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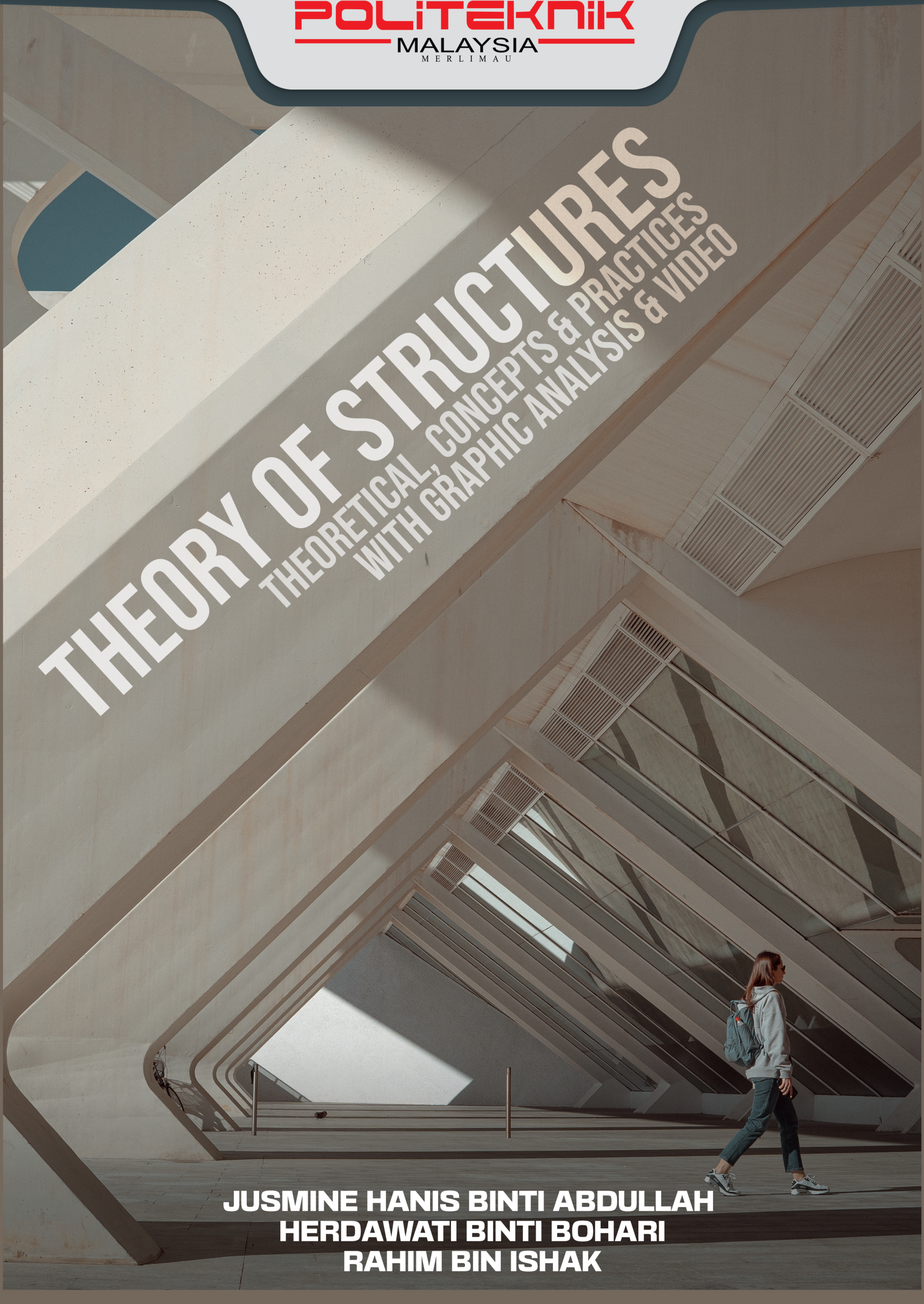

THEORY OF STRUCTURES

THEORETICAL, CONCEPTS & PRACTICES WITH GRAPHIC ANALYSIS & VIDEO

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Published in 2022

Perpustakaan Negara Malaysia Cataloguing-in-Publication Data

Jusmine Hanis Abdullah
THEORY OF STRUCTURES : Theoretical, Concepts & Practices with graphic analysis & video / JUSMINE HANIS BINTI ABDULLAH, HERDAWATI BINTI BOHARI, RAHIM BIN ISHAK.
Mode of access: Internet
eISBN 978-967-2762-16-4
1. Structural analysis (Engineering).
2. Structural engineering.
3. Structural design.
4. Government publications--Malaysia.
5. Electronic books.
I. Herdawati Bohari. II. Rahim Ishak.
III. Title.
624.17

Published by:
Politeknik Merlimau, Melaka
KB1031 Pej Pos Merlimau,
77300 Merlimau Melaka

ACKNOWLEDGEMENT

THEORY OF STRUCTURES

In the name of Allah, the Most Gracious and the Most Merciful.

We over helmed in all humbleness and gratefulness to acknowledge our depth to all those who have helped us to put these ideas, well above the level of simplicity and into something concrete. We would like to express our special thanks of gratitude to our family members as well as our organization who gave us the golden opportunity to do this beneficial project on Theory of Structures e-book, which also helped us improving our teaching and learning skills. Any attempt at any level can 't be satisfactorily completed without the support and guidance of our colleague and students who supported us in preparing this e-book. All of the conversations and advisory services have been used to improve the writing of this book. At last, but not in least, we would like to thank everyone who helped and motivated us to work on this book.

We hereby declare that this book is our original work. To the best our knowledge it contains no materials previously written or published by another person. However, if there is any, due acknowledgement and credit are mentioned accordingly in the e-book.

PREFACE

THEORY OF STRUCTURES

This e-book is intended to provide students with a clear and thorough presentation of structural analysis theory and application as it applies to beams, frames and trusses. The content is designed in line with the Malaysia Polytechnics syllabus that covers all topics. We aim to help students calculate statically indeterminate beams and portal frame using appropriate method, analyze joint displacement in statically determinate trusses and internal forces for statically indeterminate trusses correctly and evaluate the influence lines for statically determinate beams correctly. This e-book contains 8 chapters, and each chapter is loaded with examples and easy solutions with video for students completed understanding. Furthermore, explanation in YouTube using QR code were provided to show the right way in finding solutions. Also, problem given which content in this e-book to generate problem-solving skills are further developed when the various techniques are thought out and applied in a clear and orderly way. By solving problems in this way one can better grasp the way loads are transmitted through a structure and obtain a more complete understanding of the way the structure deforms under load. Finally, we hope this e-book enable students and readers better understanding in theory of structures and enjoy learning in digital notes thus making learning structures a lot easier.

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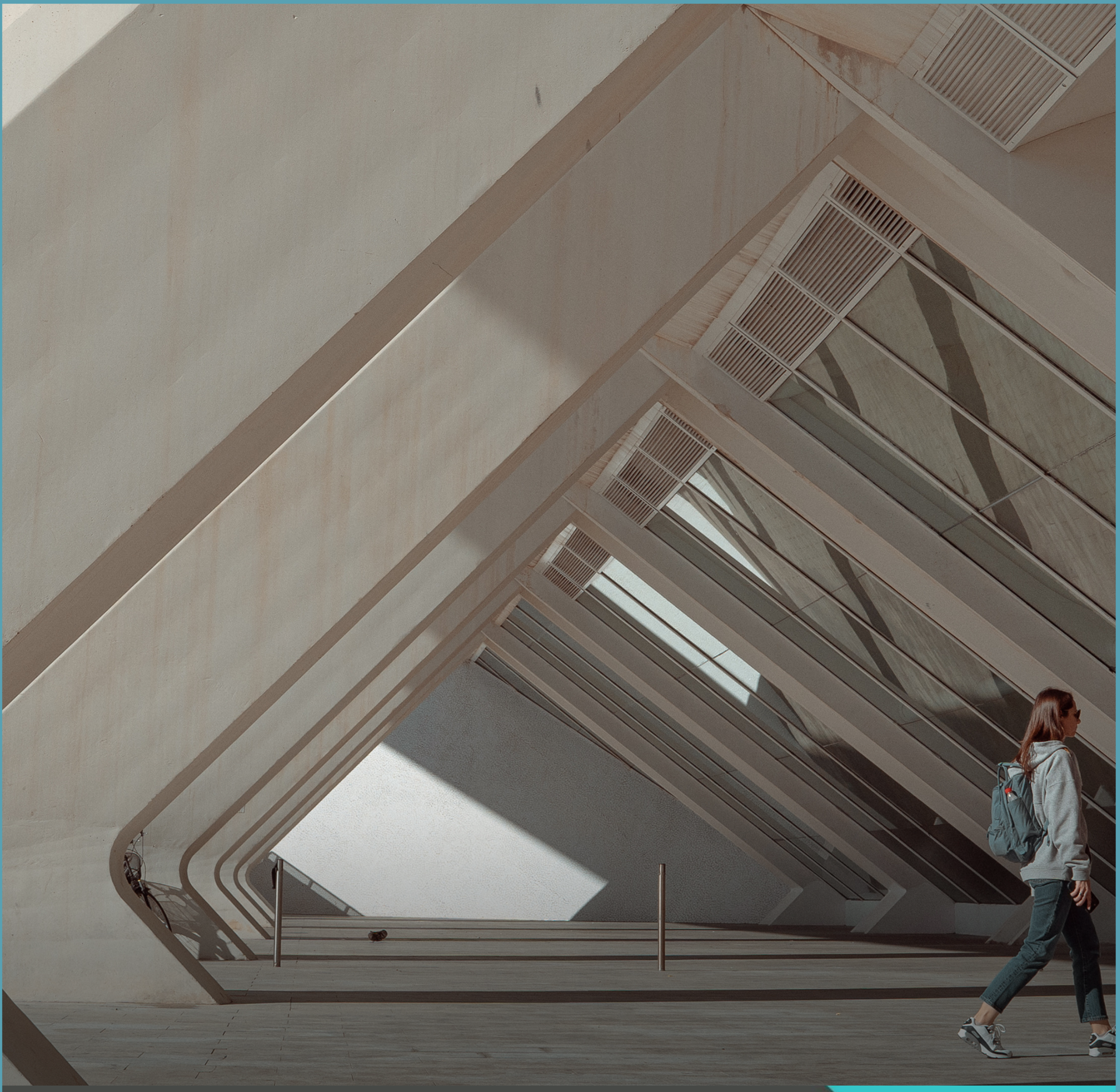


CHAPTER 1

SLOPE DEFLECTION METHOD FOR STATICALLY INDETERMINATE BEAM

1.1 Introduction to slope deflection

Statically indeterminate structures are structures that cannot be statically analyzed using only equilibrium equation. They required other material properties such as deformation in order to analyze them. As a general rule, a structure can be identified as being either statically determinate or statically indeterminate by drawing free-body diagrams of all its members, or selective parts of its members, and then comparing the total number of unknown reactive force and moment components with the total number of available equilibrium equations. So that if there is a total of n parts and r force and moment reaction.

$r = 3n$, statically determinate
$r > 3n$, statically indeterminate

where;

n = number of parts
r = number of reactions

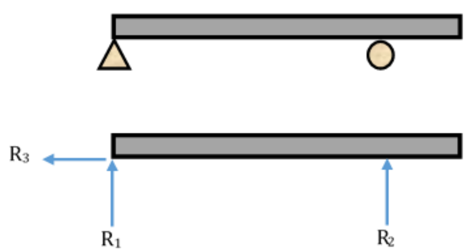
Quiz

How many types of support are there in structures?

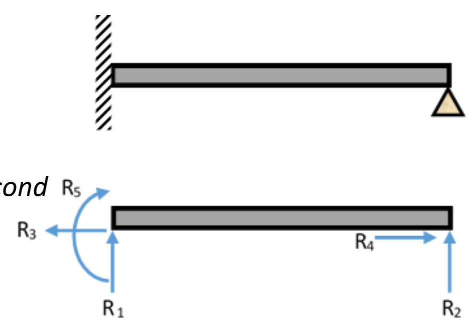
Ans. 3

Example 1.1

Classify the beam below whether it is determinate or indeterminate beam.



r = 3, n = 1
Therefore; 3 = 3(1)
[Statically determinate]



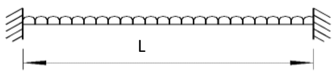
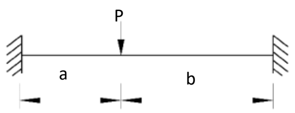
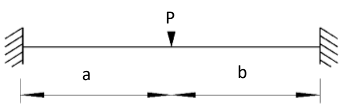
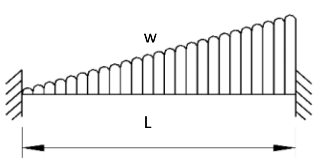
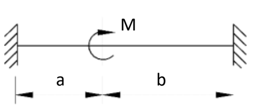
r = 5, n = 1
Therefore; 5 > 3(1)
[statically indeterminate to second degree]

OBJECTIVE

- *Able to identify indeterminate and determinate beam*

1.2 Basic concept of slope deflection

Table 1.0 : Fixed End Moments for different load configurations.

Moment left	Configuration	Moment right
$FEM_{AB} = -\frac{wL^2}{12}$		$FEM_{BA} = \frac{wL^2}{12}$
$FEM_{AB} = -\frac{wab^2}{L^2}$		$FEM_{BA} = \frac{wba^2}{L^2}$
$FEM_{AB} = -\frac{wL}{8}$		$FEM_{BA} = \frac{wL}{8}$
$FEM_{AB} = -\frac{wL^2}{30}$		$FEM_{BA} = \frac{wL^2}{30}$
$FEM_{AB} = -\frac{Mb}{L^2}(2a-b)$		$FEM_{BA} = \frac{Mb}{L^2}(2b-a)$

The moment on support A and B for member AB due to the Fixed End Moment, slope and support settlement is:

$$M_{AB} = -FEM_{AB} + \frac{2EI}{L}\left(2\theta_A + 2\theta_B - 3\frac{\Delta}{L}\right)$$

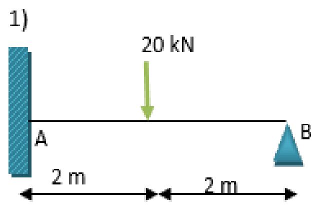
$$M_{BA} = +FEM_{BA} + \frac{2EI}{L}\left(2\theta_B + 2\theta_A - 3\frac{\Delta}{L}\right)$$

OBJECTIVE

- *Able to use formula and moment equation*

1.3 Fixed End Moment

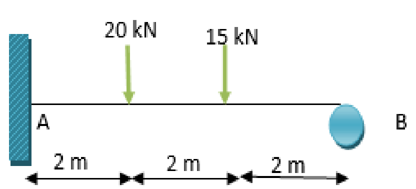
1)



FEM $_{AB}$ = - $-\frac{WL}{8} = -\frac{20(4)}{8}$ = **-10 kNm**

FEM $_{BA}$ = - $+\frac{WL}{8} = +\frac{20(4)}{8}$ = **+10 kNm**

2)



FEM $_{AB}$ = $-\frac{Wab^2}{l^2} - \frac{Wab^2}{l^2}$

$= -\frac{20(2)(4)^2}{6^2} - \frac{15(4)(2)^2}{6^2}$

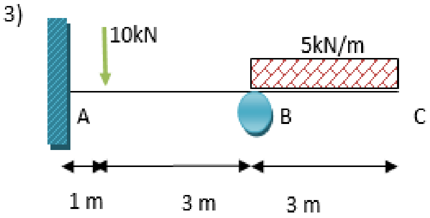
= **-24.44 kNm**

FEM $_{BA}$ = $+\frac{Wab^2}{l^2} + \frac{Wab^2}{l^2}$

$= +\frac{20(2)^2(4)}{6^2} + \frac{15(4)^2(2)}{6^2}$

= **22.22 kNm**

3)



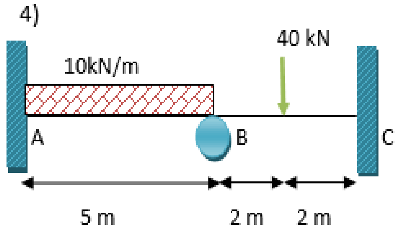
FEM $_{AB}$ = $-\frac{Wab^2}{l^2} = -\frac{10(1)(3)^2}{4^2}$ = **-5.625 kNm**

FEM $_{BA}$ = $+\frac{Wa^2b}{l^2} = +\frac{10(1)^2(3)}{4^2}$ = **1.875 kNm**

M_{BC} + 5(3)(3/2) = 0

M_{BC} = **-22.5 kNm**

4)



FEM $_{AB}$ = $-\frac{Wl^2}{12} = -\frac{10(5)^2}{12}$ = **-20.83 kNm**

FEM $_{BA}$ = $+\frac{Wl^2}{12} = +\frac{10(5)^2}{12}$ = **20.83 kNm**

FEM $_{BC}$ = $-\frac{Wab^2}{l^2} = -\frac{40(2)(2)^2}{4^2}$ = **-20 kNm**

FEM $_{BA}$ = $+\frac{Wa^2b}{l^2} = +\frac{40(2)^2(2)}{4^2}$ = **+20 kNm**

YouTube

How to get fixed end moment

OBJECTIVE

- *Able to find fixed end moments*

Example 1.2

A continuous beam is supported and loaded as shown in Figure 1.1. Analyze the beam for support moments and reactions. The EI's value is constant

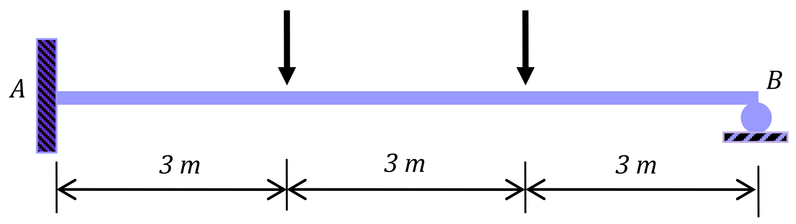


Figure 1.1

Solution

1. Finding the fixed end moment (FEM)

$$FEM_{AB} = -\frac{Pab^2}{L^2} = -\frac{60\times3\times6^2}{9^2} - \frac{25\times6\times3^2}{9^2} = -96.67\ kNm$$

$$FEM_{BA} = \frac{Pba^2}{L^2} = \frac{60\times6\times3^2}{9^2} + \frac{25\times3\times6^2}{9^2} = 73.33\ kNm$$

2. Boundary condition:

$\theta_A = 0$, $\theta_B = ?$ $\Delta = 0$

3. Slope Deflection Equation:

$$M_{AB} = \frac{2EI}{L}\left[2\theta_A + \theta_B - 3\frac{\Delta}{L}\right] + FEM_{AB} = \frac{2EI}{9}[2(0) + \theta_B - 3(0)] - 96$$

$$= \mathbf{0.22EI\theta_B - 96.67 \ldots\ldots\ldots (1)}$$

OBJECTIVE

- *Able to find final moment using slope deflection method*

$$M_{BA} = \frac{2EI}{L}\left[2\theta_B + \theta_A - 3\frac{\Delta}{L}\right] + FEM_{BA} = \frac{2EI}{9}[2\theta_B + (0) - 3(0)] + 7$$

$$= \mathbf{0.44EI\theta_B + 73.33 \ldots\ldots\ldots (2)}$$

4. Equilibrium equation

$\sum M_B = 0;$

$M_{BA} = 0;\quad 0.44EI\theta_B + 73.33 = 0$

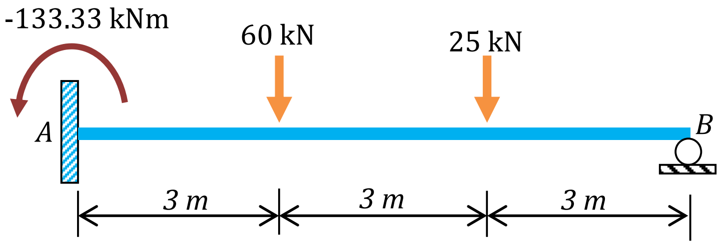
$$\boldsymbol{EI\theta_B} = -\frac{\mathbf{73.33}}{\mathbf{0.44}} = \mathbf{-166.65}$$

5. End of Moment:

$M_{AB} = -133.33\ kNm$

$M_{BA} = 0\ kNm$

6. Support reaction:



$\sum M_B = 0;$

$$-133.33 + (60 \times 3) + (25 \times 6) - (V_B \times 9) = 0$$

$$\boldsymbol{V_B = 21.85\ kN}$$

$\sum V = 0;$

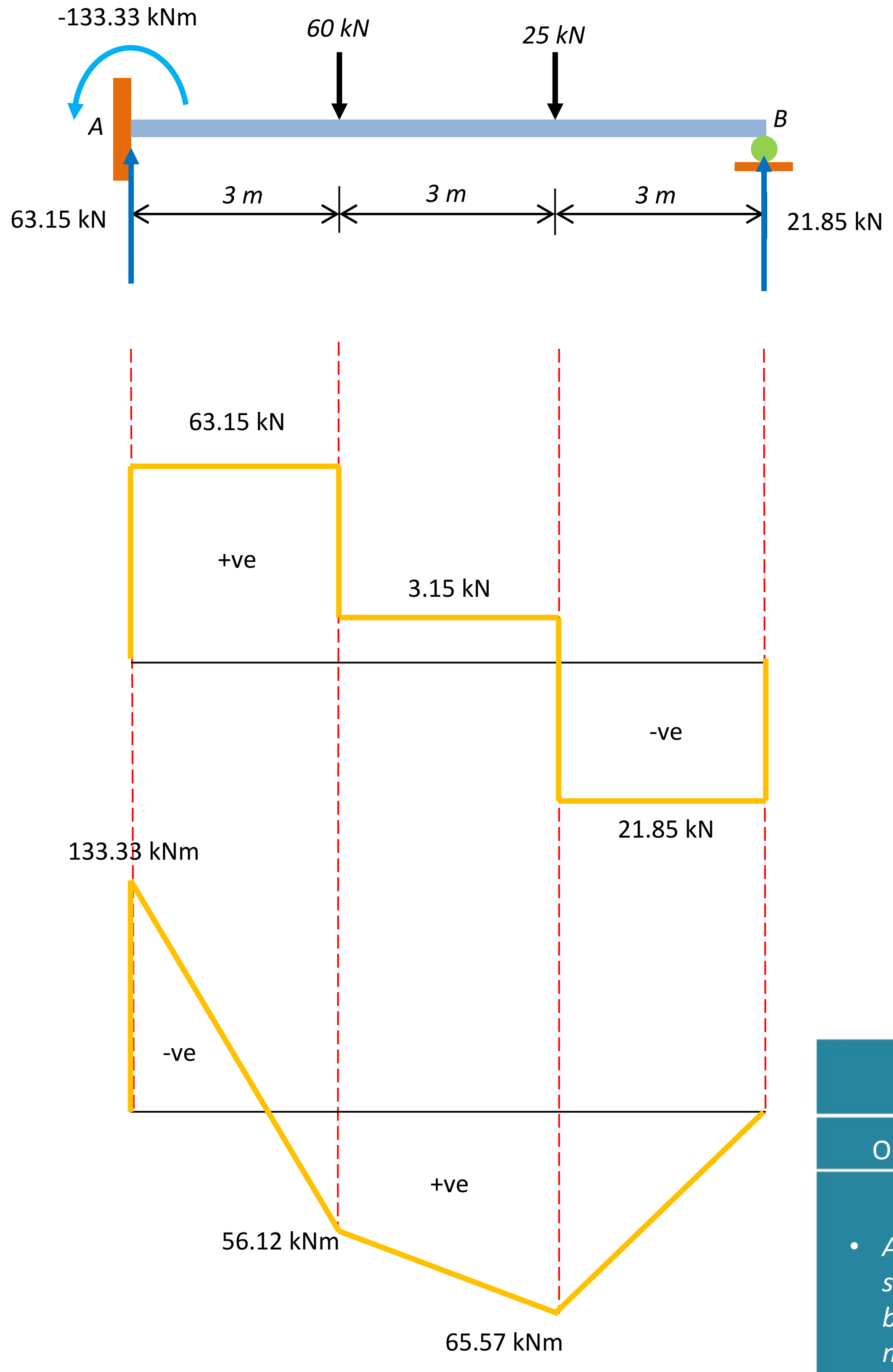
$$V_A + 21.85 - 60 - 25 = 0$$

$$\boldsymbol{V_A = 63.15\ kN}$$

OBJECTIVE

- *Able to find final moment using slope deflection method*

7. Shear force and bending moment diagram;



OBJECTIVE

- *Able to draw shear force and bending moment diagram*

Example 1.2

Find the reaction at each support as in Figure 1.2 using slope deflection method for the indeterminate beam. Then sketch the shear force diagram and bending moment diagram. Assume EI for every member is homogenous.

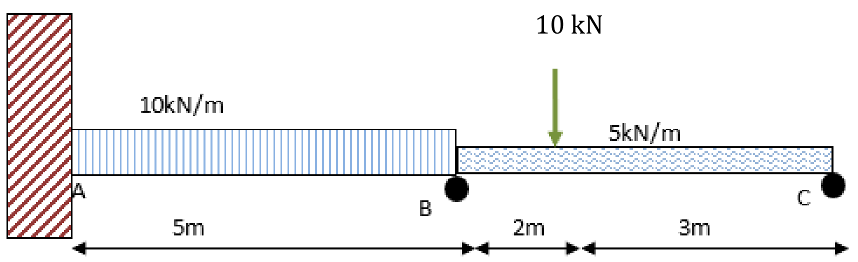


Figure 1.2

YouTube

Slope deflection for indeterminate beam

OBJECTIVE

- *Able to find final moment using slope deflection method*

Solution

1. Fixed end moment (FEM)

$$FEM_{AB} = -\frac{wl^2}{12} = -\frac{10(5)^2}{12} = -20.83 \text{ kNm}$$

$$FEM_{BA} = \frac{wl^2}{12} = \frac{10(5)^2}{12} = +20.83 \text{ kNm}$$

$$FEM_{BC} = -\frac{wl^2}{12} - \frac{wab^2}{l^2} = -\frac{5(5)^2}{12} - \frac{10(2)(3)^2}{(5)^2} = -17.62 \text{ kNm}$$

$$FEM_{CB} = +\frac{wl^2}{12} + \frac{wa^2b}{l^2} = +\frac{5(5)^2}{12} + \frac{10(2)^2(3)}{(5)^2} = +15.22 \text{ kNm}$$

2. Moment equation

$M_{AB} = \frac{2EI}{L} [\ 2\theta_A + \theta_B - \frac{3\Delta}{L}\ ] + FEM_{AB}$ *NOTE ; $\theta_A = 0$; $\delta = 0$

$M_{AB} = \frac{2}{5}EI\ [(\theta_B) - 20.83] = 0.4\ EI(\theta_B) - 20.83$

$M_{BA} = \frac{2EI}{L} [\ 2\theta_B + \theta_A - \frac{3\Delta}{L}\ ] + FEM_{BA}$

$M_{BA} = \frac{2EI}{5} [\ 2\theta_B + \theta_A - \frac{3\Delta}{L}\ ] + 20.83 = EI\ (0.8\theta_B) + 20.83$

$M_{BC} = \frac{2EI}{L} [\ 2\theta_B + \theta_C - \frac{3\Delta}{L}\ ] + FEM_{BC}$

$M_{BC} = \frac{2EI}{5} [\ 2\theta_B + \theta_C\ ] - 17.62 = EI\ (\ 0.8\theta_B + 0.4\theta_C) - 17.62$

$M_{CB} = \frac{2EI}{L} [\ 2\theta_C + \theta_B - \frac{3\Delta}{L}\ ] + FEM_{CB}$

$M_{CB} = \frac{2EI}{5} [\ 2\theta_C + \theta_B - \frac{3\Delta}{L}\ ] + 15.22 = EI\ (\ 0.8\theta_C + 0.4\theta_B) + 15.22$

3. Find value of angle θ_B & θ_C

$\sum M_B = 0$

$M_{BA} + M_{BC} =$

$= [EI\ (0.8\theta_B) + 20.83] + [\ EI\ (\ 0.8\theta_B + 0.4\theta_C) - 17.62]$

$[1.6\theta_B + 0.4\theta_C]EI + 3.21 = 0$

$[1.6\theta_B + 0.4\theta_C]EI = -3.21$ 1

$\sum M_C = 0$

$0 = M_{CB}$

$0 = EI\ (\ 0.8\theta_C + 0.4\theta_B) + 15.22$

$(\ 0.4\theta_B + 0.8\theta_C\ )EI = -15.22$ 2

$\theta_B = 3.14/EI$

$\theta_C = -20.59/EI$

OBJECTIVE

- *Able to find final moment using slope deflection method*

4. Values of free bending moment at every point

$$M_{AB} = \frac{2}{5}EI\,[(\theta_B) - 20.83] = \quad 0.4\,EI(\theta_B) - 20.83 = -19.57\text{ kNm}$$

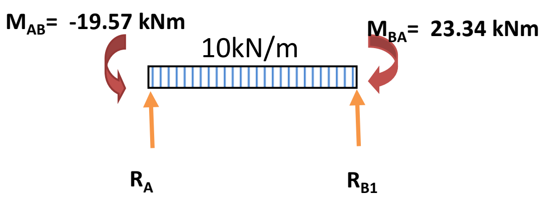
$$M_{BA} = \frac{2EI}{5}\,[\,2\theta_B + \theta_A\,] + 20.83 = EI\,(0.8\theta_B) + 20.83 = +23.34\text{ kNm}$$

$$M_{BC} = \frac{2EI}{5}\,[\,2\theta_B + \theta_C\,] - 17.62 = EI\,(\,0.8\theta_B + 0.4\theta_C) - 17.62 = -23.34\text{ kNm}$$

$$M_{CB} = \frac{2EI}{5}\,[\,2\theta_C + \theta_B\,] + 15.22 = EI\,(\,0.8\theta_C + 0.4\theta_B) + 15.22 = 0\text{ kNm}$$

NOTE * Assume EI homogenous

5. Find reaction at every support & sketch the SFD & BMD



$\sum M_B = 0$

$R_A\,(5) - 10(5)(\frac{5}{2}) - 19.57 + 23.34 = 0$

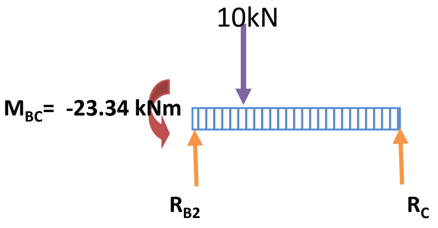
$R_A = 24.25$ kN

$\sum M_A = 0$

$-R_B(5) + 10\,(5)(2.5) - 19.57 + 23.34 = 0$

$R_{B1} = 128.77/5$

$R_{B1} = 25.75$ kN



$\sum M_B = 0$

$-R_C\,(5) \;+5(5)(\frac{5}{2}) + 10(2) - 23.34 = 0$

$R_C = 11.83$ kN

$\sum F_Y = 0$

$11.83 + R_{B2} - 5\,(5) - 10 = 0$

$R_{B2} = 35 - 11.83$

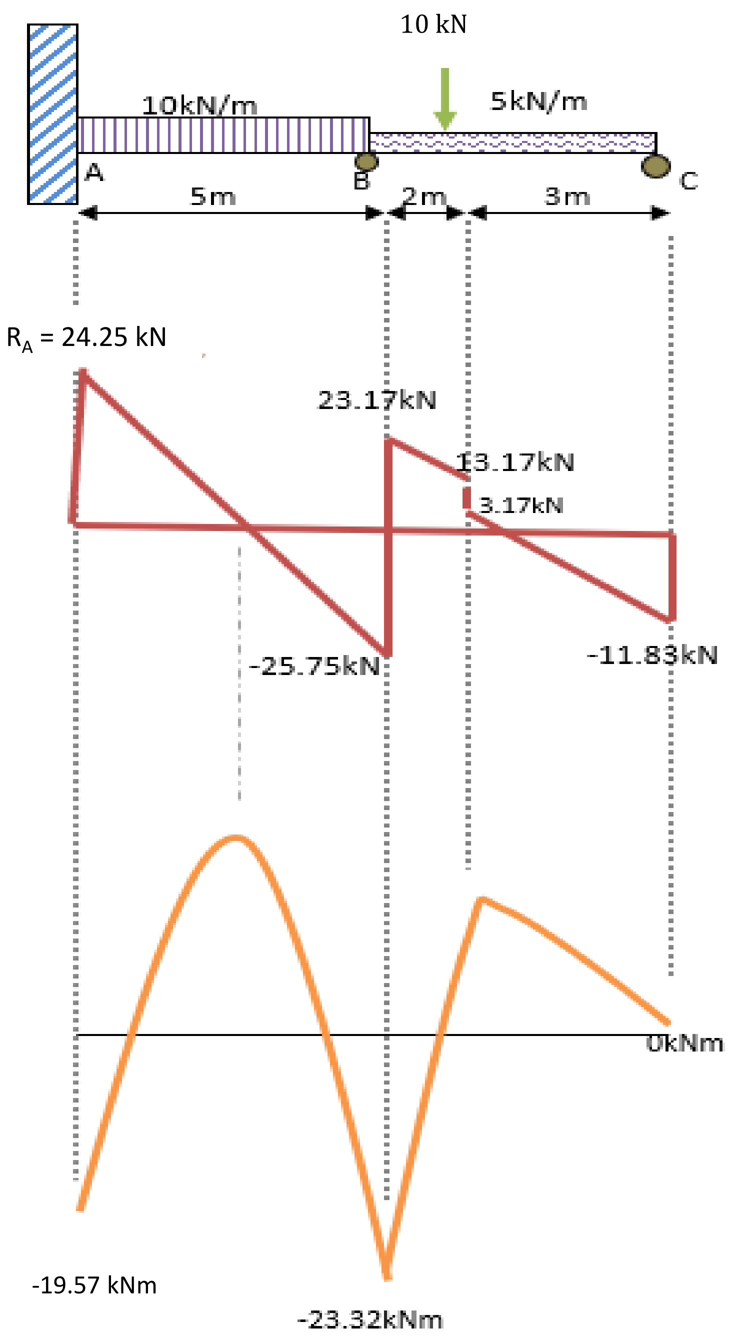
$R_{B2} = 23.17$ kN

$R_B = R_{B1} + R_{B2} = 25.75 + 23.17 = 48.92$ kN

OBJECTIVE

- *Able to find final moment using slope deflection method*

5. Find reaction at every support & sketch the SFD & BMD



M_A = -19.57 kNm

M_B = -19.57 -10(5)(2.5) + 24.25 (5)=-23.32 kNm

M_D = -19.57 -10(5)(2.5 +2) + 24.25 (7)+ 48.92(2) – 5 (2)(1)= 13.02 kNm

M_C = -19.57 -10(5)(2.5+5) + 24.25 (10) + 48.92(5) – 5 (5)(2.5) -10 (3)=-0.00 kNm

OBJECTIVE

- *Able to draw shear force and bending moment diagram*

1.4 PROBLEMS

QUESTION 1

A beam 10m long, fixed at ends A and B is continuous over joint B and is loaded as shown in Figure 1.3. Using the slope deflection method, compute the end moments and plot the shear force and bending moment diagram. The beam has constant EI for both the spans.

