POLITEKNIK MALAYSIA
PORT DICKSON

STATISTICS & PROBABILITY

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We hereby declare that this module is our original work. To the best of our knowledge, it contains no materials previously written or published by another person. However, if there is any, due acknowledgement and credit are mentioned accordingly in the e-book.

PREFACE

Assalamualaikum & Peace be Upon You.

In the name of Allah, the Almighty who give us the truth, the knowledge, the enlightment and with regards to Prophet Muhammad S.A.W. May Allah give us the ability to continue our good deeds in this field.

Alhamdulillah, finally the e-book of Engineering Mathematics 3 For Polytechnics has been successfully published. This e-book is developed based on the latest Engineering Mathematics 3, Polytechnics Course Syllabus and is written by the lecturers from the Mathematics, Science and Computer Department, Polytechnic Port Dickson.

This e-book is a useful learning material for students who have never taken additional mathematics in secondary school and as a source of reference for newly trained lecturers in this field.

We hope this attempt will be a continuous process towards achieving our vision to become an excellent academic center. Any comments and suggestion from students or other readers is welcomed to make sure this module can be improved for further editions.
Thank you so much.

TABLE OF CONTENT

TABLE OF CONTENT

STATISTICS

1.1 Demonstrate Statistical Data

STATISTICS

a set of concepts, rules, and procedures that help us to organize numerical information in the form of tables, graphs, and charts.

DATA

collection of facts, such as numbers, words, measurements, observations or even just descriptions of things.

STATISTICAL TERMINOLOGY

UNGROUPED DATA

data given as individual data points

GROUPED DATA

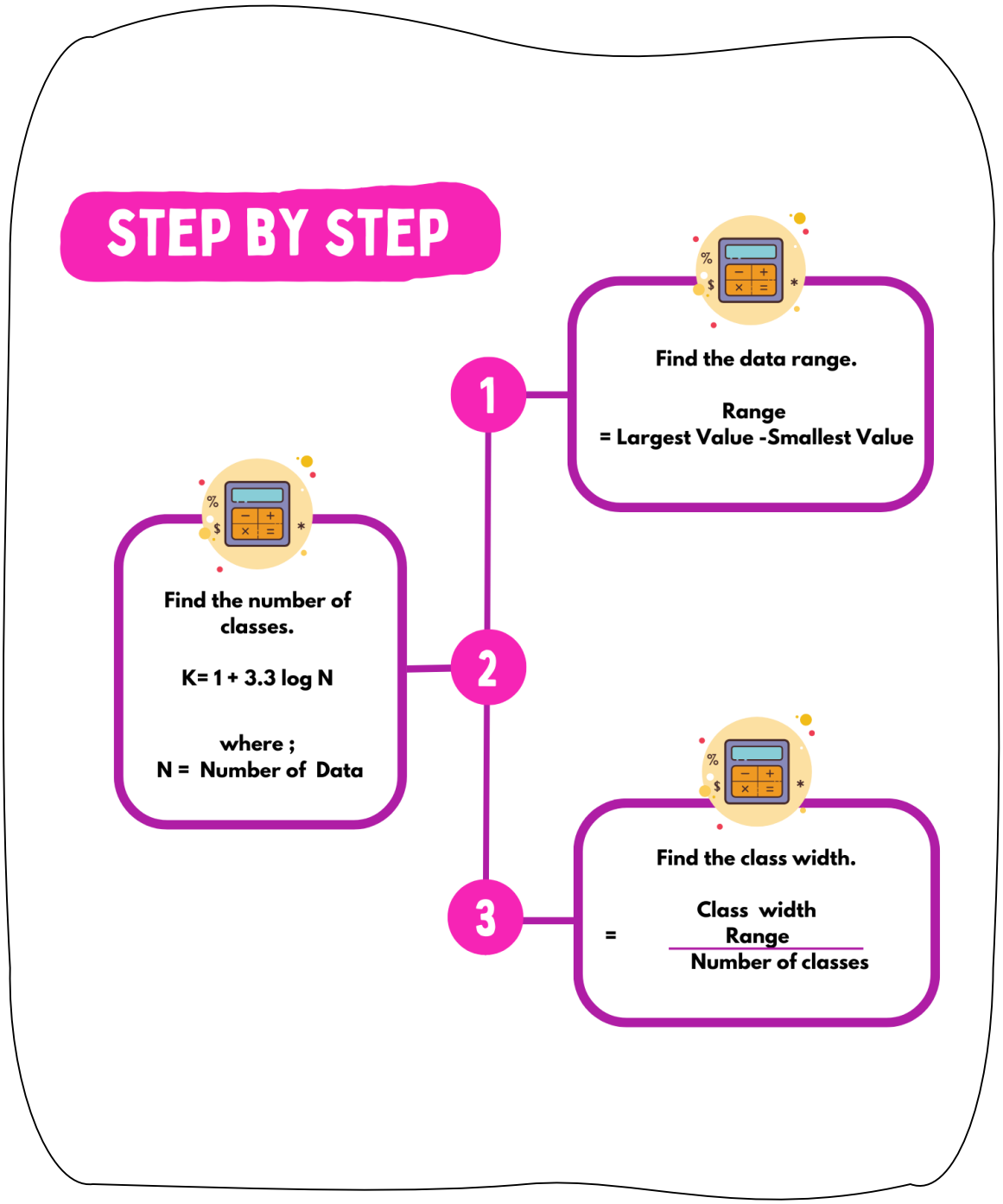
the data given in the form of class intervals

FUN FACT

Did you know that there are 2,500,000 rivets in the Eiffel Tower.

Frequency Distribution Table

A frequency distribution describes the number of observations for each possible value of a variable. Frequency distributions are depicted using **graphs** and **frequency tables.**



EXAMPLE 1

Given below are the 30 data obtained from Air Pollutant Index (API) collected at Larkin, Johor. Determine a suitable class interval for this data and construct the frequency distribution table.

44	43	42	42	43	45	46	47	50	51
52	55	58	57	59	59	59	59	58	58
54	54	54	54	55	56	56	57	57	58

SOLUTION:

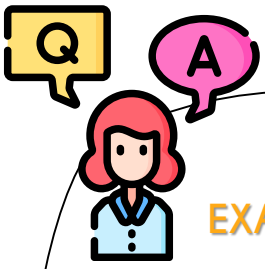
Class Limit	Tally	Frequency,f
42 – 44	/////	5
45 – 47	///	3
48 – 50	/	1
51 – 53	//	2
54 – 56	///// ///	8
57-59	///// ///// /	11

1. Determining the class intervals
Range, R
=(highest value-lowest value)
= 59-42
= 17

2. Formula for Number of class, K
K= 1+3.3 log N
N = the total of data which is 30 recorded API
= 1 + 3.3 log 30
= 5.87 Round off K = 6

3. Class interval/width
= Range/Number of class, K
= 17/6
= 2.83
= 3

Class Width = 3
(57-59) = 57,58,59



EXAMPLE 2

Given below are the database from Sheffield Solar Farm (UK).

34	45.9	38.8	63.3	54.9	51.7	37.6	36.7	39	32.7
49.1	48.8	53.9	50.9	55.5	53.6	71	63.7	69.2	57.7
52.3	76.2	74.9	69.4	66	65.7	37.8	46.1	49.1	31.8

Construct a frequency distribution table with 31.8-39.2 as first class interval. Then, calculate the cumulative frequency for this data.

SOLUTION:

Class Limit	Tally	Frequency ,f	Cumulative Frequency, F
31.8-39.2	///// ///// /	11	11
39.3-46.7	/	1	12
46.8-54.2	///// /	6	18
54.3-61.7	////	4	22
61.8-69.2	////	4	26
69.3-76.7	////	4	30

Find the Class Width
39.2 – 31.8 = 7.4
Upper Limit
39.3 + 7.4 = 46.7
So, the class Width is
39.3 -46.7

Cumulative Frequency
11+ 1 = 12
12 + 6 = 18
18 +4 = 22
22+ 4 = 26
26 +4 = 30

QUICK TIP

Cumulative Frequency is the total of all frequencies so far in a frequency distribution. It is the 'running total' of frequencies.

EXAMPLE 3

Given below are the marks obtained by 30 students in English test. Construct a frequency distribution table using a class size of 10 and calculate the class boundaries.

97	99	98	97	96	96	89	88	88	88
82	88	88	82	76	76	76	76	76	74
72	74	74	72	69	69	65	60	60	52

SOLUTION:

Class Limit	Class Boundary	Frequency
50 - 59	49.5 - 59.5	1
60 - 69	59.5 - 69.5	5
70 - 79	69.5 - 79.5	10
80 - 89	79.5 - 89.5	8
90 - 99	89.5 - 99.5	6

Lower limit

Upper limit

Lower Boundary

Upper Boundary

Class Size:
10
(50,51,52,53,54
,55,56,57,58,59)

Class Boundary
= (90-89) / 2
= 0.5
∴ Lower Limit - 0.5
= 89.5
∴ Upper Limit + 0.5
= 99.5

EXERCISE TIME

1. The following data below shows the masses in kg of 40 netball players. Construct a frequency distribution table with class size of 3 kg.

62	58	71	68	65	70	56	63	66	63
69	56	72	61	64	70	67	65	55	66
57	60	68	71	67	59	65	56	61	55
72	64	69	61	67	70	58	68	69	59

Ans:

Mass (kg)	55 - 57	58 - 60	61 - 63	64 - 66	67 - 69	70 - 72
Frequency	6	5	6	7	9	7

2. The following data shows the Engineering Mathematics 3 mark of class DAD3F.

30	50	46	42	33	50	43	51	38	50
56	36	51	55	40	46	48	34	51	46
40	35	48	52	31	46	37	41	50	34

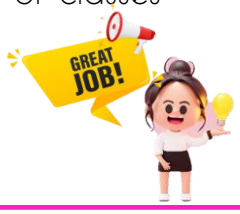
Construct a frequency distribution table with class size 30-34 as first class interval.

Ans:

Mass (kg)	30 - 34	35 - 39	40 - 44	45 - 49	50 - 54	55 - 59
Frequency	5	5	4	6	8	2

3. The following data shows the heights of a group of football players. Determine a suitable number of classes and class interval for this data.

Ans: K=5, Class interval=4



1.2 Compute of Central Tendency and Dispersion

MIND MAP

Measures of Central Tendency

- Mean
- Mode
- Median

Mean

Grouped Data

$$\bar{x} = \frac{\sum fx}{\sum f}$$

x = midpoint of class
f = class frequency

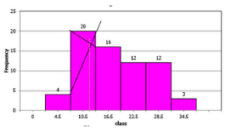
Ungrouped Data

$$\bar{x} = \frac{\sum x}{N}$$

x = the sum of all observations
f = number of observations

Mode

Grouped Data



mode

Ungrouped Data

the observation with the highest frequency in set of data. A set of data may have more than one mode or none at all.

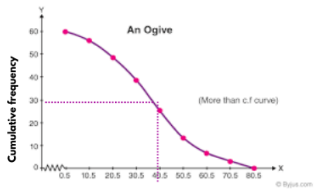
Median

Grouped Data

$$m = L + \left(\frac{\frac{N}{2} - F}{f_m} x\ c\right)$$

L =lower boundary of class median
N=Total frequency
F=Total cumulative frequency before class median
fm=frequency of class median
c=size of class median

Grouped Data



Median = value of the (N/2)th observation in an ogive

Ungrouped Data

the median is an observation which divides a set of data arranged in order two equal parts with each having equal number of observations.

REMEMBER

1.2.1 Calculate Mean, Median and Mode for Ungrouped Data

EXAMPLE 1

Determine the mean of the following data set:

a) 45, 56, 61, 48, 47, 43, 61

b) 12, 17, 11, 8, 5

c) 100, 88, 74, 84

SOLUTION:

a) Mean,

$$\bar{x} = \frac{\Sigma x}{N}$$

$$\bar{x} = \frac{45+56+61+48+47+43+61}{7}$$

$$= \frac{361}{7}$$

b) Mean,

$$\bar{x} = \frac{\Sigma x}{N}$$

$$\bar{x} = \frac{12+17+11+8+5}{5}$$

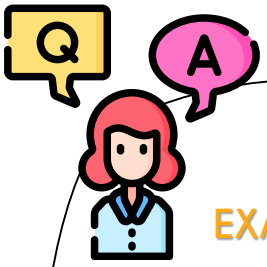
$$= \frac{53}{5}$$

c) Mean,

$$\bar{x} = \frac{\Sigma x}{N}$$

$$\bar{x} = \frac{100+88+74+84}{4}$$

$$= \frac{173}{2}$$



EXAMPLE 2

Determine the mode of the following data set:

a) 45, 56, 61, 48, 47, 48, 61

b) 12, 17, 11, 8, 5

c) 100, 88, 74, 84

SOLUTION:

a) 45, 47, 48, 48, 56, 61, 61

61 and 48 occurs 2 times

Mode : 61 and 48

b) 12, 17, 11, 8, 5

Mode : None

c) 74, 84, 88, 88, 100

88 occurs 2 times

Mode : 88

EXAMPLE 3

Determine the median of the following data set:

a) 45, 56, 61, 48, 47, 43, 58

b) 12, 17, 11, 8, 5,18

SOLUTION:

a) 45, 56, 61, 48, 47, 43, 58

43, 45, 47, 48, 56, 58, 61

Data is rearranged in ascending order

Median is 48

b) 12, 17, 11, 8, 5, 18

5, 8, 11, 12, 17, 18

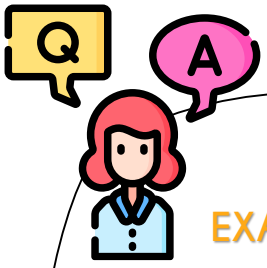
Data is rearranged in ascending order

$$\text{Median} = \frac{11+12}{2}$$

$$= 11.5$$

FUN FACT

0.3% of solar energy from the Sahara is enough to power the whole of Europe.



EXAMPLE 4

The table shows the frequency distribution of the scores obtained by a group students in a game.

Scores	1	2	3	4
Frequency	1	10	5	2

Calculate the mean, mode and median scores obtained by the group of student.

