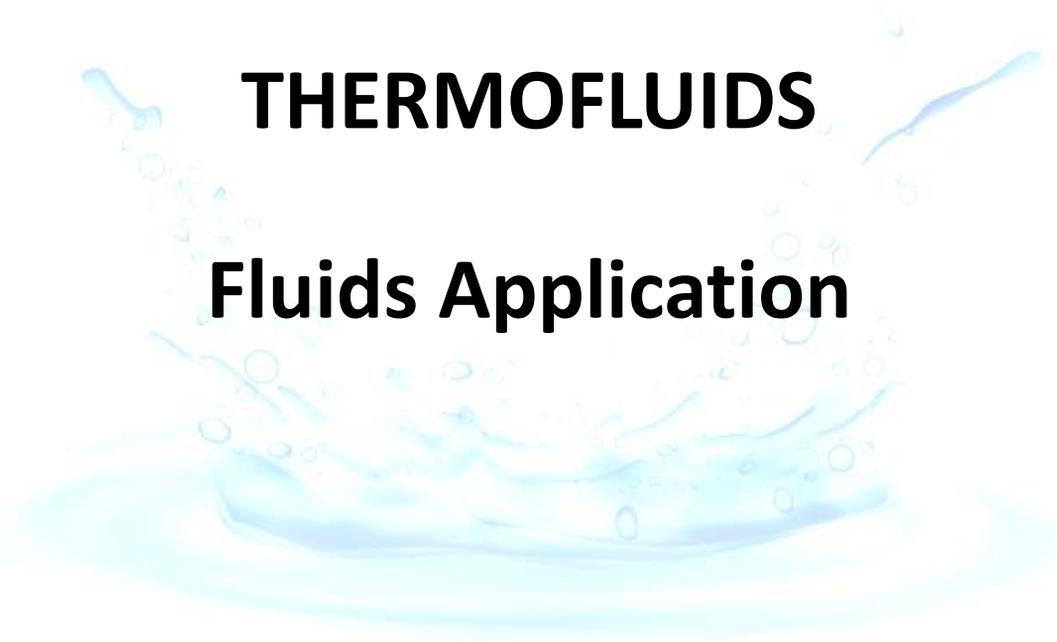


# THERMOFLUIDS

—◆————◆—  
Fluids Application

**CHE AZLINA BINTI CHE NORHOSENI  
RAMLATUL RAHILAH BINTI MOHD HUSIN  
AZMI BIN HJ UDI**



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We hereby declare that this module is our original work. To the best of our knowledge, it contains no materials previously written or published by another person. However, if there is any, due acknowledgement and credit are mentioned accordingly in the e-book.

# PREFACE

The thermofluids e-book prepares mechatronics students the basic concepts of thermodynamics and fluid mechanics into one integrated course. The fluid application of the focus topics of this e-book only.

The contents in this e-book are relevant to the syllabus for Diploma in Mechatronics Engineering student. There are the note, diagrams, example of problem solution and tutorial of the topic to ease for student to refer to. This e-book is very helpful for polytechnic students as a reference in their studies.

Hopefully, the student get benefits form this e-book and succeed in their life.



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# CHAPTER 1

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## Pressure





Figure 1.1 Blood pressure

## 1.1 What is Pressure?



In engineering, pressure is related to force and surface area.

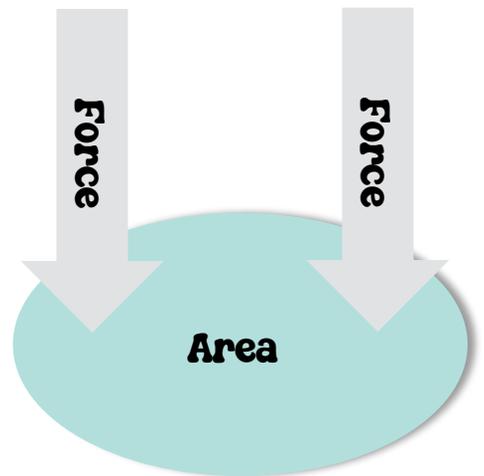


Pressure is the result of a force applied in a perpendicular fashion to a surface.



SI unit : Pascal (Pa) or (N/m<sup>2</sup>)

$$1 \text{ Pa} = \frac{1 \text{ Newton}}{1 \text{ m}^2}$$



$$\text{Pressure} = \frac{\text{Force}}{\text{Area}}$$

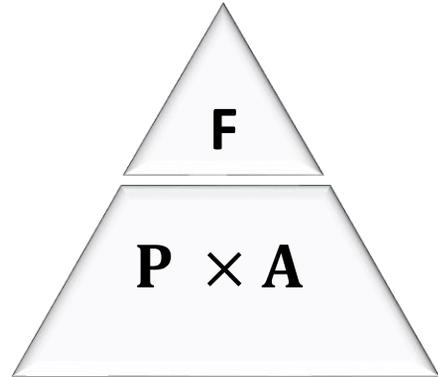
### Note

$$\begin{aligned} 1 \text{ bar} &= 10^5 \text{ N/m}^2 \\ &= 10^3 \text{ kN/m}^2 \end{aligned}$$

## 1.2 Formula for Pressure

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}}$$

$$P = \frac{F}{A}$$



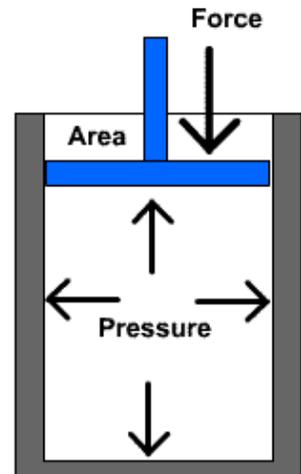
### NOTE :

P = Pressure (N/m<sup>2</sup>) or Pascal (Pa)

F = Force (N)

A = Area of pressure m<sup>2</sup>

Unit of pressure :Pascal, mmHg, atm



### HIGH PRESSURE:

- Weight from heeled shoes is spread over a smaller area.
- This exerts a higher pressure on the ground.



### LOW PRESSURE:

- Weight from flat shoes is spread over a larger area.
- This exerts a lower pressure on the ground.



### 1.3 Types of Pressure



**Atmospheric Pressure,  $P_{atm}$**

The atmospheric pressure of the earth's surface depends on the air above the surface.



**Gauge Pressure,  $P_G$**

The pressure difference between absolute pressure and atmospheric pressure.

**Absolute Pressure,  $P_{Abs}$**

Addition of atmospheric pressure and gauge pressure.



$$P_{Abs} = P_G + P_{Atm}$$



**Vakum Pressure,  $P_v$**

The pressures below atmospheric pressure.

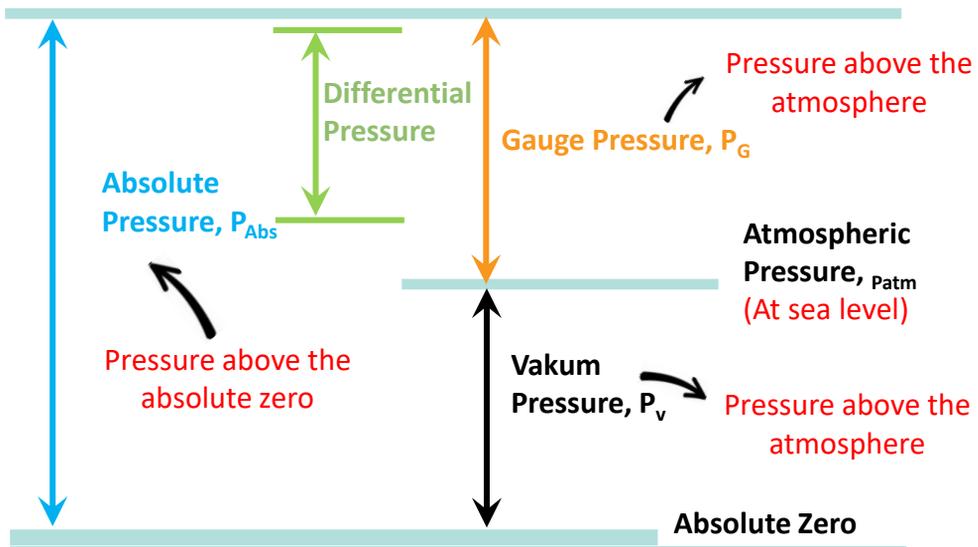


Figure 1.2: Relationships between Absolute, Gauge and Atmospheric Pressure



### 1.3.1 Atmospheric Pressure, $P_{atm}$

- ❑ Atmospheric pressure is also known as barometric pressure.
- ❑ At sea level, used as a reference pressure or as a datum line (zero gauge pressure).
- ❑ The average value of one atmosphere and gradually decreases as an altitude increases.
- ❑ Atmospheric pressure at sea level is approximately **1 Atm = 101.3 kN/m<sup>2</sup>**



Figure 1.3: Sea level

**Note**

1 Atm

Equivalent to

760 mmHg

14.7 Psi

101 kPa

1.013 bar

### 1.3.2 Gauge Pressure, $P_g$

- ❑ Gauge pressure is the pressure measuring instrument that takes atmosphere pressure as the datum.
- ❑ The gauge pressure can be a **positive value (+)** if is above atmospheric pressure or can be a **negative value (-)** if below atmospheric pressure.

Hence pressure gauge reading is showing at atmosphere

0 Psi<sup>g</sup>

**Note**

- Sometimes added with to indicate gauge pressure.
- HOWEVER WITHOUT 'g' ALSO, UNDERSTOOD AS GAUGE PRESSURE



Figure 1.4: Pressure Gauge



### 1.3.3 Vacuum Pressure, $P_v$

- Pressure below the atmospheric pressure is called vacuum pressure.
- Also known as **negative** gauge pressure.

**Note**  
Vacuum pressure is usually indicated with the **negative sign**



Figure 1.5: Pressure Gauge

$$P_v = P_{Abs} + P_{Atm}$$



Figure 1.6 : Galaxy

A perfect vacuum never possible in practice it is not observed anywhere in universe.

**Note**  
SPACE CAN HAVE VACUUM OF  $10^{-10}$  atm

### 1.3.4 Absolute Pressure, $P_{abs}$

□ Pressure above absolute zero is called **ABSOLUTE PRESSURE**

$$P_{Abs} = P_G + P_{Atm}$$

$$44.7 \text{ Psi} = 14.7 \text{ Psi} + 30 \text{ Psi}$$

**EXAMPLE**  
 Absolute pressure is always indicated with pressure gauge



Figure 1.7: Pressure Gauge

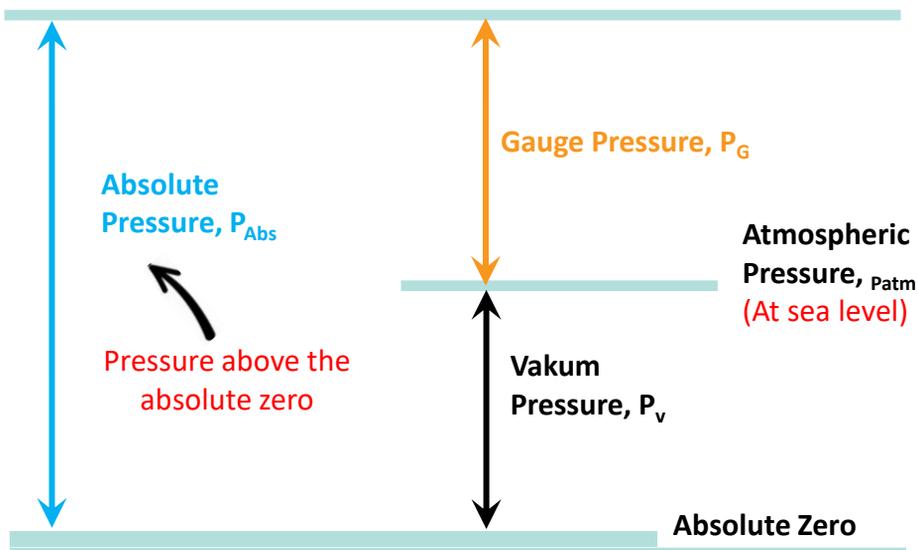


Figure 1.8: Relationships between Absolute, Gauge and Atmospheric Pressure

**Example 1.1**

A bourdon gauge is attached to a boiler which is located at sea level with a reading pressure of 12 bar. If atmospheric pressure is 1.013 bar, calculate the absolute pressure in that boiler (in  $\text{kN/m}^2$ ).

**Solution For Example 1.1**

**Given data :**

$$P_G = 12 \text{ bar} = 12 \times 10^5 \text{ N/m}^2$$

$$P_{atm} = 1.013 \text{ bar} = 1.013 \times 10^5 \text{ N/m}^2$$

**NOTE:**

Convert pressure  
bar to  $\text{N/m}^2$

$$\begin{aligned} P_{abs} &= P_G + P_{atm} \\ &= (12 \times 10^5) + (1.013 \times 10^5) \\ &= 1301300 \text{ N/m}^2 \\ P_{abs} &= \mathbf{1301.3 \text{ kN/m}^2} \end{aligned}$$

**Example 1.2**

Calculate the pressure gauge of air in the cylinder if the absolute pressure is  $225 \text{ kN/m}^2$  and atmospheric gauge is  $99.8 \text{ kN/m}^2$ .

**Solution For Example 1.2**

**Given data :**

$$P_{Abs} = 225 \text{ kN/m}^2$$

$$P_{atm} = 99.8 \text{ kN/m}^2$$

$$P_G = ?$$

$$P_{abs} = P_G + P_{atm}$$

Therefore,

$$\begin{aligned} P_G &= P_{abs} - P_{atm} \\ &= (225 \times 10^3) - (99.8 \times 10^3) \\ &= 125.2 \times 10^3 \text{ N/m}^2 \end{aligned}$$

$$P_G = \mathbf{125.2 \text{ kN/m}^2}$$



### Tutorial 1.1

The gauge pressure is 4.35 bar and the atmospheric pressure is  $100.5 \text{ kN/m}^2$ , calculate the absolute pressure.

Answer :  $535.5 \text{ kN/m}^2$



### Tutorial 1.2

Calculate the pressure gauge of water in the cylinder if the absolute pressure is 6.5 bar and atmospheric gauge is  $101.5 \text{ kN/m}^2$ .

Answer :  $548.5 \text{ kN/m}^2$



### Tutorial 1.3

Calculate the absolute pressure of air in the compressor cylinder if the atmospheric gauge is 756 mmHg and the pressure gauge is 37 kN/m<sup>2</sup>.

Answer : 37.1 kN/m<sup>2</sup>

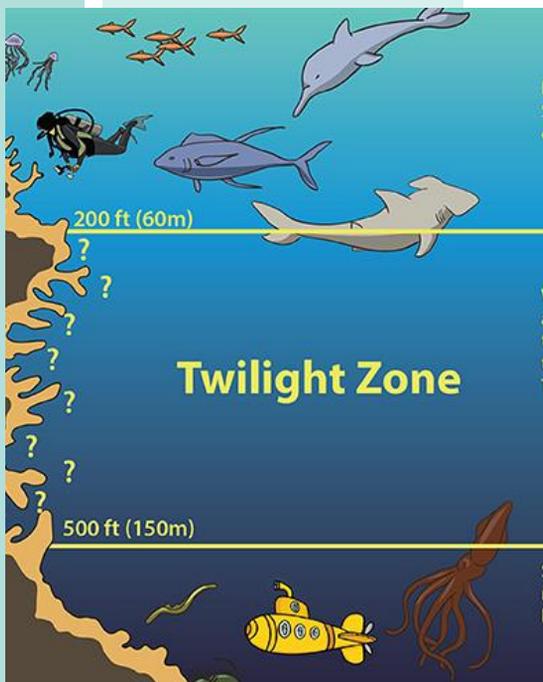


### Tutorial 1.4

A bourdon pressure gauge attached to a boiler located at sea level shows a reading pressure 25 bar, if the atmospheric pressure is 1.013 bar, calculate :

- i) The absolute pressure in N/m<sup>2</sup>
- ii) The pressure head of water in m

Answer : 2601.3 kN/m<sup>2</sup>, 136.595m



# CHAPTER 2

## Relationship Between Pressure & Depth

## 2.1 Pressure In A Liquid

- Fluid as a substance of can flow easily and has not fixed on the shape.
- The pressure in the fluid will be measured according to the depth of the object.
- The deeper the object is in the fluid the higher the pressure value.
- The pressure of an object in a fluid is influenced by depth.

$$P = \rho gh$$

Where

$P$  = Pressure of fluid ( $\text{N/m}^2$ )

$\rho$  (*rho*) = the density of fluid ( $\text{kg/m}^3$ )

$g$  = the acceleration of gravity ( $g = 9.81 \text{ m/s}^2$ )

$h$  = the height of the fluid above the object (m)