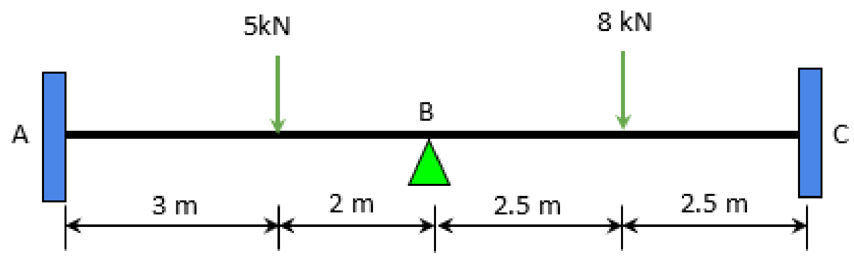


Figure 1.3

QUESTION 2

Using the slope deflection method, compute the end moments and plot the shear force and bending moment diagram as in Figure 1.4 below.

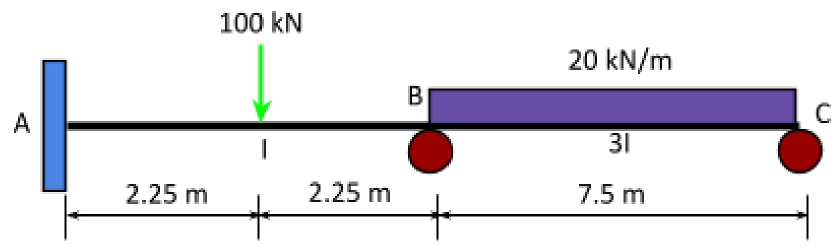


Figure 1.4

OBJECTIVE

- *Able to solve problems using slope deflection method*

1.4 PROBLEMS

QUESTION 3

A continuous beam is subjected to a loads as shown below. Using slope deflection method, draw the shear force and bending moment diagram as Figure 1.5.

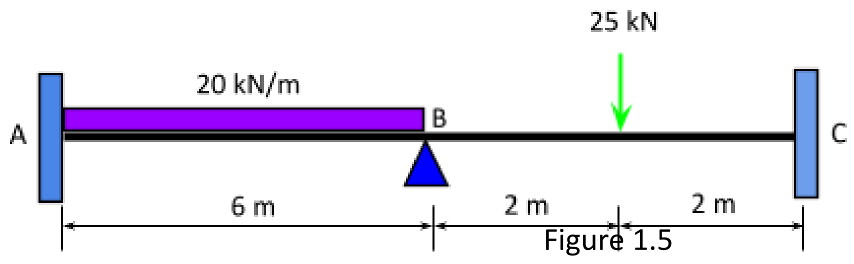


Figure 1.5

QUESTION 4

Using the slope deflection method, compute the end moments as indeterminate beam shown in Figure 1.6.

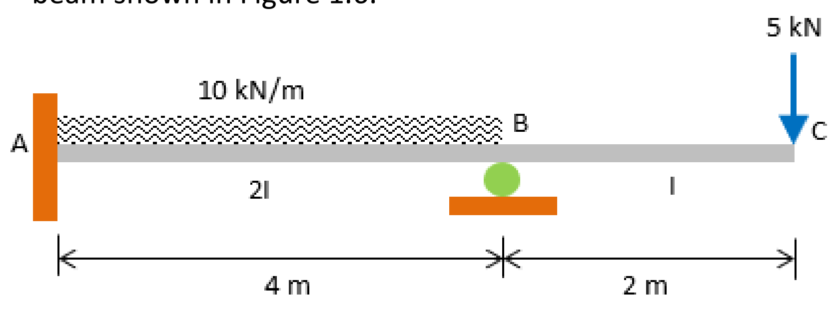


Figure 1.6

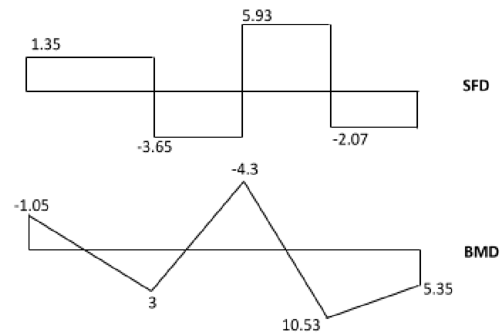
OBJECTIVE

- *Able to solve problems using slope deflection method*

1.5 ANSWERS

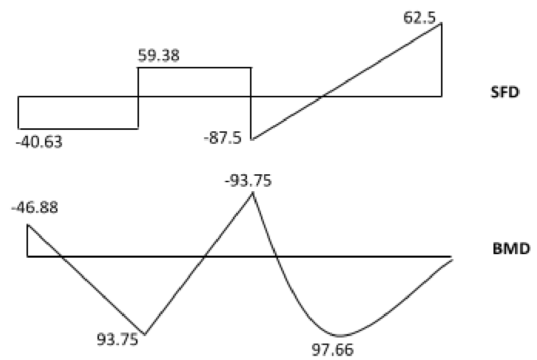
Question 1

M_{AB} =-1.05 kNm
M_{BA} =4.30 kNm
M_{Bc} =-4.30 kNm
M_{CB} =-5.35 kNm



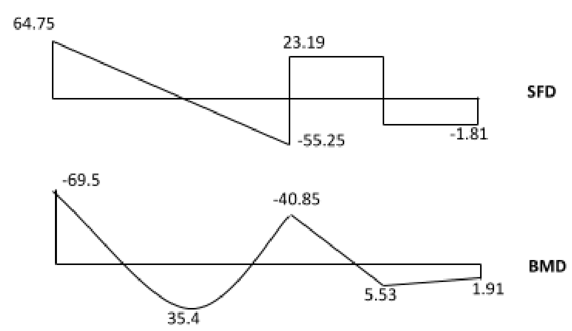
Question 2

M_{AB} =-46.88 kNm
M_{BA} =93.75 kNm
M_{Bc} =-93.75 kNm
M_{CB} =0 kNm



Question 3

M_{AB} =-69.5 kNm
M_{BA} =41 kNm
M_{Bc} =-41kNm

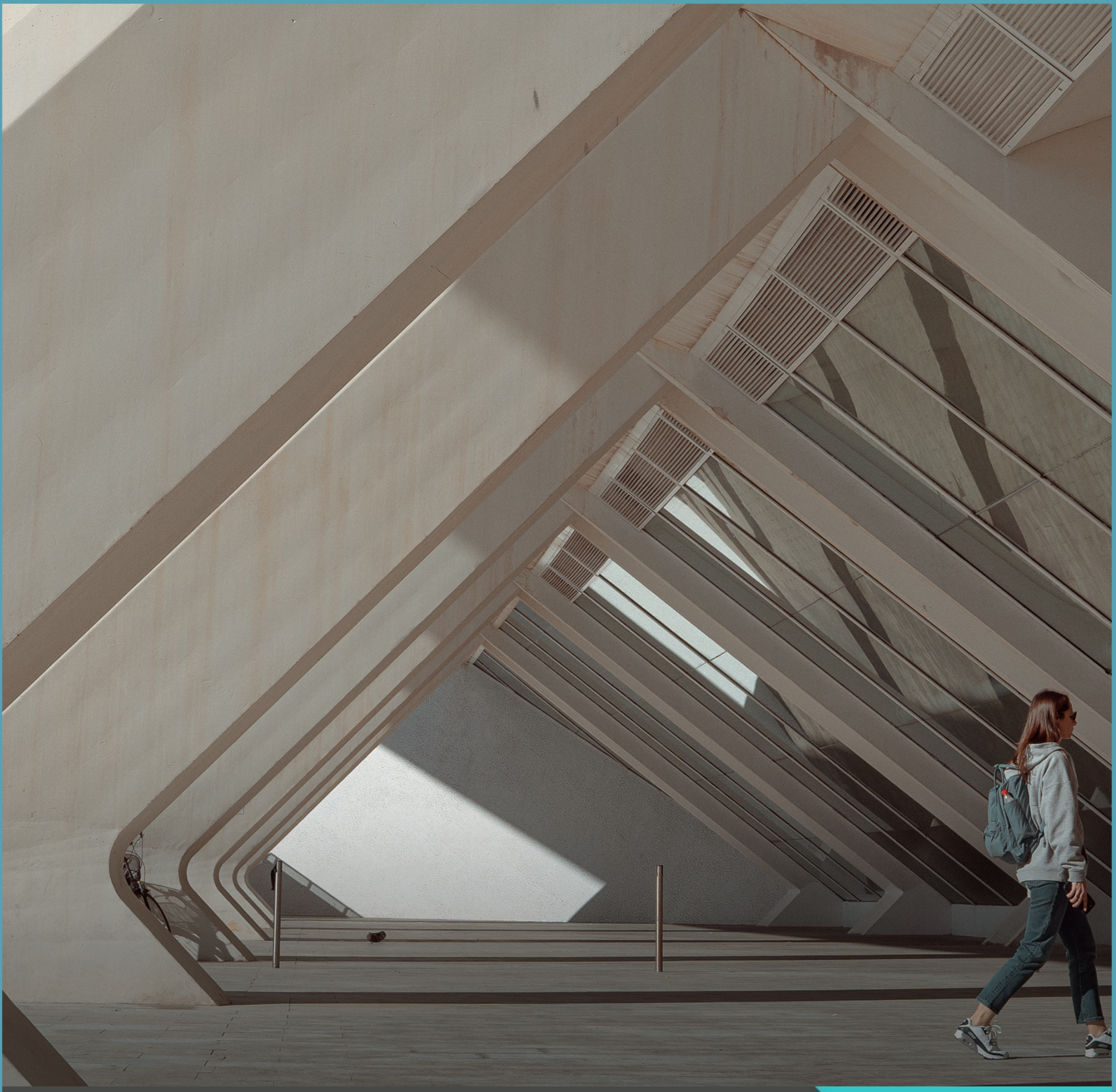


Question 4

$M_{AB} = -15 \text{ kNm}$
$M_{BA} = 10\text{kNm}$
$M_{Bc} = -10 \text{ kNm}$
$M_{CB} = 0 \text{ kNm}$

OBJECTIVE

- *Able to solve problems using slope deflection method*



CHAPTER 2

SLOPE DEFLECTION METHOD FOR STATICALLY INDETERMINATE PORTAL FRAME

2.1 Analysis of portal Frames

Portal frames are frequently used over the entrance of a bridge and as a main stiffening element in building design in order to transfer horizontal forces applied at the top of the frame to the foundation. Portals can be pin supported, fixed supported, or supported by partial fixity.

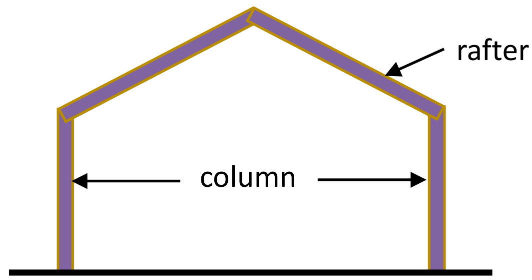


Figure 2.1: Portal Frames

A frame will not side sway, or be displaced to the left or right, provided it is properly restrained. Also, no side sway will occur in an unrestrained frame provided it is symmetric with respect to both loading and geometry. The term Δ in the slope-deflection equation is equal to zero.

While, frames that are nonsymmetrical or subjected to nonsymmetrical loadings have tendency to side sway. It can take place due to following causes:

a) Length of different columns in a frame structure is different.
b) Sections of columns having different cross-sectional properties. For example, if moment of inertia of one vertical member is different from the other.
c) Due to un-symmetrical loading.
d) Lateral loads are acting.

In general, the approach used and the steps involved in analyzing the portal framework are same as on the beam.

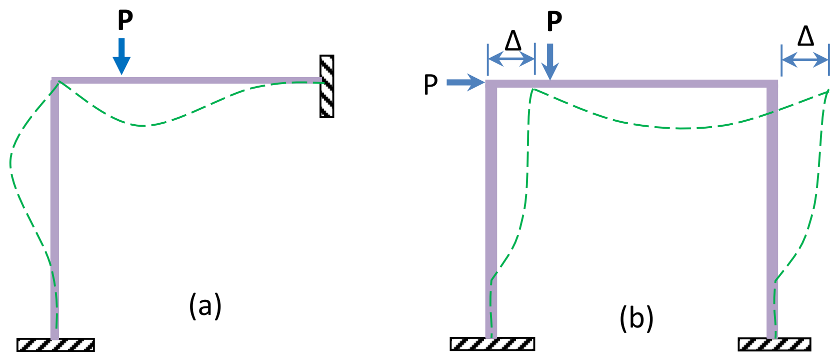


Figure 2.2: (a) No Side Sway Frame (b) Side sway Frame

Quiz

How many axis involved in portal frames?

Ans. 2

OBJECTIVE

- *Able to understand a portal frames system.*

Example 2.1

Draw shear force and bending moment diagram for structure below. The EI's value is constant.

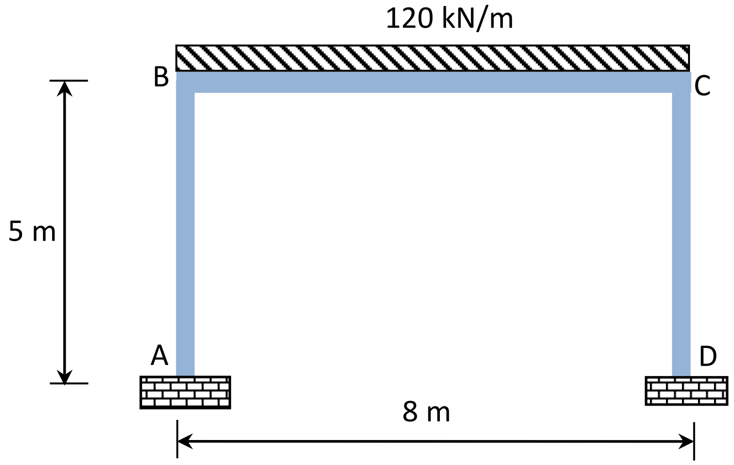


Figure 2.3

Solution

1. Finding the fixed end moment (FEM)

$$FEM_{AB} = 0\ kNm$$

$$FEM_{BA} = 0\ kNm$$

$$FEM_{BC} = \frac{-(120)(8)^2}{12} = -640\ kNm$$

$$FEM_{CB} = \frac{(120)(8)^2}{12} = 640\ kNm$$

$$FEM_{CD} = 0\ kNm$$

$$FEM_{DC} = 0\ kNm$$

2. Boundary condition:

$\theta_A = \theta_D = 0$, $\theta_B = \theta_C = ?$, $\Delta = 0$

OBJECTIVE

- *Able to find final moment using slope deflection method*

Solution

3. Slope Deflection Equation:

$$M_{AB} = \frac{2EI}{5}(2\theta_A + \theta_B) = \frac{2EI}{5}(2(0) + \theta_B) = 0.4EI\theta_B \ldots\ldots\ldots\ldots (1)$$

$$M_{BA} = \frac{2EI}{5}(2\theta_B + \theta_A) = \frac{2EI}{5}(2\theta_B + 0) = 0.8EI\theta_B \ldots\ldots\ldots\ldots (2)$$

$$M_{BC} = \frac{2EI}{8}(2\theta_B + \theta_C) - 640 = \frac{2EI}{8}(2\theta_B + \theta_C) - 640 = 0.5EI\theta_B + 0.25EI\theta_C - 640 \ldots\ldots (3)$$

$$M_{CB} = \frac{2EI}{8}(2\theta_C + \theta_B) + 640 = \frac{2EI}{8}(2\theta_C + \theta_B) + 640 = 0.5EI\theta_C + 0.25EI\theta_B + 640 \ldots\ldots (3)$$

$$M_{CD} = \frac{2EI}{5}(2\theta_C + \theta_D) = \frac{2EI}{5}(2\theta_C + 0) = 0.8EI\theta_C \ldots\ldots\ldots\ldots (5)$$

$$M_{DC} = \frac{2EI}{5}(2\theta_D + \theta_C) = \frac{2EI}{5}(2(0) + \theta_C) = 0.4EI\theta_C \ldots\ldots\ldots\ldots (6)$$

4. Equilibrium equation:

$$\sum M_B = M_{BA} + M_{BC} = 0;$$

$$0.8EI\theta_B + 0.5EI\theta_B + 0.25EI\theta_C - 640 = 0$$

$$\theta_C = \frac{2560}{EI} - 5.2\theta_B \ldots\ldots\ldots\ldots (7)$$

$$\sum M_C = M_{CB} + M_{CD} = 0;$$

$$0.5EI\theta_C + 0.25EI\theta_B + 640 + 0.8EI\theta_C = 0$$

$$1.3EI\theta_C + 0.25EI\theta_B = -640 \ldots\ldots (8)$$

Substituting (7) into (8)

$$1.3EI\theta_C + 0.25EI\theta_B = -640$$

$$1.3EI\left(\frac{2560}{EI} - 5.2\theta_B\right) + 0.25EI\theta_B = -640$$

$$EI\theta_B = 609.52$$

OBJECTIVE

- *Able to find final moment using slope deflection method*

Solution

Substituting $EI\theta_B$ into (7)

$$EI\theta_C = 2560 - 5.2\theta_B$$

$$= \frac{2560}{EI} - 5.2(609.52)$$

$$EI\theta_C = -609.5_C$$

5. The end moments :

$M_{AB} = 243.8\ kNm$

$M_{BA} = 487.6\ kNm$

$M_{BC} = -487.6\ kNm$

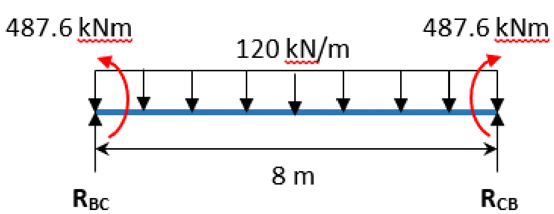
$M_{CB} = 487.6\ kNm$

$M_{CD} = -487.6\ kNm$

$M_{DC} = -243.8\ kNm$

6. Support reactions:

Span BC



$\Sigma M_C = 0$:

$-487.6 + 487.6 - (120 \times 8 \times 4) + R_{BC}(8) = 0$

R_{BC} = 480 kN (↑)

$\Sigma R_V = 0$:

$480 - (120 \times 8) + R_{CB} = 0$

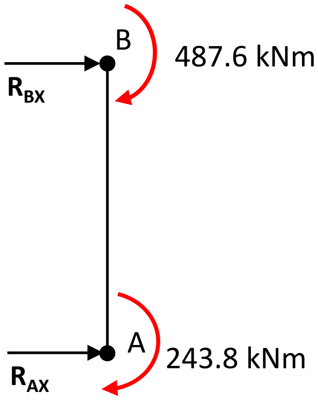
$R_{CB} = 480$ kN (↑)

OBJECTIVE

- *Able to calculate support reactions*

Solution

Span AB



$\Sigma M_B = 0$:

$487.6 + 243.8 + R_{AX}(5) = 0$

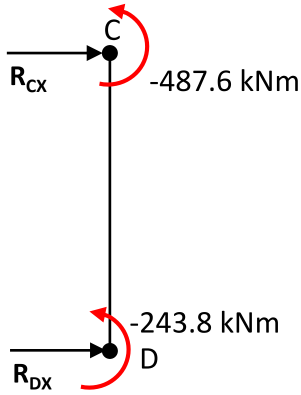
$R_{AX} = -146.28$ kN (←)

$\Sigma F_X = 0$:

$-146.28 + R_{BX} = 0$

$R_{BX} = 146.28$ kN (→)

Span CD



$\Sigma M_B = 0$:

$-487.6 - 243.8 + R_{DX}(5) = 0$

$R_{DX} = 146.28$ kN (→)

$\Sigma F_X = 0$:

$146.28 + R_{CX} = 0$

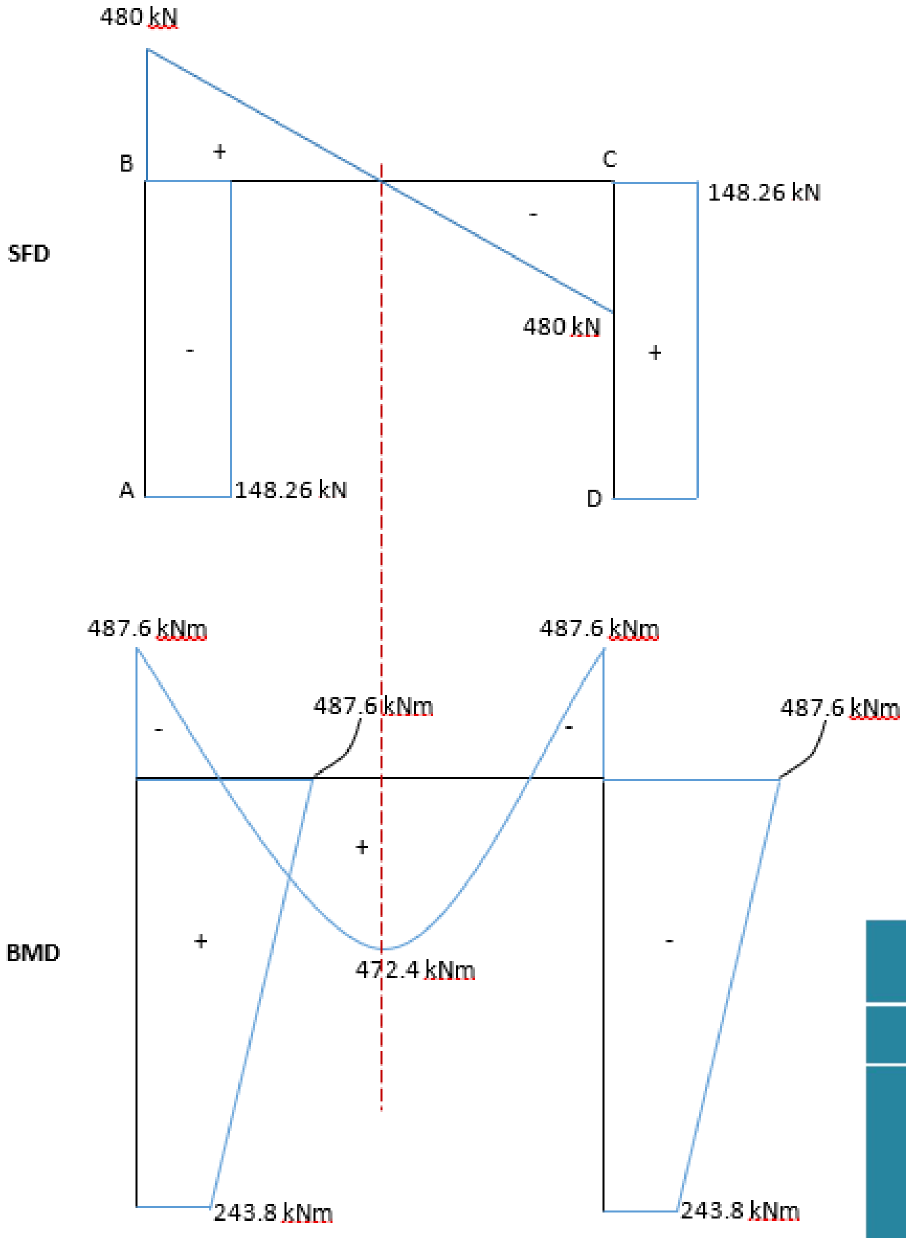
$R_{CX} = -146.28$ kN (←)

OBJECTIVE

- *Able to calculate support reactions*

Solution

7. Shear Force & Bending Moment Diagrams



OBJECTIVE

- *Able to draw a shear force and bending moment diagrams*

Example 2.2

Draw shear force and bending moment diagram for structure below. The EI's value is constant.