SOLUTION:

Scores, x	Frequency, f	fx	Cumulative Frequency, F
1	1	1	1
2	10	20	11
3	5	15	16
4	1	4	17
	Σf = 17	Σfx = 40	

8.5^{th}

Mean value, $\bar{x} = \frac{\Sigma fx}{\Sigma f}$

$= \frac{40}{17}$

Median data- $m = \frac{1}{2}n$

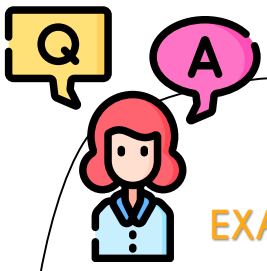
$= \frac{1}{2}(17)$

$= 8.5^{th}$ data (refer to F)

$= 2$

Mode = 2

Value of data with the highest frequency



EXAMPLE 5

The table shows the frequency distribution of the marks obtained by a group students in a Mathematics quiz.

Marks	5	6	8	9
Frequency	2	10	5	2

Calculate the mean, mode and median scores obtained by the group of student.

SOLUTION:

Scores, x	Frequency, f	fx	Cumulative Frequency, F
5	2	10	2
6	10	60	12
8	5	40	17
9	2	18	19
	Σf = 19	Σfx = 128	

9.5^{th}

Mean value, $\bar{x} = \frac{\Sigma fx}{\Sigma f}$

$= \frac{128}{19}$

$= 6.74$

Median, data- $m = \frac{1}{2}n$

$= \frac{1}{2}(19)$

$= 9.5^{th}$ data (refer to F)

$= 6$

Mode = 6

Value of data with the highest frequency

EXAMPLE 6

The table shows the frequency distribution of the time taken, in seconds, to complete a race obtained by a group students in a competition.

Time taken, (s)	16	17	19	20
Frequency	1	6	5	2

Calculate the mean, mode and median scores obtained by the group of student.

SOLUTION:

Scores, x	Frequency, *f*	*fx*	Cumulative Frequency, F
16	1	16	1
17	6	102	7
19	5	90	12
20	2	40	14
	Σf = 14	Σ fx = 248	

7^{th}

Mean value, $\bar{x} = \frac{\Sigma fx}{\Sigma f}$

$= \frac{248}{14}$

$= 17.71$

Median, data- $m = \frac{1}{2}n$

$= \frac{1}{2}(14)$

$= 7^{th}$ data (refer to F)

$= 17$

Mode = 17

Value of data with the highest frequency

EXERCISE TIME

1. The following data shows the number of students that borrow books from the library in 8 days.

   24, 70, 33, 17, 32, 45, 30, 67

   Find :

   a. Mean

   b. Median

   Ans: a. = 39.75 b. Mean = 32.5

2. 2, 2, 3, 5, 2, 8, 8, 9, 10, 10, 10, 8

   is a series of number. Find:

   a. Mean

   b. Mode

   c. Median

   Ans: a. Mean = 6.417 b. Mode = 2,8 &10 c. Median = 8

3. The table shows the number of students in a piano classes.

Number of classes	8	9	10	11
Number of students	20	21	15	14

   Find :

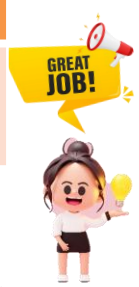
   a. Mean

   b. Mode

   Ans: a. 163.25, b. 9

4. The table shows the number of goals scored in 15 matches.

Scores	0	1	2	3
Number of goal	3	3	5	4

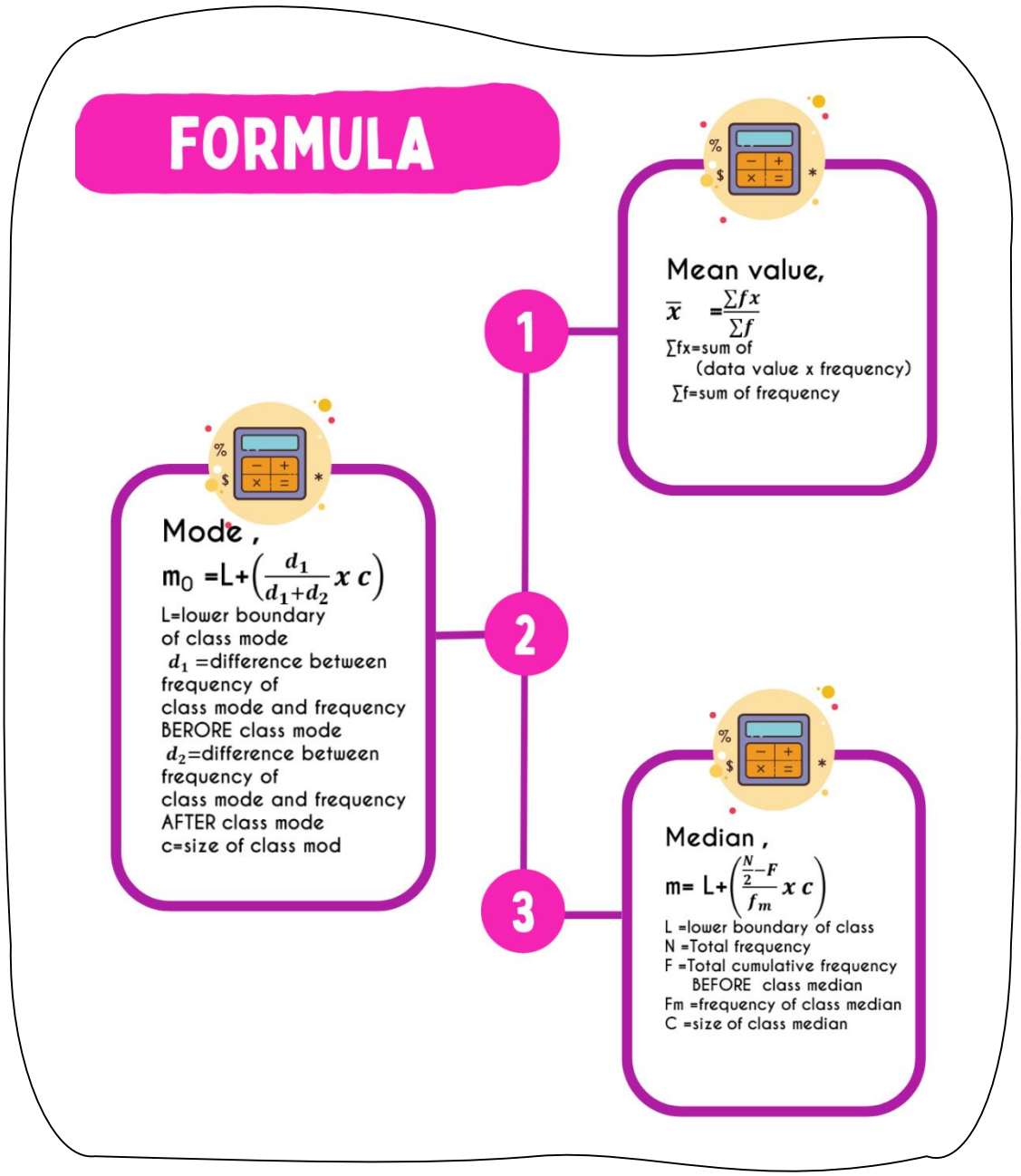


   Find :

   a. Mean b. Median

   Ans: a. 3.75, b. 2

1.2.2 Calculate Mean, Median and Mode for Grouped Data by using formula



WATCH THIS FOR MORE INFO

https://www.youtube.com/watch?v=0ifDuw-Qgvo

EXAMPLE 7

Determine the mean of the following data set:

Class	5-10	11-16	17-22	23-28	29-34
Frequency	4	20	16	12	12

SOLUTION:

[1] Class	[2] Frequency	[3] Boundaries	[4] Midpoint, x	[5] fx	[6] F
5-10	4	4.5-10.5	7.5	30	4
11-16	20	10.5-16.5	13.5	270	24
17-22	16	16.5-22.5	19.5	312	40
23-28	12	22.5-28.5	25.5	306	52
29-34	2	28.5-34.5	31.5	378	64
	$\Sigma f = 64$			$\Sigma fx = 1296$	

FROM COL [1]
$\frac{17+22}{2}$
$= 19.5$

FROM COL [2] X [4]
4 x 7.5=30

Mean value, $\bar{x} = \frac{\Sigma fx}{\Sigma f}$

$= \frac{1296}{64}$

$= \frac{81}{44}$

QUICK TIP

Midpoint = Middle value of classes

STEP BY STEP

To find the mode using FORMULA

1. Prepare the cumulative frequency table for given distribution
2. Determine the mode class
3. Use the formula to determine the mode

WATCH THIS FOR MORE INFO

https://www.youtube.com/watch?v=mk8tOD0t8M0

EXAMPLE 8

Determine the mode of the following data set:

Class	5-10	11-16	17-22	23-28	29-34
Frequency	4	20	16	12	12

SOLUTION:

Class	Frequency	Boundaries	Cumulative Frequency, F
5-10	4	4.5-10.5	4
11-16	20	10.5-16.5	24
17-22	16	16.5-22.5	40
23-28	12	22.5-28.5	52
29-34	2	28.5-34.5	64
	$\Sigma f = 64$		

Modal Class

Mode, $m_0 = L + \left(\frac{d_1}{d_1+d_2} x\ c\right)$

$= 10.5 + \left(\frac{(4-20)}{(4-20)+(16-20)} x6\right)$

$= 15.3$

QUICK TIP

Modal Class is referring to the highest frequency of class

STEP BY STEP

To find the median using FORMULA

1. Prepare the cumulative frequency table for given distribution
2. Determine the median class
3. Use the formula to determine the median

WATCH THIS FOR MORE INFO

https://youtu.be/zjHfAhcU6kE

EXAMPLE 9

Determine the median of the following data set:

Class	5-10	11-16	17-22	23-28	29-34
Frequency	4	20	16	12	12

SOLUTION:

Class	Frequency	Boundaries	Midpoint x	fx	F
5-10	4	4.5-10.5	7.5	30	4
11-16	20	10.5-16.5	13.5	270	24
17-22	16	16.5-22.5	19.5	312	40
23-28	12	22.5-28.5	25.5	306	52
29-34	2	28.5-34.5	31.5	378	64
	$\Sigma f = 64$			$\Sigma fx = 1296$	

32^{th}

Median , m

$$= L + \left(\frac{\frac{N}{2} - F}{f_m} x\, c\right)$$

$$= 16.5 + \left(\frac{\frac{64}{2} - 24}{16} x6\right)$$

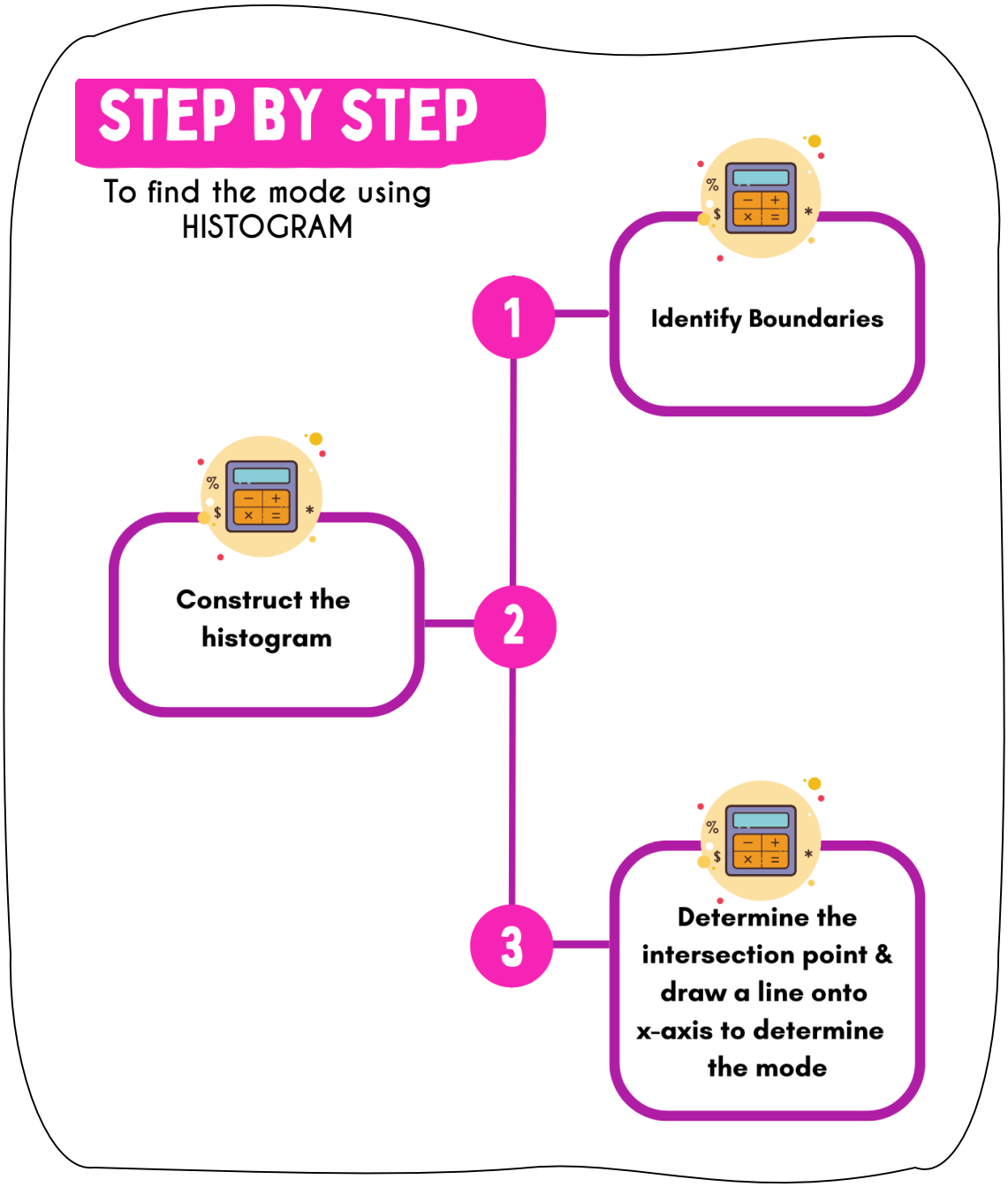
$$= 19.5$$

Median Class, $m = (\frac{\Sigma f}{2})$

$$= \frac{1}{2}(64)$$

$$= 32^{th} \text{ data (refer to F)}$$

1.2.3 Calculate Mode and Median for Grouped Data by using graph



WATCH THIS FOR MORE INFO

https://youtu.be/YLPDPglvePY

EXAMPLE 10

Determine the mode (by using a graph) of the following data set:

Class	5-8	9-12	13-16	17-20	21-24
Frequency	2	6	16	3	3

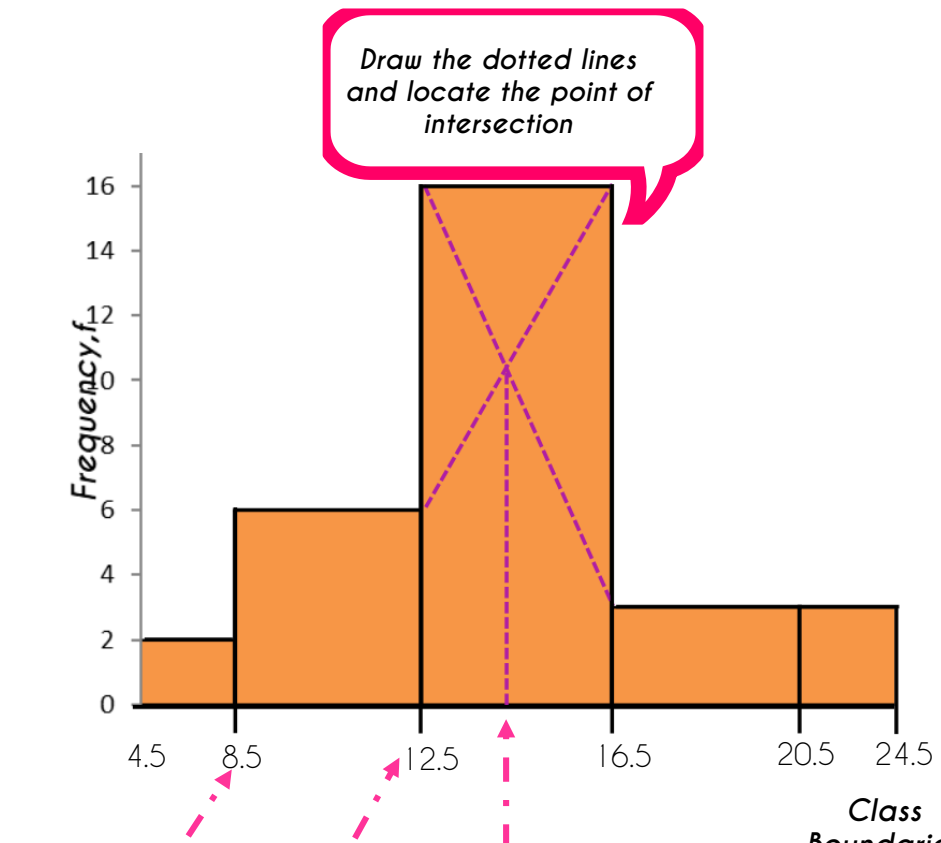
SOLUTION:

Class	Frequency	Boundaries
5-8	2	4.5-8.5
9-12	6	8.5-12.5
13-16	16	12.5-16.5
17-20	3	16.5-20.5
21-24	3	20.5-24.5
	$\Sigma f = 30$	

Histogram

Histogram is a graphical display of data using bars of different heights. The histogram is form when frequencies are plotted against the class boundaries or class interval or class midpoint.

Histogram of the given data set



QUICK TIP

Do not leave any space between bars & the size of each bar must in same size.

STEP BY STEP

To find the median using OGIVE

1. Prepare the cumulative frequency table for given distribution
2. Plot the ogive
3. The median is the value of the (N/2)th observation.

WATCH THIS FOR MORE INFO

https://youtu.be/Lxy_JfrlaMA

EXAMPLE 11

Determine the median (by using a graph) of the following data set:

Class	5-8	9-12	13-16	17-20	21-24
Frequency	2	6	16	3	3

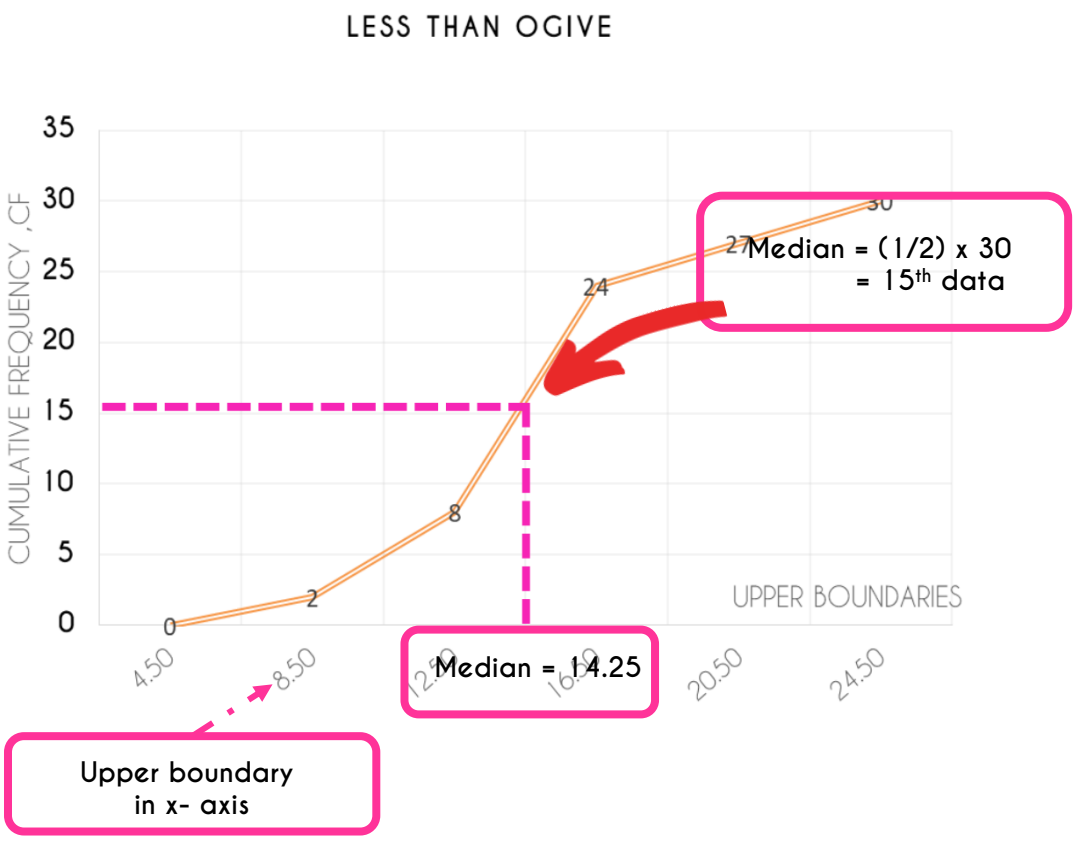
SOLUTION:

Class	Frequency	Boundaries	Cumulative Frequency	
			Less than	More than
1-4	0	0.5-4.5	0	30
5-8	2	4.5-8.5	2	28
9-12	6	8.5-12.5	8	22
13-16	16	12.5-16.5	24	6
17-20	3	16.5-20.5	27	3
21-24	3	20.5-24.5	30	0
	$\Sigma f = 30$			

Add a new class with frequency 0

Ogive

Ogive is known as cumulative frequency curve. The curve is form when cumulative frequencies are plotted against the upper class boundaries. The are wo types of ogives which **are less than ogive and more than ogive.**



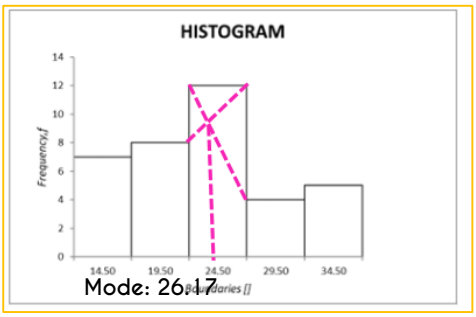
EXERCISE TIME

1. The table shows the ages of a group of softball players in

Ages	15 - 19	20 - 24	25 - 29	30 - 34	35 - 39
Frequency	7	8	12	4	5

Draw a histogram. Use the histogram to find the mode.

Ans:

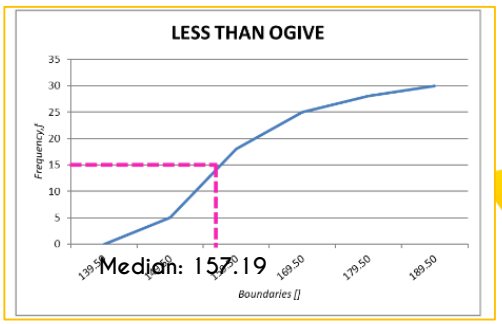


Mode: 26.17

2. The following data shows the frequency distribution of the heights of a group of football player.

Height (cm)	140 - 149	150 - 159	160 - 169	170 - 179	180 - 189
Number of players	5	13	7	3	2

By using the ogive, find the median.



Median: 157.19

MIND MAP

Measures of Central Dispersion

Range

Grouped Data

Range
= midpoint of the highest class - midpoint of the lowest class

Ungrouped Data

Range
= largest value-smallest value

Variance

Grouped Data

$$s^2 = \frac{\sum[(x-\bar{x})^2 f]}{\sum f}$$

$$s^2 = \frac{\sum fx^2}{\sum f} - \left[\frac{\sum fx}{\sum f}\right]^2$$

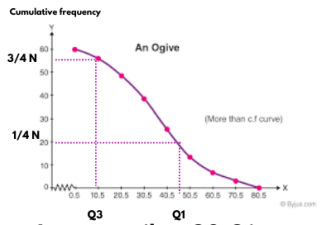
Ungrouped Data

$$s^2 = \frac{\sum(x-\bar{x})^2}{n}$$

$$s^2 = \frac{\sum_{i=1}^{n} x_i^2 - n\bar{x}^2}{n}$$

Interquartile Range

Grouped Data



Interquartile: Q3-Q1

Ungrouped Data

Interquartile:
Third Quartile - First Quartile

Grouped Data & Ungrouped Data

Standard Deviation
= $\sqrt{variance}$

REMEMBER

1.2.4 Calculate mean deviation, variance, standard deviation

FORMULA

Without frequency

1

Mean Deviation, E

$$\frac{\sum|x-\bar{x}|}{n}$$

$\bar{x}$ = Mean
x = Data Value
n = Number of data

2

Variance, s^2 @ σ^2

$$\frac{\sum(x-\bar{x})^2}{n}$$

$\bar{x}$ = Mean
x = Data Value
n = Number of data

3

Standard deviation,
s @σ

$$\sqrt{variance}$$

WATCH THIS FOR MORE INFO

https://www.youtube.com/watch?v=9CJltYX10fY

EXAMPLE 15

The data below shows the ages, in years, of 5 teenagers who have Covid 19 in a hospital recently. Calculate the mean deviation, variance and standard deviation.

18, 20, 19, 16, 15

SOLUTION:

Age, x	$\lvert x-\bar{x}\rvert$	$\lvert x-\bar{x}\rvert^2$
15	2.6	6.76
16	1.6	2.56
18	0.4	0.16
19	1.4	1.96
20	2.4	5.76
	$\Sigma\lvert x-\bar{x}\rvert$= 8.4	$\Sigma\lvert x-\bar{x}\rvert^2$ =17.2

Mean,

$\bar{x} = \frac{\Sigma x}{N}$

$\bar{x} = \frac{18+20+19+16+15}{5}$

$= 17.6$

Variance,

$S^2 = \frac{\Sigma|x-\bar{x}|^2}{N}$

$S^2 = \frac{17.2}{5}$

$= 3.44$

Mean deviation,

$E = \frac{\Sigma|x-\bar{x}|}{N}$

$E = \frac{8.4}{5}$

$= 4.2$

Standard Deviation,

$S = \sqrt{\text{Variance}}$

$S = \sqrt{3.44}$

$= 1.85$

FORMULA

With frequency

1

Mean Deviation, E

$$\frac{\sum(f|x-\bar{x}|)}{\sum f}$$

$\bar{x}$ = Mean

x = Midpoint

f = Frequency

2

Variance, s^2 @ σ^2

$$\frac{\sum[f(x-\bar{x})^2]}{\sum f}$$

$\bar{x}$ = Mean

x = Midpoint

f = Frequency

3

Standard deviation,

s @σ

$$\sqrt{variance}$$

WATCH THIS FOR MORE INFO

https://youtu.be/YEd7hDzmneU

EXAMPLE 16

Calculate the mean deviation, variance and standard deviation for the following data set:

Marks	5	10	15	20	25
Frequency	1	3	2	6	8

SOLUTION:

Marks (x)	f	fx	$f\lvert x-\bar{x}\rvert$	$f(x-\bar{x})^2$
5	1	5	24.6	603.69
10	3	30	58.7	1148.95
15	2	30	29.1	424.57
20	6	120	57.4	549.51
25	8	200	36.6	167.08
	$\Sigma f=20$		$\Sigma f(x-\bar{x})=179.4$	$\Sigma f(x-\bar{x})^2=2893.8$

Mean,

$$\bar{x}=\frac{\Sigma fx}{\Sigma f}$$
$$=\frac{385}{20}$$
$$=19.25$$

Variance,

$$s^2=\frac{\Sigma(f(x-\bar{x})^2)}{\Sigma f}$$
$$=\frac{2893.8}{20}$$
$$=38.188$$

Mean deviation,

$$E=\frac{\Sigma[f(x-\bar{x})}{\Sigma f}$$
$$=\frac{179.4}{20}$$
$$=8.97$$

Standard Deviation,

$$\sigma=\sqrt{variance}$$
$$=\sqrt{124.81}$$
$$=6.17$$

EXAMPLE 17

Calculate the mean deviation, variance and standard deviation for the following data set

Marks	1-4	5-8	9-12	13-16	17-20
Frequency	16	20	28	24	16

SOLUTION:

Marks	f	x	fx	$f[x-\bar{x}]$	$f[x-\bar{x}]^2$
1-4	8	2.5	20	62.7	492
5-8	10	6.5	65	38.4	147
9-12	18	10.5	189	2.9	0
13-16	4	14.5	58	16.6	69.2
17-20	10	18.5	185	81.6	666
	$\Sigma f = 50$		$\Sigma fx = 517$	$\Sigma f[x-\bar{x}] = 202.2$	$\Sigma f[x-\bar{x}]^2 = 1374.2$

Mean,
$\bar{x} = \frac{\Sigma fx}{\Sigma f}$
$= \frac{517}{50}$
$= 10.34$

Mean deviation
$E = \frac{\Sigma[f(x-\bar{x})}{\Sigma f}$
$= \frac{202.2}{50}$
$= 4.04$

Variance
$s^2 = \frac{\Sigma(f(x-\bar{x})^2)}{\Sigma f}$
$= \frac{1374.2}{50}$
$= 27.48$

Standard Deviation
$\sigma = \sqrt{\text{variance}}$
$= \sqrt{27.48}$
$= 5.24$

EXERCISE TIME

1. Given a series of number:

   5.567, 5.888, 5.321, 5.431, 5.121, 5.689, 5.002

   Find variance and standard deviation.

