FADHLIANA MOHAMOD
RUHANA MAT KIA
NOORAZLINA ABD.KARIM

ENGINEERING MATHEMATICS 1 SELF LEARNING

DEPARTMENT OF MATHEMATICS, SCIENCE & COMPUTER
POLYTECHNIC PORT DICKSON

POLITEKNIK MALAYSIA
PORT DICKSON

ENGINEERING MATHEMATICS 1 SELF LEARNING

FADHLIANA BINTI MOHAMOD
RUHANA BINTI MAT KIA
NOORAZLINA BINTI ABD.KARIM

Published by:
Mathematics, Science & Computer Department
Politeknik Port Dickson
Km 14, Jalan Pantai, 71050 Si Rusa
Port Dickson, Negeri Sembilan
http://www.polipd.edu.my
FEBRUARY 2022

Perpustakaan Negara Malaysia Cataloguing-in-Publications Data

Fadhliana Mohamod, 1982-
Engineering Mathematics 1, Self Learning eBook/ FADHLIANA BINTI MOHAMO RUHANA BINTI MAT KIA, NOORAZLINA BINTI ABD.KARIM
Mode of access: Internet
eISBN 978-967-2897-47-7

1. Engineering Mathematics
2.Government publication—Malaysia
3. Electronic books.
I.Ruhana Mat Kia, 1980-, II. Noorazlina Abd.Karim, 1981-
III.Title
620.00151

Published by:
Mathematics, Science & Computer Department
Politeknik Port Dickson
Km 14, Jalan Pantai, 71050 Si Rusa
Port Dickson, Negeri Sembilan
http://www.polipd.edu.my
FEBRUARY 2022

PREFACE

- Foremost, Alhamdulillah Grateful to Allah because with his grace, this Mathematics 1 Self Learning Module has come to fruition. Thanks to all of the team members for giving full commitment to complete this module according to the set plan. Hopefully, the time and energy poured in yields good results
- We would also like to thank the Head of the Department of Mathematics, Science and Computer for the continuous support towards a better way of learning process. A lot of appreciation and thanks to Engineering Mathematics lecturers that gave us a helping hand and support while we finished written this module.
- This module is developed to help students to do self-learning to achieve course learning outcome for Engineering Mathematics 1. Besides, we also welcome all new lecturer to use it as one of the guidelines in a way of teaching and learning.
- This module contains three first topic in Engineering Mathematics 1 Polytechnics which are Basic Algebra, Trigonometry and Complex Numbers. Each topic not only contain notes and example. It’s also had link to YouTube video for students to watch and understand lecturer’s explanations on the example step by step. Students will feel like learning in the class. Together with it, we also provide online games in every topic for students to try out their knowledge in a fun way. Last but not least, students do not need lecturer help to solve the “Test Your Knowledge” questions. It's because, the module already provides full scheme for the students to refer.
- We hope that this module will encourage Engineering Mathematics 1 students to enjoy learning this course and can understand the content very well. It is a way to prepare students for their next step towards diploma students.

Why You Can Self Learning With This Book?

Objective:

Students will be able to use mathematical statement to describe relationship between various physical phenomena that include:

1. Simplify, solving and show procedure on Basic Algebra
2. Solve Trigonometry and apply sine and cosine rules
3. Explain and demonstrate of Complex Number

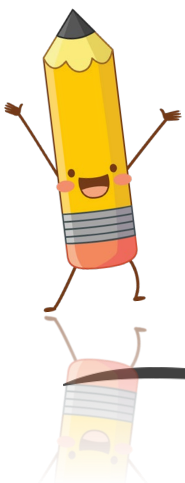
Topic:

1. Session 1 (Basic Algebra)
2. Session 2 (Trigonometry)
3. Session 3 (Complex Number)

Book Content

1. Complete notes for three topics
2. Complete example for each of the subtopics
3. Complete Exercise (Test Your Knowledge) for each subtopic
4. Teaching & learning video for each example
5. Live games after each topic
6. Full schema for **Test Your Knowledge** are provided

TABLE OF CONTENT



BASIC ALGEBRA

SUBTOPIC :-

1.1 BASIC ALGEBRA

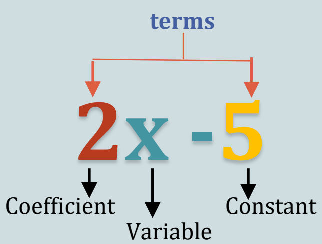
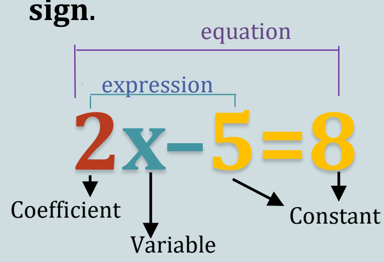
1.2 QUADRATIC EQUATION

- Factorization
- Quadratic Formula
- Completing the Square

1.3 PARTIAL FRACTION

1.1 BASIC ALGEBRA

Algebra is a division of mathematics designed to help solve certain types of problems quicker and easier. Algebra is based on the concept of *unknown* values called **variables**.

Algebraic Expression	Algebraic Equation
✓ An **expression** which is made up **of variables and constants**, along with **algebraic operations** (addition, subtraction, etc.) ✓ Expressions are made up of **terms.**	✓ An algebraic **equation** can be defined as **a mathematical statement in which two expressions are set equal to each other.** ✓ In simple words, equations mean equality **i.e. the equal sign**.
	

- A **variable**, any *symbol or alphabet* that being used to represent any *unknown value*.
- A **term** is an expression involving letters and/or numbers (called **factors**) exist in a mathematical expression/equation.
- A **coefficient** is a *numerical value* used to multiply variable.

Remember:-
An **equation** must have " = " in the mathematics statement, without this symbol, it is an mathematical expression.

1.1 BASIC ALGEBRA

Algebraic Rules

Commutative, Associative and Distributive Laws	Order of Arithmetic's operation Rules
Commutative Property: The order of elements does **not** make any difference in the outcome. This only true of **multiplication and addition.**	Order of Operations **BODMAS**
Distributive Property: The process of distributing a number on the outside of the parentheses to each number on the inside; $a(b+c)=ab+ac$	1. Work within parentheses ( ), **B**rackets [ ], and braces { } from innermost and work outward. 2. Simplify exponents/p**O**wer and roots working from left to right. 3. Do **D**ivision and **M**ultiplication, whichever comes first; from *left to right.* 4. Do **A**ddition and **S**ubtraction, whichever comes first; from *left to right.*
Associative Property: Grouping does **not** make any difference: $(a+b)+c=a+(b+c)$, $(ab)c=a(bc)$	

Algebraic Operations Rules

$a(b+c)=ab+ac$ $a(b-c)=ab-ac$	$\left(\frac{a}{b}\right)+\left(\frac{c}{d}\right)=\frac{ad+bc}{bd}$
$\frac{(ab+ac)}{a}=b+c$ a≠ 0	$\left(\frac{a}{b}\right)-\left(\frac{c}{d}\right)=\frac{ad-bc}{bd}$
$\left(\frac{a-b}{c-d}\right)=\left(\frac{b-a}{d-c}\right)$ **Multiply both numerator and denominator by* **(-1)**	$\frac{a}{\left(\frac{b}{c}\right)}=\frac{ac}{b}$
$\frac{(a+b)}{c}=\frac{a}{c}+\frac{b}{c}$	$\frac{\left(\frac{a}{b}\right)}{c}=\frac{a}{bc}$
$a\left(\frac{c}{d}\right)=\frac{ab}{c}$	$\frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{d}\right)}=\frac{ad}{bc}$

Example 1.1

Simplify each of the following expressions:

a. $7 + 4x + 13 - 2x$ *b.* $\frac{y}{x+3}\left(\frac{3(x+3)}{y^2(x-3)}\right)$

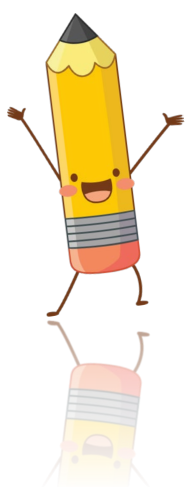
Solution

a. $7+4x+13-2x$

$= 2x+20$

Combine unknown x with unknown x, constant with constant

CLICK ME.....
https://youtu.be/ydTKGMI4cSU

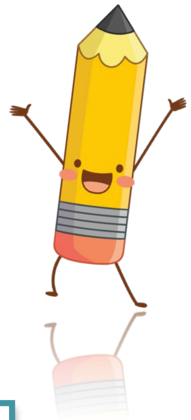


Solution

b. $\frac{y}{x+3}\left(\frac{3(x+3)}{y^2(x-3)}\right)$

$= \frac{\cancel{y}}{\cancel{x+3}}\left(\frac{3\cancel{(x+3)}}{y^{\cancel{2}}(x-3)}\right)$

$= \frac{3}{y\,(x-3)}$



CLICK ME.....
https://youtu.be/w_wQcd7Aj4Y

1.1 BASIC ALGEBRA

Solve Equation by "Cancelling"		
Cancel addition with subtraction (vice versa)	$r+5=12$ $r+5-\mathbf{5}=12-\mathbf{5}$ $r=12-5$ $r=7$	$s-3=8$ $s-3+\mathbf{3}=8+\mathbf{3}$ $s=8+3$ $s=11$
Cancel multiplication with division (vice versa)	$4m=48$ $m=\frac{48}{4}$ $m=24$	$\frac{n}{3}=7$ $n=7x3$ $n=21$
Cancel exponent by taking the root (vice versa)	$x^2=9$ $x=\sqrt{9}$ $x=3$	$\sqrt[4]{y}=2$ $y=2^4$ $y=8$

Transposition Of Formulae

This is also known as **Changing the Subject of the Formula** or some may say **Conversion of Formula**.
As simple example**,** $\boldsymbol{y=mx+c}$, in this case, **y** is the subject of the formula.

RULES – to make transposition of the formula, we applied **reverse** process of **BODMAS.** Ensure yourself to :-

- **Remove** all fractions, brackets and roots.
- Put **ALL** the terms containing the "**requested new subject**" on one side of the formula, and the others on another side.
- **Simplify** both sides.
  Lets said if the "requested new subject" appears in more than one term, then treat it as a common factor.
  Then divide both sides of the formula by the terms in front of the new subject.

Remember:-
Transposition of Formulae is totally **solve** once the "requested new subject" only appear once in the formula as a new subject of the formula.

Example 1.2

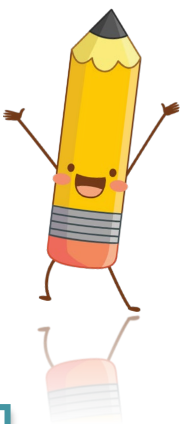
Simplify the following expression :

a. $\frac{3x}{4y}+\frac{7-5x}{8y}$

b. $\frac{2st}{3xy^4}\div\frac{st^2}{x^3y^2}$

Solution

a.
$$\frac{3x}{4y}+\frac{7-5x}{8y}$$
$$=\frac{3x}{4y}\left(\frac{2}{2}\right)+\frac{7-5x}{8y}$$
$$=\frac{6x+7-5x}{8y}$$
$$=\frac{x+7}{8y}$$



Solution

b.
$$\frac{2st}{3xy^4}\div\frac{st^2}{x^3y^2}$$
$$=\frac{2st}{3xy^4}\times\frac{x^3y^2}{st^2}$$
$$=\frac{2x^2}{3y^2t}$$

CLICK ME…..
https://youtu.be/DMPQEbm4TWc

Example 1.3

Given $t = 2\pi\sqrt{\frac{1}{g}}$. Make **g** the subject of the formula.

Solution

$$t = 2\pi\sqrt{\frac{1}{g}}$$

$$(t)^2 = \left(2\pi\sqrt{\frac{1}{g}}\right)^2$$

*****Square** all the terms to remove the square root sing

$$t^2 = 4\pi^2\left(\frac{1}{g}\right)$$

$$g \times (t^2) = \left(\frac{4\pi^2}{g}\right) \times g$$

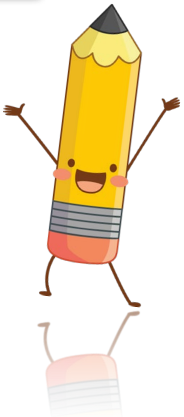
*****Multiply** both sides by g

$$gt^2 = 4\pi^2$$

$$\frac{1}{t^2} \times (gt^2) = (4\pi^2) \times \frac{1}{t^2}$$

*****Divide** both sides by t^2

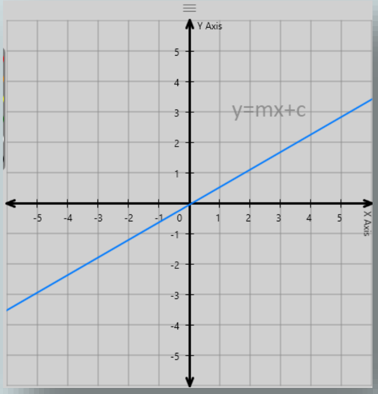
$$g = \frac{4\pi^2}{t^2}$$



CLICK ME…..
https://youtu.be/W5J-SWPTb7M

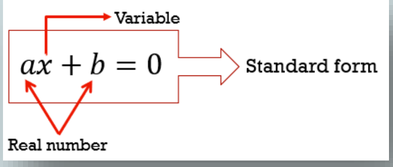
1.1 BASIC ALGEBRA

Linear Equation



The graph of **y = mx + c** is a straight line

- ✓ A **linear** equation is an equation for a straight **line.**
- ✓ **Power** for *unknown* always 1. (***m*** is representing gradient of the line while ***c*** is representing *y*-intercept.)
- ✓ Can be written in the form



1.1 BASIC ALGEBRA

Linear Equation

✓ $y = 3x + 7$ or $x - 3y + 2z = 10$

The highest degree of terms in above equations are 1. So both of the equations are linear equation.

Left side = Right side

LEFT SIDED EXPRESSION		RIGHT SIDED EXPRESSION	
Subtract	$-x$	$+x$	Addiction
Addiction	$+x$	$-x$	Subtract
Division	$\div x$	$\times x$	Multiple
Multiple	$\times x$	$\div x$	Division
Power root	$\sqrt{x}$	x^2	Roots
Roots	x^2	$\sqrt{x}$	Power root

Table above show what happened when an operation from left side being move to the right side (vice versa).

Example 1.4

Solve the following equation:

a. $6s-4=3s+5$

b. $\frac{y+2}{y-3}=\frac{y-3}{y+2}$

Solution

a.
$$6s-4=3s+5$$
$$6s-3s=5+4$$
$$3s=9$$
$$s=\frac{9}{3}$$
$$s=3$$

Solution

b.
$$\frac{y+2}{y-3}=\frac{y-3}{y+2}$$
$$(y+2)(y+2)$$
$$=(y-3)(y-3)$$
$$y^2+2y+2y+4$$
$$=y^2-3y-3y+9$$
$$y^2+4y+4=y^2-6y+9$$
$$y^2-y^2+4y+6y=9-4$$
$$10y=5$$
$$y=\frac{5}{10}$$
$$y=\frac{1}{2}$$

CLICK ME…..
https://youtu.be/StoQZK4HMTQ

Test Your Knowledge 1.1

1. Simplify the following expression :

a. $6(m-3n)-5m(n+4)-2n(1-n)$

b. $(2-v)-\frac{3}{2+v}$

c. $\frac{2v^2+8v^3}{5w} \times \frac{(10w-15)}{1+4v}$

d. $\frac{v+3}{m^2-n^2} \div \frac{3v+9}{m+n}$

2. $v=\sqrt{t^2+w}$ make w the subject of the formulae.

Answer :

1.

a. $2n^2-5mn-20n-14m$

b. $\frac{1-v^2}{2+v}$

c. $\frac{(2v^2)(2w-3)}{w}$

d. $\frac{1}{3(m-n)}$

2. $w=t^2+v^2$

1.2 QUADRATIC EQUATION

Quadratic Equation

- **Factorization**
- **Quadratic Formula**
- **Completing the Square**

Quadratic equation always has index = 2.

Standard Form of Quadratic Equation

$$ax^2 + bx + c = 0$$

a, b and c are knowns values
a can't be 0

Remember:-

1. **Quadratic equations** always have two solutions, either same value or different value.
2. **Linear equations** have only 1 value for solution.

Tips:

1. **Factorization Method** – **ONLY** for **integers** and **fractions.**
2. **Quadratic Formula and Completing the Square Method** – for all types of value.
3. Solve by **suitable method** – defined the value by using calculator, if integer and fraction, most suitable is Factorization. For any decimal values, Factorization is totally NOT SUITABLE, use either Quadratic Formula or Completing the Square Method.

1.2 QUADRATIC EQUATION

Some equations will not state as equal to 0. Therefore, in order to solve quadratic equations, you must **simplify** or **rearrange** the equation to make it equal to 0.

$3x^2 + 7x + 2 = 0$	$\boldsymbol{a = 3, b = 7, c = 2}$
$x^2 + 5x - 3 = 0$	$\mathbf{a = 1, b = 5, c = -3}$ • The value of $\mathbf{a = 1}$, in mathematical statement, we do not write any coeeficient $= 1$. • The value of $\mathbf{c = -3}$ cause there is minus operation.
$x^2 + 9 = 0$	$\mathbf{a = 1, b = 0, c = 9}$ • The value of $\mathbf{b = 0}$, that is why x is not exist in the equation.

$x^2 + 6x = -3$	Move all terms to one side only, make it *equal to 0.*	$x^2 + 6x + 3 = 0$	$a = 1,$ $b = 6,$ $c = 3$
$(x + 2)(x - 3) = 0$	**Expand** the bracket.	$x^2 - x - 6 = 0$	$a = 1,$ $b = -1,$ $c = -6$
$2(x^2 + 2) = 7$	**Expand** bracket. Then, move all terms to one side only, make it *equal to 0*.	$2x^2 + 4 = 7$ $2x^2 + 4 - 7 = 0$	$a = 2,$ $b = 4,$ $c = -7$

1.2 QUADRATIC EQUATION

Factorization Method (ONLY for integers and Fraction.)	
Step 1	Ensure to rearrange or simplify your equation to be in the standard form of quadratic equations. $\boxed{ax^2 + bx + c = 0}$
Step 2	Factorize the left side i.e. $\boxed{ax^2 + bx + c}$ of the equation. Apply this step only your equation = 0. $(x - x_1)(x - x_2) = 0$
Step 3	Assign each factor equal to zero. $x - x_1 = 0 \text{ and } x - x_2 = 0$
Step 4	Now solve the equation in order to determine the values of *x*. $x = x_1 \text{ and } x = x_2$ x_1 and x_2 is any numerical value.

Remember:-

1. Always ensure to set your ***equation = 0*** before solving.
2. Sort your equation starting with ***terms of index equal to 2*** and sort it in descending order.

