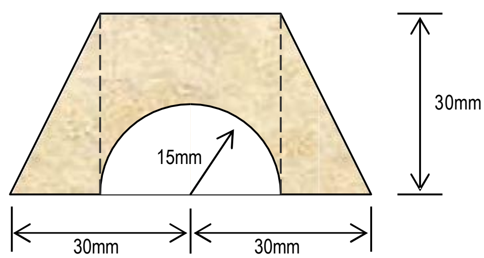
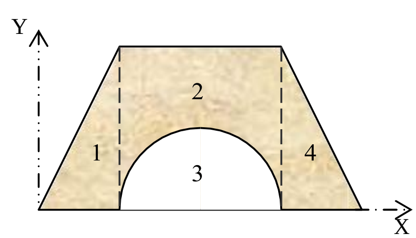


Figure 8.9 : Section With One Symmetrical Axis

Solution

> *Tips....*
> *The section is divided into four basic geometrical shape.*



(i) Distance of centroid from y-axis, $\bar{x}$ = 30mm# (symmetrical section with y-axis)

(ii) Distance of centroid from x-axis, $\bar{y}$, solved as in table below:.

Component	A (mm^2)	y (mm)	A y (mm^3)
Triangle, 1	0.5 x 15 x 30 = 225 (+)	30/3 = 10	2250
Rectengle, 2	30x30 = 900 (+)	30/2 = 15	13 500
Semicircle, 3	$(\pi \times 15^2) \div 2$ = 353.4 (-)	$(4 \times 15)/3\pi$ = 6.37	- 2251.2
Triangle, 4	0.5 x 15 x 30 = 225 (+)	30/2 = 15	2250
Σ	996.6		15 748.8

$$\therefore \bar{y} = \frac{\sum A\bar{y}}{\sum A} = \frac{15748.8}{996.6} = 15.8\text{mm}\#$$

4.2.1 SECOND MOMENT OF AREA

Second moment of area is also known as Moment of Inertia. Second moment of area symbolizes the stiffness of a sectional area. It is defines as the moment of the first moment of area. In other words, second moment of area is the product of area multiplies with the squares of the distance of the area to the reference axis. Its magnitude is different for a different axis.

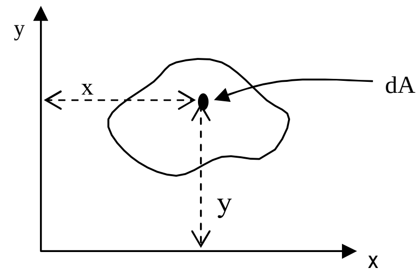


Figure 8.11 : Element of Area A

Second moment of area about the x-axis;

$I_x = \int_A y^2 dA$

Second moment of area about the y-axis;

$I_y = \int_A x^2 dA$

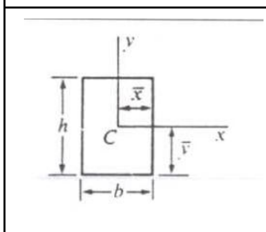
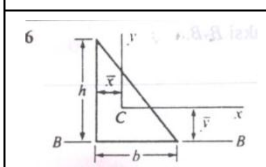
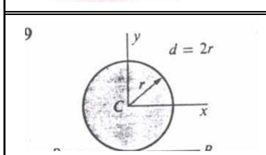
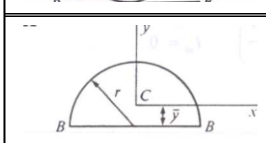
Bentuk	A (mm^2)	I_{pg}	I_{xx}
	bh	$\frac{bh^3}{12}$	$\frac{bh^3}{3}$
	$\frac{bh}{2}$	$\frac{bh^3}{36}$	$\frac{bh^3}{12}$
	πr^2	$\frac{\pi d^4}{64}$	$\frac{\pi d^4}{32}$
	$\frac{\pi r r^2}{2}$	$0.11r^4$	$\frac{\pi r^4}{8}$

Figure 8.12: Second Moment of Area for Basic Shapes

Note :
I_{ga} = Origin of axes is at the centroid, *I_{xx} = Origin of axes is at the corner.*

In designing structural members, the ability of these members in supporting loads is an important factor. The value of second moment of area is one of the factor to ensure the capability of the members to support the applied loads. Consider the following two retangular beam sections (Figure 8.11);

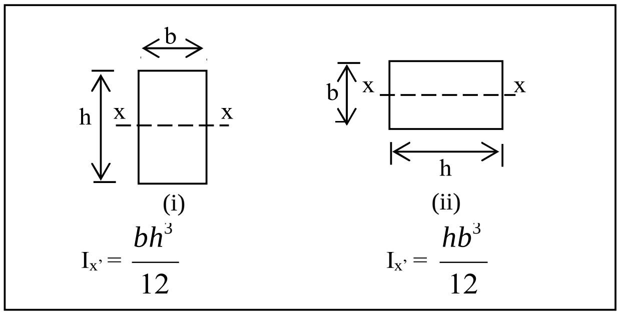


Figure 8.13: Perfect Rectangular Sections

Both beams are having the same cross-sectional area, however, the magnitude of second moment area, I_{pg} are different. The first section gives greater second moment of area, as such it is stiffer compared to the second section. As a comparision, consider the shape of sections as shown in Figure 8.14.

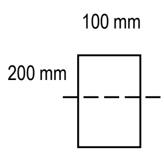
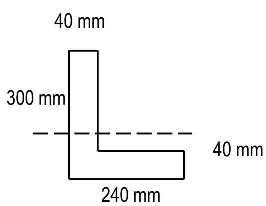
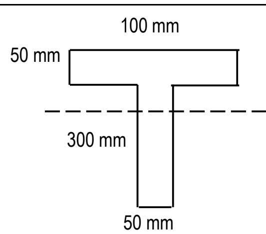
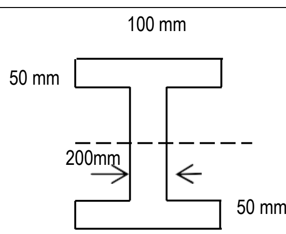
Bentuk rasuk	Luas keratan rentas	Momen luas kedua
	$A = 20 \times 10^3$ mm^2	$I = 66.7 \times 10^6$ mm^4
	$A = 20 \times 10^3$ mm^2	$I = 172.2 \times 10^6$ mm^4
	$A = 20 \times 10^3$ mm^2	$I = 228.4 \times 10^6$ mm^4
	$A = 20 \times 10^3$ mm^2	$I = 191.7 \times 10^6$ mm^4

Figure 4.14: Sections of the Same Area

From Figure 4.14 it is found that the same cross-sectional area of beam but having different value of second moment of area. This clearly shows that shape of section influences the value of second moment of area hence the stiffness of beam to support the loadings.

??? Which sectional area is the stiffest????

4.2.1 Theorem of Parrallel Axes

The theorem of parallel axes says that the second moment of area of a section about an axis is retaled to the second moment of area about its centroidal axis. The second moment of area about an arbitrary axis in x direction is given by;

$$I_x = \int_A y^2 \, dA \text{ --------- (i)}$$

Consider an area as shown bellow;

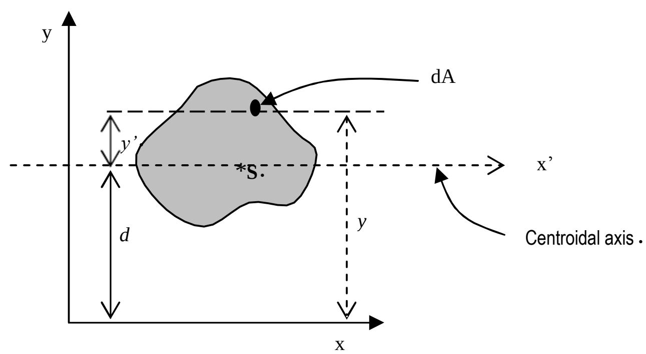


Figure 8.15 : Element of Area A

- Position of centroidal axis x' passing through the centroid, s is parallel to the x-axis.
- d, is distance between two axes *i.e* x-axis and centroidal axis.
- Distance of an elementary area dA from x-axis is y = y' + d
- From equation (i), and y is substituted by y' + d , then;

$$I_x = \int_A (y' + d)^2 \, dA$$

$$= \int_A y'^2 \, dA + 2d \int_A y' \, dA + d^2 \int_A dA \text{ --------(ii)}$$

From equation (ii) :-

- First integration - Second moment of area about its centroidal axis
- Second integration - Sum of moment of area about its centroidal axis is zero.
- Kamiran ketiga - Sum of area A

Therefore:-

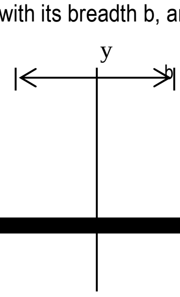
$$\mathbf{I_x = I_{ga} + Ad^2}$$

**Note : Theorem of parallel axes can be used if one of the axes is a centroidal axis.*

4.3 SECTION MODULUS

Difinition: The ratio of second moment of area of a section about its centroidal axis to the maximum distance between the centroid and the top/bottom most of the section. Normally it carries a symbol of Z.

Consider rectengular section with its breadth b, and height d as in Figure 8.16.



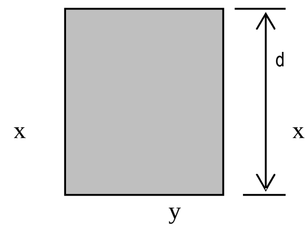


Figure 8.16 : Beam Cross-Section

Assume I_x as the second moment of area about x-axis.

$$\therefore \; Z_x = \frac{\text{Second moment of area about the centroidal axis}}{\text{Furthest distance of section from centroidal axis}}$$

$$= \frac{I_{xx}}{b/2}$$ (Where b/2 is the distance of centroid to the furthest end of section @ y_{mak})

@ $$Z_x = \frac{I_{xx}}{y_{max}}$$

Calculate the second moment of area for a perfect rectangle about its x-axis (Figure 4.17)

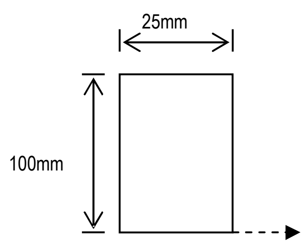


Figure **4.17**

Solution

For a perfect rectangle, second moment of area about its centroidal axis;

$I_{pg} = bh^3 / 12 = 25 \times 10^3 / 12 = 2.08 \times 10^6 \text{ mm}^4$

Using theorem of parallel axis;

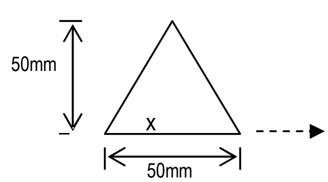
$I_x = I_{pg} + Ad^2 = (2.08 \times 10^6) + (100 \times 25 \times 50^2) = 8.33 \times 10^6 \text{ mm}^4$ #

SOLUTION OF PROBLEM

To calculate second moment of area of abasic geometrical seection

Calculate second moment of area for a triangle (kakisama) about its x-axis.

Figure 4.18



Solution

$I_{pg} = bh^3 / 36 = 50 \times 50^3 / 36 = 1.74 \times 10^5 \text{ mm}^4$

With the theorem of parallel axis;

$I_x = I_{pg} + Ad^2$

$= 1.74 \times 10^5 + \{(50x50)/2\}x(50/3)^2$

$= 5.21 \times 10^5$ mm^4 #

Calculate the second moment of area of the T-Section about its centroidal axis (Figure 4.19)

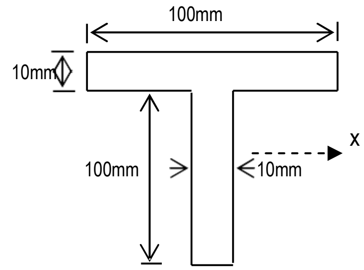
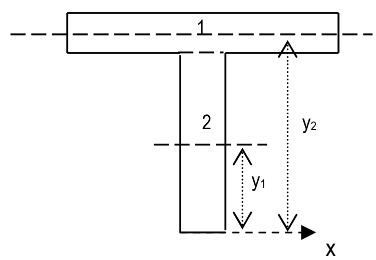


Figure 4.19: 'T' Section

Solution

(i) Divide the T-section into two rectengular components.
(ii) Determine the position of its centroid.



> *Tips…*
> *To simplify the calculation of centroid,, it is suggested to use table.*
> *T-Section has a symmetrical axis about y-axis hence $\bar{x}$ need not to be calculated*

Komponen	A (mm^2)	y (mm)	Ay
1	100 x 10 = 1000	100 + 10/2 = 105	1000 x 105 = 105 000
2	100 x 10 = 1000	100 4 2 = 50	1000 x 50 = 50 000
Σ	2000		155 000

$$\therefore \bar{y} = \frac{\Sigma Ay}{\Sigma A} = \frac{155000}{2000} = \mathbf{77.5\ mm}$$

(iii) To determine the second moment of area for the two components.

<u>*Component 1*</u>

$I_{pg} = bh^3 / 12 = 100 \times 10^3 / 12 =$ **8.33×10^3 mm^4**

From theorem of parallel axis;

$I_{pg1} = I_{pg} + Ad^2 = 8.33 \times 10^3 + [1000 \times (105 – 77.5)^2] =$ **7.65×10^5 mm^4**

<u>*Component 2*</u>

$I_{pg} = bh^3 / 12 = 10 \times 10^3 / 12 =$ **8.33×10^5 mm^4**

From theorem of parallel axis;

$I_{pg2} = I_{pg} + Ad^2 = 8.33 \times 10^5 + [1000 (77.5 – 50)^2] =$ **1.59×10^6 mm^4**

Momen luas kedua sekitar paksi sentroid;

$I_{pg} = I_{pg1} + I_{pg2} = 7.65 \times 10^5 + 1.59 \times 10^6 =$ **23.55×10^5 mm^4 #**

SOLUTION OF PROBLEM

To determine the second moment of area of an I-Section'

Calculate the second moment of area about its centroidal axis of the I-Section as shown in Figure 4.20.

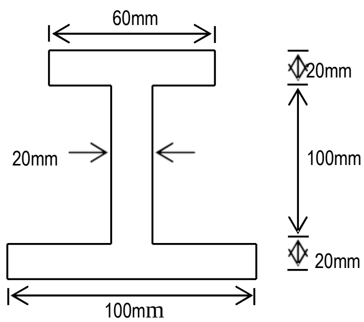
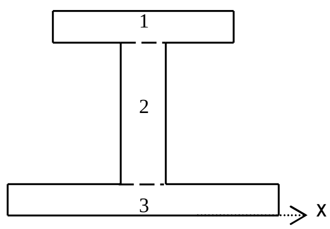


Figure 4.20 : I - Section

Solution

(i) Devide the section into three rectangular components.
(ii) Determine the position of the centroid.



- Assume y, as the distance of centroid of each component from base of section, the x-axis.

Component 1
$A_1 = 60 \times 20$ $= 1200\ mm^2$
$y_1 = 20 + 100 + 20/2$ $= 130\ mm$

Component 2
$A_2 = 100 \times 20$ $= 2000\ mm^2$
$y_2 = 20 + 100/2$ $= 70\ mm$

Component 3
$A_3 = 100 \times 20$ $= 2000\ mm^2$
$y_3 = 20/2$ $= 10\ mm$

Hence;

$$\bar{y} = \frac{A_1Y_1 + A_2Y_2 + A_3Y_3}{A_1 + A_2 + A_3}$$

$$= \frac{(1200x130) + (2000x70) + (2000x10)}{1200 + 2000 + 2000}$$

$= 60.77$ mm

(iii) Second Moment of Area

Component 1

$$I_{pg1} = \frac{bd^3}{12} = \frac{60x20^3}{12} = 40 \times 10^3\ mm^4$$

From the parallel axis theorem;

$I_{x1} = I_{pg} + Ad^2$

$= 40 \text{ x } 10^3 + [1200 \text{ x } (130 – 60.77)^2]$

$= 5.79 \text{ x } 10^6 \text{ mm}^4$

Component 2

$$I_{pg2} = \frac{20x100^3}{12} = 1.67 \text{ x } 10^6 \text{mm}^4$$

From the parallel axis theorem;

$I_{x2} = I_{pg} + Ad^2$

$= 1.67 \text{ x } 10^6 + [2000 \, (70-60.77)^2]$

$= 1.84 \text{ x } 10^6 \text{ mm}^4$

Component 3

$$I_{pg3} = \frac{100x20^3}{12} = 6.67 \text{ x } 10^4 \text{mm}^4$$

From the parallel axis theorem;

$I_{x3} = I_{pg} + Ad^2$

$= 6.67 \text{ x } 10^4 + [2000(60.77-10)^2]$

$= 5.22 \text{ x } 10^6 \text{ mm}^4$

Second Moment of areaof the I-section about its centroidal axis;

$I_x = I_{x1} + I_{x2} + I_{x3}$

$= (5.79 \text{ x } 10^6) + (1.84 \text{ x } 10^6) + (5.22 \text{ x } 10^6)$

$= 1.285 \text{ x } 10^7 \text{ mm}^4$ #

4.2.3 INTRODUCTION

In unit 2, you have been exposed to a beam, when loaded, it will experience bending moment and shear force to the entire section of the beam. In this unt however, we will see in more detail on the effect of bending moment compared to shear force. When a beam is subjected to a load along its longitudial axis, it tends to bend to bend and the analysis of the magnitude of bending has been discussed in topic 2. What will be determined in this unit is the internal stress that axists in the beam section to counter the bending moment. Bending theory is a theory that explains about bending stress at a beam section.

SIMPLE BENDING THEORY

Simple bending theory is a theory that enable the analysis of the magnitude and distribution of bending stress at any section of the beam. As has been discussed in unit 8 that the bending level at a section are different and varies, and depends on the distance from the neutral axis. Bending along the longitudinal axis of beam tends to course compression and tension at the longitudinal section of the beam.

However, there are several assumptions should be followed in order to get the bending theory, that is;

(i) cross-section of the beam remains plane before and after loading.
(ii) Material of beam is homogeneous and obeys Hooke's Law.
(iii) Modulus of elasticity for compression and tension is equal and the same.
(iv) Beam is straight and having uniform cross-section along the beam section.
(v) Load acts parallel with the vertical axis of beam cross-section and perpendicular *(serenjang)* to the longitudinal axis of beam.

To determine the bending stress, consider a beam AB subjected to an external load P. Bagi mendapatkan tegasan lentur, pertimbangkan satu rasuk AB yang dibebankan dengan daya luaran sebesar P. Rasuk akan melentur dengan permukaan atas rasuk mengalami mampatan dan permukaan bawah mengalami tegangan.

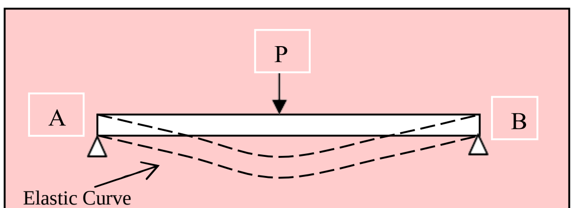


Figure 9.1 : Pure Bending

4.2.3.1 Derivation of Bending Stress Formula

Consider a small element of the beam, *dx* that experiences bending moment (Figure 9.2). This causes the lement to bend with O as the centre of the curvature.