   Ans: Variance: 0.0977 and Standard Deviation:0.313

2. The table shows the ages of a group of softball players in Selangor.

Marks	15	18	19	20	21
Frequency	1	3	2	6	8

   Find the variance and standard deviation for the following data set.

   Ans: Variance: 0.0977 and Standard Deviation:0.313

3. The following data shows the frequency distribution of the heights of a group of football player.

Height (cm)	140 - 149	150 - 159	160 - 169	170 - 179	180 - 189
Number of players	5	13	7	3	2

   Find the mean deviation, variance and standard deviation for the following data set.

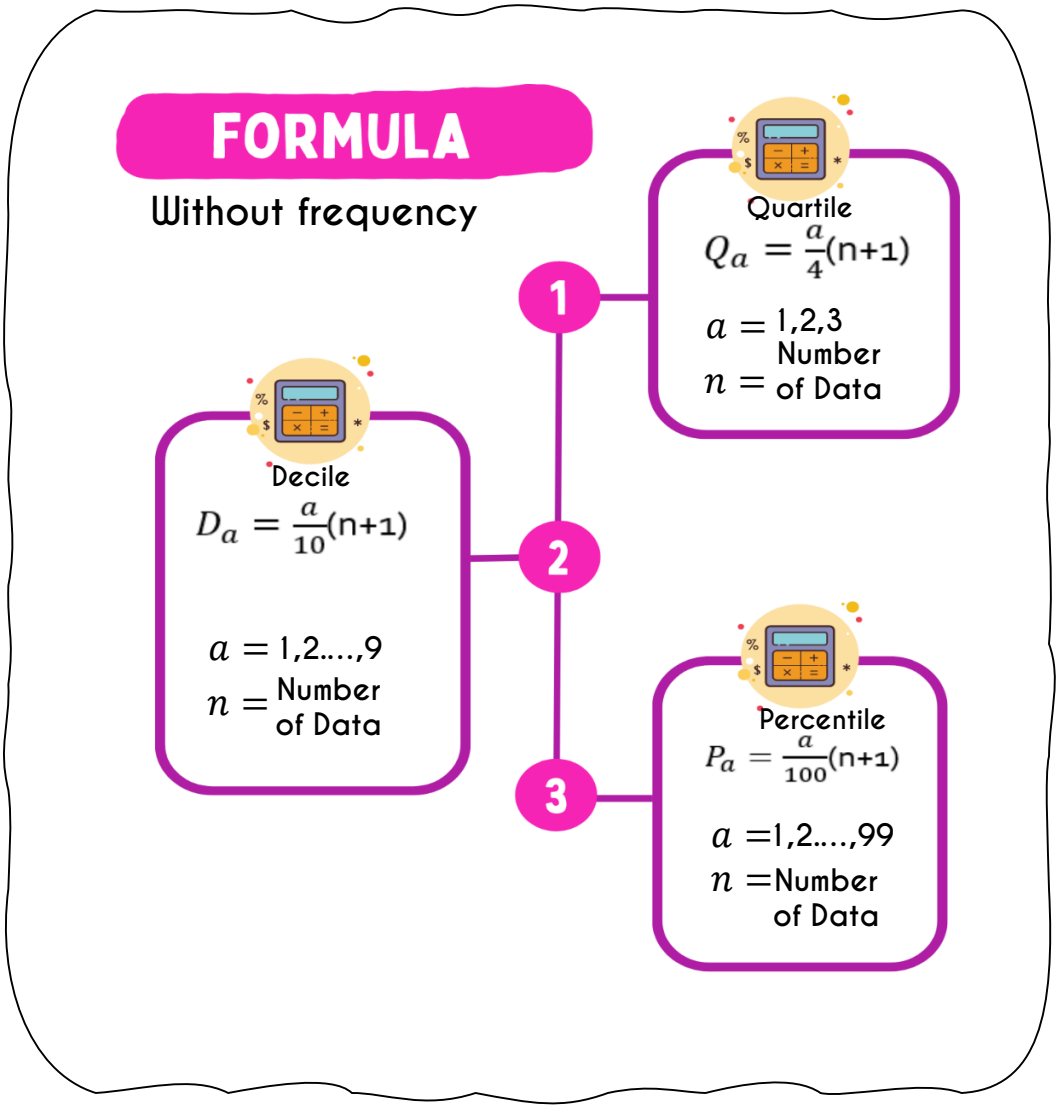
   Ans: Mean deviation: 8.93, Variance:118.21 and Standard Deviation:10.87

1.2.5 Calculate quartiles, deciles and percentiles by using formula and graph

Quartiles
Quartiles are the 3 values that splits data set into 4 equal parts.
a = 1,2,3

Deciles
Deciles are the 9 values that split the data set into 10 equal parts.
a = 1,2……..9

Percentiles
Percentiles are the 99 values that split the data set into 100 equal parts.
a = 1,2……..99



EXAMPLE 18

Determine Q_1, D_8, P_{20} and interquartile range for the data series below:

57, 64, 63, 45, 56, 61, 48, 47, 43, 61

SOLUTION:

Sort this data in ascending order

P_{20} Q_1 Q_3 D_8

43, 45, 47, 48, 56, 57, 61, 61, 63, 64

$Q_a = \frac{a}{4}(n+1)$

$Q_1 = \frac{1}{4}(10+1)$

$= 2.75^{th}$ data (placing)

$\approx 3^{th}$ data

So, $Q_1 = 47$

$D_a = \frac{a}{10}(n+1)$

$D_8 = \frac{8}{10}(10+1)$

$= 8.8^{th}$ data (placing)

$\approx 9^{th}$ data

So, $D_8 = 63$

$P_a = \frac{a}{100}(n+1)$

$P_{20} = \frac{20}{100}(10+1)$

$= 2.2^{th}$ data (placing)

$\approx 2^{th}$ data

So, $P_{20} = 45$

Interquartile Range $= Q_3 - Q_1$

$= 61 - 47$

$= 14$

$Q_a = \frac{a}{4}(n+1)$

$Q_3 = \frac{3}{4}(10+1)$

$= 8.25^{th}$ data (placing)

$\approx 8^{th}$ data

So, $Q_3 = 61$

QUICK TIP

Interquartile Range is the difference between the third quartile and the first quartile in an ordered data of sets.

FORMULA

With frequency

1

Quartile

$Q_a = \frac{a}{4}(\Sigma f)$

$a = 1,2,3$

$f =$ Frequency

2

Decile

$D_a = \frac{a}{10}(\Sigma f)$

$a = 1,2....9$

$f =$ Frequency

3

Percentile

$P_a = \frac{a}{100}(\Sigma f)$

$a = 1,2...99$

$f =$ Frequency

WATCH THIS FOR MORE INFO

https://youtu.be/tZigRvc8LD8

FORMULA

With frequency

Quartile

$$=L+\left(\frac{\frac{a}{4}(\Sigma f)-F_b}{f_q}\ x\ c\right)$$

L = lower boundary of class quartile
∑f = total frequency
F_b= cumulative freq BEFORE class quartile
f_m= freq of class quartile
C = size of class

Decile

$$=L+\left(\frac{\frac{a}{10}(\Sigma f)-F_b}{f_d}\ x\ c\right)$$

L = lower boundary of class decile
∑f = total frequency
F_b= cumulative freq BEFORE class decile
f_m= freq of class decile
C = size of class

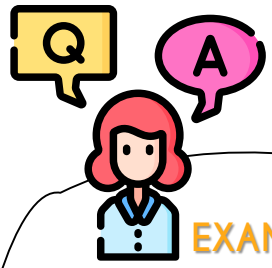
Percentile

$$=L+\left(\frac{\frac{a}{100}(\Sigma f)-F_b}{f_p}\ x\ c\right)$$

L = lower boundary of class percentile
∑f = total frequency
F_b= cumulative freq BEFORE class percentile
f_m= freq of class percentile
C = size of class

WATCH THIS FOR MORE INFO

https://youtu.be/2hAmtEFL9Jo



EXAMPLE 19

Determine the Q_3(by using a formula) of the following data set:

Class	5-8	9-12	13-16	17-20	21-24
Frequency	2	6	16	3	3

SOLUTION:

Class	Frequency	Boundaries	Cumulative Frequency
5-8	2	4.5-8.5	2
9-12	6	8.5-12.5	8
13-16	16	12.5-16.5	24
17-20	3	16.5-20.5	27
21-24	3	20.5-24.5	30
	Σf = 30		

Quartile Class

$Q_a = \frac{a}{4}(\Sigma f)$

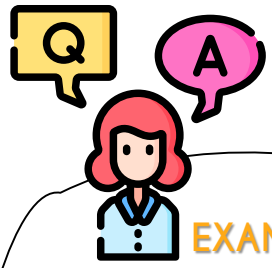
$Q_3 = \frac{3}{4}(30)$

$= 22.5^{th}$ data (placing)

$$\text{Quartile} = L + \left(\frac{\frac{a}{4}(\Sigma f) - F_b}{f_q}\ x\ c\right)$$

$$= 12.5 + \left(\frac{\mathbf{22.5 - 8}}{\mathbf{16}}\ \boldsymbol{x\ 4}\right)$$

$$= 16.13$$



EXAMPLE 20

Determine the D_4 (by using a formula) of the following data set:

Class	5-8	9-12	13-16	17-20	21-24
Frequency	2	6	16	3	3

SOLUTION:

Class	Frequency	Boundaries	Cumulative Frequency
5-8	2	4.5-8.5	2
9-12	6	8.5-12.5	8
13-16	16	12.5-16.5	24
17-20	3	16.5-20.5	27
21-24	3	20.5-24.5	30
	Σf =30		

Decile Class

$D_a = \frac{a}{10}(\Sigma f)$

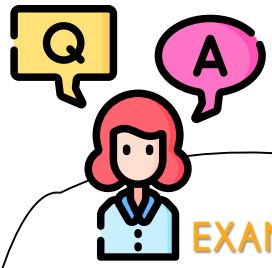
$D_4 = \frac{4}{10}(30)$

$= 12^{th}$ data (placing)

$\text{Decile} = L + \left(\frac{\frac{a}{10}(\Sigma f) - F_b}{f_d} \; x \; c\right)$

$= 12.5 + \left(\frac{12-8}{16} \; x \; 4\right)$

$= 13.5$



EXAMPLE 21

Determine the P_{20} (by using a formula) of the following data set:

Class	5-8	9-12	13-16	17-20	21-24
Frequency	2	6	16	3	3

SOLUTION:

Class	Frequency	Boundaries	Cumulative Frequency
5-8	2	4.5-8.5	2
9-12	6	8.5-12.5	8
13-16	16	12.5-16.5	24
17-20	3	16.5-20.5	27
21-24	3	20.5-24.5	30
	$\Sigma f = 30$		