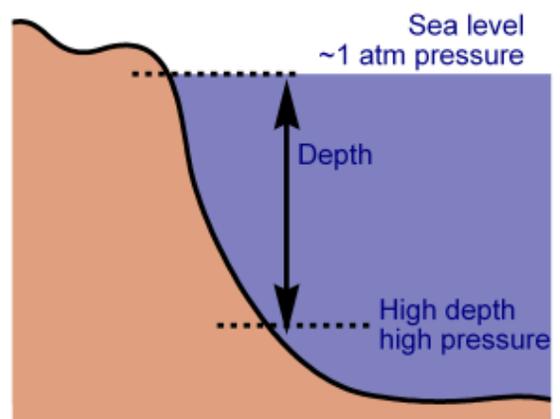


Figure 2.1: Depth on sea level

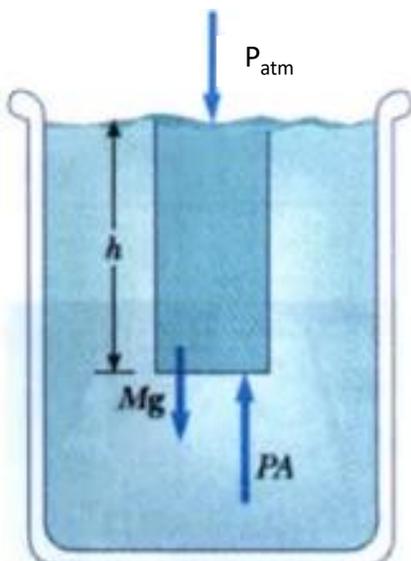


Figure 2.2: The pressure atmospheric in open container.

$$P_{total} = P_{Atm} + P_G$$

$$P_{total} = P_{Atm} + \rho gh$$

### NOTE:

The initial pressure in this case is atmospheric, which is a **CONSTANT**.

$$P_{atm} = 101.3 \text{ kN/m}^2$$

- ❑ If the container is open to the atmosphere above, the added pressure must be included if one is to find the total pressure on an object.
- ❑ The total pressure is the same as absolute pressure ( $P_{abs}$ ) on pressure gauge ( $P_{gauge}$ ) readings, while the gauge pressure is the same as the fluid pressure alone, not including atmospheric pressure ( $P_{atm}$ ).
- ❑ Atmospheric pressure at sea level is about  $101.3 \text{ kN/m}^2$ , which is equivalent to a head of 10.35 m of water or 760 mm of mercury approximately, and it decreases with altitude.

$$P_{total} = P_{Atm} + P_G$$

$$P_{total} = P_{Atm} + \rho gh$$

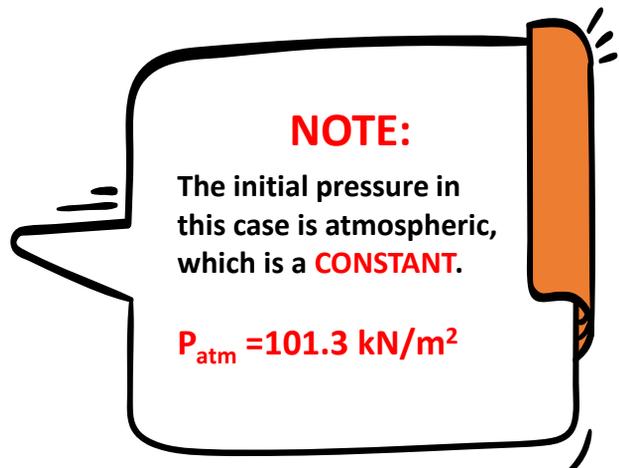
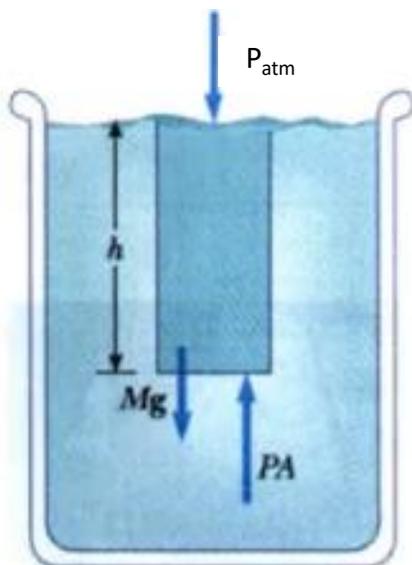


Figure 2.3: The pressure atmospheric in open container.

## 2.2 Relationship Fluid Pressure And Depth

- ❑ Consider a vessel containing some liquid.
- ❑ The liquid will exert pressure on all sides and the bottom of the vessel.
- ❑ The weight of liquid contained in the cylinder is  $\omega hA$ .

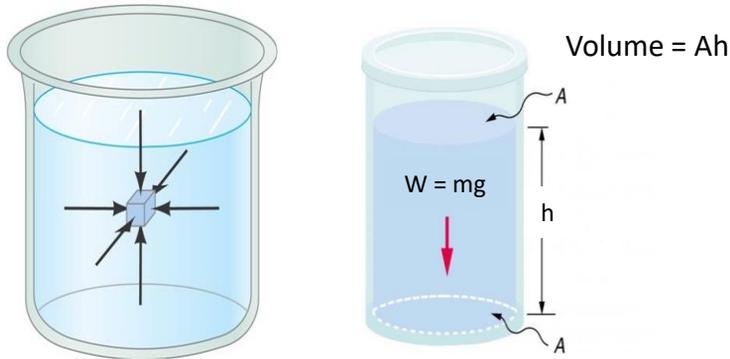


Figure 2.4: Pressure of the fluid

### Where

$W$  = weight of liquid contained in the cylinder

$\omega$  = Specific weight of the liquid

(where is  $\omega_{\text{water}} = 1000 \text{ kg/m}^3 \times 9.81 \text{ m/s}^2$ )

$h$  = the height of liquid in the cylinder (m)

$A$  = Area of the cylinder base

$$W = \omega hA$$

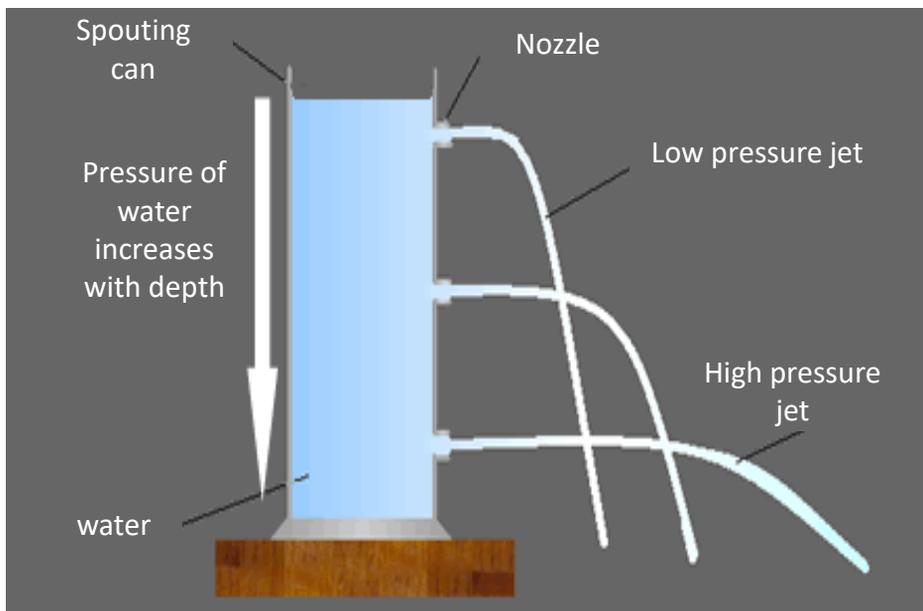


Figure 2.5: Pressure & depth

- ❑ Pressure and depth have a directly proportional relationship.
- ❑ This is due to the greater column of water that pushes down on an object submersed.
- ❑ Conversely, as objects are lifted and the depth decreases, pressure is reduced.

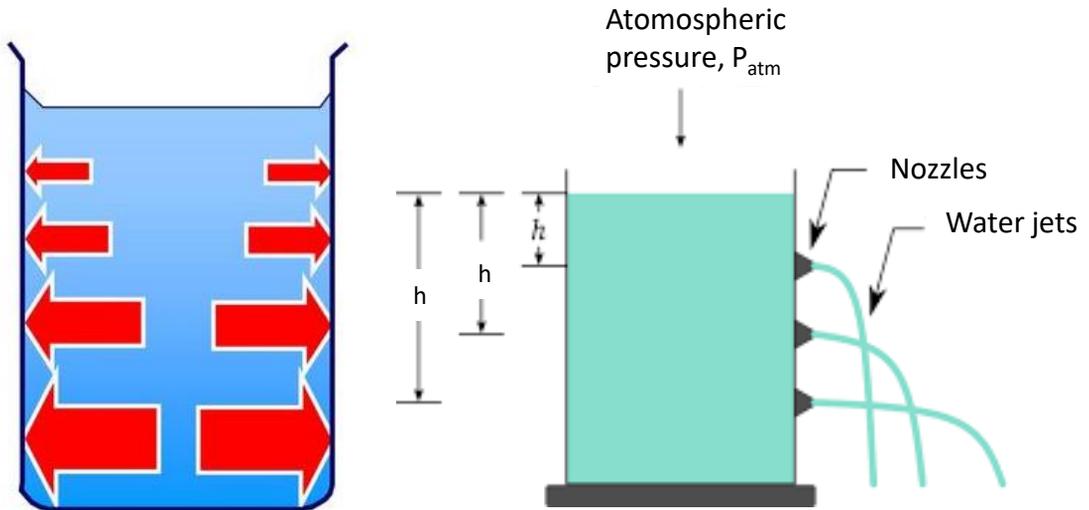


Figure 2.6: Relationships between pressure & depth

- ❑ The pressure, at the bottom of the cylinder, will be due to the weight of the liquid contained in the cylinder. Let this pressure be  $P$ .

$$P = \frac{\text{weight of liquid in the cylinder}}{\text{Area of the cylinder base}}$$

$$P = \frac{\omega h A}{A}$$

$$= \rho g h$$

- ❑ This equation shows that the intensity of pressure at any point, in a liquid, is proportional to its depth as measured from the surface (as  $\rho$  is constant for the given liquid).

## 2.2 Pressure Depends on Shape not Shape

- ❑ The pressure of a fluid is independent of the shape of the container in which the fluid is stored.
- ❑ From the formula of the pressure, can conclude that the pressure is only dependent on depth or height, the density of the fluid, and gravity.
- ❑ Therefore, in all the vessels below the pressure is same at the same depth.

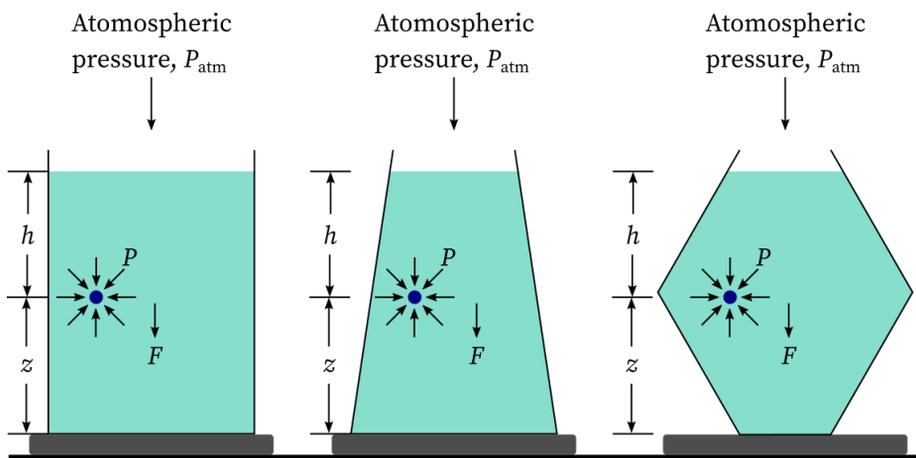


Figure 2.7: Pressure is independent of shape of a container.

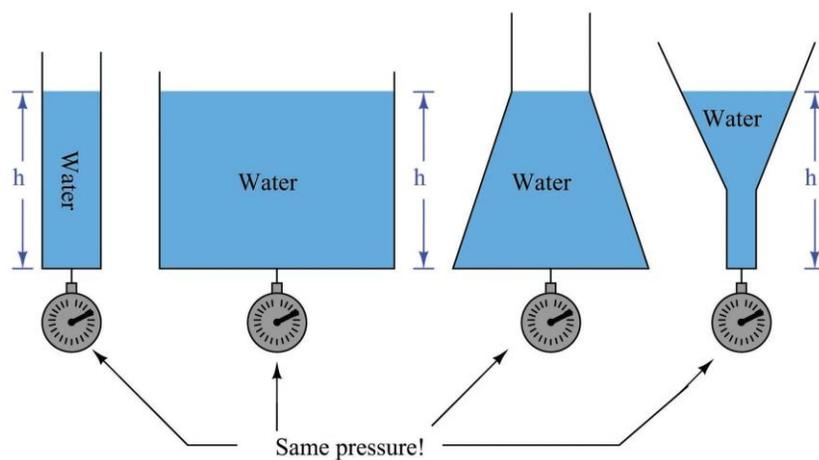


Figure 2.8: Relationships between pressure and depth



### Example 2.1

Calculate the height of a water column which has the pressure of  $20 \text{ N/m}^2$ .

(Take into consideration specific weight of water,  $\omega_{\text{water}} = 1000 \text{ kg/m}^3 \times 9.81 \text{ m/s}^2$ )

### Solution for Example 2.1

**Given data :**

$$P = 20 \text{ N/m}^2$$

$$\omega = \rho \times g$$

$$\omega = 1000 \times 9.81$$

$$= 9810 \text{ N/m}^3$$

$$P = \rho gh$$

$$P = \omega h$$

$$20 = (9810) \times h$$

$$h = \frac{20}{9810}$$

$$h = 2.039 \times 10^{-3} \text{ m}$$



### Example 2.2

Calculate the pressure at a point of 1km depth in the sea bed in Pascal and bar units. Given the density of seawater is  $1025 \text{ kg/m}^3$ .

### Solution for Example 2.2

**Given data :**

$$\rho = 1025 \text{ kg/m}^3$$

$$g = 9.81 \text{ m/s}^2$$

$$h = 1 \text{ km} = 1000 \text{ m}$$

$$P = \rho gh$$

$$= 1025 \times 9.81 \times 1000$$

$$= 10055250 \text{ Pa}$$

$$= 100.553 \text{ bar}$$

**OR**

$$\begin{aligned} \omega &= \rho g \\ &= 1025 \times 9.81 \\ &= 10055.25 \end{aligned}$$

$$\begin{aligned} P &= \omega h \\ &= 10055.25 \times 1000 \\ &= 10055250 \text{ Pa} \\ &= 100.553 \text{ bar} \end{aligned}$$



### Tutorial 2.1

Find the height of a water column which is equivalent to the pressure of  $0.5 \text{ kN/m}^2$ . (Take into consideration specific weight of water,  $\omega_{\text{water}} = 1000 \text{ kg/m}^3 \times 9.81 \text{ m/s}^2$ ).

*Answer:  $h = 0.357 \text{ m}$*



### Tutorial 2.2

The pressure at a point in the sea bed is 100.55 bar. Find height:

- Express this pressure as a head of freshwater of density  $1000 \text{ kg/m}^3$ .
- What is this pressure as a head of mercury of specific gravity 13.6?

*Answer:  $h = 1025 \text{ m}$ ,  $h = 75.37 \text{ m}$*



# CHAPTER 3

## Pascal's Law & Hydraulic Jack



### 3.1 Pascal's Law

- ❑ Pascal's principle, also called **Pascal's law**, in fluid (gas or liquid) mechanics, statement that in a fluid at rest in a closed container, a pressure change in one part is transmitted without loss to every portion of the fluid and to the walls of the container.
- ❑ If external pressure is applied to a confined fluid, the pressure at every point within the fluid increases by that amount.
- ❑ The principle was first enunciated by the French scientist **Blaise Pascal**.
- ❑ Applications: Hydraulic lift and brakes.