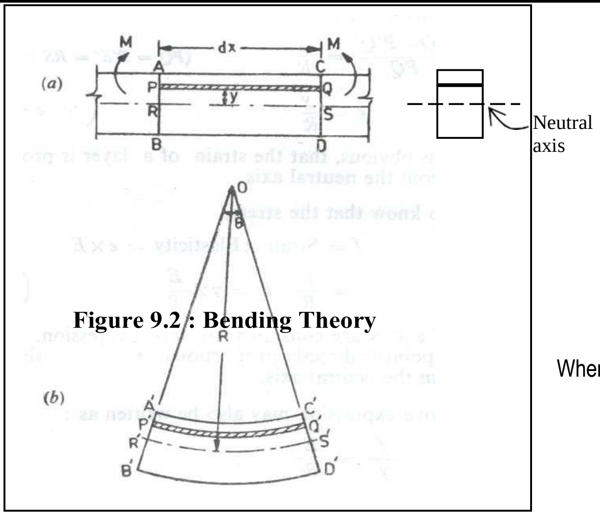


Figure 9.2 : Bending Theory

Where;

M = Moment
Θ = Angle of curve
R = Radius of curve

Consider;

- Filament PQ at a distance y from RS, neutral axis ofbeam
- This filament will not compressed to P'Q' after bending

∴ Reduction in length of the filement PQ:-

$$\delta l = PQ - P'Q'$$

∴ Strain, $\varepsilon = \dfrac{\delta l}{l}$

$$= \frac{PQ - P'Q'}{PQ} \text{ -------- (i)}$$

Consider the geometrical shape of the curve (Figure 9.2(b), it found out that two parts OP'Q' adn OR'S' are the same;

$$\therefore \quad \frac{P'Q'}{R'S'} = \frac{R-y}{R}$$

$$@ \quad 1 - \frac{P'Q'}{R'S'} = 1 - \frac{R-y}{R}$$

And simplified as :

$$\frac{R'S' - P'Q'}{PQ} = \frac{y}{R}$$

$$\frac{PQ - P'Q'}{PQ} = \frac{y}{R}$$

From (i) : $\therefore \ \varepsilon = \dfrac{y}{R}$ ✍

From here it shows clearly that strain is proportional to distance from neutral

We know that;

Stress, σ = Strain x Modulus of Elasticity

$$= \frac{y}{R} \times E = y \times \frac{E}{R}$$

or

$$\frac{\sigma}{y} = \frac{E}{R}$$

4.2.3.2 Resistance Moment

In the earlier part of this unit, it has been discussed that bending in beam is caused by the reaction moment at both ends of the beam section. There axists bending stress in the beam section that produces resistance moment of equal magnitude with the external moment. Consider beam section below (Figure 9.3)

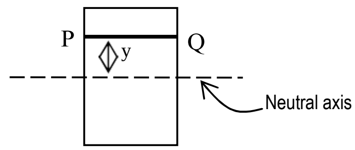


Figure 9.3 : Element of a beam section

Consider filament PQ at a distance of y from the neutral axis;

Assume; δa = area of filement PQ

Known that; $\sigma = y \times \frac{E}{R}$

$\therefore$ Total stress in this filement

$$\sigma_{PQ} = y \times \frac{E}{R} \times \delta a$$

Moment of force in the filament PQ about the neutral axis,

$$M = y \times \frac{E}{R} \times \delta a \times y$$

$$= \frac{E}{R} \cdot y^2 \cdot \delta a \text{ ---------(i)}$$

the term $y^2 \cdot \delta a$ represent the second moment of area of filement about the neutral axis;

$$\therefore \quad M = \frac{E}{R} \times I$$

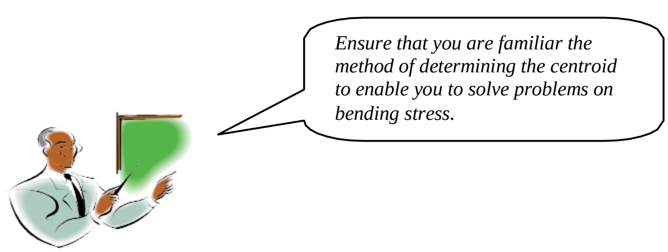
$$\frac{M}{I} = \frac{E}{R}$$

$$\therefore \quad \frac{M}{I} = \frac{E}{R} = \frac{\sigma}{y}$$

POSITION OF THE NEUTRAL AXIS

It has been discussed earlier that the filement at the neutral axis does not experience any deformation deu to bending. As if this filement does not being subjected to bending moment force. As being proved from the bending theory, bending stress is proportional to distance of filement from the neutral axis. The further the filement from the neutral axis the greater the bending stress owuld be.

The question is, where is the position of the neutral axis? Glance through the discussion in unit 8, where any moment of area about the neutral axis is equal to zero. This shows that neutral axis of a section is across or passed through the centroid of that section.



DISTRIBUTION OF BENDING STRESS

For a simply supported beam at booth ends, compressive bending stress occurs above the neutral axis and the tensile bending stress occurs below the neutral axis. Maximum bending stress, σ_m situated at the furhtest distance from the neutral axis (Figure).

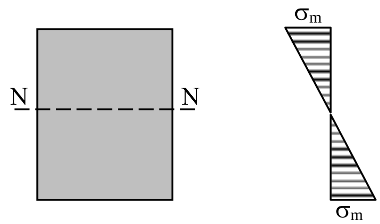


Figure: Bending Stress Distribution

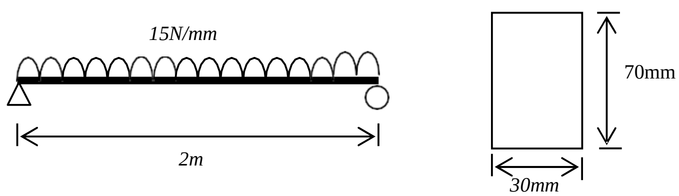
Lets go to the following that shows method of solving problems on bending stress. Turn to the next pages.

SOLUTION OF PROBLEM

To determine the load that can be supported by the beam

To calculate maximum bending stress and to sketch the stress distribution diagram

A 2m beam simply supported at both ends, is subjected to a uniformly distributed load of 15N/mm as shown below. Determine the maximum bending stress that ooccurs in the beam section which has a dimension of 30mm wie and 70mm depth. Sketch the stress distribution diagram to the section.



Solution

(i) Moment of inertia of the section;

$$I_{pg} = \frac{bh^3}{12} = \frac{30 \, x \, 70^3}{12} = 8.58 \times 10^5 \text{ mm}^4$$

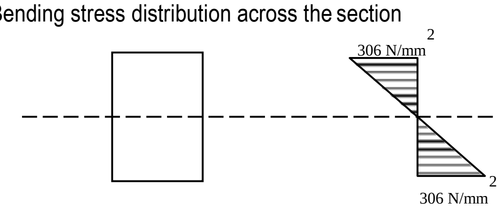
(ii) Maximum bending moment for simply supported beam loaded with a udl;

$$M_{mak} = \frac{wL^2}{8} = \frac{15 \, x \, 2000^2}{8} = 7.5 \times 10^6 \text{ Nmm}$$

(iii) Maximum bending stress occurs at a distance 35mm from the neutral axis;

$$\sigma_{mak} = \frac{My}{I} = \frac{(7.5 x 10^6) 35}{8.58 \, x \, 10^5} = 305.94 \text{ N/mm}^2$$

(iv) Bending stress distribution across the section



SOLUTION OF PROBLEM

To determine the load that can be supported by the beam

A beam section has a dimension of 300mm deep is simply supported with a span of 4m long. What is the magnitude of a uniformly distributed load that can be applied to the beam if the bending stress is not to exceed 120 N/mm^2. [Given I = 8 x 10^6 mm^4]

Solution

maximum bending moment of simply supported beam under a udl is given by;

$$M_{max} = \frac{wl^2}{8} = \frac{w(4^2)}{8} = 2w \quad \text{------------- (i)}$$

Also known that; $\frac{M}{I} = \frac{\sigma}{y}$

$$M = \frac{\sigma}{y} \times I = \frac{120}{150} \times 8 \times 10^6 = 6.4 \times 10^6 \text{ Nmm} = 6.4 \times 10^3 \text{ Nm}$$

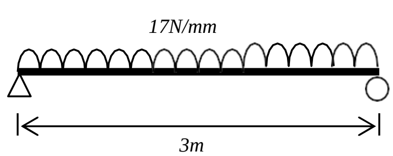
From (i); $2w = 6.4 \times 10^3$

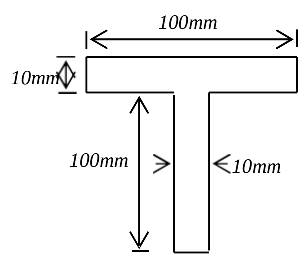
$$\therefore \quad w = \frac{6.4 x 10^3}{2} = 3.2 \times 10^3 \text{ N/m \#}$$

SOLUTION OF PROBLEM

To determine maximum bending stress of a T-section

A steel beam of a T-section with 3m long is simply supported at both ends is subjected to a uniformly distributed load of 17N/mm through its span. Calculate the maximum bending stress of the section.





Solution

<u>*Steps of solution*</u>

(i) Determine position of centroid.
(ii) Determine second moment of area.

> *Please refer solution of problem 8g in unit 8 in determining the centroid and the calculation of second moment of area.*

y_{mak} = 77.5 mm and I = 23.55 x 10^5 mm^4

(iii) To calculate maximum bending stress.

Given that;

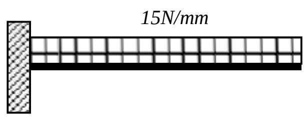
$$M_{mak} = \frac{wL^2}{8} = \frac{17 x 3000^2}{8} = 19.13 \times 10^6 \text{ Nmm}$$

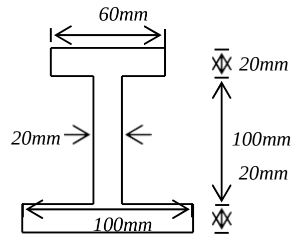
Also;

$$\sigma_{mak} = \frac{M_{mak}}{I} \times y_{mak} = \frac{19.13\,x10^6}{23.55\,x10^5} \times 77.5 = 629.54\ N/mm^2\ \#$$

SOLUTION OF PROBLEM

A 4m long centilever beam is of an I-section. The beam carries a unifomly distributed load of 15N/mm through its entire length. Calculate the maximum bending stress and sketch the stres distribution.





Solution

Steps of solution

(i) Determine position of centroid.
(ii) Determine second moment of area.

Please refer solution of problem 8g in unit 8 in determining the centroid and the calculation of second moment of area.

$y_{tegangan}$ = 79.23 mm ; $y_{mampatan}$ = 60.77 mm ; $I = 1.285 \times 10^7\ mm^4$

(iii) To calculate the maximum bending stress.

Given that;

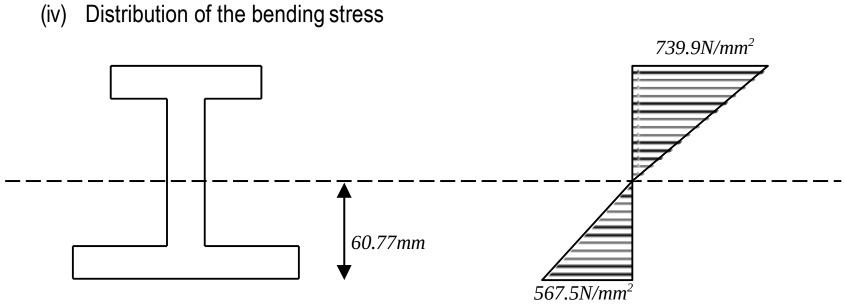
$$M_{mak} = -\frac{wl^2}{2} = -\frac{15\,x\,4000^2}{2} = 120 \times 10^6\ Nmm$$

Also;

$$\sigma_{maxcomp} = \frac{M_{max}}{I} \times y_{maxcomp} = \frac{120\,x10^6}{1.285\,x10^7} \times 60.77 = 567.5\ N/mm^2$$

$$\sigma_{maxten} = \frac{M_{max}}{I} \times y_{maxten} = \frac{120\,x10^6}{1.285\,x10^7} \times 79.23 = 739.9\ N/mm^2$$

(iv) Distribution of the bending stress



SHEAR STRESS

GENERAL OBJECTIVE: At end of the unit, students should be able to learn, understand and solve problems on **shear stress, shear strain and modulus of rigidity**.

SPECIFIC OBJECTIVES

After following unit 5, the student should be able to:

- Understand the basic knowledge of shear stress in bolt or rivet and beam.
- Apply the formula of shear stress in bolt or rivet and beam.

5.1 INTRODUCTION

Joint is a component that joined frameworks or members of a structure. Joints functioned to transfer various types of forces and moments. Joints should be designed to ensure it is able to sustain loads and moments. Theoretically, a joint will experience actions of one or more than one combined, of the following forces;

i. Axial load
ii. Shear load
iii. Tensile load
iv. Moment

Types of joint that are commonly used in construction are bolt and nuts, rivets and welding. However, in this unit only bolt and rivet joints that are under shear loadings will be discussed.

5.1.1 SHEAR STRESS

Consider two wooden blocks of about an inch thick are placed overlapped to each other on a table as shown in figure 1.1. Use a force F to push the top block of wood to the left. It is found out that the top block tends to slide on top of the lower block.

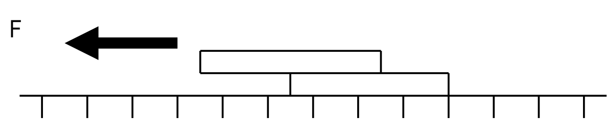


Figure 1.1: Two blocks subjected to force, F

Figure 1.2 however shows the first block is glued to the table and the second block is glued to the top surface of the first block. A thin plate is glued to the second plate. If a horizontal force, F is applied to the plate, it is found out that the plate tends to slide on the top surface of the second block and the second block tends to slides on the top surface of the first block. As long as these blocks and plate are rigidly glued to each other, they would not be able to slide on the surfaces.

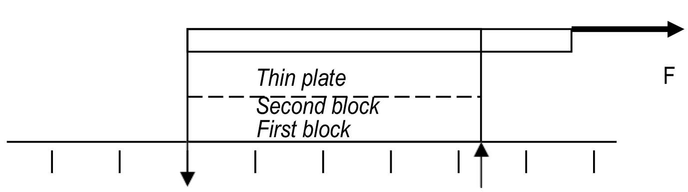


Figure 1.2: Shear stress caused by shear force

The tendency of the section to slide is called the **shear action**.

Consider a block subjected to **two non-linear forces of the same magnitudes in opposite directions**. Assumed this block is divided into two parts by an imaginary horizontal plane $x_1 - x_2$. The top part of block tends to slide at the bottom of the plane. The sections on both imaginary plane is to be under the **shear action**. The stress derived is known as the **shear stress**. Hence, the direction of the shear stress is the same as the direction of the applied force.

The imaginary horizontal plane x_1 - x_2 is the **shear plane**.

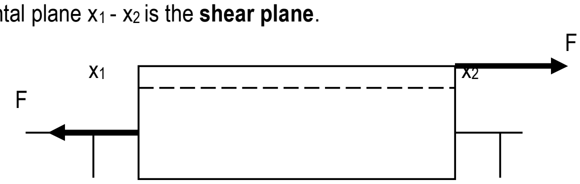


Figure 1.3: Shear action due to force F

In the Civil Engineering field, shear stress arises in various conditions, but in this unit will stress only on plane, bolt and rivet.

5.1.2 Unit for Shear Stress

Shear stress is defined as the intensity of load ating tangentially to a surface that will experience sliding or shearing. Example in figure 1.3, if the sliding plane x_1 - x_2 having an area A, then the shear stress is the ratio of load F parallel to the surface area of shear;

Shear stress = Force /Area

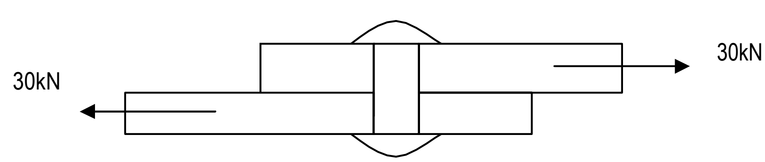
τ **= F/A**
(Unit = N/mm^2)

5.1.2.1 Single ShearStress

Single shear stress occurs when there is a pair of forces acting in opposite direction at a joint.

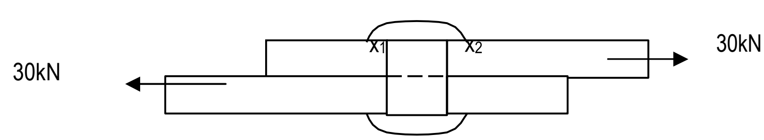
Example 5.1

Two steel plates are joined together using rivet of 1.5 cm diameter. If a tensile load applied is 30kN, estimate the shear stress in the rivet.



Solution

Force 30kN applied will cause the top shearing plane x_1x_2 of rivet to slide on the lower part of plane. The shear force 30kN is transferred to x_1x_2 surface that is marked with dotted line that causes the shear stress.



Shear stress = Force / Area

$$\tau = F / A$$

$$= 30\,kN \,/\, \frac{\pi\,(1.5)^2}{4}$$

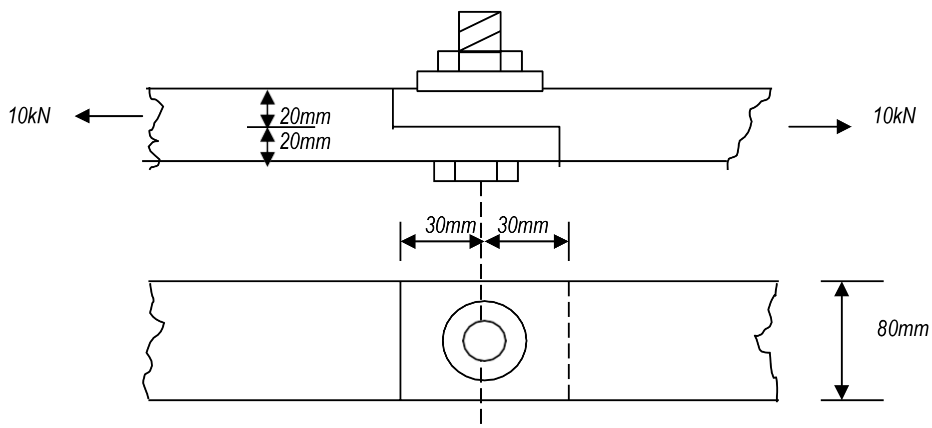
$$= 16.98 kN/cm^2$$

(Change to kN/m^2 Unit)

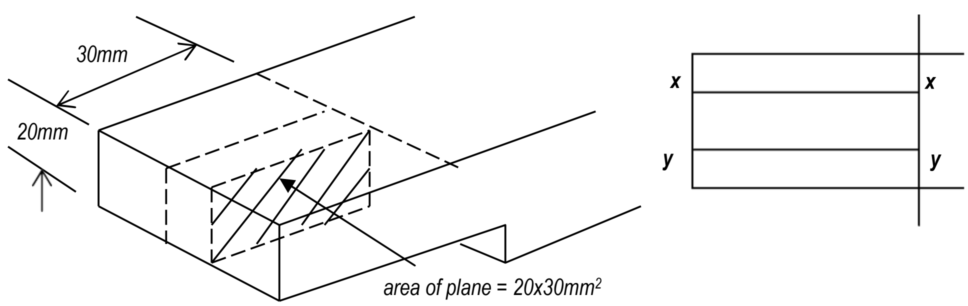
Example 5.2

Two timber are joined together using a bolt of 13mm diameter as shown bellow. Determine:

i. Shear stress in timber
ii. Shear stress in bolt



Solution



Shearing occurs on plane xx and yy as in figure above.

i. Shear area occurring on timber is the area of the xx and yy planes.
Hence, area of shear = $2 \times 20 \times 10^{-3} \times 30 \times 10^{-3}\ m^2$
$= 1.2 \times 10^{-3}\ m^2$

$$\tau_{kayu} = \frac{10 \times 10^{3} N}{1.2 \times 10^{-3} m^{2}}$$

$$\tau_{kayu} = 8.33 \times 10^{6} N/m^{2} \longrightarrow \tau_{kayu} = 8.33 MN/m^{2}$$

ii. Shear stress in bolt = $\frac{F}{A}$

$$\tau_{bolt} = \frac{10 \times 10^{3}}{\frac{p(13 \times 10^{-3})^{2}}{4}} N/m^{2}$$

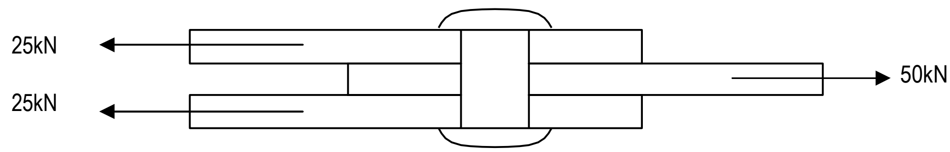
$$\tau_{bolt} = 75.34 \times 10^{6} N/m^{2} \longrightarrow \tau_{bolt} = 75.34 MN/m^{2}$$

5.1.2.2 Double Shear Stress

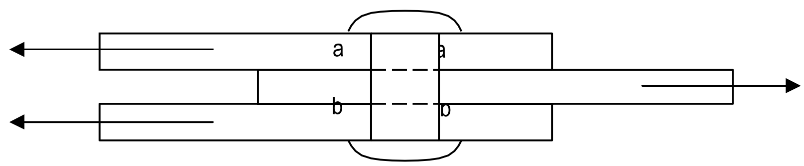
Twin shear stress occurs when there exists more than a pair of forces acting in opposite directions at a joint. This situation exists when more than two members of a structure are joined.

Example 5.3

Three steel plates are lapped and riveted together using a 1.5cm rivet. If the jointed member carries a tensile load of 50kN, determine the shear stress in rivet.



Solution



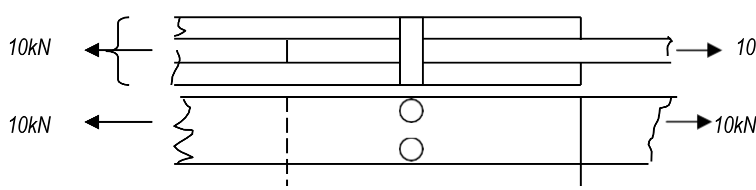
Due to the tensile load applied, the tendency of the rivet to shear across the plane a-a and b-b as shown in figure. The area that will resist shear is twice the cross sectional area of rivet where area of rivet is:

$$A = \frac{\pi}{4}(0.0015)^2 = 0.177\ x10^{-3}\,m^2$$

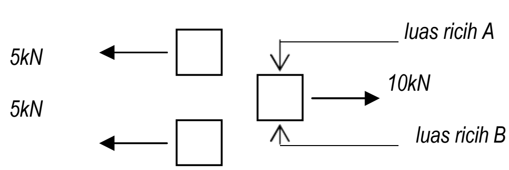
Thus the average shear stress in rivet, $\tau = \frac{F}{A}$, $= \frac{25\ x10^3}{0.177\ x10^{-3}}$ =141 MN/m^2

Example 5.4

Three plates are joined together by two rivets of the same diameter as shown in figure below. If the shear stress in rivet is not to exceed 80MN/m^2, determine the diameter of the rivet used.



solution



Above shear experiences double shear and there are two rivets, therefore shear area, A = $\frac{2 \times 2 \times \pi\, d^2}{4}$

But $\tau = \frac{F}{A}$

Thus, $80 \times 10^6 = \dfrac{10 \times 10^3}{\dfrac{2 \times 2 \times \pi \times d^2}{4}}$

$d^2 = \frac{10 \times 10^3 m}{80 \times \pi \times 10^6}$, **d = 6.31mm**

5.2 SHEAR STRAIN

Under the action of shear stress, there is no tendency to elongate or shortened in the x, y or z direction. In other words, the length of the side of the element does not changed instead **shear stress causes the change in shape.**

As shown in figure 1.4b initial cube shaped of an element changes into a rhombus. Angle γ is a measurement of the distortion or change of shape of the element and is called shear strain. Angle γ is measured in radians.