Percentile Class

$P_a = \frac{a}{100}(\Sigma f)$

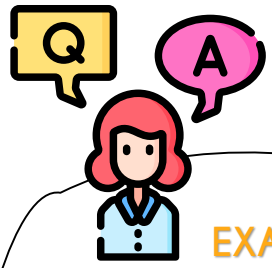
$P_{20} = \frac{20}{100}(30)$

$= 6^{th}$ data (placing)

$$\text{Percentile} = L + \left(\frac{\frac{a}{100}(\Sigma f) - F_b}{f_p} \; x \; c\right)$$

$$= 8.5 + \left(\frac{6-2}{6} \; x \; 4\right)$$

$= 11.17$



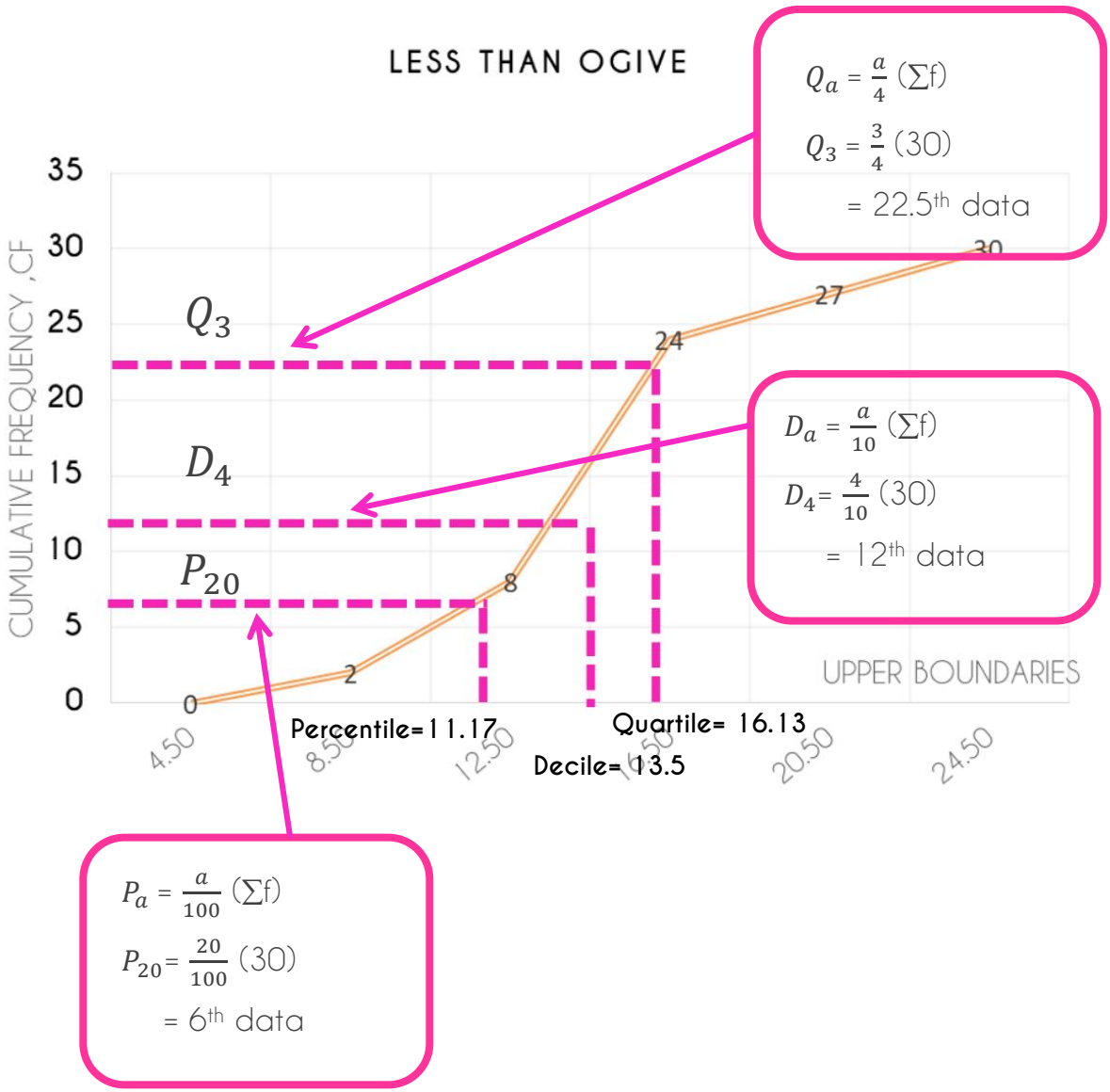
EXAMPLE 22

Determine the Q_3, D_4, P_{20} (by using a graph) of the following data set:

Class	5-8	9-12	13-16	17-20	21-24
Frequency	2	6	16	3	3

SOLUTION:

Class	Frequency	Boundaries	Cumulative Frequency
5-8	2	4.5-8.5	2
9-12	6	8.5-12.5	8
13-16	16	12.5-16.5	24
17-20	3	16.5-20.5	27
21-24	3	20.5-24.5	30


LESS THAN OGIVE
CUMULATIVE FREQUENCY ,CF
UPPER BOUNDARIES
35
30
25
20
15
10
5
0
Q_3
D_4
P_{20}
0
2
8
24
27
30
4.50
8.50
12.50
16.50
20.50
24.50
Percentile= 11.17
Decile= 13.5
Quartile= 16.13
$Q_a = \frac{a}{4}(\Sigma f)$
$Q_3 = \frac{3}{4}(30)$
$= 22.5^{th}$ data
$D_a = \frac{a}{10}(\Sigma f)$
$D_4 = \frac{4}{10}(30)$
$= 12^{th}$ data
$P_a = \frac{a}{100}(\Sigma f)$
$P_{20} = \frac{20}{100}(30)$
$= 6^{th}$ data

EXERCISE TIME

1. Determine Q_1, D_8, P_{20} and interquartile range for the data series below:

   14, 19, 20, 27, 17, 24, 22, 18 , 28, 29

   Ans: Q_1= 18, $D_{8=}$ 28 and P_{20}=17

2. The following data shows the frequency distribution of the heights of a group of football player.

Height (cm)	140 - 149	150 - 159	160 - 169	170 - 179	180 - 189
Number of players	5	13	7	3	2

   By using the formula, find the Q_1, D_2, and P_{50}.

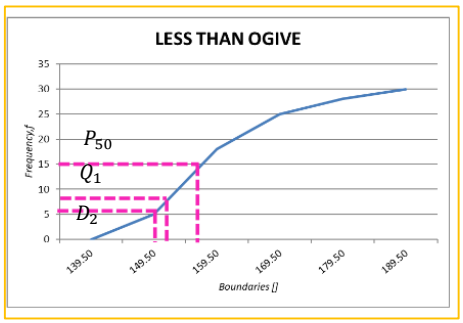
   Ans: Q_1= 151.423, $D_{2=}$ 150.269 and $P_{50.}$ = 157.92

3. The following data shows the frequency distribution of the heights of a group of football player.

Height (cm)	140 - 149	150 - 159	160 - 169	170 - 179	180 - 189
Number of players	5	13	7	3	2

   By using the ogive, find theQ_1, D_2, and P_{50}.

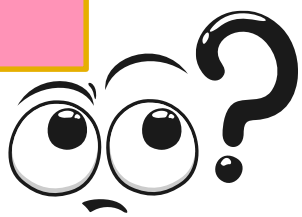
   Ans:



PROBABILITY

1.3 Probability

What is probability?



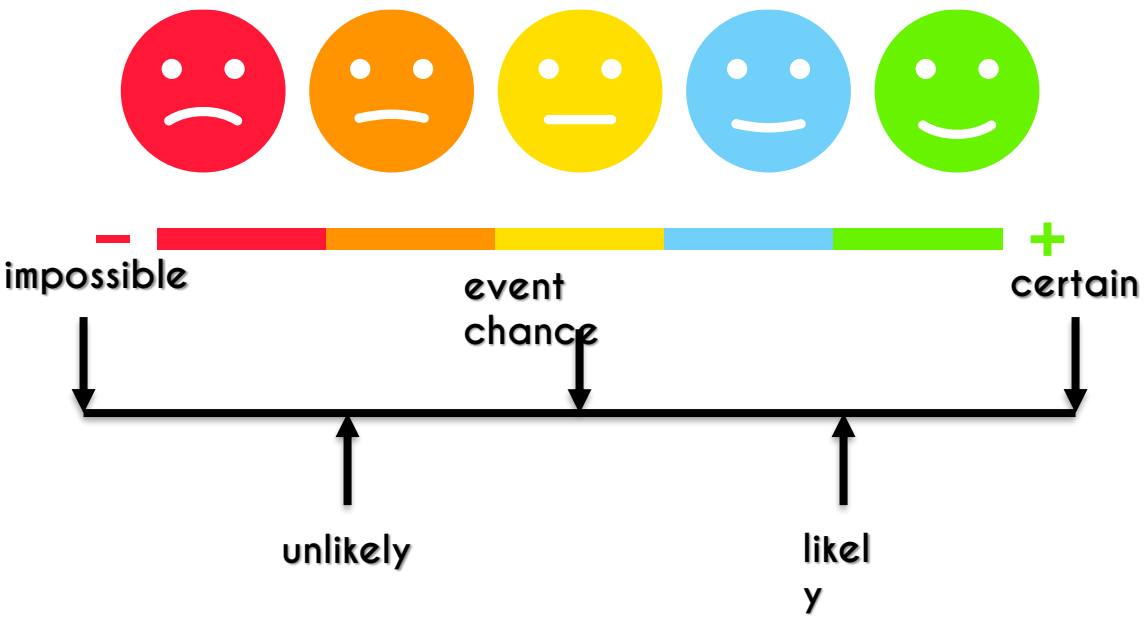
Measure of the likelihood that an event will occur.

(Webster Dictionary, 1913)

Quantified as a number between 0 (impossible) and 1 (certain)

Example: "It is unlikely to rain tomorrow"

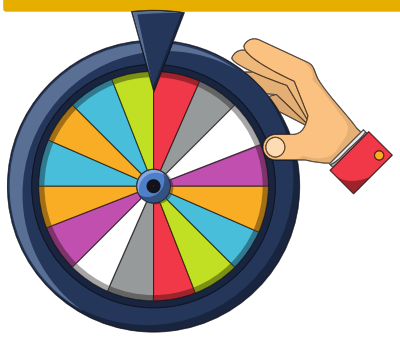
Probability Line

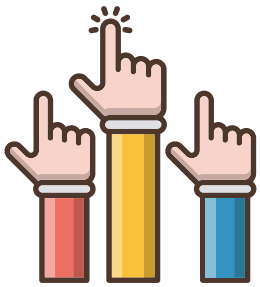


Probability Words

Problem: A spinner has 4 equal sectors coloured yellow, blue, green and red. What are the chances of landing on blue after spinning the spinner?

Solution: The chances of landing on blue are 1 in 4, or one fourth.





Some definitions and examples from the problem above

Definition	Example
Experiment A repeatable procedure with a set of possible results	Spinning the spinner
Trials Repetition of experiment	Spinning the spinner 3 times
Outcome A possible result of an experiment	Landing on yellow, blue, green and red
Event One or more outcomes of an experiment	Landing on blue
Sample space A set of all possible outcomes of a probability experiment	Yellow, blue, green or red

EXERCISE TIME

1. What is the probability of choosing a vowel from the alphabet?

$Ans: \frac{5}{26}$

2. A number from 1 to 11 is chosen at random. What is the probability of choosing an odd number?

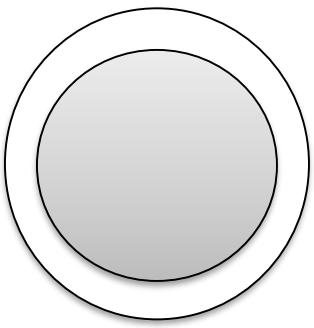
$Ans: \frac{6}{11}$

3. A dice is thrown once. Calculate the probability that the score is:
   a. 7
   b. 1
   c. Prime no

$Ans: a)\ 0, b)\frac{1}{6}, c)\frac{1}{2}$

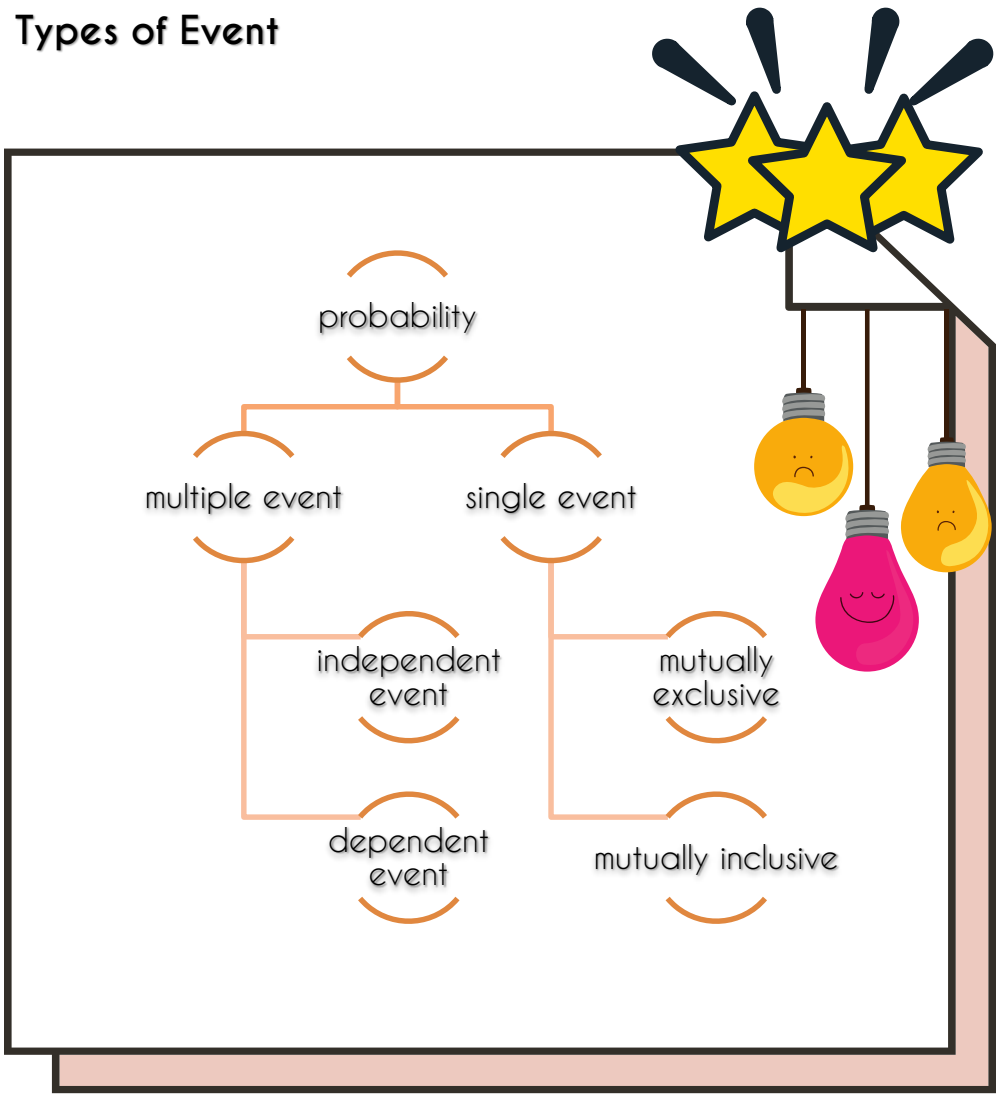
4. The figure shows two concentric circles such that the radius of the smaller shaded circle is 42 cm and the radius of the bigger circle is 63 cm. If an arrow is thrown at the circles, determine that the arrow strike the shaded circle. $\left(\pi = \frac{22}{7}\ and\ A_{circle} = \pi r^2\right)$

$Ans: \frac{4}{9}$



$$Probability, P(X) = \frac{the\ number\ of\ event}{the\ number\ of\ sample\ space}$$

1.3.1 Types of Event

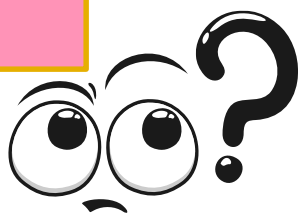


Multiple Event

Happened more than once or pick more than one

Independent Event

What is independent event?