Example 1.5

Solve the following quadratic equation :

a. $3x^2 - 6x = 0$ b. $x^2 + 2x = 8$

Solution

a. $3x^2 - 6x = 0$

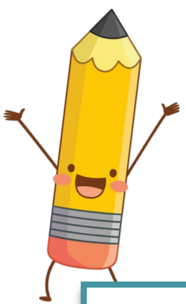
Common factor for this quadratic equation is $3x$, so we take out $3x$ as

$3x(x-2) = 0$

Find roots for the equation,

$3x = 0 \qquad x - 2 = 0$

$x = 0 \qquad x = 2$



Solution

b. $x^2 + 2x = 8$

Step 1: $x^2 + 2x - 8 = 0$

Step 2: $(x+4)(x-2) = 0$

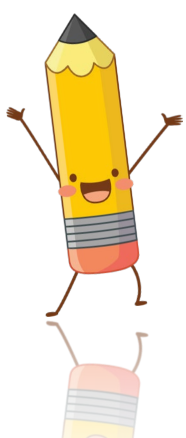
Step 3:

$x + 4 = 0 \quad \textit{and} \quad x - 2 = 0$

Step 4:

$x + 4 = 0 \qquad x - 2 = 0$

$x = -4 \qquad x = 2$



CLICK ME.....
https://youtu.be/U0A-hZbetcE

Example 1.6

Solve the following quadratic equation :

a. $5x^2 + 35x = 0$ b. $x^2 + 11x + 24 = 0$

Solution

a. $5x^2 + 35x = 0$

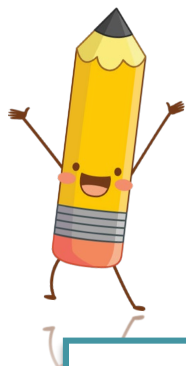
Common factor for this quadratic equation is $5x$, so we take out $5x$ as

$5x(x+7) = 0$

Find roots for the equation,

$5x = 0$ $x + 7 = 0$

$x = 0$ $x = -7$



Solution

b. $x^2 + 11x + 24 = 0$

Step 1: $x^2 + 11x + 24 = 0$

Step 2: $(x+3)(x+8) = 0$

Step 3:

$x + 3 = 0$ *and* $x + 8 = 0$

Step 4:

$x + 3 = 0$ $x + 8 = 0$

$x = -3$ $x = -8$

CLICK ME…..
https://youtu.be/FncGHxhQ-hg

1.2 QUADRATIC EQUATION

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

1. $\pm$ (one plus and one minus) represent two distinct roots for the Quadratic Equation.

2. Used to solve any quadratic equation that cannot be solved by Factorization. (*Even can be used to solve Quadratic Equation that can be solved by Factorization.*)

3. To use this Quadratic Formula, always state out the value of **a, b** and **c** in order to avoid mistake.

Step 1	Bring the equation to the form $ax^2 + bx + c = 0$ where ***a, b*** and ***c*** are **numerical value**.
Step 2	Identify the values of ***a, b*** and ***c.***
Step 3	Then substitute these values into the quadratic formula. $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Step 4	Use the order of operations (**BODMAS**) to simplify the quadratic formula.
Step 5	Simplify till you get the single term of square root.
Step 6	Solve for the value of ***x***.

Example 1.7

Solve the following $x^2 - 6x + 2 = 0$ using quadratic formula:

Solution

Step 1:

$$x^2 - 6x + 2 = 0$$

Step 2:

$$x^2 - 6x + 2 = 0$$

$$a = 1 \quad b = -6 \quad c = 2$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Step 3:

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(2)(1)}}{2(1)}$$

Step 4:

$$x = \frac{6 \pm \sqrt{36 - 8}}{2}$$

Step 5:

$$x = \frac{6 \pm \sqrt{28}}{2}$$

Step 6:

$$x = \frac{6 + \sqrt{28}}{2} \qquad x = \frac{6 - \sqrt{28}}{2}$$

$$x = 5.647 \qquad x = 0.3524$$

CLICK ME…..
https://youtu.be/Fluzm9NrPqc

Example 1.8

Solve the following $2x^2 - 7x = 6$ using quadratic formula:

Solution

Step 1: $2x^2 - 7x - 6 = 0$

Step 2: $2x^2 - 7x - 6 = 0$

$a = 2 \quad b = -7 \quad c = -6$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Step 3:
$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(2)(-6)}}{2(2)}$$

Step 4:
$$x = \frac{7 \pm \sqrt{49 + 48}}{4}$$

Step 5:
$$x = \frac{7 \pm \sqrt{97}}{4}$$

Step 6:
$$x = \frac{7 + \sqrt{97}}{4} \qquad x = \frac{7 - \sqrt{97}}{4}$$

$$x = 4.2122 \qquad x = -0.7122$$

CLICK ME…..
https://youtu.be/AMRk_Kgskao

Example 1.9

Solve the following $x^2 + 5x = -6$ using quadratic formula:

Solution

Step 1:

$$x^2 + 5x + 6 = 0$$

Step 2:

$$x^2 + 5x + 6 = 0$$

$$a = 1 \quad b = 5 \quad c = 6$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Step 3:

$$x = \frac{-5 \pm \sqrt{(5)^2 - 4(1)(6)}}{2(1)}$$

Step 4:

$$x = \frac{-5 \pm \sqrt{25 - 24}}{2}$$

Step 5:

$$x = \frac{-5 \pm \sqrt{1}}{2}$$

Step 6:

$$x = \frac{-5 + 1}{2} \qquad x = \frac{-5 - 1}{2}$$

$$x = -2 \qquad x = -3$$

1.2 QUADRATIC EQUATION

Completing The Square Method

1. In this method, we have to convert the given equation into a perfect square.
2. **7 step** to complete the square in **ANY** quadratic equation .

$$ax^2 + bx + c = 0$$

Step 1	Isolate the number or variable c to the right side of the equation	$ax^2 + bx = c$
Step 2	Divide all terms by a; You must ensure that coefficient of x^2 must 1.	$x^2 + bx = c$
Step 3	Multiply coefficient b by $\frac{1}{2}$ and then square.	$\left(b \times \frac{1}{2}\right)^2$
Step 4	Add this value to both sides of the equation	$x^2 + bx + \left(\frac{b}{2}\right)^2 = c + \left(\frac{b}{2}\right)^2$
Step 5	**Factorize** left side of the equation and **expand** the right side.	$(x+d)^2 = c + d^2$
Step 6	Square root both sides of the equation; This will eliminate square on the left side and give you $x + d$.	$x + d = \pm\sqrt{(c + d^2)}$
Step 7	Subtract whatever number remains on the left side of the equation to yield x and complete the square.	$x = +\sqrt{(c + d^2)} - d$ or $x = -\sqrt{(c + d^2)} - d$

Example 1.10

Solve the following $x^2 + 2x - 8 = 0$ using completing the square.

Solution

Step 1: $x^2 + 2x = 8$

Step 2: $x^2 + 2x = 8$

Since **a = 1**, you can directly go to step 3

Step 3 & 4: $x^2 + 2x + (1)^2 = 8 + (1)^2$

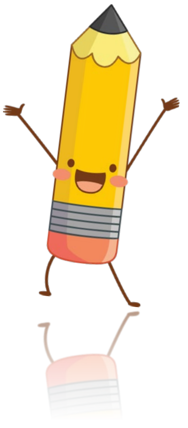
$b = 2, \therefore \left(\mathbf{2} \times \frac{\mathbf{1}}{\mathbf{2}}\right)^2 = (1)^2$

Step 5: $(x+1)^2 = 8 + (1)^2$

Step 6: $x + 1 = \pm 3$

Step 7: $x = +3 - 1 \quad or \quad x = -3 - 1$

$x = 2 \qquad\qquad x = -4$



CLICK ME…..
https://youtu.be/jBOEAtvfsLc

Example 1.11

Solve the following $4x^2 + 6x - 12 = 0$ using completing the square.

Solution

Step 1: $4x^2 + 6x = 12$

Step 2: $\left(\frac{4}{4}\right)x^2 + \left(\frac{6}{4}\right)x = \left(\frac{12}{4}\right)$

Since **a = 4**, divide whole equation with 4.

$$x^2 + \frac{3}{2}x = 3$$

Step 3 & 4: $x^2 + \left(\frac{3}{2}\right)x + \left(\frac{3}{4}\right)^2 = 3 + \left(\frac{3}{4}\right)^2$

$b = \frac{3}{2}, \therefore \left(\frac{\mathbf{3}}{\mathbf{2}} \times \frac{\mathbf{1}}{\mathbf{2}}\right)^2 = \left(\frac{3}{4}\right)^2$

Step 5: $\left(x + \frac{3}{4}\right)^2 = 3 + \left(\frac{3}{4}\right)^2$

Step 6: $x + \frac{3}{4} = \pm\sqrt{\frac{57}{16}}$

Step 7: $x = +\sqrt{\frac{57}{16}} - \frac{3}{4}$ *or* $x = -\sqrt{\frac{57}{16}} - \frac{3}{4}$

$x = 1.1375$ $\quad$ $x = -2.6375$

CLICK ME…..
https://youtu.be/j-7WSlrZDa8

Test Your Knowledge 1.2

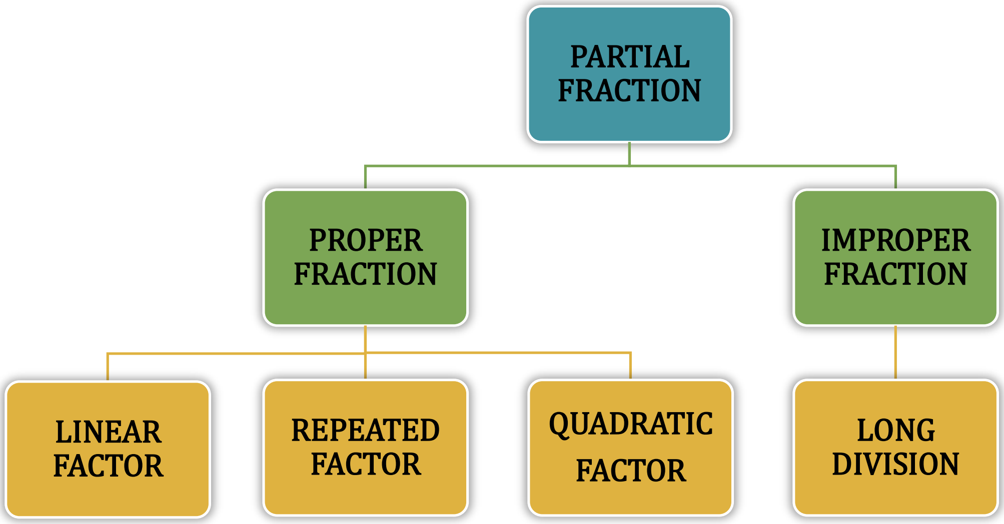
Determine the roots for quadratic equations below by using the stated method:

1.	$x^2 + 2x - 63 = 0$	Factorization
2.	$x^2 - 16x + 48 = 0$	Factorization
3.	$3x(x-1) = 3x$	Quadratic Formula
4.	$4(x^2 - 64) = 0$	Quadratic Formula
5.	$6(x^2 - 2) = x$	Completing the Square
6.	$3x^2 = 5 + 8x$	Completing the Square

Answer :

1. $x = 7$ $\quad x = 9$
2. $x = 12$ $\quad x = 4$
3. $x = 2$ $\quad x = 0$
4. $x = 8$ $\quad x = -8$
5. $x = 1.3@\frac{3}{2}$ or $x = -1.3333@\left(-\frac{4}{3}\right)$
6. $x = 3.1892$ $\quad or \quad x = -0.5226$

1.3 PARTIAL FRACTION



Partial Fraction : one of the simpler fractions into the sum of which the quotient of two polynomials may be **decomposed.**

Algebraic fraction

$$\frac{\text{numerator(n)}}{\text{denominator(d)}} = \frac{\text{polynomial expression}}{\text{polynomial expression}}$$

When presented with a fraction, we can note the degree of the numerator , say **n**, and the degree of the denominator , say **d.**

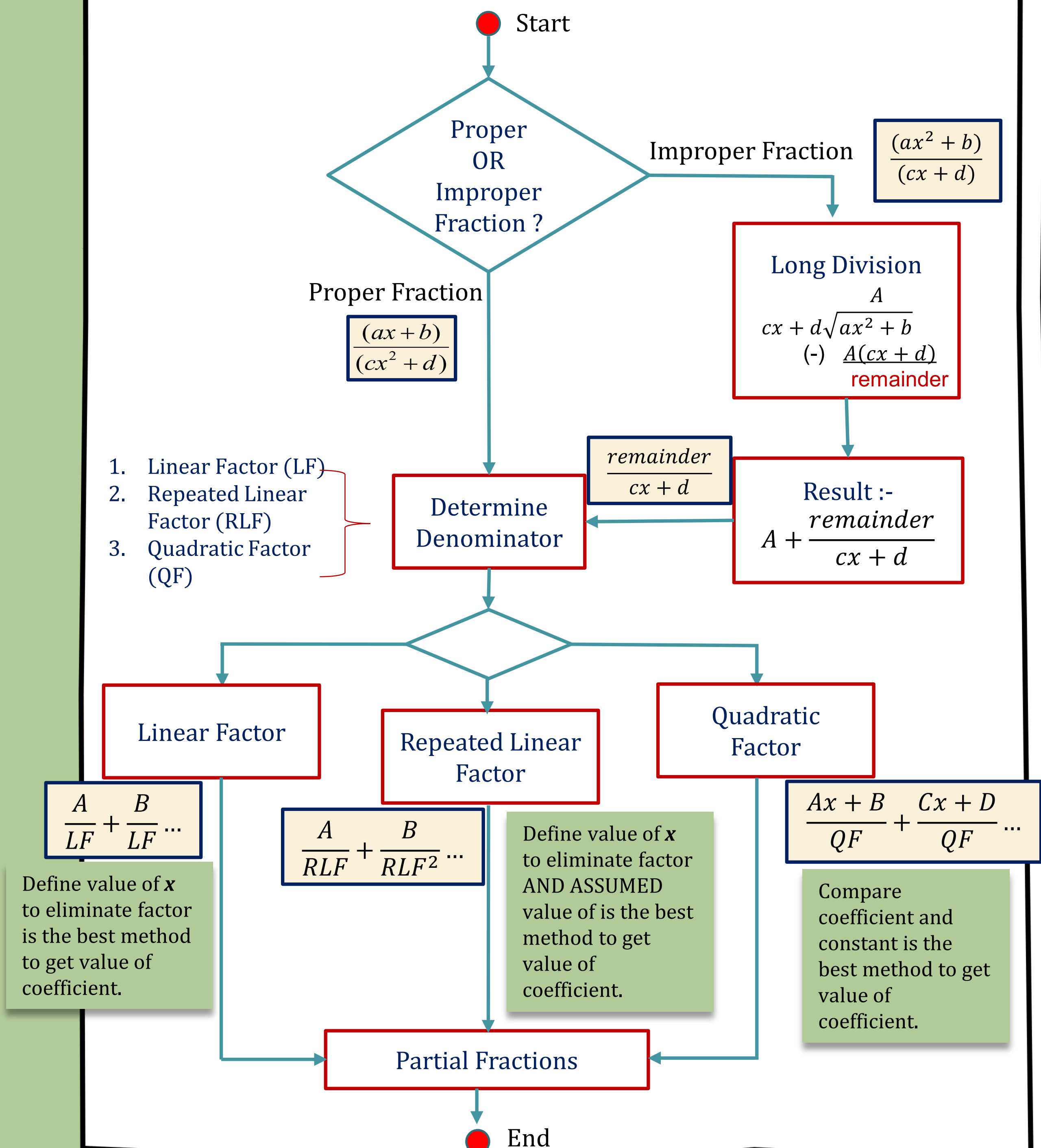
✓ If **n<d** , the fraction is **proper**. Some examples are

$$\frac{2}{(1-2x)(1+x)}, \frac{4x+1}{x^2+3x+1}, \frac{2x}{(x+5)(x-2)}, \text{etc}$$

✓ If **n≥ *d*** the fraction is **improper**. Some examples are

$$\frac{2x+1}{x-1}, \frac{x^3+2}{x^2+3x+1}, \frac{x^4+x^2-3}{(x+5)(x-2)}, \text{etc}$$

1.3 PARTIAL FRACTION



1.3 PARTIAL FRACTION

Algebraic Fraction can be expressed as Partial Fraction as following

Case	Fraction $\frac{n(x)}{d(x)}$	Form of denominator d(x)	Partial Fraction Form
1	$\frac{n(x)}{(ax+b)(cx+d)}$	Linear Factors **(n<d)**	$\frac{A}{ax+b}+\frac{B}{cx+d}$
2	$\frac{n(x)}{(ax+b)^2}$	Repeated Linear Factors **(n<d)**	$\frac{A}{ax+b}+\frac{B}{(ax+b)^2}$
	$\frac{n(x)}{(ax+b)(cx+d)^2}$	Linear and Repeated Linear Factors **(n<d)**	$\frac{A}{ax+b}+\frac{B}{cx+d}+\frac{C}{(cx+d)^2}$
3	$\frac{n(x)}{(ax+b)(cx^2+d)}$	Quadratic Factors **(n<d)**	$\frac{A}{ax+b}+\frac{Bx+C}{cx^2+d}$
4	$\frac{(ax^2+b)}{(cx+d)}$	Long Division Method **(n≥ *d*)**	A $cx+d\overline{)ax^2+b}$ (-) $\underline{A(cx+d)}$ remainder **Tips:** once you get, **highest power** of **remainder** less than Denominator, then only, you can stop doing Long Division.

1.3 PARTIAL FRACTION

Step to solve Partial Fraction as following

Step 1	**Check** your **fraction** either Proper Fraction or Improper Fraction. (If highest index of Numerator ≥ highest index of Denominator, so it is Improper Fraction.) If Improper Fraction, do Long Division to convert it to Mixed Number. [*Mixed number for Partial Fraction is combination of algebraic terms + fraction of remainder.*] If Proper Fraction go to step 2.
Step 2	**Check factors** of denominator. Use your calculator for fast checking. Bear in mind; **Linear equation** = **1** value of x. **Quadratic equation** = **2** values of x (if you get **ONLY 1** value of **Integers or Fractions**, it is indicated **Repeated Linear Factor**. If you get **decimal values or complex number**, it is indicated **Quadratic Factor**.) **Cubic Equation** = **3** values of x.
Step 3	**Write down partial fraction** for each factor obtained by using suitable formula accordingly to the type of your factor. (*You may use any Alphabet, but starting with **A** is the best practice.*)
Step 4	**Remove denominator** by cross-multiply.
Step 5	**Find** the **value** of your variable (Alphabet), different factor may have different method.
Step 6	Finally, **substitute** the **values** of **Variable** into your formula of partial fractions.

Example 1.12

Determine the partial fraction decomposition of the following :

a. $\dfrac{5}{x^2+x-6}$ b. $\dfrac{2x+16}{x^2-4}$ c. $\dfrac{2x+3}{(x-1)^2}$

Solution

a. $\dfrac{5}{x^2+x-6}$

Proper Fraction (n<d) with Linear Factor (LF)

Step 1: Proper Fraction

$$\frac{5}{x^2+x-6}=\frac{5}{(x+3)(x-2)}$$

Step 2: Linear Factor (LF)

Step 3 : Partial Fraction Formula

$$\frac{5}{(x+3)(x-2)}=\frac{A}{(x+3)}+\frac{B}{(x-2)}$$

Step 4 : Remove Denominator.

$$5=A(x-2)+B(x+3)$$

Step 5: Find value of **A** and **B** by defining value of x to eliminate factor.