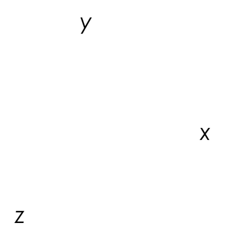


Figure 1.4 a

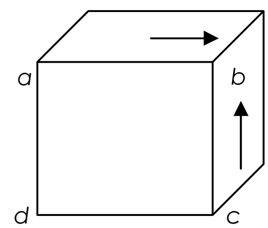
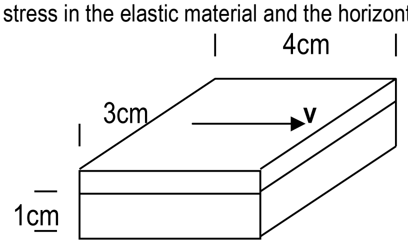
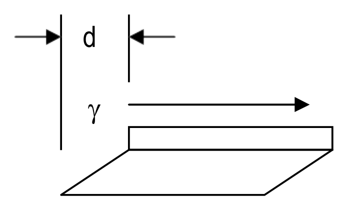


Figure 1.4 b

Example 5.6

A bearing plate as shown below comprises of a straight elastic material of 1cm thick is attached to a rigid steel plate with dimensions of 3mm x 4mm. this steel palte is subjected to a horizontal shear load of 45kN. Determine the average shear stress in the elastic material and the horizontal shift of the steel plate.





Solution

Average shear stress is the load divided with the area it acts.

$$\tau_{purata} = \frac{45kN}{4cm x 3cm}$$

$= 3.75 kN/cm^2$

5.3 MODULUS OF RIGIDITY

Modulus of rigidity, G is defined as the ratio of the shear stress, τ to the shear strain, γ. Hence :

$G=\tau/\gamma$

G is also known as the Modulus of Elasticity in Shear or just the Shear Modulus. The shear modulus is the same as Modulus Young, E, for the direct compression and tension. Most of the construction materials the value of E ia approximately 2.5 times the value of G.

SLOPE AND DEFLECTION OF BEAM DUE TO SYMMETRICAL BENDING

GENERAL OBJECTIVE: Students should be able to understand the meaning of slope and maximum deflection of a simply supported beam and cantilever beam.

SPECIFIC OBJECTIVES:

After following Unit 4, students should be able to:-

- Understand the determination of slope and deflection of beam due to point loads and uniformly distribution loads using Macaulay and Moment Area Method
- Apply the formula of slope and deflection in beam

6.0 Introduction

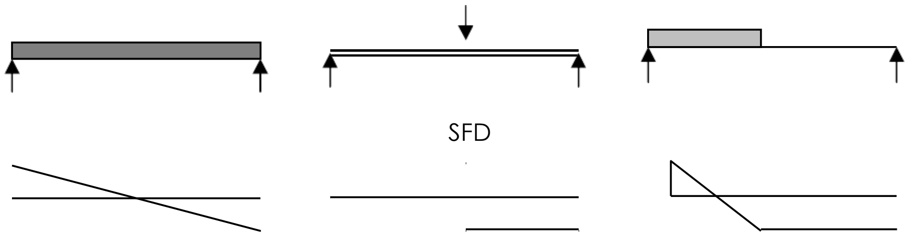
In general, a beam is fully designed to sustain failure due to either in bending or shear. Deflection of beam exceeding the allowable limit will cause damage or failure to adjacent member structure (example column, slab etc)

6.1 Deflection of Beam

A beam, when subjected to a load, the initial straight longitudinal axis is deformed to a curve known as deflection curve of beam. Distinct curvature and a beam that is not functioning well will cause failure or damage to the structure as a whole. To ensure this from occurring the beam should be designed in such a way that when loaded, it will not deflect greater than the allowable limit as stated in the code in example :-

$$\frac{1}{360} \text{ x span of beam}$$

Generally, a beam supports various type of loadings, hence parameter such as shear load, bending moment, slope and deflection do not have specific linear functions to the entire beam. However, an expression may be deduced for the entire beam without dividing the beam into a number of sections using non-linear function. Several examples for linear and non-linear functions may be seen from shear force diagrams of a beam loaded as shown in Figure 6.1.



Linear Function | Non-Linear Function | Non-Linear Function

Figure 6.1: Linear and Non-Linear Beam

6.1.1 Mathematical Relation Between Bending Moment, Slope and Deflection

When load is applied, the beam will sag or deform making an arc of unknown radius. Take a small section of the arc assuming P and Q (closed together) as the end points of the arc on the longitudinal axis of the beam. Shape of deflection is of a small curve with radius R and centre O. Position of P and Q is x and x + δx respectively from their initial positions.

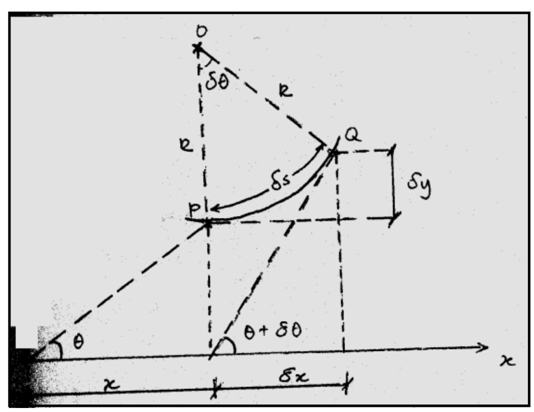


Figure 6.1.1(a): Small Section of Beam

δs = Length of arc PQ
R = radius of curvature of arc
O = centre of arc
θ = tangential angle at P with axial line ox
$\theta + \delta_\theta$ = tangential angle at Q with axial line ox

Geometrically from Figure 4.2(a), it is found that

$$\angle POQ = \delta_\theta$$

$$\delta s = R\delta_\theta$$

$$R = \frac{\delta_s}{\delta_\theta}$$

If curve δs is very small, it may be assumed that

$$\delta s = \delta x$$

Hence $$R = \frac{\delta_x}{\delta_\theta}$$

$$\frac{1}{R} = \frac{\delta_\theta}{\delta_s}$$

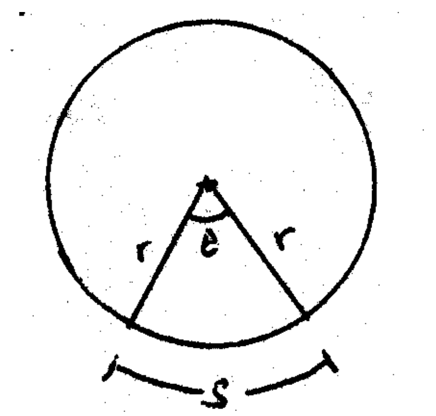


Figure 6.1.1 (b)

The coordinate of P is (x,y), hence

$\tan\theta = \dfrac{dy}{dx}$ ------------------ equation 4.1

as θ is very small, then $\tan\theta = \theta$, hence

$$\theta = \frac{dy}{dx}$$

Differentiate this equation with respect to x, gives

$$\frac{d\theta}{dx} = \frac{d^2y}{dx^2}$$

$\theta = \tan\theta = \dfrac{dy}{dx}$ (insert it in equation 4.1)

$$\frac{1}{R} = \frac{d\theta}{dx} = \frac{d\left(\frac{dy}{dx}\right)}{dx}$$

$\dfrac{1}{R} = \dfrac{d^2y}{dx^2}$ --------------------- equation 4.2

From bending equation,

$\dfrac{M}{I} = \dfrac{E}{R} = \dfrac{\sigma}{Y}$ --------------------- equation 4.3

$$\frac{M}{I} = \frac{E}{R}, \quad \frac{I}{R} = \frac{M}{E}$$

$\dfrac{1}{R} = \dfrac{M}{EI}$ --------------------- equation 4.4

Insert equation 4.2 in equation 4.4

The general equation of differentiation for deflection is,

$\boxed{\dfrac{M}{EI} = \dfrac{d^2y}{dx^2}}$ ----------------- **equation 4.5**

Standard Signs and Units

Table 6.1.1(c) gives standard signs for bending moment, slopes and deflection that will used in this unit. This signs is in line with the sign convention for shear force and bending moment.

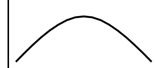
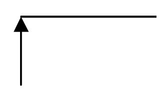
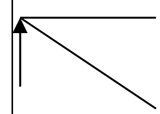
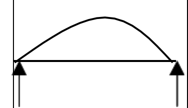
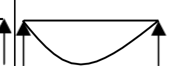
Effects	Symbols	Differential Coefficient	Integration	Unit	Sign Procedure	
					Positive	Negative
Bending Moment	M	$EI\,\dfrac{d^2y}{dx^2}$	M	Nm KNm		
Slope	$\dfrac{dy}{dx}$	$EI\,\dfrac{dy}{dx}$	$\int \dfrac{M}{EI}dx$	Radian		
Deflection	y	EI y	$\iint \dfrac{M}{EI}dx.dx$	mm		

Table 6.1.1(c): Sign Convention for Bending Moment, Slope and Deflection

6.1.2 slope and deflection of simply supported beam and cantilever beam subjected to point loads, distribution loads and moment

Simply Supported Beam with Point Load

Using Macaulay's method, only one sectioning is required to build the general expression of moment that is at an area closest to the last load (example at section s-s as in figure 6.1.2(a) and section s'-s' as in figure 6.1.2(b).

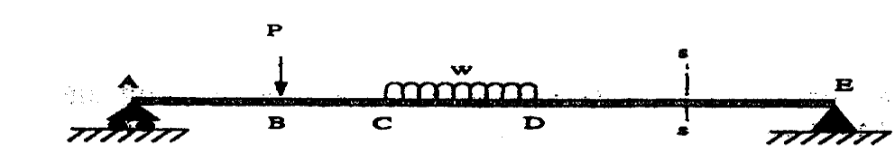


Figure 6.1.2(a)

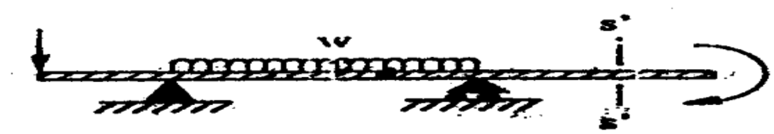


Figure 6.1.2(b)

SLOPE AND DEFLECTION OF BEAM DUE TO SYMMETRICAL BENDING

As it's already known that when a beam is subjected to loads it will sag and in equilibrium position as long as the supports able to sustain the loads. These supports may be column or any other types of supports. The reactions at supports can be calculated by applying the equations of equilibrium.

Solving problems using Macaulay's method, it requires certain conditions to be obeyed as follows:

i. Equation of moment for the entire beam should be determined at a point to the right most before end of beam with a distance measurement made from the left end of it (Origin is from left end of beam). Refer figure 4.5.

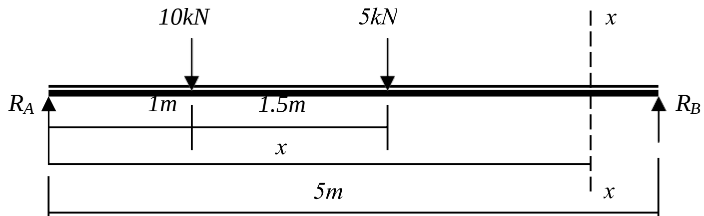


Figure 6.1.2(c)

From figure 4.5 and applying condition (i) will produce the equation of moment as follows:

$M_x = R_A(x) - 10(x - 1) - 5(x - 2.5)$

ii. To determine the derived equation can be used for the entire beam regardless where the point is, the functions (x-1) and (x-2.5) that were derived due to change in first (10kN) and second (5kN) loadings should be converted into Macaulay's function as shown below.

$(x - 1)$ into $[x - 1]$
$(x - 2.5)$ into $[x - 2.5]$

Square brackets [ ] is recognized as Macaulay's brackets and its main characteristic is that it will become zero whenever the content becomes negative.
Thus, the equation of moment derived from first condition should be modified in the form as follows,

$M_x = R_A[x] - 10[x - 1] - 5[x - 2.5]$ -------- equation 4.6

As a check, if the value of x is x_1, with $x_1<1$, equation 4.6 will become

$$M_x = R_A[x_1] - 10[\underset{\uparrow (a)}{x_1 - 1}] - 5[\underset{\uparrow (b)}{x_1 - 2.5}]$$

Hence $x_1 < 1$, value of the phrase will become negative, and the value in Macaulay's bracket will become zero. This will deduce an equation as shown below with $0 \le x \le 1$

$M_x = R_A x_1$

Thus, it shows that Macaulay's equation is valid for the entire beam.

iii. Integration of functions in Macaulay's brackets should be carried out as follows,

Integration of [x – 1] becomes $\frac{[x-1]^2}{2}$

Integration of [x – 2.5] becomes $\frac{[x-2.5]^2}{2}$

Example

A beam simply supported is subjected to a point load of 20kN as shown in figure 4.6. Determine the slope and deflection at point C.

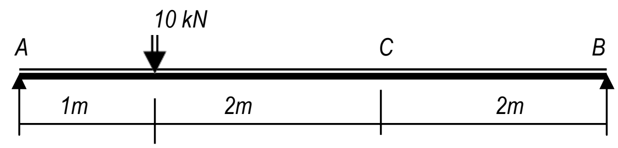


Figure 6.1.2 (d)

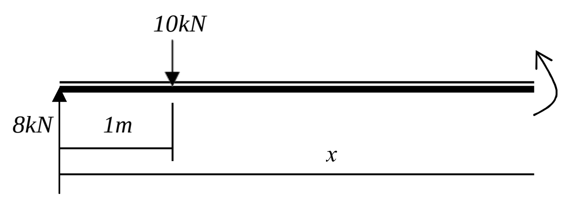
Solution

Solving by Macaulay's method, the reactions at supports should be firstly determined using the equation of moment. For a simply supported beam at both ends only reaction at support A is required.

$$\sum M_B = \sum M_B$$

$10(4) = R_A (5)$

$R_A = \frac{10 \times 4}{5} = 8kN$



Equation of moment

$M_x = 8x – 10 [x – 1]$ ----------------------------------- equation 4.7

From equation $EI\frac{d^2y}{dx^2} = M$ -------------------- equation 4.8

Insert equation 4.7 in equation 4.8 , it becomes

$$EI\frac{d^2y}{dx^2} = 8x - 10[x-1]$$

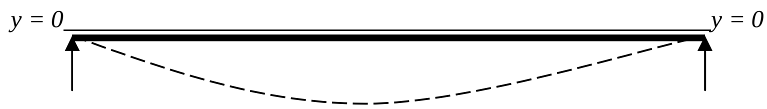
Integrate the above equation to get the equation for slope, thus,

$$EI\frac{dy}{dx} = \frac{8x^2}{2} - \frac{10[x-1]^2}{2} + C_1$$

Integrate once again to get the equation for deflection, hence,

$$EIy = \frac{8x^3}{6} - \frac{10[x-1]^3}{6} + C_1x + C_2$$

To determine the magnitudes of C_1 and C_2 the **boundary conditions** should be applied, that is for a simply supported beam the deflection is maximum when slope is zero whereas deflection is zero at both supports.



At A, x = 0, y = 0

$$EI(0) = \frac{8[0]^3}{6} - \frac{10[0-1]^3}{6} + C_1(0) + C_2$$

$$C_2 = 0$$

At B, x = 5m, y = 0

$$EI(5) = \frac{8[5]^3}{6} - \frac{10[5-1]^3}{6} + C_1(5) + 0$$

$$C_1 = -12$$

Complete Solution for

i. **Slope**, $EI\frac{dy}{dx} = \frac{8x^2}{2} - \frac{10[x-1]^2}{2} - 12$

ii. **Deflection**, $EIy = \frac{8x^3}{6} - \frac{10[x-1]^3}{6} - 12x$

Slope at point C, x = 3m
Substitute x = 3m in the slope equation

$$EI\frac{dy}{dx} = \frac{8(3)^2}{2} - \frac{10[3-1]^2}{2} - 12$$

$$\frac{dy}{dx} = \frac{4}{EI}$$

Deflection at point C, x = 3m

$$EI\,y = \frac{8[3]^3}{6} - \frac{10[3-1]^3}{6} - 12(3)$$

$$y = -\frac{13.33}{EI}$$

*** If magnitude of EI is given, then the **slope is given in radians** and **deflection in mm**.*

Simply Supported Beam with UDL

For a uniformly distributed load covering part of the beam but not through the entire span to right of the beam, the load should be extended to the end and then should be subtracted with the same extended load of the magnitude but of opposite direction (upward direction), applied at bottom for the extended section of the beam. This is explained by referring to figure 4.10(a).

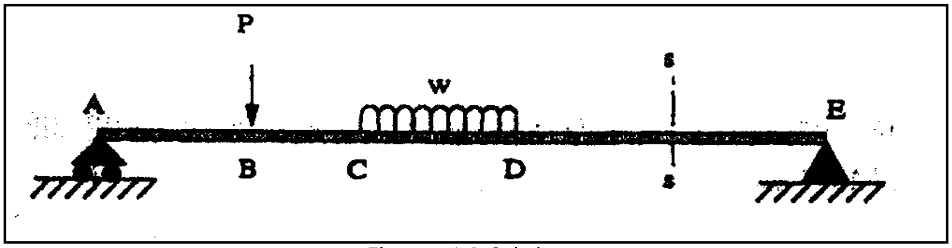


Figure 6.1.2 (e)

Load w applied on area CD should be extended till point E. Then a load with the same magnitude with an upward direction should be induced for the area DE at lower part of beam. In this condition, the beam will experience the same load as its initial condition. Refer figure 4.10(b).

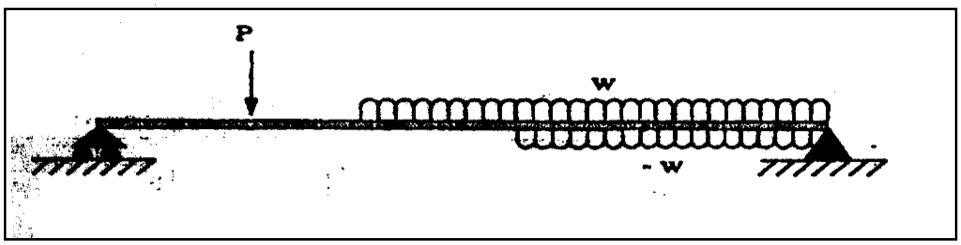


Figure 6.1.2 (e)

Moment expression for a uniformly distributed load is given by

$M_x = R(x) - \frac{w(x-a)^2}{2}$, see figure 4.10(c) below.