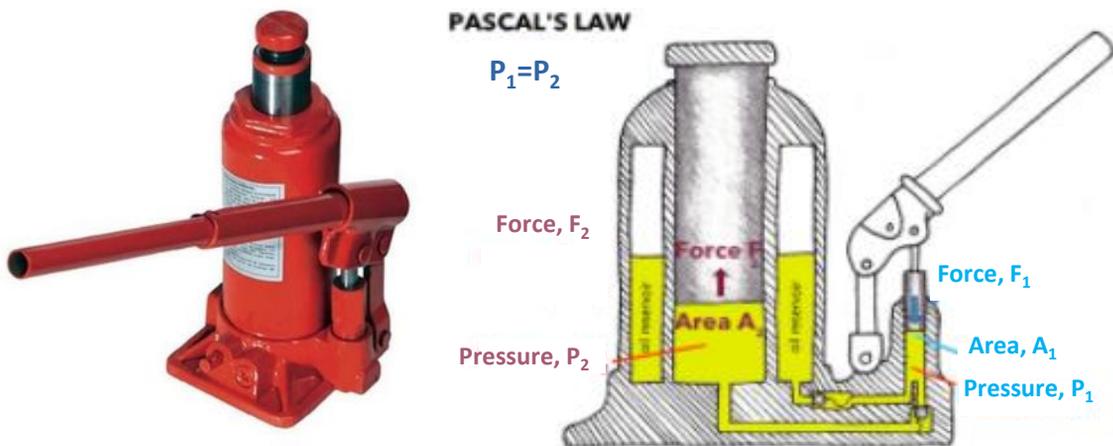


Figure 3.1: Hydraulic Jack

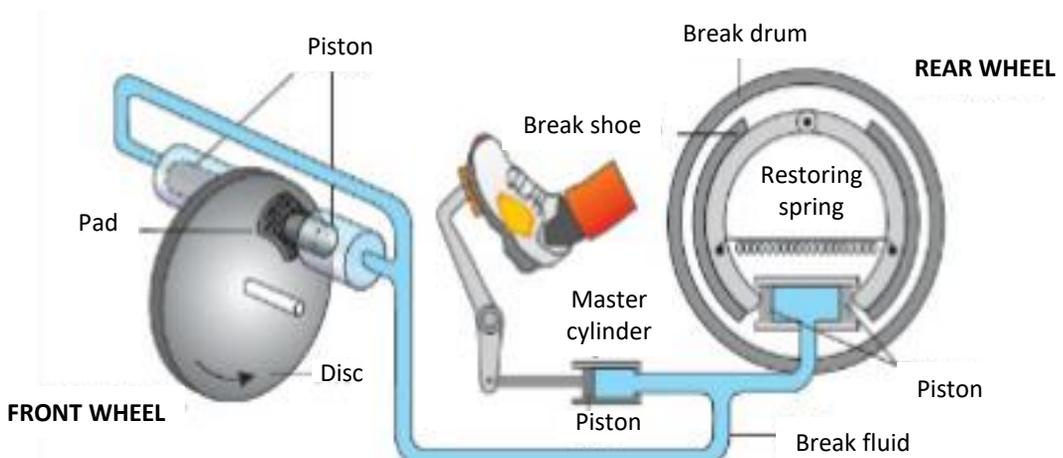


Figure 3.2: System break

## 3.2 Hydraulic Jack

- Hydraulic systems use the principle that pressure is transmitted throughout a liquid to allow a force applied in one part of the system to be transferred to another.
- All hydraulic systems use two-piston linked via a pipe carrying a special oil called hydraulic fluid.
- According to Pascal's principle, in a hydraulic system, the pressure exerted on a piston produces an equal increase in pressure on another piston in the system.
- If the second piston has an area 10 times that of the first, the force on the second piston is 10 times greater, though the pressure is the same as that on the first piston.
- This effect is exemplified by the hydraulic press, based on Pascal's principle, which is used in such applications as hydraulic brakes.

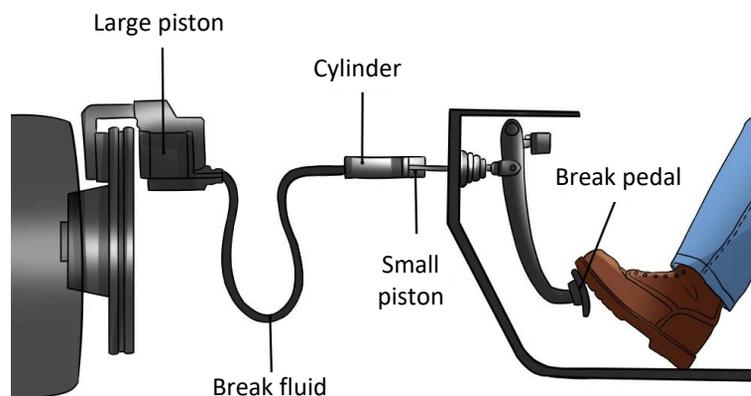


Figure 3.3 System hydraulic break car

- All hydraulic brake systems (eg: in a car) use a small master piston and a bigger slave piston.
- The matter piston is used to apply a force. This puts the liquid under pressure.
- The pressure is transmitted to the piston on all four wheels of the car.

### 3.3 Application & Example



Figure 3.4: Hydraulic lift

#### Hydraulic Lift

- ❑ A system used to lift heavy objects.
- ❑ Using the “lift’ and “work” methods of force is used in this principle.

#### Hydraulic Jack

- ❑ Used in the automotive industry to lift cars during maintenance and repairs.
- ❑ Hydraulic angles are used to lift objects.
- ❑ The hydraulic handle moves up and down repeatedly so that the hydraulic corners are lifted.



Figure 3.5: Hydraulic jack

#### Hydraulic Brakes

- ❑ Used in motorcycle, car and truck brake systems.
- ❑ The force applied to the brake pedal that sends pressure to the wheels of the vehicle.
- ❑ The result of this frictional force on the brake discs or brake drums causes the vehicle to stop.

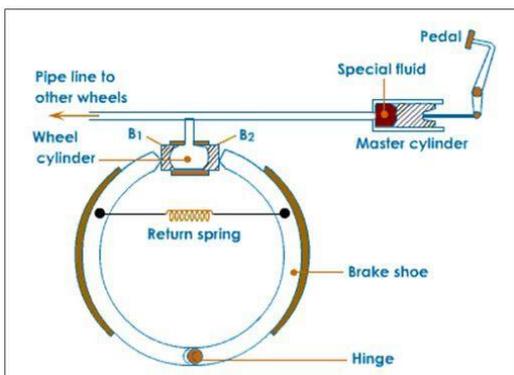


Figure 3.6: Hydraulic break

### 3.3 Application & Example

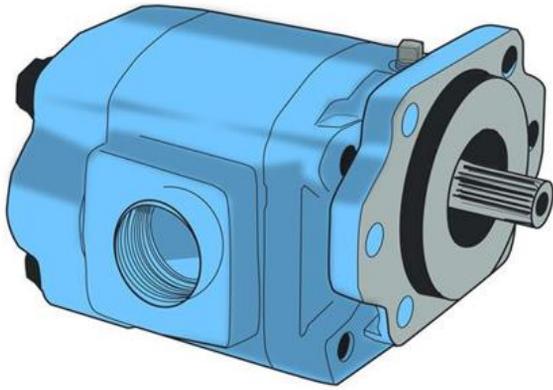


Figure 3.7: Hydraulic pump

### Hydraulic Pumps

- ❑ Hydraulic pumps, which convert mechanical energy into hydraulic energy, facilitate the movement of a fluid.
- ❑ Hydraulic pumps help in the discharge of fluid.
- ❑ It is equipped with a small cylinder connected to a large cylinder, and both the cylinders are filled with oil.
- ❑ Compressed air introduced to the small cylinder exerts pressure on the surface of the oil.
- ❑ This pressure is transmitted by the oil to the large cylinder where the pressure acts on a large piston to produce a force large enough to lift a car.

### Aircraft Hydraulic System

- ❑ A few components form the braking system in cars.
- ❑ When force is applied on the brake pedal, there is a movement of the piston and rod in the master cylinder.
- ❑ A liquid which is known as brake or hydraulic fluid, enclosed in the container, is used to transmit the pressure from the brake pedal to the wheels of the vehicle against the brake discs or brake drums.
- ❑ The frictional force between these force components causes the vehicle to stop.
- ❑ Hydraulic brakes are used in cars, motorcycles and lorries.

#### HYDRAULIC SYSTEM

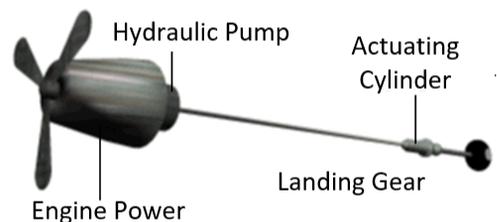


Figure 3.8: Hydraulic System

### 3.4 Formula Pascal's Law

#### PRINCIPLE

- The pressure applied to an enclosed fluid is transmitted equally in all directions and to all parts of the container.

#### FORMULA

$$P_1 = P_2$$

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

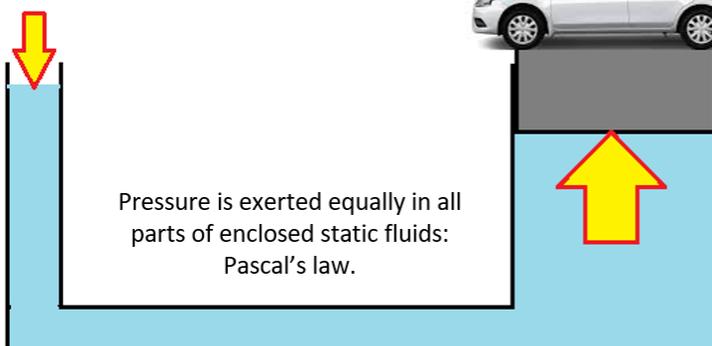
#### When

$P$  = Pressure

$F$  = Action Force

$A$  = Area which the force is applied.

Pressure is exerted on fluid in small cylinder, usually by a compressor.



Though the pressure is same, it is exerted over a much larger area, giving a multiplication of force that lift the car.

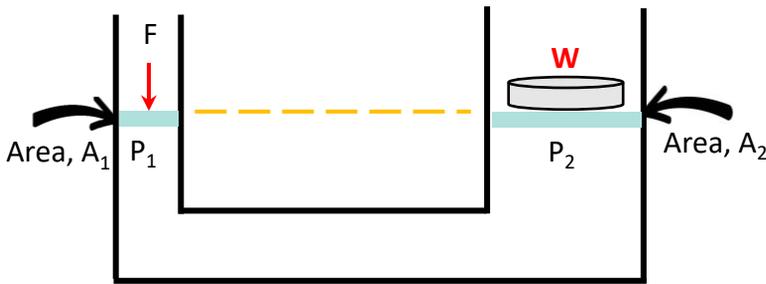
The force in the small cylinder must be exerted over a much larger distance. A small force exerted over a large distance is traded for a large force over a small distance.

Figure 3.9: System Pascal

### 3.5 The Situations of Hydraulic Jack

1

The small and the large piston are at the same level



Equation for the piston the same level

$$P_1 = P_2$$

$$\frac{F}{A_1} = \frac{W}{A_2}$$

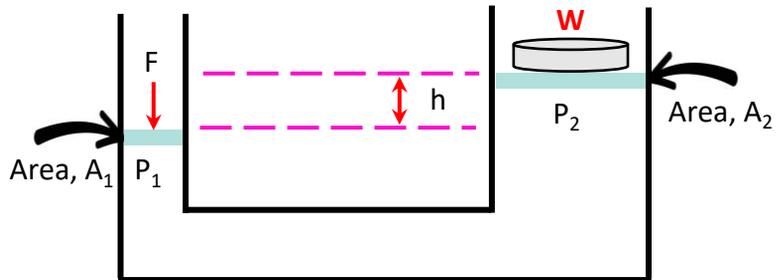
2

The small piston is below the large piston

Equation for the small piston is below the large piston

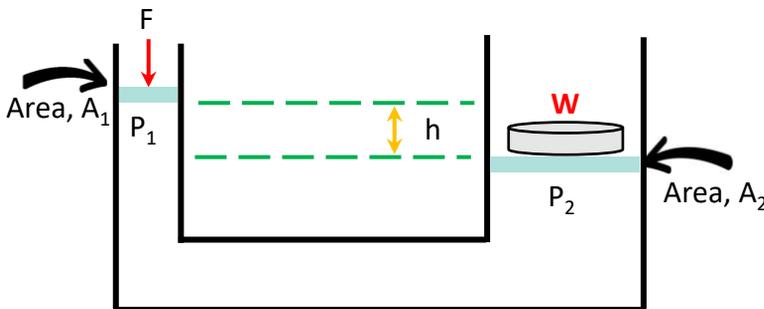
$$P_1 = P_2 + \rho gh$$

$$\frac{F}{A_1} = \frac{W}{A_2} + \rho gh$$



3

The large piston is below the small piston



Equation for the small piston is below the large piston

$$P_2 = P_1 + \rho gh$$

$$\frac{W}{A_2} = \frac{F}{A_1} + \rho gh$$

**Example 3.1**

A force of 400 N is applied on the small cylinder which has an area 20 cm<sup>2</sup>. Determine the load that can be lifted by the hydraulic jack if the area of the larger cylinder is 180 cm<sup>2</sup>.



**Solution for Example 3.1**

**Given data :**

$$F = 400 \text{ N}$$

$$A_1 = 20 \text{ cm}^2 = 20 \times 10^{-4} \text{ m}^2$$

$$A_2 = 180 \text{ cm}^2 = 180 \times 10^{-4} \text{ m}^2$$

$$P_1 = P_2$$

$$\frac{F}{A_1} = \frac{W}{A_2}$$

$$\frac{400}{20 \times 10^{-4}} = \frac{W}{180 \times 10^{-4}}$$

$$W = 3600 \text{ N}$$

 **Example 3.2**

A force, ( $F$ ) at 850N is applied to hydraulic jack system. The area of the small piston is 18  $\text{cm}^2$  and the larger piston is 180  $\text{cm}^2$ . Consider the mass density,  $\rho_{\text{water}}$  of the liquid in the jack is  $1000 \text{ kg/m}^3$ . Calculate the load,  $W$  if:

- i. The pistons are located at the same level.
- ii. The small piston is 85 cm below the large piston.
- iii. The large piston is 50 cm below the small piston.

**Solution for Example 3.2**

**Given data :**

$$F = 850 \text{ N}$$

$$A_1 = 18\text{cm}^2 = 18 \times 10^{-4}\text{m}^2$$

$$A_2 = 180\text{cm}^2 = 180 \times 10^{-4}\text{m}^2$$

- i. The pistons are located at the same level.

$$P_1 = P_2$$

$$\frac{F}{A_1} = \frac{W}{A_2}$$

$$\frac{850}{18 \times 10^{-4}} = \frac{W}{180 \times 10^{-4}}$$

$$W = 8500\text{N}$$

- ii. The small piston is 85 cm below the large piston.

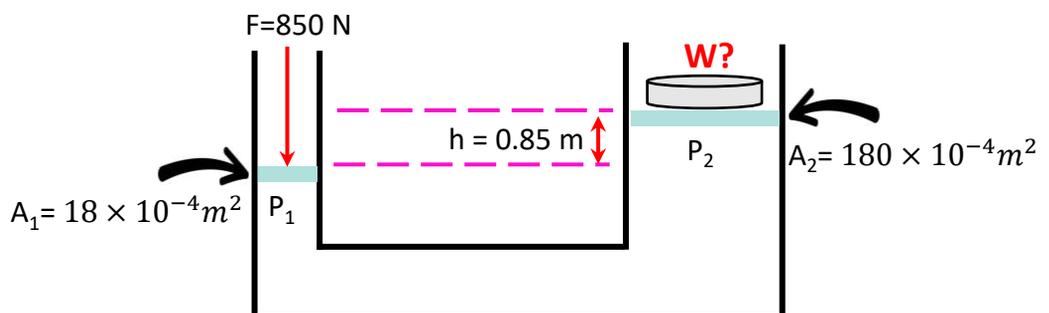
$$P_1 = P_2 + \rho gh$$

$$\frac{F}{A_1} = \frac{W}{A_2} + \rho gh$$

$$\frac{850}{18 \times 10^{-4}} = \frac{W}{180 \times 10^{-4}} + (1000 \times 9.81 \times 0.85)$$

$$472222.222 = \frac{W}{180 \times 10^{-4}} + 8338.5$$

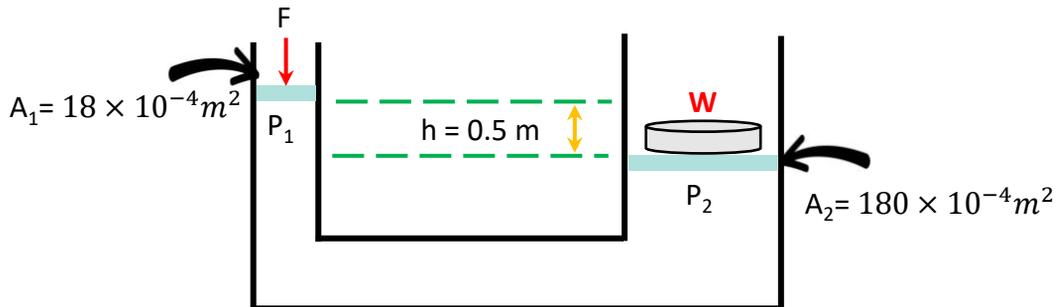
$$W = 8349.91\text{N}$$





### Solution for Example 3.2

iii. The large piston is 50 cm below the small piston.



$$P_2 = P_1 + \rho gh$$

$$\frac{W}{A_2} = \frac{F}{A_1} + \rho gh$$

$$\frac{W}{180 \times 10^{-4}} = \frac{850}{18 \times 10^{-4}} + (1000 \times 9.81 \times 0.5)$$

$$\frac{W}{180 \times 10^{-4}} = 472222.222 + 4905$$

$$W = 8588.29N$$



### Tutorial 3.1

A hydraulic jack with an 8 cm and 25 cm diameter cylinder. To lift the load of 1100 kg at the larger cylinder, a force has been applied to the little cylinder. Calculate the small cylinder's force  $F$ .