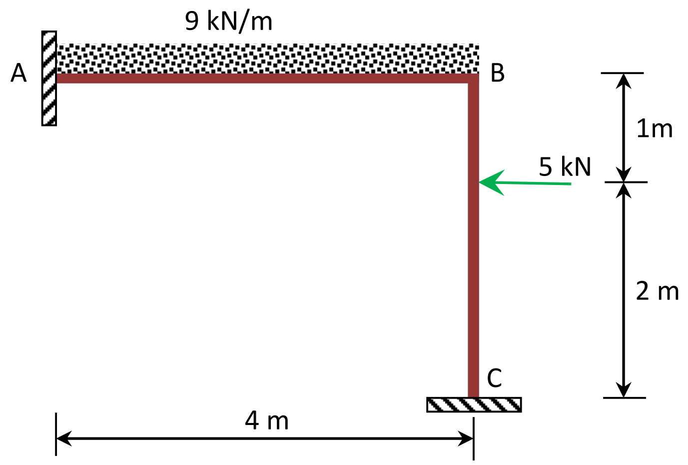


Figure 2.4

Solution

OBJECTIVE

- *Able to find final moment using slope deflection method*

1. Finding the fixed end moment (FEM)

$$FEM_{AB} = -\frac{(9)(4^2)}{12} = -12\ kNm$$

$$FEM_{BA} = \frac{(9)(4^2)}{12} = 12\ kNm$$

$$FEM_{BC} = -\frac{(5)(1)(2^2)}{3^2} = -2.22\ kNm$$

$$FEM_{CB} = -\frac{(5)(2)(1^2)}{3^2} = 1.11\ kNm$$

2. Boundary condition:

$\theta_A = \theta_c = 0, \theta_B = ?, \Delta = 0$

Solution

3. Slope Deflection Equation:

$$M_{AB} = \frac{2EI}{4}(2(0) + \theta_B) - 12 = 0.5EI\theta_B - 12 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$M_{BA} = \frac{2EI}{4}(2\theta_B + 0) + 12 = EI\theta_B + 12 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

$$M_{BC} = \frac{2EI}{3}(2\theta_B + 0) - 2.22 = 1.33EI\theta_B - 2.22 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

$$M_{CB} = \frac{2EI}{3}(2(0) + \theta_B) + 1.11 = 0.67EI\theta_B + 1.11 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

4. Equilibrium equation:

$$\sum M_B = M_{BA} + M_{BC} = 0;$$

$$EI\theta_B + 12 + 1.33EI\theta_B - 2.22 = 0$$

$$EI\theta_B = -4.19$$

5. The end moments :

Substituting $EI\theta_B$ into (1) – (4)

$M_{AB} = -14.10\ kNm$

$M_{BA} = 7.81\ kNm$

$M_{BC} = -7.81 kNm$

$M_{CB} = -1.69\ kNm$

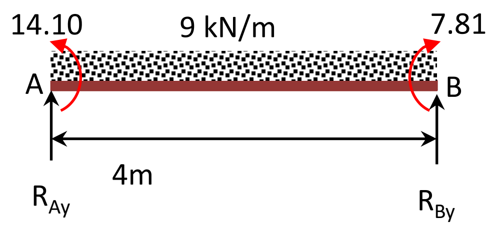
OBJECTIVE

- *Able to find final moment using slope deflection method*

Solution

6. Support reactions:

Span AB



$\Sigma M_B = 0$:

$-14.10 + 7.81 - (9 \times 4 \times 2) + R_{Ay} (4) = 0$

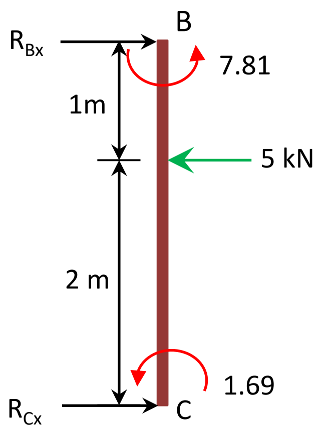
$R_{Ay} = 19.57 kN (\uparrow)$

$\Sigma R_Y = 0$:

$19.57 - (9 \times 4) + R_{By} = 0$

$R_{By} = 16.43 kN (\uparrow)$

Span BC



$\Sigma M_B = 0$:

$-7.81 - 1.69 + (5 \times 1) - R_{CX} (3) = 0$

$R_{CX} = 1.5 kN (\leftarrow)$

$\Sigma F_X = 0$:

$-1.5 - 5 + R_{CX} = 0$

$R_{CX} = 6.5 kN (\rightarrow)$

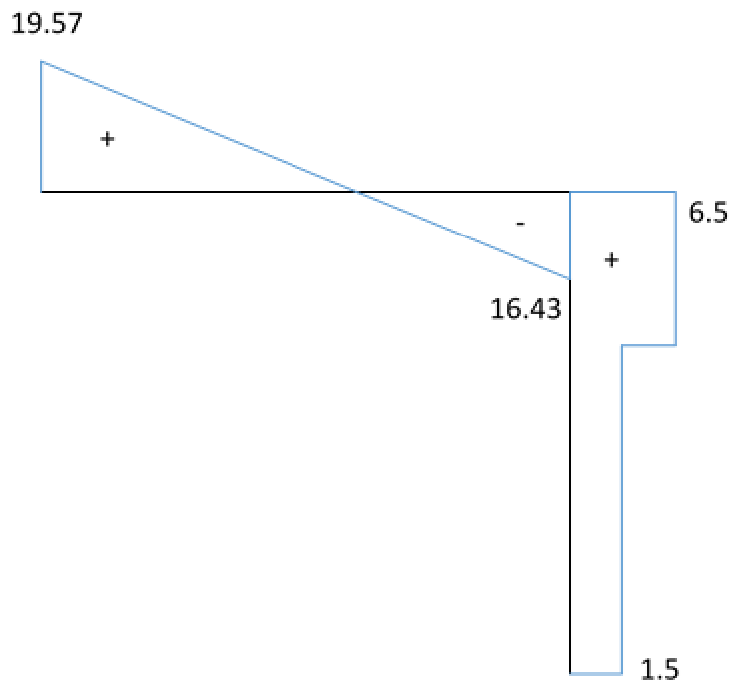
OBJECTIVE

- *Able to calculate support reactions*

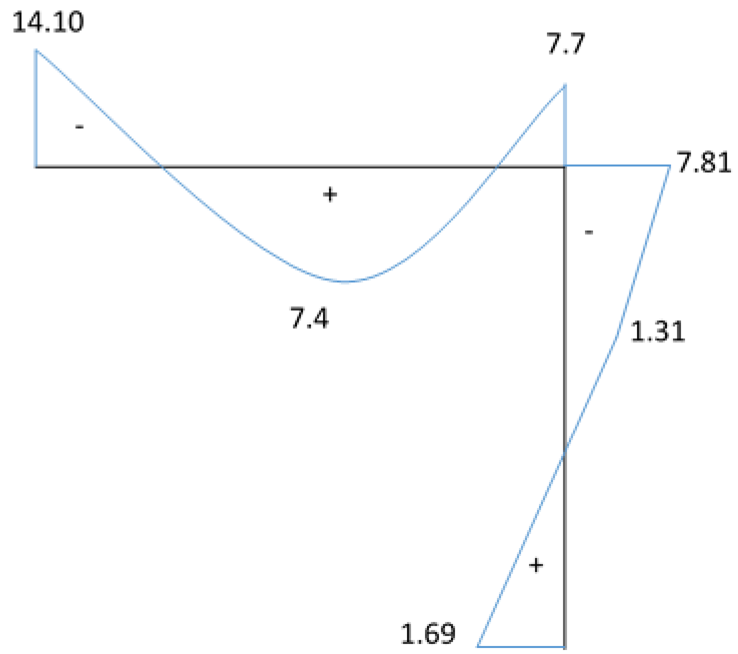
Solution

7. Shear Force & Bending Moment Diagrams

SFD



BMD



OBJECTIVE

- *Able to draw a shear force and bending moment diagrams*

2.2 PROBLEMS

QUESTION 1

The Statically Indeterminate Non-Sway Frame as shown below. Using moment distribution method, determine the end moments and sketch the shear force and bending moment diagrams

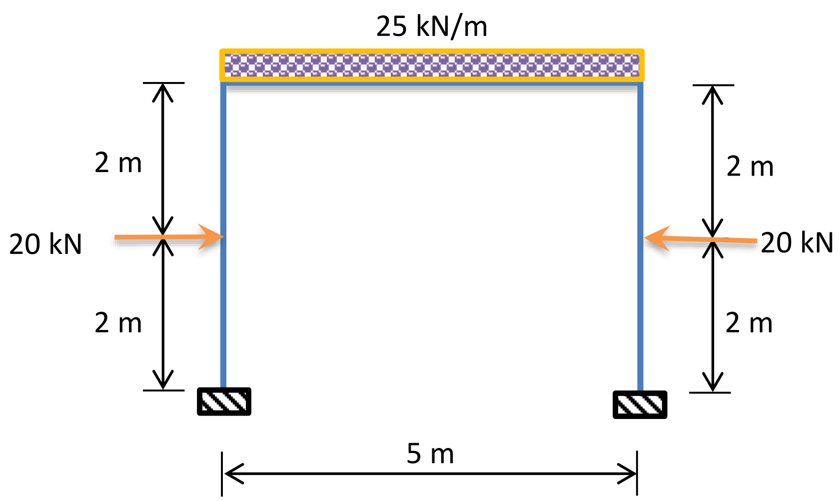


Figure 2.5

QUESTION 2

Using moment distribution method, determine the end moments and sketch the shear force and bending moment diagrams

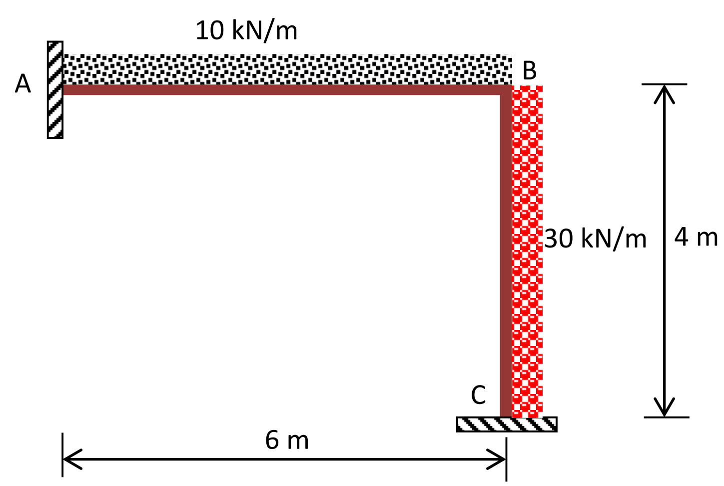


Figure 2.6

OBJECTIVE

- *Able to solve problems using moment distribution method*

2.3 ANSWERS

Question 1

$$M_{AB} = 5.08\ kNm$$

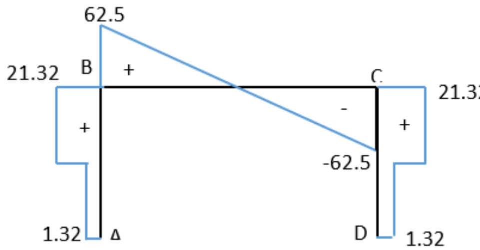
$$M_{BA} = 40.2\ kNm$$

$$M_{BC} = -40.2kNm$$

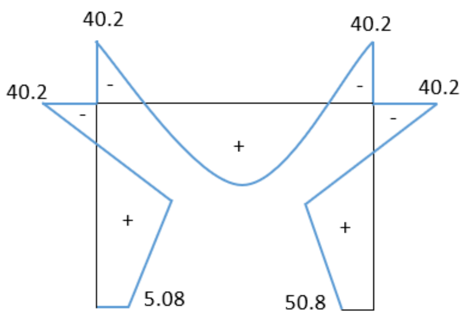
$$M_{CB} = 40.2\ kNm$$

$$M_{CD} = -40.2\ kNm$$

$$M_{DC} = -5.08kNm$$



SFD



BMD

OBJECTIVE

- *Able to calculate the end moments and draw the SFD & BMD*

2.3 ANSWERS

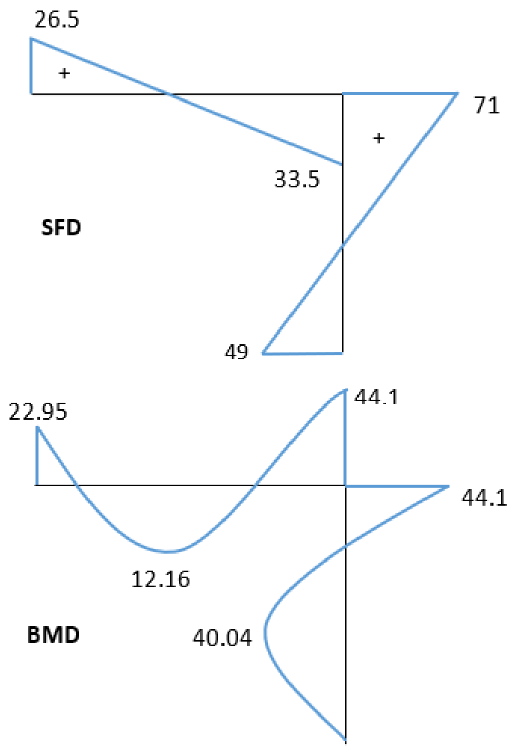
Question 2

$$M_{AB} = -22.95\ kNm$$

$$M_{BA} = 44.10\ kNm$$

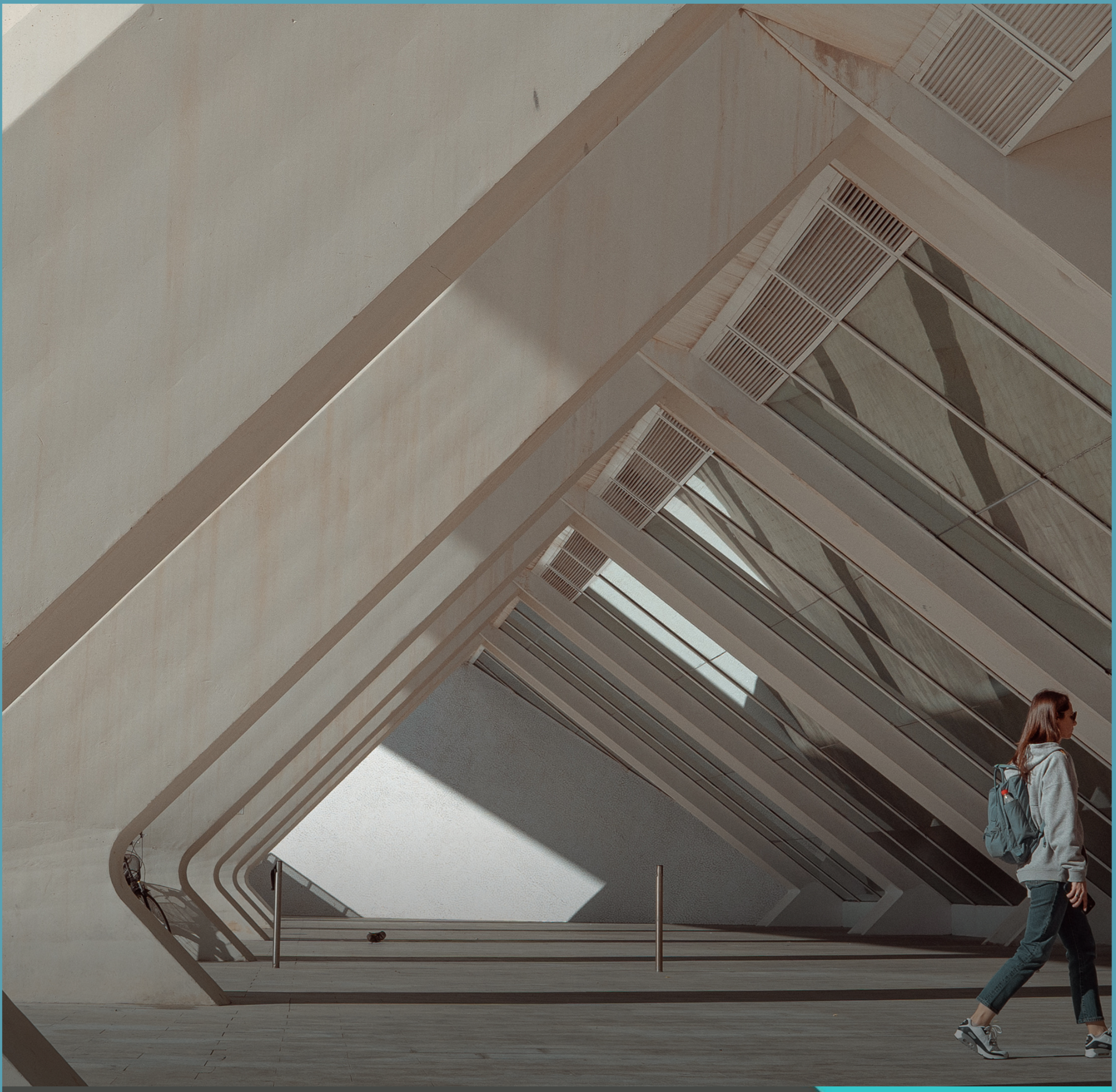
$$M_{BC} = -44.10kNm$$

$$M_{CB} = 0\ kNm$$



OBJECTIVE

- *Able to calculate the end moments and draw the SFD & BMD*



CHAPTER 3

MOMENT DISTRIBUTION METHOD FOR STATICALLY INDETERMINATE BEAM

3.1 Introduction to moment distribution

The moment-distribution method proposed by Hardy Cross in 1932, solves these equations by the method of successive approximations. In this method, the results may be obtained to any desired degree of accuracy. Until recently, the moment-distribution method is very popular among engineers. It is very simple and is being used even today for preliminary analysis of small structures. It is still being taught in the classroom for the simplicity and physical insight it gives to the analyst even though stiffness method is being used more and more. Had the computers not emerged on the scene, the moment-distribution method could have turned out to be a very popular method.

The accuracy of moment distribution method is dependent to the number repeat which does and usually more than 5 repeat real enough. Right value will be acquired when no more moments that need distributed.

Quiz

Who proposed the moment distribution method?

Ans. Hardy Cross

3.2 General procedure

1. Calculate fixed end moments due to applied loads following the same sign convention and procedure, which was adopted in the slope-deflection method.
2. Calculate relative stiffness
3. Determine the distribution factors for various members framing into a particular joint.
4. Distribute the net fixed end moments at the joints to various members by multiplying the net moment by their respective distribution factors in the first cycle.
5. In the second and subsequent cycles, carry-over moments from the far ends of the same member (carry-over moment will be half of the distributed moment).

OBJECTIVE

- *Able to understand the moment distribution method*
- *Understand steps in moment distribution method*

3.2 General procedure

6

Consider this carry-over moment as a fixed end moment and determine the balancing moment. This procedure is repeated from second cycle onwards till convergence.

Stiffness Factor	Distribution Factor
$K = \frac{4EI}{L}$	$DF = \frac{K}{\sum K}$

YouTube

WATCH NOW

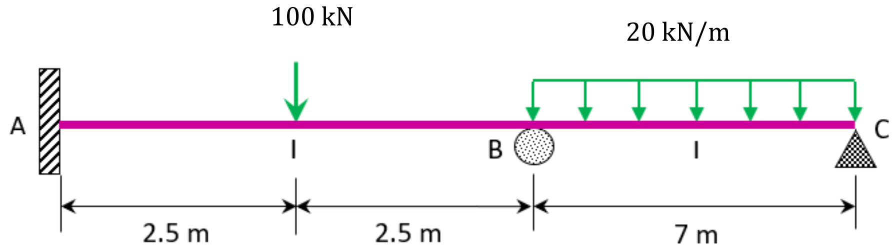
Moment Distributions for indeterminate beam

OBJECTIVE

- *Able to find final moment using moment distribution method*

Example 3.1

Determine the moment value and shear strength to each support and draw shear force and bending moment diagram for structure beam below. The EI's value is constant.



Solution

1. Calculate a Distribution Factor (DF)

DF_{AB} = **0** *(Fixed End Support at end span)*

DF_{CB} = **1** *(Pin Support at end span)*

JOINT	MEMBER	K	ΣK	DF
B	BA	4EI/5 = 0.8EI	0.8EI + 0.57EI = 1.37EI	0.8EI/1.37EI = 0.58
	BC	4EI/7 = 0.57EI		0.57EI/1.37EI = 0.42

2. Finding the fixed end moment (FEM)

$$FEM_{AB} = -\frac{Pab^2}{L^2} = -\frac{(100)(2.5)(2.5)^2}{5^2} = -62.5\ kNm$$

$$FEM_{BA} = \frac{Pa^2b}{L^2} = -\frac{(100)(2.5)(2.5)^2}{5^2} = 62.5\ kNm$$

$$FEM_{BC} = -\frac{wL^2}{12} = -\frac{(20)(7)^2}{12} = -81.67\ kNm$$

$$FEM_{CB} = \frac{wL^2}{12} = \frac{(20)(7)^2}{12} = 81.67\ kNm$$

OBJECTIVE

- *Able to find final moment using moment distribution method*

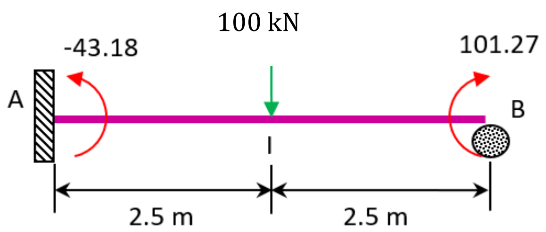
Solution

3. Calculate a Moment Distribution

MEMBER	AB	BA	BC	CB
DF	0.00	0.58	0.42	1.00
CO	0.00	0.50	0.50	0.50
FEM	-62.50	62.50	-81.67	81.67
DIS.	0.00	11.18	7.99	-81.67
CO	5.59	0.00	-40.83	3.99
DIS.	0.00	23.82	17.01	-3.99
CO	11.91	0.00	-2.00	8.51
DIS.	0.00	1.16	0.83	-8.51
CO	0.58	0.00	-4.25	0.42
DIS.	0.00	2.48	1.77	-0.42
CO	1.24	0.00	-0.21	0.89
DIS.	0.00	0.12	0.09	-0.89
End Moment	**-43.18**	**101.27**	**-101.27**	**0.00**

4. Finding the support reaction

Span AB



$\sum M_B = 0$;

$-43.18 + 101.27 - (100)(2.5) + (5)R_{Ay} = 0$

$R_{Ay} = 38.38\ kN$

$\sum F_Y = 0$;

$R_{By} + 38.38 - 100 = 0$

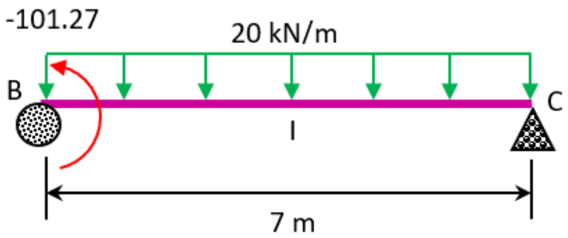
$R_{By} = 61.62\ kN$

OBJECTIVE

- *Able to find support reaction*

Solution

Span BC



$$\sum M_C = 0\,;$$

$$-101.27 - (20)(7)(3.5) + (7)R_{By} = 0$$

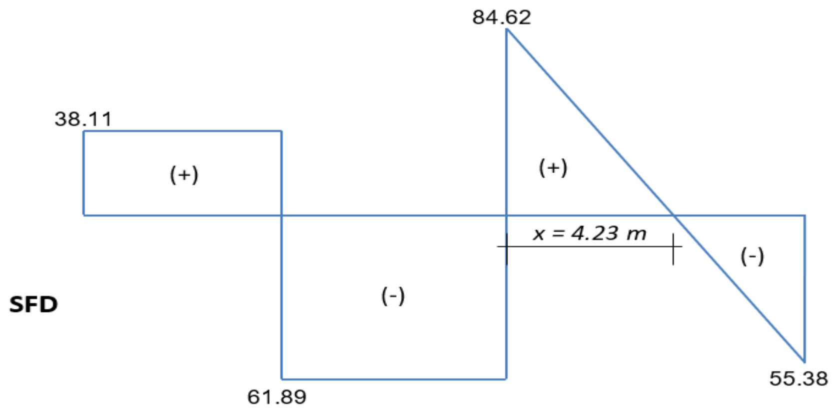
$$R_{By} = 84.47\ kN$$

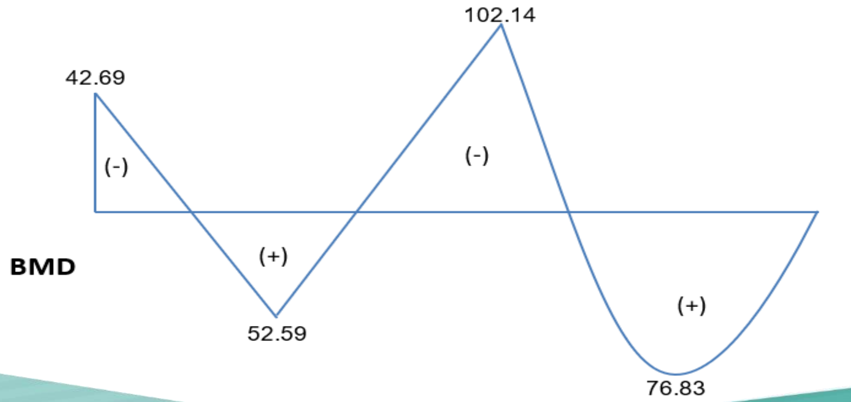
$$\sum F_Y = 0\,;$$

$$R_{Cy} + 84.47 - (20)(7) = 0$$

$$R_{Cy} = 55.53\ kN$$

5. Shear force and bending moment diagram





OBJECTIVE

- *Able to draw shear force and bending moment diagram*

Example 3.2

Determine the moment value and shear strength to each support and draw shear force and bending moment diagram for structure beam below. The EI's value is constant.

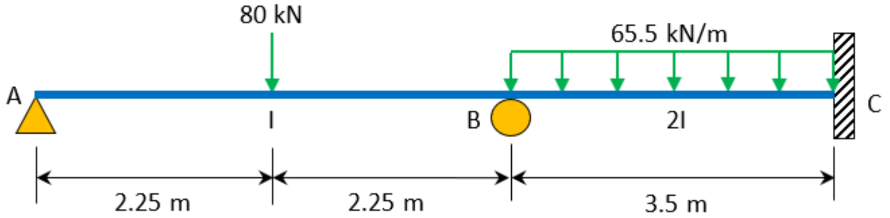


Figure 3.2

Solution

1. Calculate a Distribution Factor (DF)

DF_{AB} = **0** *(Pin Support at end span)*

DF_{CB} = **1** *(Fixed End Support at end span)*

JOINT	MEMBER	K	ΣK	DF = $^{K}/_{\sum K}$
B	BA	4EI/4.5 = 0.89EI	0.89EI + 2.29EI = 3.18EI	0.28
	BC	(4)(2)EI/3.5 = 2.29EI		0.72

2. Finding the fixed end moment (FEM)

$$FEM_{AB} = -\frac{Pab^2}{L^2} = -\frac{(80)(2.25)(2.25)^2}{4.5^2} = -45\ kNm$$

$$FEM_{BA} = \frac{Pa^2b}{L^2} = -\frac{(80)(2.25)(2.25)^2}{4.5^2} = 45\ kNm$$

$$FEM_{BC} = -\frac{wL^2}{12} = -\frac{(65.5)(3.5)^2}{12} = -66.86\ kNm$$

$$FEM_{CB} = \frac{wL^2}{12} = \frac{(65.5)(3.5)^2}{12} = 66.86\ kNm$$

OBJECTIVE

- *Able to find final moment using moment distribution method*

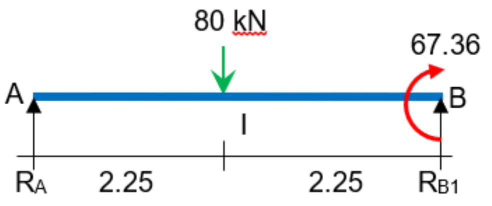
Solution

3. Calculate a Moment Distribution

MEMBER	AB	BA	BC	CB
DF	1	0.28	0.72	0
CO	0.50	0.50	0.50	0
FEM	-45	45	-66.86	66.86
DIS.	45	6.12	15.74	0
CO	3.06	22.5	0	7.87
DIS.	-3.06	-6.3	-16.2	0
CO	-3.15	-1.53	0	-8.1
DIS.	3.15	0.43	1.1	0
CO	0.22	1.58	0	0.55
DIS.	-0.22	-0.44	-1.14	0
CO	-0.22	-0.11	0	-0.57
DIS.	0.22	0.03	0.08	0
CO	0.22	0.11	0	0.04
DIS.	-0.22	-0.03	-0.08	0
End Moment	**0**	**67.36**	**-67.36**	**66.65**

4. Finding the support reaction

Span AB



$$\sum M_B = 0\ ;$$

$$67.36 - (80)(2.25) + (4.5)R_{Ay} = 0$$

$$R_{Ay} = 25.03\ kN$$

$$\sum F_Y = 0\ ;$$

$$R_{By} + 25.03 - 80 = 0$$

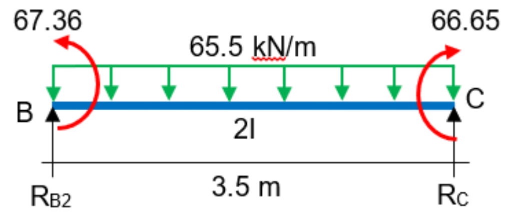
$$R_{By} = 54.97\ kN$$

OBJECTIVE

- *Able to find final moment using moment distribution method*
- *Able to find support reaction*

Solution

Span AB



$$\sum M_C = 0\ ;$$

$$-67.36 + 66.65 - (65.5)(3.5)(3.5/2) + (3.5)R_{By} = 0$$

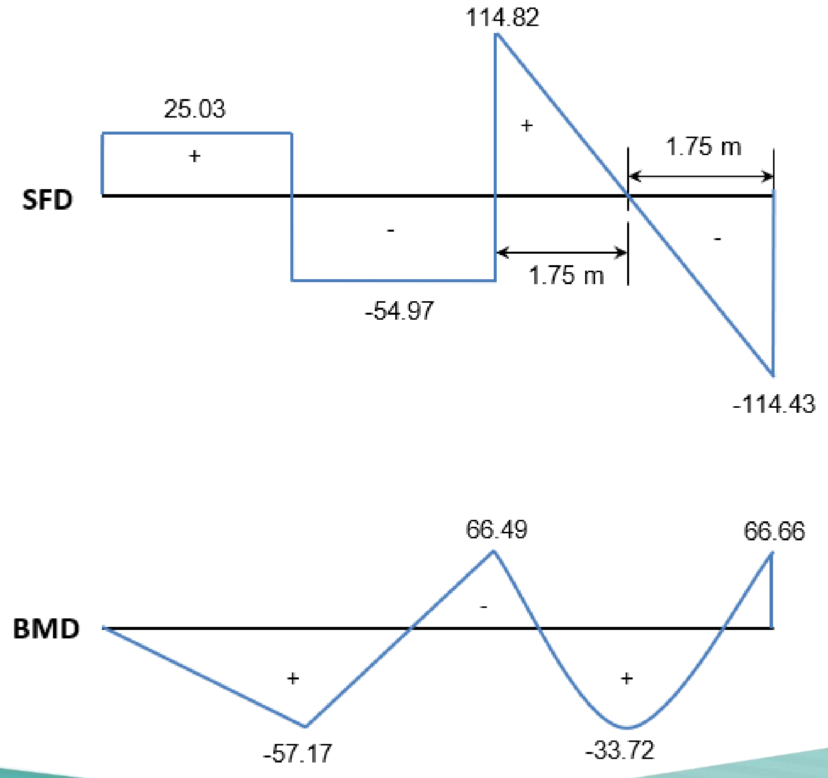
$$R_{By} = 114.65\ kN$$

$$\sum F_Y = 0\ ;$$

$$R_{Cy} + 114.65 - (6.5)(3.5) = 0$$

$$R_{Cy} = 114.6\ kN$$

5. Shear force and bending moment diagram;



OBJECTIVE

- *Able to find support reaction*
- *Able to draw shear force and bending moment diagram*

3.3 Modify stiffness for pinned or role support at the end case

To facilitate moment distribution process so that focus more precisely, supply beam continuous with either or both of them support pin or roller, distribution method this moment may be modified.

Moment to pin or roller which is located at the end beam must be zero. Therefore, there is no need to do the process of bringing a side on a support that cannot bear/withstand the moment.

For a beam span having one fixed end support and the other end connected to a pin, the stiffness taken for the span having the pin connected is 3/4 from the original stiffness during the calculation of the distribution factor.

Quiz

What is the symbol of stiffness factor?

Ans. K

Example 3.3

Determine the moment value and shear strength to each support and draw shear force and bending moment diagram for structure beam below. The EI's value is constant.

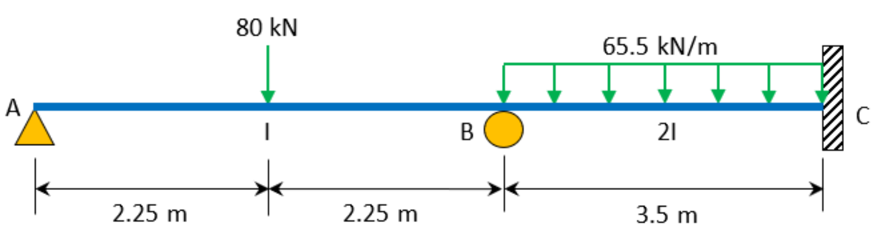


Figure 3.3

OBJECTIVE

- *Able to find final moment using modify stiffness for pin or role support*

Solution

1. Calculate a Distribution Factor (DF)

DF_{AB}	=	**0** *(Pin Support at end span)*
DF_{CB}	=	**1** *(Fixed End Support at end span)*

Solution

JOINT	MEMBER	K	ΣK	DF = $^{K}/_{\sum K}$
B	BA	(3/4)(4EI/L) = 12EI/18= 0.67EI	0.67EI + 2.29EI = 2.96EI	0.23
	BC	(2)4EI/L = 8EI/3.5 = 2.29E		0.77

2. Finding the fixed end moment (FEM)

$$FEM_{AB} = -\frac{Pab^2}{L^2} = -\frac{(80)(2.25)(2.25)^2}{4.5^2} = -45\ kNm$$

$$FEM_{BA} = \frac{Pa^2b}{L^2} = -\frac{(80)(2.25)(2.25)^2}{4.5^2} = 45\ kNm$$

$$FEM_{BC} = -\frac{wL^2}{12} = -\frac{(65.5)(3.5)^2}{12} = -66.86\ kNm$$

$$FEM_{CB} = \frac{wL^2}{12} = \frac{(65.5)(3.5)^2}{12} = 66.86\ kNm$$

So, with make comparison between Example 2 and 3, distribute moment with using the modified method are more disposed and quicker compared to with use the means ordinary stiffness.

3. Calculate a Moment Distribution

MEMBER	AB	BA	BC	CB
DF	1	0.28	0.72	0
CO	0.50	0.50	0.50	0
FEM	-45	45	-66.86	66.86
DIS.	45	5.03	16.83	0
CO	0	2.25	0	8.42
DIS.	0	-5.18	-17.32	0
CO	0	0	0	0
DIS.	0	0	0	0
End Moment	**0**	**67.35**	**-67.35**	**66.68**

OBJECTIVE

- *Able to find final moment using modify stiffness for pin or role support*

3.4 PROBLEMS

QUESTION 1

Determine end moment for all members and reaction at the support, and then draw SFD and BMD for the beam below.