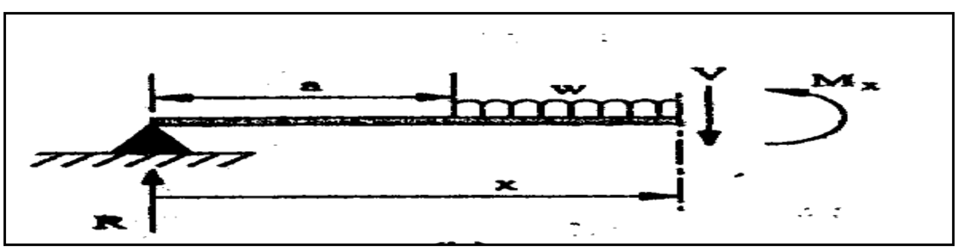


Figure 6.1.2 (f)

<u>Example</u>

Determine the slope and deflection at point C for the beam shown in figure 4.11, given the value of E = 200kN/mm^2 and I = 10^8 mm^4

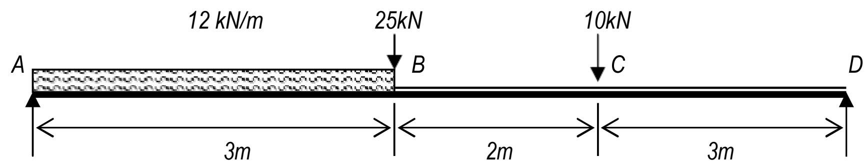


Figure 6.1.2 (g)

<u>Solution</u>

Reaction at support A , equilibrium of moment

$$\circlearrowleft \sum M_D = \sum M_D \circlearrowright$$

$R_A(8) = 12(3)(6.5) + 25(5) + 10(3)$

$\mathbf{R_A = 48.63\ kN}$

Equation of moment

$$M_x = EI\frac{d^2y}{dx^2} = 48.63(x) - \frac{12x^2}{2} + \frac{12[x-3]^2}{2} - 25[x-3] - 10[x-5]$$

(The uniformly distributed load should be extended till y-y and needed be balanced)

Integrate equation of moment to get the equation of slope

$$EI\frac{dy}{dx} = \frac{48.63x^2}{2} - \frac{12x^3}{6} + \frac{12[x-3]^3}{6} - \frac{25[x-3]^2}{2} - \frac{10[x-5]^2}{2} + C_1$$

Integrate once again to get equation of deflection

$$EIy = \frac{48.63x^3}{6} - \frac{12x^4}{24} + \frac{12[x-3]^4}{24} - \frac{25[x-3]^3}{6} - \frac{10[x-5]^3}{6} + C_1x + C_2$$

Boundary Conditions,

Deflection is zero at supports; from equation of deflection

At point A, x = 0, y = 0 hence $C_2 = 0$

At point D, x = 8, y = 0 hence;

$$EI(0) = \frac{48.63(8)^3}{6} - \frac{12(8)^4}{24} + \frac{12[8-3]^4}{24} - \frac{25[8-3]^3}{6} - \frac{10[8-5]^3}{6} + C_1(8) + 0$$

$C_1 = -231.05$

The complete equation for

i. Slope

$$EI\frac{dy}{dx} = \frac{48.63x^2}{2} - \frac{12x^3}{6} + \frac{12[x-3]^3}{6} - \frac{25[x-3]^2}{2} - \frac{10[x-5]^2}{2} - 231.05$$

ii. Deflection

$$EI\,y = \frac{48.63x^3}{6} - \frac{12x^4}{24} + \frac{12[x-3]^4}{24} - \frac{25[x-3]^3}{6} - \frac{10[x-5]^3}{6} - 231.05x$$

Slope at point C, x = 5m

$$EI\frac{dy}{dx} = \frac{48.63(5)^2}{2} - \frac{12(5)^3}{6} + \frac{12[5-3]^3}{6} - \frac{25[5-3]^2}{2} - \frac{10[5-5]^2}{2} - 231.05$$

$$\frac{dy}{dx} = \frac{94.825x10^6}{200x10^8} = \mathbf{4.74 \times 10^{-3}\ radians}$$

Deflection at point C, x = 5m

$$EI\ y = \frac{48.63(5)^3}{6} - \frac{12(5)^4}{24} + \frac{12[5-3]^4}{24} - \frac{25[5-3]^3}{6} - \frac{10[5-5]^3}{6} - 231.05(5)$$

$$y = \frac{-479.96x10^9}{200x10^8} = \mathbf{-24\ mm}$$

(a) Point moment applied on the beam as shown in figure 4.6(a) should be details as follows,

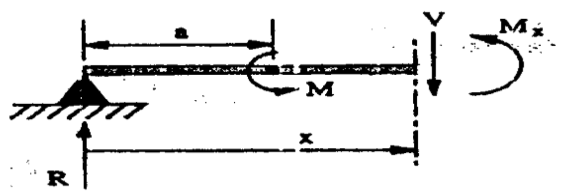


Figure 6.1.2 (h)

$\mathbf{M_x = R_A x - M_1 [x - a]^0}$

The phrase in Macaulay's brackets $[x - a]^0$ has a value of 1, and this does not affect the actual equation for moment, that is;

$M_x = R_A x - M_1$

However the following conditions should be adhered, that is:

If $x - a \leq 0$ then $(x - a) = 0$

If $x - a \geq 0$ then $(x - a) = (x - a)$
a is the distance from left support to load or moment under consideration.

Simply Supported Beam with Moment

<u>Example</u>

A beam simply supported at both ends is loaded as shown in figure 4.16(a). Determine the slope and deflection at a point 5m to the right of support A. Given the value of $E = 5 \times 10^6\ N/mm^2$ and $I = 9 \times 10^6\ mm^4$

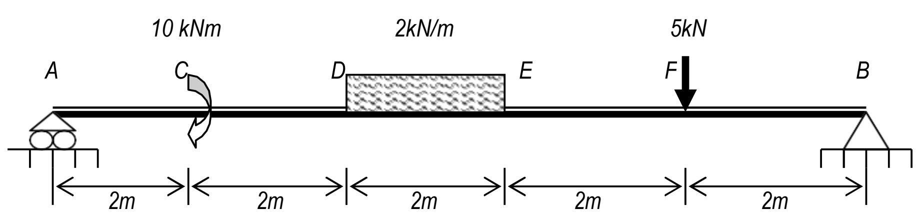


Figure 6.1.2 (i)

Solution

$\sum M_A = 0$ ↻ +ve, $\quad$ **R_B** $= \dfrac{10 + 2(2)(5) + 5(8)}{10}$ **= 7kN**

$\sum F_y = 0$ ↑ +ve, $\quad$ **R_A** $= 2\times2 + 5 - 7$ **= 2kN**

Sectioning should be made between point F and B as in figure 4.16(b).

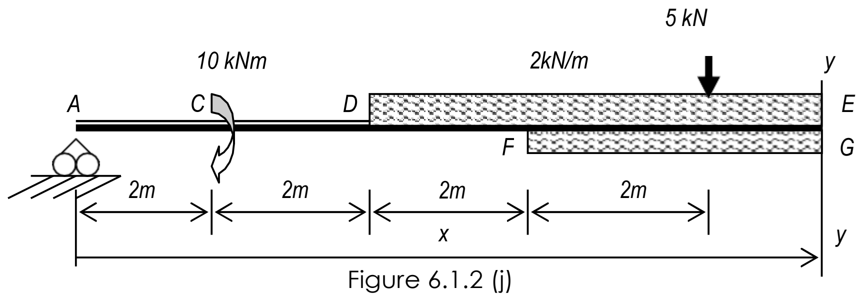


Figure 6.1.2 (j)

$\sum M = 0$

$$M = EI\frac{d^2y}{dx^2} = 2[x] + 10[x-2]^0 - \frac{2[x-4]^2}{2} + \frac{2[x-6]^2}{2} - 5[x-8]$$

Integrate to get the equation of slope,

$$EI\frac{dy}{dx} = \frac{2x^2}{2} + 10[x-2] - \frac{2[x-4]^3}{6} + \frac{2[x-6]^3}{6} - \frac{5[x-8]^2}{2} + C_1$$

Integrate once again to get the equation of deflection,

$$EI\,y = \frac{2x^3}{6} + \frac{10[x-2]^2}{2} - \frac{2[x-4]^4}{24} + \frac{2[x-6]^4}{24} - \frac{5[x-8]^3}{6} + C_1x + C_2$$

Boundary conditiions

$x = 0,\ y = 0,$ $\quad$ **$C_2 = 0$**

$x = 10,\ y = 0$

$$0 = \frac{2(0)^3}{6} + \frac{10[10-2]^2}{2} - \frac{2[10-4]^4}{24} + \frac{2[10-6]^4}{24} - \frac{5[10-8]^3}{6} + C_1(10) + 0$$

$C_1 = -56$

Hence the complete equations for

Slope, $EI\dfrac{dy}{dx} = \dfrac{2x^2}{2} + 10[x-2] - \dfrac{2[x-4]^3}{6} + \dfrac{2[x-6]^3}{6} - \dfrac{5[x-8]^2}{2} - 56$

Deflection, $EI\,y = \dfrac{2x^3}{6} + \dfrac{10[x-2]^2}{2} - \dfrac{2[x-4]^4}{24} + \dfrac{2[x-6]^4}{24} - \dfrac{5[x-8]^3}{6} - 56x$

Deflection at a distance x = 5m from A,

$$EI\,y = \frac{2(5)^3}{6} + \frac{10[5-2]^2}{2} - \frac{2[5-4]^4}{24} + \frac{2[5-6]^4}{24} - \frac{5[5-8]^3}{6} - 56(5)$$

$= -193.4$ kNm^3

$$y = -\frac{193.4(1000)(1000)^3}{(5\times10^6)(9\times10^6)} = \mathbf{-4.3mm}$$

(Negative sign shows the deflection is downward)

Cantilever Beam with Point Load

For a cantilever beam, there is no slope and deflection at fixed end. Maximum deflection occurs at the free end where x = L. Hence the position of the support is important.

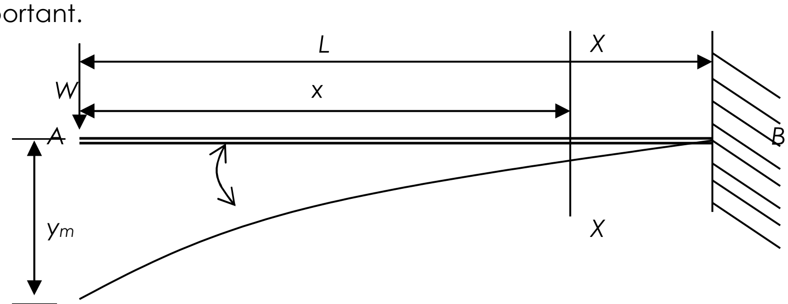


Figure 6.1.2 (k)

At XX section, the bending moment

$M_x = -Wx$

$$EI\frac{d^2y}{dx^2} = -Wx \qquad \text{---------------- equation 5.1}$$

Integrate equation 5.1 to get the equation of slope

$$EI\frac{dy}{dx} = \frac{-Wx^2}{2} + C_1 \qquad \text{---------------- equation 5.2}$$

Boundary condition

When x =L, $\frac{dy}{dx}$ = 0. Substitute these values in equation 5.2 to give

$$EI(0) = \frac{-WL^2}{2} + C_1$$

$$C_1 = \frac{WL^2}{2}$$

Equation 5.2 becomes,

$$EI\frac{dy}{dx} = \frac{-Wx^2}{2} + \frac{WL^2}{2}$$

Second integration gives,

$$EIy = \frac{-Wx^3}{6} + \frac{WL^2x}{2} + C_2 \qquad \text{------------------ equation 5.3}$$

When x = L, y = 0. Substitute these values in equation 5.3, gives

$$0 = \frac{-WL^3}{6} + \frac{WL^3}{2} + C_2$$

Thence $C_2 = \frac{W L^3}{3}$

Maximum deflection occurs at x – 0m

$$y_m = \frac{-\left(WL^3\right)}{3\,EI}$$

Example (i)

Determine the slope and deflection at the free end of the beam below.

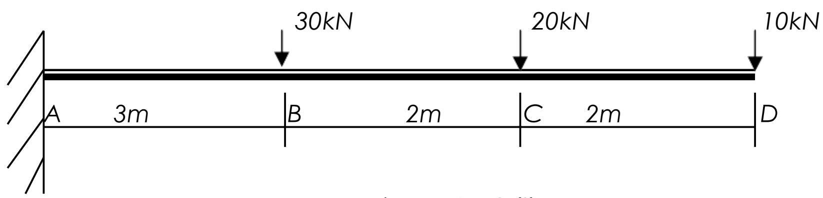


Figure 6.1.2 (I)

Solution:

Ra = 30 + 20 + 10
= 60kN

$\Sigma M_A = 0$
$M_A + 30(3) + 20(5) + 10(7) = 0$
$\mathbf{M_A = -260kNm}$

The equation for moment,

$M_{y-y} = 60\,x - 260x^0 - 30(x-3) - 20(x-5)$

Change it to the Macaulay's equation

$$EI\,\frac{d^2y}{dx^2} = 60\,x - 260\,x^0 - 30\left[x-3\right] - 20\left[x-5\right]$$

$$EI\,\frac{dy}{dx} = \frac{60\,x^2}{2} - 260\,x - \frac{30\left[x-3\right]^2}{2} - \frac{20\left[x-5\right]^2}{2} + C_1$$

$$EIy = \frac{60\,x^3}{6} - \frac{260\,x^2}{2} - \frac{30\left[x-3\right]^3}{6} - \frac{20\left[x-5\right]^3}{6} + C_1 x + C_2$$

Applying the boundary condition to determine the magnitude of integral constants C_1 and C_2;

At A, $\frac{dy}{dx} = 0$, x =0

$$EI\,(0) = \frac{60\,(0)^2}{2} - 260\,(0) - \frac{30\left[0-3\right]^2}{2} - \frac{20\left[0-5\right]^2}{2} + C_1$$

$C_1 = 0$

At A, y = 0, x = 0

$C_2 = 0$

Thus the complete equation for

Slope, $$EI\frac{dy}{dx} = \frac{60x^2}{2} - 260x - \frac{30[x-3]^2}{2} - \frac{20[x-5]^2}{2}$$

Deflection, $$EIy = \frac{60x^3}{6} - \frac{260x^2}{2} - \frac{30[x-3]^3}{6} - \frac{20[x-5]^3}{6}$$

Slope at free end is when, x = 7m, substituting it in the slope equation

$$EI\frac{dy}{dx} = \frac{60(7)^2}{2} - 260(7) - \frac{30[7-3]^2}{2} - \frac{20[7-5]^2}{2}$$

$$\frac{dy}{dx} = \frac{-414}{EI}$$ radians

Deflection at free end, x = 7m and substituting it in the deflection equation

$$EIy = \frac{60(7)^3}{6} - \frac{260(7)^2}{2} - \frac{30[7-3]^3}{6} - \frac{20[7-5]^3}{6}$$

$$y = \frac{-3286.7}{EI}$$

<u>Example ii)</u>

Determine the slope and deflection of the cantilever shown in figure 5.3 in term of EI.

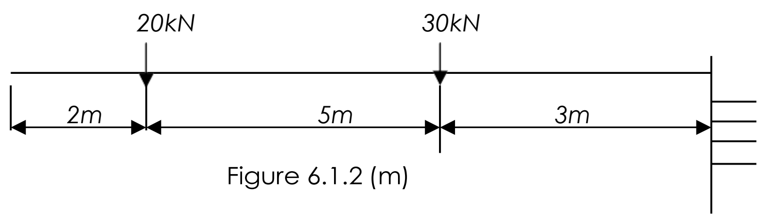


Figure 6.1.2 (m)

With Macaulay's method, sectioning y-y should be made at the very right end of the beam.

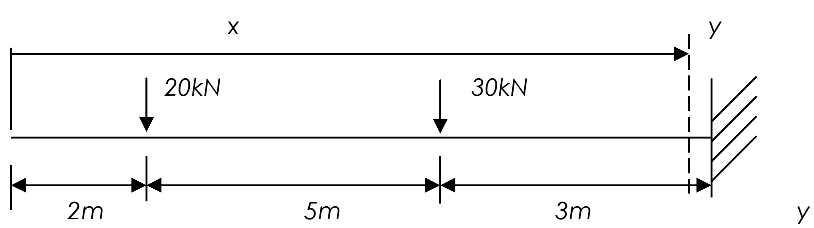


Figure 6.1.2 (n)

<u>Solution:</u>

Moment at section y-y

M_{yy} = - 20(x- 2) - 30(x - 7)

In this condition (when the fixed end is on right hand side) reactions forces are not required.

From the original equation of moment that is;

$$EI\frac{d^2y}{dx^2} = M$$

Substitute the equation of moment at section y-y in this original moment equation that becomes

$$EI\frac{d^2y}{dx^2} = -20(x-2) - 30(x-7)$$

To get the equation for slope integrate the equation of moment

$$EI\frac{dy}{dx} = \frac{-20(x-2)^2}{2} - \frac{30(x-7)^2}{2} + C_1$$

From equation for slope integrate once again to get the equation for deflection.

$$EIy = \frac{-20(x-2)^3}{6} - \frac{30(x-7)^3}{6} + C_1x + C_2$$

Applying the boundary conditions, that is by substituting

$\frac{dy}{dx} = 0$, x = 10 in the equation for slope

$$EI(0) = \frac{-20(10-2)^2}{2} - \frac{30(10-7)^2}{2} + C_1$$

$0 = -775 + C_1$

$C_1 = 775$

y = 0 , x = 10 in the equation for deflection

$$EI(0) = \frac{-20(10-2)^3}{6} - \frac{30(10-7)^3}{6} + 775(10) + C_2$$

$C_2 = 5908.33$

Then, the complete equation for

Slope, $$EI\frac{dy}{dx} = \frac{-20(x-2)^2}{2} - \frac{30(x-7)^2}{2} + 775$$

Deflection, $$EIy = \frac{-20(x-2)^3}{6} - \frac{30(x-7)^3}{6} + 775x + 5908.33$$

Maximum slope and deflection occur at free end that is when x = 0
Therefore;

Slope, $$EI\frac{dy}{dx} = \frac{-20(0-2)^2}{2} - \frac{30(0-7)^2}{2} + 775$$

$$\frac{dy}{dx} = \frac{775}{EI}$$

Deflection, $$EIy = \frac{-20(0-2)^3}{6} - \frac{30(0-7)^3}{6} + 775(0) + 5908.33$$

$$y = \frac{5908.33}{EI}$$

Cantilever Beam with UDL and Moment

In the case of a uniformly distributed load that covers part of beam and not till to the right end of the cantilever, the load should be extended as if it covers the entire span. This added load then should be subtracted with the same magnitude but of opposite direction (upwards) denoted under the cantilever (refer unit 4).

SLOPE AND DEFLECTION OF BEAM DUE TO SYMMETRICAL BENDING

Example i)

Determine the slope and deflection of the cantilever shown in figure below in term of EI

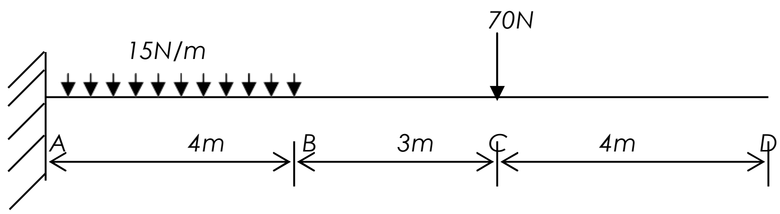


Figure 6.1.2 (o)

Solution

Applying the equation of equilibrium,

determine the reaction at fixed end, A

↑ $\Sigma F_y = 0$

$R_A = 15(4) + 70$ **= 130N**

Sum of positive moment equals to sum of negative moment

$\Sigma M_A = \Sigma M_A$

$15(4)(2) + M_A + 70(7) = 0$
$M_A = -610$ Nm

Equation of moment

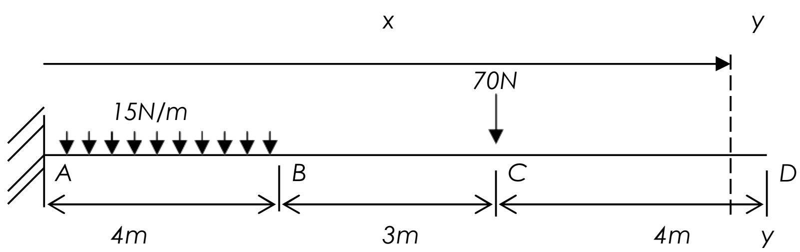


Figure 6.1.2 (p)

Sectioning at y-y

$$EI\frac{d^2y}{dx^2} = 130x - 610x^0 - \frac{15(x)^2}{2} + \frac{15(x-4)^2}{2} - 70(x-7)$$

(When uniformly distributed load should be extended till section y-y and should be counter balanced)

Integrate the equation of moment to get the equation for slope

$$EI\frac{dy}{dx} = \frac{130x^2}{2} - 610x - \frac{15x^3}{6} + \frac{15(x-4)^3}{6} - \frac{70(x-7)^2}{2} + C_1$$

Integrate once again the same equation to get the equation for deflection

$$EIy = \frac{130x^3}{6} - \frac{610x^2}{2} - \frac{15x^4}{24} + \frac{15(x-4)^4}{24} - \frac{70(x-7)^3}{6} + C_1x + C_2$$

Boundary conditions

At A, x = 0, $\frac{dy}{dx} = 0$

(When the support is on the left side, and if x = 0 slope and deflection are zero)

$$EI(0) = \frac{130(0)^2}{2} - 610(0) - \frac{15(0)^3}{6} + \frac{15(0-4)^3}{6} - \frac{70(0-7)^2}{2} \square C_1$$

$$C_1 = 0$$

At A, x = 0, y = 0

$$EI(0) = \frac{130(0)^3}{6} - \frac{610(0)^2}{2} - \frac{15(0)^4}{24} + \frac{15(0-4)^4}{24} - \frac{70(0-7)^3}{6} + C_1(0) \square C_2$$

$$C_2 = 0$$

The complete equation for

i. Slope, $EI\frac{dy}{dx} = \frac{130x^2}{2} - 610x - \frac{15x^3}{6} + \frac{15(x-4)^3}{6} - \frac{70(x-7)^2}{2}$ $_1$

ii. Deflection, $EIy = \frac{130x^3}{6} - \frac{610x^2}{2} - \frac{15x^4}{24} + \frac{15(x-4)^4}{24} - \frac{70(x-7)^3}{6}$

Slope at point D, x = 11m (at free end)

$$EI\frac{dy}{dx} = \frac{130(11)^2}{2} - 610(11) - \frac{15(11)^3}{6} + \frac{15(11-4)^3}{6} - \frac{70(11-7)^2}{2}$$

$$\frac{dy}{dx} = \frac{-1875}{EI} rad$$

Deflection at point D , x = 11 (at free end)

$$EIy = \frac{130(11)^3}{6} - \frac{610(11)^2}{2} - \frac{15(11)^4}{24} + \frac{15(11-4)^4}{24} - \frac{70(11-7)^3}{6}$$

$$y = \frac{-16463.33}{EI} mm$$

Example ii)

Determine the slope and deflection at free end, taking value of E = 200 kN/mm^2 and I = 10^8 mm^4

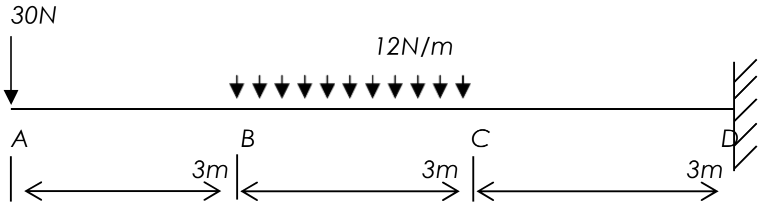


Figure 6.1.2 (q)

Solution

Reactions and reaction moment at supports should first be determined.