$x-2=0$

$assume\ x=2$

$5=A(2-2)+B(2+3)$

$5=5B$

$B=1$

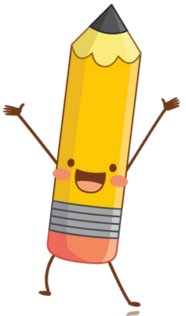
$x+3=0$

$assume\ x=-3$

$5=A((-3)-2)+B((-3)+3)$

$5=-5A$

$A=-1$



Step 6:

$$\frac{5}{(x+3)(x-2)}=\frac{-1}{(x+3)}+\frac{1}{(x-2)}$$

$$=-\frac{1}{(x+3)}+\frac{1}{(x-2)}$$

CLICK ME.....
https://youtu.be/N1d5Wevo72Q

Example 1.12 : Continue

Determine the partial fraction decomposition of the following :

a. $\dfrac{5}{x^2+x-6}$ b. $\dfrac{2x+16}{x^2-4}$ c. $\dfrac{2x+3}{(x-1)^2}$

Solution

b. $\dfrac{2x+16}{x^2-4}$

Proper Fraction (n<d) with Linear Factor (LF)

Step 1: Proper Fraction

$$\frac{2x+16}{x^2-4}=\frac{2x+16}{(x-2)(x+2)}$$

Step 2: Linear Factor (LF)

$$\frac{2x+16}{(x-2)(x+2)}=\frac{A}{(x-2)}+\frac{B}{(x+2)}$$

Step 3 : Partial Fraction Formula

Step 4 : Remove Denominator.

$$2x+16=A(x+2)+B(x-2)$$

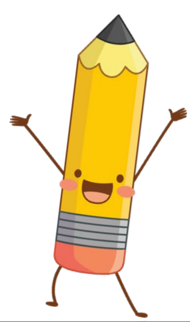
Step 5: Find value of **A** and **B** by defining value of ***x*** to eliminate factor.

$$x-2=0$$
$$assume\ x=2$$
$$2(2)+16=A(2+2)+B(2-2)$$
$$20=4A$$
$$A=5$$

$$x+2=0$$
$$assume\ x=-2$$
$$2(-2)+16=A((-2)+2)+B((-2)-2)$$
$$-4+16=-4B$$
$$12=-4B$$
$$B=-3$$

Step 6:

$$\frac{2x+16}{x^2-4}=\frac{5}{(x-2)}-\frac{3}{(x+2)}$$



CLICK ME.....
https://youtu.be/be31p7zoS6l

Example 1.12 : Continue

Determine the partial fraction decomposition of the following :

a. $\frac{5}{x^2+x-6}$ b. $\frac{2x+16}{x^2-4}$ c. $\frac{2x+3}{(x-1)^2}$

Solution

c. $\frac{2x+3}{(x-1)^2}$

Proper Fraction (n<d) with Repeated Linear Factor (RLF)

Step 1: Proper Fraction

Step 2: Repeated Linear Factor (RLF)

Step 3 : Partial Fraction Formula

$$\frac{2x+3}{(x-1)^2}=\frac{A}{(x-1)}+\frac{B}{(x-1)^2}$$

Step 4 : Remove Denominator.

$$2x+3=A(x-1)+B$$

Step 5: Find value of **A** and **B** by defining value of ***x*** to eliminate factor.

$x-1=0$

$assume\ x=1$

$2(1)+3=A(1-1)+B$

$5=B$

$assume\ x=0\ and\ substitution B=5$

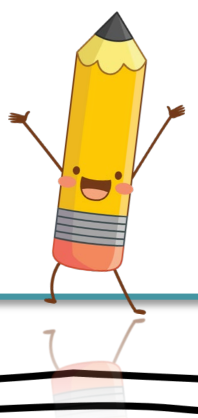
$2(0)+3=A(0-1)+5$

$3=-A+5$

$-2=-A$

Step 6:

$$\frac{2x+3}{(x-1)^2}=\frac{2}{(x-1)}+\frac{5}{(x-1)^2}$$



CLICK ME…..

https://youtu.be/V26BXd5SNtU

Example 1.13

Determine the partial fraction decomposition for the following :

$$\frac{4x^2}{(x-1)(x-2)^2}$$

Solution

$$\frac{4x^2}{(x-1)(x-2)^2}$$

Proper Fraction (n<d) with Repeated Linear Factor (RLF)

Step 1: Proper Fraction

Step 2: Repeated Linear Factor (RLF)

Step 3 : Partial Fraction Formula

$$\frac{4x^2}{(x-1)(x-2)^2} = \frac{A}{(x-1)} + \frac{B}{(x-2)} + \frac{C}{(x-2)^2}$$

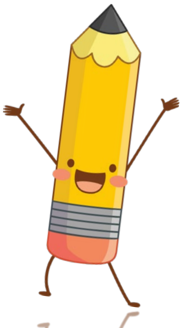
Step 4 : Remove Denominator.

$$4x^2 = A(x-2)^2 + B(x-1)(x-2) + C(x-1)$$

Step 5: Find value of **A** and **B** by defining value of ***x*** to eliminate factor.

$$assume\ x-2=0$$
$$x=2$$
$$4(2)^2 = A(2-2)^2 + B(2-2)(2-1) + C(2-1)$$
$$16 = C$$

$$assume\ x-1=0$$
$$x=1$$
$$4(1)^2 = A(1-2)^2 + B(1-2)(1-1) + C(1-1)$$
$$4 = A$$



Example 1.13 : Continue

Determine the partial fraction decomposition for the following :

$$\frac{4x^2}{(x-1)(x-2)^2}$$

Solution

Step 5: Find value of **C** by assuming any value of $\boldsymbol{x}$, as long the value not yet being used and substitute value of **A** and **B.**

$$\begin{aligned} &assume\ x = 0 \\ &4(0)^2 = A(0-2)^2 + B(0-2)(0-1) + C(0-1) \\ &0 = 4A + 2B - C \end{aligned}$$

Substitution A=4 and C=16 in equation

$$\begin{aligned} &0 = 4(4) + 2B - 16 \\ &0 = 2B \\ &B = 0 \end{aligned}$$

Step 6:

$$\begin{aligned} \frac{4x^2}{(x-1)(x-2)^2} &= \frac{4}{(x-1)} + \frac{0}{(x-2)} + \frac{16}{(x-2)^2} \\ &= \frac{4}{(x-1)} + \frac{16}{(x-2)^2} \end{aligned}$$

CLICK ME.....
https://youtu.be/E1SdQdENe5k

Example 1.14

Determine the partial fraction decomposition for the following :

$$\frac{2x^2+x+1}{(x+1)(x^2+1)}$$

Solution

$$\frac{2x^2+x+1}{(x+1)(x^2+1)}$$

Proper Fraction (n<d) with combination of Linear Factor (LF) and Quadratic Factor (QF)

Step 1: Proper Fraction

Step 2: Linear Factor (LF) and Quadratic Factor (QF)

Step 3 : Partial Fraction Formula

$$\frac{2x^2+x+1}{(x+1)(x^2+1)}=\frac{A}{(x+1)}+\frac{Bx+C}{(x^2+1)}$$

Step 4 : Remove Denominator.

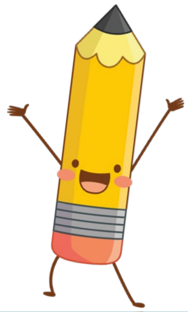
$$2x^2+x+1=A\,(x^2+1)+(Bx+C)(x+1)$$

Expand to remove any brackets, then regroup according to the similar terms of *x*.

$$2x^2+x+1=A\,x^2+A+Bx^2+Bx+Cx+C$$

$$2x^2+x+1=(A+B)x^2+(B+C)x+A+C$$

Remember:-
A **coefficient** is a multiplicative factor in the terms of a polynomial, a series, or any expression. It is generally a number.



CLICK ME…..
https://youtu.be/xOlg9FRkQTo

Example 1.14 : Continue

Determine the partial fraction decomposition for the following :

$$\frac{2x^2+x+1}{(x+1)(x^2+1)}$$

Solution

Step 5: Find value of **A, B** and **C.**

It is much easier to define value of x to eliminate factor of LF.

$2x^2+x+1 = A\,(x^2+1) + (Bx+C)(x+1)$

For $x+1=0$
$\therefore x = -1$

$2(-1)^2 + (-1) + 1 = A\left((-1)^2+1\right) + (B(-1)+C)\left((-1)+1\right)$
$2 = 2A + 0$
$A = 1$

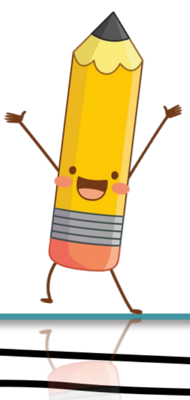
For values of **B** and **C**, use compared coefficient method.
$2x^2+x+1 = (A+B)x^2 + (B+C) + A + C$

Compare ***coefficient x^2*** and substitute value of A.
$2 = A + B$
$2 = 1 + B$
$B = 1$

Compare ***coefficient x*** and substitute value of B. (**option** : *you can also compare constant to get value of C*)
$1 = B + C$
$1 = 1 + C$
$C = 0$

Step 6:

$$\frac{2x^2+x+1}{(x+1)(x^2+1)} = \frac{1}{(x+1)} + \frac{x}{(x^2+1)}$$



Example 1.15

Determine the partial fraction decomposition of the following :

$$\frac{8}{(x^2+1)(2x-3)}$$

Solution

$$\frac{8}{(x^2+1)(2x-3)}$$

Proper Fraction (n<d) with combination of Linear Factor (LF) and Quadratic Factor (QF)

Step 1: Proper Fraction

Step 2: Linear Factor (LF) and Quadratic Factor (QF)

Step 3 : Partial Fraction Formula

$$\frac{8}{(x^2+1)(2x-3)} = \frac{Ax+B}{(x^2+1)} + \frac{C}{(2x-3)}$$

Step 4 : Remove Denominator.

$$8 = (Ax+B)(2x-3) + C(x^2+1)$$

Expand to remove any brackets, then regroup according to the similar terms of *x*.

$$8 = 2A\,x^2 - 3Ax + 2Bx - 3B + Cx^2 + C$$

$$8 = (2A+C)x^2 + (-3A+2B)x - 3B + C$$

CLICK ME…..
https://youtu.be/Yi06PQvtk-E

Example 1.15 : Continue

Determine the partial fraction decomposition of the following :

$$\frac{8}{(x^2+1)(2x-3)}$$

Solution

Step 5: Find value of **A, B** and **C.**

Define value of x to eliminate factor of LF.

$$8 = (Ax+B)(2x-3) + C(x^2+1)$$

For $2x - 3 = 0;\ \therefore x = \frac{3}{2}$

$$8 = \left(A\left(\frac{3}{2}\right) + B\right)\left(2\left(\frac{3}{2}\right) - 3\right) + C\left(\left(\frac{3}{2}\right)^2 + 1\right)$$

$$8 = \frac{13}{4}C$$

$$C = \frac{32}{13}$$

For values of **B** and **C**, use compared coefficient method.

$$8 = (2A + C)x^2 + (-3A + 2B)x - 3B + C$$

Compare ***coefficient x^2*** and substitute value of C.

$$0 = 2A + C$$

$$0 = 2A + \frac{32}{13} \qquad A = -\frac{16}{13}$$

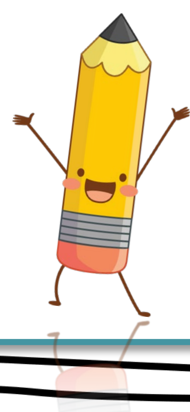
Compare ***constant*** and substitute value of C.

$$8 = -3B + C$$

$$8 = -3B + \frac{32}{13} \qquad B = -\frac{24}{13}$$

Step 6:

$$\frac{8}{(x^2+1)(2x-3)} = -\frac{16x+24}{13(x^2+1)} + \frac{32}{13(2x-3)}$$



Example 1.16

Determine the partial fraction decomposition of the following :

$$\frac{x^3 + 3x^2 + 6x + 7}{x^2 + 3x + 2}$$

Solution

$$\frac{x^3 + 3x^2 + 6x + 7}{x^2 + 3x + 2}$$

Improper Fraction (d<n) so do Long Division to convert it to **Mixed Number**.

Step 1: Improper Fraction

$$\frac{x^3 + 3x^2 + 6x + 7}{x^2 + 3x + 2}$$

$$\begin{array}{r} x \\ x^2+3x+2\,\overline{)\,x^3+3x^2+6x+7} \\ (-)\;\; x^3+3x^2+2x+0 \\ \hline 0x^3+0x^2+4x+7 \end{array}$$

Once you get index lower than 2, stop do Long Division.

Mixed Number from Long Division

A: x, B: $\frac{4x+7}{x^2+3x+2}$

$$x + \frac{4x+7}{x^2+3x+2}$$

B should be a Proper Fraction. So now, lets solve it. Solution for partial fraction is ONLY for fraction, leave A for a while.

Step 2: Linear Factor (LF)

$$\frac{4x+7}{x^2+3x+2} = \frac{4x+7}{(x+1)(x+2)}$$

Step 3 : Partial Fraction Formula

$$\frac{4x+7}{(x+1)(x+2)} = \frac{A}{(x+1)} + \frac{B}{(x+2)}$$

Example 1.16 : Continue

Determine the partial fraction decomposition of the following :

$$\frac{x^3 + 3x^2 + 6x + 7}{x^3 + 3x + 2}$$

Solution

Step 4 : Remove Denominator.

$$4x + 7 = A(x + 2) + B(x + 1)$$

Step 5: Find value of **A** and **B** by defining value of *x*.

$assume \quad x + 2 = 0$

$x = -2$

$4(-2) + 7 = A(-2 + 2) + B(-2 + 1)$

$-1 = -B$

$B = 1$

$assume \quad x + 1 = 0$

$x = -1$

$4(-1) + 7 = A(-1 + 2) + B(-1 + 1)$

$3 = -B$

$B = -3$

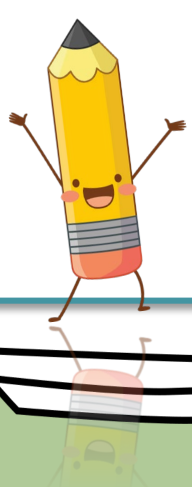
Step 6:

$$\frac{x^3 + 3x^2 + 6x + 7}{x^3 + 3x + 2} = x + \left(\frac{1}{x + 1} + \frac{-3}{x + 2}\right)$$

$$= x + \frac{1}{x + 1} - \frac{3}{x + 2}$$

Remember:-

1. Your **final answer** should in form of **Mixed Number** (product of Long Division).
2. **Solution** for partial fraction applied ONLY for **remainder** of Long Division.



CLICK ME…..
https://youtu.be/vvTODx7Uxws

Test Your Knowledge 1.3

Construct the partial fraction for the following equations:

1. $\dfrac{5x}{x^2-x-2}$

2. $\dfrac{x^2}{(x+1)(x-2)^2}$

3. $\dfrac{2(1+x)}{x(x^2+4)}$

4. $\dfrac{x^3+2x^2-3}{x^2-3x}$

Answer :

1. $\dfrac{5x}{x^2-x-2}=\dfrac{10}{3(x-2)}+\dfrac{5}{3(x+1)}$

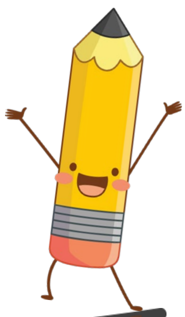
2. $\dfrac{x^2}{(x+1)(x-2)^2}=\dfrac{1}{9(x+1)}+\dfrac{8}{9(x-2)}+\dfrac{4}{3(x-2)^2}$

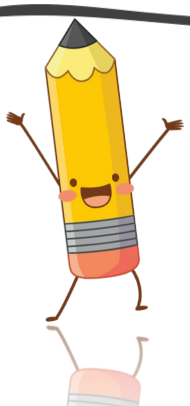
3. $\dfrac{2(1+x)}{x(x^2+4)}=\dfrac{1}{2x}-\dfrac{x}{2(x^2+4)}+\dfrac{2}{(x^2+4)}$

4. $\dfrac{x^3+2x-3}{x^2-3x}=x+3+\dfrac{1}{x}+\dfrac{10}{(x-3)}$

Let's play games…………..

Basic Algebra





Basic Algebra

Quadratic Equation

TRIGONOMETRY

SUBTOPIC :-

2.1 FUNDAMENTAL OF TRIGONOMETRIC FUNCTION

- Introduction
- Quadrants for Trigonometry Ratio
- Special Angle

2.2 TRIGONOMETRY EQUATIONS AND IDENTITIES

- Trigonometric Identities and Formulae

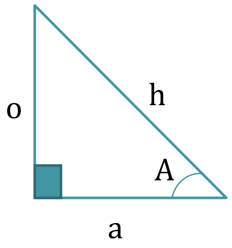
2.3 SINE AND COSINE RULES

- Introduction
- Sine and Cosine Rules

2.1 FUNDAMENTAL OF TRIGONOMETRIC FUNCTIONS

2.1.1 INTRODUCTION

***TRI**gonometry* – study of measurement of sides and angles for **TRIangles**.



hypotenuse – opposite side to right-angled

adjacent – adjacent side to A

opposite – opposite side to A

Theorem of Pythagoras? – a theorem used to find measurement of sides for right-angled triangle.

$$h^2 = o^2 + a^2$$

To solve trigonometry problems, Theorem of Pythagoras frequently used when 2 sides of right-angled are given without any angle.

Tips : Once you draw right-angled triangle, 1st determine your acute angle.

Trigonometric ratio for acute angle:-

1. $sin\ A = \frac{o}{h}$
2. $cos\ A = \frac{a}{h}$
3. $tan\ A = \frac{o}{a}$
4. $cosecant\ A = \frac{1}{sin\ A}$ or $\frac{h}{o}$
5. $secant\ A = \frac{1}{cos\ A}$ or $\frac{h}{a}$
6. $cotangent\ A = \frac{1}{tan\ A}$ or $\frac{a}{o}$

Tips:-

1. **Acute angle** – angle with value range $0^o \leq angle \leq 90^o$.
2. **Cosecant** also can be written as csc instead of cosec
3. **Secant** frequently write as sec.
4. **Cotangent** frequently write as cot.

Example 2.1

1. If $\tan A = \frac{18}{23}$, determine the value of the other five trigonometry ratio.

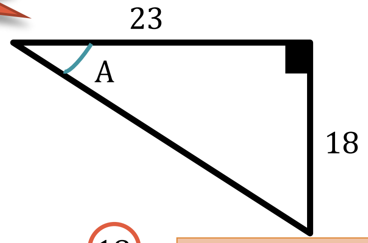
Method 1

Solution

1^{st} : Draw your right-angled triangle.
- Sketch and determine the right-angle then hypotenuse side
- Then, label angle A follow by both adjacent and opposite side.

2^{nd} : Used Theorem of Pythagoras to get value of hypotenuse side:-

$$h^2 = o^2 + a^2$$
$$h = \sqrt{o^2 + a^2}$$
$$h = \sqrt{18^2 + 23^2}$$
$$h = \sqrt{853} \; or \; \approx \; 29.21$$



$$\tan A = \frac{18}{23}$$

Value of **O**pposite side

Value of **A**djacent side

Solution :- continue

3^{rd} : calculate other 5 trigonometry ratio.

Method 1 – without using calculator, use right-angled triangle only.

According to above triangle :-

1. $\sin A = \frac{18}{\sqrt{853}}$
2. $\cos A = \frac{23}{\sqrt{853}}$
3. $\cot A = \frac{23}{18}$
4. $\csc A = \frac{\sqrt{853}}{18}$
5. $\sec A = \frac{\sqrt{853}}{23}$

Tips:-
Refer to previous page for formula of trigonometry ratio.

CLICK ME.....
https://youtu.be/G7ix_M90Bm8

Example 2.1

1. If $\tan A = \frac{18}{23}$, determine the value of the other five trigonometry ratio.

Method 2

Solution

1st : Used calculator to get the value of angle A.

$$\tan A = \frac{18}{23}$$
$$A = \tan^{-1}\left(\frac{18}{23}\right)$$
$$A \approx 38.05^o$$

Solution :- continue

2nd : calculate other 5 trigonometry ratio.

Method 2 – by using calculator, use value of angle A to calculate all trigonometry ratio.

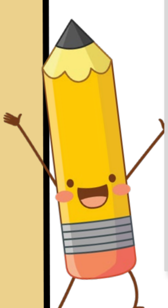
According to above triangle :-

1. $\sin 38.05^o \approx 0.6163$
2. $\cos 38.05^o = 0.7875$
3. $\cot 38.05^o = \frac{1}{\tan 38.05^o} \approx 1.2776$
4. $\csc 38.05^o = \frac{1}{\sin 38.05^o} \approx 1.6225$
5. $\sec 38.05^o = \frac{1}{\cos 38.05^o} \approx 1.2699$

Tips:-

1. Make sure calculator is in **D**egree Mode.
2. The **Value** must as follow:

- sine and cosine $(-1 \leq value \leq 1)$
- cosecant and secant can be either
  - $-1 < value$
  - $value > 1$



CLICK ME…..
https://youtu.be/5CC4xd5A10A

Example 2.2

2. If $\sec\theta = \frac{25}{23}$, determine the value of the other five trigonometry ratio.

Method 1

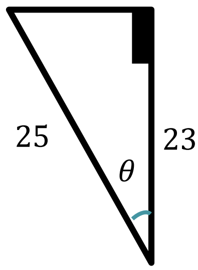
Solution

1st : Draw your right-angled triangle.