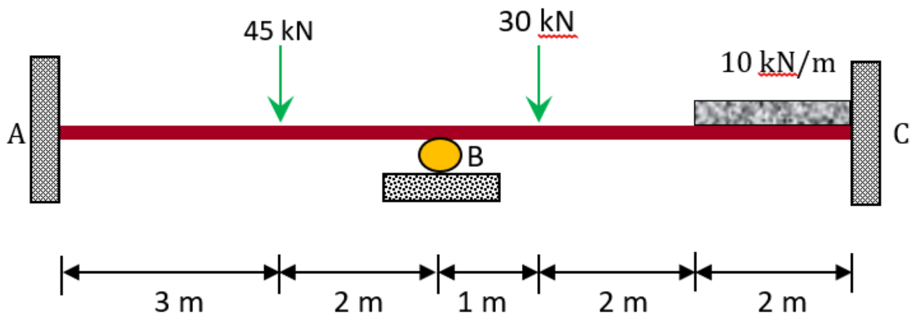


Figure 3.4

QUESTION 2

Determine end moment for all members and reaction at the support, and then draw SFD and BMD for the beam below.

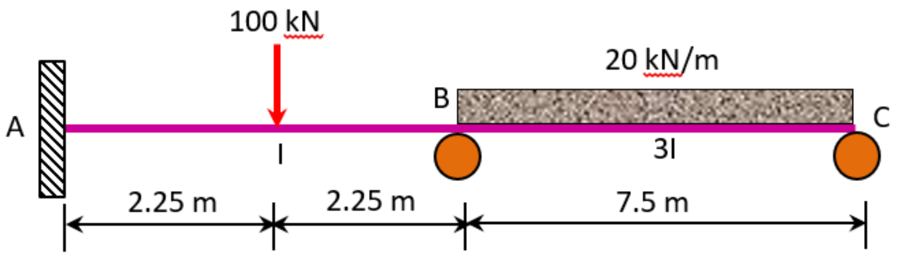


Figure 3.5

OBJECTIVE

- *Able to solve problems using moment distribution method*

3.5 ANSWERS

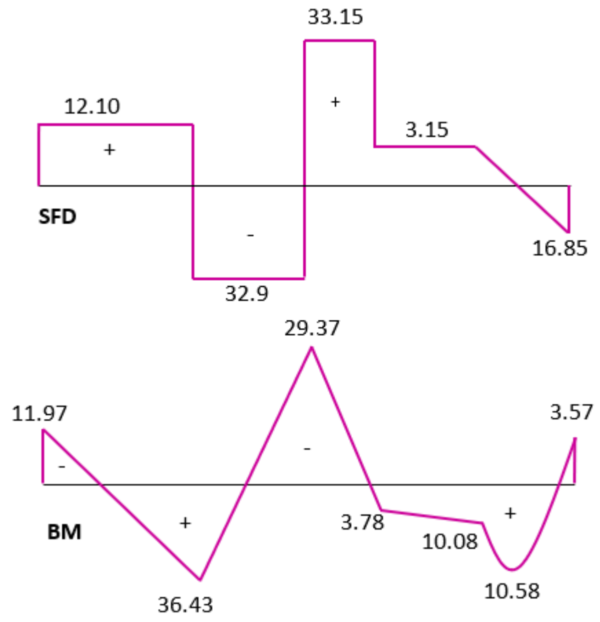
Question 1

$$M_{AB} = -11.97\ kNm$$

$$M_{BA} = 29.39\ kNm$$

$$M_{BC} = -29.39\ kNm$$

$$M_{CB} = 3.64\ kNm$$



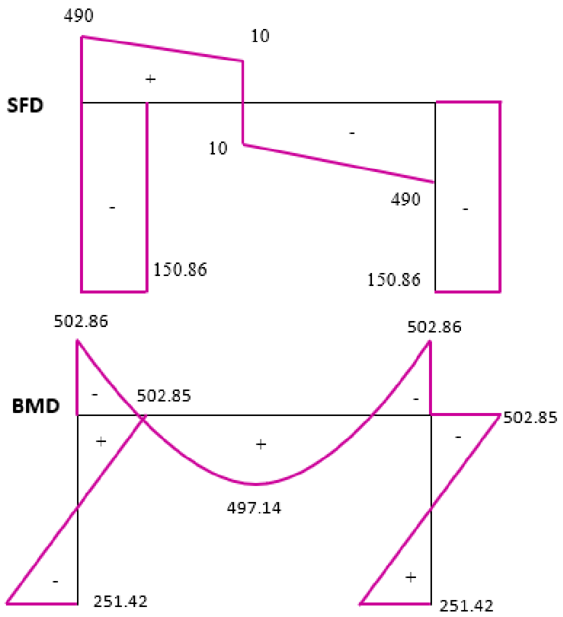
OBJECTIVE

- *Able to solve problems using slope deflection method*

3.5 ANSWERS

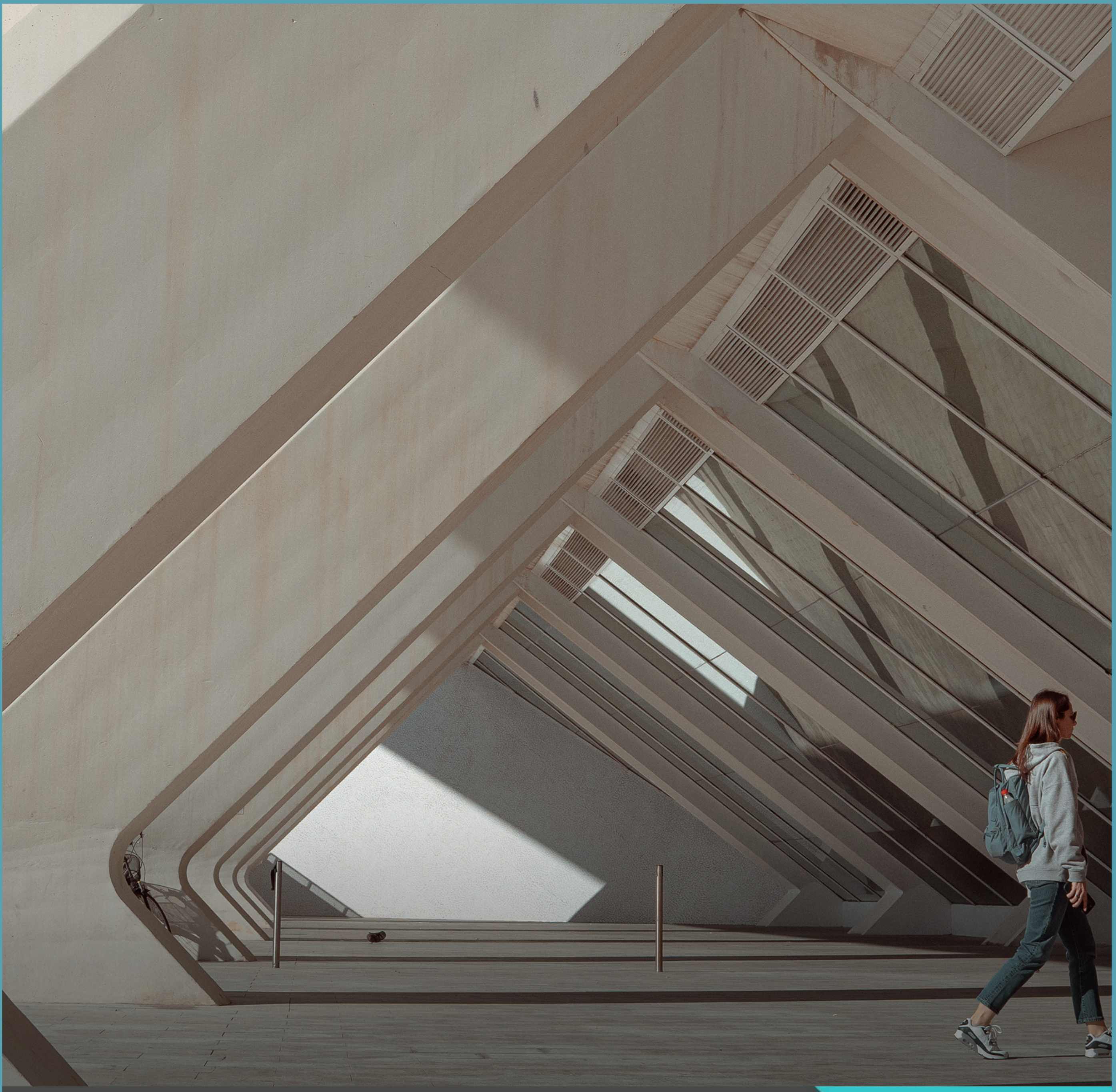
Question 2

$$M_{AB} = -46.88\ kNm$$
$$M_{BA} = 93.75\ kNm$$
$$M_{BC} = -93.75\ kNm$$
$$M_{CB} = 0\ kNm$$



OBJECTIVE

- *Able to solve problems using slope deflection method*



CHAPTER 4

MOMENT DISTRIBUTION METHOD FOR STATICALLY INDETERMINATE PORTAL FRAME

4.1 Introduction to moment distribution portal frame

Application of the moment distribution method for frames having no side sway follows the same procedure as for beams. To minimize the chance for errors, it is suggested that the analysis be arranged in a tabular form as shown in previous examples. Also, the distribution of moments can be shortened if the stiffness factor of span can be modified as indicated in previous section.

Quiz

How many unknowns at pin support ?

Ans. 2

WATCH THIS VIDEO!

YouTube

Moment Distributions for frames

OBJECTIVE

- *Able to understand the moment distribution method*
- *Understand steps in moment distribution method*

Example 4.1

Draw shear force and bending moment diagram to rigid frame structure as below.

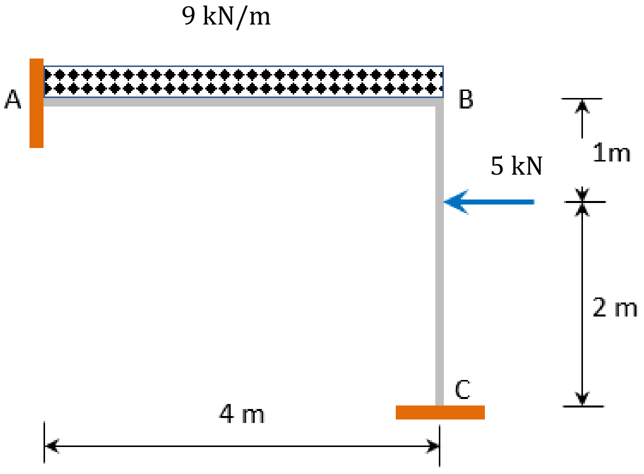


Figure 4.1

Solution

1. Calculate the Distribution Factor (DF)

DF_{AB} = DF_{CB} = 0 *(Fixed End Support at end span)*

JOINT	MEMBER	K	ΣK	DF
B	BA	4EI/4 = 1EI	(1EI) + (1.33EI)	0.43
	BC	4EI/3= 1EI	= 2.33EI	0.57

2. Finding the fixed end moment (FEM)

$$FEM_{AB} = -\frac{(9)(4^2)}{12} = -12\ kNm$$

$$FEM_{BA} = \frac{(9)(4^2)}{12} = 12\ kNm$$

$$FEM_{BC} = -\frac{(5)(1)(2^2)}{3^2} = -2.22\ kNm$$

$$FEM_{CB} = -\frac{(5)(2)(1^2)}{3^2} = 1.11\ kNm$$

OBJECTIVE

- *Able to find final moment using moment distribution method*

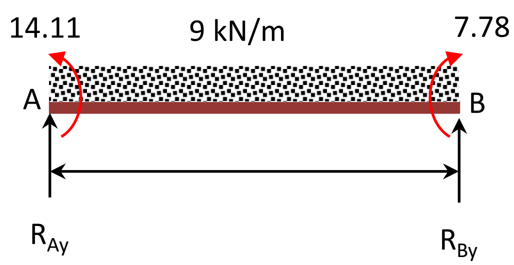
Solution

3. Calculate a Moment Distribution

MEMBER	AB	BA	BC	CB
CO	0	0.5	0.5	0
DF	0	0.43	0.57	0
FEM	-12	12	-2.22	1.11
DIS.	0	-4.21	-5.58	0
CO	-2.11	0	0	-2.79
DIS.	0	0	0	0
End Moment	**-14.11**	**7.78**	**-7.8**	**-1.68**

4. Find the support reaction

Span AB



$\Sigma M_B = 0$:

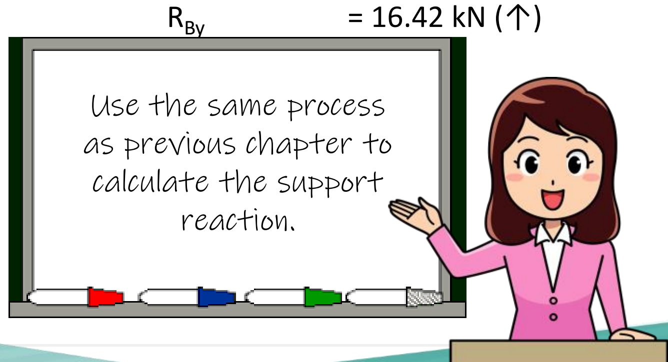
$-14.11 + 7.78 - (9 \times 4 \times 2) + R_{Ay}(4) = 0$

$R_{Ay} =$ 19.58 kN (↑)

$\Sigma R_Y = 0$:

$19.58 - (9 \times 4) + R_{By} = 0$

$R_{By} = 16.42$ kN (↑)

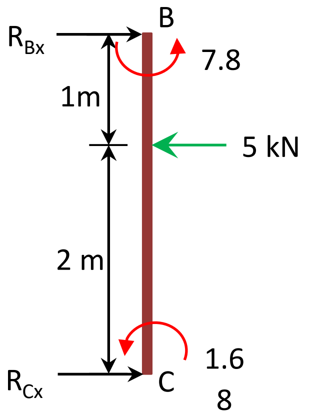


OBJECTIVE

- *Able to find final moment using moment distribution method*

Solution

Span BC



$\Sigma M_B = 0$:

$-7.8 - 1.68 + (5 \times 1) - R_{CX}(3) = 0$

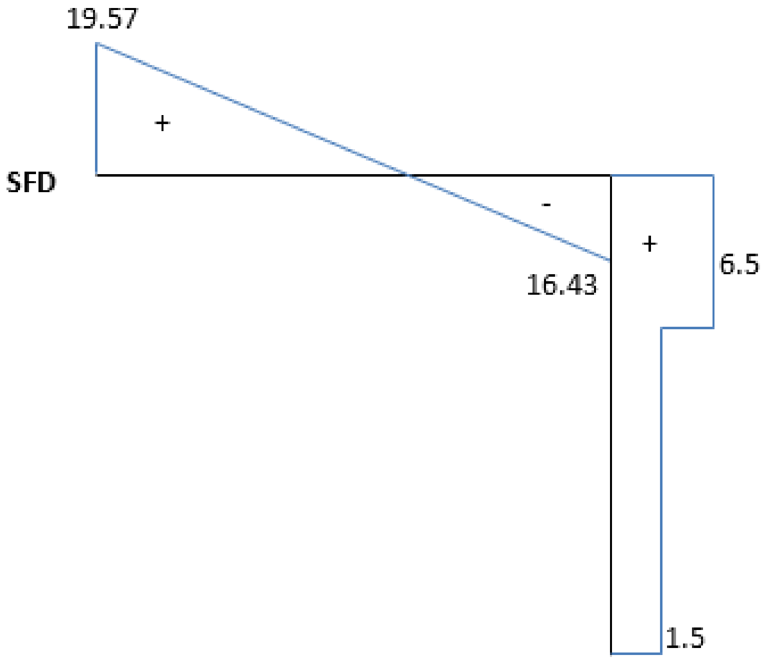
$R_{CX} = 1.49$ kN (←)

$\Sigma F_X = 0$:

$-1.49 - 5 + R_{CX} = 0$

$R_{CX} = 6.49$ kN (→)

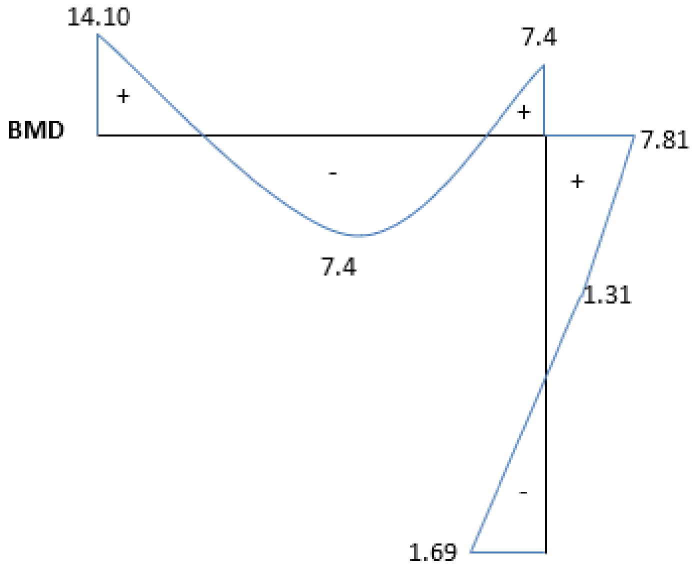
7. Shear Force & Bending Moment Diagrams



OBJECTIVE

- *Able to find support reaction*
- *Able to draw shear force and bending moment diagram*

Solution



Example 3.2

Determine the moment value and shear strength to each support and draw shear force and bending moment diagram for structure below. The EI's value is constant.

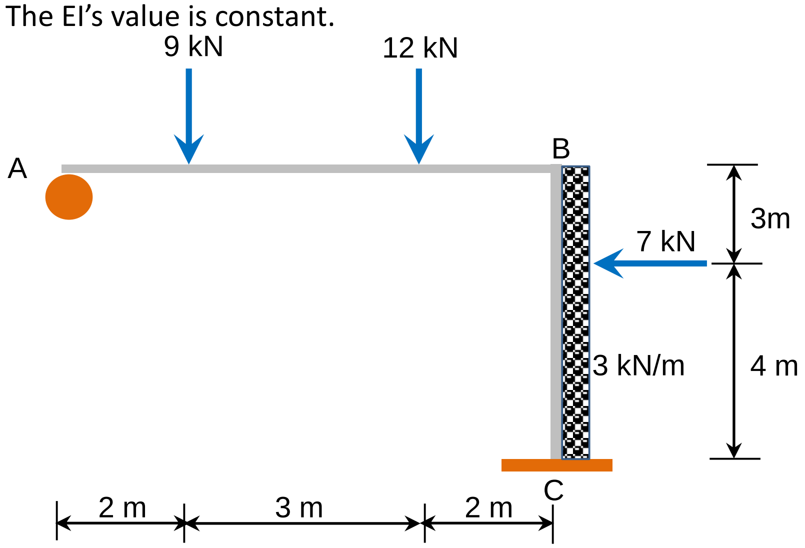


Figure 4.2

OBJECTIVE

- *Able to draw shear force and bending moment diagram*
- *Able to solve the problem*

Solution

1. Calculate the Distribution Factor (DF)

DF$_{AB}$ = **1** *(Pin Support at end span)*

DF$_{CB}$ = **0** *(Fixed End Support at end span)*

JOINT	MEMBER	K	ΣK	DF = $K/\sum K$
B	BA	4EI/L = 4EI/7 =0.57EI	(0.57EI) + (0.57EI) = 1.14EI	**0.5**
	BC	4EI/L = 4EI/7= 0.57EI		**0.5**

2. Finding the fixed end moment (FEM)

$$FEM_{AB} = -\frac{(9)(2)(5^2)}{7^2} = -14.08\ kNm$$

$$FEM_{BA} = \frac{(9)(5)(2^2)}{7^2} = 15.91kNm$$

$$FEM_{BC} = -\frac{(3)(7^2)}{12} - \frac{(7)(3)(4^2)}{7^2} = -19.11kNm$$

$$FEM_{CB} = \frac{(3)(7^2)}{12} - \frac{(7)(4)(3^2)}{7^2} = 17.39\ kNm$$

OBJECTIVE

- *Able to find final moment using moment distribution method*

Solution

3. Calculate the Moment Distribution

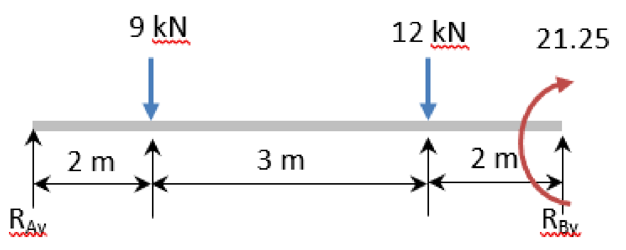
MEMBER	AB	BA	BC	CB
CO	0.5	0.5	0.5	0
DF	1	0.50	0.50	0.00
FEM	-14.08	15.91	-19.11	17.39
DIST.	14.08	1.60	1.60	0.00
CO	0.80	7.04	0.00	0.80
DIST.	-0.80	-3.52	-3.52	0.00
CO	-1.76	-0.40	0.00	-1.76
DIST.	1.76	0.20	0.20	0.00
CO	0.10	0.88	0.00	0.10
DIST.	-0.10	-0.44	-0.44	0.00
CO	-0.22	-0.05	0.00	-0.22
DIST.	0.22	0.03	0.03	0.00
End Moment	**0.00**	**21.25**	**-21.25**	**16.31**

OBJECTIVE

- *Able to find final moment using moment distribution method*
- *Able to find support reaction*

4. Find the support reaction

Span AB



Solution

$\sum M_B = 0;$

$21.25 - (9 \times 5) - (12\ x\ 2)\ + R_{Ay}(7) = 0$

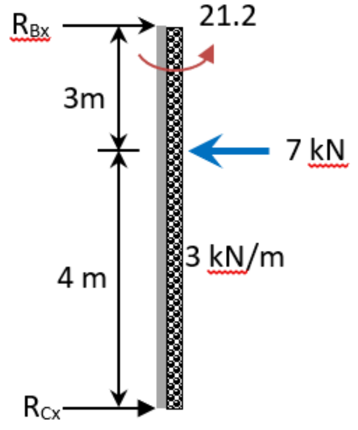
$R_{Ay} = 6.82\ kN$

$\sum F_Y = 0;$

$6.82 + R_{By} - 9 - 12 = 0$

$R_{By} = 14.18\ kN$

Span BC



$\sum M_B = 0;$

$R_{Cx}(7) + (7x3) + (3x7x3.5) - 21.25 = 0$

$R_{Cx} = 12.77\ kN$

$\sum F_X = 0;$

$12.77 + R_{Bx} - 7 - (3x7) = 0$

$R_{Bx} = 15.23\ kN$

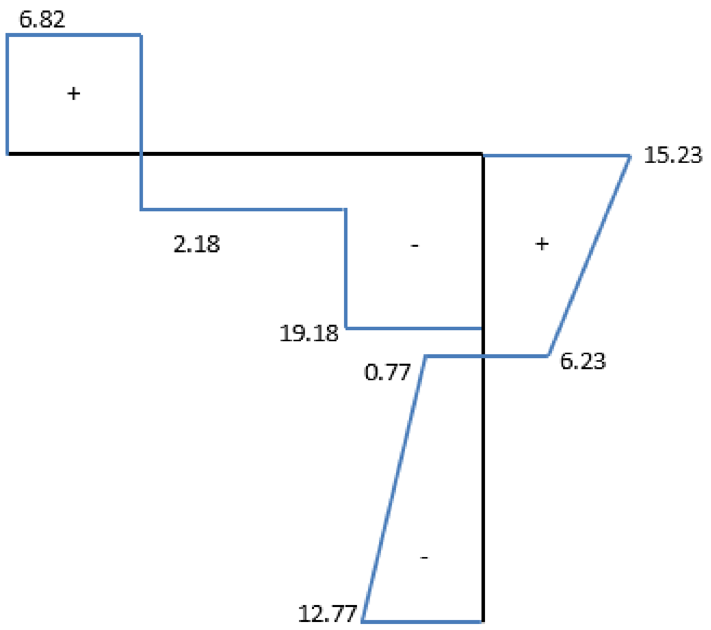
OBJECTIVE

- *Able to find support reaction*

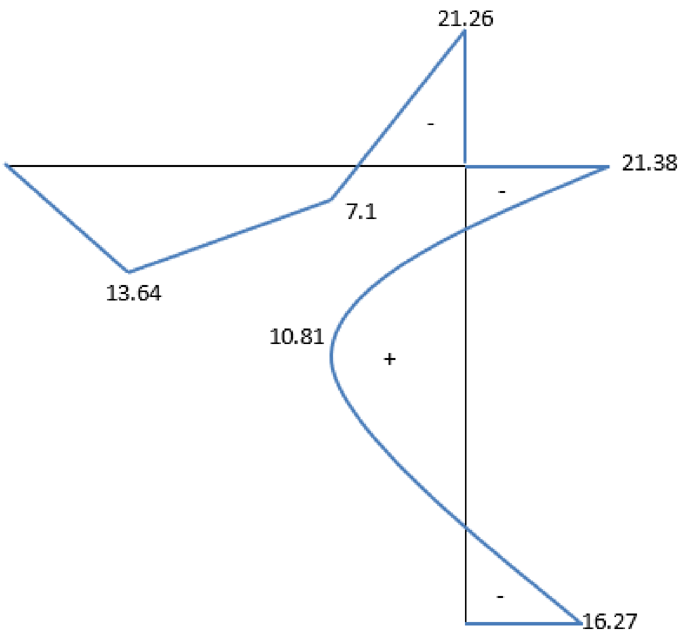
Solution

5. Shear force and bending moment diagram;

SFD



BMD



OBJECTIVE

- *Able to draw shear force and bending moment diagram*

4.2 PROBLEMS

QUESTION 1

Determine end moment for all members and reaction at the support, and then draw SFD and BMD for the beam below.

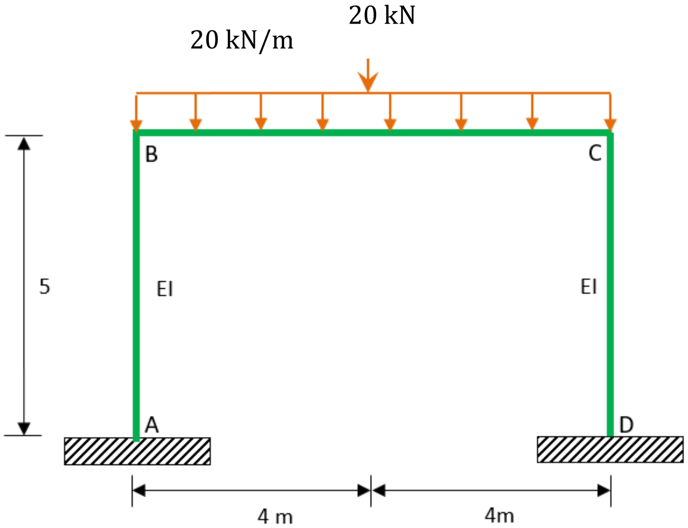


Figure 4.3

QUESTION 2

Determine end moment for all members and reaction at the support, and then draw SFD and BMD for the beam below.

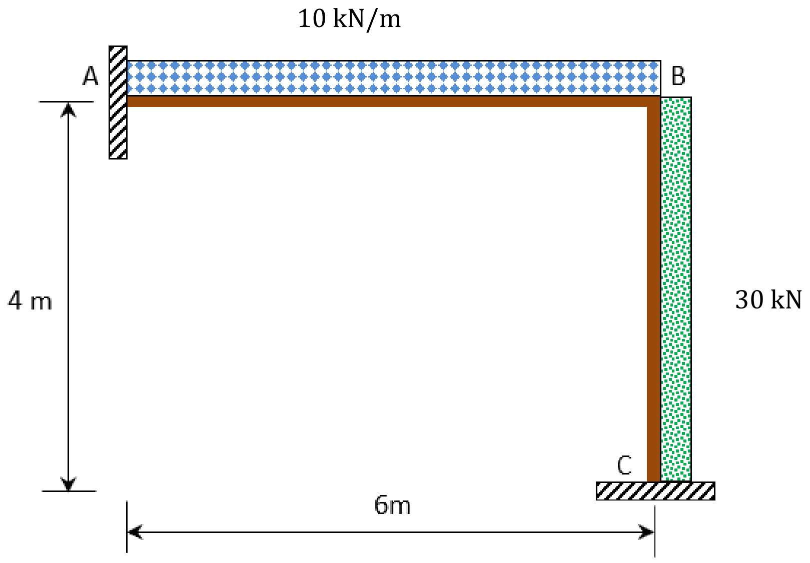


Figure 4.4

OBJECTIVE

- *Able to solve problems using moment distribution method*

4.3 ANSWERS

Question 1

$M_{AB} = 251.42 kNm$

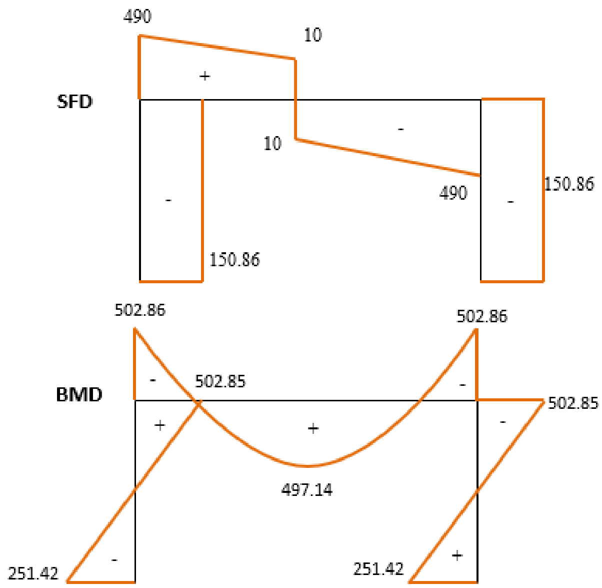
$M_{BA} = 502.86\ kNm$

$M_{BC} = -502.86\ kNm$

$M_{CB} = 502.86\ kNm$

$M_{CD} = -502.85\ kNm$

$M_{DC} = -251.42\ kNm$



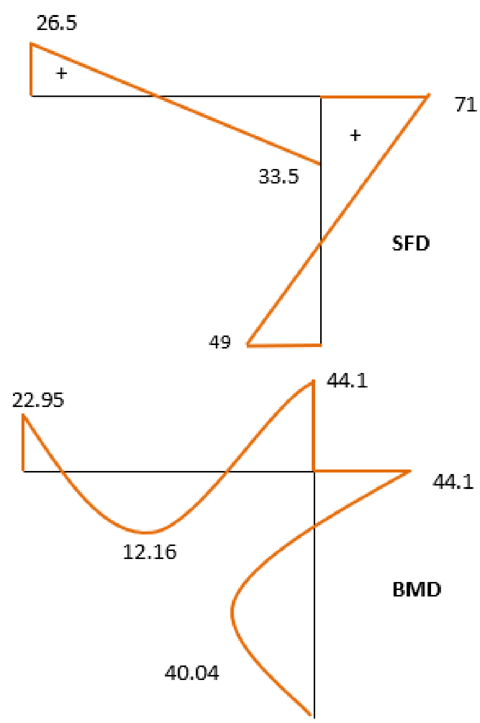
Question 2

$M_{AB} = -22.95\ kNm$

$M_{BA} = 44.10\ kNm$

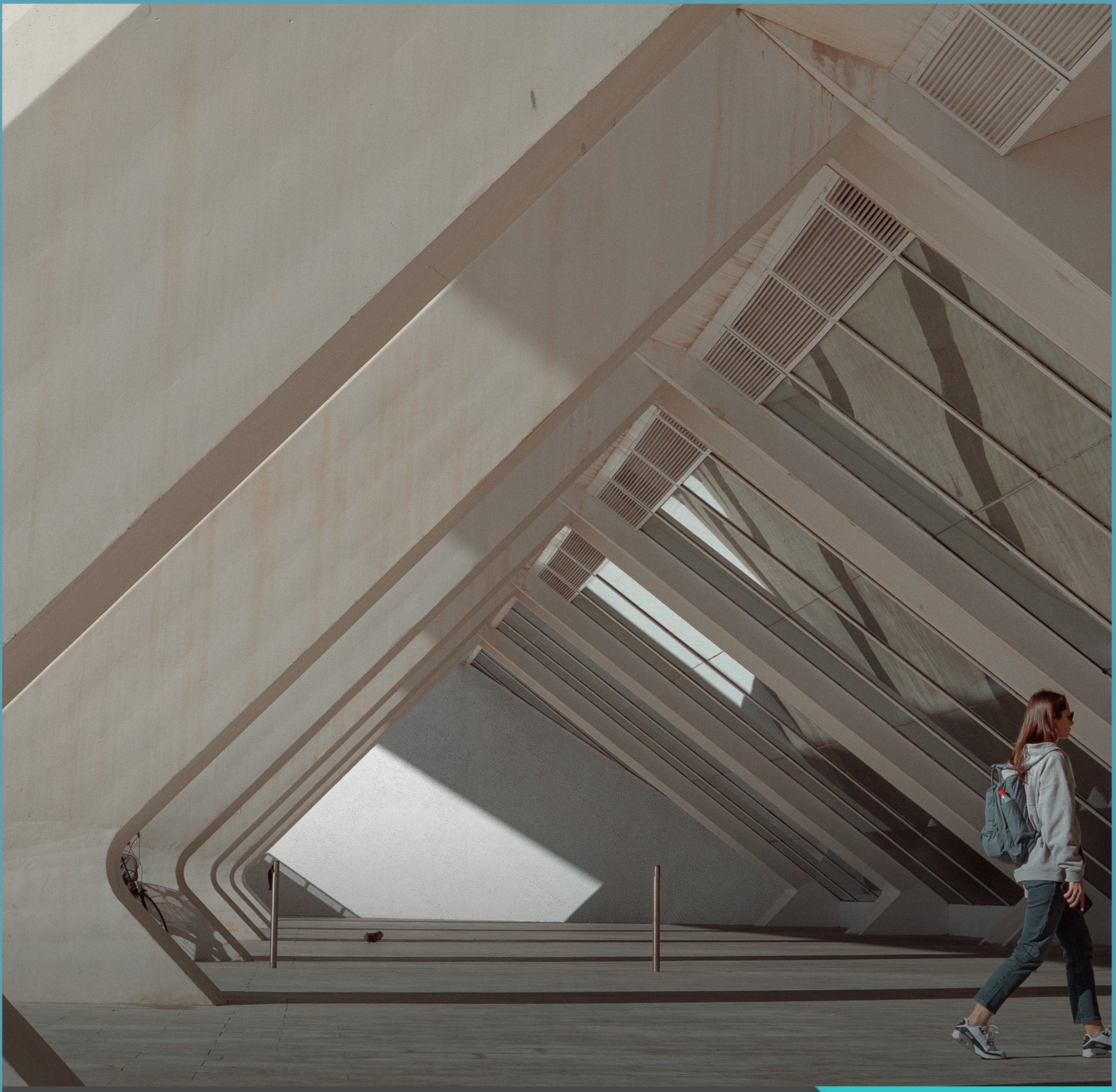
$M_{BC} = -44.10 kNm$

$M_{CB} = 0\ kNm$



OBJECTIVE

- *Able to solve problems using moment distribution method*



CHAPTER 5

ANALYSIS OF STATICALLY DETERMINATE 2D PIN-JOINTED TRUSSES

5.1 Introduction to statically determinate truss

Truss is a structure composed of slender member jointed together at their end points. The members commonly used in construction consist of wooden struts, metal bars, angles or channels. The joint connections are usually formed by bolting or welding the ends of the members to a common plate, called a gusset plate by bolt or pin through each of the members. A structure of truss commonly used in bridge deck construction and roof building construction. A truss is a structure composed of slender members joined together at their end points. The members commonly used in construction consist of wooden struts, metal bars, angles, or channels.