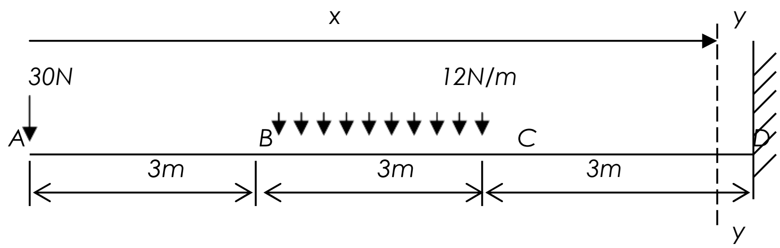


Figure 6.1.2 (r)

Solution

Equation of moment, $M_{yy} = EI\frac{d_2 y}{dx^2}$

$$EI\frac{d^2 y}{dx^2} = -30x - \frac{12(x-3)^2}{2} + \frac{12(x-6)^2}{2}$$

(Uniformly distributed load should be extended till section y-y and be counter balanced)

Integrate equation of moment to get the equation for slope.

$$EI\frac{dy}{dx} = \frac{-30x^2}{2} - \frac{12[x-3]^3}{6} + \frac{12[x-6]^3}{6} + C_1 \quad \text{.................................. (i)}$$

Integrate equation of slope to get the equation of deflection

$$EIy = \frac{-30x^3}{6} - \frac{12[x-3]^4}{24} + \frac{12[x-6]^4}{24} + C_1 x + C_2 \quad \text{.......................... (ii)}$$

Boundary conditions

At point D, x= 9m, $\frac{dy}{dx} = 0$, from (i)

$$EI(0) = \frac{-30(9)^2}{2} - \frac{12[9-3]^3}{6} + \frac{12[9-6]^3}{6} + C_1$$

$C_1 = 1593$

At point D, x = 9m, y = 0, from (ii)

$$EI(0) = \frac{-30(9)^3}{6} - \frac{12[9-3]^4}{24} + \frac{12[9-6]^4}{24} + 1593$$

(9) $\square C_2$

$C_2 = -10084.5$

The complete equation for

Slope, $EI\dfrac{dy}{dx} = \dfrac{-30x^2}{2} - \dfrac{12[x-3]^3}{6} + \dfrac{12[x-6]^3}{6} + 1593$

Deflection, $EIy = \dfrac{-30x^3}{6} - \dfrac{12[x-3]^4}{24} + \dfrac{12[x-6]^4}{24} + 1593x - 10084.5$

Slope and deflection at point A (the free end)

At point A, x = 0mm

Slope, $EI\dfrac{dy}{dx} = \dfrac{-30(0)^2}{2} - \dfrac{12[0-3]^3}{6} + \dfrac{12[0-6]^3}{6} + 1593$

$\dfrac{dy}{dx} = \dfrac{1593\,x10^6}{2x10^{10}}$ **= 0.08 rad**

Deflection, $EIy = \dfrac{-30(0)^3}{6} - \dfrac{12[0-3]^4}{24} + \dfrac{12[-6]^4}{24} + 1593(0) - 10084.5$

$y = \dfrac{-10084.5\,x10^9}{2\,x10^{10}}$ **= 504.2mm**

Example (iii)

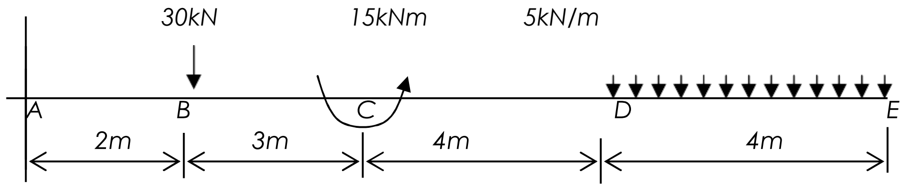


Figure 6.1.2 (s)

Solution

Reaction at support (fixed end)

Shear force

↑ $\Sigma F_y = 0$

$R_A = 30 + 5(4) = 50kN$

Moment

$\Sigma M_A = \Sigma M_A$

$M_A + 30(2) + 5(4)(11) = 15$

$M_A = -265$ kNm

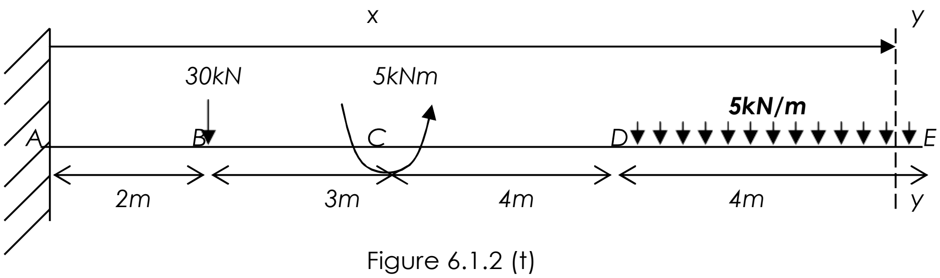


Figure 6.1.2 (t)

Equation of moment

$$M = EI\frac{d^2y}{dx^2} = 50x - 30[x-2] - 265x^0 - 15[x-5]^0 - \frac{5[x-9]^2}{2}$$

Integrate equation of moment to get the equation for slope.

$$EI\frac{dy}{dx} = \frac{50x^2}{2} - \frac{30[x-2]^2}{2} - 265x - 15[x-5] - \frac{5[x-9]^3}{6} + C_1$$

Integrate equation of slope to get the equation for deflection.

$$EIy = \frac{50x^3}{6} - \frac{30[x-2]^3}{6} - \frac{265x^2}{2} - \frac{15[x-5]^2}{2} - \frac{5[x-9]^4}{24} + C_1x + C_2$$

Boundary conditions

At point A, x = 0 , $\frac{dy}{dx} = 0$

$$EI(0) = \frac{50(0)^2}{2} - \frac{30[0-2]^2}{2} - 265(0) - 15[0-5] - \frac{5[0-9]^3}{6} + C_1$$

$$C_1 = 0$$

At point A,x = 0, y = 0

$$EI(0) = \frac{50(0)^3}{6} - \frac{30[0-2]^3}{6} - \frac{265(0)^2}{2} - \frac{15[0-5]^2}{2} - \frac{5[0-9]^4}{24} + (0)x + C_2$$

$$C_2 = 0$$

The complete equation for

i. Slope, $$EI\frac{dy}{dx} = \frac{50x^2}{2} - \frac{30[x-2]^2}{2} - 265x - 15[x-5] - \frac{5[x-9]^3}{6}$$

ii. Deflection,

$$EIy = \frac{50x^3}{6} - \frac{30[x-2]^3}{6} - \frac{265x^2}{2} - \frac{15[x-5]^2}{2} - \frac{5[x-9]^4}{24}$$

Slope at point x = 13m

$$EI\frac{dy}{dx} = \frac{50(13)^2}{2} - \frac{30[13-2]^2}{2} - 265(13) - 15[13-5] - \frac{5[13-9]^3}{6}$$

$$\frac{dy}{dx} = \frac{-1208.33}{EI}rad$$

Deflection at point x = 13

$$EIy = \frac{50(13)^3}{6} - \frac{30[13-2]^3}{6} - \frac{265(13)^2}{2} - \frac{15[13-5]^2}{2} - \frac{5[13-9]^4}{24}$$

$$y = \frac{-11272.5}{EI}mm$$

Example (iv)

Determine the slope and deflection at free end. Given value of E = 10GN/m² and I_{xx} = 45 x 10⁸ mm⁴

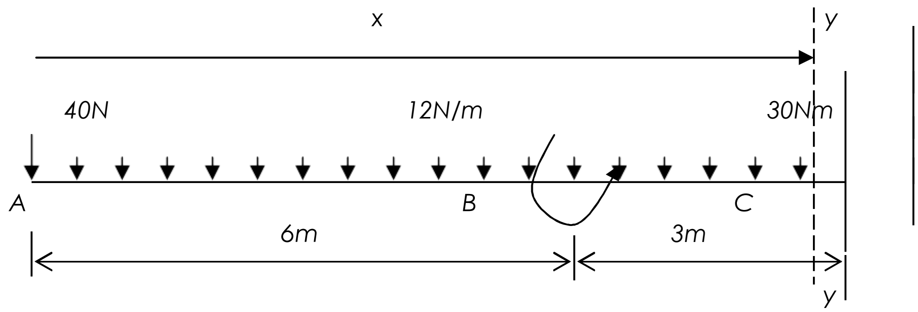


Figure 6.1.2 (u)

(Magnitude of reaction is not required to be determined)

Equation of moment at section y-y

$$EI\frac{d^2y}{dx^2} = M_{yy}$$

$$M_{yy} = -40x - \frac{12x^2}{2} - 30[x-6]^0$$

$$EI\frac{d^2y}{dx^2} = -40x - \frac{12x^2}{2} - 30[x-6]^0$$

Integrate equation of moment to get the equation for slope.

$$EI\frac{dy}{dx} = \frac{-40x^2}{2} - \frac{12x^3}{6} - 30[x-6] + C_1$$

Integrate equation of slope to get the equation for deflection.

$$EIy = \frac{-40x^3}{6} - \frac{12x^3}{24} - \frac{30[x-6]^2}{2} + C_1x + C_2$$

Boundary conditions

At point C, x = 9m , $\frac{dy}{dx} = 0$

$$EI(0) = \frac{-40(9)^2}{2} - \frac{12(9)^3}{6} - 30[0-6] \square C_1$$

$$C_1 = 3168$$

At point C, x = 9m, y =0

$$EI(0) = \frac{-40(9)^3}{6} - \frac{12(9)^3}{24} - \frac{30[9-6]^2}{2} + 3168(9) \square C_2$$

$$C_2 = -20236.5$$

The complete equation for

Slope, $EI\frac{dy}{dx} = \frac{-40x^2}{2} - \frac{12x^3}{6} - 30[x-6] + 3168$

Deflection, $EIy = \frac{-40x^3}{6} - \frac{12x^3}{24} - \frac{30[x-6]^2}{2} + 3168x - 20236.5$

Slope and deflection at point A, x = 0m

Equation of slope,

$$EI\frac{dy}{dx} = \frac{-40(0)^2}{2} - \frac{12(0)^3}{6} - 30\,[0-6] + 3168$$

$$\frac{dy}{dx} = \frac{3168\,x10^{6}}{4.5x10^{10}} = 0.07 \text{ rad}$$

Equation for deflection,

$$EIy = \frac{-40(0)^3}{6} - \frac{12(0)^3}{24} - \frac{30\,[0-6]^2}{2} + 3168\,{}_1x - 20236.5$$

$$y = \frac{-20236.5\,x10^{9}}{4.5\,x10^{10}} = 449.7\text{mm}$$

6.2 MOMENT AREA METHOD

To explain the moment area method, lets consider an area C and D at a deflection of a beam at a section where deflection is positive. Figure 6.1(a) is the bending moment diagram of a beam and figure 6.1(b) is the shape of deflection of beam subjected under moment.

Point C and D are too close together as such magnitude of moment acting at both points may be assumed the same. Bending distance, δs between the two points may also be assumed equal to their horizontal distance.

Let say:

A = Area of bending moment diagram
M = moment at point C and D
δ x = horizontal distance between C and D
δS = length of curve between C and D
J = radius of curvature of beam between C and D
δυ = difference of angle between tangents at C and D

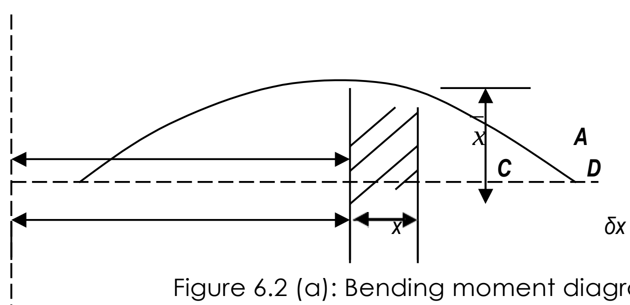
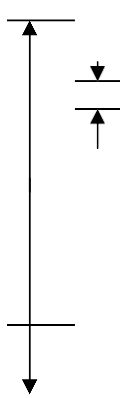


Figure 6.2 (a): Bending moment diagram



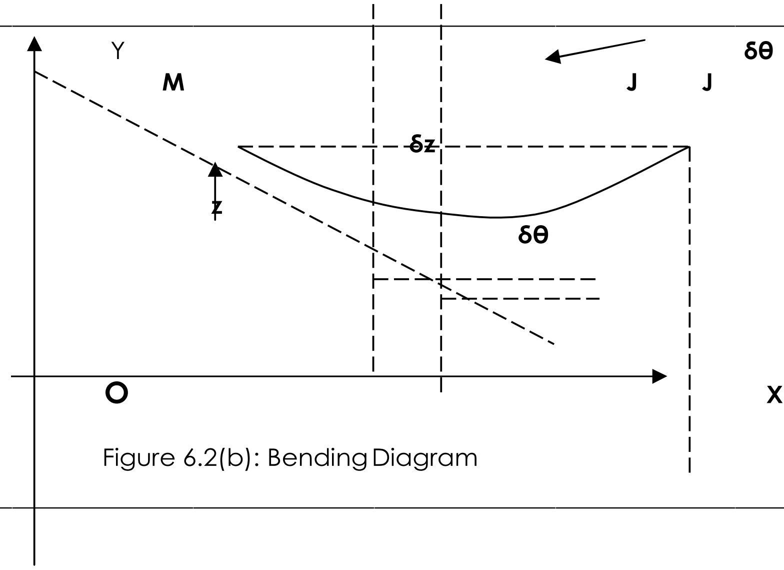


Figure 6.2(b): Bending Diagram

Thus, distance of curve between C and D is:

$\delta x = J\ \delta a$ ------ **equation 6.1**

Tangent at C and D will cause horizontal distance of tangent to the line almost identical that is:

$\delta \upsilon = x\ \delta a$ ------**equation 6.2**

From equation 6.1, substitute $\delta a = \delta x / J$ in equation 6.2.

Therefore,

$$\delta v = x\ \frac{dx}{J}$$

$$\delta v = \frac{x}{J} dx$$

From bending equation in unit 4, $\dfrac{d^2 y}{dx^2} = \dfrac{M}{EI}$

Integrate between C and D, $\left[\dfrac{dy}{dx}\right]_C^D = \int \dfrac{M dx}{EI}$

If EI is constant, then $\left(\dfrac{dy}{dx}\right)_Q - \left(\dfrac{dy}{dx}\right)_P = \dfrac{A}{EI}$ ------- **equation 6.3**

To determine the slope

Moment of Area Theorem I

This shows change of slope (angle θ_{dc}) between any two points on the beam is equal to the algebraic sum of the bending moment diagrams between the said points divided by EI.

Change in slope = area of bending moment diagram/EI

Relative angle θ_{ba} will have positive s θ_b, algebraically is larger than θ_a

Bending moment M will h positive when M cau compression at the top part of the beam

$\theta_{dc} = A / EI$

Area of diagram M/EI with ± sign depends on the sign of the bending moment

If J = radius of curvature of beam between point C and D

$\delta\theta = \delta x$ = angle of tangent

then, $\delta z = x\ \delta\theta$

$= x\ \delta x / R$

$= Mx\,\delta x / EI$

$$z = \int \frac{Mx.dx}{EI}$$

$$\boxed{z = \frac{A\bar{x}}{EI}}$$ when EI constant --------- **equation 6.4**

To determine the deflection

where z = vertical change of deflection
$\bar{x}$ = distance of centroid

Moment of Area Theorem 2

Offset Δ_{ba} point D from C is equal to the first negative of A/EI between C and D taken from D.

6.2.1 To determine the slope of an identified point

In using the first and second moment area theorems, it is useful to sketch the shape of the deflection of beam followed with the moment. In brief, the procedures below may be followed in sequence.

PROCEDURE

i. Determine the reactions beam. Reaction on the most right may be omitted.

ii. Sketch the curvature diagram of beam. This diagram should be consistent with the known conditions at supports.

iii. Sketch the bending moment diagram. It is easier to sketch this diagram section by section (pat by part).

In the case of beam having fixed cross-section, calculation can be done direct using the bending moment diagram and then divided by the area or bending moment by EI.

iv. At a suitable point A and B draw a tangent line to the deflection of curve at one of these points, say point A.

v. Deflection point B from the tangent line at A is then being calculated using the theorem of second moment of area.

6.2.2 Area and centroid

Using the moment area method, it is important to have the knowledge of determining the area and centroid of any shape drawn. Table 1 shows the formulae for shapes common moment diagrams.

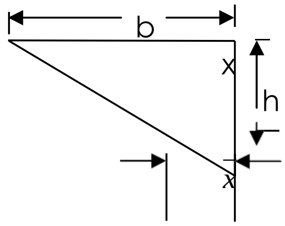
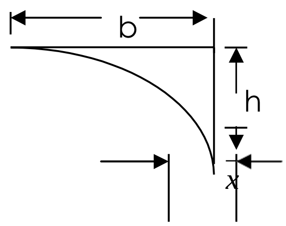
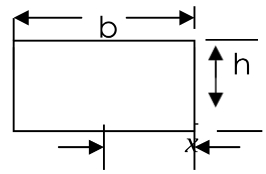
	Triangle	**Parabola**	**Rectangle**
Shape			
Area, A	$\frac{1}{2}bh$	$\frac{1}{3}bh$	$\frac{1}{4}bh$
Centroid, $\bar{x}$	$\frac{1}{3}b$	$\frac{1}{4}b$	$\frac{1}{2}b$

Figure 6.2.2: Area and centroid of basic shapes of bending moment diagram

6.2.3 Simply Supported Beam with Point Load

Below is an example of a simply supported beam under the action of point loads P_1 and P_2.

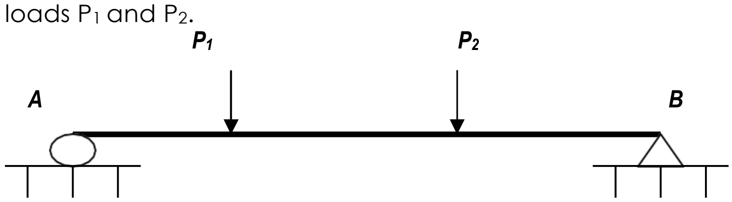
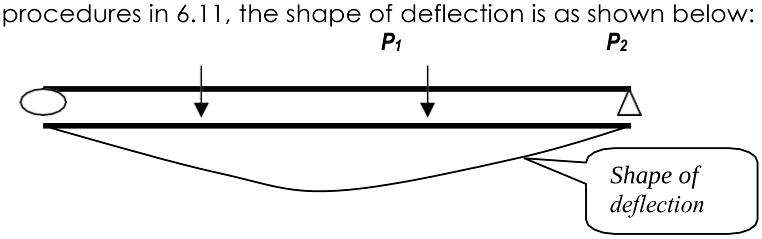


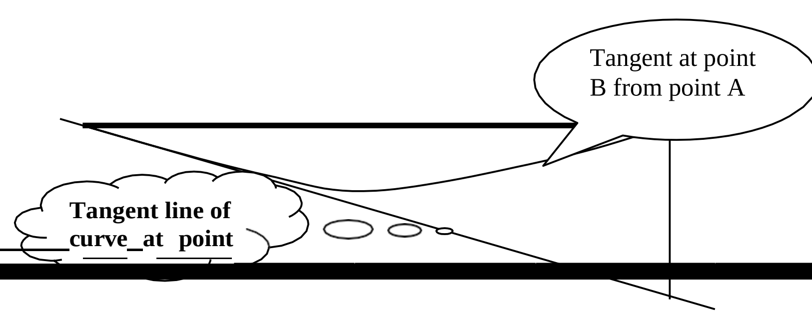
Figure 6.2: Simply supported beam under point loads

Following the procedures in 6.11, the shape of deflection is as shown below:



i. Guided from the point on the most left, draw a line that is tangent to the left most support. Then draw a tangent from the right mostsupport.

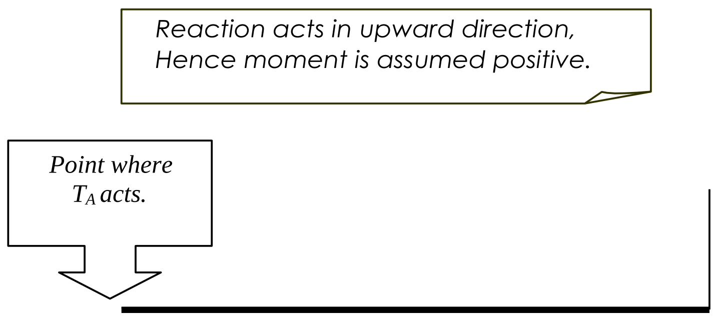
ii.



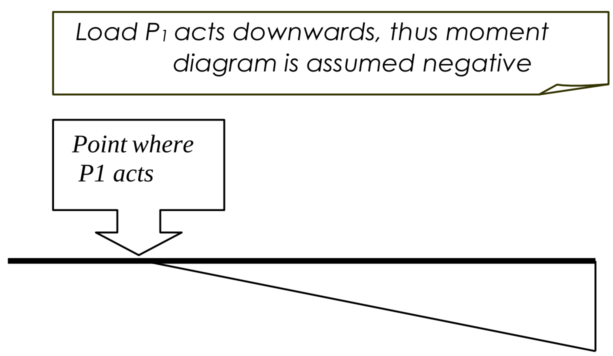
iii. Reactions at supports should first be determined. Then sketch the bending moment diagram for each load and reaction separately, that is one moment diagram for each load and reaction.