- Sketch and determine a right-angle then hypotenuse side
- Then, label angle θ follow by both adjacent and opposite side.

2nd : Used Theorem of Pythagoras to get value of opposite side:-

$$h^2 = o^2 + a^2$$
$$o = \sqrt{h^2 - a^2}$$
$$o = \sqrt{25^2 - 23^2}$$
$$o = \sqrt{96} \ or \ \approx \ 9.80$$



$\sec\theta = \frac{25}{23}$

Value of **H**ypotenuse side (25)

Value of **A**djacent side (23)

Solution :- continue

3rd : calculate other 5 trigonometry ratio.

Method 1 – without using calculator, use right-angled triangle only.

According to above triangle :-

1. $\sin\theta = \frac{\sqrt{96}}{25}$
2. $\cos\theta = \frac{23}{25}$
3. $\tan\theta = \frac{\sqrt{96}}{23}$
4. $\csc\theta = \frac{25}{\sqrt{96}}$
5. $\cot\theta = \frac{23}{\sqrt{96}}$

Tips:-

1. Always use at least 2 decimal places (2 d.p)
2. Instead of decimal values, root or fraction value is more accurate.

CLICK ME…..
https://youtu.be/NeUQMWEFm28

Example 2.2

2. If $\sec\theta = \frac{25}{23}$, determine the value of the other five trigonometry ratio.

Solution

Method 2

1st : Used trigonometry ratio to get the value of angle θ.

$$\sec\theta = \frac{25}{23}$$

$$\frac{1}{\cos\theta} = \frac{25}{23}$$

$$23 = 25\cos\theta$$

$$\cos\theta = \frac{23}{25}$$

$$\theta = \cos^{-1}\left(\frac{23}{25}\right)$$

$$\theta \approx 23.07^o$$

Tips:-
Value of $\tan 90^o = \infty$, not math error.

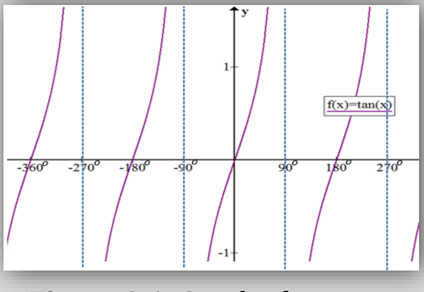


Figure 2.1: Graph of tangent

CLICK ME.....
https://youtu.be/noQcaLwVVqM

Solution :- continue

2nd : calculate other 5 trigonometry ratio.
Method 2 – by using calculator, use value of angle, θ to calculate all trigonometry ratio.

According to above triangle :-

1. $\sin 23.07^o \approx 0.3919$
2. $\cos 23.07^o \approx 0.9200$
3. $\tan 23.07^o \approx 0.4259$
4. $\csc 23.07^o = \frac{1}{\sin 23.07^o} \approx 2.5520$
5. $\cot 23.07^o = \frac{1}{\tan 23.07^o} \approx 2.3479$

Test Your Knowledge 2.1

1. If $\sin A = \frac{18}{23}$, determine the value of $\tan A$ and $\sec A$ without using calculator. (State your answer in Fraction Form.)

2. If $\cot A = \frac{29}{40}$, determine the value of $\sin B$ and $\cos B$. (State your answer in Fraction Form.)

3. Find the acute angle of $\csc D = \frac{35}{30}$ in degrees and minutes.

4. A man drive 10km due north and then 15km due west. Another woman, starting at the same time with the same speed with the man, drive 20km due west and then 5km due north. Find the distance between two of them.

5. A straight bamboo stick 10 m long is placed against a perpendicular tree with its foot 4.5 m from the tree. How far up to the tree does the bamboo stick can reach? (state your answer in 3 decimal places.)

Answer :

1. $\tan A = \frac{18}{\sqrt{205}}$ *and* $\sec A = \frac{23}{\sqrt{205}}$
2. $\sin B = \frac{40}{\sqrt{2441}}$ *and* $\cos B = \frac{29}{\sqrt{2441}}$
3. $D = 28^o 59' 50''$
4. Distance $= 7.07\ km$
5. Only 8.930 metre

2.1 FUNDAMENTAL OF TRIGONOMETRIC FUNCTIONS

QUADRANTS FOR TRIGONOMETRY RATIO

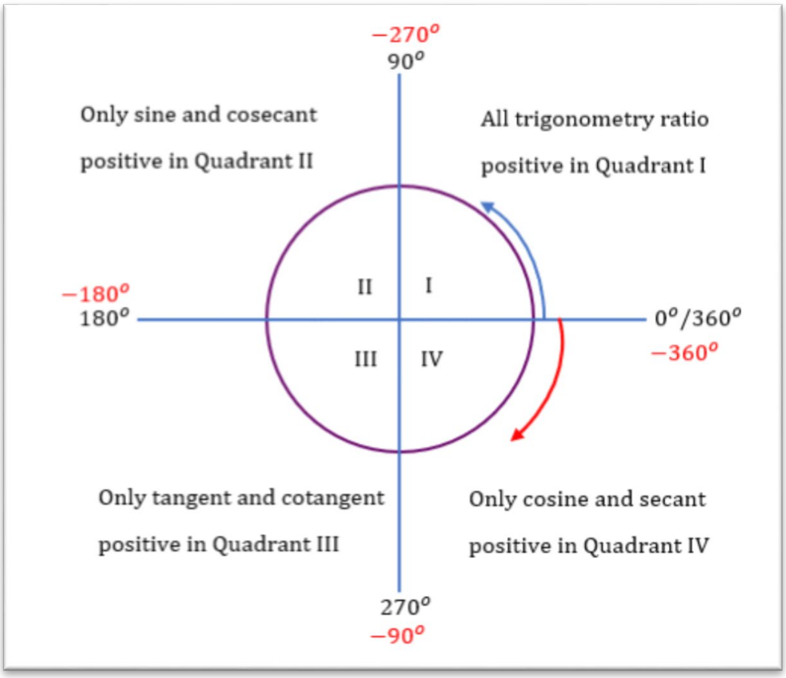


Figure 2.2: Trigonometric Function's Quadrants.

Read the following explanation carefully:-

- → indicate **positive** angle movement (anti-clockwise) and → indicate **negative** angle movement (clockwise).
- Each trigonometry ratio has **2 positive** quadrants and **2 negative** quadrants.
- Each quadrant's value is 90^o representing value of acute angle.
- **Reference angle (RA)** (always start from x-axis or axis of $0^o/180^o$) is an acute angle. Therefore, to calculate reference angle refer formula below :-

Quadrant I; RA = θ	Quadrant II; RA = $180^o - \theta$	Quadrant III; RA = $\theta - 180^o$	Quadrant IV; RA = $360^o - \theta$

Example 2.3

State the following trigonometric functions either positive or negative and in which quadrant.

i. $\tan 45^o$ ii. $\cos 120^o$ iii. $\sin 270^o$ iv. $\sec 410^o$ v. $\csc -210^o$

Solution

i. $\tan 45^o$

- Quadrant I – $(0^o \leq angle \leq 90^o)$
- $\tan 45^o$ is positive – All trigonometry ratio are positive in Quadrant I.

ii. $\cos 120^o$

- Quadrant II – $(90^o \leq angle \leq 180^o)$
- $\cos 120^o$ is negative – ONLY sine and cosecant are positive in Quadrant II.

iii. $\sin 270^o$

- Quadrant III and Quadrant IV - 270^o is border value for Quadrant III and Quadrant IV. (you may refer to Figure 2.2)
- $\sin 270^o$ is negative – sine is negative in BOTH Quadrant III and Quadrant IV.

Tips:-

1. For any angle which is greater than 360^o $(angle > 360^o)$ or less than -360^o $(angle < -360^o)$, you can **add** or **subtract** with 360^o until you can get a value less that $\pm 360^o$.
2. For any negatives angle, you can find its positive angle by adding with 360^o.

Solution

iv. $\sec 410^o$

- Quadrant I $(360^o \leq angle \leq 450^o)$
- $\sec 410^o$ is positive All trigonometry ratio are positive in Quadrant I.

v. $\csc(-210^o)$

- Quadrant II $(-180^o \leq angle \leq -270^o)$
- $\csc(-210^o)$ is positive Sine and cosecant positive in Quadrant II.

CLICK ME…..
https://youtu.be/tEAwM0tfGLM

2.1 FUNDAMENTAL OF TRIGONOMETRIC FUNCTIONS

Special Angle

1. Only **3** angle are special angle :- 30^o, 45^o and 60^o.
2. All special angles are **reference angle**.
3. Most of the question for special angle frequently need to be solved without using calculator.

Special Angle (α)	30^o	45^o	60^o
$\sin\alpha$	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
$\cos\alpha$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
$\tan\alpha$	$\frac{1}{\sqrt{3}}$	$\frac{1}{1}=1$	$\frac{\sqrt{3}}{1}=\sqrt{3}$
$\csc\alpha$	$\frac{2}{1}=2$	$\frac{\sqrt{2}}{1}=\sqrt{2}$	$\frac{2}{\sqrt{3}}$
$\sec\alpha$	$\frac{2}{\sqrt{3}}$	$\frac{\sqrt{2}}{1}=\sqrt{2}$	$\frac{2}{1}=2$
$\cot\alpha$	$\frac{\sqrt{3}}{1}=\sqrt{3}$	$\frac{1}{1}=1$	$\frac{1}{\sqrt{3}}$

Table 2.1 : Value of Special Angle for Trigonometric Function.

4. According to Table 2.1, values circled show exact value of the trigonometric functions, while value in fractions show the value of right-angled side that being used accordingly.
5. Final answer for trigonometric with special angle must in fraction and/or with roots included values of special angle use in solutions. Avoid decimal values.[Refer to Table 2.1]

Example 2.4

Find the value for each of the following without using calculator :-

i. $\tan 45^o$ ii. $\cos 120^o$ iii. $\sin 210^o$ iv. $\sec 405^o$ v. $\csc -210^o$

Solution

i. $\tan 45^o$
- 45^o is reference angle
- Quadrant I : tangent is positive

  $\tan 45^o = 1$

ii. $\cos 120^o$
- Reference angle for 120^o :-

  Quadrant II;

  $RA = 180^o - 120^o$

  $RA = 60^o$

- Quadrant II : cosine is negative.

  $\cos 120^o = -\cos 60^o$

  $= -\frac{1}{2}$

iii. $\sin 210^o$
- Reference angle for 210^o :-

  Quadrant III;

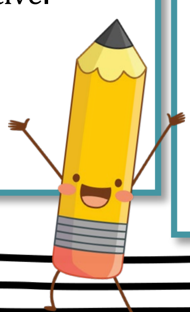
  $RA = 210^o - 180^o$

  $RA = 30^o$

- Quadrant III : sine is negative.

  $\sin 210^o = -\sin 30^o$

  $= -\frac{1}{2}$



Tips:-
To solve Special Angle, always determine its Reference Angle according to the Quadrants.

Solution

iv. $\sec 405^o$
- Reference angle for 405^o :-

  Quadrant I;

  $RA = 405^o - 360^o$

  $RA = 45^o$

- Quadrant I : sec is positive.

  $\sec 405^o = \sec 45^o$

  $= \sqrt{2}$

v. $\csc -210^o$
- positive angle for -210^o :-

  $-210^o = 360^o - 210^o$

  $= 150^o$

  Quadrant II;

  $RA = 180^o - 150^o$

  $RA = 30^o$

- Quadrant II : sine is positive.

  $\csc -210^o = \csc 30^o$

  $= 2$

CLICK ME…..
https://youtu.be/7j1aBm_iUXc

Example 2.5

2. Find the value for each of the following without using calculator:-

i. $\tan 45^o + \cos 120^o$ ii. $\sin 210^o - \cot 240^o$ iii. $\sec 405^o - \csc 240^o$

Solution

i. $\tan 45^o + \cos 120^o$

$$= \tan 45^o + [-\cos(180^o - 120^o)]$$
$$= \tan 45^o + [-\cos 60^o]$$
$$= 1 - \frac{1}{2}$$
$$= \frac{1}{2}$$

ii. $\sin 210^o - \cot 240^o$

$$= [-\sin(210^o - 180^o) - \cot(240^o - 180^o)]$$
$$= -\sin 30^o - \cot 60^o$$
$$= -\frac{1}{2} - \frac{1}{\sqrt{3}}$$
$$= -\frac{\sqrt{3}}{2\sqrt{3}} - \frac{2}{2\sqrt{3}}$$
$$= \frac{-\sqrt{3} - 2}{2\sqrt{3}}$$
$$= -\frac{\sqrt{3} + 2}{2\sqrt{3}}$$

Tips:-

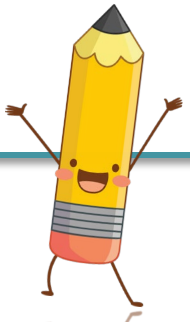
For operations of Special Angle, make sure :-

1. Determine the quadrant for each of the trigonometry function.
2. Determine either trigonometric function is positive or negative in the quadrant.
3. Find its Reference Angle.
4. Substitute the value and complete the solution.

Solution

iii. $\sec 405^o - \csc 240^o$

$$= \sec(405^o - 360^o) - [-\csc(240^o - 180^o)]$$
$$= \sec 45^o + \csc 60^o$$
$$= \sqrt{2} + \frac{2}{\sqrt{3}}$$
$$= \frac{\sqrt{2}\sqrt{3}}{\sqrt{3}} + \frac{2}{\sqrt{3}}$$
$$= \frac{\sqrt{6}}{\sqrt{3}} + \frac{2}{\sqrt{3}}$$
$$= \frac{\sqrt{6} + 3}{\sqrt{3}}$$



CLICK ME…..
https://youtu.be/GSMTr91WF-M

Test Your Knowledge 2.2

Find the value for each of the following without using calculator:-

1. $\tan 30^o + \cos 60^o$
2. $\sin 330^o - \cot 45^o$
3. $\csc 405^o - \sec 240^o$
4. $\tan -45^o + \sin\ -60^o$
5. $\cos 870^o - \cot 570^o$

Answer :

1. $\dfrac{2+\sqrt{3}}{2\sqrt{3}}$
2. $-\dfrac{3}{2}$
3. $\sqrt{2}+2$
4. $-\dfrac{2+\sqrt{3}}{2}$
5. $\dfrac{\sqrt{3}+2\sqrt{3}}{2}$

2.2 TRIGONOMETRIC EQUATIONS & IDENTITIES

TRIGONOMETRY IDENTITIES AND FORMULAE

Basic Identities

1. $\sin^2\alpha + \cos^2\alpha = 1$
2. $1 + \tan^2\alpha = \sec^2\alpha$
3. $1 + \cot^2\alpha = \csc^2\alpha$

Quotient Identities

1. $\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$
2. $\cot\alpha = \frac{\cos\alpha}{\sin\alpha}$

Double Angle Formulae

1. $\sin 2\alpha = 2\sin\alpha\cos\alpha$
2. $\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha}$
3. $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$
   $= 2\cos^2\alpha - 1$
   $= 1 - 2\sin^2\alpha$

Compound Angle Formulae : Addition Angle

1. $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$
2. $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$
3. $\tan(\alpha+\beta) = \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

Compound Angle Formulae : Subtraction Angle

1. $\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$
2. $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$
3. $\tan(\alpha-\beta) = \frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$

Do not expand compound angle, this is not algebraic expression, this is trigonometric function.

Example : $\sin(\alpha-\beta) \neq \sin\alpha - \sin\beta$

2.2 TRIGONOMETRIC EQUATIONS & IDENTITIES

Why we need trigonometric identities and formulae?

1. To solve trigonometric equations, the equations should meet 2 conditions i.e. have **SAME** trigonometry function and **SAME** angle.

> **Quadratic** and **Linear** of Trigonometric Equations with the same trigonometric ratio are considered as the same Trigonometric Function.
> Example : $\mathbf{sin}^2\ A$ and $\mathbf{sin}\ A$, there are still function of sine but with different index.

2. Therefore, we need trigonometry Identities and Formulae just to convert existing/given equation to meet point number 1.

Extra Note:-

a) Basic Identities also known as Pythagoras Identity.

b) Quotient Identities is used either to convert tangent into combination of sine and cosine functions or vice versa.

Example 2.6

Find value of α for each of the following equations which satisfy $0^o \leq \alpha \leq 360^o$;

i. $\sin\alpha = 0.5$ ii. $\cos\alpha = -0.1234$ iii. $\tan 2\alpha = 3.1203$

iv. $\csc\frac{\alpha}{2} = 2.431$

Solution

i. $\sin\alpha = +0.5$

Check the value either positive or negative to determine the angle's quadrant.

$\alpha = \sin^{-1} 0.5$

$\alpha = 30^o$

Reference Angle

Sine positive in Quadrant I and Quadrant II

Quadrant I :- $\alpha = 30^o$

Quadrant II :- $\alpha = 180^o - 30^o$

$\alpha = 150^o$

$\alpha = 30^o$ *and* 150^o.

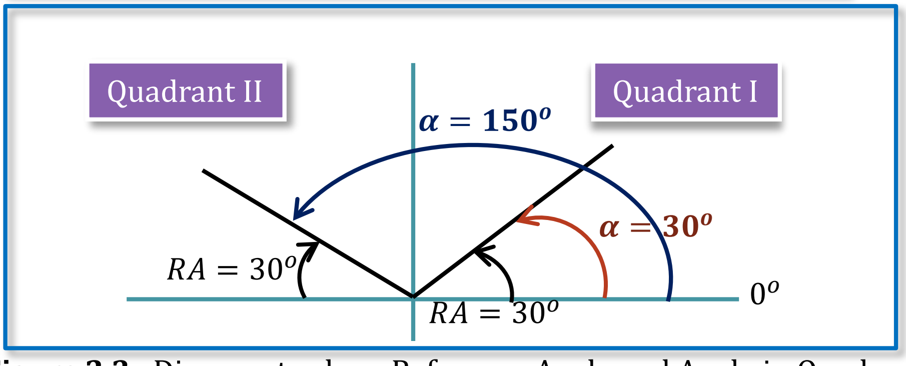


Figure 2.3 : Diagram to show Reference Angle and Angle in Quadrants.

Tips:-

1. Reference Angle is an acute angle and exist in quadrant.
2. Angle for trigonometric equation should be calculate start from 0^o.
3. You may check your answer by using calculator.
   Example: $\sin 30^o$ and $\sin 150^o$ is equal to 0.5.

Example 2.6 :- continue

Find value of α for each of the following equations which satisfy $0^o \leq \alpha \leq 360^o$;

i. $\sin \alpha = 0.5$ ii. $\cos \alpha = -0.1234$ iii. $\tan 2\alpha = 3.1203$

iv. $\csc \frac{\alpha}{2} = 2.431$

Solution

ii. $\cos \alpha = -0.1234$

$\alpha = \cos^{-1} 0.1234$

$\alpha = 82.91^o$

Cosine negative in Quadrant II and Quadrant III.

Quadrant II :-

$\alpha = 180^o - 82.91^o$

$\alpha = 97.09^o$

Quadrant III :-

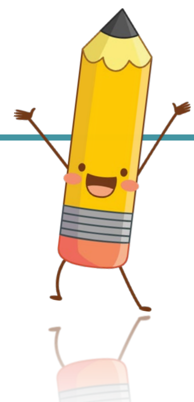
$\alpha = 180^o + 82.91^o$

$\alpha = 262.91^o$

$\alpha = 97.09^o$ and 262.91^o.