5.2 Common type of trusses

Generally, the form selected for truss is depends on the purpose for which is required. For example, types of truss are shown in Figure 5.1 would be used to support bridge.

Quiz

What are component in truss ?

Ans. Member, joints &support

OBJECTIVE

- *Understand the basic element in trusses*

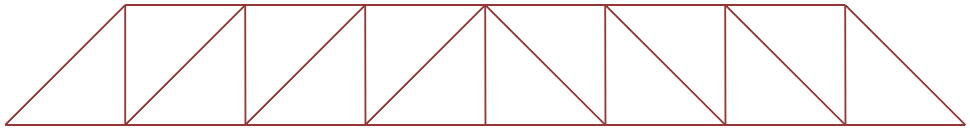


Figure 5.1a : Howe Truss

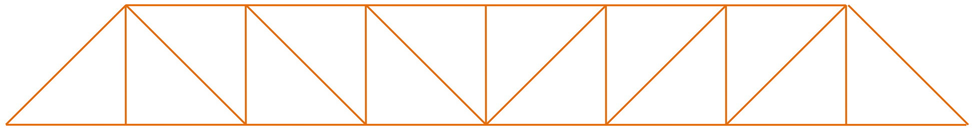


Figure 5.1b : Pratt Truss

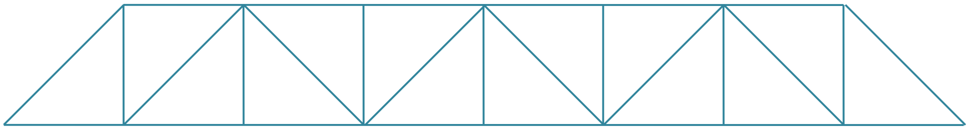


Figure 5.1c : Warren Truss

5.2 Common type of trusses

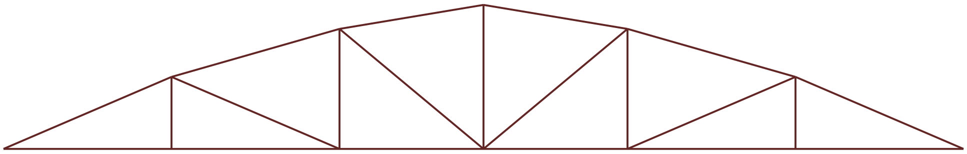


Figure 5.1d : Parker Truss

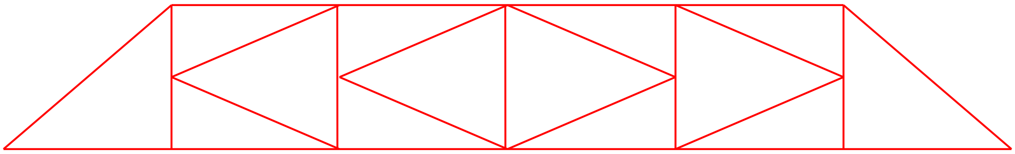


Figure 5.1e : K Truss

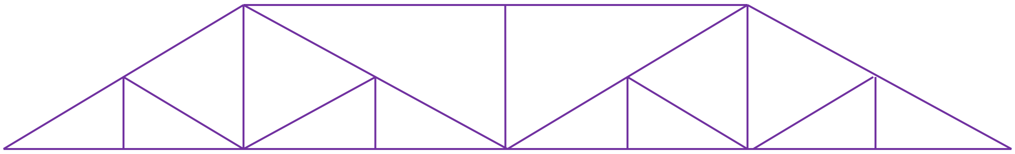


Figure 5.1f :Baltimore Truss

5.3 Assumptions in analysis of plane truss

The assumptions on which the analysis of trusses is based as follows:

(i) The members of truss are connected at their both end by frictionless pins.
(ii) The truss is loaded and supported only at its joints.
(iii) The centroidal axis of each member coincides with the line connecting the centers of adjacent joints.

OBJECTIVE

- *Identify types of trusses*

5.4 Static determinacy

A structure of truss is called statically determinate if the forces in all its members and all of its support reactions can be determined by solving using equations of equilibrium.

Generally,

$m < 2j-3$: Truss is statically unstable
$m = 2j-3$: Truss is stable and statically determinate internally.
$m > 2j-3$: Truss is stable and statically indeterminate internally.

where: m = number of members
j = number of joints

5.5 Method of joints

Method of Joints is used when we analyze all force in each member of truss. This method can only be applied if the unknown force in member is not more than two. In the method of joints, the axial forces in the members of statically determinate truss are determined by considering the equilibrium of its joints.

Example 5.1

A plane truss subject a load as shown in Figure 5.2. Determine reactions and force in each member of truss using method of joint.

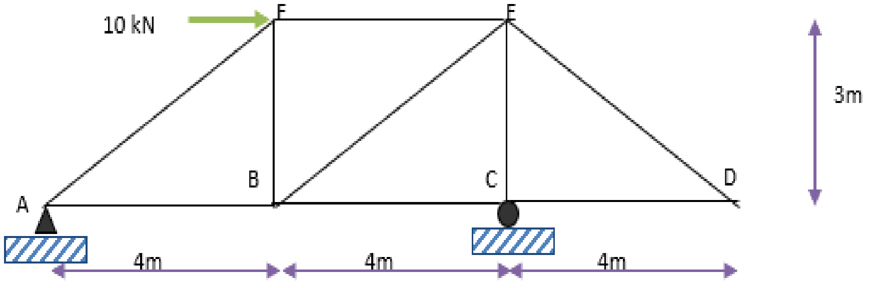


Figure 5.2

OBJECTIVE

- *Identify determinate and indeterminate trusses*

Solution

r = 3
m=9
j = 6

$m = 2j - 3$

$m = 2(6) - 3$

$m = 9$

Stable and internal statically determinate structure

$\Sigma M_A = 0$

$-R_{Cy}(8) + 10(3) = 0$

$R_{Cy} = +3.75kN \uparrow$

$\Sigma F_y = 0 \uparrow +ve$

$R_{Ay} + 3.75 = 0$

$R_{Ay} = -3.75kN$

$\sum F_x = 0 \rightarrow +ve$

$H_A + 10 = 0$

$H_A = -10kN \leftarrow$

Do you remember?

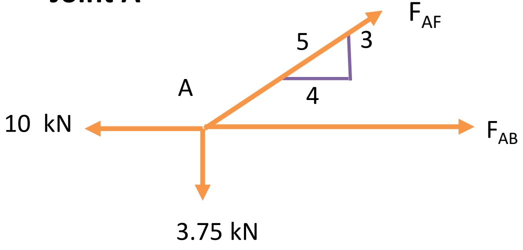
Simply supported structure

$\Sigma F_y = 0 \uparrow +ve$

$F_{AF}\left(\frac{3}{5}\right) - 3.75 = 0$

$F_{AF} = +6.25kN(T)$

Joint A



$\sum F_x = 0 \rightarrow +ve$

$-10 + F_{AB} + F_{AF}\left(\frac{4}{5}\right) = 0$

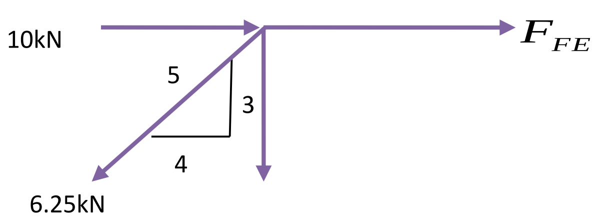
$-10 + F_{AB} + 6.25\left(\frac{4}{5}\right) = 0$

$F_{AB} = +5kN(T)$

OBJECTIVE

- *Able to use method of joint in finding internal force*

Joint F



$$\Sigma F_y = 0 \quad \uparrow +ve$$

$$-F_{FB} - 3.75 = 0$$

$$F_{FB} = -3.75kN(C)$$

$$\sum F_x = 0 \rightarrow +ve$$

$$F_{FE} + 10 - 5 = 0$$

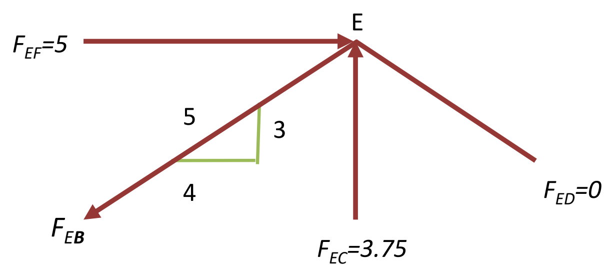
$$F_{FE} = -5kN(C)$$

Joint E

$$\Sigma F_y = 0 \quad \uparrow +ve$$

$$-F_{EB}\left(\frac{3}{5}\right) + 3.75 = 0$$

$$F_{EB} = +6.25kN(T)$$

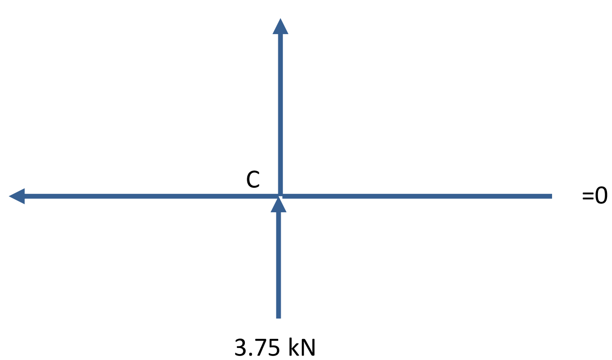


Joint C

$$\Sigma F_y = 0 \quad \uparrow +ve$$

$$+F_{CE} + 3.75 = 0$$

$$F_{CE} = -3.75kN(C)$$



$$\sum F_x = 0 \rightarrow +ve$$

$$-F_{CB} + 0 = 0$$

$$F_{CB} = 0$$

OBJECTIVE

- *Able to use method of joint in finding internal force*

5.6 Method of section

The method of sections is used to determine force in only certain members. However, this method can still be used to determine all force in each member. The method of sections enables us to determine forces in the specific members of trusses directly without calculating many unnecessary members force. To analyze a plane truss using this method, we imagine that the truss is divided into two free bodies by imaginary cutting plane through the structure. There are only three equilibrium equations available hence, they cannot be used to determine more than three unknown forces.

Procedure to analyze plane truss using method of sections.

•There is no restriction on the number of bars that can be cut, we can use sections that can be cut three bar since three equation of static equilibrium are available to analyze a free body diagram.

•Determine unknown force in members using three equation of static equilibrium below:

$$\sum Fx = 0$$

$$\sum Fy = 0$$

$$\sum M = 0$$

Example 5.2

A simply supported plane truss subjected to loads as shown in Figure 5.3. Classify the truss as stable and determinate. Then, determine force in member DC, DG, FG using Method of Sections.

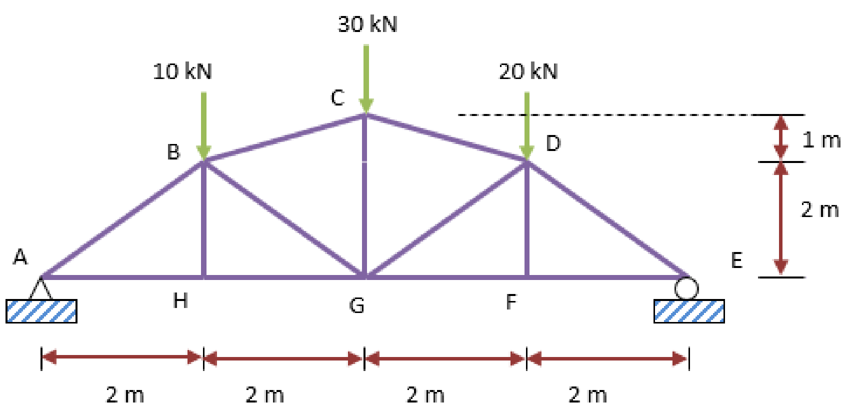
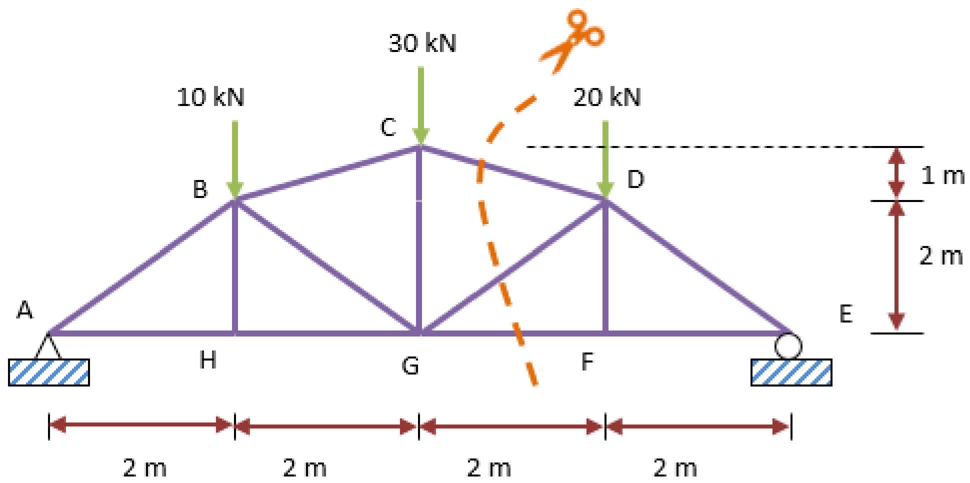


Figure 5.3

OBJECTIVE

- *Able to use method of section in finding internal force*

Solution



$m = 13, \qquad j = 8$

$m = 2j - 3$

$m = 2(8) - 3$

$m = 13$ *Stable & internal statically determinate*

$$\sum M_A = 0$$

$$10(2) + 30(4) + 20(6) - R_{Ey}(8) = 0$$

$$R_{Ey} = 32.5kN$$

$$\sum F_y = 0 \uparrow +ve$$

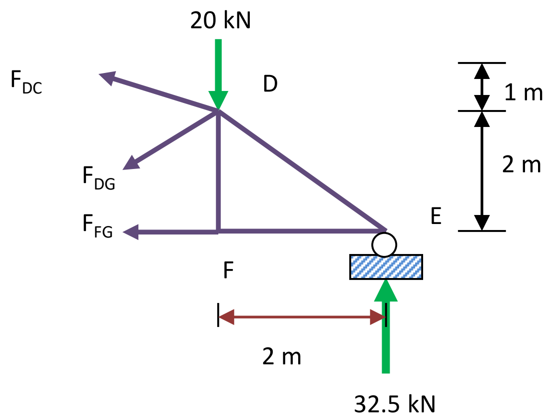
$$R_{A_y} + 32.5 - 10 - 30 - 20 = 0$$

$$R_{Ay} = 27.5kN$$

OBJECTIVE

- *Able to use method of section in finding internal force*

Right side section x-x



$$\sum F_x = 0 \rightarrow +ve$$

$$-F_{FG} + 3.54(\frac{1}{\sqrt{2}}) + 33.54(\frac{2}{\sqrt{5}}) = 0$$

$$F_{FG} = +32.5kN(T)$$

$$\sum M_F = 0 \curvearrowright +ve$$

$$-32.5(2) - F_{DG}\left(\frac{1}{\sqrt{2}}\right)(2) - F_{DC}\left(\frac{2}{\sqrt{5}}\right)(2) = 0$$

$$-65 - \left(\frac{2}{\sqrt{2}}\right)F_{DG} - \left(\frac{4}{\sqrt{5}}\right)F_{DC} = 0 \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \; equation(1)$$

$$\sum F_y = 0 \uparrow +ve$$

$$32.5 - 20 - F_{DG}\left(\frac{1}{\sqrt{2}}\right) + F_{DC}\left(\frac{1}{\sqrt{5}}\right) = 0 \quad \cdots\cdots\cdots\cdots\cdots\cdots \; equation(2)$$

$$equation(2)x2$$

$$25 - \left(\frac{2}{\sqrt{2}}\right)F_{DG} - \left(\frac{2}{\sqrt{5}}\right)F_{DC} = 0 \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \; equation(3)$$

$$equation(1) - equation(3)$$

$$-65 - 25 - \left(\frac{2}{\sqrt{2}}\right)F_{DG} + \left(\frac{2}{\sqrt{2}}\right)F_{DG} - \left(\frac{4}{\sqrt{5}}\right)F_{DC} - \left(\frac{2}{\sqrt{5}}\right)F_{DC} = 0$$

$$F_{DC} = -33.54kN(C)$$

$$substituting \quad F_{DC} = -33.54kN \quad in \quad to \quad equation(1)$$

$$F_{DG} = -3.53kN(C)$$

OBJECTIVE

- *Able to use method of section in finding internal force*

5.7 PROBLEMS

QUESTION 1

Using the method of joints, indicate all the members of the truss as shown in Figure 5.4.

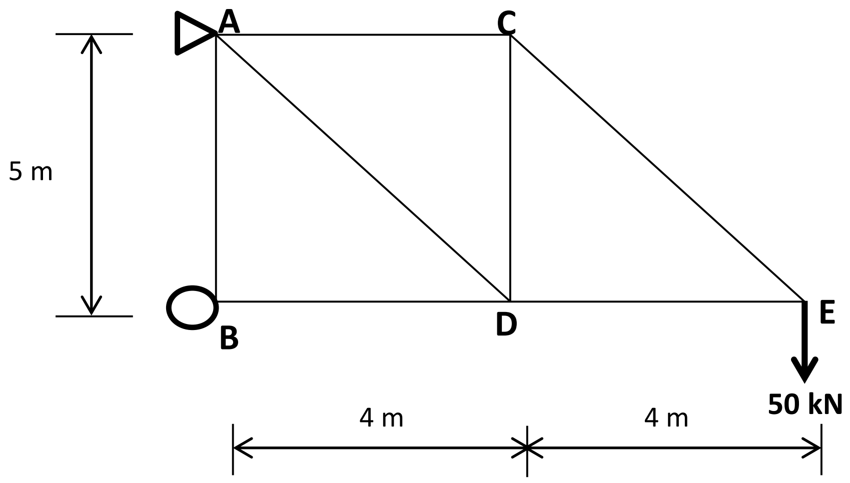


Figure 5.4

QUESTION 2

Using the method of joints, indicate all the members of the truss as shown in Figure 5.5.

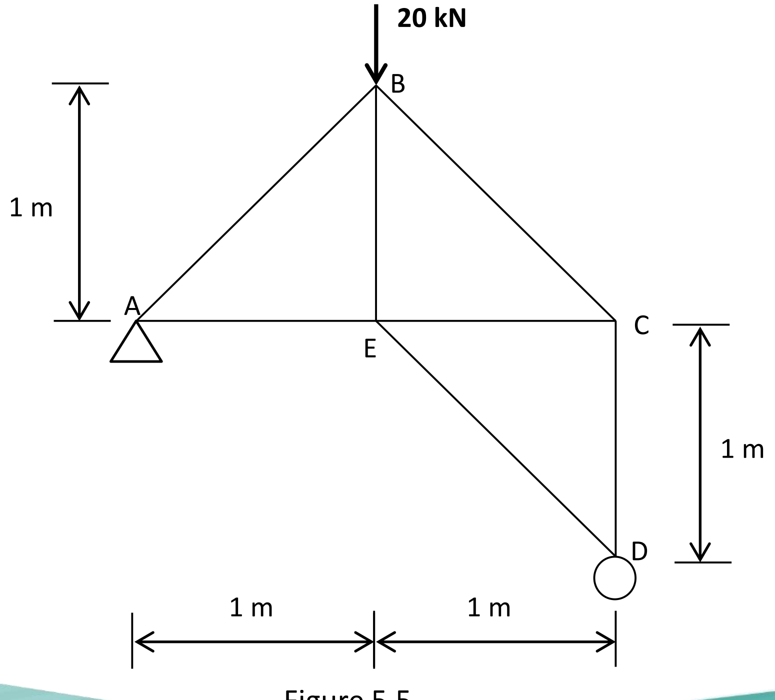


Figure 5.5

OBJECTIVE

- *Problem solving in finding internal forces*

5.7 PROBLEMS

QUESTION 3

A simply supported plane truss subjected loads as shown in Figure 5.6. Determine reactions and force in each member of truss using Method of Joints.

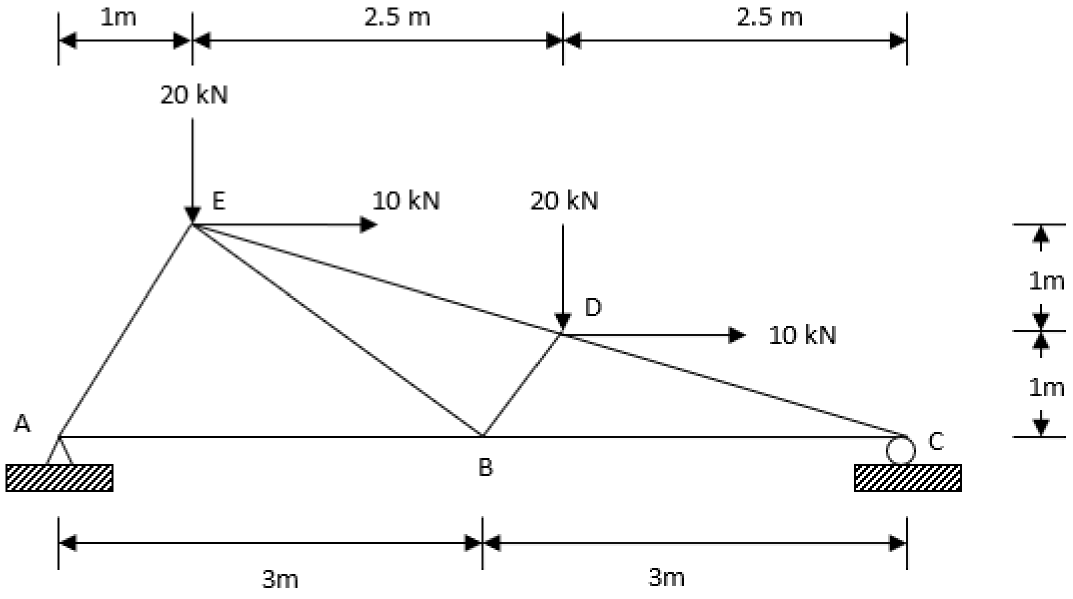


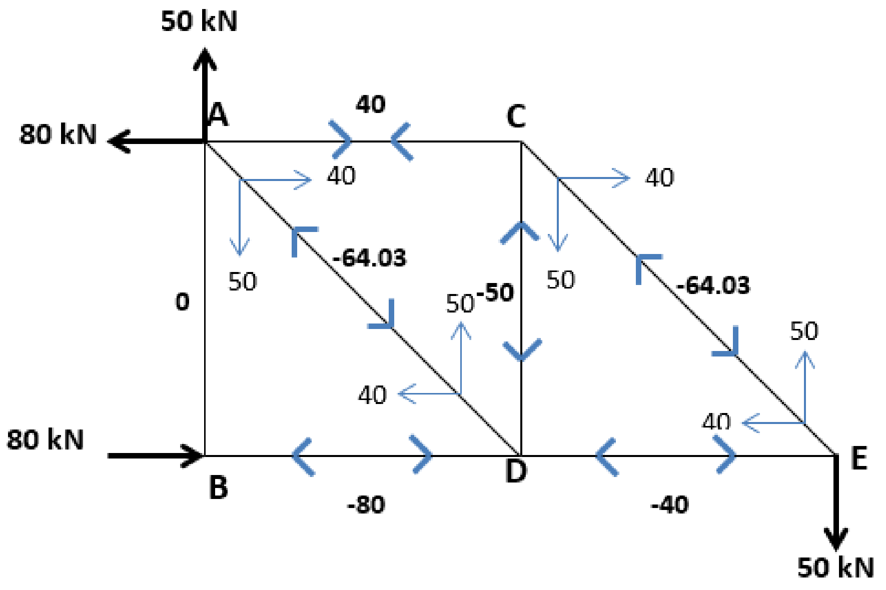
Figure 5.6

OBJECTIVE

- *Problem solving in finding internal forces*

5.8 ANSWERS

Question 1

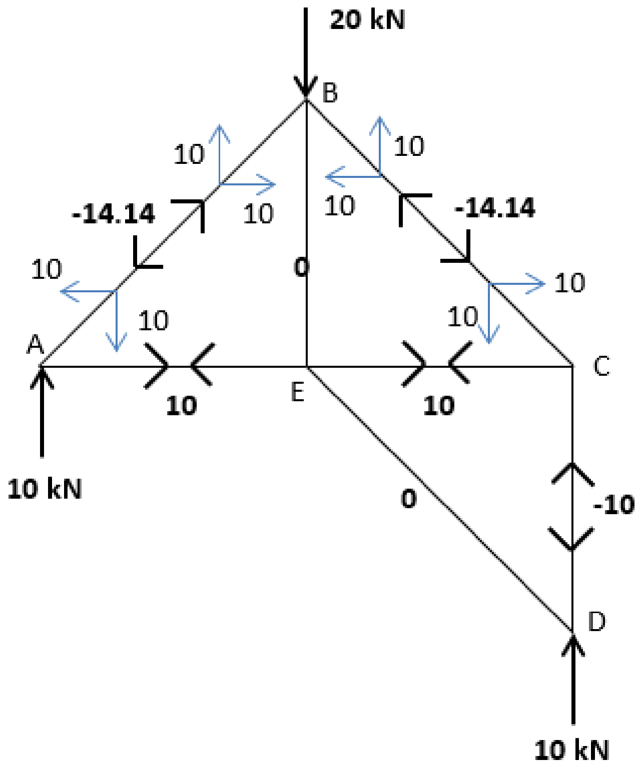


OBJECTIVE

- *Problem solving in finding internal forces*

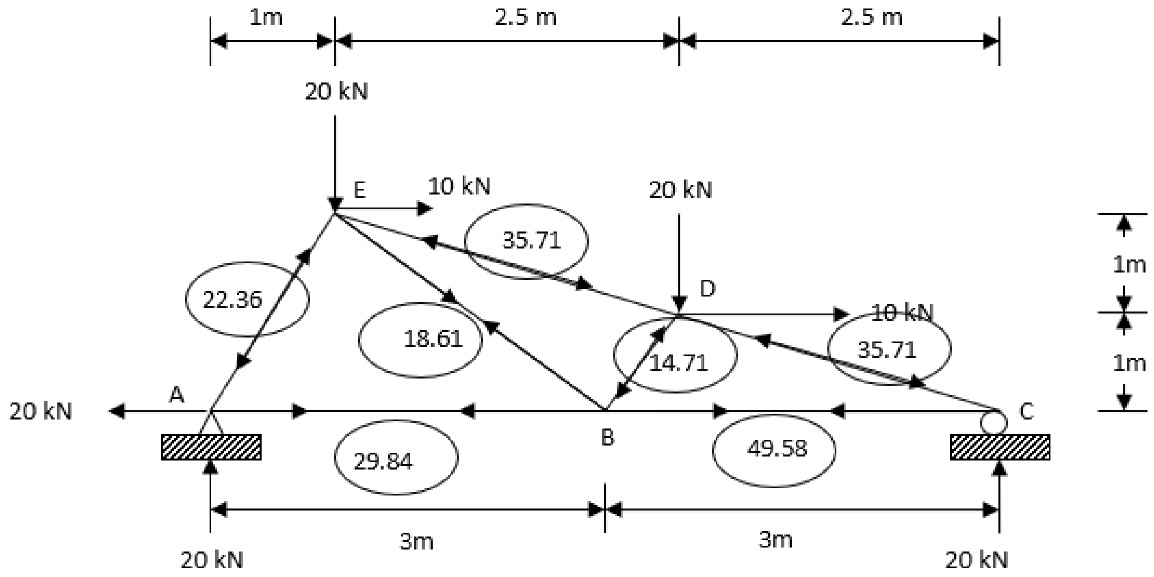
5.8 ANSWERS

Question 2



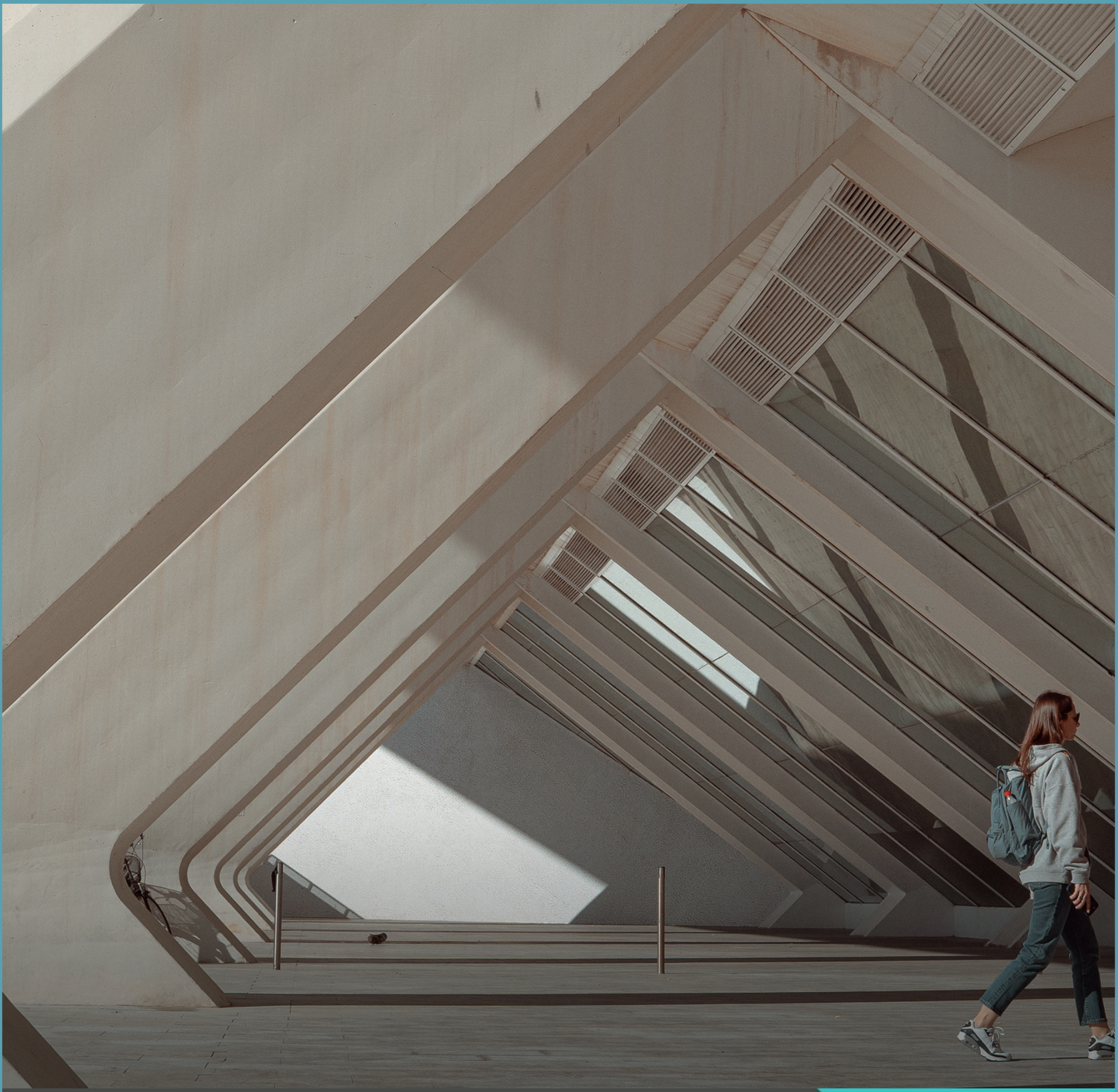
5.8 ANSWERS

Question 3



OBJECTIVE

- *Problem solving in finding internal forces*

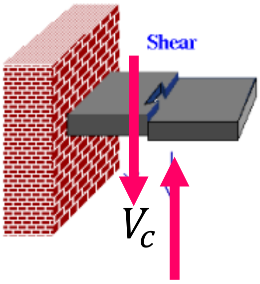


CHAPTER 6

JOINT DISPLACEMENT OF STATICALLY DETERMINATE 2D PIN-JOINTED TRUSSES

6.1 Introduction to joint displacement

Do you remember?