Let say reactions at point A and B are T_A and T_B respectively. Hence, in total there are two loads and two reactions. However it is required to sketch three bending moment diagrams only that is excluding bending moment diagram for reaction at point B (bending moment diagram for reaction at B is not required as it is at the right most point).

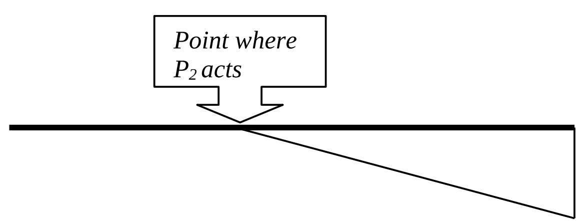
Bending moment diagram for T_A from B



Bending moment diagram of point load P_1 from B



Bending moment diagram of point load P_2 from B



Example (a)

A simply supported beam spanning 6m long is subjected to point loads of 20kN and 15kN as shown in figure 6.3. Determine the deflection at point C.

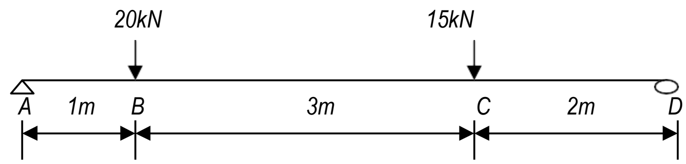


Figure 6.2.3 (a): Simply supported beam under point loads

Solution

Determine the reaction at point A. Taking moment from point D, then reaction at D is not required.

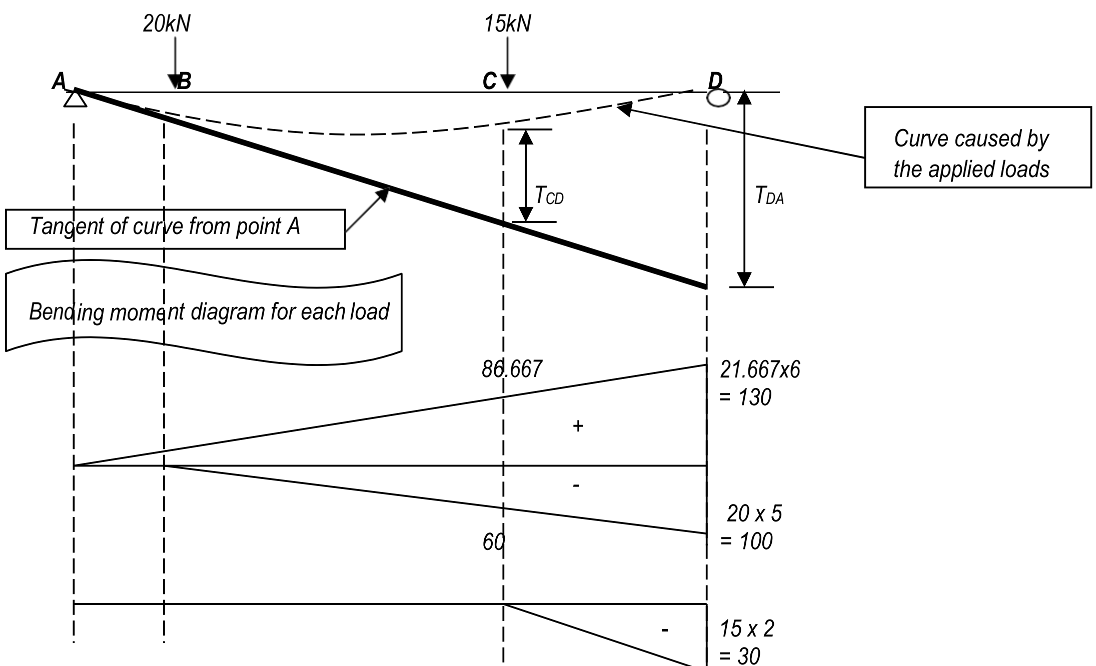
$\Sigma M = 0$

$R_A(6) - 5(20) - 15(2) = 0$

$R_A = 21.67kN$

Then follow the steps (as example 6.1) below:-

i. Sketch the probable deflection of beam that may occur.
ii. Draw tangent of at specific curve point from one support (in this unit select right support)
iii. Sketch bending moment diagrams for each load and reaction at A.
iv. Use equation 6.4 to get the value of tangent.



Applying equation of change in vertical deflection (equation 6.4)

$$z = \frac{A\bar{x}}{EI}$$

$$T_{DA} = \Sigma \frac{A\bar{x}}{EI} = \frac{1}{EI}\Sigma A\bar{x}$$

$$T_{DA} = \frac{1}{EI}\left(\left[\frac{1}{2}\times 6\times 130\right]\left[\frac{1}{3}\times 6\right] - \left[\frac{1}{2}\times 5\times 100\right]\left[\frac{1}{3}\times 5\right] - \left[\frac{1}{2}\times 2\times 30\right]\left[\frac{1}{2}\times 2\right]\right)$$

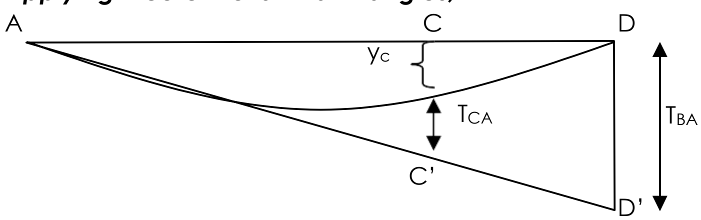
$$T_{DA} = \frac{1}{EI}(780 - 416.667 - 30)$$

$$T_{DA} = \frac{1226.667}{EI}$$

$$T_{CA} = \frac{1}{EI}\left[\left(\frac{1}{2}\times 4\times 86.667\right)\left(\frac{1}{3}\times 4\right) - \left(\frac{1}{2}\times 3\times 60\right)\left(\frac{1}{3}\times 3\right)\right]$$

$$T_{CA} = \frac{1}{EI}(231.113 - 90), \qquad T_{CA} = \frac{141.113}{EI}$$

Applying theorem of similar triangles;



$$\frac{CC'}{4} = \frac{T_{DA}}{6}$$

$$CC' = \frac{T_{DA}\times 4}{6} = \frac{1226.667\times 4}{EI\times 6}$$

$$CC' = \frac{817.778}{EI}$$

Knowing that $T_{CA} = \frac{141.113}{EI}$ and $y_c = CC' - T_{CA}$

Thus deflection at point C, $y_c = \frac{817.778 - 141.113}{EI}$

$$y_c = \frac{676.665}{EI}$$

Change of slope between point D and point A

$\theta_A = \tan\theta_A$

$\theta_A = \frac{T_{DA}}{6}$, $\theta_A = \frac{1226.667}{EI\times 6}$, $\theta_A = \frac{204.445}{EI}$

Change of slope between point A and point C is

$$\theta_{AC} = \frac{\sum area.BMD_{AC}}{EI}$$

$$\theta_{AC} = \frac{\left(\frac{1}{2}\times 4\times 86.667\right) - \left(\frac{1}{2}\times 3\times 60\right)}{EI}, \qquad \theta_{AC} = \frac{83.334}{EI}$$

Slope at point C, $\theta_C = \theta_{AC} - \theta_A$

$$\theta_C = \frac{83.334 - 204.445}{EI}$$

Simply Supported Beam with UDL

For cases of simply supported beam with uniformly distributed load, position of the distributed load should be examined as follows:

i. Load is distributed until the right end of beam as in figure 6.7 below,

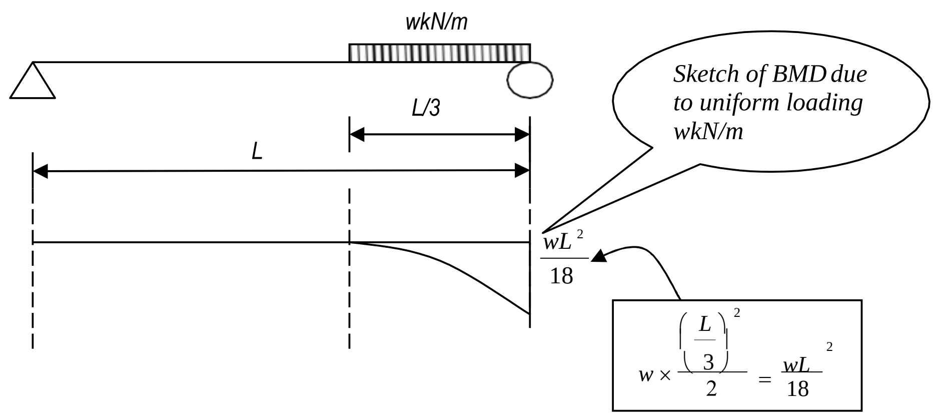
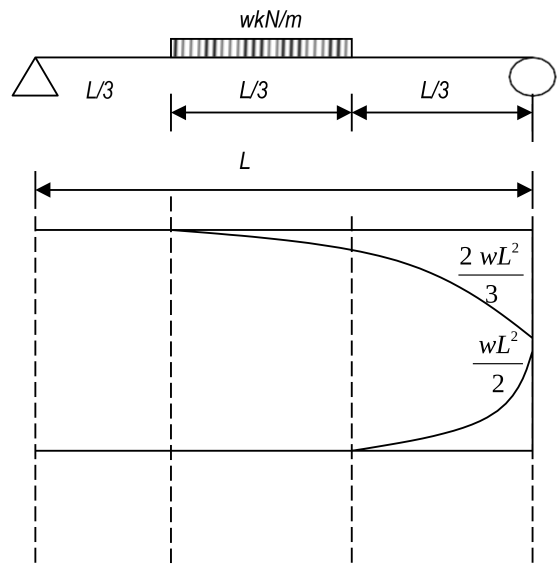


Figure 6.2.3 (b): Beam with distributed load at one end

ii. When distributed load is in middle of span, then it is necessary to do some balancing as follows.

Figure 6.8: Simply supported beam with distributed load at mid-span



When load distributed at mid-span, extend this load to the right end of beam

$$w\frac{\left(\frac{2L}{3}\right)^2}{2} = \frac{2wL^2}{3}$$

The extended load is counter balanced by applying the same load under beam at a span of L/3 to the right end of beam, hence with +ve sign.

Example 6.3

Example (b)

Determine the slope and deflection at point C of a simply supported beam subjected to a uniformly distributed load as shown in figure 6.9 below.

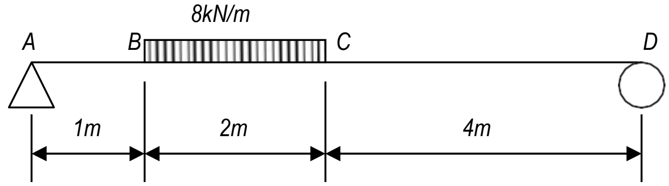


Figure 6.2.3 (c): Simply supported beam under distributed loading

SLOPE AND DEFLECTION OF BEAM DUE TO SYMMETRICAL BENDING

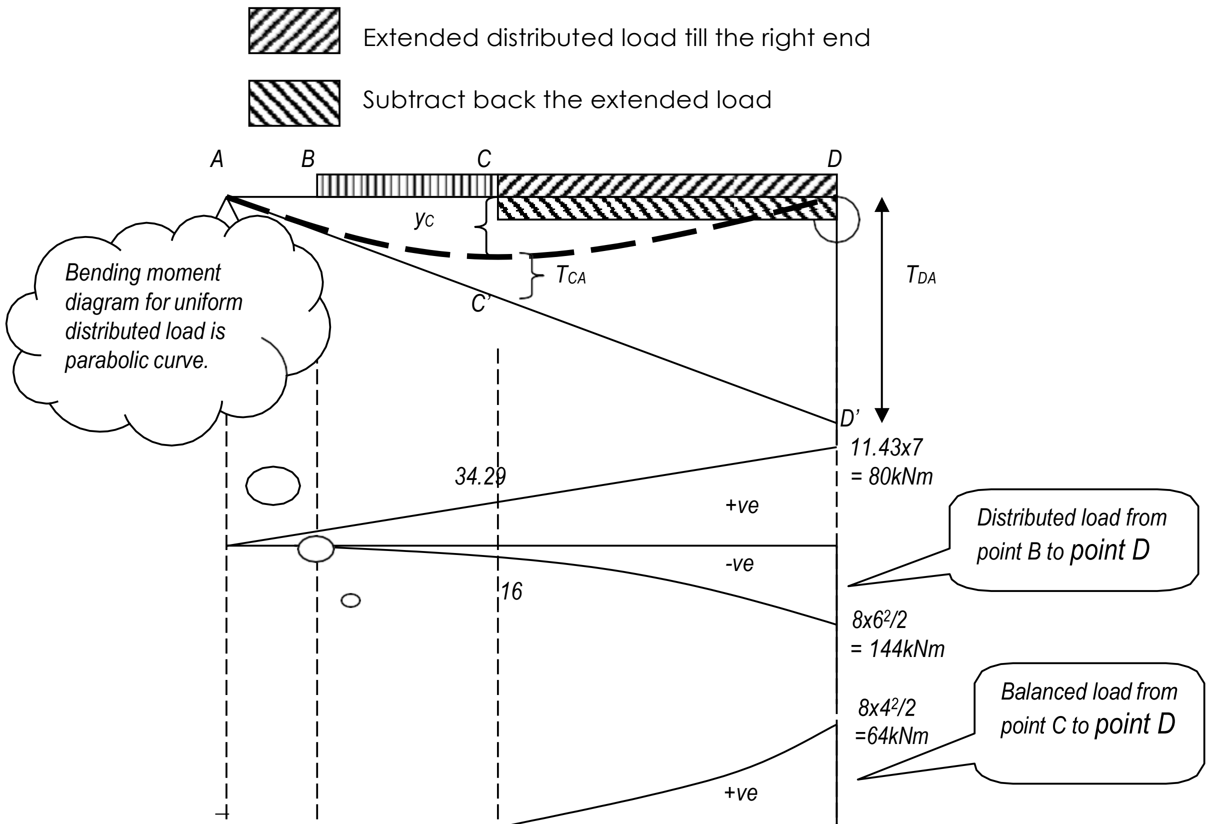
Solution

Determine the reaction at support A

$\Sigma M_F = 0$

$7R_A - 8(2)(5) = 0, \qquad R_A = 11.43kN$

Similar to the Macaulay's method for uniformly distributed loading, counter balancing of loading should be introduced, that is by extending the load to the right end of beam and then balanced by subtracting the same amount of the load extended.



$$T_{DA} = \Sigma \frac{Ax}{EI}$$

$$T_{DA} = \frac{1}{EI}\left[\left(\frac{1}{2}\times 7\times 80\right)\left(\frac{1}{3}\times 7\right) - \left(\frac{1}{3}\times 6\times 44\right)\left(\frac{1}{4}\times 6\right) + \left(\frac{1}{3}\times 4\times 64\right)\left(\frac{1}{4}\times 4\right)\right]$$

$$T_{DA} = \frac{653.333 + 132 + 85.333}{EI}$$

$$T_{DA} = \frac{870.666}{EI}$$

$$T_{CA} = \frac{1}{EI}\left[\left(\frac{1}{2}\times 3\times 34.29\right)\left(\frac{1}{3}\times 3\right) - \left(\frac{1}{3}\times 2\times 16\right)\left(\frac{1}{4}\times 2\right)\right]$$

$$T_{CA} = \frac{51.435 - 5.333}{EI}$$

$$T_{CA} = \frac{46.102}{EI}$$

Applying theorem of similar triangles ADD' and ACC'

$$\frac{CC'}{AC} = \frac{DD'}{AD}$$

$$CC' = \frac{T_{DA} \times AC}{AD}$$

$$CC' = \frac{870.666 \times 3}{EI \times 7}$$

$$CC' = \frac{373.14}{EI}$$

Deflection at point C

$y_C = CC' - T_{CA}$

$y_C = \frac{373.14 - 46.102}{EI}$, $\quad y_C = \frac{327.04}{EI}$

Slope at point C

$$\theta_A = Tan\,\theta_A = \frac{T_{DA}}{AD}$$

$$\theta_A = \frac{870.666}{EI \times 7}$$

$$\beta = \frac{124.38}{EI}$$

Change of slope between point C and point A

$$\theta_{CA} = \frac{1}{EI}\left[\left(\frac{1}{2} \times 3 \times 34.29\right) - \left(\frac{1}{3} \times 2 \times 16\right)\right]$$

$$\theta_{CA} = \frac{40.768}{EI}$$

Slope at point C

$$\theta_C = \theta_{CA} - \theta_A$$

$$\theta_C = \frac{40.768 - 124.38}{EI}$$

$$\theta_C = \frac{83.612}{EI}$$

Example (c)

Figure 6.10 shows a simply supported beam subjected to point and distributed loads. Determine the slope and deflection at point C.

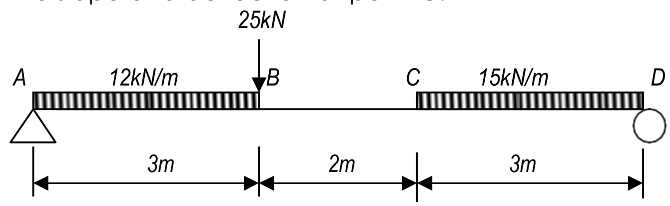


Figure 6.2.3 (d): Beam with point and distributed loads

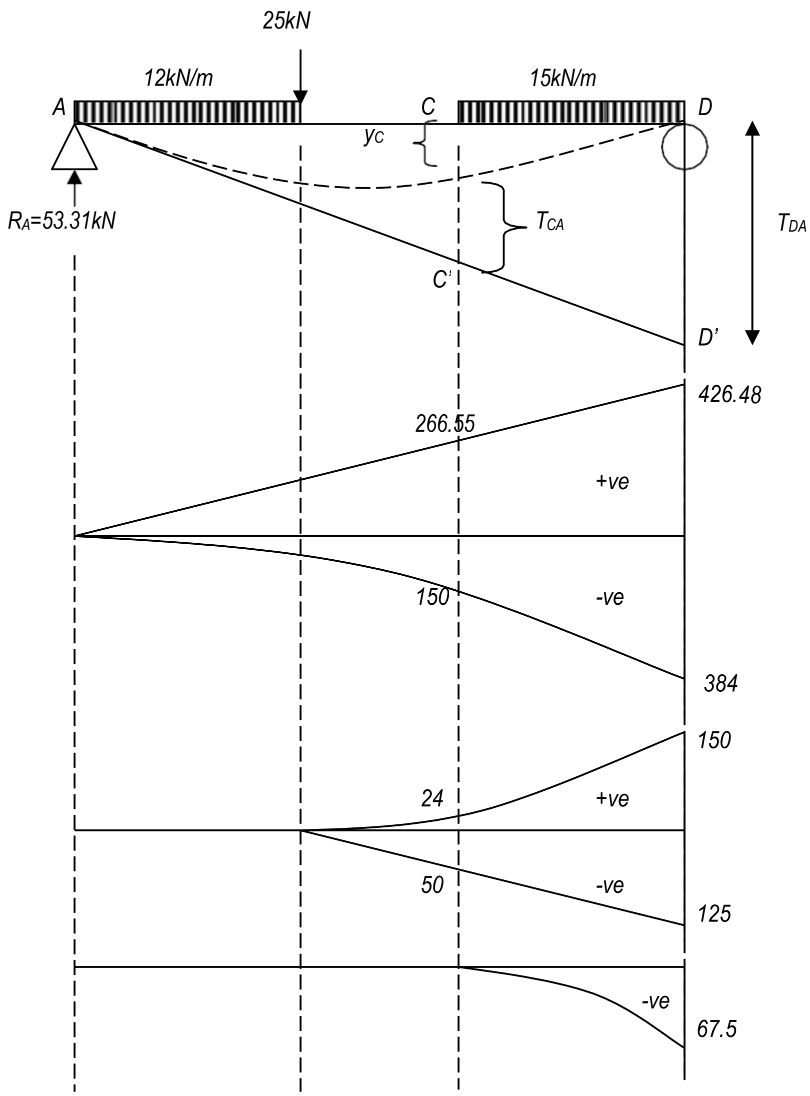
Determine the reaction at support A

$\Sigma M_D = 0$

$8R_A - 12(3)(6.5) - 25(5) - 15(3)(1.5) = 0$

$R_A = 53.31\text{kN}$

Sketch the deflection and the moment diagrams



$$T_{DA} = \Sigma \frac{A\bar{x}}{EI}$$

$$T_{DA} = \frac{1}{EI}\left[\left(\frac{1}{2}\times 8\times 426.48\right)\left(\frac{1}{3}\times 8\right)-\left(\frac{1}{3}\times 8\times 384\right)\left(\frac{1}{4}\times 8\right)+\left(\frac{1}{3}\times 5\times 150\right)\left(\frac{1}{4}\times 5\right) -\left(\frac{1}{2}\times 5\times 125\right)\left(\frac{1}{3}\times 5\right)-\left(\frac{1}{3}\times 3\times 67.5\right)\left(\frac{1}{4}\times 3\right)\right]$$