Answer : 1107.07 N



## Tutorial 3.2

A jack hydraulic cylinder is applied a force of 900 N on a small piston having a surface area of 22 cm<sup>2</sup> and a surface area on a large piston of 250 cm<sup>2</sup>. Find the value of the load, W if

- i. Position the piston at the same level.
- ii. Position the large piston 80cm below the level of the small piston.

Consider that the liquid in the jack has a mass density,  $\rho_{water}$  is 1000 kg/m<sup>3</sup>.

Answer : 10.227 kN, 10.423 kN



### Tutorial 3.3

A hydraulic jack has a large piston diameter three times that of a small piston. The diameter of the small piston is 400mm and the force applied to the small piston is 40kN. If the specific gravity for oil is 0.85. Calculate the force required to raise a large piston as high as 1.5m from the small piston.

Answer : 420.7 N



# CHAPTER 4

## Manometer, Piezometer & Barometer



## 4.1 Instrument For Pressure Measurement

- ❑ A manometer gauge is an extremely basic but very effective device that is used to measure pressure.
- ❑ The most basic measure is gas/liquid pressure against atmospheric pressure.
- ❑ Manometers come in a variety of shapes and sizes, and while the principle for measuring pressure differential is the same as the degree of accuracy can be enhanced.

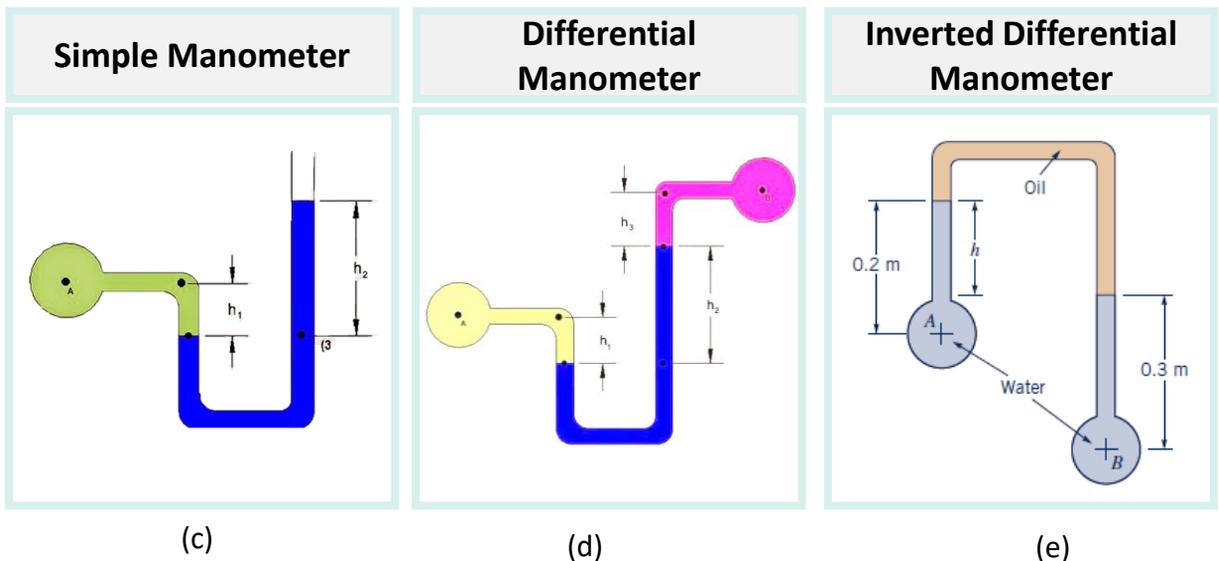
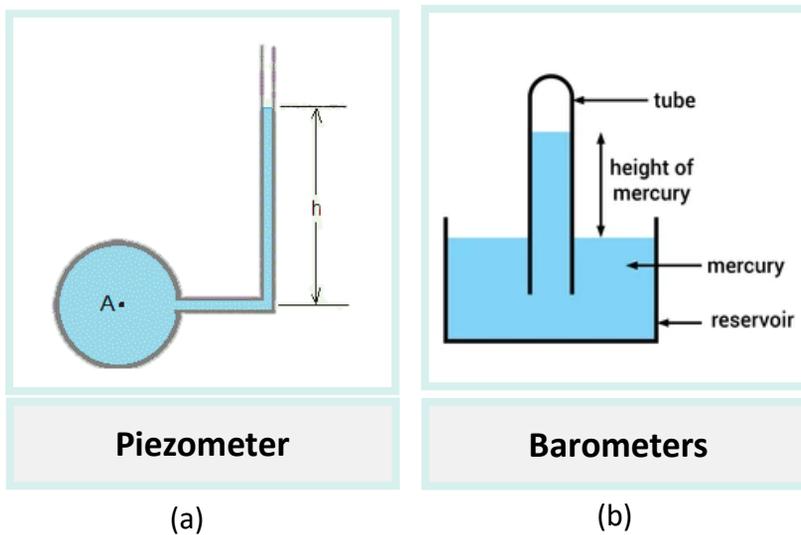


Figure 4.1: Type of Pressure Measurement

## 4.2 Piezometer Tube

- ❑ A piezometer is an open tube that is used to measure pressure via the rise of a fluid column.
- ❑ In its simplest form, it consists of a transparent tube open from other ends.
- ❑ The diameter of the tube should be  $> \frac{1}{2}$ " to avoid capillarity action.
- ❑ Piezometers may be connected to the side or bottom of the pipe to avoid eddies that are produced in the top region of pipe.

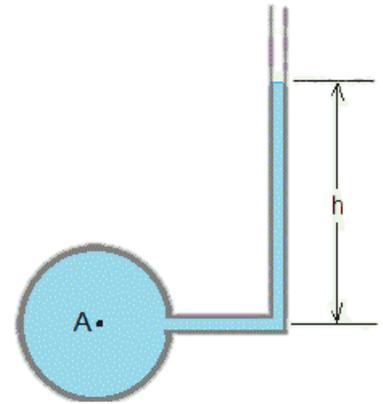


Figure 4.2 : Piezometer tube

### When

$P_A =$  Pressure at point A  
 $\rho =$  density  
 $g =$  gravity (9.81 m/s)  
 $h =$  height of liquid

$$P_A = \rho gh$$



Figure 4.3 Piezometer tube

### LIMITATIONS:

- i. It must only be used for liquids.
- ii. It should not be used for high pressure.
- iii. It cannot measure vacuum (-ve) pressure.

## 4.2 Barometers

- ❑ An instrument used to measure changes in air pressure.
- ❑ Used for measuring atmospheric pressure.
- ❑ The pressure of the outside air forces the mercury in the cistern upward into the vacuum chamber.

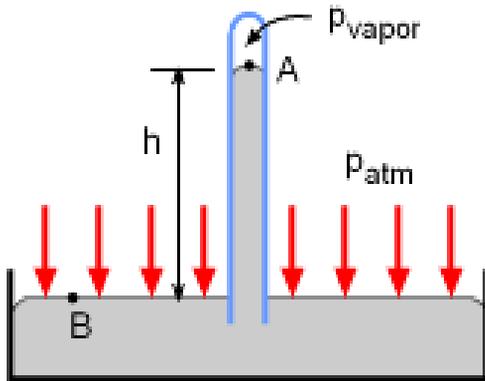


Figure 4.4 : Barometer

The vapor pressure in the glass tube is, "negligibly small" hence the atmospheric pressure is simply given by the height of the mercury column.

$P_{vapor} \approx \text{negligibly small}$

$$P_{Atm} = \rho gh + P_{vapor}$$

$$P_{Atm} = \rho gh$$

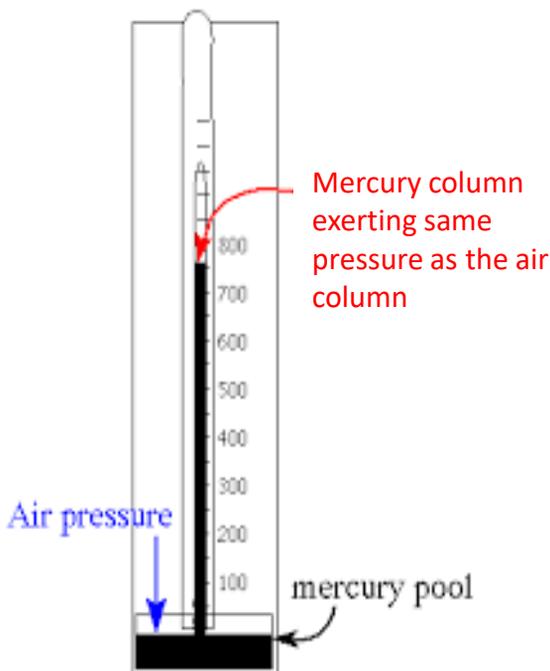


Figure 4.5 : Thermometer

When

$P = \text{Pressure}$   
 $\rho = \text{density}$   
 $g = \text{gravity (9.81 m/s)}$   
 $h = \text{height of liquid}$



<https://www.youtube.com/watch?v=EkDhlzA-lwI>

## 4.2.1 Types of Barometers

- Absolute atmospheric pressure can be read off the tube by suitably calibrating it.
- There are two types of Barometer.
  - i. Mercury Barometer
  - ii. Aneroid Barometer

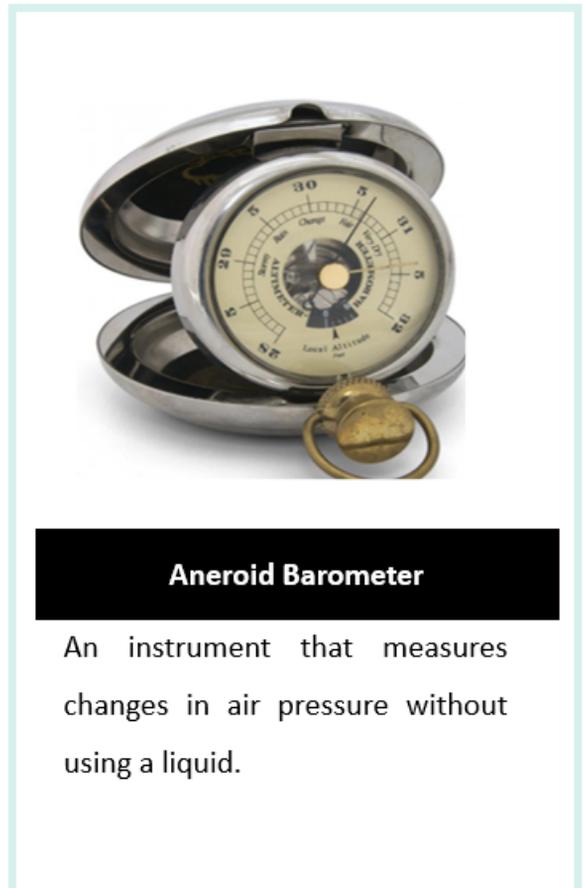
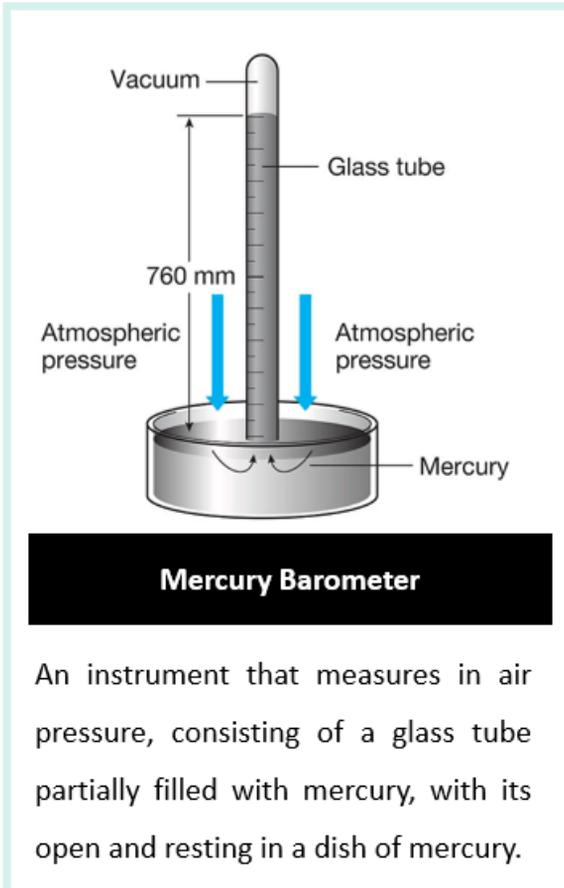
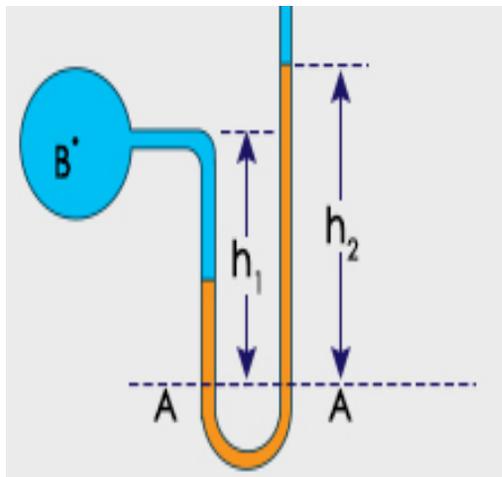


Figure 4.6 : Types of Barometer

### 4.3 Manometer



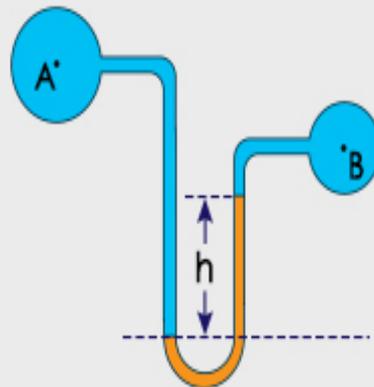
Simple Manometer

- ❑ It consists of a U shaped tube, part of which is filled with manometric fluids.
- ❑ One end of the tube is connected with the pipe whose pressure is required to be determined.
- ❑ Due to pressure, the level of manometric fluid rises on one side while it falls on other side.
- ❑ The difference in levels is measured to estimate the pressure.
- ❑ Application : Piezometer, U-Tube and Single Column.

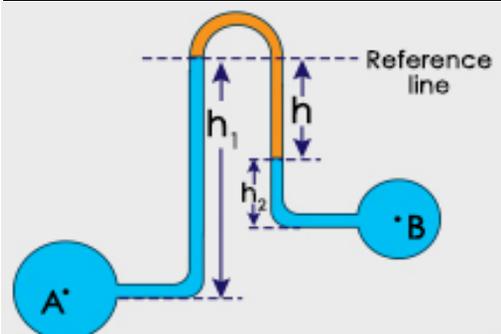
- ❑ Differential Manometers are the devices used for measuring the difference of pressure between two points in a pipe or two different pipes.
- ❑ Similar to the previous type, A and B are points at different levels with liquids having different specific gravity.



Differential Manometer



Inverted Differential Manometer



### 4.3 Differences Between Barometers & Manometers

- Manometers and barometers both measure atmospheric pressure, but manometers can measure the pressures of other gases more accurately.
- All barometers are manometers, but not all manometers are barometers.
- Barometers have one open end and a closed end with a vacuum.
- Manometers can be open on both ends or closed on one end.
- Manometers may have a high-pressure gas attached to one end to be measured.
- Both barometers and manometers now come in digital forms.

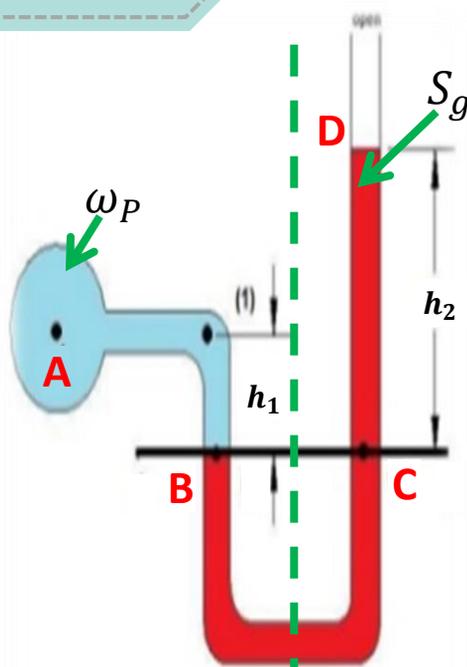
**Barometers**

**VS**

**Manometers**

Differences	Barometers	Manometers
Close-ended with Vacuum	✓	✓
Open-ended tube	✗	✓
Modern digital sensors	✓	✓
Mercury Inside	✓	✓
Light Liquids Inside	✗ (Unusual)	✓
Calculate Atmospheric Pressure	✓	✓
Calculate Other Pressures	✗	✓

### 4.3 Simple U Manometer



#### Pressure In Left

$P_B$  = Pressure, at  $A$  + Pressure due to depth,  $h_1$  of fluid  $P$

$$P_B = P_A + \omega_P h_1$$

$$= P_A + \rho_P g h_1$$

#### Pressure In Right

$P_C$  = Pressure  $P_D$  at  $D$  + Pressure due to depth  $h_2$  of fluid  $Q$

**\*\*But,**

$P_D$  = Atmospheric pressure  
= Zero gauge pressure

$$P_C = 0 + \omega_Q h_2$$

$$= 0 + \rho_Q g h_2$$

Since

$$P_B = P_C$$

$$P_A + \rho_P g h_1 = \rho_Q g h_2$$

$$P_A = \rho_Q g h_2 - \rho_P g h_1$$

$$= \omega_Q h_2 - \omega_P h_1$$

**\*\*Pressure at point B and C is the same.**

### Example 4.1

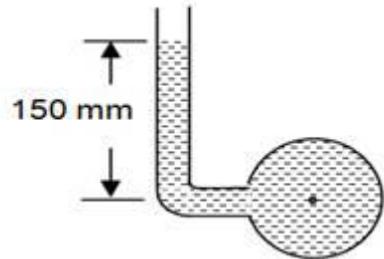
A piezometer is used to measure the pressure of water connected to a pipe. The water level on the piezometer increased by 150mm. Find the value of water pressure.