Each event is **not affected** by any other events

How to calculate probability of an dependent event?

$$P\ (A\ and\ B) = P(A)\ \times P(B)$$

Problem: Toss a coin three times and it comes up "Heads" each time. What is the chance that the next toss will also be a "Head"?

Solution: The chance is simply $\frac{1}{2}$ or $\mathbf{50\%}$ just like ANY OTHER toss of the coin

EXAMPLE 1

A code consists of a digit chosen from 0 to 9 followed by a letter of the alphabet. What is the probability the code is 9Z?

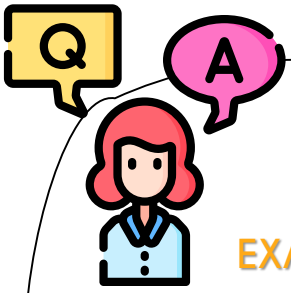
SOLUTION

$$P(9) = \frac{1}{10}$$

$$P(Z) = \frac{1}{26}$$

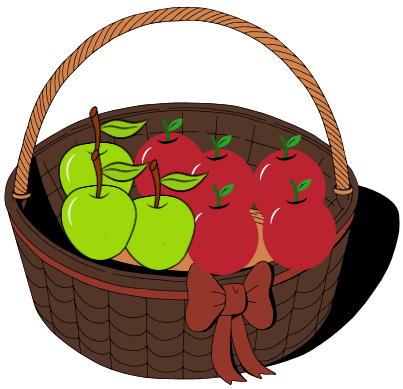
$$P(9Z) = \frac{1}{10} \times \frac{1}{26}$$

$$= \frac{1}{260}$$



EXAMPLE 2

5 red apples and 3 green apples are place in a basket. Find the probability of choosing a red apple and a green apple if the apple is replaced.



SOLUTION

$$P(R) = \frac{5}{8}$$

$$P(G) = \frac{3}{8}$$

$$P(R \text{ and } G) = \frac{5}{8} \times \frac{3}{8}$$

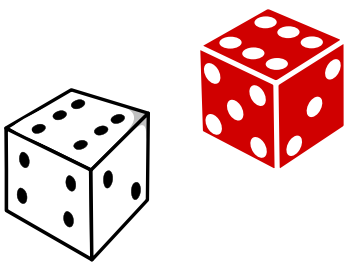
$$= \frac{15}{64}$$

EXAMPLE 3

Two fair dice, one colored white and one colored red, are thrown. Find the probability that:

a. The score on the red dice is 2 and white dice is 5.
b. The score on the white dice is 1 and red dice is even number.

SOLUTION



a. $P(2\ and\ 5) = \frac{1}{6} \times \frac{1}{6}$

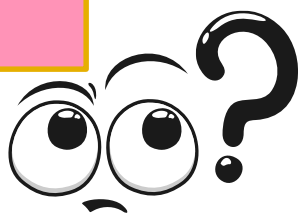
$= \frac{1}{36}$

b. $P(1\ and\ EVEN) = \frac{1}{6} \times \frac{3}{6}$

$= \frac{1}{12}$

Dependent Event

What is dependent event?



One event **can be affected** by the previous events

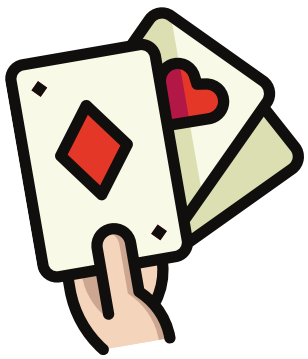
How to calculate probability of a dependent event?

$$P\ (A\ and\ B) = P(A)\ \times P(B|A)$$

Problem: Drawing 2 Kings from a deck of card without replacement

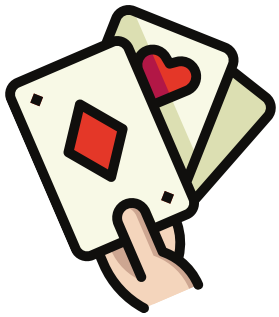
Solution: Event A is drawing the 1st King is $\frac{4}{52}$

Event B is drawing the 2nd King is $\frac{3}{51}$



EXAMPLE 1

What is the probability for you to choose 2 red cards in a deck of cards?



SOLUTION

A deck of cards has 26 black cards and 26 red cards.

$$P(RED) = \frac{26}{52} = \frac{1}{2}$$

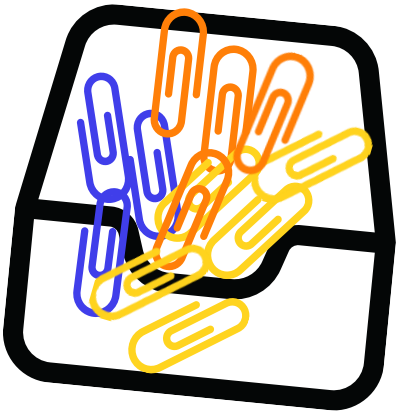
$$P(RED|RED) = \frac{25}{51}$$

$$P(RED\ THEN\ RED) = \frac{1}{2} \times \frac{25}{51}$$

$$= \frac{25}{102}$$

EXAMPLE 2

A drawer contain 3 blue paperclips, 4 oranges paperclips and 5 yellow paperclips. One paperclip is taken from the drawer and NOT replaced. Another paperclip is taken from the drawer. What is the probability that the first paperclip is blue, and the second paperclip is yellow?



SOLUTION

$$P(BLUE) = \frac{3}{12}$$

$$P(YELLOW|BLUE) = \frac{5}{11}$$

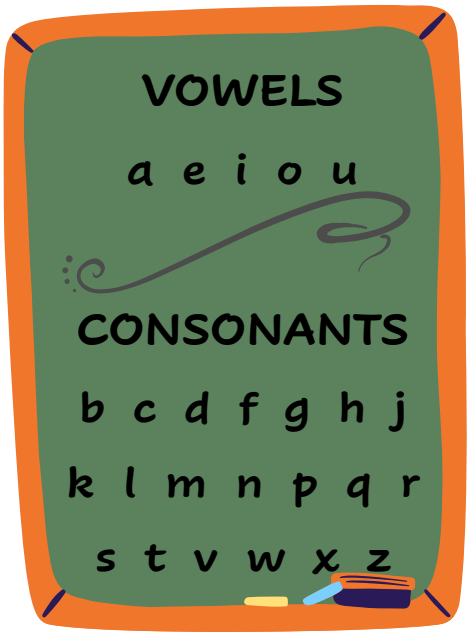
$$P(BLUE\ THEN\ YELLOW) = \frac{3}{12} \times \frac{5}{11}$$

$$= \frac{15}{132}$$

$$= \frac{15}{132}$$

EXAMPLE 3

The letters of alphabet are written on 26 cards. 2 cards are chosen at random. What is the probability that at least one of them is a consonant and one more is a vowel?



SOLUTION

$$P(CONSONANT) = \frac{21}{26}$$

$$P(VOWEL|CONSONANT) = \frac{5}{25}$$

$$P(CONSONANT\ THEN\ VOWEL) = \frac{21}{26} \times \frac{5}{25}$$

$$= \frac{21}{130}$$

More Information on "AND" and "OR"

In some experiments of multiple event, probability can be solve in several methods.

Problem: Tossing 1 coin twice, what is the probability of getting !st coin TAIL, 2nd coin HEAD.

Solution:

$$p(T) = \frac{1}{2}$$

$$p(H) = \frac{1}{2}$$

$$p(TH) = \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{1}{4}$$

Problem: Tossing 1 coin twice, what is the probability of getting TAIL and HEAD.

Solution:

$$p(HT) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

or

$$p(TH) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

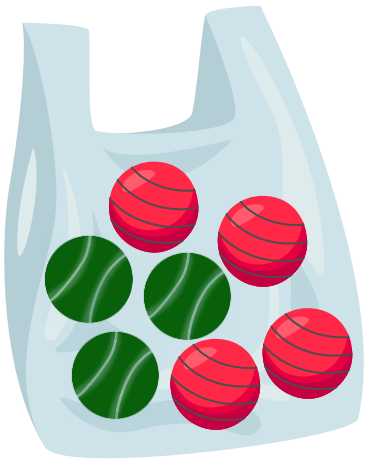
$$p(HT \text{ or } TH) = \frac{1}{4} + \frac{1}{4}$$

$$= \frac{1}{2}$$

EXAMPLE 1

A bag contains 4 red balls and 3 green balls. A ball is taken at random and then put back. Second ball is taken from the bag. What is the probability of choosing:

a. Both balls are the same color
b. At least one of the balls is green



SOLUTION

a. $P(RR) + P(GG) = \left(\frac{4}{7} \times \frac{4}{7}\right) + \left(\frac{3}{7} \times \frac{3}{7}\right)$

$$= \frac{16}{49} + \frac{9}{49}$$

$$= \frac{25}{49}$$

b. $P(RG) + P(GR) + P(GG) = \left(\frac{4}{7} \times \frac{3}{7}\right) + \left(\frac{3}{7} \times \frac{4}{7}\right) + \left(\frac{3}{7} \times \frac{3}{7}\right)$

$$= \frac{12}{49} + \frac{12}{49} + \frac{9}{49}$$

$$= \frac{33}{49}$$

EXAMPLE 2

A problem is given to three persons P, Q and R whose respectively chances of solving it are $\frac{2}{7}, \frac{4}{7}$ *and* $\frac{4}{9}$ respectively. What is the probability that:

a. All of them can solve the problem?.
b. Only one of them can solve the problem?

SOLUTION

$P(P)=\frac{2}{7}$ $P(\bar{P})=\frac{5}{7}$
$P(Q)=\frac{4}{7}$ $P(\bar{Q})=\frac{3}{7}$
$P(R)=\frac{4}{9}$ $P(\bar{R})=\frac{5}{9}$

a. $P(ALL\ CAN\ SOLVE) = \frac{2}{7} \times \frac{4}{7} \times \frac{4}{9}$

$= \frac{32}{441}$

b. $P(ONLY\ P\ CAN\ SOLVE) = \frac{2}{7} \times \frac{3}{7} \times \frac{5}{9} = \frac{10}{189}$

$P(ONLY\ Q\ CAN\ SOLVE) = \frac{5}{7} \times \frac{4}{7} \times \frac{5}{9} = \frac{100}{343}$

$P(ONLY\ R\ CAN\ SOLVE) = \frac{5}{7} \times \frac{3}{7} \times \frac{4}{9} = \frac{20}{147}$

$P(ONLY\ ONE\ CAN\ SOLVE) = \frac{10}{189} \times \frac{100}{343} \times \frac{20}{147} = \frac{4450}{9261}$

EXAMPLE 3

A drawer contains 6 black socks and 8 grey socks. At 6.00 am without switch on the light, two socks are taken at random from the drawer, one after the other without replacement. Calculate the probability that one grey sock and one black sock are taken.

SOLUTION

$$P(B \cap G) = \left(P(B) \times P(G|B)\right) or \left(P(G) \times P(B|G)\right)$$

$$= \left(\frac{6}{14} \times \frac{8}{13}\right) + \left(\frac{8}{14} \times \frac{6}{13}\right)$$

$$= \frac{96}{182}$$

$$= \frac{48}{31}$$

EXERCISE TIME

1. A couple has three children. Find each probability:
   a. Of all boys.
   b. Of exactly two boys or two girls.
   c. Of at least one child of each gender

$Ans: a) \frac{1}{8}, b) \frac{3}{4}, c) \frac{3}{4}$

2. The probability of three events happening are 1/8 for event A, 1/5 for event B and 2/7 for event C. Determine the probability of:
   a. All three events happening.
   b. Event A and B but event C not happening.
   c. Only event B happening
   d. Event A or event B happening but not event C.

$Ans: a) \frac{1}{140}, b) \frac{1}{56}, c) \frac{1}{8}, d) \frac{13}{56}$

3. Actuarial tables show that the life expectancy of three men A, B and C. Over a twenty-year period depends on their age and is given by P(A) = 4/15, P(B) = 11/15 and P(C) = 14/15. Determine the probability that in twenty years:
   a. All three men will be alive.
   b. A will be alive, but B and C will be dead.
   c. At least one man will be alive.

$Ans: a) \frac{616}{3375}, b) \frac{16}{3375}, c) \frac{331}{3375}$

EXERCISE TIME

4. Adam goes to the grocery store to buy cereal. The shelves contain 9 boxes of Brand A and 6 boxes of Brand B. He selects one brand at random and then put it back. Another person does the same thing. What is the probability they both select Brand A?

$Ans: \frac{81}{825}$

5. Adam goes to the grocery store to buy cereal. The shelves contain 9 boxes of Brand A and 6 boxes of Brand B. He selects one brand at random and then not replacing it back. Another person does the same thing. What is the probability that Adam select Brand A and another person select Brand B?

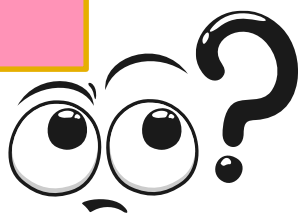
$Ans: \frac{54}{210}$

6. Arissa has six 20 cent coins and five 50 cent coins in her purse. She takes out two coins at random from her purse, one after the other, without replacement. Calculate the probability that the total value of two coins are:
   a. 40 cent.
   b. 70 cent
   c. RM 1

$Ans: a) \frac{3}{11}, b) \frac{6}{11}, c) \frac{2}{11}$

Complement Event

What is complement event?