We can use the method of virtual work to determine the displacement of a truss joint when the truss is subjected to an external loading, temperature change, or fabrication errors. .

Generally, the displacement due the axial forces in each member of truss can be determined by apply the equation:

Where : $$\Delta = \sum \mu \left(\frac{FL}{AE}\right)$$

F= Force of member in real system
μ= Force of member in virtual system
L=Length of a member
A= Cross sectional area of a member
E= modulus elasticity of member of a member

Quiz

What are the formula for stress?

Ans. Stress= $^{P}/_{A}$

Procedure for analysis displacement by using virtual work method

1. Real system
If the displacement of truss to be measured is caused by external loads, then apply the method of joints to calculate the real forces (F) in each member of the truss by using the method of joints.

2. Virtual System
Remove all the given external axial loads from truss, then apply a unit load at the joint where the displacement is required and in the direction of required displacement to form the virtual force system. Calculate the forces (μ) in each member of the truss by using the method of joints.

3. The desired displacement of truss can now be determined by the formula.

OBJECTIVE

- *Able to calculate the displacement truss by virtual work method.*

Example 6.1

Calculate the horizontal displacement at joint F of the truss shown in Figure 6.1 by using the virtual work method. Given modulus elasticity, E=210kN/mm2 and cross sectional area, A= 1000mm2.

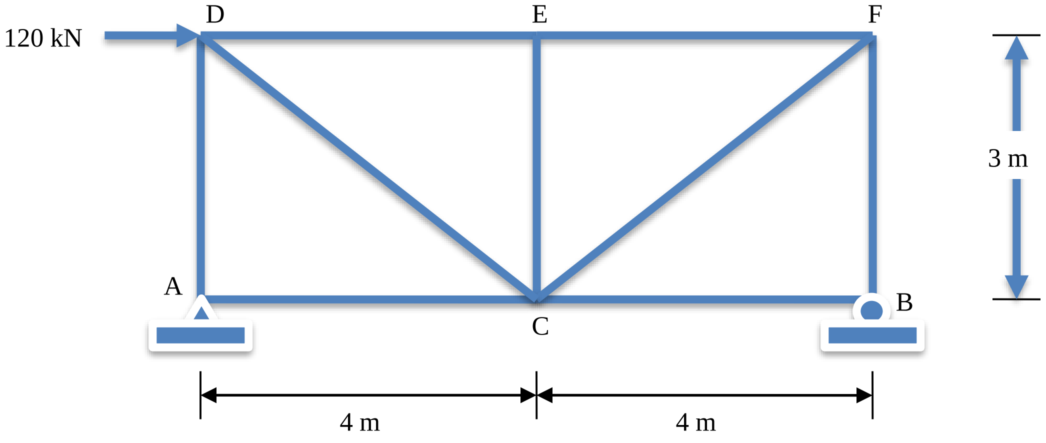


Figure 6.1

Quiz

How unknowns in roller support?

Ans. 1

Solution

a) Real System

r= 3
m = 9
j=6

OK! Stable and external statically determinate

m = 2j -3
m= 2(6) – 3
m= 9

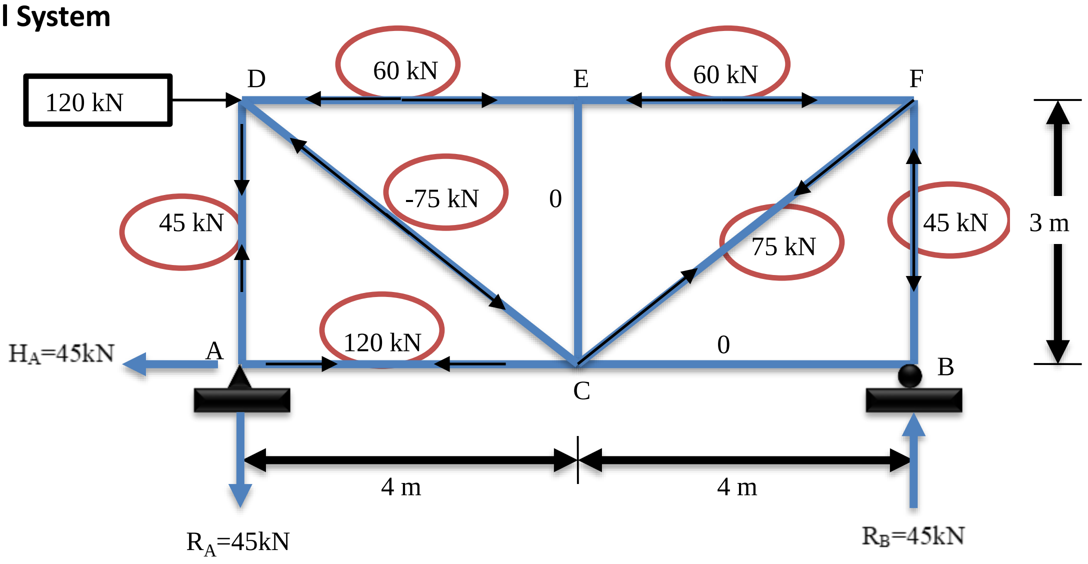
OK! Stable and internal statically determinate

OBJECTIVE

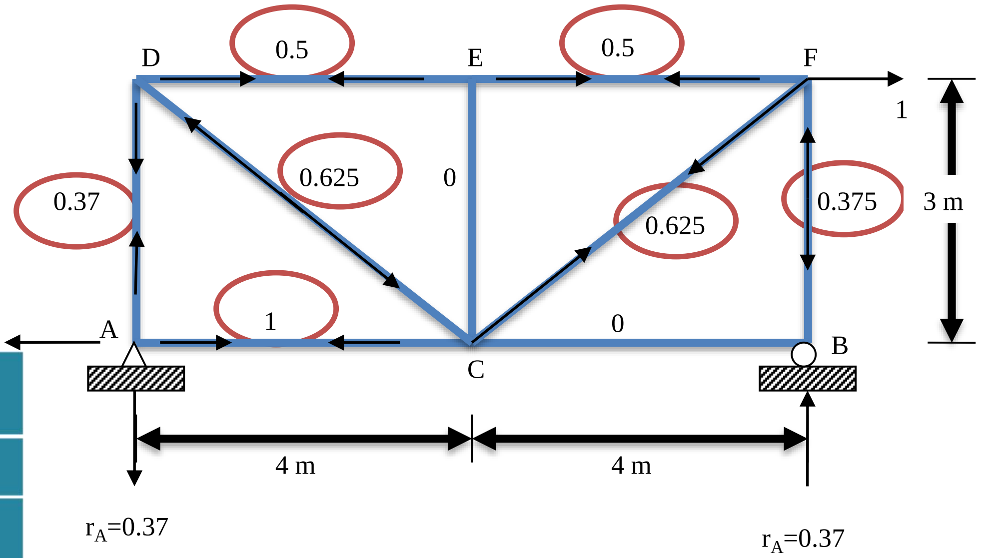
- *Able to calculate the displacement truss by virtual work method.*

Solution

a) Real System



b) Virtual System



OBJECTIVE

- *Able to calculate the displacement truss by virtual work method.*

Solution

c) Displacement of truss

Member	L($x10^3$) mm	AE ($x10^3$)kN	F (kN)	M	Δ=F μ L/AE
AC	4	210	+120	+1	**+2.29**
AD	3	210	+45	+0.375	**+0.24**
BC	4	210	0	0	**0**
BF	3	210	-45	-0.375	**+0.24**
CD	5	210	-75	-0.625	**+1.12**
CE	3	210	0	0	**0**
CF	5	210	+75	+0.625	**+1.12**
DE	4	210	-60	+0.5	**-0.57**
EF	4	210	-60	+0.5	**-0.57**
				Σ	**+3.87 mm**

Horizontal displacement at joint F, Δ_{HF}= +3.87mm (⟶)

OBJECTIVE

- *Able to calculate the displacement truss by virtual work method.*

Example 6.2

Determine the vertical displacement at joint C of the steel truss shown. Take value E = 200 Gpa. The cross sectional area of each member is indicating in the Figure 6.2

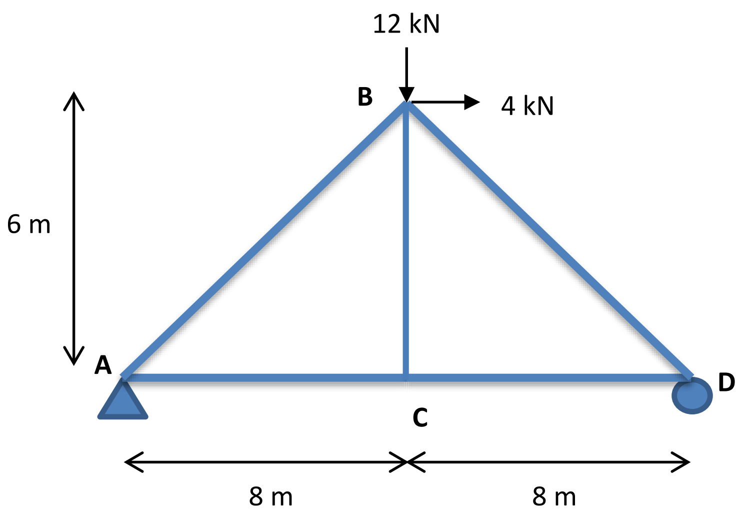
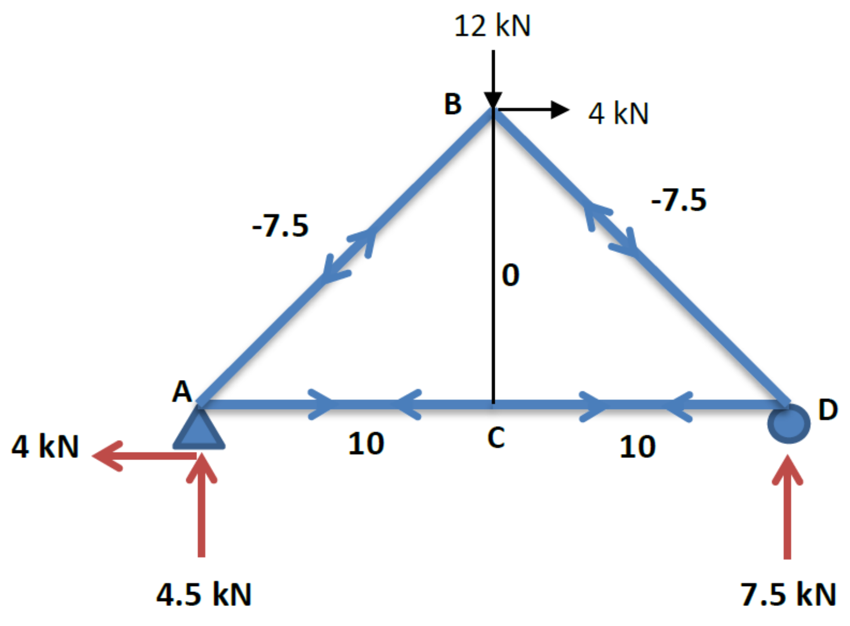


Figure 6.2

Solution

a) Real System

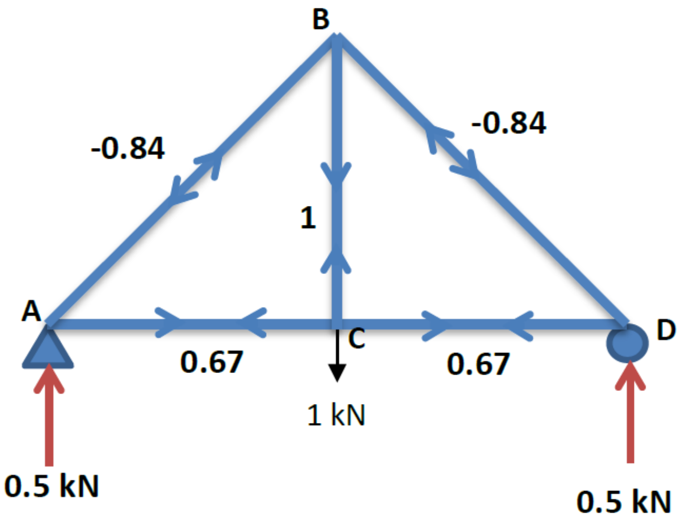


OBJECTIVE

- *Able to calculate the displacement truss by virtual work method.*

Solution

b) Virtual System



Do you remember?

σ = stress, kN/m^2 or N/m^2
P = load, kN or N
A = area over which stress develops, m^2 or mm^2

YouTube

Example to find displacement on determinate truss

OBJECTIVE

- *Able to calculate the displacement truss by virtual work method.*

Solution

c) Displacement of truss

Member	F	µ	L (m)	A (m^2)x10^{-6}	FµL/A
AB	-7.5	-0.84	10	200	315000
AC	10	0.67	8	100	536000
BC	0	1	6	200	0
BD	-12.5	-0.84	10	200	525000
CD	10	0.67	8	100	536000
					Σ = 1912000

$$\Delta = \sum \frac{P\mu L}{AE} = \frac{1912000}{(200x10^6)}$$

Vertical displacement at joint C = 9.56 mm ↓

OBJECTIVE

- *Able to calculate the displacement truss by virtual work method.*

6.2 PROBLEMS

QUESTION 1

Determine the horizontal displacement at joint F of the truss as shown in Figure 6.3 by using the virtual work method. Given the cross sectional area, A=1000mm2 and modulus elasticity, E=210 kN/mm^2.

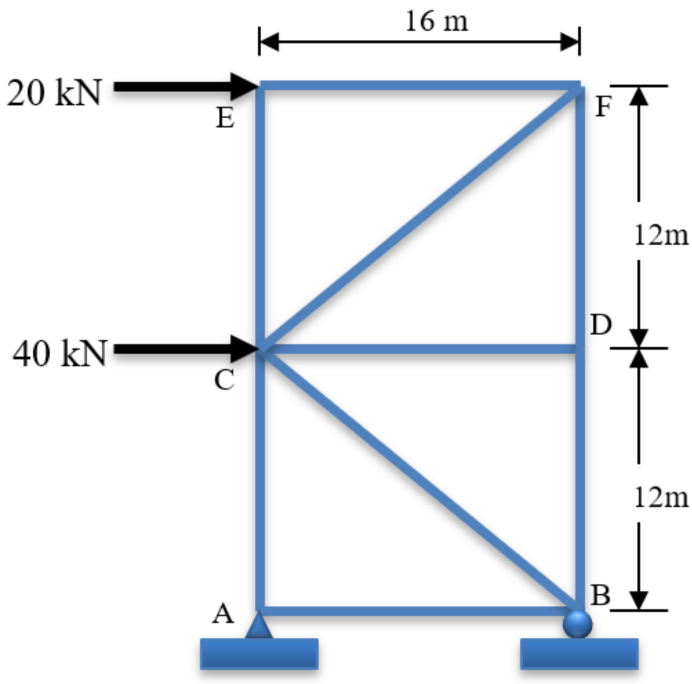


Figure 6.3

OBJECTIVE

- *Able to calculate the displacement truss by virtual work method*

6.2 PROBLEMS

QUESTION 2

Determine the vertical displacement at joint C of the truss as shown in Figure 6.4 by using the virtual work method. Given cross sectional area for each member, A=1000 mm2 and modulus elasticity, E=250 kN/mm^2.

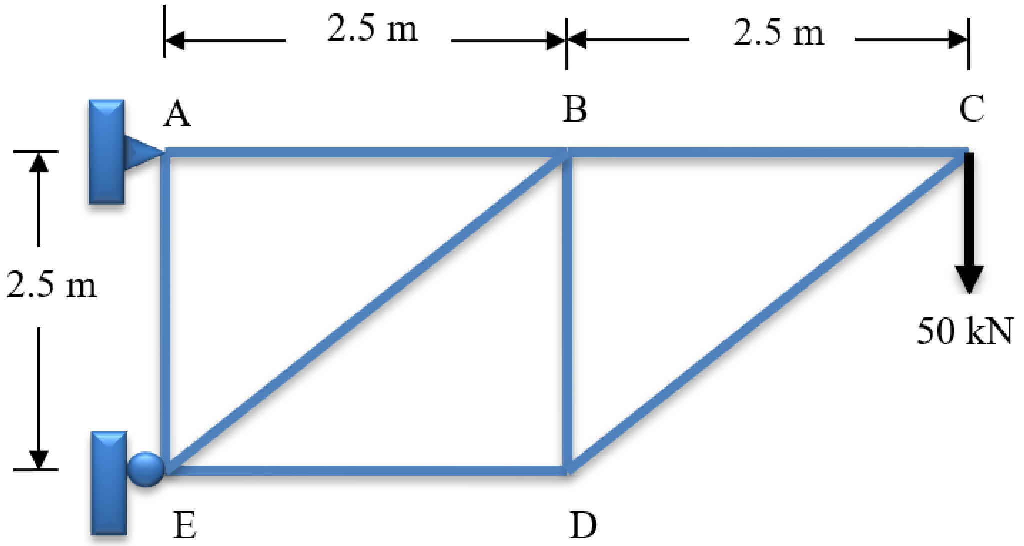


Figure 6.4

OBJECTIVE

- *Able to calculate the displacement truss by virtual work method*

6.3 ANSWERS

Question 1

Member	Length, L(mm)	AE (kN)	F (kN)	M	Δ=F µ L/AE
AB	16,000	210,000	-60	-1	**+4.57**
AC	12,000	210,000	-60	-1.5	**+5.14**
BC	20,000	210,000	-75	-1.25	**+8.93**
BD	12,000	210,000	-15	-0.75	**+0.64**
CD	16,000	210,000	0	0	**0**
CF	20000	210,000	-25	-1.25	**+2.98**
CE	12,000	210,000	-60	0	**0**
EF	16,000	210,000	-20	0	**0**
DF	12,000	210,000	-15	-0.75	**+0.64**
				ΣΔ	**+22.9**

OBJECTIVE

- *Able to calculate the displacement truss by virtual work method*

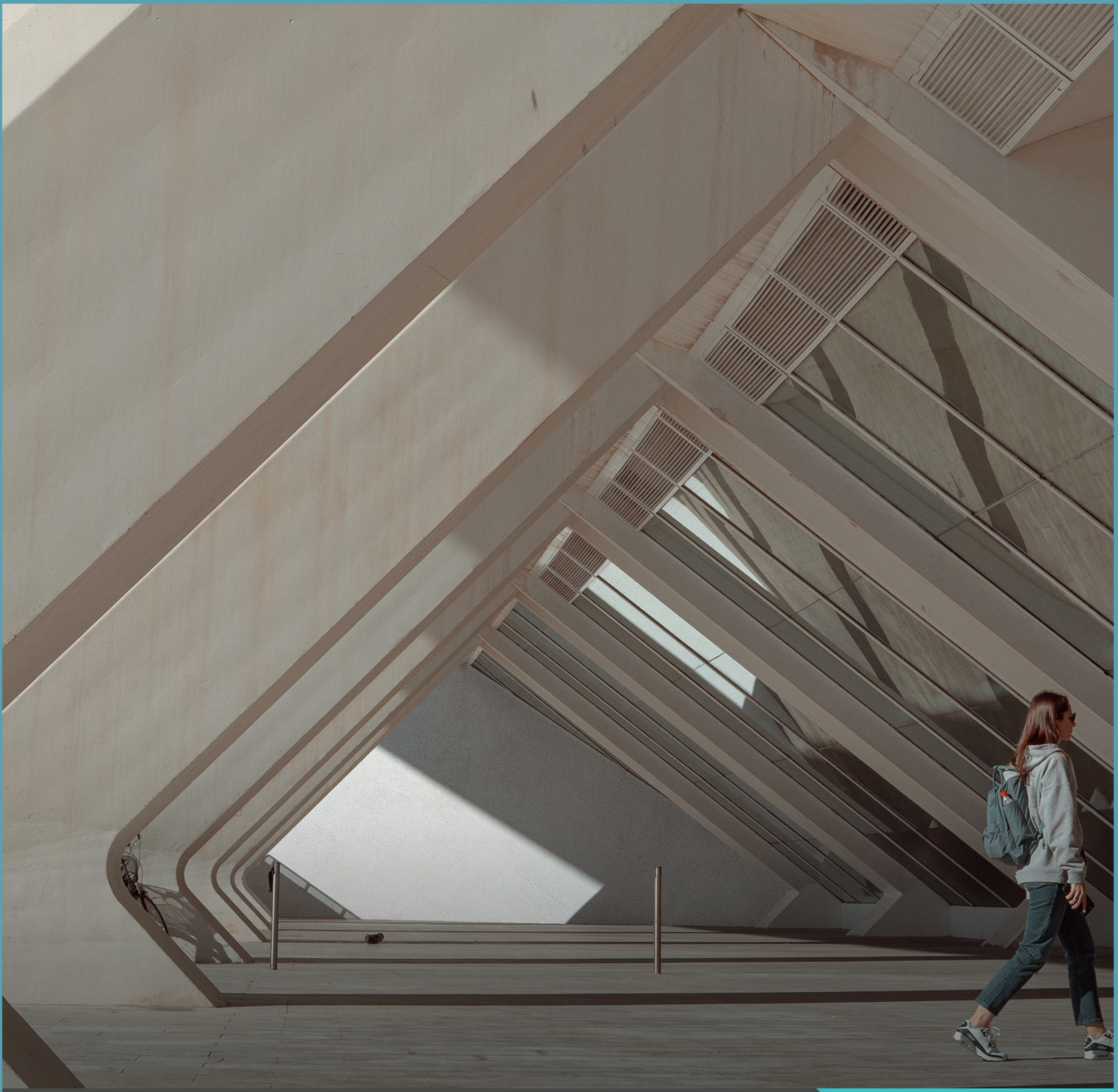
6.3 ANSWERS

Question 2

Member	L(x10^3) mm	AE (x10^3)kN	F (kN)	µ	F µ L/AE
AB	2.5	250	+100	+2	**+2**
AE	2.5	250	+50	+1	**+0.5**
BE	3.54	250	-70.71	-1.414	**+1.414**
BD	2.5	250	+50	+1	**+0.5**
BC	2.5	250	+50	+1	**+0.5**
ED	2.5	250	-50	-1	**+0.5**
CD	3.54	250	-70.71	-1.414	**+1.414**
				Σ	**+6.83 mm**

OBJECTIVE

- *Able to calculate the displacement truss by virtual work method*



CHAPTER 7

ANALYSIS OF STATICALLY INDETERMINATE 2D PIN-JOINTED TRUSSES

7.1 Introduction to indeterminate truss

Do you remember?

Gravity = 9.81 m/s^2

As discussed previously, the equation equilibrium can be used to calculate the support reactions and internal forces of statically determinate structures. Indeterminate structures, on the other hand, have more support reactions and/or members than are required for static stability.

In general, truss could be classified as follows:

If m = 2j-3 and r>3, truss is external indeterminate.

If m > 2j-3 and r=3, truss is internal indeterminate.

If m > 2j-3 and r>3, truss is external indeterminate and internal indeterminate.

Where,
m = number of members
j = number of joints
r = number of reactions

Quiz

How to classify truss indeterminate or determinate?

Ans. m = 2j-3

Compute the deflections of primary structure at the location of redundant due to the external loading and due to the unit value of the redundant.

$$\Delta = \frac{F\mu L}{AE}$$

Compute the flexibility coefficient by applying the formula,

$$f = \frac{\mu^2 L}{AE}$$

Compute the magnitude of redundant by applying the formula.

$$R = -\frac{\Sigma \frac{F\mu L}{AE}}{\frac{\mu^2 L}{AE}}$$

Compute the actual force in each members by apply the formula

$$P_1 = F_1 + \mu_1 R$$

OBJECTIVE

- *Able to calculate the internal forces in truss members using the unit load method*

Example 7.1

Determine reaction and force in each member of truss as shown in Figure 3.1. Assume cross sectional area, A= 20cm2 and Modulus Young=210kN/mm^2.

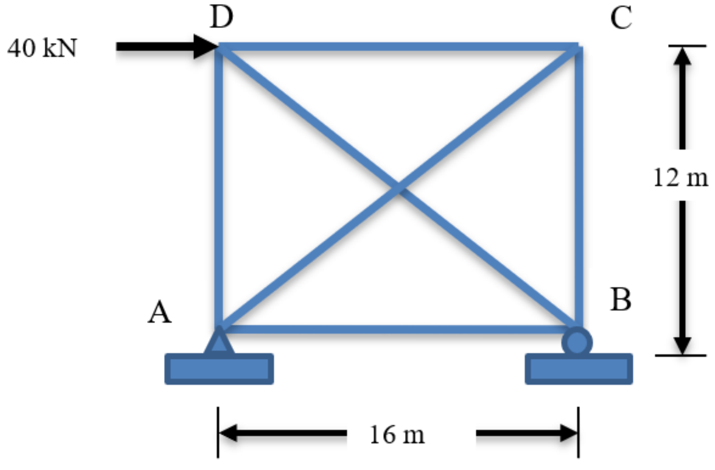


Figure 7.1

YouTube

Example to find internal force members for indeterminate truss

Solution

r= 3
m = 6 OK! Stable and external statically determinate
j=4

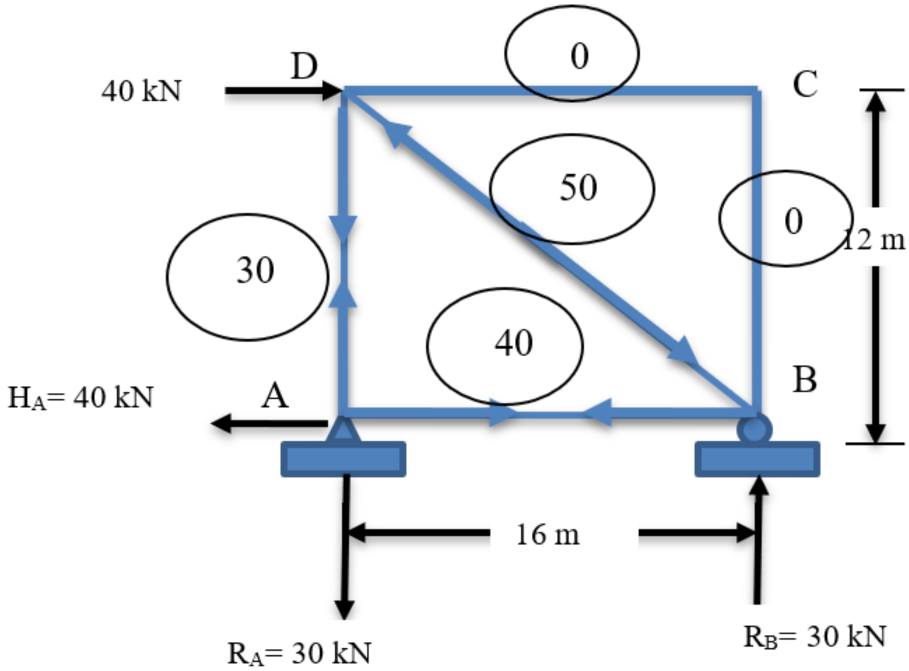
m = 2j -3
m= 2(4) – 3 OK! Stable and internal indeterminate
m > 5 Degree of indeterminacy = 1

OBJECTIVE

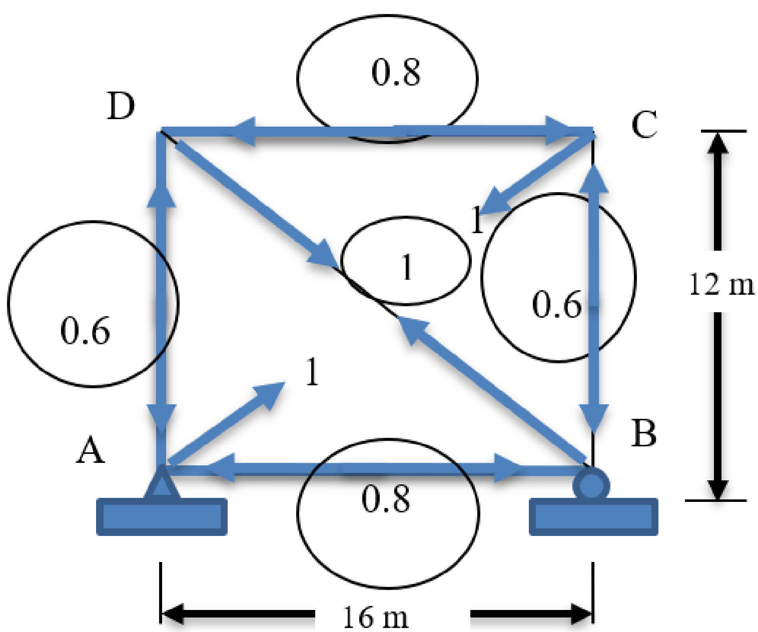
- *Able to calculate the internal forces in truss members using the unit load method*

Solution

a) Real System



b) Virtual System



OBJECTIVE

- *Able to calculate the internal forces in truss members using the unit load method*

Solution

c) Actual force of truss

Member	L(mm)	F(kN)	μ	F μL/AE	μ^2L/AE	P=F+ μR
AB	16000	+40	-0.8	-1.219	+0.024	+19.94
AC	20000	0	+1	0	+0.048	+25.08
AD	12000	+30	-0.6	-0.514	+0.010	+14.95
BC	12000	0	-0.6	0	+0.010	-15.05
BD	20000	-50	+1	-2.380	+0.048	-24.92
CD	16000	0	-0.8	0	+0.024	-20.06
			∑	-4.113	+0.164	

$$R = -\frac{\sum \frac{F\mu L}{AE}}{\sum \frac{\mu^2 L}{AE}}$$

$$= -\frac{-4.113}{+0.164}$$

= +25.08

OBJECTIVE

- *Able to calculate the internal forces in truss members using the unit load method*
-

Example 7.2

Determine the reactions and internal forces of the trusses. The modulus of elasticity (E) of each member is constant.