#### Solution for Example 4.1

Given data :

$$h = 150 \text{ mm} = 0.15 \text{ m}$$

$$\rho_{\text{water}} = 1000 \text{ kg/m}^3$$



$$\begin{aligned} P_A &= \rho_{\text{water}}gh \\ &= 1000 \times 9.81 \times 0.15 \\ &= 1471.5 \text{ N/m}^2 \end{aligned}$$

### Example 4.2

What is the difference in pressure head, measured by a mercury-oil differential manometer for a 20 cm difference of mercury level? (Specific Gravity of oil = 0.8)

#### Solution for Example 4.2

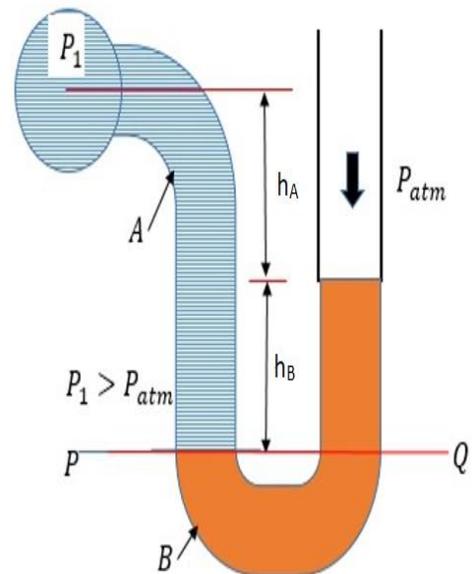
Given data :

$$h = 20 \text{ cm} = 0.2 \text{ m}$$

$$Sg_{\text{oil}} = 0.8$$

$$Sg_{\text{Hg}} = 13.6$$

$$\begin{aligned} h_A S_A &= h_B S_B \\ 0.2 \times 13.6 &= h_B \times 0.8 \\ 2.72 &= h_B \times 0.8 \\ h_B &= 3.4 \text{ m} \end{aligned}$$



**Example 4.3**

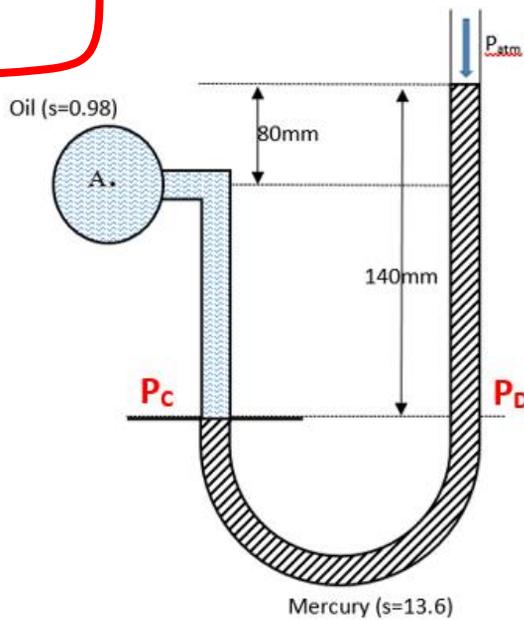
A U-tube manometer was used to measure oil pressure with a specific gravity value of 0.98. On the right side the tube is open to the atmosphere and on the left side the tube is connected to the pipe. Referring to the diagram below, mercury is used in a tube 80cm above the oil level. If the total height difference is 140mm, what is the value of the pressure at point A?

**Solution for Example 4.3**

**Given data :**

$$Sg_{oil} = 0.98$$

$$Sg_{Hg} = 13.6$$



**Pressure at Point C**

$$P_C = P_A + \rho_{oil}gh_1$$

**Pressure at Point D**

$$P_D = P_{Atm} + \rho_{Hg}gh_2$$

$$P_C = P_D$$

$$P_A + \rho_{oil}gh_1 = P_{Atm} + \rho_{Hg}gh_2$$

$$P_A + [980 \times 9.81 \times (0.14 - 0.08)] = P_{Atm} + (13600 \times 9.81 \times 0.14)$$

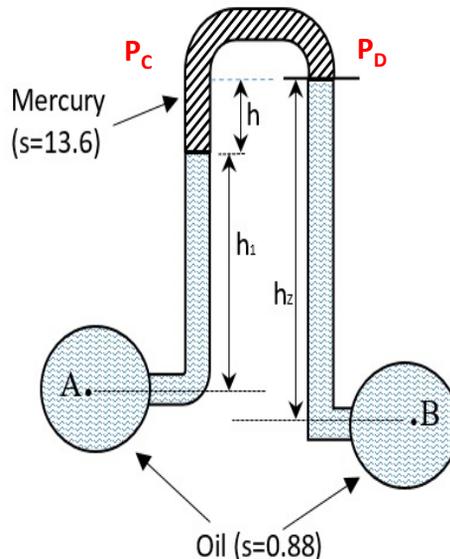
$$P_A + 576.828 = 0 + 18678.24$$

$$P_A = 18101.412 \text{ N/m}^2$$

 Example 4.4

An inverted U manometer is used to measure the pressure between two points as in the diagram below. If the value of the pressure difference between points A & B is  $45.3 \text{ kN/m}^2$ ,  $h = 38 \text{ cm}$  and  $h_2 = 95 \text{ cm}$ . Calculate the value of the height at  $h_1$ .

**Solution for Example 4.4**



**Pressure at Point C**

$$P_C = P_A - \rho_{oil}gh_1 - \rho_{Hg}gh$$

**Pressure at Point D**

$$P_D = P_B - \rho_{oil}gh_2$$

$$P_C = P_D$$

$$P_A - \rho_{oil}gh_1 - \rho_{Hg}gh = P_B - \rho_{oil}gh_2$$

$$P_A - (880 \times 9.81 \times h_1) - (13600 \times 9.81 \times 0.38) = P_B - (880 \times 9.81 \times 0.95)$$

$$P_A - 8632.8h_1 - 50698.08 = P_B - 8201.16$$

$$(P_A - P_B) - 8632.8h_1 = -8201.16 + 50698.08$$

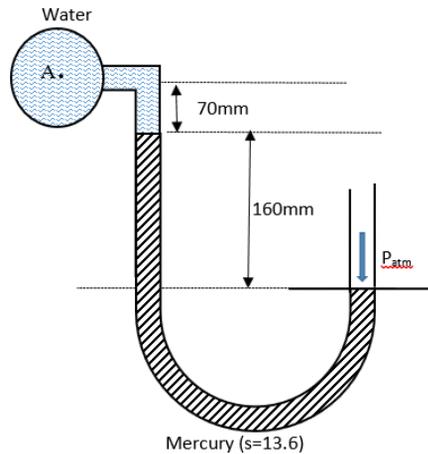
$$(45.3 \times 10^3) - 8632.8h_1 = 42496.92$$

$$h_1 = \frac{42496.92 - (45.3 \times 10^3)}{-8632.8}$$

$$h_1 = 0.325 \text{ m}$$

 Tutorial 4.1

The negative pressure in the pipe was measured using a U-tube manometer using mercury. If the difference in mercury levels in the two pipes is 160 mm and the height of water in the left limb from the pipe's center is 70 mm and the right limb is open to the atmosphere, calculate the vacuum pressure in the pipe.

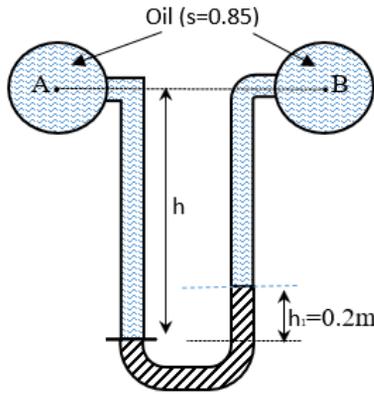


*Answer:  $P_A = -686.7 \text{ N/m}^2$*



## Tutorial 4.2

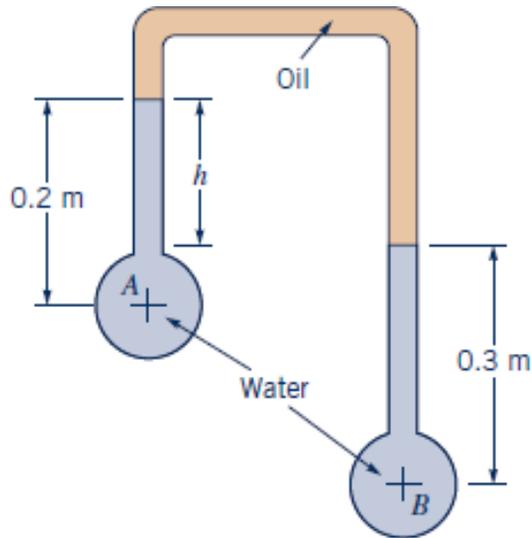
The difference in pressure in pipe at two points A and B were measured using a differential manometer. The specific gravity of oil in the pipe is 0.85. If the difference in the mercury level in the two limbs is 200 mm, calculate the pressure difference.



Answer:  $P_A - P_B = 25015.5 \text{ N/m}^2$

Tutorial 4.3

The inverted U-tube manometer in the figure below contains oil and water as shown. The pressure differential between pipes A and B,  $P_B - P_A$  is 8 kPa. Determine the differential reading,  $h$ .



Answer:  $h = 0.795 \text{ m}$



# CHAPTER 5

## Concept OF Buoyancy



## 5.1 Concept of Buoyancy

- ❑ Buoyancy is a force exerted by a fluid, that opposes an object's weight.
- ❑ In a column of fluid, pressure increases with depth as a result of the weight of the overlying fluid.
- ❑ An object in a fluid has an upward force equal to the weight of the fluid it displaces.

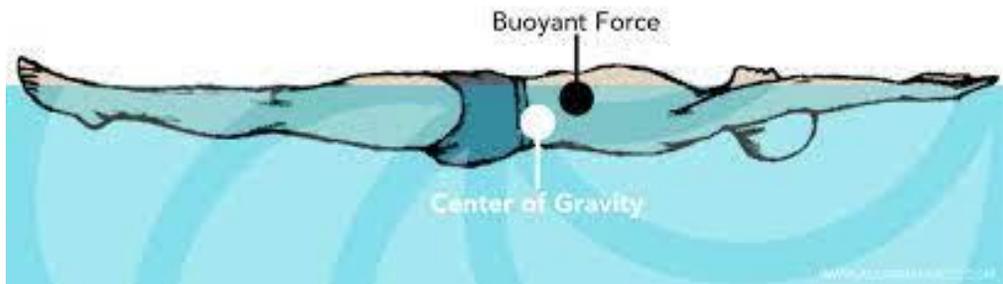


Figure 5.1 : Buoyancy force

**Buoyant float must be equal to the force of gravity**

Buoyancy Force = Weight of the Displaced Water

$$F_B = W_{object} = W_{fluid\ displaced}$$

$$F_B = W = \rho g v$$

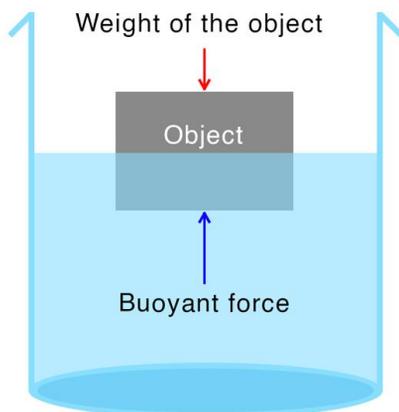


Figure 5.2 : Buoyancy force float

**When**

$F_B$  = Buoyancy Force

$\rho$  = density of fluid

$g$  = gravity ( $9.81 \text{ m/s}^2$ )

$v$  = volume

## 5.2 Why Do Objects Float or Sink In Water?

### Sink

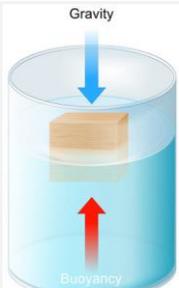


Iron

### Relative density $>1$

- Object sink in water
- If the density of an object is **more than water**
- An object sinks in water because **buoyant force is less** than the force exerted due to weight of on object

### Neutral



Wood

### Relative Density =1

- An object floats but it is full submerged
- If the density of an object is **the same as water**
- An object sinks in water because **buoyant force is equal** the force exerted due to weight of on object

### Floats



Cork

### Relative density $<1$

- An object floats in water
- If the density of an object is **smaller than water**
- An object floats in water because **buoyant force is more** than the force exerted due to weight of on object

## 5.2 Buoyancy and Archimedes Principle

- The Archimedes principle follows the law of buoyancy.
- According to this principle, when an object is immersed in a fluid, partially or wholly, it displaces the fluid.
- The weight lost by the object is equal to the weight of an equal volume of the displaced fluid, according to Archimedes' principle.
- If the object has a volume of  $V$ , then it displaces a volume  $V$  of the liquid when it is fully submerged.
- If only a part of the volume is submerged, the object can only displace that much liquid.

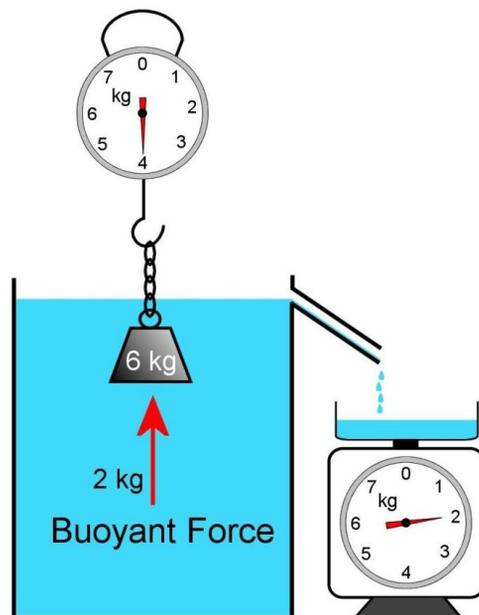


Figure 5.3 : Archimedes Principle

$$\text{Apparent weight} = \text{Weight of object (in the air)} - \text{Thrust force (buoyancy)}$$

 **Example 5.1**

Calculate the resulting force, if a steel ball of radius 6cm is immersed in water.

### Solution for Example 5.1

**Given data :**

$$r = 6 \text{ cm} = 0.06 \text{ m}$$

$$\text{Volume of steel ball, } V = \frac{4}{3}\pi r^3$$

$$V = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3}\pi \times (0.06^3)$$

$$= 9.05 \times 10^{-4} \text{ m}^3$$

$$\rho_{\text{water}} = 1000 \text{ kg/m}^3$$

**Archimedes Principle**

$$F_B = \rho g V$$

$$= 1000 \times 9.81 \times 9.05 \times 10^{-4}$$

$$= 8.87 \text{ N}$$

 **Example 5.2**

Calculate the buoyant force, if a floating body is 95% submerged in water. The density of water is  $1000 \text{ kg/m}^3$ .

### Solution for Example 5.2

**Given data :**

$$\rho_{\text{water}} = 1000 \text{ kg/m}^3$$

**Archimedes Principle**

$$F_B = \rho g V$$

$$V_B \rho_B g = \rho g V$$

$$\rho_B = \frac{V \rho}{V_B}$$

**Since 95% of the body is immersed**

$$0.95 \times V_B = V$$

$$\rho_B = 950 \text{ kg/m}^3$$

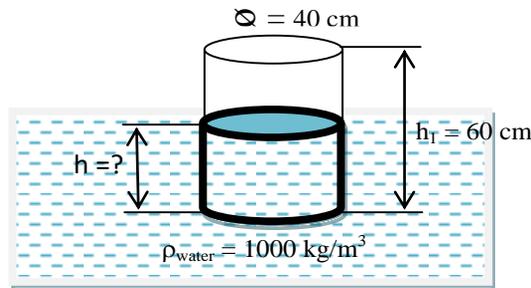
**Where**

- $\rho, g$  and  $V$  are density, acceleration due to gravity, and volume of the water.
- $V_B, \rho_B$  and  $g$  are the volume, density, and acceleration due to gravity of body immersed.