Tips:-

- **Ignore** negative sign $(-)$ when you calculate inverse trigonometry functions.
- Positive and negative just to indicate quadrants of angle.
- **Fail** to ignore, you may get reference angle $> -90^o$.



Solution

ii. $\tan 2\alpha = 3.1203$

$2\alpha = \tan^{-1} 3.1203$

$2\alpha = 72.23^o$

Tangent positive in Quadrant I and Quadrant III.

Quadrant I :-

$2\alpha = 72.23^o$ $\quad$ $2\alpha = 360^o + 72.23^o = 432.23^o$

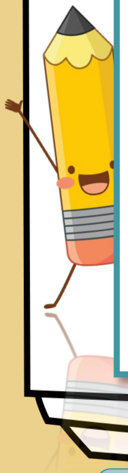
$\alpha = 36.115^o$ $\quad$ $\alpha = 216.115^o$

Quadrant III :-

$2\alpha = 180^o + 72.23^o = 252.23^o$ $\quad$ $2\alpha = 360^o + 252.23^o = 612.23^o$

$\alpha = 126.115^o$ $\quad$ $\alpha = 306.115^o$

$\alpha = 36.115^o, 126.115^o, 216.115^o$ and 306.115^o.



CLICK ME…..
https://youtu.be/KY_nUQXJS2c

Example 2.6 :- continue

Find value of α for each of the following equations which satisfy $0^o \leq \alpha \leq 360^o$;

i. $\sin \alpha = 0.5$ ii. $\cos \alpha = -0.1234$ iii. $\tan 2\alpha = 3.1203$

iv. $\csc \frac{\alpha}{2} = 2.431$

Solution

iv. $\csc \frac{\alpha}{2} = 2.431$

$$\frac{1}{\sin \frac{\alpha}{2}} = 2.431$$

$$\frac{1}{2.431} = \sin \frac{\alpha}{2}$$

$$0.4114 = \sin \frac{\alpha}{2}$$

$$\frac{\alpha}{2} = \sin^{-1} 0.4114$$

$$\frac{\alpha}{2} = 24.29^o$$

Sine positive in Quadrant I and Quadrant II.

Quadrant I :-

$$\frac{\alpha}{2} = 24.29^o$$

$$\alpha = 48.58^o$$

Quadrant II :-

$$\frac{\alpha}{2} = 180^o - 24.29^o = 155.71^o$$

$$\alpha = 311.42^o$$

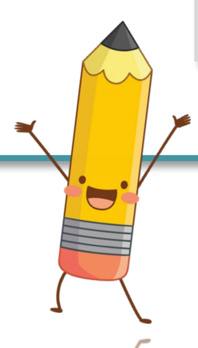
$\alpha = 48.28^o$ and 311.42^o.

Tips:-

$0^o \leq \alpha \leq 360^o$

Always check the range as below:

- $\mathbf{0^o \leq \alpha \leq 360^o}$ means:
  - i. α is **1** complete rotation (4 quadrants)
  - ii. 2α is **2** complete rotation (each quadrant will have 2 values.)
  - *iii.* $\frac{\alpha}{2}$ is **half rotation** (only quadrant I and II)
- $\mathbf{0^o \leq \alpha \leq 180^o}$ means **ONLY** for Quadrant I and II.
- $\mathbf{0^o \leq \alpha \leq 720^o}$ means **2 completed rotation**.
- $\mathbf{180^o \leq \alpha \leq 360^o}$ means **ONLY** Quadrant III and Quadrant IV.



Test Your Knowledge 2.3

Find value of θ for each of the following equations which satisfy $0^o \leq \theta \leq 360^o$;

1. $\sin\theta = 0.4756$
2. $\tan\theta = -3.0204$
3. $\cos 2\theta = 0.1203$
4. $\sec\frac{\theta}{2} = -2.431$

Answer :

1. $\theta = 28.40^o\ and\ 151.60^o$
2. $\theta = 108.32^o\ and\ 288.32^o$
3. $\theta = 41.545^o, 138.455^o, 221.545^o\ and\ 318.455^o$
4. $\theta = 228.52^o$

Example 2.7

Find value of β for each of the following equations which satisfy $0^o \leq \beta \leq 360^o$;

i. $\tan^2\beta + 3\tan\beta - 5 = 0$ ii. $6\sin^2\beta - \sin\beta = 2$

Solution

i. $\tan^2\beta + 3\tan\beta - 5 = 0$

$\tan\beta = 1.1926$ and $\tan\beta = -4.1926$

(Note :- *I used calculator to solve this Quadratic Trigonometric Equation*)

For $\tan\beta = 1.1926$

$\beta = \tan^{-1} 1.1926$

$\beta = 50.02^o$ ← (*value of reference angle*)

Tangent Positive in Quadrant I and Quadrant III.

Quadrant I :-

$\beta = 50.02^o$

Quadrant III :-

$\beta = 180^o + 50.02^o$

$\beta = 230.02^o$

For $\tan\beta = -4.1926$ ← (*sign to show quadrants of reference angle exist*)

$\beta = \tan^{-1} 4.1926$

$\beta = 76.58^o$

Tangent Negative in Quadrant II and Quadrant IV.

Quadrant II :-

$\beta = 180^o - 76.58^o$

$\beta = 103.42^o$

Quadrant IV :-

$\beta = 360^o - 76.58^o$

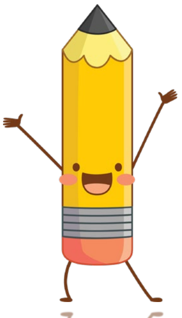
$\beta = 283.42^o$

$\beta = 50.02^o, 103.42^o, 230.02^o$ and 283.42^o.

Tips:-

$\tan^2\beta = (\tan\beta)^2$

- If trigonometric function with index of 2, please **apply Quadratics Equation** solution.
- **Linear** trigonometric equation, only have **1** reference angle.
- **Quadratic** trigonometric equation have **2** reference angle.



CLICK ME…..
https://youtu.be/L6aAkOyulLA

Example 2.7 :- continue

Find value of β for each of the following equations which satisfy $0^o \leq \beta \leq 360^o$;

i. $\tan^2\beta + 3\tan\beta - 5 = 0$ ii. $6\sin^2\beta - \sin\beta = 2$

Solution

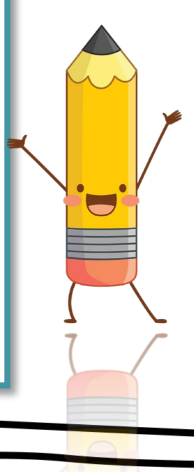
ii. $6\sin^2\beta - \sin\beta = 2$

$$6\sin^2\beta - \sin\beta - 2 = 0$$
$$(3\sin\beta - 2)(2\sin\beta + 1) = 0$$
$$\sin\beta = \frac{2}{3} \text{ and } \sin\beta = -\frac{1}{2}$$

For $\sin\beta = \frac{2}{3}$

$$= \sin^{-1}\frac{2}{3}$$

$\beta = 41.81^o, 138.19^o, 210^o$ and 330^o.



CLICK ME…..
https://youtu.be/AoO2f-XEsnE

Example 2.8

Find value of θ for each of the following equations which satisfy $0^o \leq \theta \leq 360^o$;

i. $3\sin^2\theta + 4\cos\theta - 2 = 0$ ii. $2\csc^2\theta - 5\tan\theta = 3$

Solution

i. $3\sin^2\theta + 4\cos\theta - 2 = 0$

(Note :- *Use Basic Identities to convert sine to cosine.*
WHY?
It is much easier to convert quadratic trigonometric function compare to convert linear trigonometric function.)

Choose $\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta$

$\therefore\ 3(1 - \cos^2\theta) + 4\cos\theta - 2 = 0$

$3 - 3\cos^2\theta + 4\cos\theta - 2 = 0$

$-3\cos^2\theta + 4\cos\theta + 1 = 0$

$\cos\theta = -0.2153$ and $\cos\theta = 1.5486$

For $\cos\theta = -0.2153$

$\theta = \cos^{-1} 0.2153$

$\theta = 77.57^o$

Cosine Negative in Quadrant II and Quadrant III.

Quadrant II :-

$\theta = 180^o - 77.57^o$

$\theta = 102.43^o$

Quadrant III :-

$\theta = 180^o + 77.57^o$

$\theta = 257.57^o$

For $\cos\theta = 1.5486$

$\theta = \cos^{-1} 1.5486$

$\theta =$ **NO SOLUTION**

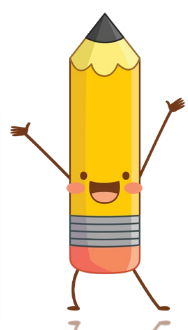
$\theta = 102.43^o$ and 257.57^o.

NEVER state your answer as MATH ERROR. It is **unacceptable** answer.

Tips:-

To **SOLVE** trigonometric Equation, the equation must meet 2 conditions:-

- Have **ONLY 1** trigonometric function.
- Have **THE SAME** angle



CLICK ME…..
https://youtu.be/PGykmnj1Nck

Example 2.8 :- continue

Find value of θ for each of the following equations which satisfy $0^o \leq \theta \leq 360^o$;

i. $3\sin^2\theta + 4\cos\theta - 2 = 0$

ii. $2\csc^2\theta - 5\cot\theta = 3$

Solution

ii. $2\csc^2\theta - 5\cot\theta = 3$

$2\csc^2\theta - 5\cot\theta - 3 = 0$

(Note :- *Use Basic Identities to convert cosecant to cotangent.*)

Choose $1 + \cot^2\theta = \csc^2\theta$

$\csc^2\theta = 1 + \cot^2\theta$

$\therefore 2(1 + \cot^2\theta) - 5\cot\theta - 3 = 0$

$2 + 2\cot^2\theta - 5\cot\theta - 3 = 0$

$2\cot^2\theta - 5\cot\theta - 1 = 0$

$$\frac{2}{\tan^2\theta} - \frac{5}{\tan\theta} - 1 = 0$$

$$\frac{2}{\tan^2\theta} - \frac{5\tan\theta}{\tan^2\theta} - \frac{\tan^2\theta}{\tan^2\theta} = 0$$

$$\frac{2 - 5\tan\theta - \tan^2\theta}{\tan^2\theta} = 0$$

$-\tan^2\theta - 5\tan\theta + 2 = 0$

$\tan\theta = -5.3723$ and $\tan\theta = 0.3723$

For $\tan\theta = -5.3723$

$\theta = \tan^{-1} 5.3723$

$\theta = 79.46^o$

Tangent Negative in Quadrant II and Quadrant IV.

Quadrant II :-

$\theta = 180^o - 79.46^o$

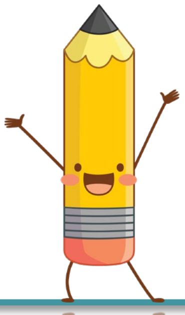
$\theta = 100.54^o$

Quadrant IV :-

$\theta = 360^o - 79.46^o$

$\theta = 280.54^o$

Solution continue on the next page.



CLICK ME…..
https://youtu.be/hr-FysDinwg

Example 2.8 :- continue

Find value of θ for each of the following equations which satisfy $0^o \leq \theta \leq 360^o$;

i. $3\sin^2\theta + 4\cos\theta - 2 = 0$ ii. $2\csc^2\theta - 5\cot\theta = 3$

Solution ii continue.

$\tan\theta = -5.3723$ and $\tan\theta = 0.3723$

For $\tan\theta = 0.3723$

$\theta = \tan^{-1} 0.3723$

$\theta = 20.42^o$

Tangent positive in Quadrant I and Quadrant III.

Quadrant I :-

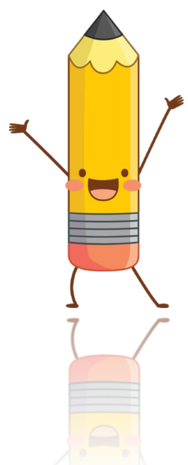
$\theta = 20.42^o$

Quadrant III :-

$\theta = 180^o + 20.42^o$

$\theta = 200.42^o$

$\theta = 20.42^o, 100.54^o, 200.42^o$ and 280.54^o.



Test Your Knowledge 2.4

Find value of θ for each of the following equations which satisfy $0^o \leq \theta \leq 360^o$: –

1. $5\sin^2\theta - 2\sin\theta - 2 = 0$
2. $2\tan^2\theta - 3\tan\theta = 5$
3. $4\sin^2\theta = 5\cos\theta - 1$
4. $8 - 2\cot^2\theta = 4\,csc^2\,\theta - 3$

Answer :

1. $\theta = 59.69^o,\ 120.31^o, 207.60^o\ and\ 332.40^o$
2. $\theta = 68.20^o,\ 135^o, 248.20^o\ and\ 315^o$
3. $\theta = 49.01^o\ and\ 310.99^o$
4. $\theta = 42.79^o,\ 137.21^o, 222.79^o\ and\ 317.21^o$

Example 2.9

Find value of A for each of the following equations which satisfy $0^o \leq A \leq 360^o$;

i. $2\cos 2A - 5\sin A = 3$ ii. $5\cos^2 A + 2\sin 2A = 0$

Solution

Double Angle (2A) versus **Single Angle (A)**

i. $2\cos 2A - 5\sin A = 3$

(Note :- *Use Double Angle Formulae to convert cosine to sine.*)

Choose $\cos 2A = 1 - 2sin^2 A$

$2(1 - 2sin^2 A) - 5\sin A = 3$

$2 - 4sin^2 A - 5\sin A = 3$

$-4sin^2 A - 5\sin A - 1 = 0$

Now, only sine functions in the question.

$\sin A = -1$ and $\sin A = -0.25$

(Note :- *I used calculator to solve this Quadratic Trigonometric Equation*)

For $\sin A = -1$

$A = \sin^{-1} 1$

$A = 90^o$

Sine Negative in Quadrant III and Quadrant IV.

Quadrant III :-

$A = 180^o + 90^o$

$A = 270^o$

Quadrant IV :-

$A = 360^o - 90^o$

$A = 270^o$

For $\sin A = -0.25$

$A = \sin^{-1} 0.25$

$A = 14.48^o$

Sine Negative in Quadrant III and Quadrant IV.

Quadrant III :-

$A = 180^o + 14.48^o$

$A = 194.48^o$

Quadrant IV :-

$A = 360^o - 14.48^o$

$A = 345.52^o$

$A = 194.48^o, 270^o$ and 345.52^o.

Tips:-

- Used **Double Angle Formulae** for a question consists of *combination Double angle and Single angle.*
- Convert trigonometry with double angle to trigonometry with single angle.
- Choose a formulae that will give you only 1 trigonometry function in a question.

Double Angle Formulae for cosine:-

$\mathbf{\cos 2\alpha} = cos^2\alpha - sin^2\alpha$

$= 2cos^2\alpha - 1$

$\mathbf{= 1 - 2sin^2\alpha}$

Choose $\cos 2A = 1 - 2sin^2 A$ to solve the question.

CLICK ME…..
https://youtu.be/rFiomqi0Lg0

Example 2.9 : Continue

Find value of A for each of the following equations which satisfy $0^o \leq A \leq 360^o$;

i. $2\cos 2A - 5\sin A = 3$ ii. $5\cos^2 A + 2\sin 2A = 0$

Solution

ii. $5\cos^2 A + 2\sin 2A = 0$

(Note :- *Use Double Angle Formulae to convert Double Angle to Single Angle.*)

Choose $\sin 2A = 2\sin A\cos A$

$$5\cos^2 A + 2(2\sin A\cos A) = 0$$

$$5\cos^2 A + \boxed{4\sin A\cos A} = 0$$

We have no choice for this, but do not worry. I will use Quotient Law later to solve it.

$$\cos A\ (5\cos A + 4\sin A) = 0$$

$\cos A = 0$ and $\mathbf{5\cos A + 4\sin A = 0}$

For $\cos A = 0$

$A = \cos^{-1} 0$

$A = 90^o$

0 is neutral value. Easier by refer to the graph of cosine to get the value of cosine = 0.

While for quadrants, you must calculate for all quadrants.

Quadrant I :-

$A = 90^o$

$A = 90^o$

Quadrant II :-

$A = 180^o - 90^o$

$A = 90^o$

Quadrant III :-

$A = 180^o + 90^o$

$A = 270^o$

Quadrant IV :-

$A = 360^o - 90^o$

$A = 270^o$

Solution continue on the next page.

Tips:-

Refer to the graph of cosine, identify the value for *cosine = 0* and get the value of the angle. (refer to the circle)

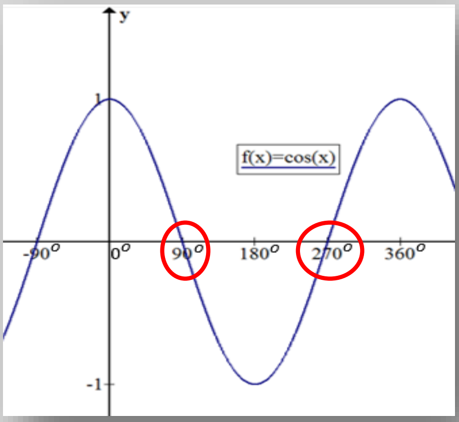


Figure 2.4: Graph of cosine

Double Angle Formulae for sine:-

$$\sin 2\alpha = 2\sin\alpha\cos\alpha$$

I have no options to choose, so just use it.

Example 2.9 : Continue

Find value of A for each of the following equations which satisfy $0^o \leq A \leq 360^o$;

i. $2\cos 2A - 5\sin A = 3$ ii. $5\cos^2 A + 2\sin 2A = 0$

Solution ii continue.

For $\boldsymbol{5\cos A + 4\sin A = 0}$

$5\cos A = -4\sin A$

$$-\frac{5}{4} = \frac{\sin A}{\cos A}$$

$$-\frac{5}{4} = \tan A$$

$$\tan A = -\frac{5}{4}$$

$$A = \tan^{-1}\frac{5}{4}$$

$$A = 51.34^o$$

Tangent Negative in Quadrant II and Quadrant IV.

Quadrant II :-

$A = 180^o - 51.34^o$

$A = 128.66^o$

Quadrant IV :-

$A = 360^o - 51.34^o$

$A = 308.66^o$

$A = 90^o, 128.66^o, 270^o$ and 308.66^o.

Used **Quotient Law** to simplify the trigonometry equation:-

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$$

Tips:-

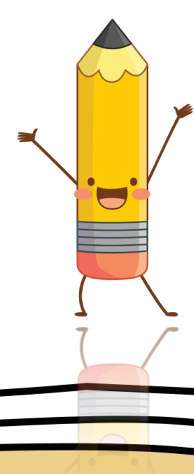
If we need to use **Quotient Law**, please use

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$$

Avoid using

$$\cot\alpha = \frac{\cos\alpha}{\sin\alpha}$$

To avoid longer steps



Test Your Knowledge 2.5

Find value of θ for each of the following equations which satisfy $0^o \leq \theta \leq 360^o: -$

1. $3\cos 2\theta = 4\cos\theta + 5$
2. $2\cos 2\theta - 5\,sin^2\theta = 4\sin\theta - 3$
3. $6\cos^2\theta = 5\sin 2\theta$

Answer :

1. $\theta = 150.28^o\ and\ 209.72^o$
2. $\theta = 10.28^o, 169.72^o, 218.52^o\ and\ 321.48^o$
3. $\theta = 30.96^o, 90^o, 210.96^o\ and\ 270^o$

Example 2.10

Find value of B for each of the following equations which satisfy $0^o \leq B \leq 360^o$;

i. $\sin(50^o + B) = 0$ ii. $\tan(B - 35^o) = 4$

Solution

i. $\sin(50^o + B) = 0$

$\sin\,(50^o + B) = \sin 50^o \cos B + \cos 50^o \sin B$

Therefore;

$\sin 50^o \cos B + \cos 50^o \sin B = 0$

$0.7660 \cos B + 0.6428 \sin B = 0$

$0.7660 \cos B = -0.6428 \sin B$

$\frac{0.7660}{-0.6428} = \frac{\sin B}{\cos B}$

$-1.1917 = \tan B$

$\tan B = -1.1917$

For $\tan B = -1.1917$

$B = \tan^{-1} 1.1917$

$B = 50.00^o$

Tangent Negative in Quadrant II and Quadrant IV.