$$T_{DA} = \frac{1}{EI}\left[4549.12 - 2048 + 312.5 - 520.833 - 50.625\right]$$

$$T_{DA} = \frac{2242.162}{EI}$$

$$T_{CA} = \frac{1}{EI}\left[\left(\frac{1}{2}\times 5\times 266.55\right)\left(\frac{1}{3}\times 5\right)-\left(\frac{1}{3}\times 5\times 150\right)\left(\frac{1}{4}\times 5\right)+\left(\frac{1}{3}\times 2\times 24\right)\left(\frac{1}{4}\times 2\right)-\left(\frac{1}{2}\times 2\times 50\right)\left(\frac{1}{3}\times 2\right)\right]$$

$$T_{CA} = \frac{1}{EI}\left[1110.625 - 312.5 + 8 - 33.333\right]$$

$$T_{CA} = \frac{772.792}{EI}$$

Theorem of similar triangles ACC' and ADD'

$$\frac{CC''}{AC} = \frac{DD'}{AD}$$

$$CC' = \frac{T_{DA}\times AC}{AD}, \qquad CC' = \frac{2242.162\times 5}{EI\times 8}$$

$$CC' = \frac{1401.35}{EI}$$

Deflection at point C

$$y_C = CC' - T_{CA}$$

$$y_C = \frac{1401.35 - 772.792}{EI} \qquad y_C = \frac{628.558}{EI}$$

Slope at point C

$$\theta_A = Tan\,\theta_A = \frac{T_{DA}}{8}$$

$$\theta_A = \frac{2242.162}{EI\times 8} \qquad \theta_A = \frac{280.27}{EI}$$

Change of slope between point C and point A

$$\theta_{CA} = \frac{1}{EI}\left[\left(\frac{1}{2}\times 5\times 266.55\right)-\left(\frac{1}{3}\times 5\times 150\right)+\left(\frac{1}{3}\times 2\times 24\right)-\left(\frac{1}{2}\times 2\times 50\right)\right]$$

$$\theta_{CA} = \frac{1}{EI}\left[666.375 - 250 + 16 - 50\right]$$

$$\theta_{CA} = \frac{382.375}{EI}$$

Thus, slope at point C

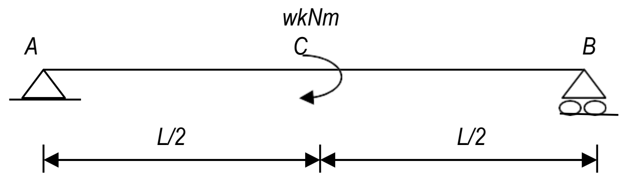
$$\theta_C = \theta_{CA} - \theta_A$$

$$\theta_C = \frac{382.373 - 280.27}{EI} \qquad \theta_C = \frac{102.103}{EI}$$

Simply Supported Beam with Moment

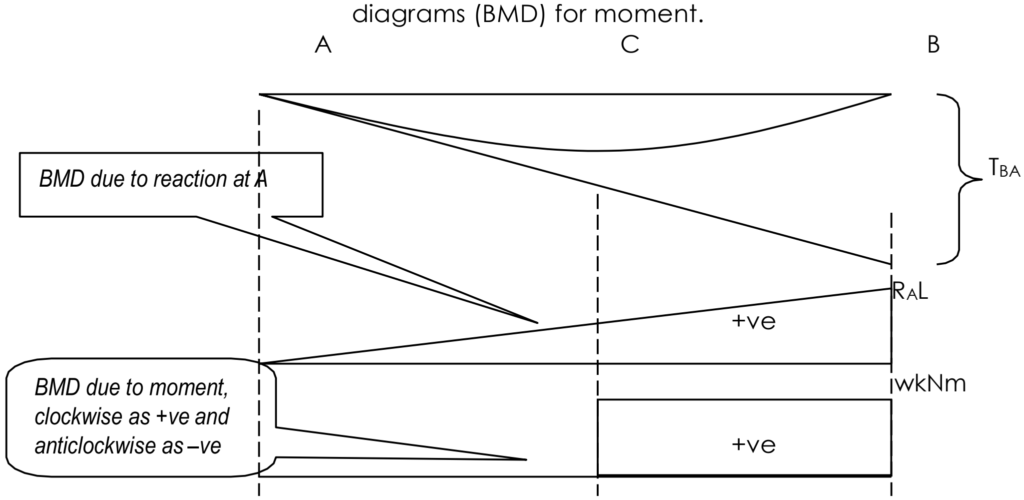
Let consider a beam simply supported as shown in figure 6.11 subjected to a moment wNm.

Figure 6.11: Simply supported Beam under moment



As input 1 and 2, follow the procedures as outlined in 6.11

What is being emphasis in unit 3 is the method of sketching bending moment diagrams (BMD) for moment.



Example (d)

A simply supported beam is subjected to loadings as shown in figure 6.12. Determine the slope and deflection at point C.

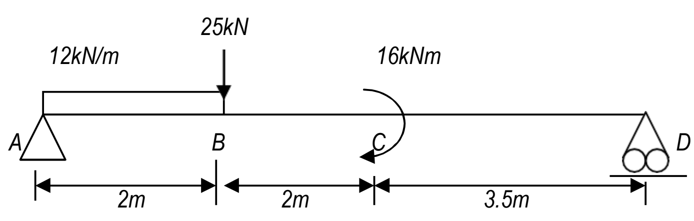
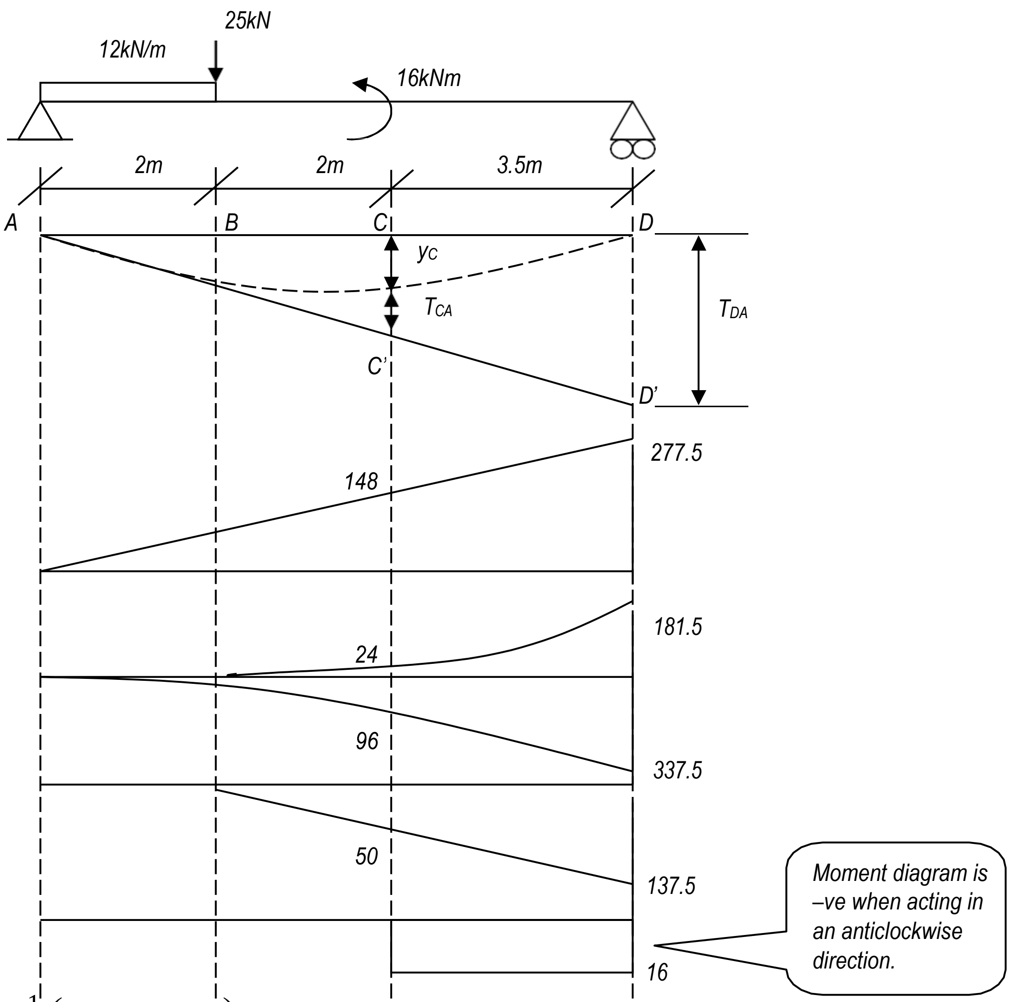


Figure 6.2.3 (e): Simply supported beam under various loadings

Solution

Determine reaction at support A

$$\sum M_d = 0$$
$$R_A(7.5) - 12(2)(6.5) - 25(5.5) + 16 = 0$$
$$R_A = 37\ kN$$



$$T_{DA} = \frac{1}{EI}\left(LuasG.M. \times x\right)$$

$$T_{DA} = \frac{1}{EI}\left[\left(\frac{1}{2}\times 277.5\times 7.5\times \frac{7.5}{3}\right)+\left(\frac{1}{3}\times 181.5\times 5.5\times \frac{5.5}{4}\right)-\left(\frac{1}{3}\times 337.5\times 7.5\times \frac{7.5}{4}\right)-\left(\frac{1}{2}\times 137.5\times 5.5\times \frac{5.5}{3}\right)-\left(16\times 3.5\times \frac{3.5}{2}\right)\right]$$

$$T_{DA} = \frac{755.833}{EI}$$

$$T_{CA} = \frac{1}{EI}\left[\left(\frac{1}{2}\times 148\times 4\times \frac{4}{3}\right)+\left(\frac{1}{3}\times 24\times 2\times \frac{2}{4}\right)-\left(\frac{1}{3}\times 96\times 4\times \frac{4}{4}\right)-\left(\frac{1}{2}\times 50\times 2\times \frac{2}{3}\right)\right]$$

$$T_{CA} = \frac{241.333}{EI}$$

Applying theorem of similar triangles ACC' and ADD'

$$\frac{AC}{CC'} = \frac{AD}{DD'}$$

$$CC' = \frac{AC \times DD'}{AD}$$

$$CC' = \frac{4 \times 755.833}{7.5EI}$$

$$CC' = \frac{403.111}{EI}$$

$$Known \quad CC' = y_c + T_{CA}$$

$$\therefore \quad y_c = CC' - T_{CA}$$

$$y_C = \frac{403.111 - 241.333}{EI}$$

$$y_c = \frac{161.778}{EI}$$

Slope at point A,

$$\theta_A = Tan\,\theta$$

$$= \frac{T_{DA}}{7.5}$$

$$= \frac{755.833}{7.5EI}$$

$$= \frac{100.777}{EI}$$

Change of slope between point A and point C,

$$\theta_{AC} = \frac{1}{EI} \times area \quad BM$$

$$\theta_{AC} = \frac{1}{EI}\left[\left(\frac{1}{2}\times 148 \times 4\right) + \left(\frac{1}{3}\times 24 \times 2\right) - \left(\frac{1}{3}\times 96 \times 4\right) - \left(\frac{1}{2}\times 50 \times 2\right)\right]$$

$$\theta_{AC} = \frac{134}{EI}$$

Slope at point C,

$$\theta_C = \theta_{AC} - \theta_A$$

$$= \frac{134 - 100.77}{EI}$$

$$= \frac{33.223}{EI}$$

Example (e)

Based on figure 6.13 determine the slope and deflection at point D using the moment area method.

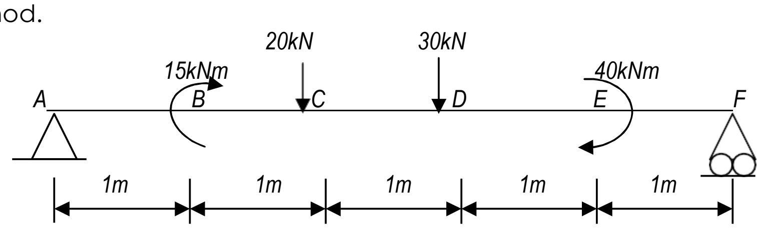


Figure 6.2.3 (f): Simply supported beam with various loadings

Solution

Determine reaction at support A

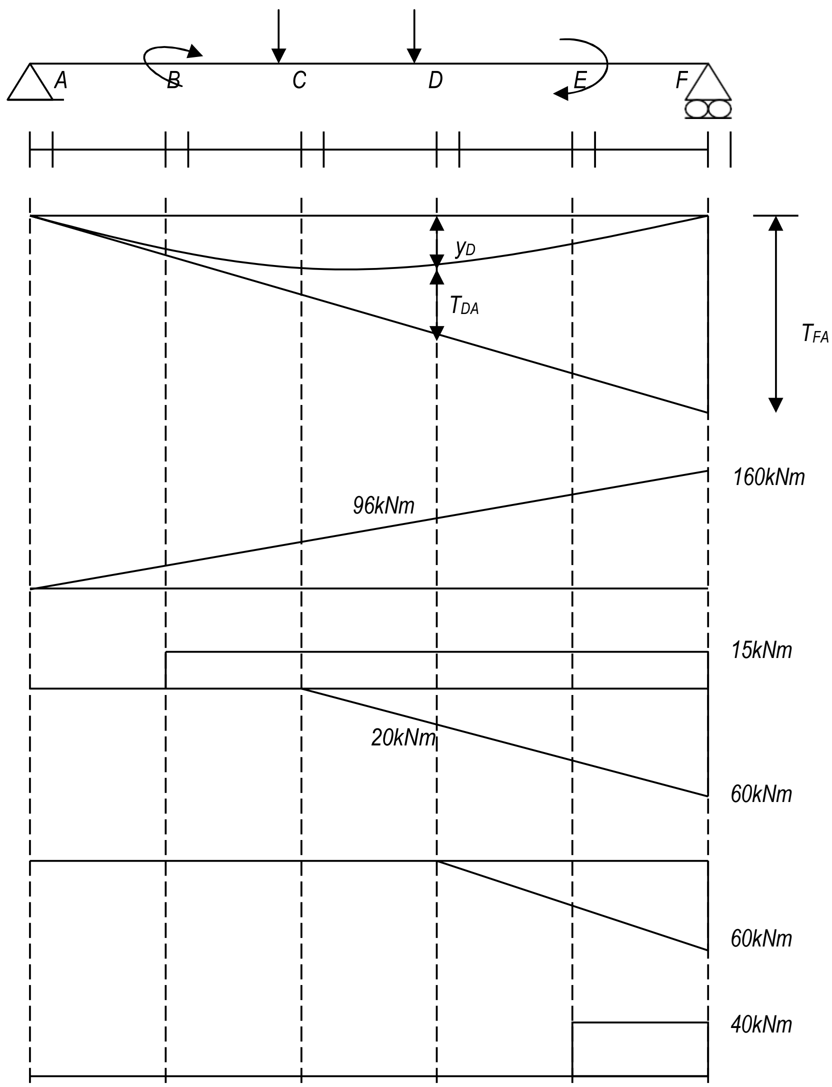
$\Sigma M_F = 0$

$5F_A + 15 - 20(3) - 30(2) - 40 = 0$

$F_A = 32kN.$

Then,

i. Sketch the deflection curve of beam
ii. Sketch bending moment diagram separately for each loadings and reaction.



SLOPE AND DEFLECTION OF BEAM DUE TO SYMMETRICAL BENDING

$$T_{FA} = \frac{1}{EI}\left[\left(\frac{1}{2}\times 5\times 160\times \frac{5}{3}\right)+\left(4\times 15\times \frac{4}{2}\right)-\left(\frac{1}{2}\times 3\times 60\times \frac{3}{3}\right)-\left(\frac{1}{2}\times 2\times 60\times \frac{2}{3}\right)-\left(1\times 40\times \frac{1}{2}\right)\right]$$

$$T_{FA} = \frac{1303.333}{EI}$$

$$T_{DA} = \frac{1}{EI}\left[\left(\frac{1}{2}\times 3\times 96\times \frac{3}{3}\right)+\left(2\times 15\times \frac{2}{5}\right)-\left(\frac{1}{2}\times 1\times 20\times \frac{1}{2}\right)\right]$$

$$T_{DA} = \frac{151}{EI}$$

Applying theorem of similar triangles ADD' and AFF', it is found out that

$$\frac{AD}{DD'} = \frac{AF}{FF'}$$

$$DD' = \frac{AD\times FF'}{AF}$$

$$DD' = \frac{3\times 1303.333}{5EI}$$

$$DD' = \frac{782}{EI}$$

$$Known \quad DD' = T_{DA} + y_C$$

$$Then \quad y_C = DD' - T_{DA}$$

$$y_C = \frac{732-151}{EI}$$

$$y_C = \frac{581}{EI}$$

$$\theta_A = \tan\theta_A$$

$$\theta_A = \frac{T_{FA}}{5}$$

$$= \frac{1303.33}{5EI}$$

$$= \frac{260.666}{EI}$$

Change in slope A and D

$$\theta_{AD} = \frac{1}{EI}\left[\sum area \ GM_{AD}\right]$$

$$\theta_{AD} = \frac{1}{EI}\left[\left(\frac{1}{2}\times 3\times 96\right)+(2\times 15)-\left(\frac{1}{2}\times 1\times 20\right)\right]$$

$$\theta_{AD} = \frac{164}{EI}$$

Slope at point C,

$$\theta_D = \theta_{AD} - \theta_A$$

$$\theta_D = \frac{164-260.666}{EI}$$

$$\theta_D = -\frac{96.666}{EI}$$

Cantilever Beam with Point Load

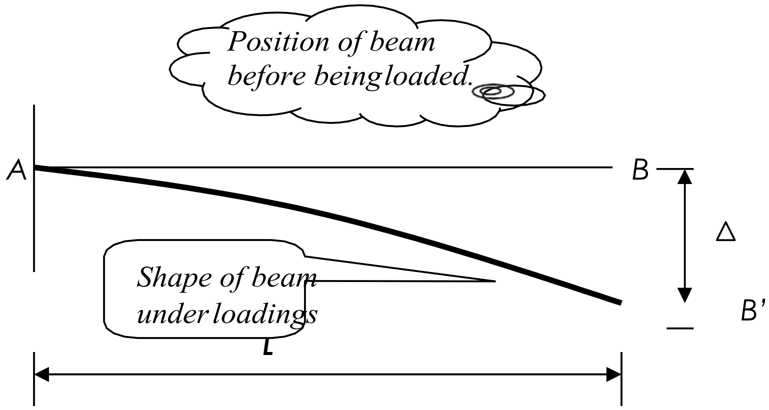


Figure 6.2.3 (g): Diagram of Deflection of Beam

For simplicity in determining slope and deflection of a specific point on the loaded beam, it is easier to apply the following sequences:

i. Determine the reactions at supports. It is ignored if the fixed end is on the right hand side.

ii. Sketch the curvature of beam under loadings. This curve must be consistent with the known conditions at support that is zero slope or zero deflection.

iii. Sketch the bending moment diagrams. It is easier if the bending moment diagram is sketched section by section as discussed in unit 6.

iv. In the case of a beam having a fixed cross-section, calculations may be performed directly using the bending moment diagram and then dividing the calculated area of bending moment diagrams by EI.

v. Choose a suitable point A and B and tangent is drawn to the curve of the deflection at one of the two points, say A.

vi. Deflection at point B from tangent at A is then calculated by using theorem of second moment of area.

In the case of a cantilever, the determination of reaction at support depends on its position whether on the left or right hand side of the cantilever. It is defined, for a cantilever, that the ***slope and deflection at fixed end is zero.***

Consider a cantilever beam as shown in figure 7.2. Determine the slope and deflection when subjected to a point P at the free end.

Figure 6.2.3 (h): Beam with a point load P

To determine the slope and deflection of the beam above, follow procedure 7.1.

i. Sketch the actual deflection shape that will occur at the free end A.

ii.

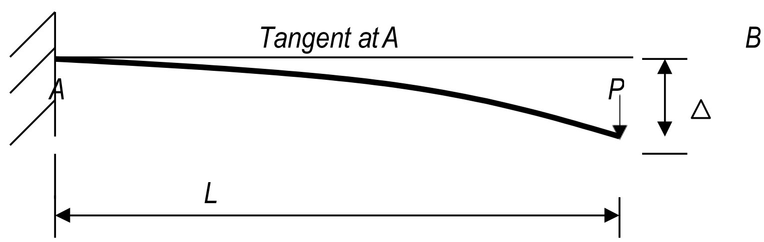


Figure 6.2.3 (i): Shape of deflection of beam due to Loading P

iii. Sketch a tangent line of deflection at point A. This tangent is in place with beam's position without deflection and is represented with a straight line (refer figure 7.1). The deflection at point B from tangent at A is the actual deflection (Δ). Deflection at free end B is determined using the second moment area theorem.

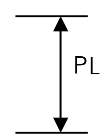


Figure 6.2.3 (j): Bending Moment Diagram due to Point Load, P

iv. It is easiest to sketch bending moment diagram starting from the left to the right as shown in figure 7.4. The centroid of this diagram is situated at 1/3 the distance from the right end. Area of the triangular shape of the bending moment diagram is given by (L/2)(-PL), negative sign is used when the bending moment is negative.

iv. From the second moment of area theorem,

$$EI\,\Delta = \left(\frac{L}{2}\right)(-PL)\left(\frac{L}{3}\right)$$

$$= \frac{-PL^3}{6}$$

or $\Delta = \dfrac{-PL^3}{6EI}$

The negative sign shows that final position of point B is below the tangent line drawn at point A.

vii. Draw tangent at point B to determine the slope at free end.