 **Example 5.3**

A wooden cylinder as in figure below has diameter of 40 cm and 60 cm height, cylindrical wood has a density of  $600 \text{ kg/m}^3$  was floated in the water. Determine;

- Buoyancy force.
- The height of the submerged in water ( $h$ ).



**Solution for Example 5.3**

**Given data :**

$$d = 40 \text{ cm} = 0.4 \text{ m}$$

$$h = 60 \text{ cm} = 0.6 \text{ m}$$

$$\rho = 600 \text{ kg/m}^3$$

**i. Buoyancy force.**

$$\begin{aligned} \text{Area, } A &= \frac{\pi d^2}{4} \\ &= \frac{\pi \times (0.4^2)}{4} \\ &= \mathbf{0.1257 \text{ m}^2} \end{aligned}$$

$$\begin{aligned} \text{Volume wood, } V &= Ah \\ &= 0.1257 \times 0.6 \\ &= \mathbf{0.0754 \text{ m}^3} \end{aligned}$$

$$\begin{aligned} \text{Mass wood, } m &= \rho_{\text{wood}} \times V_{\text{wood}} \\ &= 600 \times 0.0754 \\ &= \mathbf{45.24 \text{ kg}} \end{aligned}$$

$$\begin{aligned} F_B &= \text{Buoyancy Force} \\ &= mg \\ &= 45.24 \times 9.81 \\ &= \mathbf{443.8 \text{ N}} \end{aligned}$$

**ii. The height of the submerged in water (h).**

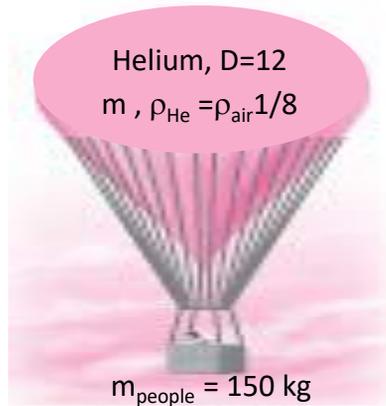
$$\begin{aligned} F_B &= \rho g V \\ \text{Volume displaced, } V &= \frac{F_B}{\rho g} \\ &= \frac{443.8}{9810} \\ &= \mathbf{0.045 \text{ m}^3} \end{aligned}$$

The height in water is,

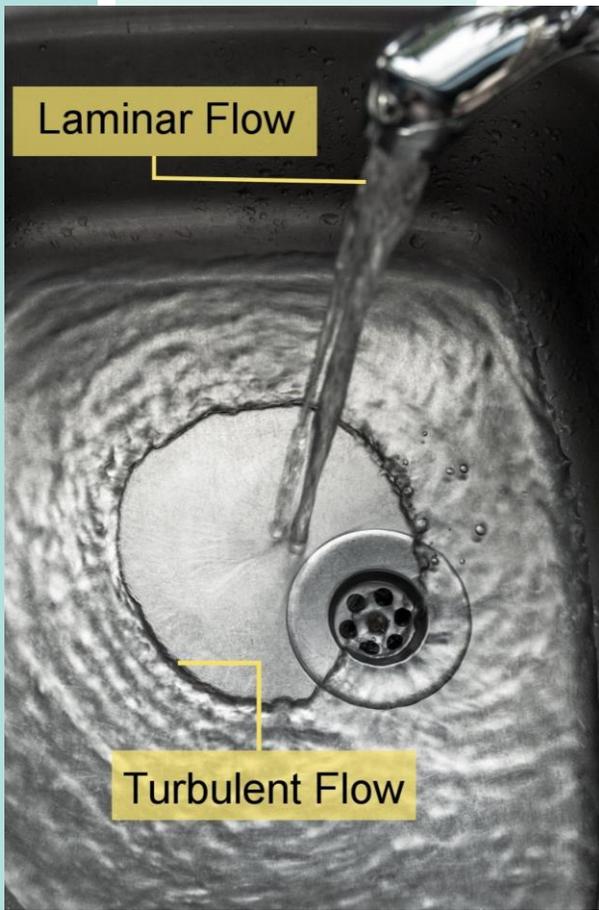
$$\begin{aligned} V &= Ah \\ h &= \frac{V}{A} \\ &= \frac{0.045}{0.1257} = \mathbf{0.36 \text{ m}} \end{aligned}$$

 Tutorial 5.1

A helium balloon tied to the ground carries 2 people will be 150 kg. The density of helium gas is  $1/8$  density of air and volume of helium equal  $4\pi r^3/3$ . Where the diameter balloon is 12 m and the density of air is  $1.2 \text{ kg/m}^3$ . Determine the buoyancy force acting on the balloon and the weight of the cage.



Answer:  $F_B = 10.65 \text{ kN}, W = 2.803 \text{ kN}$



# CHAPTER 6

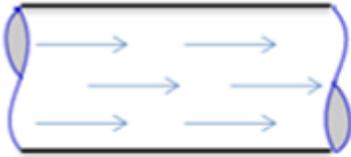
## Types Of Flow



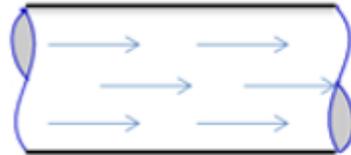


## 6.1 Types of Flow

Before



After

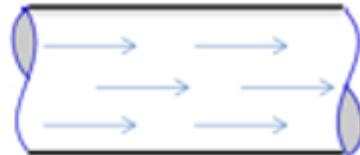


### Steady Flow

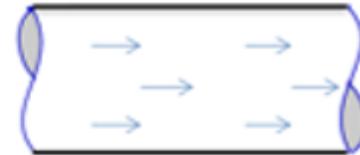
Steady flow is a characteristic of a fluid such as velocity, temperature, pressure, density and others that does not change with time.

### Un-steady Flow

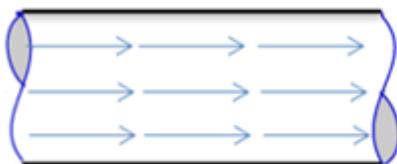
Unsteady flow in which characteristics of fluid change with time at the same point such as velocity or pressure.



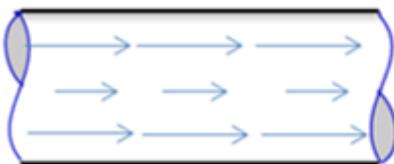
Before



After



Uniform flow

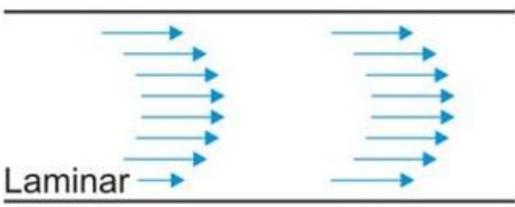


Non Uniform Flow

### Uniform Flow

The velocity of a fluid has magnitude and direction and moves uniformly with time.





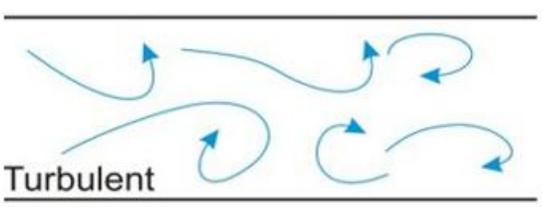
**Laminar**

### Laminar Flow

The fluid flow, in which the adjacent layers do not cross each other and move along we define path is known as laminar flow. In this flow, fluid flows along the straight line.

### Turbulent Flow

The flow in which adjacent layers cross each other and do not move along well-defined path is known as turbulence flow.



**Turbulent**

## 6.2 WHAT IS THE DIFFERENCE BETWEEN LAMINAR AND TURBULENT FLOW?

### Laminar Flow VS Turbulent Flow

Laminar Flow	Turbulent Flow
Low velocity	High velocities
Small length scales	Large length scales
High kinematic viscosities	Low kinematic viscosities
Viscous forces are dominant	Inertial forces are dominant
The value of Reynolds number (Re) is less than 2000	The value of Reynolds number (Re) is greater than 2000



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# CHAPTER 7

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## Continuity Equation Law

## 7.1 Flow Rate

- ❑ **Flow rate** is the total rate of volume flowing per time.
- ❑ The symbol for flow rate is **Q**

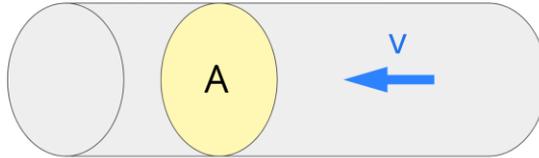


Figure 7.1 : Flow rate, Q

$$Q = Av$$

Where

$Q = \text{Flow rate, (m}^3/\text{s)}$

$A = \text{Area, (m}^2)$

$V = \text{Velocity, (m/s)}$

## 7.2 Mass Flow Rate

- ❑ **Mass flow rate** is simply a measurement of the amount of mass (weight) passing by a single point over a length of time.
- ❑ The symbol for **mass flow rate** is an  **$\dot{m}$** .



Figure 7.2 : Flow of fluid in pipes.

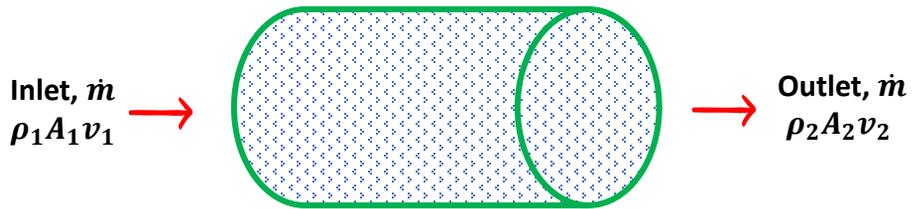


Figure 7.3 : Mass flow rate,  $\dot{m}$

## Where

$\dot{m}$  = mass flow rate,

$\rho$  = density, ( $\text{kg}/\text{m}^3$ )

$A$  = Area, ( $\text{m}^2$ )

$V$  = Velocity, ( $\text{m}/\text{s}$ )

$Q$  = Flow rate, ( $\text{m}^3/\text{s}$ )

$$\begin{aligned}\dot{m} &= Av \\ &= \rho Q\end{aligned}$$

- Therefore, if water enters a pipe at 1 kg/s it must leave the pipe at 1 kg/s as well, if there are no leaks!
- However, the volume flow rate can change and will do so if the density changes either through a change in pressure or temperature.

## 7.3 Continuity Equation Law

- The continuity equation describes the transport of some quantities like fluid or gas.
- The equation explains how a fluid conserves mass in its motion. Many physical phenomena like energy, mass, momentum, natural quantities, and electric charge are conserved using the continuity equations.
- The continuity equation provides beneficial information about the flow of fluids and their behavior during their flow in a pipe or hose.

- ❑ Continuity Equation is applied on tubes, pipes, rivers, ducts with flowing fluids or gases and many more.
- ❑ Continuity equation can be expressed in an integral form and is applied in the finite region or differential form, which is applied at a point.

$$\dot{m}_1 = \dot{m}_2$$

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

$$\rho_1 Q_1 = \rho_2 Q_2$$

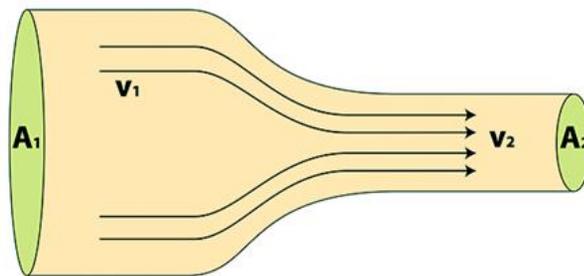


Figure 7.4 : Single pipe with different diameter.

**Note**

For continuity of flow in any system of fluid flow, the **total amount of the fluid entering the system must equal the amount leaving the system**. This occurs in the case of uniform flow and steady flow.

### 7.3.1 Principle of Continuity Equation in Single Pipe With Different Diameter

- The principle of continuity to pipes with cross-sections that have changed along their length. Consider the diagram below of a pipe with a contraction.

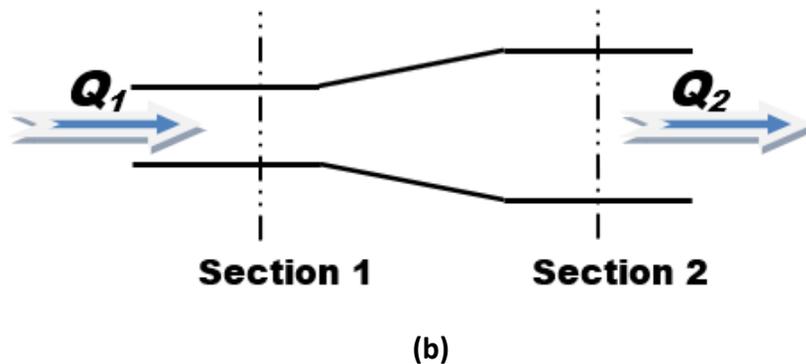
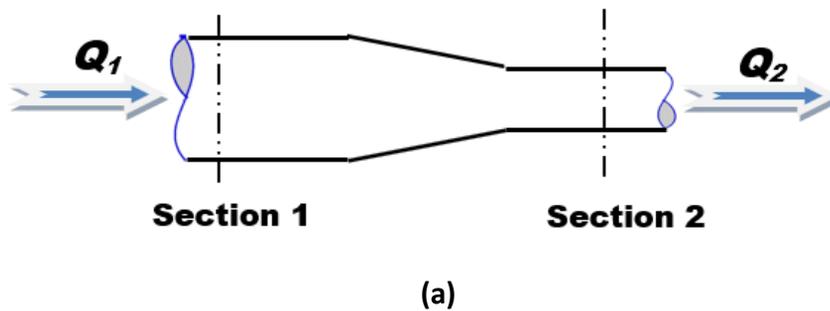


Figure 7.5 : Single pipe with different diameters.

Discharge at section 1 = Discharge at section 2

$$Q_1 = Q_2$$

$$A_1 v_1 = A_2 v_2$$

### 7.3.2 Principle of Continuity Equation in Branch Pipe

- The continuity principle can also be used to determine the velocities in pipe coming from a junction.

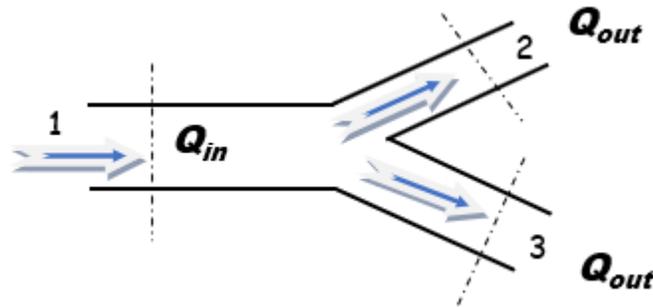


Figure 7.5 : Branch pipe

Total discharge into the junction = Total discharge out of the junction

$$\rho_1 Q_1 = \rho_2 Q_2 + \rho_3 Q_3$$

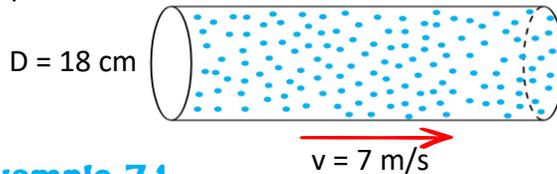
When the flow is incompressible (eg: water)  $\rho_1 = \rho_2 = \rho$

$$Q_1 = Q_2 + Q_3$$

$$A_1 v_1 = A_2 v_2 + A_3 v_3$$

### Example 7.1

A pipe has a diameter,  $D = 18 \text{ cm}$  and has a velocity,  $v = 7 \text{ m/s}$ . Determine the value of the flow rate,  $Q$  in the pipe.



#### Solution for Example 7.1

**Given data :**

$$d = 18 \text{ cm} = 0.18 \text{ m}$$

$$v = 7 \text{ m/s}$$

i. Calculate area,  $A$

$$\begin{aligned} A &= \frac{\pi d^2}{4} \\ &= \frac{\pi \times 0.18^2}{4} \\ &= \mathbf{0.0254 \text{ m}^2} \end{aligned}$$

ii. Calculate discharge,  $Q$

$$\begin{aligned} Q &= Av \\ &= 0.0254 \times 7 \\ &= \mathbf{0.1778 \text{ m}^3/\text{s}} \end{aligned}$$

### Example 7.2

Oil flows through a pipe at a velocity of  $3 \text{ m/s}$ . The diameter of the pipe is  $7.5 \text{ cm}$ . Calculate discharge and mass flow rate of oil. Take into consideration  $s_{\text{oil}} = 0.8$ .