Quadrant II :-	Quadrant IV :-
$B = 180^o - 50^o$	$B = 360^o - 50^o$
$B = 130^o$	$B = 310^o$

$B = 130^o$ and 360^o.

Tips:-

DO NOT expand $\mathbf{\sin(50^o + B)}$

$\boldsymbol{\sin(50^o + B) \neq \sin 50^\circ + \sin B}$

Because *sin* is the main function with $\mathbf{(50^o + B)}$ is the angle of it

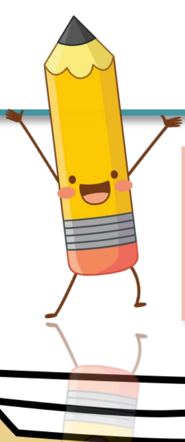
Check your answer:

Used your calculator and check

$\sin(50^o + \mathbf{130^o}) = 0$

$\sin(180^o) = 0$

Do you get $0 = 0$?



Compound Angle Formulae for sine:-

$\boldsymbol{\sin\,(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta}$

CLICK ME…..

https://youtu.be/-kbMbqH3gr0

Example 2.10 : Continue

Find value of B for each of the following equations which satisfy $0^o \leq B \leq 360^o$;

i. $\sin(50^o + B) = 0$ ii. $\tan(B - 35^o) = 4$

Solution

ii. $\tan(B - 35^o) = 4$

$$\tan(B - 35^o) = \frac{\tan B - \tan 35^o}{1 + \tan B \tan 35^o}$$

Therefore;

$$\frac{\tan B - \tan 35^o}{1 + \tan B \tan 35^o} = 4$$

$\tan B - \tan 35^o = 4(1 + \tan B \tan 35^o)$

$\tan B - \tan 35^o = 4 + 4 \tan B \tan 35^o$

$\tan B - 0.7002 = 4 + 2.8008 \tan B$

$\tan B - 2.8008 \tan B = 4 + 0.7002$

$-1.8008 \tan B = 4.7002$

$$\tan B = \frac{4.7002}{-1.8008}$$

$\tan B = -2.6101$

For $\tan B = -2.6101$

$B = \tan^{-1} 2.6101$

$B = 69.04^o$

Tangent Negative in Quadrant II and Quadrant IV.

Quadrant II :-

$B = 180^o - 69.04^o$

$B = 110.96^o$

Quadrant IV :-

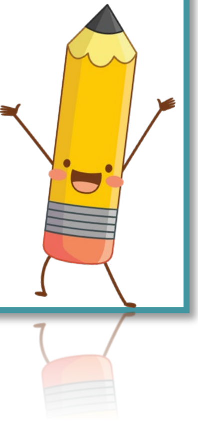
$B = 360^o - 69.04^o$

$B = 290.96^o$

$B = 110.96^o$ and 290.96^o.

Compound Angle Formulae for tangent:-

$$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha \tan\beta}$$



Test Your Knowledge 2.6

Find value of θ for each of the following equations which satisfy $0^o \leq \theta \leq 360^o$: –

1. $2\sin(30^o + \theta) = 0$
2. $\tan(\theta + 35^o) = 4$
3. $3\cos(\theta - 15^o) = 0$
4. $5\tan(20^o - \theta) = 3$

Answer :

1. $\theta = 150^o \text{ and } 330^o$
2. $\theta = 40.96^o \text{ and } 220.96^o$
3. $\theta = 75.00^o \text{ and } 255^o$
4. $\theta = 169.04^o \text{ and } 349.04^o$

2.3 SINE & COSINE RULES

INTRODUCTION

1. Sine and Cosine rules can be used to all types of triangle.

2. Used formulae of area for triangle, $A = \frac{1}{2}ab \sin C$ if the question stated value of Area.

3. **Never** use Theorem of Pythagoras for this subtopic unless the question stated it is right-angled triangle.

4. Frequently, labelling triangle consist of pairing alphabet; capital letter and small letter. Example; **A** and **a**.

   - Capital letter – to label angle
   - Small letter – to label side
   - Angle and side is opposite to one another.

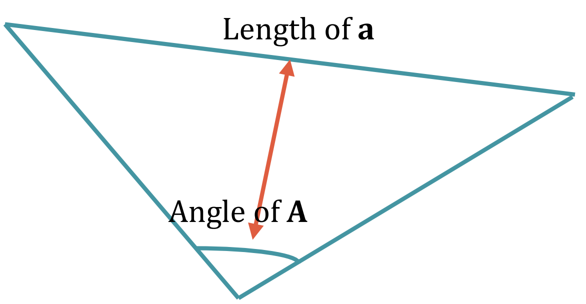


Figure 2.5 : Labelling Angle and Length Using Pairing Alphabet.

5. Formulae of **Interior Angles for Triangles** i.e. $180^o = A + B + C$ always being used to find the value of third angle once value of first and second angles are found.

6. For non-right angled triangle, trigonometric ratio **cannot** be used.

2.3 SINE & COSINE RULES

SINE AND COSINE RULES

Sine Rules

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Or

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Note :-

1. Used only 2 parts on Sine Rule formulae not 3 parts.
2. You need to know at least 1 pair of angle and its side to used this rule.

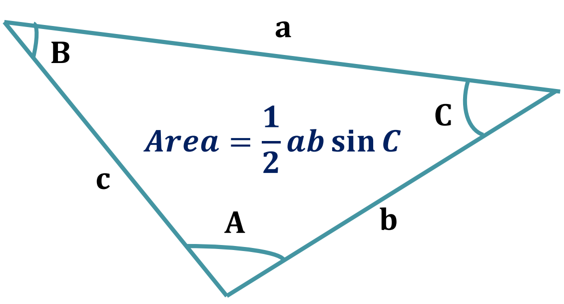


Figure 2.6 : Triangle ABC.

Cosine Rules

- $a^2 = b^2 + c^2 - 2bc\cos A$, or
- $b^2 = a^2 + c^2 - 2ac\cos B$

Note :-

1. You must know how to **re-create** cosine rules to meet your need.
2. Above formulae can be used to find length of ***a*** or angle ***A*** (*first formulae*) and length of ***b*** or angle ***B*** (*second formulae*).
3. This law mostly suitable for a question which:-
   - **All** values of **side are given** without value of angle.
   - **2 sides are given** and **1 angle BUT** there are **not opposite** to each other. And
   - To calculate second angle if first angle is an acute angle since Law of Cosine can trace obtuse angle.

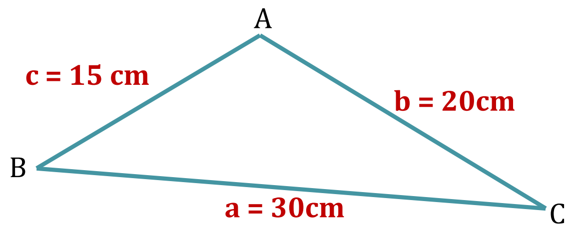
Tips:-
Obtuse angle is angle with range $90^o < angle < 180^o$.

Example 2.11

In triangle ABC, AB is 15cm, BC is 30 cm and AC is 20cm. Sketch the triangle, solve the triangle and find its area.

Solution

First : Sketch your triangle completed with relevant label.



A = 117.28^o	a = 30cm
B = 36.33^o	b = 20cm
C = ______	c = 15cm

This table is optional but I always used it for fast checking. You can get the data from your triangle.

Second : Calculate all angle for triangle ABC.

1. Used **Law of Cosine** to calculate angle A.

$$\boldsymbol{a^2 = b^2 + c^2 - 2bc\cos A}$$

$$30^2 = 20^2 + 15^2 - 2(20)(15)\cos A$$

$$\cos A = \frac{30^2 - 20^2 - 15^2}{-2(20)(15)}$$

$$\cos A = \frac{275}{-600}$$

$$A = \cos^{-1}\left(-\frac{275}{600}\right)$$

$$A = 117.28^o$$

2. Used **Law of Sine** to calculate angle B.

$$\boldsymbol{\frac{\sin A}{a} = \frac{\sin B}{b}}$$

$$\frac{\sin 117.28^o}{30} = \frac{\sin B}{20}$$

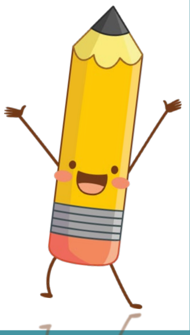
$$\sin B = \frac{20 \times \sin 117.28^o}{30}$$

$$\sin B = 0.5925$$

$$B = \sin^{-1} 0.5925$$

$$B = 36.33^o$$

Solution continue on the next page.



Example 2.11

In triangle ABC, AB is 15cm, BC is 30 cm and AC is 20cm. Sketch the triangle, solve the triangle and find its area

Solution : Continue

3. Used **Interior Angle of Triangle Formula** to calculate angle C.

$$180^o = 117.28^o + 36.33^o + C$$
$$C = 180^o - 117.28^o - 36.33^o$$
$$C = 26.39^o$$

A = 117.28^o	a = 30cm
B = 36.33^o	b = 20cm
C = 26.39^o	c = 15cm

My triangle data is now completed.

Third : Calculate area of triangle ABC.

$$Area = \frac{1}{2}ab\sin C$$
$$Area = \frac{1}{2}(30)(20)\sin 26.39^o$$
$$Area = 133.34cm^2$$ **Or**

$$Area = \frac{1}{2}bc\sin A$$
$$Area = \frac{1}{2}(20)(15)\sin 117.28^o$$
$$Area = 133.32cm^2$$ **Or**

$$Area = \frac{1}{2}ac\sin B$$
$$Area = \frac{1}{2}(30)(15)\sin 36.33^o$$
$$Area = 133.30cm^2$$

As you might have seen, three formula with different data of angles and sides give different value of area, however, do not worry, all of the answer are acceptable accordingly to the formulae that being used.

Note :- Just calculate the area once. I did three times just to show you that even which formulae you used, the answer almost same.

CLICK ME…..
https://youtu.be/EZuln8hFdWc

Example 2.12

Solve the triangle PQR and find its area. Given that p = 8.5 m, q = 10 m and $\angle R = 85^o$.

Solution

Calculate side ***r*** by using **Law of Cosine.**

$$\boldsymbol{r^2 = p^2 + q^2 - 2pq\cos R}$$
$$r^2 = 8.5^2 + 10^2 - 2(8.5)(10)\cos 85^o$$
$$r^2 = 157.43$$
$$r = \sqrt{157.43}$$
$$r = 12.55\ m$$

P = 42.43^o	p = 8.5 m
Q = ______	q = 10 m
R = 85^o	r = 12.55 m

Tips:-

The longest side of triangle will have **largest angle opposite** to it.

According to the above data, I'm not using Law of Cosine to calculate second angle since the longest side is 12.55 m and its angle is 85^o. So there is no Obtuse angle in this triangle.

Used **Law of Sine** to calculate angle P.

$$\boldsymbol{\frac{\sin R}{r} = \frac{\sin P}{p}}$$
$$\frac{\sin 85^o}{12.55} = \frac{\sin P}{8.5}$$
$$\sin P = \frac{8.5 \times \sin 85^o}{12.55}$$
$$\sin P = 0.6747$$
$$P = \sin^{-1} 0.6747$$
$$P = 42.43^o$$

Lets check angle P by using Law of Cosine.

$$\boldsymbol{p^2 = q^2 + r^2 - 2pq\cos P}$$
$$8.5^2 = 10^2 + 12.55^2 - 2(10)(12.55)\cos P$$
$$\cos P = \frac{8.5^2 - 10^2 - 12.55^2}{-2(10)(12.55)}$$
$$\cos P = \frac{-185.25}{-251}$$
$$\cos P = 0.7380$$
$$A = \cos^{-1}\left(-\frac{275}{600}\right)$$
$$A = 42.44^o$$

Solution continue on the next page.

Example 2.12 : Continue

Solve the triangle PQR and find its area. Given that p = 8.5 m, q = 10 m and $\angle R = 85^o$.

Solution

P = 42.43^o	p = 8.5 m
Q = 52.57^o	q = 10 m
R = 85^o	r = 12.55 m

Used **Interior Angle of Triangle Formula** to calculate angle Q

$$180^o = 42.43^o + Q + 85^o$$
$$Q = 180^o - 42.43^o - 85^o$$
$$Q = 52.57^o$$

Lets check angle Q by using Law of Sine.

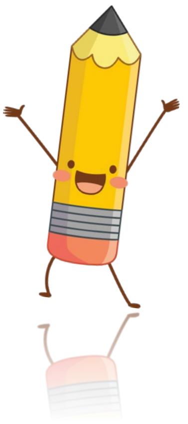
$$\frac{\sin Q}{q} = \frac{\sin R}{r}$$
$$\frac{\sin Q}{10} = \frac{\sin 85^o}{12.55}$$
$$\sin Q = \frac{10 \times \sin 85^o}{12.55}$$
$$\sin Q = 0.7938$$
$$Q = \sin^{-1} 0.7938$$
$$Q = 52.54^o$$

Tips:-

It is normal to have some differences in value due to numbers of significant figure and decimal places being used.
More significant figure/decimal places being used, more accurate answer will be.

Calculate area of triangle *PQR*.

$$Area = \frac{1}{2} pq \sin R$$
$$Area = \frac{1}{2}(8.5)(10) \sin 85^o$$
$$Area = 42.34\ m^2$$



Test Your Knowledge 2.7

1. Solve the triangle XYZ and find its area. Given $\angle YXZ = 36^o$, XY = 20 cm and XZ = 30 cm.

2. A triangle RST has sides r = 7.8 m, s = 8.3 m and t = 9.5m. Determine its three angles and its area.

3. An area of triangle ABC is 144 cm^2 with side a = 18 cm and b = 23 cm. Solve the triangle.

Answer :

1. $x = 16.148\ cm, Y = 111.74^o, Z = 38.26^o$, Area $=$ 150 cm^2

2. $R = 51.45^o, S = 56.27^o, T = 72.28^o$, Area $=$ 30.83 m^2

3. $A = 51.19^o, B = 84.73^o, C = 44.08^o$, and c $=$ 16.068 cm

Let's play games…………..

Fundamental of Trigonometry

Trigonometric Equations

COMPLEX NUMBER

SUBTOPIC :-

3.1 CONCEPT OF COMPLEX NUMBER

3.2 OPERATION OF COMPLEX NUMBER

- Addition and Subtraction of Complex Number
- Multiplication of Complex Number
- Division of Complex Number

3.3 ARGAND DIAGRAM

- Draw argand Diagram
- Modulus and argument of complex number

3.4 OTHER FORM OF COMPLEX NUMBER

- Other Forms of Complex Number
- Multiplication and division of Complex Number in Polar and Trigonometric form

3.1 CONCEPT OF COMPLEX NUMBER

What is Complex Number?

A combination of real part and imaginary parts

$$z = a + bi$$

i here is imaginary parts

a is real number

b is imaginary number

Example of complex number:

	Real number	Imaginary number	Imaginary part
$z = 2 + 3i$	2	3	$3i$
$w = \sqrt{5} + \frac{1}{4}i$	$\sqrt{5}$	$\frac{1}{4}$	$\frac{1}{4}i$
$z = -4i$	0	-4	$-4i$

Notes: $b \neq 0$

Why complex number are created?

To find the square root of negative numbers

$$\sqrt{-1} = \sqrt{i^2}$$
$$\sqrt{-1} = i$$

It happened because we assume $\mathbf{-1 = i^2}$

Example 3.1

Simplify the following In terms of complex number:

a) $\sqrt{-49}$

b) $-\sqrt{-25}$

Solution 3.1 (a)

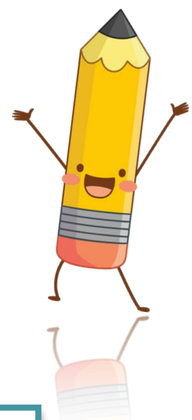
$\sqrt{-49}$

$= \sqrt{(-1)(49)}$ — separate number with (-1)

$= \sqrt{(i^2)(49)}$ — Change -1 to i^2

$= \sqrt{i^2}\sqrt{49}$

$= i7 = 7i$



Solution 3.1 (b)

$-\sqrt{-25}$

$= -\sqrt{(-1)(25)}$

$= -\sqrt{(i^2)(25)}$

$= -\sqrt{i^2}\sqrt{25}$

$= -i5 = -5i$

CLICK ME…..
https://youtu.be/J7iTEt9gsN4

Example 3.1

Simplify the following In terms of complex number:

c) $\sqrt{-4}+3$

d) $5-\sqrt{-8}$

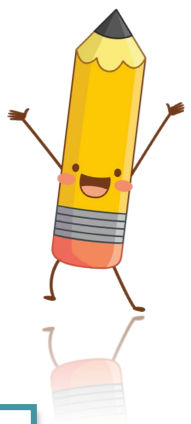
Solution 3.1 (c)

$\sqrt{-4}+3$

$=\sqrt{(-1)(4)}+3$

$=\sqrt{i^2(4)}+3$

$=2i+3$



Solution 3.1 (d)

$5-\sqrt{-8}$

$=5-\sqrt{(-1)(8)}$

$=5-\sqrt{(-1)(8)}$

$=5-\sqrt{i^2(8)}$

$=5-\sqrt{8}i$

CLICK ME…..
https://youtu.be/EvdLOG0w18w

Example 3.1

Simplify the following In terms of complex number:

e) i^5

f) i^8

Solution 3.1 (e)

i^5

$= i^4 \cdot i$

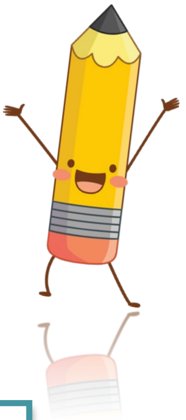
$= (i^2)^2 \cdot i$

$= (-1)^2 \cdot i$

$= (1)i = i$

If the index/power is odd number, take out one i to make it even number

Then change *i* into power of 2 (i^2) so that we can get (-*1*)



Solution 3.1 (f)

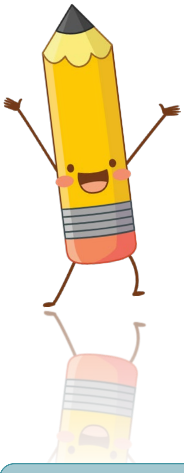
i^8

$= (i^2)^4$

$= (-1)^4$

$= 1$

If the index/power is even number, we can directly change *i* into power of 2 (i^2)



CLICK ME.....
https://youtu.be/-lWzA9HoP08

Example 3.1

Simplify the following In terms of complex number:

g) $5i^7 + 6$

h) $3i^{10} - 4i$

Solution 3.1 (g)

$5i^7 + 6$

$= 5i^6 \cdot i + 6$

$= 5(i^2)^3 \cdot i + 6$

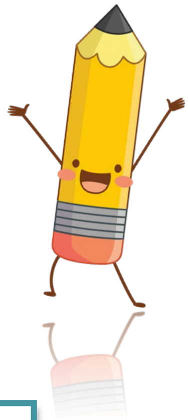
$= 5(-1)^3 \cdot i + 6$

$= 5(-1)i + 6$

$= -5i + 6$

$(-1)^{\text{odd number}} = (-1)$

$(-1)^{\text{even number}} = (+1)$



Solution 3.1 (h)

$3i^{10} - 4i$

$= 3(i^2)^5 - 4i$

$= 3(-1)^5 - 4i$

$= 3(-1) - 4i$

$= -3 - 4i$

Test Your Knowledge 3.1

Simplify the following In terms of complex number:

a) $\sqrt{-9} + 8$

b) $3 - \sqrt{-41}$

c) $\sqrt{4} + \sqrt{-5}$

d) $5i^6 + 2i^9$

e) $3i^{13} - 7i^4$

f) $4 + i^{11}$

Answer :

a) $3i + 8$

b) $3 - \sqrt{41}i$

c) $2 + \sqrt{5}i$

d) $-5 + 2i$

e) -4

f) $4 - i$

3.2 OPERATION OF COMPLEX NUMBER

3.2.1 Addition and Subtraction of Complex Number

Remember, adding or subtracting complex number is like add or subtract normal algebraic term.