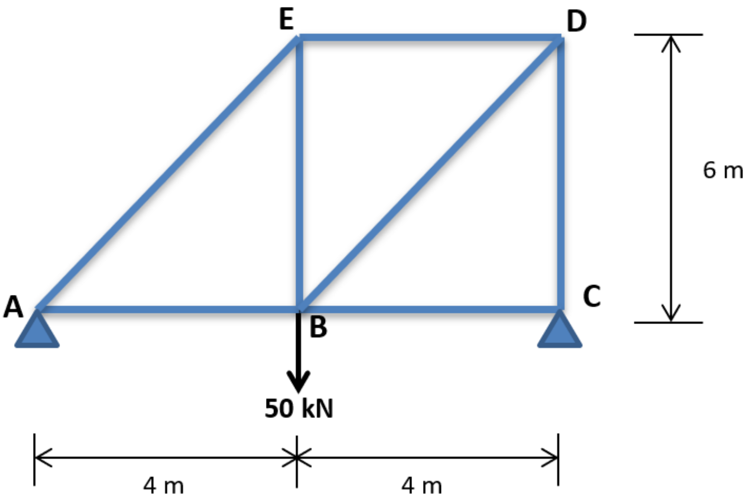


Figure 7.2

Solution

m = 7 2j – 3 = 2(5) -3 = 7

r = 4 **>** **3**

j = 5

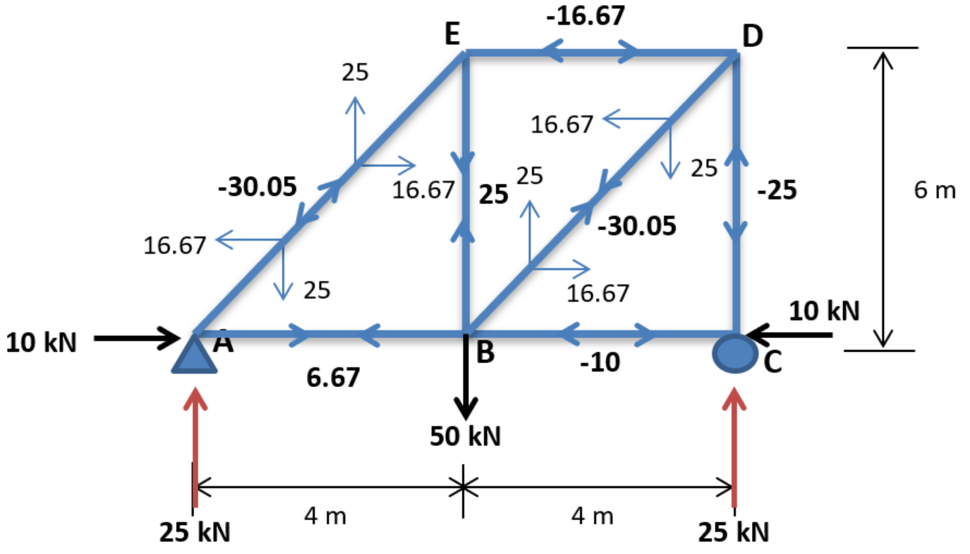
∴ ***m = 2j – 3 (statically external indeterminate)***

OBJECTIVE

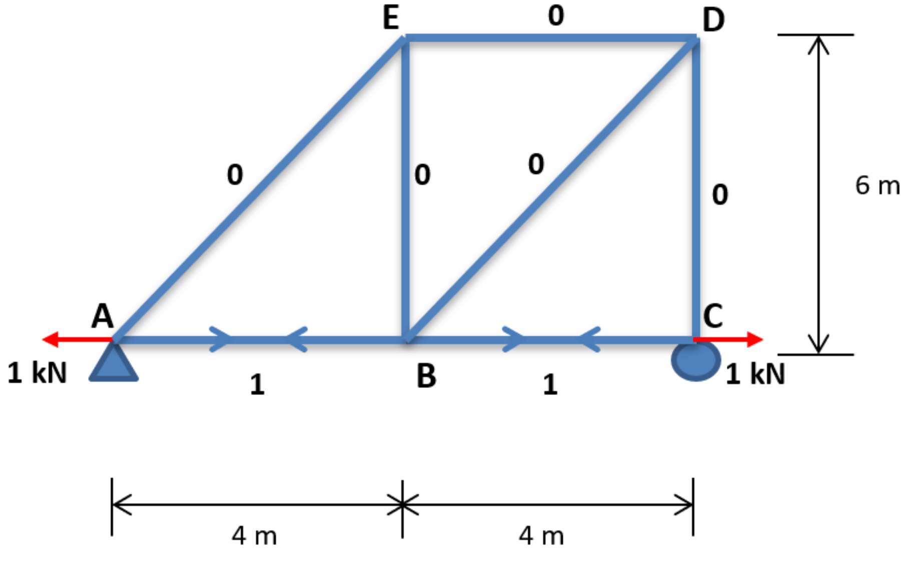
- *Able to calculate the internal forces in truss members using the unit load method .*

Solution

a) Real system



a) Virtual system



OBJECTIVE

- *Able to calculate the internal forces in truss members using the unit load method*
-

Solution

c) Actual force of truss

Member	P	μ	L (m)	μPL	μ^2L	F = P + μR
AB	6.67	1	4	26.68	4	7.41
AE	-30.05	0	7.21	0	0	-30.05
BC	-10	1	4	0	-40	-9.26
BD	30.05	0	7.21	0	0	30.05
BE	25	0	6	0	0	25
CD	-25	0	6	0	0	-25
DE	-16.67	0	4	0	0	-16.67
				Σ 26.68	**Σ -36**	

$$R = -\left(\frac{\frac{\mu PL}{AE}}{\frac{\mu^2 L}{AE}}\right) = -\left(\frac{26.68}{-36}\right) = 0.74\ kN$$

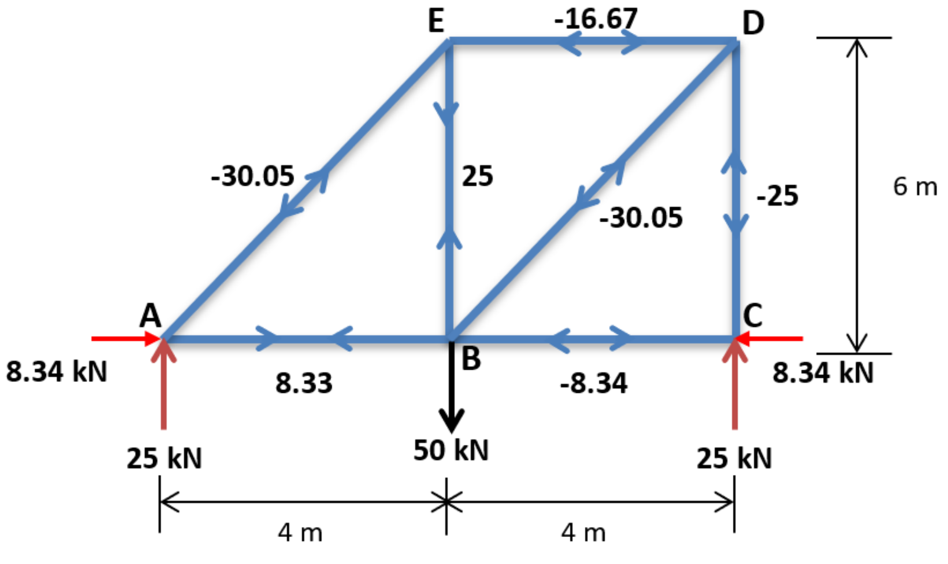
OBJECTIVE

- *Able to calculate the internal forces in truss members using the unit load method*

Solution

c) Actual force of truss

F	$F = P + \mu R$	
F_{Ay}	25 + (0)(0.74) = 25	↑
F_{Ax}	0 + (-1)(0.74) = -0.74	←
F_{Cy}	25+(0)(0.74) = 25	↑
F_{Cx}	0 + (1)(0.74) = 0.74	→



OBJECTIVE

- *Able to calculate the internal forces in truss members using the unit load method*

7.2 PROBLEMS

QUESTION 1

Determine the force in each member of the truss as shown in Figure 7.3. Cross sectional areas in each member are shown in brackets. Assume modulus elasticity in each member, E=200Gpa

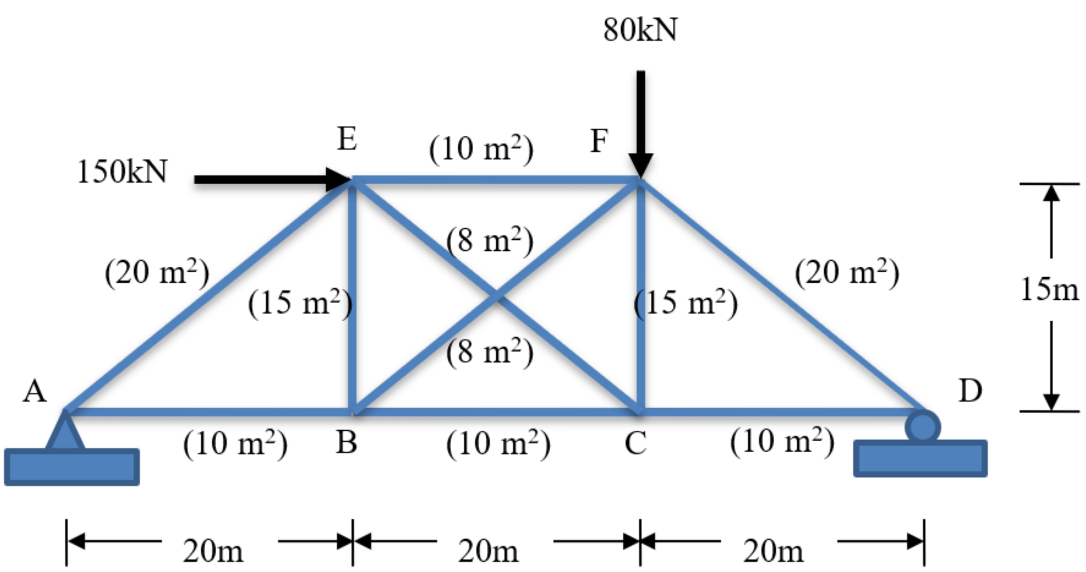


Figure 7.3

OBJECTIVE

- *Able to calculate the internal forces in truss members using the unit load method*

7.2 ROBLEMS

QUESTION 2

Determine the reaction and the force in each member of the truss shown Figure 7.4 by the Force Method. Assume EA constant (E = 250 kN/mm^2 and A=1000mm^2).

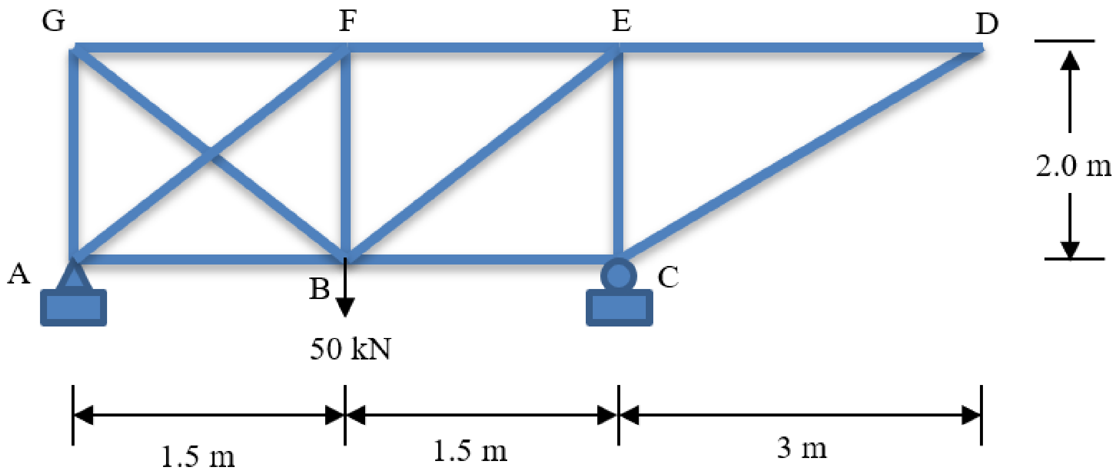


Figure 7.4

QUESTION 3

Classify the trusses below and determine the internal forces in Figure 7.5. The modulus of elasticity (E) of each member is constant.

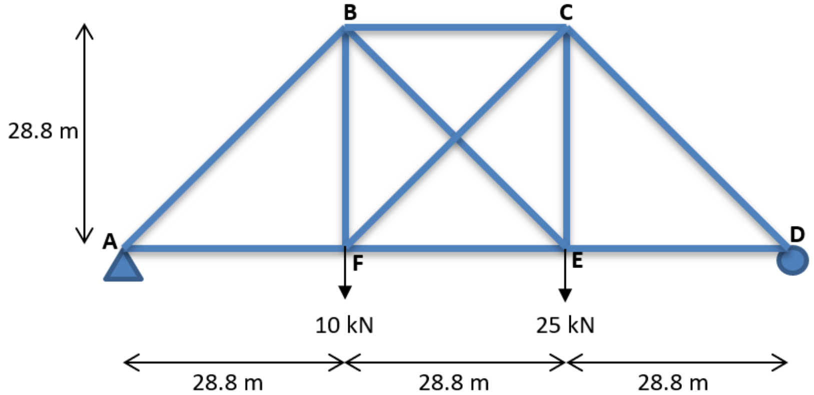


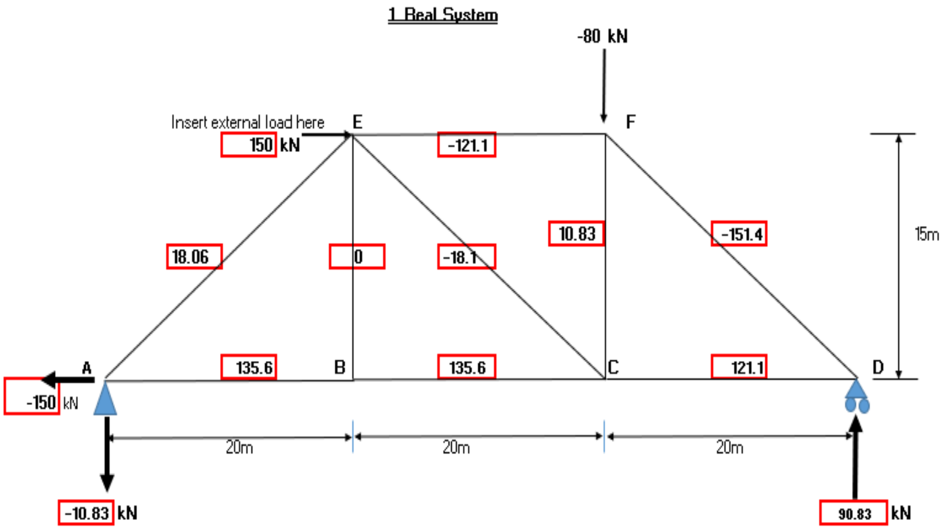
Figure 7.5

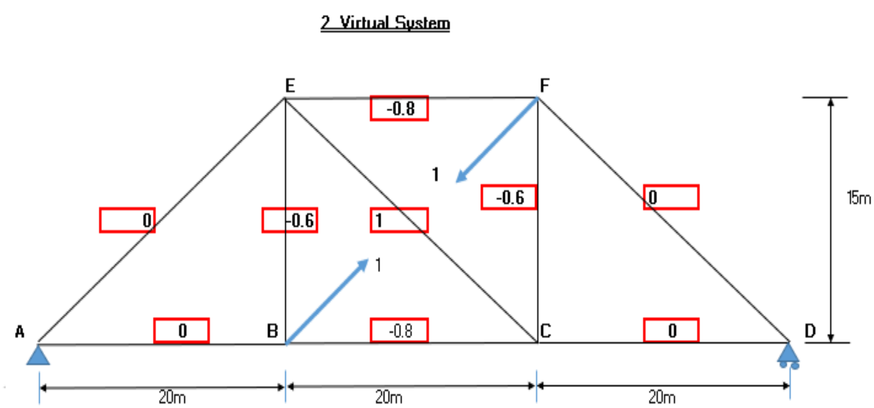
OBJECTIVE

- *Able to calculate the internal forces in truss members using the unit load method*

7.3 ANSWERS

Question 1





OBJECTIVE

- *Able to calculate the displacement truss by virtual work method*

7.3 ANSWERS

Question 1

3. Actual Force(kN)

Member	Length (mm)	AE(kN)	F(kN)	μ	Δ=FμL/AE	μ^2L/AE	P=F+μR
AB	16000	2,000,000,000.00	135.56	0	0.0000000	0.0000000	135.56
BC	16000	2,000,000,000.00	135.56	-0.8	-0.0008676	0.0000051	128.33
CD	16000	2,000,000,000.00	121.11	0	0.0000000	0.0000000	121.11
DF	20000	4,000,000,000.00	-151.39	0	0.0000000	0.0000000	-151.39
FE	16000	2,000,000,000.00	-121.11	-0.8	0.0007751	0.0000051	-128.33
AE	20000	4,000,000,000.00	18.06	0	0.0000000	0.0000000	18.06
EB	12000	3,000,000,000.00	0.00	-0.6	0.0000000	0.0000014	-5.42
EC	20000	1,600,000,000.00	-18.06	1	-0.0002257	0.0000125	-9.03
FC	12000	3,000,000,000.00	10.83	-0.6	-0.0000260	0.0000014	5.42
BF	20000	1,600,000,000.00	0.00	1	0.0000000	0.0000125	9.03
∑					-0.0003441	0.0000381	

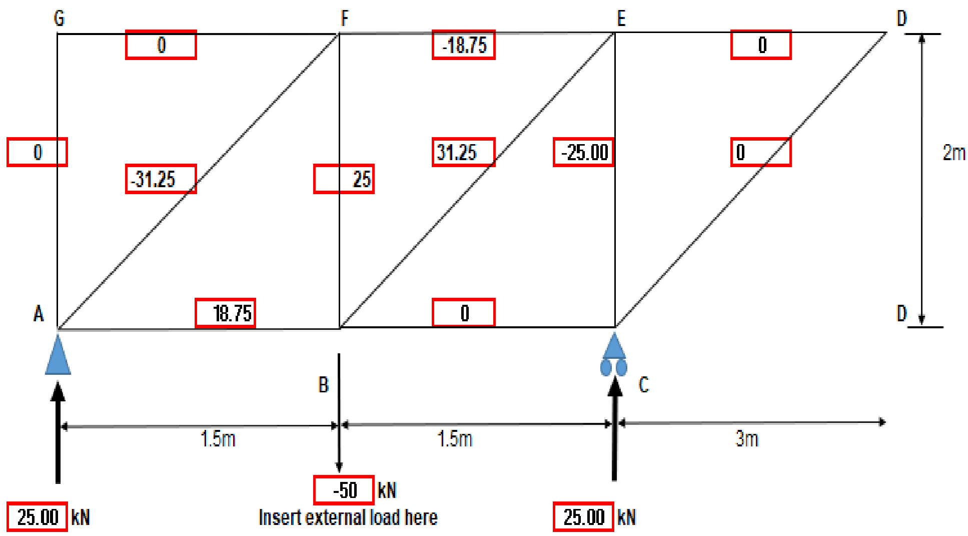
Magnitude of Redundant, R= 9.03

OBJECTIVE

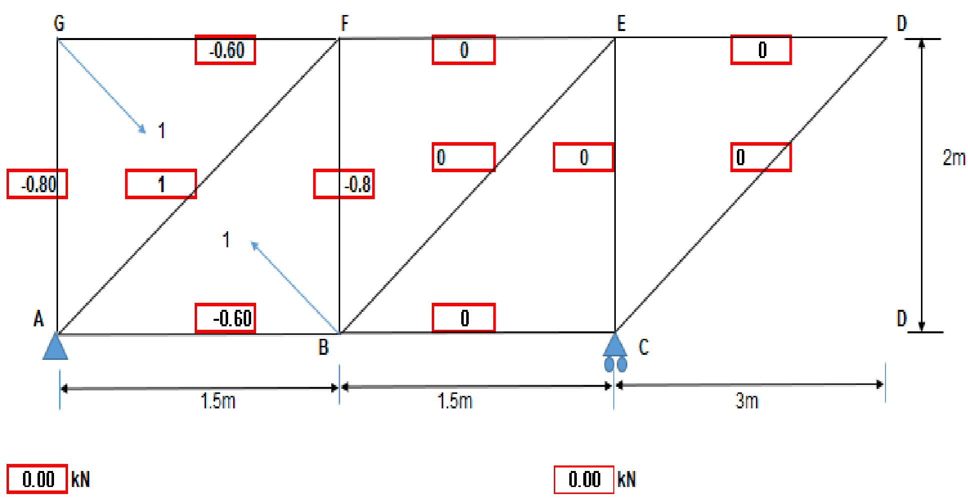
- *Able to calculate the displacement truss by virtual work method*

7.3 ANSWERS

Question 2



2. Virtual System



OBJECTIVE

- *Able to calculate the displacement truss by virtual work method*

7.3 ANSWERS

Question 2

Member	Length (mm)	AE(kN)	F(kN)	μ	Δ=FμL/AE	μ^2L/AE	P=F+μR
AB	1500	250,000.00	18.75	-0.60	-0.068	0.002	9.38
BC	1500	250,000.00	0.00	0	0.000	0.000	0.00
CD	2500	250,000.00	0.00	0	0.000	0.000	0.00
DE	1500	250,000.00	0.00	0	0.000	0.000	0.00
EF	1500	250,000.00	-18.75	0	0.000	0.000	-18.75
FG	1500	250,000.00	0.00	-0.60	0.000	0.002	-9.38
GA	2000	250,000.00	0.00	-0.80	0.000	0.005	-12.50
AF	2500	250,000.00	-31.25	1	-0.313	0.010	-15.63
FB	2000	250,000.00	25.00	-0.8	-0.160	0.005	12.50
BE	2500	250,000.00	31.25	0	0.000	0.000	31.25
GB	2500	250,000.00	0.00	1	0.000	0.010	15.63
EC	2000	250,000.00	-25.00	0	0.000	0.000	-25.00
				∑	-0.540	0.035	

OBJECTIVE

- *Able to calculate the displacement truss by virtual work method*

7.3 ANSWERS

Question 3

Bar	P	μ	L (m)	μPL	μ^2L	F = P + μR
AF	15	0	28.8	0	0	15
FE	15	-0.707	28.8	-305.42	14.4	13.89
ED	20	0	28.8	0	0	20
AB	-21.2	0	40.8	0	0	-21.2
BC	-20	-0.707	28.8	407.23	14.4	-21.11
CD	-28.3	0	40.8	0	0	-28.3
BF	10	-0.707	28.8	-203.62	14.4	8.89
BE	7.1	1	40.8	289.68	40.8	8.68
FC	0	1	40.8	0	40.8	1.58
CE	20	-0.707	28.8	-407.23	14.4	18.89
				Σ -219.36	Σ 139.18	

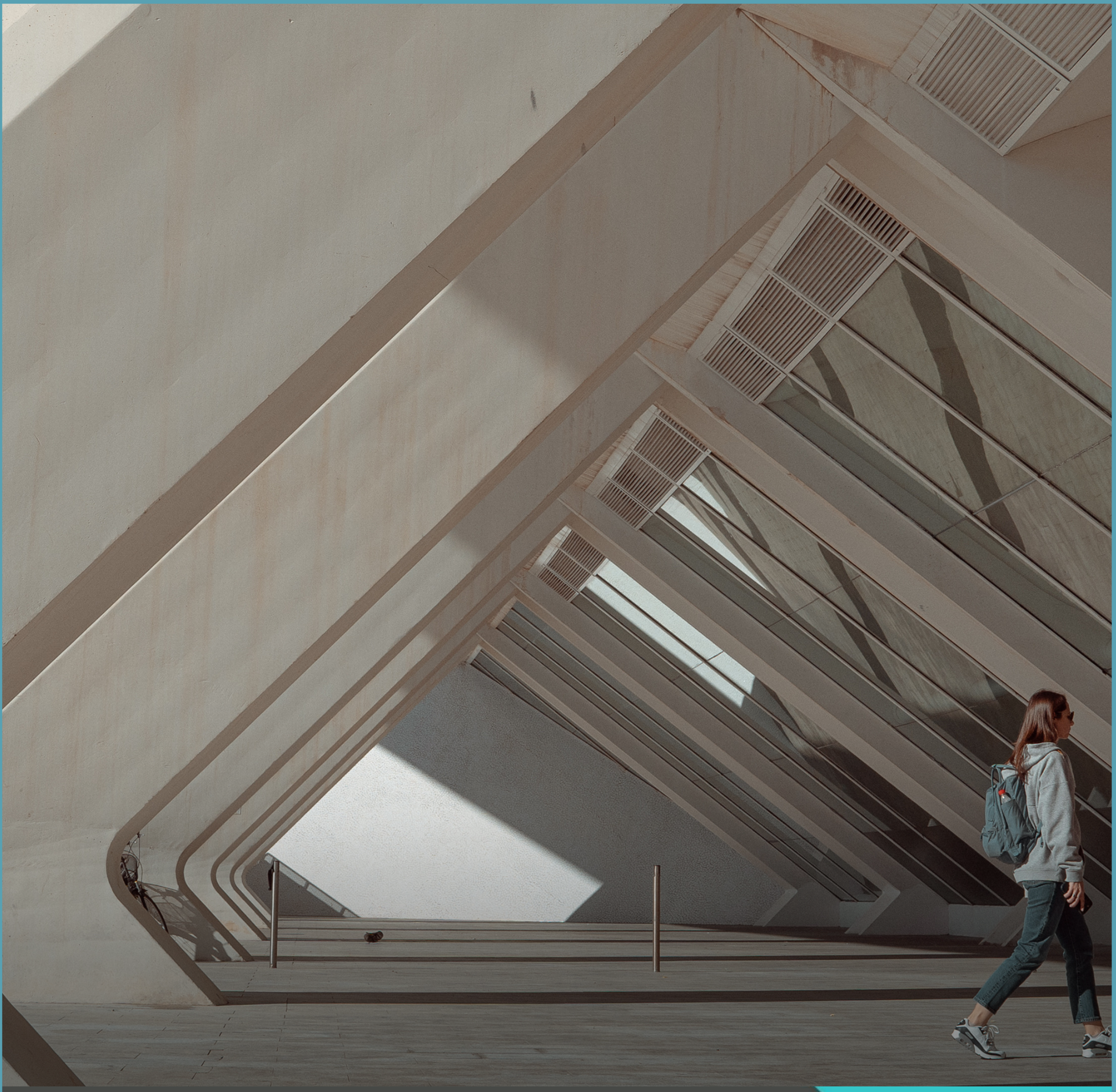
$$R = -\left(\frac{\frac{\mu PL}{AE}}{\frac{\mu^2 L}{AE}}\right)$$

$$= -\left(\frac{-219.36}{139.18}\right)$$

$$= 1.58\ kN$$

OBJECTIVE

- *Able to calculate the displacement truss by virtual work method*



CHAPTER 8

INFLUENCE LINE FOR STATICALLY DETERMINATE BEAMS

CHAPTER 8

INFLUENCE LINE FOR STATICALLY DETERMINATE BEAMS

8.1 Introduction to influence line

For static loads on beam, the bending moment diagram and the shear force diagram show the change of moment and shear along the beam. For loads moving along the beam, the change of shear force and bending moment are illustrated using influence lines. Structures such as bridges and overhead cranes are designed to resist moving loads as well as their own weight. Since structures are designed for the critical loads that may occur in them, influence lines are used to obtain the position on a structure where a moving load will cause the largest stress.

Influence lines represent changes in response, friction, moment and even displacement at a particular point in the structure at the time of a point load of 1 unit moving along structure. Once an influence line at a point has been constructed, it is easier to determine the position of the live load that will produce the greatest effect at that point. Furthermore, the magnitude of the associated reaction, shear, moment, or deflection at the point can be calculated from the ordinates of the influence-line diagram. For these reasons, influence lines play an important part in the design of bridges, industrial crane rails, conveyors, and other structures where loads move across their span.

Procedure for analysis:

- Allow a unit load (1 kN) to move over beam from left to right.
- Find the values of reaction, shear force or bending moment at the specific point under consideration.
- Plot the values of the reaction, shear force or bending moment over the length of the beam.

Quiz

How to get maximum shear in beams?

Ans. Shear force diagram

How to draw the influence line in easy way

OBJECTIVE

- *Able to understand influence line purposes*
- *Able to apply influence line to find the greatest effect in beam*

Example 8.1

A simply supported beam with one end hanging subjected a series of moving axial loads from point A to D as shown in Figure 8.1.

a) Draw influence lines diagram of reaction at support B.

b) Draw influence lines diagram of reaction at support D.

c) Draw influence lines diagram of shear force at joint C.

d) Draw influence lines diagram of bending moment at joint C.