#### Solution for Example 7.2

**Given data :**

$$v = 3 \text{ m/s}$$

$$d = 7.5 \text{ cm}$$

$$s_{\text{oil}} = 0.8$$

i. Calculate area,  $A$

$$\begin{aligned} A &= \frac{\pi d^2}{4} \\ &= \frac{\pi \times 0.075^2}{4} \\ &= \mathbf{4.4179 \times 10^{-3} \text{ m}^2} \end{aligned}$$

ii. Calculate discharge,  $Q$

$$\begin{aligned} Q &= Av \\ &= 4.4179 \times 10^{-3} \times 3 \\ &= \mathbf{0.0133 \text{ m}^3/\text{s}} \end{aligned}$$

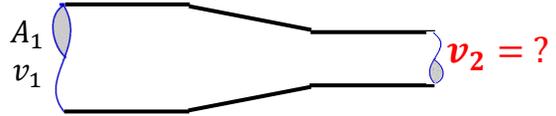
iii. Calculate mass flow rate

$$\begin{aligned} \dot{m} &= \rho Q \\ &= 0.8 \times 1000 \times 0.0133 \\ &= \mathbf{10.64 \text{ m}^3/\text{kg}} \end{aligned}$$

 **Example 7.3**

If the area  $A_1 = 15 \times 10^{-3} m^2$  and  $A_2 = 5 \times 10^{-3} m^2$  and the upstream mean velocity,  $v_1 = 3.8 m/s$ , calculate the downstream mean velocity.

**Solution for Example 7.3**



**Given data :**

$$A_1 = 15 \times 10^{-3} m^2$$

$$A_2 = 5 \times 10^{-3} m^2$$

$$v_1 = 3.8 m/s$$

**Discharge at section 1 = Discharge at section 2**

$$Q_1 = Q_2$$

$$A_1 v_1 = A_2 v_2$$

$$v_2 = \frac{A_1 v_1}{A_2}$$

$$= \frac{(15 \times 10^{-3}) \times (3.8)}{5 \times 10^{-3}}$$

$$= 11.4 m/s$$

 **Example 7.4**

The diameter at inlet is  $d_1 = 3 cm$  and at outlet is  $d_2 = 5.5 cm$  and the mean velocity at outlet is  $v_2 = 1.5 m/s$ . Calculate the velocity at inlet the diffuser.

**Solution for Example 7.4**

**Given data :**

$$d_1 = 3 cm = 0.03 m$$

$$d_2 = 5.5 cm = 0.055 m$$

$$v_2 = 1.5 m/s$$



**Discharge at section 1 = Discharge at section 2**

$$Q_1 = Q_2$$

$$A_1 v_1 = A_2 v_2$$

$$v_1 = \frac{A_2 v_2}{A_1}$$

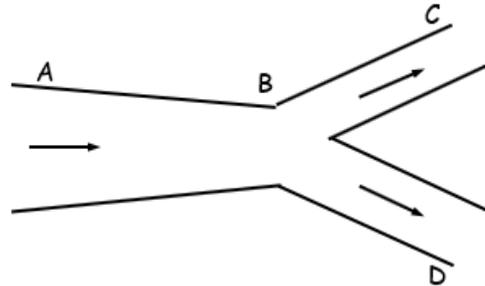
$$= \frac{(2.376 \times 10^{-3}) \times (1.5)}{7.069 \times 10^{-4}}$$

$$= 5.042 m/s$$

### Example 7.5

A pipe is split into 2 pipes which are BC and BD as shown in the figure below. The following information is given:

- Diameter pipe AB at A = 0.45 m
- Diameter pipe AB at B = 0.3 m
- Diameter pipe BC = 0.2 m
- Diameter pipe BD = 0.15 m



Calculate:

- a) Discharge at section A if  $v_A = 2 \text{ m/s}$
- b) Velocity at section B and section D if velocity at section C = 4 m/s

### Solution for Example 7.5

**Given data :**

$$v_A = 2 \text{ m/s}$$

$$v_C = 4 \text{ m/s}$$

**Discharge at A,  $Q_A$**

$$\begin{aligned} Q_A &= Av \\ &= \left( \frac{\pi \times 0.45^2}{4} \right) \times 2 \\ &= \mathbf{0.318 \text{ m}^3/\text{s}} \end{aligned}$$

a) Discharge at section A = Discharge at section B

$$\begin{aligned} Q_A &= Q_B \\ A_A v_A &= A_B v_B \\ v_B &= \frac{A_A v_A}{A_B} \\ &= \frac{(0.318) \times (4)}{\left( \frac{\pi \times 0.3^2}{4} \right)} \\ &= \mathbf{4.5 \text{ m/s}} \end{aligned}$$

**Continuity of flow**

$$Q_B = Q_C + Q_D$$

$$Q_D = Q_B - Q_C$$

$$= A_B v_B - A_C v_C$$

$$= \left[ \frac{\pi \times 0.3^2}{4} \times 4.5 \right] - \left[ \frac{\pi \times 0.2^2}{4} \times 4 \right]$$

$$= \mathbf{0.192 \text{ m}^3/\text{s}}$$

**Pipe BD**

$$Q_D = Av = \mathbf{0.192 \text{ m}^3/\text{s}}$$

$$v_D = \frac{Q_D}{A_D}$$

$$= \frac{(0.192) \times (4)}{\left( \frac{\pi \times 0.15^2}{4} \right)}$$

$$= \mathbf{10.86 \text{ m/s}}$$



## Tutorial 7.1

The raw oil flows through a pipe of diameter 40 mm and entered a pipe of diameter 25 mm. The volume flow rate is 3.75 liter /s. Calculate the velocity of both pipes and the density of raw oil if the mass flow rate is 3.23 kg/s.

$$\text{Answer: } v_1 = 2.98 \text{ m/s}, v_2 = 7.64 \text{ m/s}, \dot{m} = 860 \text{ kg/s}$$



## Tutorial 7.2

Benzene with a flow rate of  $0.0884 \text{ m}^3/\text{s}$ , flows through a 150 mm diameter pipe at a mass flow rate 77.7 kg/s. Calculate the velocity and specific gravity of benzene,  $S_{\text{benzene}}$ .

$$\text{Answer: } v = 5 \text{ m/s}, S_{\text{benzene}} = 0.879 \text{ kg/m}^3$$

**Tutorial 7.3**

A 300 mm diameter pipeline to bring water flow with an average velocity of 4.5 m / s. Two lines and then branched into two pipes each of diameter 150 mm and 200 mm. If the average velocity of 150 mm pipe is 5 / 8 of the velocity in the main line, calculate:

- (i) The average velocity of flow in a pipe is 200 mm
- (ii) The total flow rate in the piping system

*Answer:  $v = 8.54 \text{ m/s}$ ,  $Q = 0.3181 \text{ m}^3/\text{s}$*



## Tutorial 7.4

Oil flows through a pipe that narrowing from the diameter of 450 mm at A to a diameter of 300 mm at B. The pipe is split into 2 pipes and one of the branches has a diameter of 150 mm at C and the other one has a diameter of 250 mm at D. If velocity at A is 1.8 m/s and velocity at D is 3.6 m/s, calculate :

- i. Discharge at D dan C
- ii. Velocity at B and C.

$$\text{Answer: } Q_D = 0.177 \text{ m}^3/\text{s}, Q_C = 0.11 \text{ m}^3/\text{s}, v_B = 4.05 \text{ m/s}, v_C = 6.225 \text{ m/s}$$



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# CHAPTER 8

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## Bernoulli Theorem



## 8.1 Definition of Bernoulli Theorem

- Bernoulli's Theorem states that the total energy of each particle of a body of fluid is the same provided that no energy enters or leaves the system at any point.
- Considered to be a statement of **the conservation of energy principle** for flowing fluids.
- The conservation of energy principle stated that **the total energy of each particle of a body of fluids is the same provided that no energy enters or leaves the system at any point.**

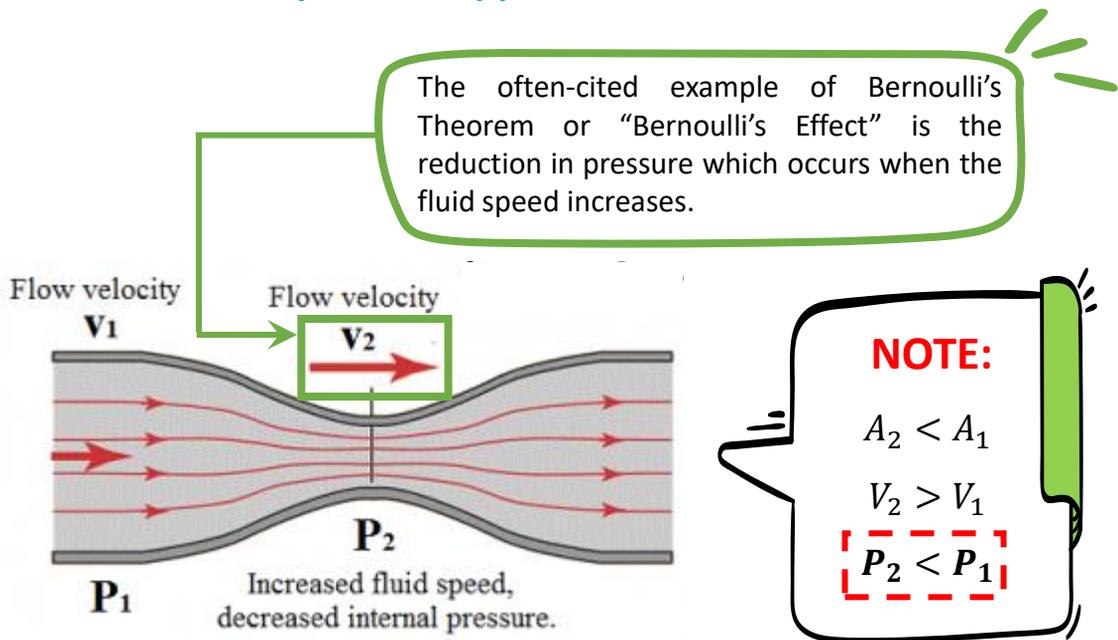


Figure 8.1 : Bernoulli Theorem Diagram

$$H = Z + \frac{P}{\omega} + \frac{v^2}{2g} = \text{constant}$$

TOTAL  
ENERGY

POTENTIAL  
ENERGY

PRESSURE  
HEAD

KINETIC  
ENERGY

Total energy per unit weight at section 1 = Total energy per unit weight at section 2

$$Z_1 + \frac{P_1}{\omega} + \frac{v_1^2}{2g} = Z_2 + \frac{P_2}{\omega} + \frac{v_2^2}{2g}$$

**When**

$Z = \text{Potential energy}$

$\frac{p}{\omega} = \text{Pressure head}$

$\frac{v^2}{2g} = \text{Kinetic energy}$

## 8.2 The limitation of Bernoulli Theorem

Bernoulli's Equation has some restrictions in its applicability, they are :

- The flow is steady.
- The density is constant (which also means the fluid is incompressible)
- Friction losses are negligible.
- The equation relates the state at two points along a single streamline (not conditions on two different streamlines).

## 8.3 Application of Bernoulli Theorem



Horizontal and Incline pipe.



Horizontal and Incline venturi meter.



Orifice.



Pitot Tube.



### 8.3.1 Horizontal Pipe

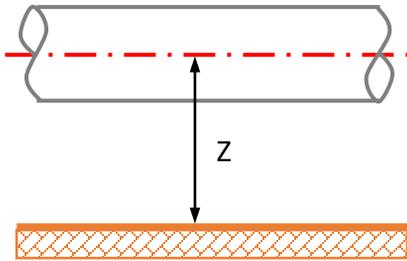


Figure 8.2 : Horizontal Pipe

$$H = Z + \frac{p}{\omega} + \frac{v^2}{2g}$$

#### When

$H =$  Total energy

$Z =$  height,  $m$

$P =$  Pressure,  $N/m^2$

$\omega =$  specific weight,  $N/m^3$

$v =$  velocity,  $m/s$

$g =$  gravity ( $9.81 m/s$ )



### Example 8.1

Water flows through a pipe 30 m from the sea level as shown in the figure below. Pressure in the pipe is  $410 \text{ kN/m}^2$  and the velocity is  $5.8 \text{ m/s}$ . Calculate the total energy of every weight of unit water above sea level.

#### Solution For Example 8.1

**Given data :**

$$Z = 30 \text{ m}$$

$$P = 410 \text{ kN/m}^2$$

$$v = 5.8 \text{ m/s}$$

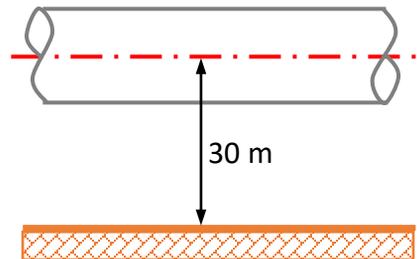
$$\rho_{\text{water}} = 1000 \text{ kg/m}^3$$

So,

$$\omega_{\text{water}} = \rho g$$

$$= 1000 \times 9.81$$

$$= 9810 \text{ N/m}^3$$



$$H = Z + \frac{p}{\omega} + \frac{v^2}{2g}$$

$$= 30 + \frac{410 \times 10^3}{9810} + \frac{5.8^2}{2 \times 9.81}$$

$$= 73.51 \text{ J/N}$$



### Example 8.2

The diameter of a pipe has changed uniformly from a diameter of 150 mm at point A with the height of 6 m from the datum, to diameter of 75 mm at point B with the height of 3 m from the first datum. Pressure and velocity at point A is 103 kN/m<sup>2</sup> and 3.6 m/s respectively. Calculate the pressure at point B.

### Solution for Example 8.2

**Given data :**

$$d_A = 15 \times 10^{-3} \text{ m}$$

$$d_B = 75 \times 10^{-3} \text{ m}$$

$$P_A = 103 \text{ kN/m}^2$$

$$v_A = 3.6 \text{ m/s}$$

$$Z = 6 \text{ m}$$

Using Bernoulli's Equation:

$$Z_1 + \frac{P_1}{\omega} + \frac{v_1^2}{2g} = Z_2 + \frac{P_2}{\omega} + \frac{v_2^2}{2g}$$

Discharge at section 1 = Discharge at section 2

$$Q_1 = Q_2$$

$$v_1 \times A_1 = v_2 \times A_2$$

$$v_2 = \frac{v_1 \times A_1}{A_2}$$

$$= \frac{3.6 \times (0.15^2)}{0.075}$$

$$= 14.4 \text{ m/s}$$

Using Bernoulli's Equation:

$$P_1 - P_2 = \left[ \frac{v_2^2 - v_1^2}{2g} + (Z_2 - Z_1) \right] \times \omega$$

$$103000 - P_2 = 67770.423$$

$$P_2 = 103000 - 67770.423$$

$$= 35.23 \text{ kN/m}^2$$



### Tutorial 8.1

A pipe measure 15 m length, supplying water to a house that located on a hill, 5.5 above sea level. The diameter of the pipe is 30 cm. if the water velocity is m/s, calculate the total energy. The water pressure is 5000 Pascal.

*Answer:  $H = 6.213$  m*



### 8.3.2 Incline Pipe

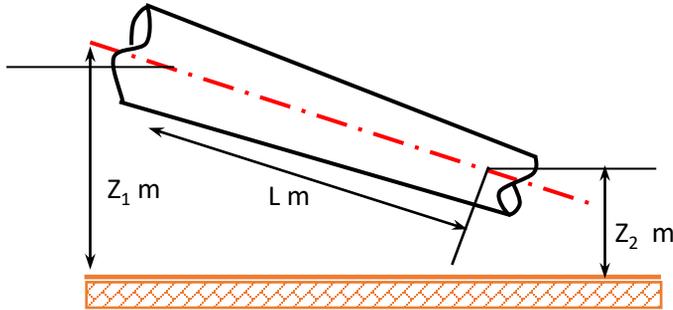


Figure 8.3 : Incline Pipe

#### Bernoulli's Equation

$$Z_1 + \frac{P_1}{\omega} + \frac{v_1^2}{2g} = Z_2 + \frac{P_2}{\omega} + \frac{v_2^2}{2g}$$

#### When

$Z$  = height,  $m$

$P$  = Pressure,  $N/m^2$

$\omega$  = specific weight,  $N/m^3$

$v$  = velocity,  $m/s$

$g$  = gravity ( $9.81 m/s$ )



### Example 8.3

A bent pipe labelled MN measures  $5 m$  and  $3 m$  respectively above the datum line. The diameter M and N are both  $20 cm$  and  $5 cm$ . The water pressure is  $5 kg/cm^2$ . If the velocity at M is  $1 m/s$ , determine the pressure at N in  $kg/cm^2$ .



## Tutorial 8.2

The diameter of a pipe has changed uniformly from a diameter of 150 mm at point A with the height of 6 m from datum, to a diameter of 75 mm at point B with the height of 3 m from the first datum. Pressure and velocity at point A is  $103 \text{ kN/m}^2$  and  $3.6 \text{ m/s}$  respectively. Calculate The pressure at point B.

*Answer:  $v_2 = 14.4 \text{ m/s}$ ,  $P_2 = 35.23 \text{ kN/m}^2$*

### 8.3.3 Horizontal Venturi Meter

Venturi meter is a type of flowmeter that works on the principle of Bernoulli's Equation.