So, what you should do?

1. You have to sort between real number and imaginary number.

2. Just add or subtract real number with real number

3. Then, add or subtract imaginary number with imaginary number

Addition:

$$(a + bi) + (c + di)$$
$$= (a + c) + (bi + di)$$

Subtraction:

$$(a + bi) - (c + di)$$

Expand the negative value to c and d

$$= a + bi - c - di$$
$$= a - c + bi - di$$

Example 3.2.1

If given $z = 3 + 4i$ and $w = 5 - i$, find

a) $z + w$

b) $z - w$

Solution 3.2.1 (a)

$z + w$

$= (3 + 4i) + (5 - i)$

$= 3 + 4i + 5 - i$

$= 3 + 5 + 4i - i$

$= 8 + 3i$

Solution 3.2.1 (b)

$z - w$

$= (3 + 4i) - (5 - i)$

$= 3 + 4i - 5 + i$

$= 3 - 5 + 4i + i$

$= -2 + 5i$

CLICK ME…..
https://youtu.be/Hd9Bji1PTaE

Example 3.2.1

If given $z = -4 + 2i$ and $w = 2 - 5i$, find

c) $2z + w$

d) $z - 3w$

Solution 3.2.1 (c)

$2z + w$

$= 2(-4 + 2i) + (2 - 5i)$

$= -8 + 4i + 2 - 5i$

$= -8 + 2 + 4i - 5i$

$= -6 - i$

Solution 3.2.1 (d)

c) $z - 3w$

$= (-4 + 2i) - 3(2 - 5i)$

$= -4 + 2i - 6 + 15i$

$= -4 - 6 + 2i + 15i$

$= -10 + 17i$

CLICK ME…..
https://youtu.be/yYdl9PvzYXg

Test Your Knowledge 3.2.1

Solve the operation of the complex number below:

a) $(3 + 2i) + (3 - 4i)$

b) $(2 + i) - (3 - 5i)$

c) $4 - 3i + 5i - 4$

d) $3i - (4 + 2i)$

e) $2(5 - 3i) + 5(1 - 2i)$

f) $3(2 - i) - (2 - 3i)$

g) $4 + 2(2 - 3i) - 5i$

h) $(-8i - 7) - 2i + (3 + 3i)$

Answer :

a) $6 - 2i$

b) $-1 + 6i$

c) $2i$

d) $-4 + i$

e) $15 - 16i$

f) 4

g) $8 - 11i$

h) $-7i - 4$

3.2 OPERATION OF COMPLEX NUMBER

3.2.2 Multiplication of Complex Number

Multiplication of complex numbers is like multiplying binomial functions.

If given $z = a + bi$ and $w = c + di$,

Hence, $z \cdot w = (a + bi)(c + di)$

We need to expand the function just like below

$$= (a + bi)(c + di)$$

$$= ac + adi + bci + bdi^2$$

$$= ac + adi + bci + bd(-1)$$
$$= ac - bd + adi + bci$$

Don't forget that $\boldsymbol{i^2 = -1}$

Combine real number with real number, imaginary number with imaginary

Example 3.2.2

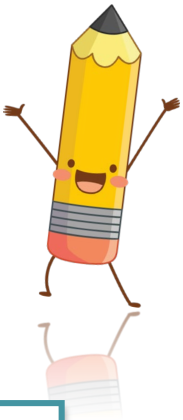
Multiply the complex numbers below

a) $2(3+4i)$
b) $5i(4-2i)$

Solution 3.2.2 (a)

$2(3+4i)$ ← Expand the function

$= 6 + 8i$



Solution 3.2.2 (b)

$5i(4-2i)$ ← Expand the function

$= 20i - 10i^2$ ← $i^2 = -1$

$= 20i - 10(-1)$

$= 20i + 10$

Example 3.2.2

Multiply the complex numbers below

c) $(4 - i)(3 + 5i)$

d) $(3 - 5i)(4 - 2i)$

Solution 3.2.2 (c)

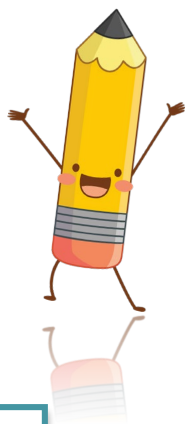
$(4 - i)(3 + 5i)$ ← Expand the function

$= 12 + 20i - 3i - 5i^2$ ← $i^2 = -1$

$= 12 + 20i - 3i - 5(-1)$

$= 12 + 20i - 3i + 5$

$= 17 + 17i$



Solution 3.2.2(d)

$(3 - 5i)(4 - 2i)$ ← Expand the function

$= 12 - 6i - 20i + 10i^2$ ← $i^2 = -1$

$= 12 - 6i - 20i + 10(-1)$

$= 12 - 6i - 20i - 10$

$= 2 - 26i$

CLICK ME.....
https://youtu.be/M3ZyXgMyxwY

Test Your Knowledge 3.2.2

Simplify the following complex number:

a) $4i(1-3i)$

b) $2(2+5i)$

c) $(4+3i)(2+7i)$

d) $(7-i)(2+6i)$

e) $(3-3i)(4-4i)$

f) $5-6i(4+2i)$

g) $2(3+4i)(2-i)$

h) $(9i-2)(4+4i)$

i) $(4+5i)^2$

Answer :

a) $4i+12$

b) $4+10i$

c) $-13+34i$

d) $20+30i$

e) $-24i$

f) $17-24i$

g) $20+10i$

h) $28i-44$

i) $-9+40i$

3.2 OPERATION OF COMPLEX NUMBER

3.2.3 Division of Complex Number

When it comes to division of complex number, make sure we solve it until the denominator become the real number.

How to do it?

1. Put the division into fraction form
2. Multiply the numerator and denominator with **conjugate of the denominator**
3. Simplify the expression

What is Conjugate?

The different sign of the imaginary part

Complex Number	Conjugate
$a + bi$	$a - bi$
$c - di$	$c + di$

3.2 OPERATION OF COMPLEX NUMBER

Steps in solving division of complex numbers

If given $z = a + bi$ and $w = c + di$, hence

Put in fraction form

$$\frac{z}{w} = \frac{a+bi}{c+di}$$

Multiply the fraction with conjugate of denominator

$$= \frac{a+bi}{c+di} \times \frac{c-di}{c-di}$$

Expand numerator and denominator

$$= \frac{ac - adi + bci - bdi^2}{c^2 - cdi + cdi - d^2i^2}$$

$i^2 = -1$

$$= \frac{ac - adi + bci - bd(-1)}{c^2 - d^2(-1)}$$

$$= \frac{ac + bd - adi + bci}{c^2 + d^2}$$

Example 3.2.3

Solve the division of complex numbers below

a) $\frac{2+3i}{4+i}$

b) $\frac{1-2i}{2-3i}$

Solution 3.2.3 (a)

$$\frac{2+3i}{4+i}$$

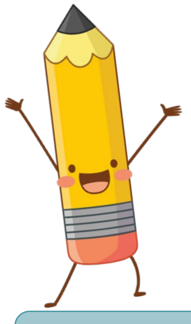
$$=\frac{2+3i}{4+i}\times\frac{4-i}{4-i}$$

$$=\frac{8-2i+12i-3i^2}{16-4i+4i-i^2}$$

$$=\frac{8+10i-3(-1)}{16-(-1)}$$

$$=\frac{11+10i}{16+1}$$

$$=\frac{11}{17}+\frac{10}{17}i$$



Solution 3.2.3 (b)

$$\frac{1-2i}{2-3i}$$

$$=\frac{1-2i}{2-3i}\times\frac{2+3i}{2+3i}$$

$$=\frac{2+3i-4i-6i^2}{4+6i-6i-9i^2}$$

$$=\frac{2-i-6(-1)}{4-9(-1)}$$

$$=\frac{2+6-i}{4+9}$$

$$=\frac{8-i}{13}$$

$$=\frac{8}{13}-\frac{1}{13}i$$

CLICK ME…..
https://youtu.be/IdEjxulo6cg

Example 3.2.3

Solve the division of complex numbers below

c) $\frac{4-4i}{5i+3}$

d) $\frac{1+2i}{4i}$

Solution 3.2.3 (c)

$$\frac{4-4i}{5i+3}$$

$$=\frac{4-4i}{5i+3}\times\frac{-5i+3}{-5i+3}$$

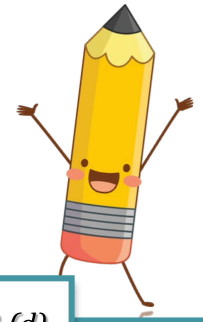
$$=\frac{-20i+12+20i^2-12i}{-25i^2+15i-15i+9}$$

$$=\frac{-32i+12+20(-1)}{-25(-1)+9}$$

$$=\frac{-32i+12-20}{25+9}$$

$$=\frac{-32i-8}{34}$$

$$=-\frac{16}{17}i-\frac{4}{17}$$



Solution 3.2.3 (d)

$$\frac{1+2i}{4i}$$

$$=\frac{1+2i}{4i}\times\frac{-4i}{-4i}$$

$$=\frac{-4i-8i^2}{-16i^2}$$

$$=\frac{-4i-8(-1)}{-16(-1)}$$

$$=\frac{-4i+8}{16}$$

$$=-\frac{1}{4}i+\frac{1}{2}$$

CLICK ME…..
https://youtu.be/rG8tCxWvl-o

Test Your Knowledge 3.2.3

Simplify the following complex number:

a) $\frac{4-3i}{4+2i}$

b) $\frac{2+5i}{2+3i}$

c) $\frac{2+6i}{7-i}$

d) $\frac{3-3i}{4-4i}$

e) $\frac{3+4i}{2-i}$

f) $\frac{9-2i}{4i}$

g) $\frac{4+5i}{4i-2}$

Answer :

a) $\frac{1}{2}-i$

b) $\frac{19}{13}+\frac{4}{13}i$

c) $\frac{4}{25}+\frac{22}{25}i$

d) $\frac{3}{4}$

e) $\frac{2}{5}$

f) $-\frac{9}{4}i-\frac{1}{2}$

g) $\frac{13}{8}i+\frac{3}{4}$

3.3 ARGAND DIAGRAM

3.3.1 Draw argand Diagram

Argand diagram is a graphical presentation of complex number.

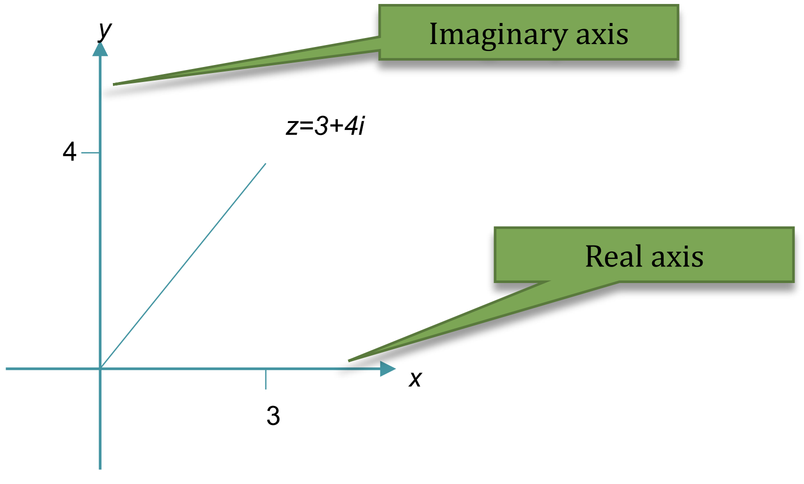
If the complex number is $z = a + bi$, then a is a real part and bi is imaginary part.

To present the complex number in Argand Diagram is just like presenting in Cartesian Plane.

Therefore, in Argand Diagram, *a* will be in horizontal axis (*x*-axis) and *bi* will be in vertical axis (*y*-axis).

Example:

If we have z= $3 + 4i$, with *3* is real part and *4i* is imaginary part,



3.3 ARGAND DIAGRAM

3.3.2 Modulus and argument of complex number

In each Argand Diagram must have the value of Modulus and Argument of complex number.

What is Modulus?

Modulus is the length from the origin to the point of complex number as we call it line segment

How to calculate modulus:

$$|z| = \sqrt{x^2 + y^2}$$

Where ***x*** is real number, and ***y*** is imaginary number

What is Argument?

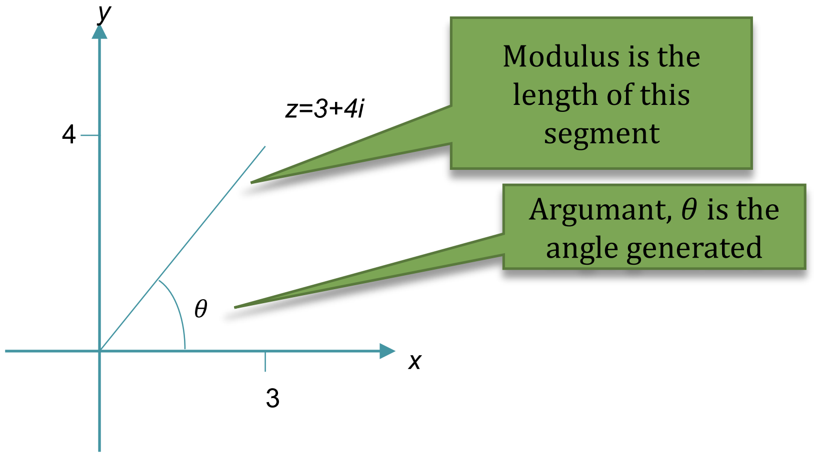
Argument is the angle generated from the positive x-axis to the line segment.

Just like the example in the page before, z= $3 + 4i$

How to calculate argument:

$$\arg z = tan^{-1}\left(\frac{y}{x}\right)$$

3.3 ARGAND DIAGRAM



By referring to the diagram above, here's how to find the value of **modulus** and **argument**:

Modulus,

$$
\begin{aligned}
|z| &= \sqrt{3^2 + 4^2} \\
&= \sqrt{9 + 16} \\
&= \sqrt{25} \\
&= 5\text{unit}
\end{aligned}
$$

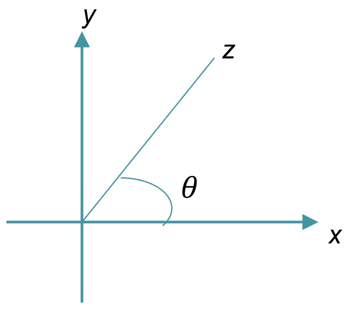
Argument,

$$\theta = \arg z = tan^{-1}\left(\frac{4}{3}\right)$$

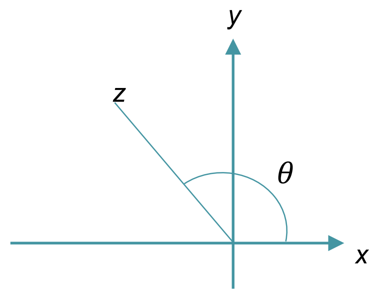
$$= 53.13^0$$

3.3 ARGAND DIAGRAM

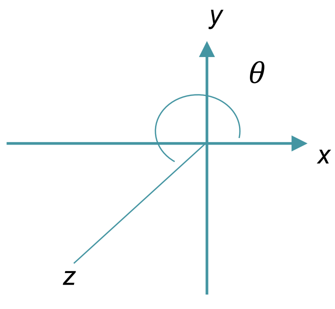
Here's we show you the **argument principles** for you to refer the value of argument in different Quadrant



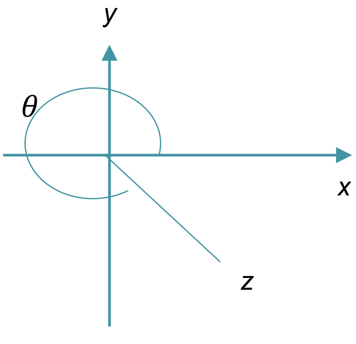
1st quadrant
$\theta = \arg z$



2nd quadrant
$\theta = 180^o - \arg z$



3rd quadrant
$\theta = 180^o + \arg z$



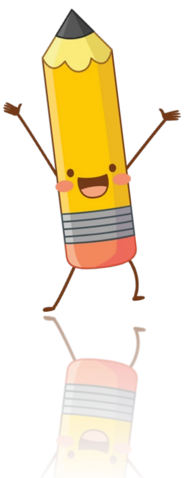
4th quadrant
$\theta = 360^o - \arg z$

θ is the argument, $\arg z$ is the value from calculator

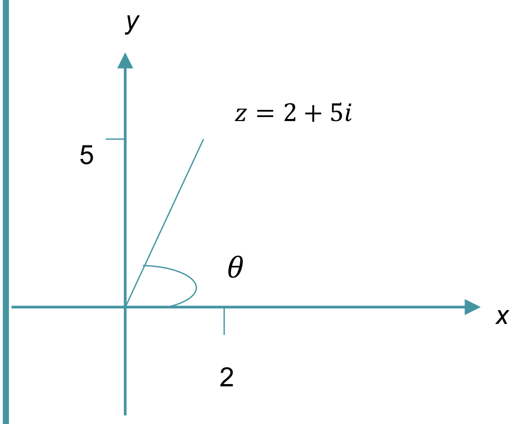
Example 3.3

1. Sketch the argand diagram for $z = 2 + 5i$, hence calculate the modulus and argument

Solution 3.3



1. $z = 2 + 5i$



Modulus $|z| = \sqrt{2^2 + 5^2}$

$= \sqrt{4 + 25}$

$= \sqrt{29}$unit

Argument,

$\arg z = tan^{-1}\left(\frac{5}{2}\right)$

$= 68.2°$

1^{st} quadrant
$\theta = \arg z$

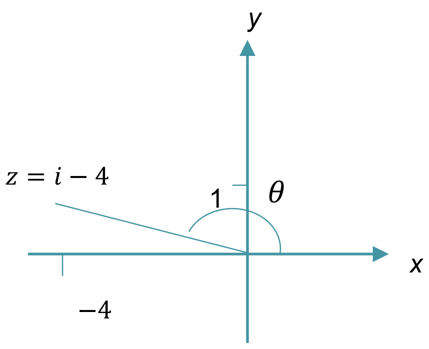
CLICK ME.....
https://youtu.be/O8P6dSyZIFA

Example 3.3

2. Sketch the argand diagram for $z = i - 4$, hence calculate the modulus and argument

Solution 3.3

2. $z = i - 4$



$$\text{Modulus } |z| = \sqrt{1^2 + (-4)^2}$$
$$= \sqrt{1 + 16}$$
$$= \sqrt{17}\text{unit}$$

$$\text{Argument, } \arg z = tan^{-1}\left(\frac{1}{-4}\right)$$
$$= -14.04°$$
$$\theta = 180^o - 14.04^o$$
$$= 165.96°$$

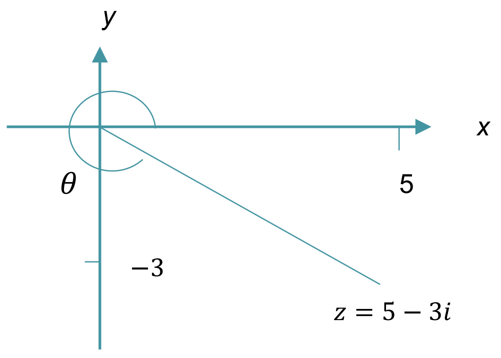
2^{nd} quadrant
$\theta = 180^o - \arg z$

Example 3.3

3. Sketch the argand diagram for $z = 5 - 3i$, hence calculate the modulus and argument

Solution 3.3

3. $z = 5 - 3i$



$$\text{Modulus } |z| = \sqrt{5^2 + (-3)^2}$$
$$= \sqrt{25 + 9}$$
$$= \sqrt{34}\text{unit}$$

$$\text{Argument, } \arg z = tan^{-1}\left(\frac{-3}{5}\right)$$
$$= -30.96°$$
$$\theta = 360^o - 30.96^o$$
$$= 329.04°$$

4^{th} quadrant
$\theta = 360^o - \arg z$

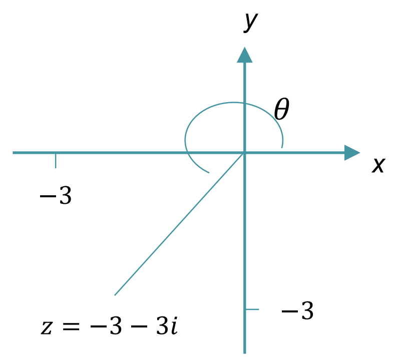
CLICK ME.....
https://youtu.be/vEcHalurIl4

Example 3.3

4. Sketch the argand diagram for $z = (5 + 3i) - (8 + 6i)$, hence calculate the modulus and argument

Solution 3.3

4. $z = (5 + 3i) - (8 + 6i)$
$= 5 + 3i - 8 - 6i$
$= -3 - 3i$



Modulus $|z| = \sqrt{(-3)^2 + (-3)^2}$
$= \sqrt{9 + 9}$
$= \sqrt{18}$unit

Argument, $\arg z = tan^{-1}\left(\frac{-3}{-3}\right)$
$= 45°$
$\theta = 180^o +$
45^o
$= 225°$

3rd quadrant
$\theta = 180^o + \arg z$

Test Your Knowledge 3.3

Sketch the argand diagram for the following, hence calculate the modulus and argument

a) $(1-3i)$

b) $(2+5i)$

c) $(-4-3i)$

d) $(7-i)$

e) $(3-3i)+(4-4i)$

f) $6i-(4+2i)$

g) $4-(2-i)$

Answer :

a) $|z|=\sqrt{10}unis;$ $\theta = 288.43$

b) $|z|=\sqrt{29}units;$ $\theta = 68.20°$

c) $|z|=5units;$ $\theta = 216.87°$

d) $|z|=\sqrt{50}units;$ $\theta = 351.87°$

e) $|z|=\sqrt{98}units;$ $\theta = 315°$

f) $|z|=\sqrt{32}units;$ $\theta = 135°$

g) $|z|=\sqrt{5}units;$ $\theta = 26.57°$

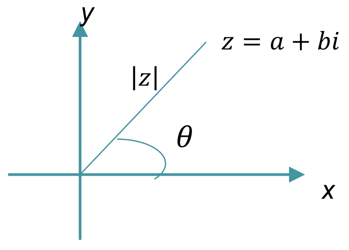
3.4 FORMS OF COMPLEX NUMBER

3.4.1 Other Forms of Complex Number

We normally know the complex number in terms of $z = a + bi$. Actually there are other forms of complex number.