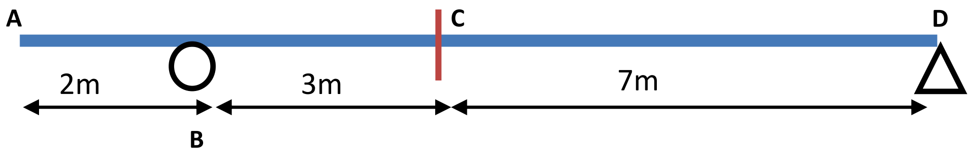
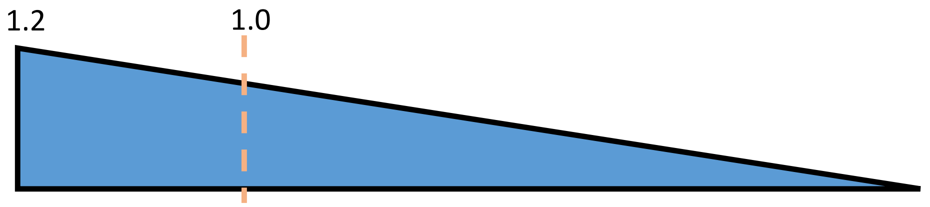


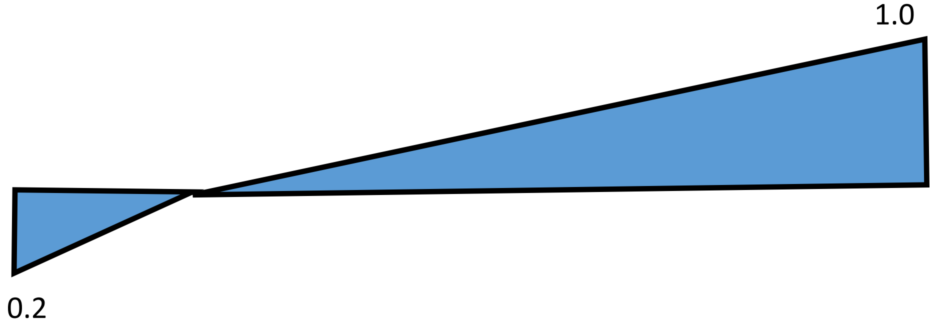
Figure 8.1

Solution

a) Draw influence lines diagram of reaction at support B.



b) Draw influence lines diagram of reaction at support D.

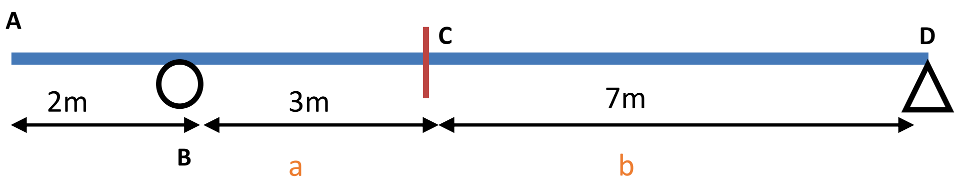


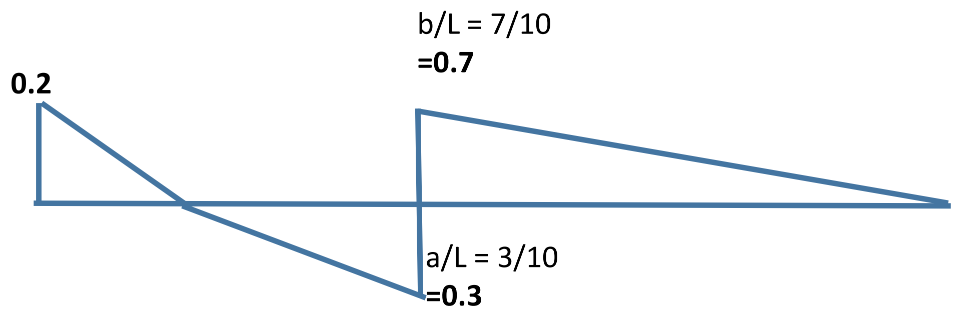
OBJECTIVE

- *Able to draw influence line for beam*

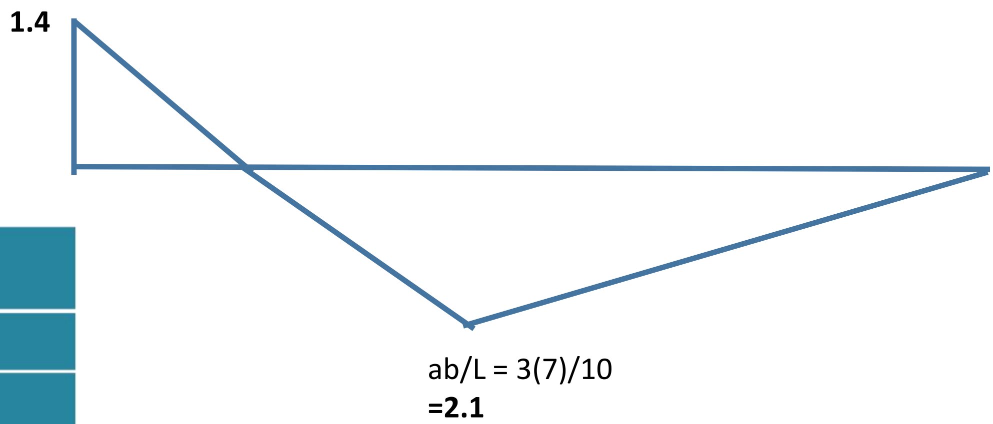
Solution

c) Draw influence lines diagram of shear force at joint C.





d) Draw influence lines diagram of bending moment at joint C.



OBJECTIVE

- *Able to draw the influence line for beam*

Example 8.2

According to simply supported beam AB with 15 mm span as shown in Figure 8.2 below find:

a) Maximum reaction at B
b) Maximum shear force at point C
c) Maximum Bending moment at point C
d) If concentrated moving load moving from A to B.

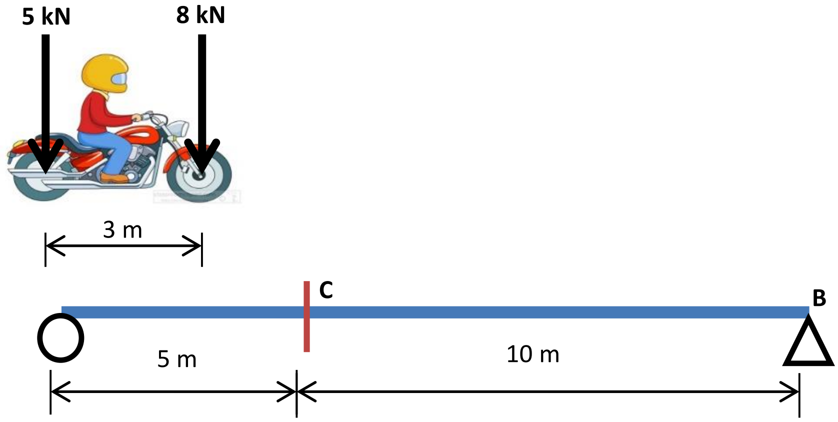
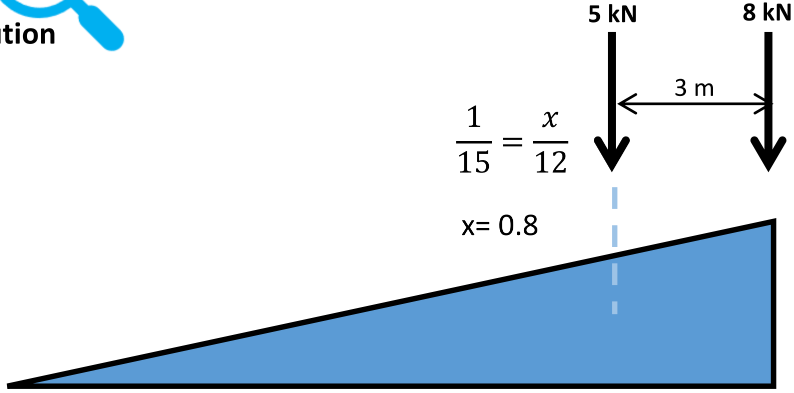


Figure 8.2

Solution



$$\frac{1}{15} = \frac{x}{12}$$

x= 0.8

V = 5 (0.8) + 8 (1) = 12 kN

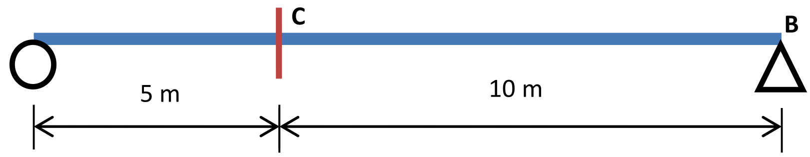
The maximum reaction at B is 12 kN.

OBJECTIVE

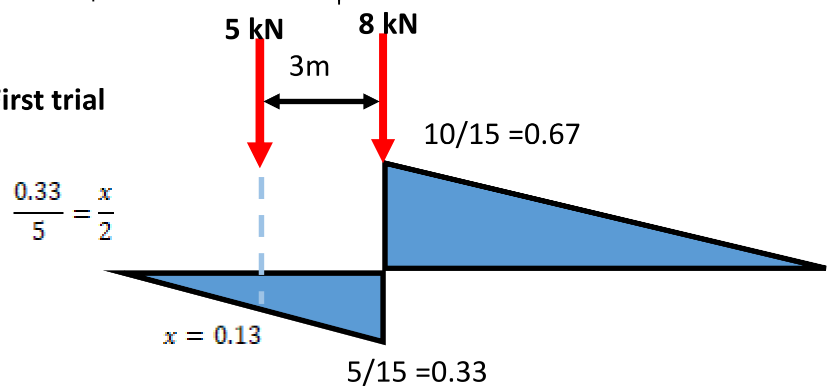
- *Able to calculate maximum shear and bending moment*

Solution

a) Shear Force at C



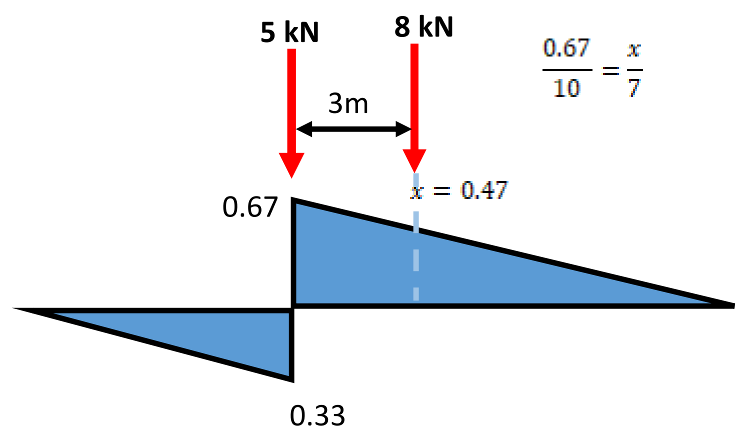
First trial



$$V_{C1} = 5(-0.13) + 8(-0.33) = -3.29\ kN$$

$$V_{C2} = 5(-0.13) + 8(0.67) = 4.71\ kN$$

Second trial



$$V_{C3} = 5(-0.33) + 8(0.47) = 2.11\ kN$$

$$V_{C4} = 5(0.67) + 8(0.47) = 7.11\ kN$$

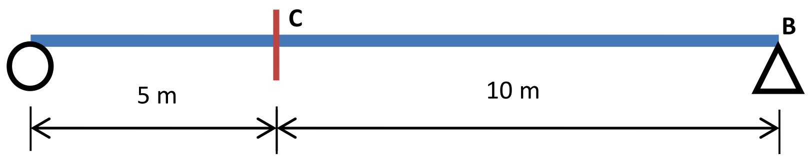
The maximum shear force is 7.11 kN.

OBJECTIVE

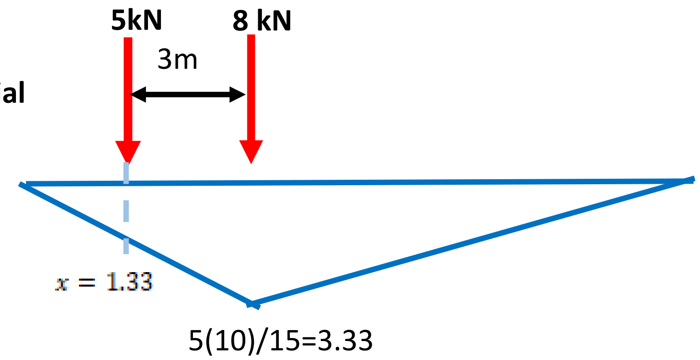
- *Able to calculate maximum shear and bending moment*

Solution

a) Bending Moment at C

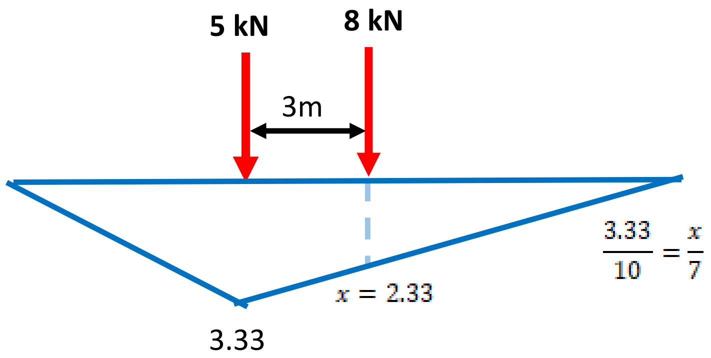


First trial



$$M_{C1} = 5(1.33) + 8(3.33) = 33.29\ kNm$$

Second trial



$$M_{C2} = 5(3.33) + 8(2.33) = 35.29\ kNm$$

The maximum bending moment is 35.29 kNm.

OBJECTIVE

- *Able to calculate maximum shear and bending moment*

Example 8.3

According to simply supported beam AB with 15 mm span as shown in Figure 8.3 below, find:

a) Maximum shear force at point C

b) Bending moment at point C

If concentrated moving load moving from A to B with 8 kN leading.

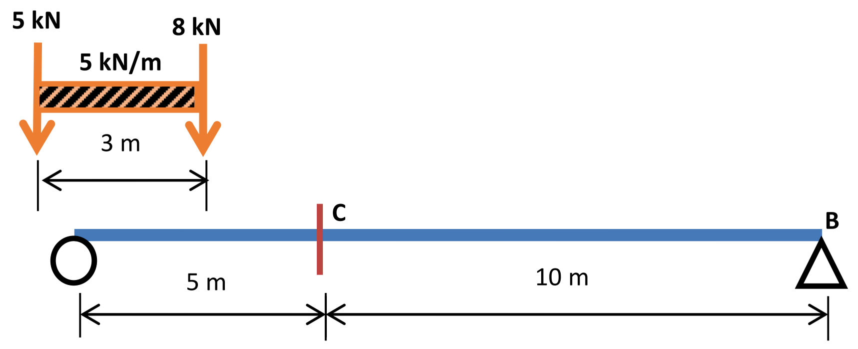


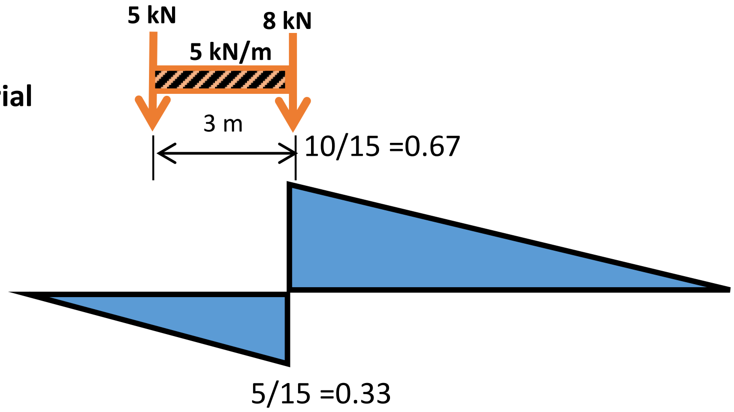
Figure 8.3

OBJECTIVE

- *Able to calculate maximum shear and bending moment*

a) Maximum shear force at point C

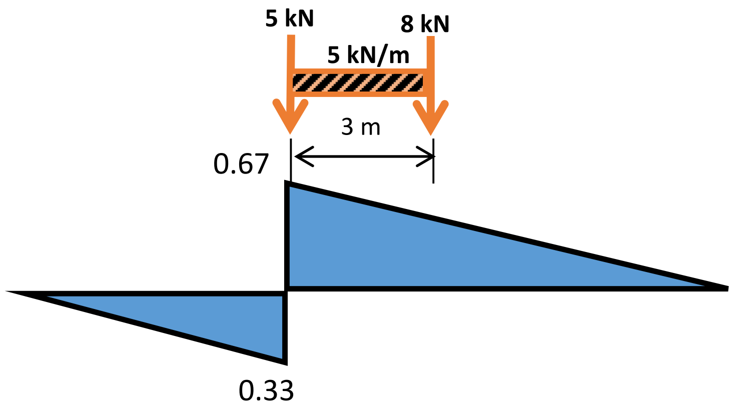
First trial



$$V_{C1} = 5(-0.13) + 8(-0.33) + \left[-5\left[\frac{1}{2}(0.13 + 0.33)(3)\right]\right] = -6.74\ kN$$

$$V_{C2} = 5(-0.13) + 8(0.67) + \left[-5\left[\frac{1}{2}(0.13 + 0.33)(3)\right]\right] = 1.42\ kN$$

Second trial



$$V_{C3} = 5(-0.33) + 8(0.47) + 5\left[\frac{1}{2}(0.67 + 0.47)(3)\right] = 10.66\ kN$$

$$V_{C4} = 5(0.67) + 8(0.47)5\left[\frac{1}{2}(0.67 + 0.47)(3)\right] = 15.66\ kN$$

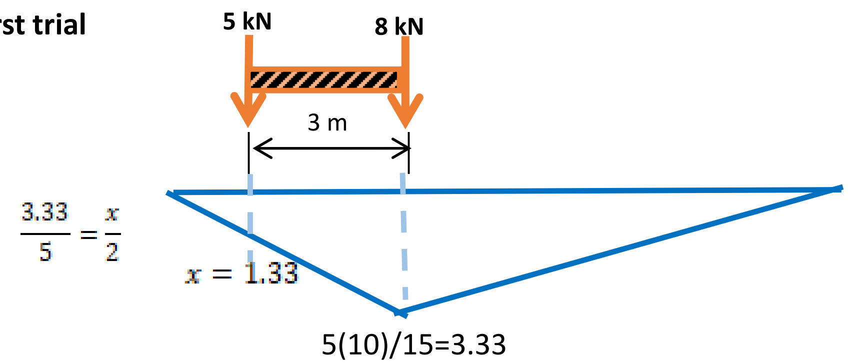
The maximum shear force is 15.66 kN.

OBJECTIVE

- *Able to calculate maximum shear and bending moment*

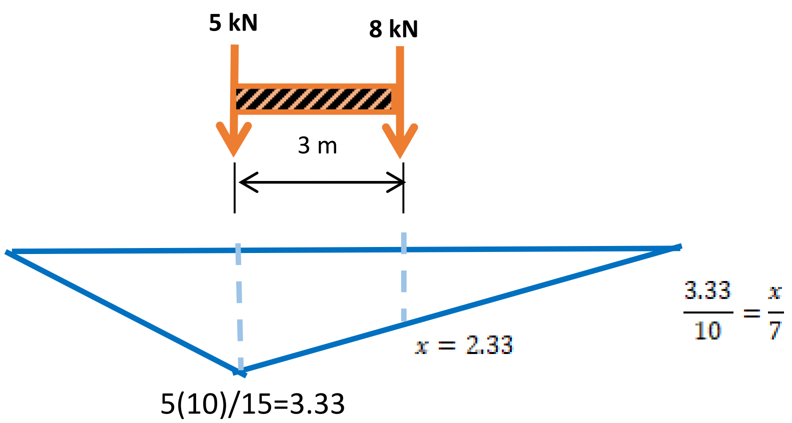
b) Maximum bending moment at point C

First trial



$$M_{C1} = 5(1.33) + 8(3.33) + 5\left[\frac{1}{2}(1.33 + 3.33)(3)\right] = 68.24\ kNm$$

Second trial



$$M_{C2} = 5(3.33) + 8(2.33) + 5\left[\frac{1}{2}(2.33 + 3.33)(3)\right] = 77.74\ kNm$$

The maximum bending moment is 77.74 kNm.

OBJECTIVE

- *Able to calculate maximum shear and bending moment*

8.2 Absolute Maximum Moment

In influence lines section, we have learned the methods for calculating the maximum shear force and moment at specific point in the beam due to concentrated moving loads. Problem involves determination of absolute maximum moment. To obtain absolute maximum moment, we must determine both location of the point in the beam and the position of the loading on the beam. We need to develop a method for computing the maximum shear and moment on beam that determines both location of the point and position of the loading.

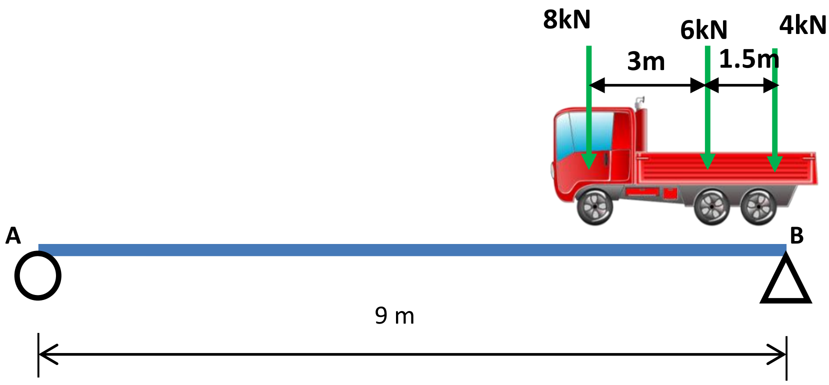
For a simply supported beam the maximum moment cannot be located by inspection. However, it can be proved that the absolute maximum moment in a simply-supported beam occurs under one of the concentrated loads. The location is determined as such that the force is positioned on the beam so that the resultant force of the system is equidistance from the beam's center line. By applying this technique to each of the concentrated forces, the absolute maximum moment can be calculated.

OBJECTIVE

- *Understand the principle of absolute maximum moment*

Example 8.4

Determine the absolute maximum bending moment in the simply supported bridge deck shown in Figure 8.4 below.



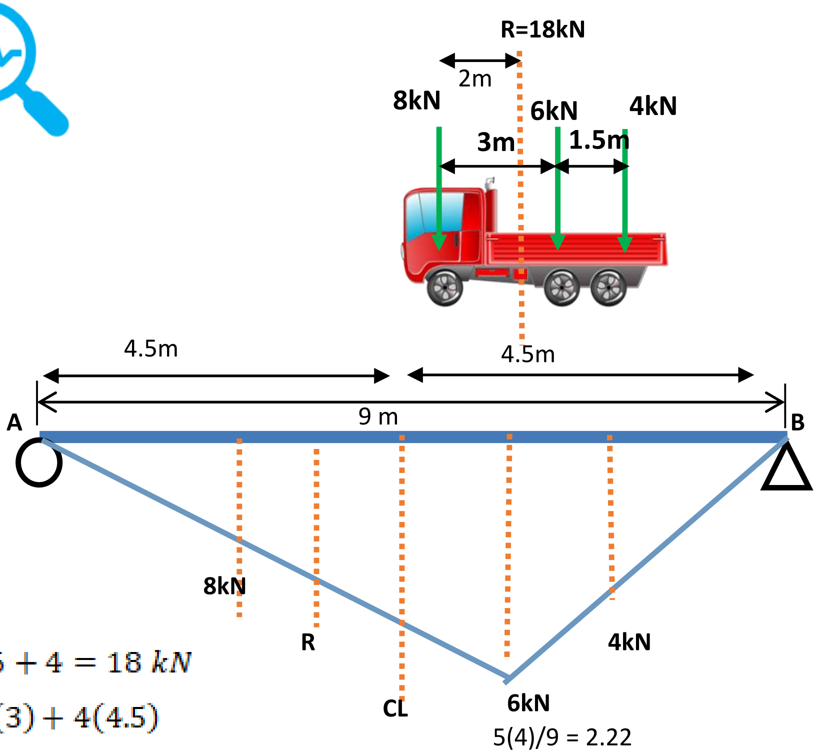
Solution

$$\frac{2.22}{5} = \frac{x_1}{2}$$

$$x_1 = 0.89$$

$$\frac{2.22}{4} = \frac{x_2}{2.5}$$

$$x_2 = 1.39$$



$$R = 8 + 6 + 4 = 18\ kN$$

$$R(x) = 6(3) + 4(4.5)$$

X=2m

$$\sum M = (8x0.89) + (6x2.22) + (4x1.39) = 26\ kNm$$

OBJECTIVE

- *Able to calculate absolute maximum moment*

8.3PROBLEMS

QUESTION 1

A beam with both ends overhang is subjected to external loads as shown in Figure 8.5. Using Influence Line Diagram Method calculate the maximum shear and bending moment at point C.

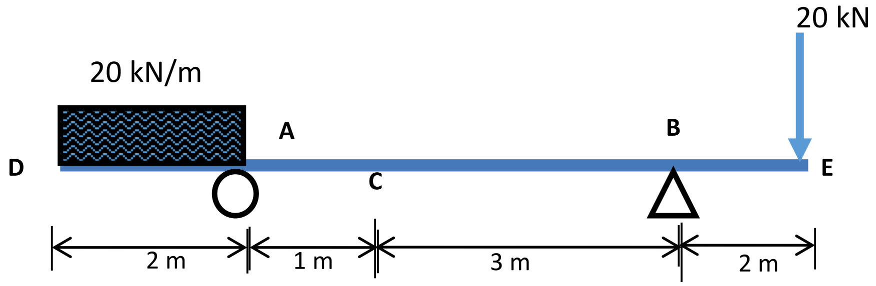


Figure 8.5.

QUESTION 2

According to simply supported beam AB with 25 mm span as shown in Figure 8.6. Find:

a) Maximum shear force at point C

b) Bending moment at point C

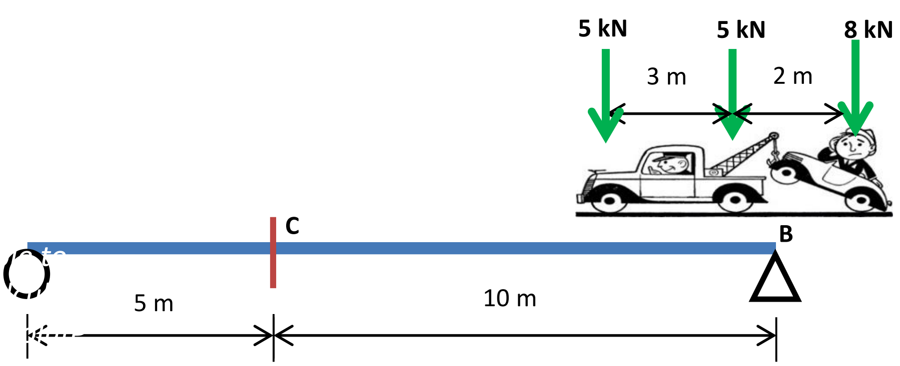


Figure 8.6

OBJECTIVE

- *Able to calculate maximum shear and bending moment and absolute maximum moment*

8.3 PROBLEMS

QUESTION 3

According to simply supported beam AB with 15 mm span as shown in Figure 8.7. Find:
a) Maximum shear force at point C
b) Bending moment at point C

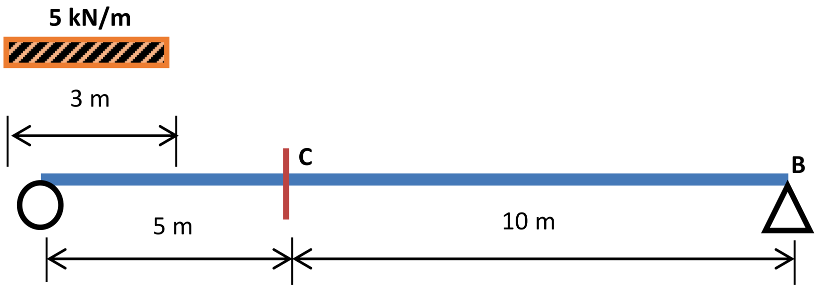


Figure 8.7

QUESTION 4

Determine the absolute maximum live moment in the girder due to the loading shown in Figure 8.8

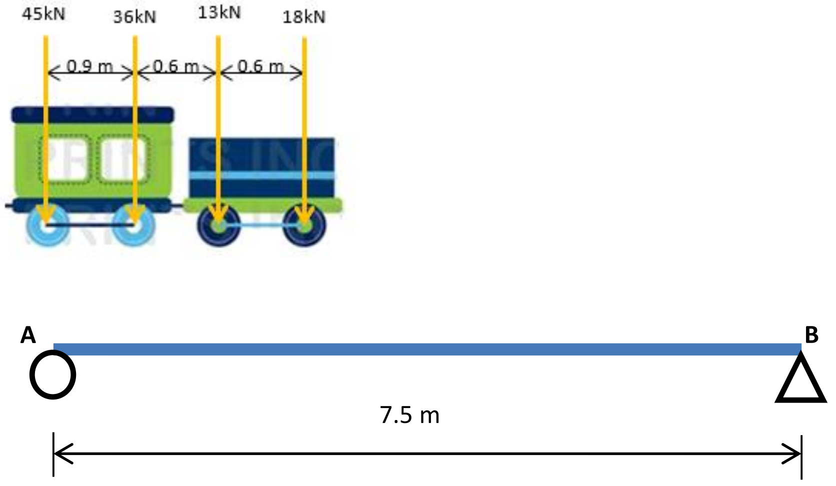


Figure 8.8

OBJECTIVE

- *Able to calculate maximum shear and bending moment and absolute maximum moment*

8.3 PROBLEMS

QUESTION 5

A simply supported beam subjected a series of moving concentrated loads as shown in Figure 8.9.

i) Determine magnitude and resultant force.

ii) Compute Absolute Maximum Moment.

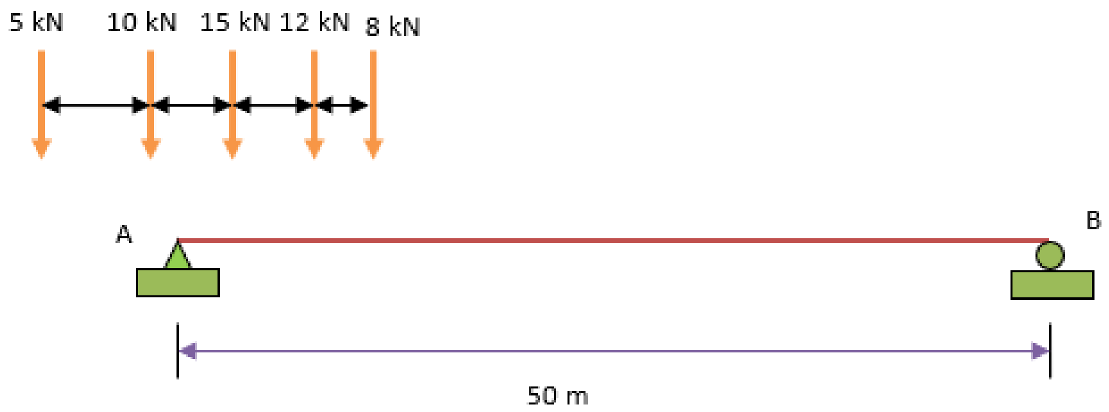


Figure 8.9

OBJECTIVE

- *Able to calculate maximum shear and bending moment and absolute maximum moment*

8.4 ANSWERS

Question 1

V_C = 22.5 kN

Mc= -47.5 kNm

Question 2

V_C = 8.6 kN

Mc= 92.6 kNm

Question 3

V_C = 8.55 kN

Mc= 44.93 kNm

Question 4

Absolute maximum moment is 173..96 kNm

Question 5

Absolute maximum moment is 506.75 kNm

OBJECTIVE

- *Able to calculate maximum shear and bending moment and absolute maximum moment*

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THEORY OF STRUCTURES

The authors is an experienced lecturer teaching over ten years in the field of structure. They are significantly involved in the formulation of questions and curriculum in the organization and are actively involved in the development of teaching and learning.

With the hope that this work will stimulate an interest in Structural Analysis and provide an acceptable guide to its understanding.

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e ISBN 978-967-2762-16-4

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THEORY OF STRUCTURES