- This device is widely used in the water, chemical, pharmaceutical, and oil & gas industries to measure the flow rates of fluids inside a pipe.
- The pipe cross-sectional area is reduced to create a pressure difference which is measured with a manometer to determine the rate of fluid flow.
- The venturi meter is a differential head type flowmeter that converts pressure energy into kinetic energy.



(a)



(b)



(c)



(d)

Figure 8.5: The Venturi Meter is used in a variety of industrial applications.

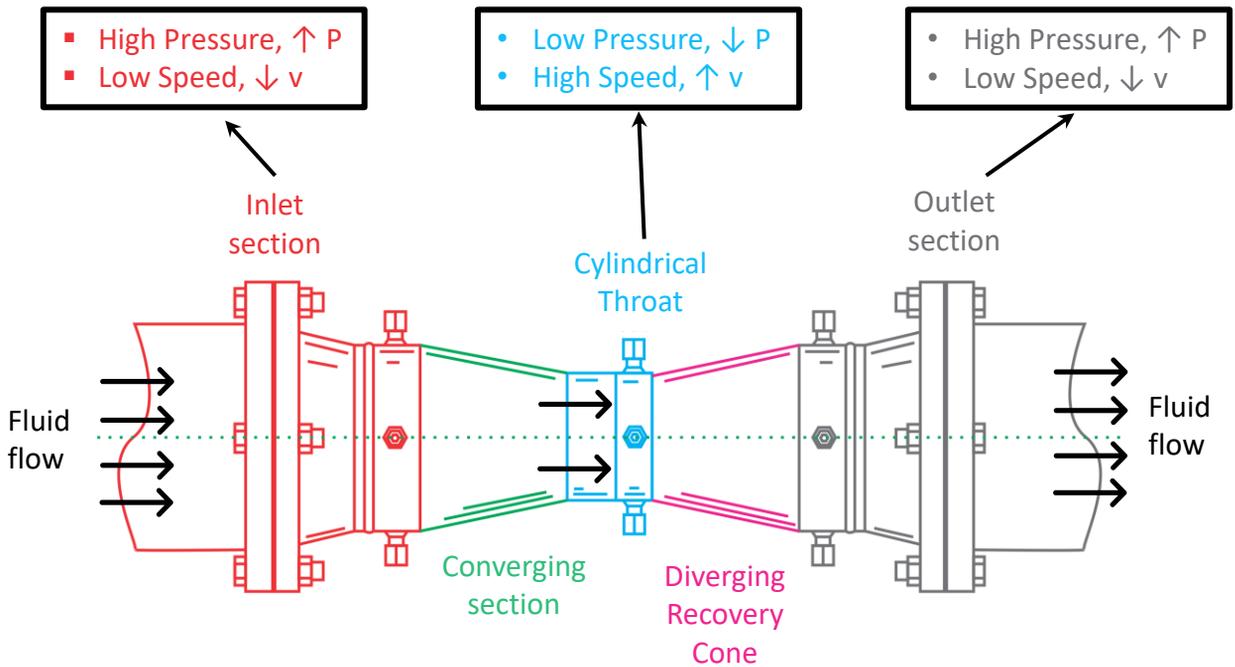


Figure 8.6: Venturi Meter diagram

□ The venturi meter is a device used for measuring the rate of flow of a non-viscous, incompressible fluid in non-rotational and steady-stream lined flow.

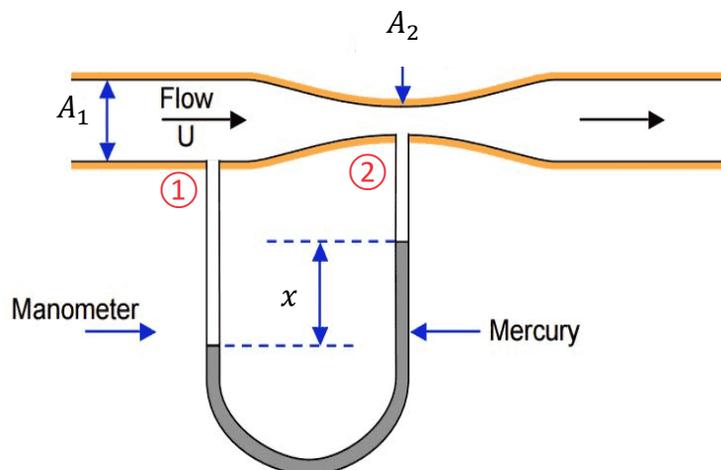


Figure 8.7: Venturi Meter diagram



Figure 8.8: Fuel Industry

## i) To Calculate Flowrate The Theory Discharge and Actual Discharge in Venturi Meter

- The theoretical discharge  $Q$  can be converted to actual discharge by multiplying by the coefficient of discharge  $C_d$  was found experimentally.

### Theory Discharge

$$Q_{theory} = A_1 \sqrt{\frac{2gH}{m^2 - 1}}$$

### Actual Discharge

$$\begin{aligned} Q_{Ac} &= C_d \times Q_{theory} \\ &= C_d A_1 \sqrt{\frac{2gH}{m^2 - 1}} \end{aligned}$$

#### When

$A_1$  = Area at entrance

$g$  = Gravity (9.81 m/s<sup>2</sup>)

$C_d$  = Coefficient of Discharge

$m$  = Area ratio (Value of  $m > 1$ )

$$m = \frac{A_1}{A_2} = \frac{d_1^2}{d_2^2}$$

$H$  = Pressure difference expressed as a head of the liquid flowing in Venturi meter (m)

$$H = \frac{P_1 - P_2}{\omega_{sub}} \quad \text{OR} \quad H = x \left( \frac{\omega_{Hg}}{\omega_{sub}} - 1 \right)$$

#### When

$x$  = Difference of level in U – tube manometer (m)

$\omega_{Hg}$  = Specific weight of mercury

$\omega_{sub}$  = Specific weight of fluid entering the venture meter.



**Step 1**

Find the value of  $A_1$

$$A = \frac{\pi d^2}{4}$$

**Step 2**

Find the value of area ratio,  $m$

$$m = \frac{A_1}{A_2} \quad \text{OR} \quad m = \frac{d_1^2}{d_2^2}$$

**Step 3**

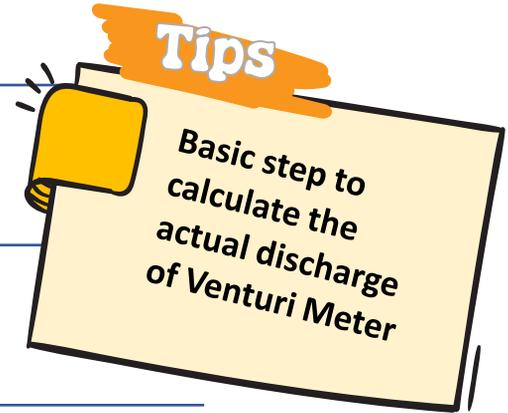
Find the value of  $H$  using

$$H = \frac{P_1 - P_2}{\omega_{sub}} \quad \text{OR} \quad H = x \left( \frac{\omega_{Hg}}{\omega_{sub}} - 1 \right)$$

**Step 4**

Insert in  $Q_{actual}$  Equation

$$Q_{Ac} = C_d A_1 \sqrt{\frac{2gH}{m^2 - 1}}$$



**Example 8.4**

A horizontal venturi meter measures the flow of oil of specific gravity 0.9 in a 75 mm diameter pipeline. If the difference of pressure between the full bore and the throat tapping is 34.5 kN/m<sup>2</sup> and the area ratio,  $m$  is 4, calculate the rate of flow, assuming a coefficient of discharge is 0.97.

**Solution for Example 8.4**

**Given data :**

$$S_g = 0.9$$

$$d_1 = 75 \times 10^{-3} \text{ m}$$

$$P = 34.5 \text{ kN/m}^2$$

$$m = 4 \quad \text{STEP (2)}$$

$$C_d = 0.97$$

**STEP (1)**

$$A_1 = \frac{\pi \times d_1^2}{4}$$

$$A_1 = \frac{\pi \times (75 \times 10^{-3})^2}{4} \\ = 4.418 \times 10^{-3} \text{ m}^2$$

**STEP (4)**

$$Q_{Ac} = C_d A_1 \sqrt{\frac{2gH}{m^2 - 1}}$$

$$Q_{Ac} = 0.97 \times 4.418 \times 10^{-3} \sqrt{\frac{2 \times 9.81 \times 3.92}{16 - 1}} \\ = 0.0106 \text{ m}^3/\text{s}$$

**STEP (3)**

$$H = \frac{P}{\omega} \\ = \frac{34.5 \times 10^3}{0.9 \times 9.81 \times 1000} \\ = 3.92 \text{ m}$$



### Tutorial 8.3

Meter venture has an inlet diameter 400 mm and a throat diameter 200 mm. If the different height of mercury in the manometer tube is 200 mm and the value of Coefficient Discharge,  $C_d$  is 0.9. Determine the flow rate in the venturi meter.

*Answer:  $Q_{Ac} = 0.206 \text{ m}^3/\text{s}$*



### Tutorial 8.4

A horizontal venture meter with a diameter of 250 mm at the inlet and 150 mm at the throat. A mercury differential manometer linked at venture meter shown at different level reading is X meter. Given the discharge coefficient of 0.9, elaborate mathematically the differential of X if the discharge of water is  $0.063 \text{ m}^3/\text{s}$ .

*Answer:  $Q_{Ac} = 0.2828 \text{ m}^3/\text{s}, x = 0.055 \text{ m}$*

### 8.3.4 Inclined Venturi Meter

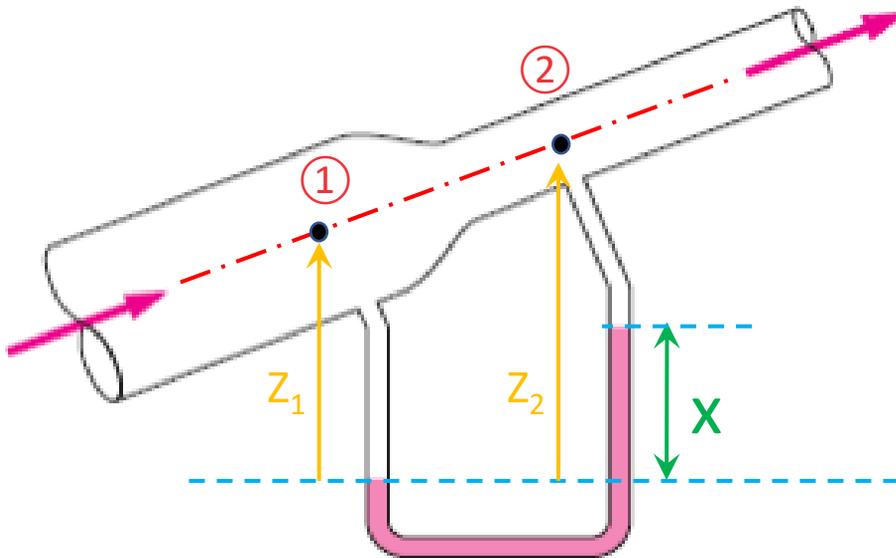


Figure 8.9: Inclined Venturi Meter

#### Actual Discharge

$$Q_{Ac} = C_d \times Q_{theory}$$

$$= C_d A_1 \sqrt{\frac{2gH}{m^2 - 1}}$$

$$Q_{Ac} = \frac{C_d \times A_1}{\sqrt{(m^2 - 1)}} \sqrt{\left[ 2g \left\{ \left( \frac{P_1 - P_2}{\omega_{sub}} \right) + (Z_1 - Z_2) \right\} \right]}$$

$$= C_d A_1 \sqrt{\frac{2gH}{m^2 - 1}}$$

#### When

$A_1$  = Area at entrance

$g$  = Gravity (9.81 m/s<sup>2</sup>)

$C_d$  = Coefficient of Discharge

$m$  = Area ratio (Value of  $m > 1$ )

$$m = \frac{A_1}{A_2} = \frac{d_1^2}{d_2^2}$$

$H$  = Pressure difference expressed as a head of the liquid flowing in inclined Venturi meter (m)

$$H = \frac{P_1 - P_2}{\omega_{sub}} + (Z_1 - Z_2) \quad \text{OR} \quad H = x \left( \frac{\omega_{Hg}}{\omega_{sub}} - 1 \right)$$

**When**

$P_1 - P_2$  = Difference of pressure in entrance and throat

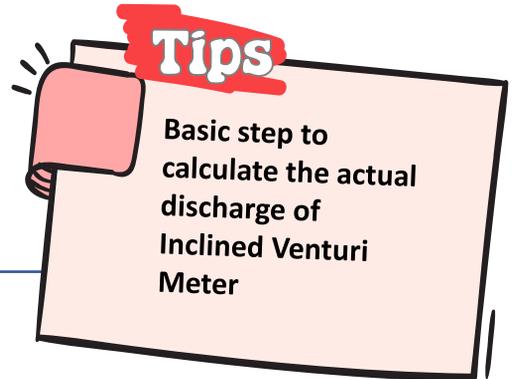
$Z_1 - Z_2$  = Difference of height between entrance and throat.

➤ (Normally throat,  $Z_2$  is above entrance,  $Z_1$  so the value of  $Z_1 - Z_2$  should be **negative**)

$x$  = Difference of level in U – tube manometer (m)

$\omega_{Hg}$  = Specific weight of mercury

$\omega_{sub}$  = Specific weight of fluid entering the venture meter.



**Step 1**

Find the value of  $A_1$

$$A = \frac{\pi d^2}{4}$$

**Step 2**

Find the value of area ratio,  $m$

$$m = \frac{A_1}{A_2} \quad \text{OR} \quad m = \frac{d_1^2}{d_2^2}$$

**Step 3**

Find the value of  $H$  using

$$H = x \left( \frac{\omega_{Hg}}{\omega_{sub}} - 1 \right) \quad \text{OR} \quad H = \frac{P_1 - P_2}{\omega_{sub}} (Z_1 - Z_2)$$

**Step 4**

Insert in  $Q_{actual}$  Equation

$$Q_{Ac} = C_d A_1 \sqrt{\frac{2gH}{m^2 - 1}} \quad \text{OR} \quad Q_{Ac} = \frac{C_d \times A_1}{\sqrt{(m^2 - 1)}} \sqrt{\left[ 2g \left\{ \left( \frac{P_1 - P_2}{\omega_{sub}} \right) + (Z_1 - Z_2) \right\} \right]}$$



 **Example 8.5**

A vertical venturi meter measures the flow of oil of specific gravity 0.82 and has an entrance of 125 mm diameter and throat of 50 mm diameter. There are pressure gauges at the entrance and at the throat, which is 300 mm above the entrance. If the coefficient for the meter is 0.97 and the pressure difference is 27.5 kN/m<sup>2</sup>, calculate the actual discharge in m<sup>3</sup>/s.

**Solution for Example 8.4**

**Given data :**

$$S_g = 0.82$$

$$d_1 = 125 \times 10^{-3} m$$

$$d_2 = 50 \times 10^{-3} m$$

$$Z_1 - Z_2 = -0.3 m$$

$$P_1 - P_2 = 27.5 \text{ kN/m}^2$$

$$C_d = 0.97$$

**STEP ①**

$$A_1 = \frac{\pi \times d_1^2}{4}$$

$$\begin{aligned} A_1 &= \frac{\pi \times (125 \times 10^{-3})^2}{4} \\ &= \mathbf{0.01227 \text{ m}^2} \end{aligned}$$

**STEP ②**

$$\begin{aligned} m &= \frac{d_1^2}{d_2^2} \\ &= \left( \frac{0.125^2}{0.05^2} \right) \\ &= \mathbf{6.25} \end{aligned}$$

**STEP ③**

$$\begin{aligned} H &= \frac{P_1 - P_2}{\omega_{sub}} + (Z_1 - Z_2) \\ &= \frac{27.5 \times 10^3}{0.82 \times 9.81 \times 1000} + (-0.3) \\ &= \mathbf{3.119 \text{ m}} \end{aligned}$$

**STEP ④**

$$\begin{aligned} Q_{Ac} &= \frac{C_d \times A_1}{\sqrt{(m^2 - 1)}} \sqrt{\left[ 2g \left\{ \left( \frac{P_1 - P_2}{\omega_{sub}} \right) + (Z_1 - Z_2) \right\} \right]} \\ &= \frac{C_d \times A_1}{\sqrt{(m^2 - 1)}} \sqrt{[2g (H)]} \\ &= \frac{0.97 \times 0.01227}{\sqrt{6.25^2 - 1}} \sqrt{2 \times 9.81 \times 3.119} \\ &= \mathbf{0.151 \text{ m}^3/\text{s}} \end{aligned}$$



### Tutorial 8.5

A vertical venturi meter measures the flow of oil of specific gravity 0.85 and has an entrance of 125 mm diameter and throat of 50 mm diameter. There are pressure gauges at the entrance and at the throat, which is 350 mm above the entrance. If the coefficient for the meter is 0.97 and the actual discharge is  $0.021 \text{ m}^3/\text{s}$ . Calculate the pressure difference in the venturi meter.

$$\text{Answer: } P_1 - P_2 = 53.2922 \text{ kN/m}^2$$



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