Forms of Complex Number	Formula
Trigonometric Form	$\lvert z\rvert(\cos\theta + i\sin\theta)$
Polar Form	$\lvert z\rvert\angle\theta$
Exponential Form	$\lvert z\rvert e^{i\theta}$ Where θmust in radian

How it's been generated into other form?



The coordinate of the Argand diagram is (a, b), then we convert the value of a and b using the angle,θ and the length, $|z|$:

$$\boldsymbol{a + bi = |z|cos\theta + (|z|\sin\theta)i}$$

Therefore $\boldsymbol{a + bi = |z|(\cos\theta + i\sin\theta)}$

And we can also write it in angle notation that we called it as polar form:

$$\boldsymbol{|z|\angle\theta}$$

The symbol of $\angle$ in polar form is called **CIS**.

Exponential form being generated by using **Euler formula**

$$\boldsymbol{e^{i\theta} = \cos\theta + i\sin\theta}$$

Example 3.4.1

1. Express $z = 5 - 3i$ in trigonometric form and polar form

Solution 3.4.1

1. $z = 5 - 3i$

To change from cartesian form to other form, we must find the value of modulus and argument.

$$|z| = \sqrt{5^2 + (-3)^2}$$
$$= \sqrt{25 + 9}$$
$$= \sqrt{34}\text{units}$$

$$\arg z = tan^{-1}\left(\frac{-3}{5}\right)$$
$$= -30.96°$$
$$\theta = 360^o - 30.96^o$$
$$= 329.04^o$$

4^{th} quadrant
$\theta = 360^o - \arg z$

Trigonometric form; $z = |z|(\cos\theta + i\sin\theta)$
$$z = \sqrt{34}(\cos 329.04° + i \sin 329.04°)$$

Polar form; $z = |z|\angle\theta$
$$z = \sqrt{34}\angle 329.04°$$

CLICK ME.....
https://youtu.be/om2b_VLd4gI

Example 3.4.1

2. Express $z = -2 - 6i$ in trigonometric form and exponential form

Solution 3.4.1

2. $z = -2 - 6i$

To change from cartesian form to other form, we must find the value of modulus and argument.

$$|z| = \sqrt{(-6)^2 + (-2)^2}$$
$$= \sqrt{36 + 4}$$
$$= \sqrt{40}\text{units}$$

$$\arg z = tan^{-1}\left(\frac{-6}{-2}\right)$$
$$= 71.56°$$
$$\theta = 180^O + 71.56^O$$
$$= 251.57^O$$

$$\theta = 251.57^O \times \frac{\pi}{180^O}$$
$$= 4.39\, rad$$

Trigonometric form; $z = |z|(\cos\theta + i\sin\theta)$
$$z = \sqrt{40}(\cos 251.57° + i \sin 251.57°)$$

Exponential form; $z = |z|e^{i\theta}$
$$= \sqrt{40}e^{4.39i}$$

The value of θ in exponential form must in radian

CLICK ME.....
https://youtu.be/umuOWEonfco

Example 3.4.1

3. Express $z = 5(\cos 125° + i \sin 125°)$ in polar form, exponential form and cartesian form

Solution 3.4.1

3. $z = 5(\cos 125° + i \sin 125°)$

$|z| = 5$
$\theta = 125°$
$\theta = 125^{o} \times \dfrac{\pi}{180^{o}}$
$\quad = 2.18\, rad$

Polar form; $z = |z|\angle\theta$
$z = 5\angle 125°$

Exponential form; $z = |z|e^{i\theta}$
$z = 5e^{2.18i}$

The value of θ in exponential form must in radian

Cartesian form; $z = 5(\cos 125° + i \sin 125°)$
$= 5(-0.57 + 0.82\text{i})$
$= -2.85 + 4.1i$

Just use the calculator to find cos 125° and sin 125°

CLICK ME…..
https://youtu.be/7fUqnvNldYk

Example 3.4.1

4. Express $z = 4e^{2.5i}$ in trigonometric form and cartesian form

Solution 3.4.1

3. $z = 4e^{2.5i}$

$|z| = 4$

$\theta = 2.5\, rad$

$$\theta = 2.5 \times \frac{180^O}{\pi}$$

$$= 143.24°$$

From radian to degree; $\theta \times \frac{180^O}{\pi}$

From degree to radian, $\theta \times \frac{\pi}{180^O}$

Trigonometric form; $z = |z|(\cos\theta + i\sin\theta)$

$$z = 4(\cos 143.24° + i\sin 143.24°)$$

Cartesian form; $z = 4(\cos 143.24° + i\sin 143.24°)$

$$z = 4(-0.80 + 0.60i)$$

$$= -3.2 + 2.4i$$

Just use the calculator to find cos 143.24° and sin 143.24°

CLICK ME…..
https://youtu.be/fExECZpQKNI

Test Your Knowledge 3.4.1

Express the following complex number according to the form stated

a)	$(1+3i)$	(Polar Form and Trigonometric Form)
b)	$(2-5i)$	(Polar Form and Exponential Form)
c)	$3.5\,(\cos 130° + \sin 130°)$	(Exponential Form and Cartesian Form)
d)	$4e^{2.5i}$	(Trigonometric Form and Cartesian Form)
e)	$1.6\angle 150°$	(Exponential Form and Cartesian Form)

Answer :

a) Polar form; $z = \sqrt{10}\angle 71.5°$

Trigonometric form; $z = \sqrt{10}(\cos 71.5° + i \sin 71.5°)$

b) Polar Form; $z = \sqrt{29}\angle 291.80°$

Exponential Form; $\sqrt{29}e^{5.09i}$

c) Exponential Form; $z = 3.5e^{2.27i}$

Cartesian Form; $z = -2.24 + 2.70i$

d) Trigonometric Form; $z = 4(\cos 143.24° + i \sin 143.24°)$

Cartesian Form; $z = -3.2 + 2.4i$

e) Exponential Form; $z = 21.6e^{2.61i}$

Cartesian Form; $z = -1.39 + 0.8i$

3.4 FORMS OF COMPLEX NUMBER

3.4.2 Multiplication and division of Complex Number in Polar and Trigonometric form

In multiplication and division of complex number for polar and trigonometric form, note that,

When we multiply,

We will **multiply** the modulus, and **add** the argument

When we divide,

We will **divide** the modulus, and **minus** the argument

Multiplication and division Polar form

If given $\boldsymbol{z = r_z \angle \theta_z}$ **and** $\boldsymbol{w = r_w \angle \theta_w}$

$$z \times w = (r_z \times r_w)\angle(\theta_z + \theta_w)$$

$$\frac{z}{w} = \frac{r_z}{r_w}\angle(\theta_z - \theta_w)$$

Multiplication and division Trigonometric form

If given $\boldsymbol{z = r_z(\cos\theta_z + i\sin\theta_z)}$ **and** $\boldsymbol{w = r_w(\cos\theta_w + i\sin\theta_w)}$

$$z \times w = (r_z \times r_w)[cos(\theta_z + \theta_w) + i\,sin(\theta_z + \theta_w)]$$

$$\frac{z}{w} = \frac{r_z}{r_w}[cos(\theta_z - \theta_w) + i\,sin(\theta_z - \theta_w)]$$

Example 3.4.2

1. If $z = 4\angle 120°$ and $w = 2\angle 85°$ find,

   a) $z \times w$

   b) $\frac{z}{w}$

Solution 3.4.2 (1)

a) $z \times w = (4 \times 2)\angle(120^O + 85^O)$
$= 8\angle 205°$

b) $\frac{z}{w} = \frac{4}{2}\angle(120^O - 85^O)$
$= 2\angle 35°$

2. If $z = 3(\cos 135° + i \sin 135°)$ and $w = 1.5(\cos 60° + i \sin 60°)$ find,

   a) $z \times w$

   b) $\frac{z}{w}$

Solution 3.4.2 (2)

a) $z \times w = (3 \times 1.5)[\cos(135^O + 60^O) + i \sin(135^O + 60^O)]$
$= 4.5[\cos 195° + i \sin 195°]$

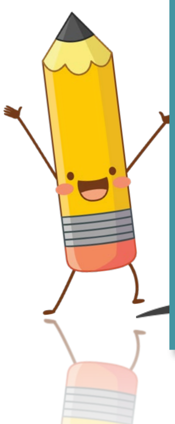
b) $\frac{z}{w} = \frac{3}{1.5}[\cos(135^O - 60^O) + i \sin(135^O - 60^O)]$
$= 2(\cos 75° + i \sin 75°)$

CLICK ME.....
https://youtu.be/Vf3n1uwGmqY

Example 3.4.2

3. If $z = 5\angle 150°$ and $w = 3(\cos 77° + i \sin 77°)$ find, $z \times w$ and give your answer in Polar form

Solution 3.4.2 (3)



Change $w = 3(\cos 77° + i \sin 77°)$ in polar form
$w = 3\angle 77°$

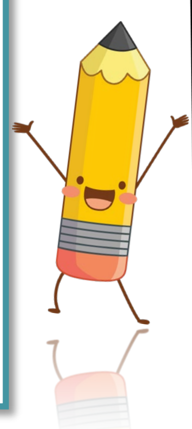
$z \times w = 5 \times 3\angle 150^{o} + 77^{o}$
$= 15\angle 227°$ *(answer in polar form)*

4. If $z = 4(\cos 30° + i \sin 30°)$ and $w = 1.2\angle 56°$ find $\frac{z}{w}$ and give your answer in trigonometric form

Solution 3.4.2 (4)

Change $w = 1.2\angle 56°$ into trigonometric form
$w = 1.2(\cos 56° + i \sin 56°)$

$$\frac{z}{w} = \frac{4}{1.2}[\cos(30^{o} - 56^{o}) + i \sin(30^{o} - 56^{o})]$$
$$= 3.33[\cos(-26^{o}) + i \sin(-26^{o})]$$

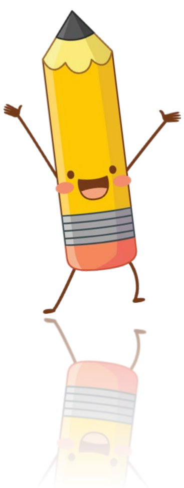


CLICK ME…..
https://youtu.be/C1yFgPxQfAQ

Example 3.4.2

3. If $z = 5e^{\frac{\pi}{2}i}$ and $w = 3(\cos 77° + i\sin 77°)$ find, $z \times w$ and give your answer in Trigonometric form

Solution 3.4.2 (3)



Step 1 : State the modulus and argument of z,

$|z| = 5$

$\theta = \frac{\pi}{2} \times \frac{180^o}{\pi}$

$= 90°$

Step 2: Change z into trigonometric form

$z = 5(\cos 90° + i\sin 90°)$

Step 3

$z \times w = (5 \times 3)(\cos(90^o + 77^o) + i\sin(90^o + 77^o))$

$= 15(\cos 167° + i\sin 167°)$

Test Your Knowledge 3.4.2

Find $z \times w$ and $\frac{z}{w}$ for the following and give the answer as stated:

a) $z = 4(\cos 100° + i \sin 100°)$ and $w = 2.3(\cos 55° + i \sin 55°)$
(give answer in Trigonometric Form)

b) $z = 1.8\angle 325°$ and $w = 5\angle 180°$
(give answer in Polar Form)

c) $z = 3.5(\cos 150° + i \sin 150°)$ and $w = 2\angle 50°$
(give answer in Polar Form)

d) $z = 2(\cos 90° + i \sin 90°)$ and $w = 4\angle 125°$
(give answer in Trigonometric Form)

e) $z = 3.8e^{1.5i}$ and $w = 1.9\angle 105°$
(give answer in Polar Form)

Answer :

a) $z \times w = 9.2(\cos 155° + i \sin 155°)$; $\frac{z}{w} = 1.74(\cos 45° + i \sin 45°)$

b) $z \times w = 9\angle 475°$; $\frac{z}{w} = 0.36\angle 175°$

c) $z \times w = 7\angle 200°$; $\frac{z}{w} = 1.75\angle 100°$

d) $z \times w = 8(\cos 215° + i \sin 215°)$; $\frac{z}{w} = 1.5(\cos(-35^{O}) + i \sin(-35^{O}))$

e) $z \times w = 7.22\angle 190.94°$; $\frac{z}{w} = 2\angle(-105°)$

Let's play games…………..

BasicComplex Numbers

Operation of Complex Numbers

Modulus, Argument & Forms of Complex Numbers

REFERENCES

1. John Bird ed.(2014), *Higher Engineering Mathematics, 7^{th}* Edition, Bell and Bain Ltd. Glasgow.
   ISBN : 978-0-315-85882-1

2. Ruhana Mat Kia et. al. (2017), *Engineering Mathematics 1 for Polytechnics, 2^{nd}* Edition, Mathematics, Science and Computer Department, Politeknik Port Dickson, Negeri Sembilan.
   ISBN : 978-967-5677-49-6

3. Tho Lai Hoong, Thum Lai Chun (2019), Kunci Emas Formula A+ SPM Tingkatan 4 & 5 KBSM Matematik Tambahan Tip Pemeriksa, Sasbadi Sdn. Bhd., Selangor.
   ISBN : 978-983-77-1409-0

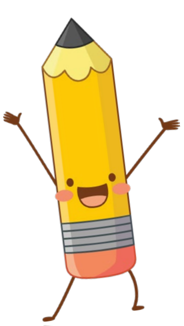
4. Chan Tze Hin(2019), Kunci Emas Formula A+ SPM Tingkatan 4 & 5 KBSM Matematik Tambahan Revisi, Sasbadi Sdn. Bhd., Selangor.
   ISBN : 978-983-77-1407-6

5. Mathcentre, 2009, *The Double Angle Formulae*, accessed 7 September 2021, <https://www.mathcentre.ac.uk/resources/uploaded/mc-ty-doubleangle-2009-1.pdf>

6. A.J. Reynolds, 2010, *Section 4 : Sine and Cosine Rule,* accessed 8 September 2021 <https://www.cimt.org.uk/projects/mepres/step-up/sect4/index.htm>

7. Algebraic Rules, Retrieved 26 August 2021, from <https://i.pinimg.com/originals/f4/80/2d/f4802d37964176dcd9dc6d4ab9caea20.png>

8. Basic of Algebra (2005-2021) Retrieved 22 August 2021, from <https://www.wyzant.com/resources/lessons/math/algebra/equation-basics/>



REFERENCES

9. Transposition of formulae , Retrieved 30 August 2021, from <https://cpb-eu-w2.wpmucdn.com/mypad.northampton.ac.uk/dist/d/6334/files/2018/10/Algebra-Formulae-Transposition-1chm7ai.pdf>

10. Math is Fun, 2020, *Definition of Variables*, accessed 13 September 2021, https://www.mathsisfun.com/definitions/variable.html

11. Math is Fun, 2018, *Definition of Coefficient*, accessed 13 September 2021, https://www.mathsisfun.com/definitions/coefficient.html

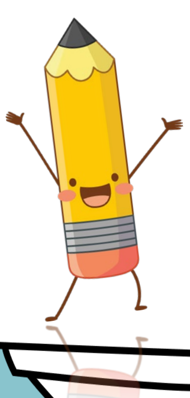
12. Pierce, Rod. (3 Aug 2019). "Complex Numbers". Math Is Fun. Retrieved 26 Sep 2021 from http://www.mathsisfun.com/numbers/complex-numbers.html

13. Complex Number, Retrieved 26 August 2021, from < https://www.cuemath.com/numbers/complex-numbers/>

14. Murray Bourne, Polar Form of Complex Numbers. . Intmath.com. Retrieved 26 Sep 2021 from <https://www.intmath.com/complex-numbers/4-polar-form.php>

15. Dawkins, p., 2021. Complex Number Primer. [online] Tutorial.math.lamar.edu. Available at: <https://tutorial.math.lamar.edu/Extras/ComplexPrimer/Forms.aspx> [Accessed 26 September 2021].

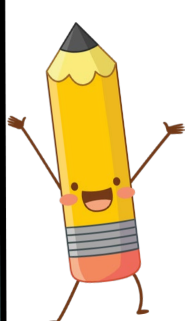
16. Courses.lumenlearning.com. 2021. Multiply and divide complex numbers | College Algebra. [online] Available at: <https://courses.lumenlearning.com/ivytech-collegealgebra/chapter/multiply-and-divide-complex-numbers/> [Accessed 26 September 2021].



ATTRIBUTION

1. Figure 2.1 (Chapter 2 : Trigonometry) is an adaption of "*Image : Graphing y = tan x*" from the Engineering Mathematics 1 for polytechnics Second Edition (2017). Image have been modified.

2. Figure 2.4 (Chapter 2 : Trigonometry) is an adaption of "*Image : Graphing f(x) = cos (x)*" from the Engineering Mathematics 1 for polytechnics Second Edition (2017). Image have been modified.



e ISBN 978-967-2897-47-7

9 7 8 9 6 7 2 8 9 7 4 7 7

ENGINEERING
MATHEMATICS 1 SELF
LEARNING