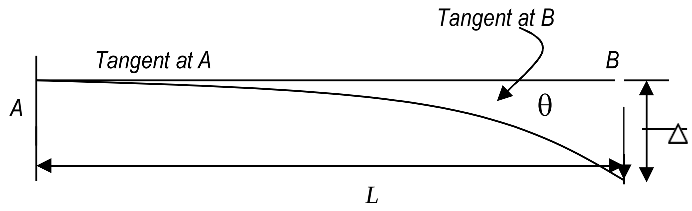


Figure 6.2.3 (k): Slope at free end, B due to point load P

According to theorem of first moment of area, ***angle θ between the two tangents is equal to the area of moment diagram between the two points divided by EI.*** That is

$$EI\theta = (L)\left(\frac{-PL}{2}\right)$$

$$\theta = \frac{-PL^2}{2EI}$$

Negative sign indicates that tangent line on right side of point B forms an angle in clockwise direction with tangent on left of point A. ***Angle θ is in radians***.

Example (f)

A cantilever beam as shown in figure 7.6 supports a point load of 23kN at the free end. Determine the slope and deflection at free end.

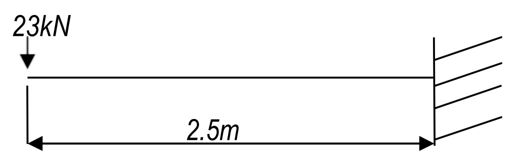
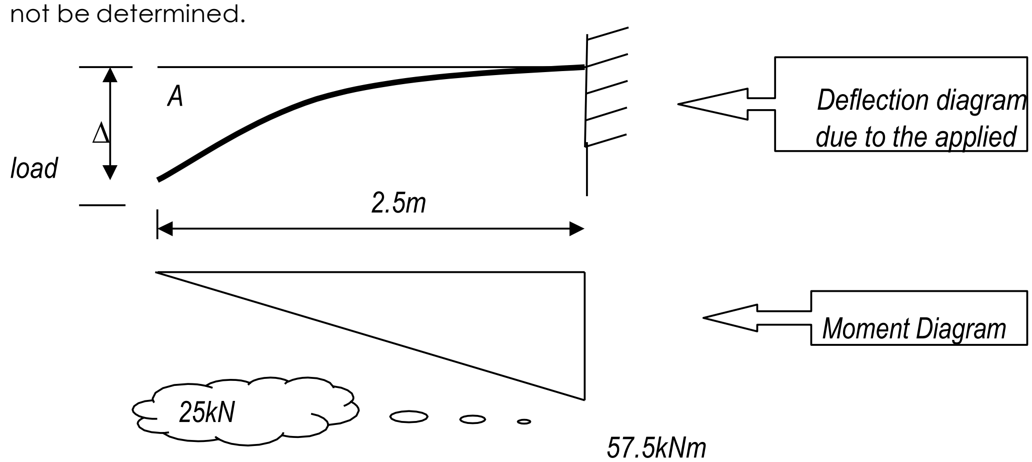


Figure 6.2.3 (l): A cantilever beam subjected to a point load

Solution

When a cantilever is fixed at the right end then the reaction at the fixed end need not be determined.



EIΔ = area of moment diagram x distance of the centroid to the right most point

$$EI\,\Delta = \left(2.5 \times 57.5 \times \frac{1}{2}\right)\left(\frac{1}{3} \times 2.5\right)$$

$$EI\,\Delta = 59.896\,, \qquad \Delta = \frac{59.896}{EI}$$

θ = Area of moment diagram ÷ EI

θ = 2.5 x 57.5 x 0.5 ÷ EI = 71.875/EI

Example (g)

A cantilever beam shown below is subjected to a 40N load at the free end, calculate the maximum slope and deflection.

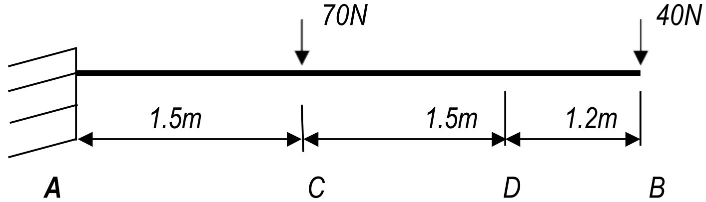


Figure 6.2.3 (m): Cantilever subjected to point load

Solution

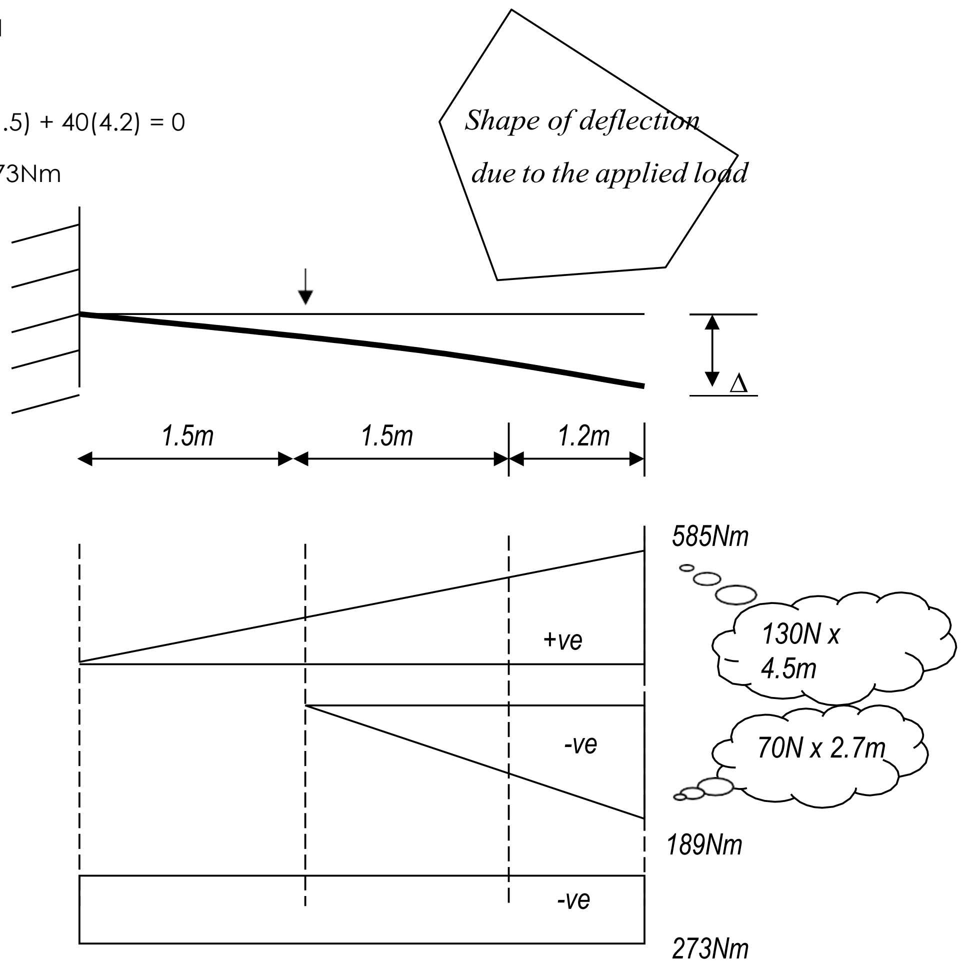
Determine the reaction at point A.

R_A = 130N

$\Sigma M_A = 0$

M_A + 70(1.5) + 40(4.2) = 0

M_A = - 273Nm



To determine the maximum deflection, then:

EIΔ = Area of moment diagram x distance of centroid to the most right point

$$EI\,\Delta = \left(\frac{1}{2}\times 4.2\times 168\right)\left(\frac{2}{3}\times 4.2\right) + \left(\frac{1}{2}\times 1.5\times 105\right)\left(\left(\frac{2}{3}\times 1.5\right) + 2.7\right)$$

$$EI\,\Delta = 1279.215\ Nm^3$$

$$\Delta = \frac{1.279\ kNm^3}{EI}$$

To determine the maximum slope, then:

θ = Area of bending moment diagram ÷ EI

$$\theta = \frac{\left(\frac{1}{2}\times 4.2\times 168\right) + \left(\frac{1}{2}\times 1.5\times 105\right)}{EI}$$

$$\theta = \frac{431.55\ Nm^2}{EI}$$

CANTILEVER BEAM WITH UNIFORMLY DISTRIBUTE LOAD

A cantilever beam supports a uniformly distributed load through its entire span as shown in figure 7.9. Determine the deflection at the free end of beam.

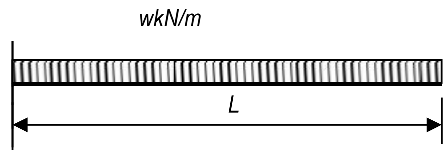


Figure 6.2.3 (o): Cantilever under uniformly distributed load

i. Reaction at fixed end should be determined.

$R_A = wL$

$$M_A = \frac{-wL}{2}$$

ii. Sketch the deflection curve of the beam. Thus the thick line A to B in figure 7.11 represents this deflection.

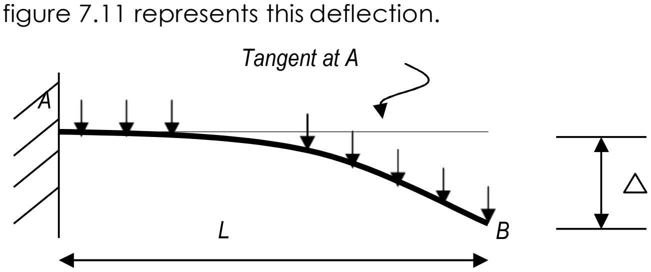


Figure 6.2.3 (p): Shape of deflection of beam under loadings

Since the beam is fixed on the left, then the slope and deflection at this end is zero.

iii. Draw a tangent line at the fixed end, A. The drop of point B from tangent at A indicates the deflection at the free end, (Δ).

iv. Sketch the moment diagram. It is better to sketch it from left to the right.

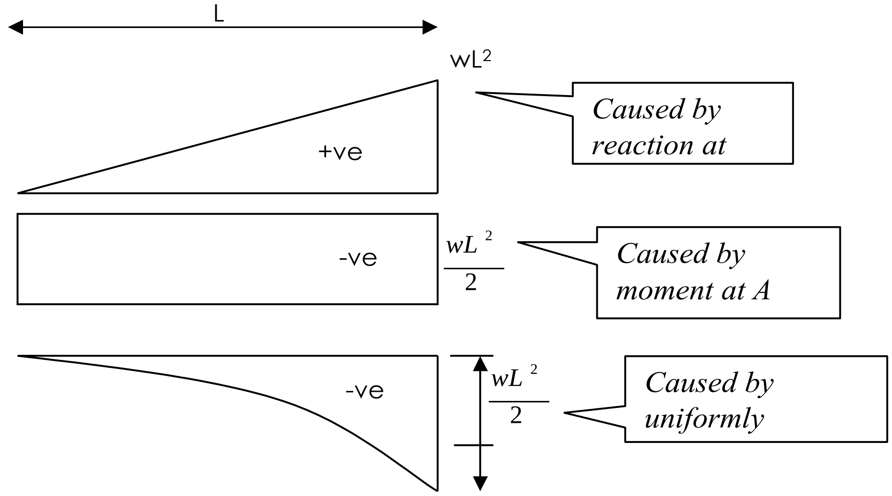


Figure 6.2.3 (q): Bending Moment Diagram

v. Maximum bending moment occurs at the fixed end with magnitude of $wL^2/2$. The diagram of this moment is shown with parabolic curve. Deflection at free end is determined by applying the second moment of area theorem. Thus, the moment of area of the bending moment diagrams between A and B should be calculated and then divided by EI. Using the relationship between area and position of centroid of area of parabola, then;

$$EI\,\Delta = \left(\frac{L}{2} \times wL^2 \times \frac{L}{3}\right) + L\left(-\frac{wL^2}{2}\right)\left(\frac{L}{2}\right) + \left(\frac{L}{3} \times \left(\frac{wL^2}{2}\right) \times \frac{L}{4}\right)$$

$$= \frac{wL^4}{6} - \frac{wL^4}{4} - \frac{wL^4}{24}$$

$$\Delta = -\frac{wL^4}{8\,EI}$$

Negative sign shows final position of point B is below the tangent line drawn through point A.

Example (h)

A cantilever beam is subjected to a uniformly distributed load from a point to the free end of beam as shown in figure 10.13 below. Determine deflection at the free end.

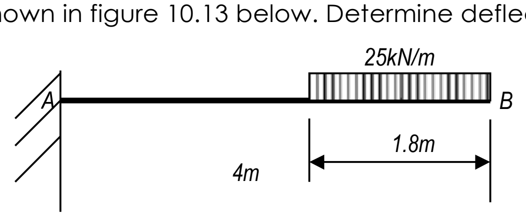


Figure 6.2.3 (r): A cantilever with uniformly distributed load

Solution

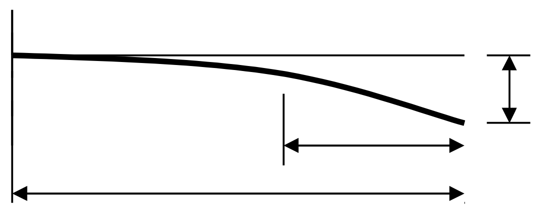
i. Determine reaction at fixed end

R_A = 25(1.8) = 45kN

$\Sigma M_A = 0$

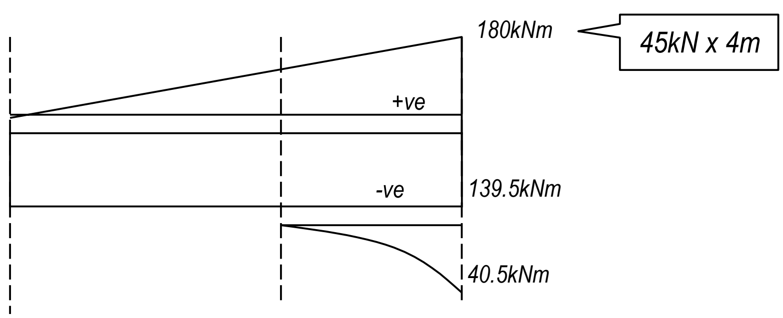
M_A + 25(1.8)(3.1) = 0, M_A = - 139.5kNm

ii. Sketch the deflection curve



iii. Bending moment diagram can be determined starting from left to right of beam. ***The bending moment diagram of uniformly distributed load should be a parabola in shape***.

As the uniformly distributed load finishes till the right end of beam counter balance of loading is not necessary.



Deflection of beam at free end,

$$EI\,\Delta = \left(\frac{1}{2} \times 4 \times 180\right)\left(\frac{1}{3} \times 4\right) - (4 \times 139.5)\left(\frac{1}{3} \times 1.8 \times 40.5\right)\left(\frac{1}{4} \times 1.8\right)$$

$$EI\,\Delta = -74.417$$

$$\Delta = \frac{-74.417\ kNm^3}{EI}$$

Slope of beam at free end,

$$\theta = \left(\frac{1}{3} \times 4 \times 50\right) + \left(\frac{1}{3} \times 1.8 \times 40.2\right)$$

$$\theta = \frac{90.787}{EI}$$

CANTILEVER BEAM WITH MOMENT

Consider a cantilever beam subjected to a moment and a uniformly distributed load as shown in figure 7.16 below.

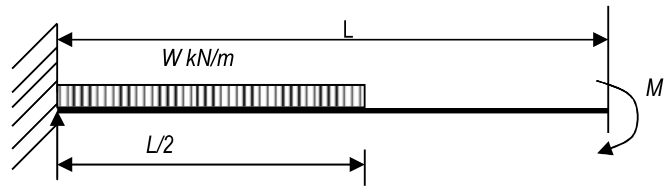
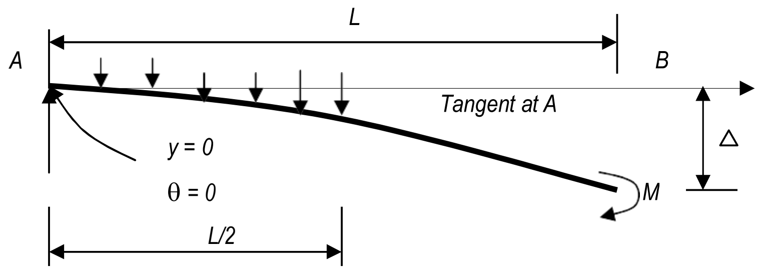


Figure 6.2.3 (s): A cantilever under moment and distributed load



Rajah 6.2.3 (t): Shape of beam under loading

In this case there are two forces acting on the beam. The simplest way drawing the bending moment diagrams is by drawing it section by section, one diagram represents the uniformly distributed load and the other represents the moment M. In constructing each diagram, it is easier by starting it from the left to the right of beam.

The tangent line against curve of the deflection is then drawn from point A. The free end of beam is marked as B. This tangent coincides with the initial position of beam and represented as a straight line as shown. Therefore, deflection of point B from tangent at point A represents the required deflection at free end. According to second moment area theorem, ***the deflection of point B from tangent at point A is given by the moment of all areas under the bending moment diagram between point A and the vertical line through point B divided by EI***.

$$EA\,\Delta = (-ML)\left(\frac{l}{2}\right) + \frac{1}{3}\left(\frac{L}{2}\right)\left(\frac{-WL^2}{8}\right)\left(\frac{L}{2} + \frac{3}{4}\frac{L}{2}\right)$$

$$\Delta = -\frac{ML^2}{2EI} - \frac{7WL^4}{384EI}$$

The negative sign shows the final position of B is below the tangent line drawn at A.

Example (i)

A cantilever is under loadings as shown in figure 7.18 below. Determine the maximum slope and deflection.

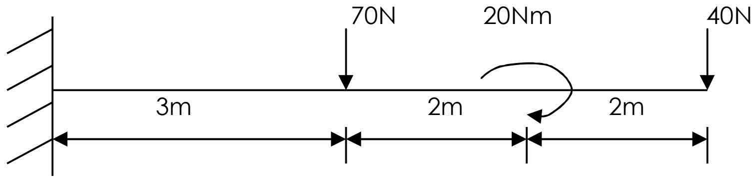


Figure 6.2.3 (u): Cantilever under moment Load

Solution

i. Reactions at fixed end should firstly be determined.

ii. Sketch the curve of deflection of beam.

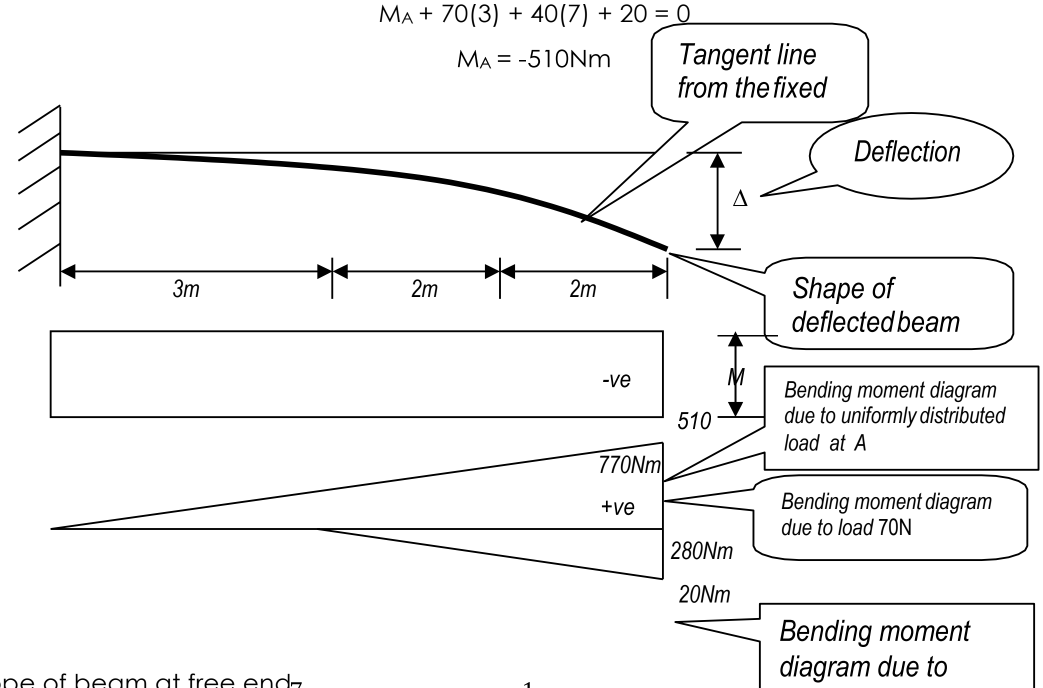
iii. Sketch the bending moment diagrams from left to right of beam.

$$R_A = 70 + 40 = 110N$$

$$\Sigma M = 0$$

$$M_A + 70(3) + 40(7) + 20 = 0$$

$$M_A = -510Nm$$



Slope of beam at free end

$$\theta = (-510 \times 7) + \left(\frac{7}{2} \times 770\right) + \left(-280 \times \frac{1}{2} \times 4\right) + (20 \times 2)$$

$$\theta = \frac{1475}{EI}$$

Deflection of beam at free end

$$EI\Delta = (-510 \times 7)\left(7 \times \frac{1}{2}\right) + \left(\frac{7}{2} \times 770\right)\left(\frac{1}{3} \times 7\right) + \left(-280 \times \frac{1}{2} \times 4\right) + \left(20 \times 2 \times \frac{1}{2}\right)$$

$$EI\Delta = -6746.67$$

$$\Delta = \frac{-6746.67\ kNm^3}{EI}$$

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