All the other outcomes (**not the one that we want**)

Symbol: $P(A')$ *or* $P(\bar{A})$

How to calculate probability of a complement event?

$$P(\bar{A}) = 1 - P(A)$$

Problem: If one day is picked from a week, what is the probability of picking weekdays?

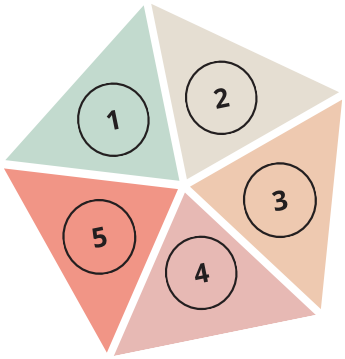
Solution:

Weekdays, (X) = {Monday, Tuesday, Wednesday, Thursday, Friday}

The complement of weekdays is weekends, $(\bar{X})$ = {Saturday, Sunday}

EXAMPLE 1

A spinner is made from a piece of card in the shape of a regular pentagon with a toothpick pushed through the center. When the spinner is spun, and it lands on an edge, each of the numbers from 1 to 5 is equally likely. If the spinner is spun twice, what is the probability the 2 scores are different?



SOLUTION

$Probability\ of\ same\ number, P(S) = \frac{5}{25}$

$P(\bar{S}) = 1 - \frac{5}{25}$

$= \frac{20}{25}$

$= \frac{4}{5}$

EXAMPLE 2

Throw two dice. What is the probability the two scores are different?

SOLUTION

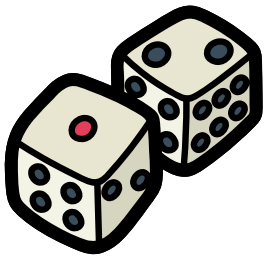
	1	2	3	4	5	6
1	1,1	1,2	1,3	1,4	1,5	1,6
2	2,1	2,2	2,3	2,4	2,5	2,6
3	3,1	3,2	3,3	3,4	3,5	3,6
4	4,1	4,2	4,3	4,4	4,5	4,6
5	5,1	5,2	5,3	5,4	5,5	5,6
6	6,1	6,2	6,3	6,4	6,5	6,6

Same score = { (1,1), (2,2), (3,3), (4,4), (5,5), (6,6) }

$$P(DIFFERENT) = 1 - P(SAME)$$

$$P(DIFFERENT) = 1 - \frac{6}{36}$$

$$= \frac{5}{6}$$



EXAMPLE 3

The probability of choosing a girl at random from a statistic class is $\frac{3}{7}$.

a. Find the probability of choosing a boy at random from the class.
b. If there are 12 boys in the class, find the number of girls in the class.

SOLUTION

a. $Probability\ of\ boy, P(\bar{G}) = 1 - \frac{3}{7}$

$= \frac{4}{7}$

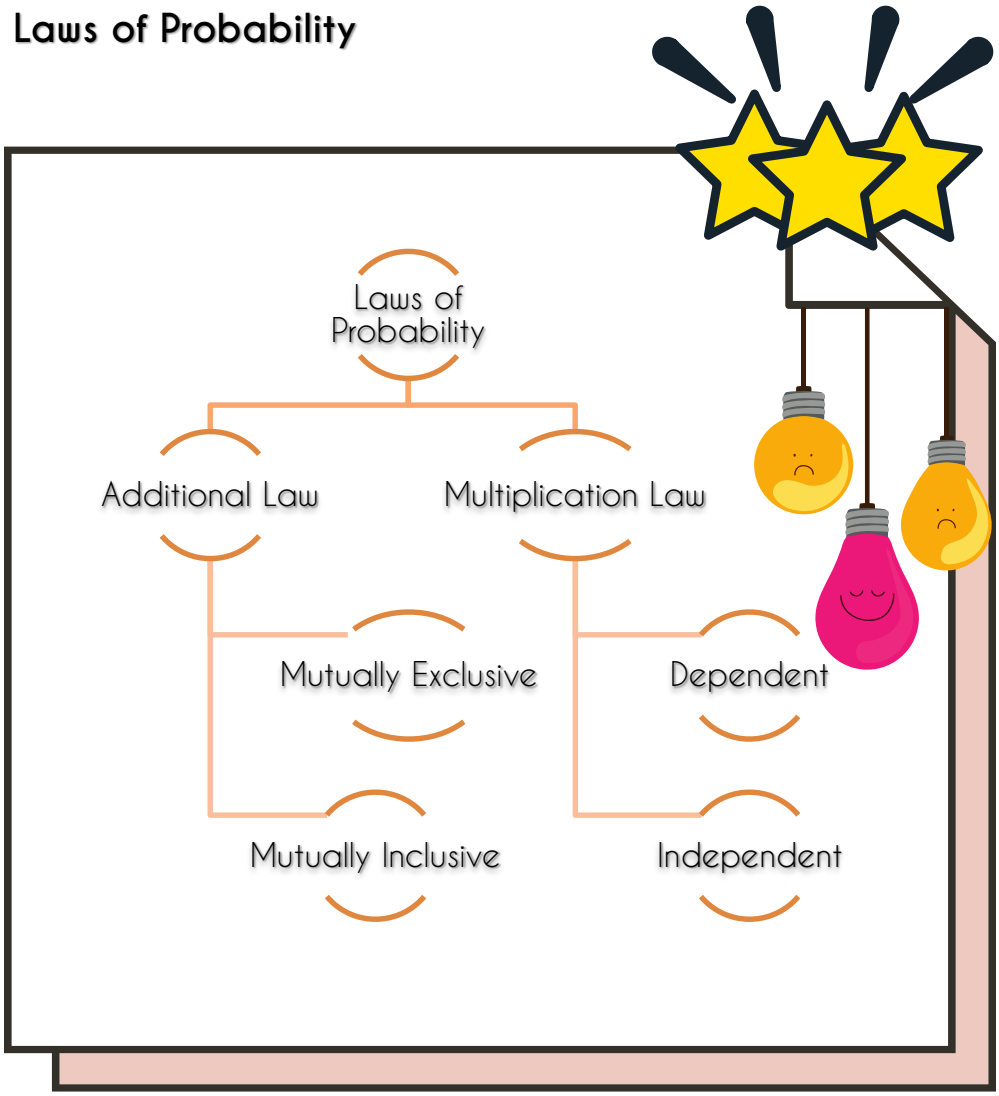
b. $Total\ number\ of\ students = 12 \times \frac{4}{7}$

$= 21$

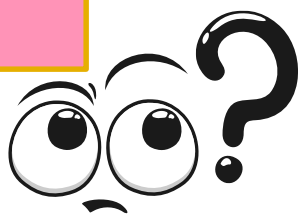
$Number\ of\ girls = 21 - 12$

$= 9$

1.3.2 Laws of Probability



What is the laws?



Additional Law

Mutually Exclusive

$$P\ (A\ or\ B) = P(A) + P(B)$$

Mutually Inclusive

$$P\ (A\ or\ B) = P(A) + P(B) - P\ (A\ and\ B)$$

Multiplication Law

Dependent event

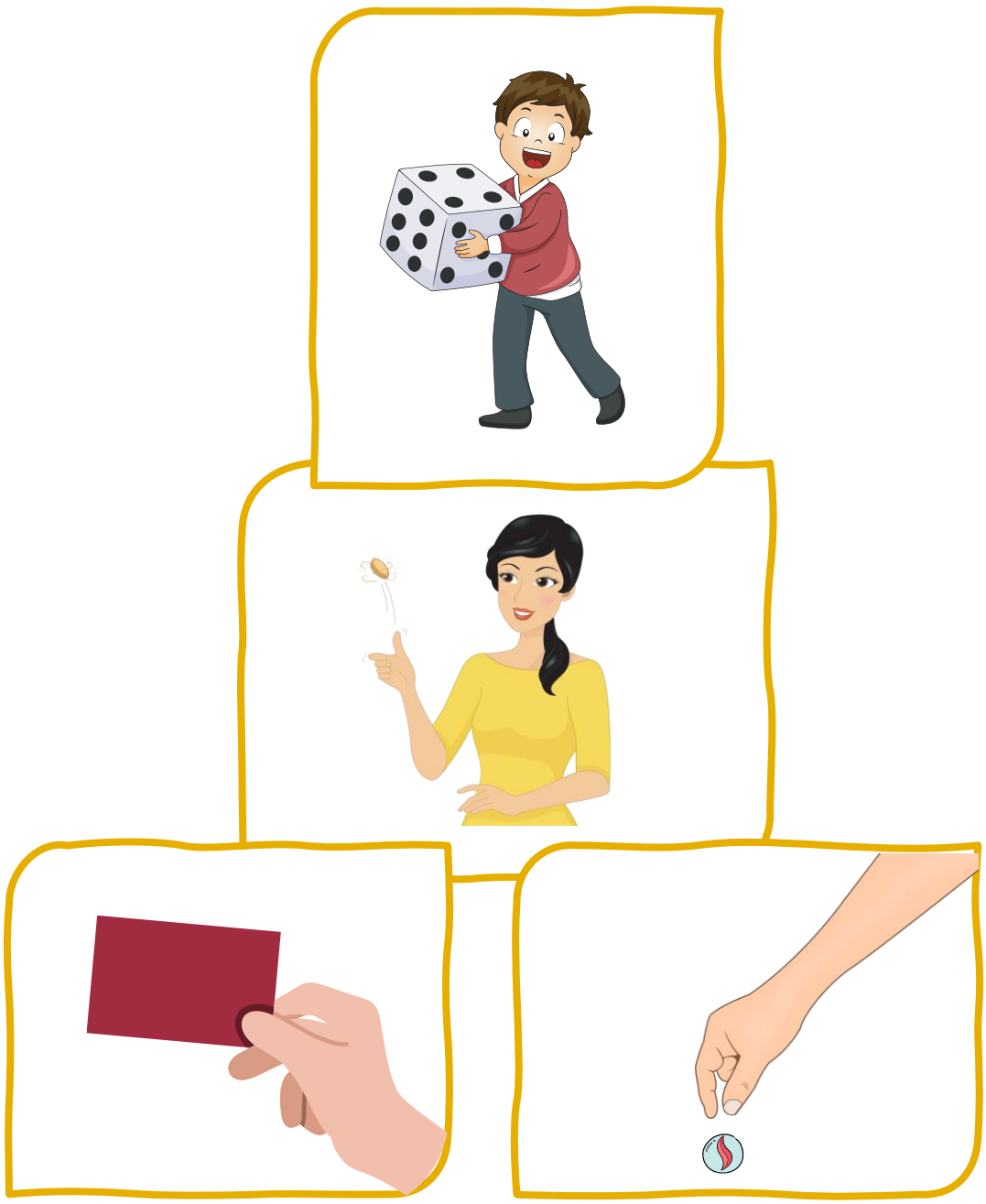
$$P\ (A\ and\ B) = P(A)\ \times P(B|A)$$

Independent event

$$P\ (A\ and\ B) = P(A)\ \times P(B)$$

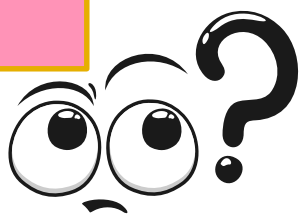
Single Event

Happened once or pick one only



Mutually Exclusive Event

What is mutually exclusive event?



Can't happen at the same time

How to calculate probability of a mutually exclusive event?

$$P(A \cap B) = 0$$

$$P(A \cup B) = P(A) + P(B)$$

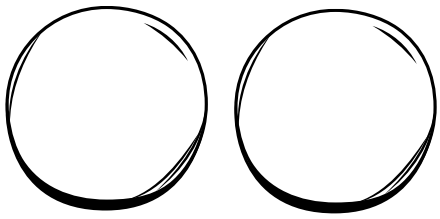
Problem: A card is drawn at random from a deck of cards. . What is the probability of getting a King and a Queen?

Solution: A card cannot be a King and a Queen at the same time.

$$P(K \text{ and } Q) = 0$$

$$P(K \text{ or } Q) = P(K) + P(Q)$$

mutually exclusive



EXAMPLE 1

A number from 1 to 10 is chosen at random. What is the probability of choosing a 5 or an even number?

SOLUTION

$$P(5) = \frac{1}{10}$$

$$P(even) = \frac{5}{10}$$

$$P(5 \text{ or } even) = \frac{1}{10} + \frac{5}{10}$$

$$= \frac{6}{10}$$

$$= \frac{3}{5}$$

EXAMPLE 2

A single letter is chosen at random from the word S C H O O L. What is the probability of choosing:

a. A letter S or an O.
b. A letter O or a vowel.

SOLUTION

a. $P(S \cup O) = \frac{1}{6} \times \frac{2}{6}$

$= \frac{3}{6}$

$= \frac{1}{2}$

b. $P(O \cup VOWEL) = \frac{2}{6}$

$= \frac{1}{3}$

EXAMPLE 3

The probabilities of an engine failing are given by: P_1 failure due to overheating, P_2 failure due to ignition problems, P_3 failure due to fuel blockage. When $P_1 = \frac{1}{7}, P_2 = \frac{2}{9}$ $and P_3 = \frac{3}{11}$. Determine the probabilities of:

a. Both P_1 and P_2 happening.
b. Either P_2 or P_3 happening.
c. Both P_1 and either P_2 or P_3 happening

SOLUTION

a. $P(P_1 \cap P_2) = \frac{1}{7} \times \frac{2}{9}$

$= \frac{2}{63}$

b. $P(P_1 \cup P_2) = \frac{2}{9} + \frac{3}{11}$

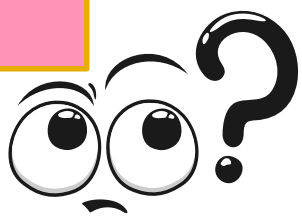
$= \frac{49}{99}$

c. $P(P_1) \cap P(P_1 \cup P_2) = \frac{1}{7} \times \frac{49}{99}$

$= \frac{7}{99}$

Mutually Inclusive Event

What is mutually inclusive event?



Can happen at the same time

How to calculate probability of a mutually inclusive event?

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

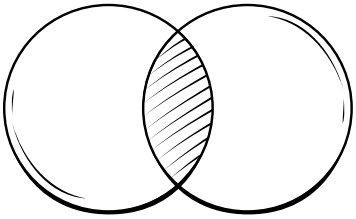
Problem: A single 6-sided dice is rolled. What is the probability of rolling a 5 or an odd number?

Solution: The number rolled can be a 5

The number rolled can be an odd number $(1, 3\ or\ 5)$

The number rolled can be a 5 and odd.

m u t u a l l y i n c l u s i v e



EXAMPLE 1

On New Year's Eve, the probability of a person having a car accident is 0.09. The probability of a person driving while intoxicated is 0.32 and probability of a person having a car accident while intoxicated is 0.15. What is the probability of a person driving while intoxicated or having a car accident?

SOLUTION

$P(I) = 0.32$

$P(A) = 0.09$

$P(I \text{ and } A) = 0.15$

$$P(I \text{ or } A) = P(I) + P(A) - P(I \text{ and } A)$$
$$= 0.32 + 0.09 - 0.15$$
$$= 0.26$$

EXAMPLE 2

In a hospital unit there are 8 nurses and 5 doctors. 7 nurses and 3 doctors are females. If a staff person is selected the probability that the subject is a nurse or a male.

SOLUTION

Staff	Female	Male	Total
Nurse	7	1	8
Doctor	3	2	5
Total	10	3	13

$$P(N) = \frac{8}{13}$$

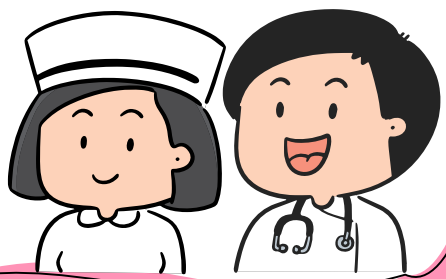
$$P(M) = \frac{3}{13}$$

$$P(N \text{ and } M) = \frac{1}{13}$$

$$P(N \text{ or } M) = P(N) + P(M) - P(N \text{ and } M)$$

$$= \frac{8}{13} + \frac{3}{13} - \frac{1}{13}$$

$$= \frac{10}{13}$$



EXAMPLE 3

50 interviewer come for an interview. 12 out of 20 male are experience worker and 20 out of 30 female are fresh graduate. What is the probability if one of the succeed interviewer is:

a. An experience female worker.
b. A fresh graduated or male.

SOLUTION

	Male	Female	Total
Fresh Graduate	8	20	28
Experience Worker	12	10	22
Total	20	30	50

$$a.\ P(EXPERIENCE\ FEMALE) = \frac{10}{50}$$
$$= \frac{1}{5}$$

$$b.\ P(FRESH\ GRADUATE\ or\ MALE) = \frac{28}{50} + \frac{20}{50} - \frac{8}{50}$$
$$= \frac{4}{5}$$

EXERCISE TIME

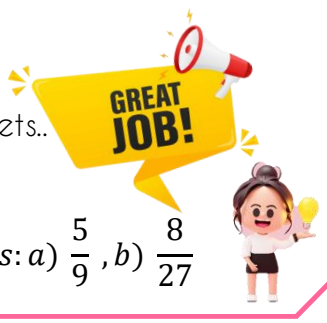
1. Jar A contains three cards numbered 1, 2 and 3. Jar B contains four cards numbered 1, 3, 4 and 6. A card is drawn at random from jar A and at the same time another card is drawn at random from jar B. Calculate the probability that two numbers drawn have the same value or the sum of 5.

$Ans: \frac{1}{3}$

2. A card is drawn at random from a box which contains nine card numbered 1 to 9. Calculate the probability that the number on the card drawn is:
   a. A prime number and an even number.
   b. An odd number or a multiple of four.
   c. A perfect square number or an odd number or both.

$Ans: a) \frac{1}{9}, b) \frac{7}{9}, c) \frac{2}{3}$

3. Azrin and Sara compete against each other in a badminton match. The match ends when any one of the two players wins two sets. The probability that Azlin wins any set is 2/3 . Calculate the probability that
   a. The match ends in two sets only.
   b. Azlin wins the match after competing in three sets..



$Ans: a) \frac{5}{9}, b) \frac{8}{27}$

4. A woman's clothing store owner buys from three companies: A, B and C. The most recent purchases are shown here:

Product	A	B	C
Dress	24	18	12
Blouse	13	36	16

If one item is selected at random, find the following probabilities.

a. It was purchased from company A or is a dress.

b. It was purchased from company B or company C.

c. It is a blouse, or it was purchased from company A.

$$Ans: a)\ \frac{67}{119}, b)\frac{82}{119}, c)\frac{89}{119}$$

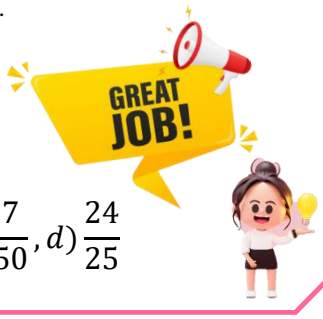
5. In a sample of 50 students, 21 had type O blood, 22 had type A blood, 5 had type B blood and 2 had type AB blood. Set up a frequency distribution and find the following probabilities:

a. A student has type O blood.

b. A person has type A or type B blood.

c. A person has neither type A nor type O blood.

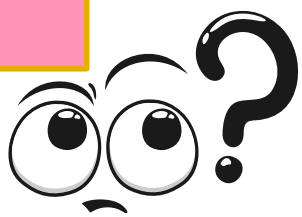
d. A person does not have type AB blood



$$Ans: a)\ \frac{21}{50}, b)\frac{27}{50}, c)\frac{7}{50}, d)\frac{24}{25}$$

Conditional Event

What is conditional event?



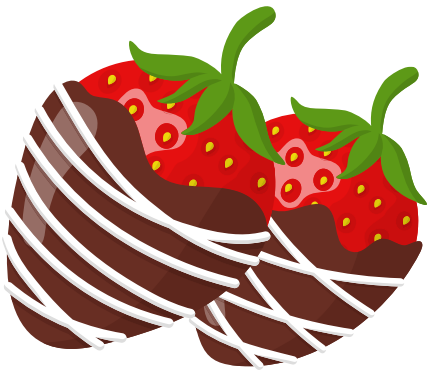
Event B happen given that event A has already happened

How to calculate probability of a conditional event?

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

Problem: 70% of your friends like chocolate, and 35% like chocolate and Strawberry? What percent of those who like chocolate also like strawberry?

Solution: Probability like chocolate and strawberry is 0.35
Probability like chocolate is 0.7
Probability like chocolate also strawberry is 0.5 or 50%



EXAMPLE 1

A math teacher gave her class two test. 25% of the class passed both tests and 42% of the class passed the first test. What percent of those who passed the first test also passed the second test?

SOLUTION

$P(1st\ and\ 2nd) = 25\% = 0.25$

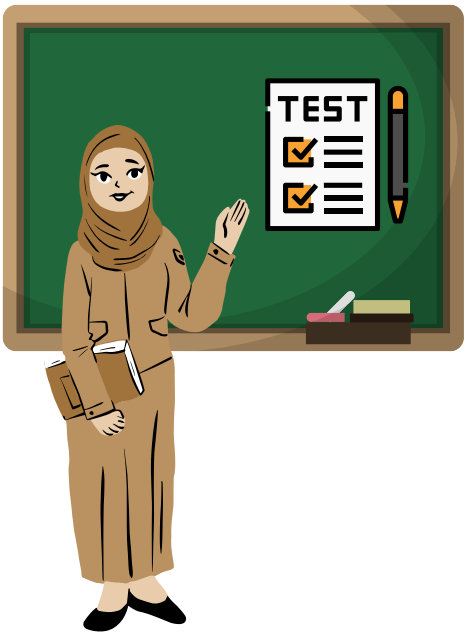
$P(1st) = 42\% = 0.42$

$$P(2nd|1st) = \frac{P(1st\ and\ 2nd)}{P(1st)}$$

$$= \frac{0.25}{0.42}$$

$$= 0.60$$

$$= 60\%$$



EXAMPLE 2

From the table below, what is the probability a randomly selected person is male, given that they own a pet?

	Have Pet	Don't Have Pet	Total
Male	0.41	0.08	0.49
Female	0.45	0.06	0.51
Total	0.86	0.14	1

SOLUTION

$P(\ OWN\ PET \cap MALE) = 25\% = 0.25$

$P(OWN\ PET) = 42\% = 0.42$

$$P(MALE|OWN\ PET) = \frac{P(OWN\ PET \cap MALE)}{P(OWN\ PET)}$$

$$= \frac{0.41}{0.86}$$

$$= 0.477$$

EXAMPLE 3

A recent survey asked 100 people if they thought Sardine X is the best sardine. The result of the survey shown in the table.

Gender	Yes	No	Total
Male	32	18	50
Female	8	42	50
Total	40	60	100

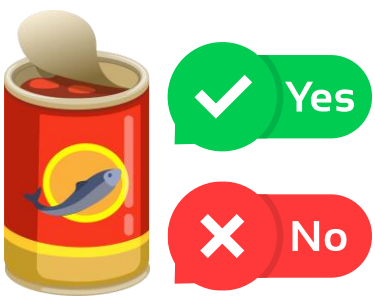
a. The respondent answered "yes" given that the respondent was a female.

b. The respondent was a male given that the respondent answered "no".

SOLUTION

$$a.\ P(Y|F) = \frac{{}^{8}/_{100}}{{}^{50}/_{100}}$$

$$= \frac{4}{25}$$



$$b.\ P(M|N) = \frac{{}^{18}/_{100}}{{}^{60}/_{100}}$$

$$= \frac{3}{10}$$

1. Afif is a very good student in Engineering Mathematics 3 class. The probability that he studies and passes her mathematics test is $\frac{18}{20}$. If the probability that Afif studies $\frac{14}{15}$, find the probability that Afif passes his mathematics test given that he has studies.?

   $Ans: \frac{27}{28}$

2. The probability that it is Friday and that a student is absent is 0.03. Since there are 5 school days in a week, the probability that it is Friday is 0.2. What is the probability that a student is absent given that today is Friday?

   $Ans: 0.15$

3. In one polytechnic, the probability that a student carry the Engineering Mathematics 1 and Engineering Science subject is 0.092. The probability that a student carry the Engineering Science subject is 0.55. Find the probability that the student is carrying the Engineering Mathematics 1 given that he/she is carrying Engineering Science subject.

   $Ans: 0.167$

4. A lecturer has noticed that even though attendance is not a component of the grade for his class, student who attend regularly obtain better grades. In fact, the probability of attending regularly and receiving an 'A' is 0.4, while the probability of not attending regularly and receiving an 'A' is 0.1. The probability of attending class regularly is 0.7. Find the probability of
   a. A student receiving an 'A' grade given that he/she attends class regularly
   b. A student receiving an 'A' grade given that he/she does not attends class regularly

$$Ans: a)\frac{4}{7}, b)\frac{1}{3}$$

5. From the table below, find the probability that the person is female, given that the person work at government sector.

Sector	Male	Female	Total
Private	65	58	123
Government	82	45	127
Total	147	103	250

$$Ans: \frac{45}{127}$$

REFERENCES

01 "*Frequency Distribution*". [online]. Haygot Technologies, Ltd. https://www.toppr.com/guides/maths/statistics/frequency-distribution/

02 Shaun Turney. (2022) . "*Frequency Distribution*". [online]. https://www.scribbr.com/statistics/frequency-distributions/

03 "*Fun Facts and Statistics*". [online]. Statistic Brain. https://www.statisticbrain.com/fun-facts /

04 "*Thinking idea*". [online]. Freepik Company S.L.https://www.freepik.com/photos/thinking-idea">Thinking idea photo created by mamewmy - www.freepik.com

05 Lan Foo Huat & Yong Kien Cheng. (2016) . Additional Mathematics SPM. Selangor : Sasbadi Sdn. Bhd.

06 Nur Hafezah binti Md Yasin & Intan Maisarah binti Osman (2014) . *Engineering Mathematics 3 : Based on Polytechnic Course Syllabus:* Sarawak: Politeknik Mukah Sarawak

REFERENCES

07 Les Bill Gates, et al. (2019). "*Maths is Fun*". [online]. https://www.mathsisfun.com/data/probability.html

08 Hyco, Inc. (2020). "*Maths Goodies*". [online]. https://www.mathgoodies.com/search/node/PROBABILITY

09 Fadhliana Binti Mohamod, et al. (2015). *Engineering Mathematics 3*. Negeri Sembilan: Politeknik Port Dickson

10 Sharifah Noor Anita Syed Jaafar, et al. (2020). *Engineering Mathematics 3*. Negeri Sembilan: Politeknik Port Dickson

11 "*Why do we need the median*". [online]. Don't Memorise. https://www.youtube.com/watch?v=0ifDuw-Qgvo

12 "*Mode, Median, Mean, Range, and Standard Deviation*". [online]. Simple Learning Pro. https://www.youtube.com/watch?v=mk8tOD0t8M0

13 "*Mean, Median, and Mode of Grouped Data & Frequency Distribution Tables Statistics*". [online]. The Organic Chemistry Tutor. https://youtu.be/zjHfAhcU6kE

14 "*What is a Histogram*". [online]. Don't Memorise. https://youtu.be/YLPDPglvePY

REFERENCES

15 *"Median (Formula & Graph"*. [online]. RCC Pune. https://youtu.be/Lxy_JfrlaMA

16 *"Mean and Standard Deviation (Casio fx-570ES)*. [online]. Ah Sing TV. https://www.youtube.com/watch?v=9CJltYX10fY

17 *"Mean, standard deviation and variance using Casio fx-570MS"*. [online]. Noratikah Abu. https://youtu.be/YEd7hDzmneU

18 *"Quartile, Decile and Percentile (Measures of Position of Grouped Data"*. [online]. Berns X. https://youtu.be/tZigRvc8LD8

19 *"Quartile, Decile and Percentile"*. [online]. Mathstorya. https://youtu.be/2hAmtEFL9Jo

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