International Journal of Mechanical Engineering and Technology (IJMET)
Volume 15, Issue 01, Jan-Feb 2024, pp. 1-9. Article ID: IJMET_15_01_001
Available online at https://iaeme.com/Home/issue/IJMET?Volume=15&Issue=1
ISSN Print: 0976-6340 and ISSN Online: 0976-6359
Impact Factor (2024): 20.99 (Based on Google Scholar Citation)

A SEMI-ANALYTICAL APPROACH TO INVESTIGATE THE VALIDITY OF THE THERMAL EQUILIBRIUM ASSUMPTION IN THE ANALYSIS OF UNSTEADY FREE CONVECTION IN POROUS MICRO-CHANNEL

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ABSTRACT

Exploring the transient thermal dynamics of a porous micro-channel, this study delves into a uniform composite substrate governed by the parabolic heat–conduction model. The composite structure comprises a fluid domain (matrix) and solid inserts, each crafted from distinct materials. The research defines the parameter range within the application of the thermal equilibrium assumption (TEA) is suitable for analyzing transient free convection flow in the porous micro-channel, as characterized by the parabolic heat-conduction model. The investigation also scrutinizes the influence of various parameters on the validity of the local thermal equilibrium assumption (LTEA)) within the framework of the parabolic heat conduction model. The most significant impact on the LTEA is observed for the volumetric Biot number, thermal conductivity ratio, Knudsen number, and total thermal capacity ratio. Specifically, large values of the Knudsen number and thermal capacity ratio, along with small values of the Biot number and thermal conductivity ratio, contribute to securing the LTEA.

Key words: Conduction; Parabolic; Porous; Semi-Analytical; Thermal Equilibrium.

Cite this Article: A. F. Khadrawi and A.M. Maqableh, A Semi-Analytical Approach to Investigate the Validity of The Thermal Equilibrium Assumption in The Analysis of Unsteady Free Convection in Porous Micro-Channel, International Journal of Mechanical Engineering and Technology (IJMET), 15(1), 2024, pp. 1-9.
https://iaeme.com/Home/issue/IJMET?Volume=15&Issue=1

1. INTRODUCTION

Over the past two decades, there has been significant research focus on convective heat transfer in porous channels, driven by its diverse applications such as groundwater pollution transport, geothermal energy engineering, waste of nuclear disposal, chemical engineering reactor, insulation of pipes and buildings. Previous studies primarily concentrated on either forced or natural convection within porous media.

The thermal behavior of porous systems is commonly described using two models found in the literature: the single-phase and two-phase Schumann models. The key distinction lies in the TEA in the single phase model, while no such assumption is made in the two-phase model. Consequently, the single-phase model simplifies to a single governing energy equation, whereas the two-phase model involves two governing two governing energy equations, each with s fluid-to-solid heat transfer term.

The concept of LTEA has been extensively utilized in modeling transport phenomena in porous media, with limited investigations challenging this assumption [1-7]. Fewer studies have conducted a comparison between both models, often focusing on specific cases and applications [8-13]. Various models of the conservation of momentum equation, such as the Darcian model, have been employed to described fluid flow in porous media [14-16].

Forced convection heat transfer in porous media has been explored by specific researchers [17-20], particularly investigating the LTEA in transient conjugated forced convection channel flow analytically [4-6]. Analysis concentrates on the operating conditions necessary for both the fluid and solid phases to approach the same temperature, ensuring the LTEA.

Addressing the gaps in existing research, it becomes imperative to identify the valid ranges for the LTEA. Previous studies have touched upon this aspect in the case of transient forced convection in porous channel flow [21]. Examination of the TEA in transient free convection in porous micro-channel is investigated numerically [22]. Consequently, the current study aims to scrutinize the LTEA of transient natural convection in micro-channel flow semi-analytically.

2. ANALYSIS

In examining the unsteady free convection fluid flow within a micro-parallel plates channel entirely occupied by porous domain, the temporal variation in thermal characteristics results from an abrupt alteration in the temperature of the wall of the channel. As depicted in Figure 1, the governing energy equations, accompanied by their respective initial and boundary conditions, are presented for both the solid and fluid domains. It's important to note that conduction within the fluid domain is neglected in this analysis. The dimensionless system of equations of the wavy heat conduction model is employed for this study is as follows:

$$\frac{\partial \theta_f}{\partial \eta} = \frac{\partial^2 \theta_f}{\partial Y^2} + Bi\left(\theta_s - \theta_f\right) \tag{1}$$

$$\frac{\partial \theta_s}{\partial \eta} = -Bi\left(\theta_s - \theta_f\right) \tag{2}$$

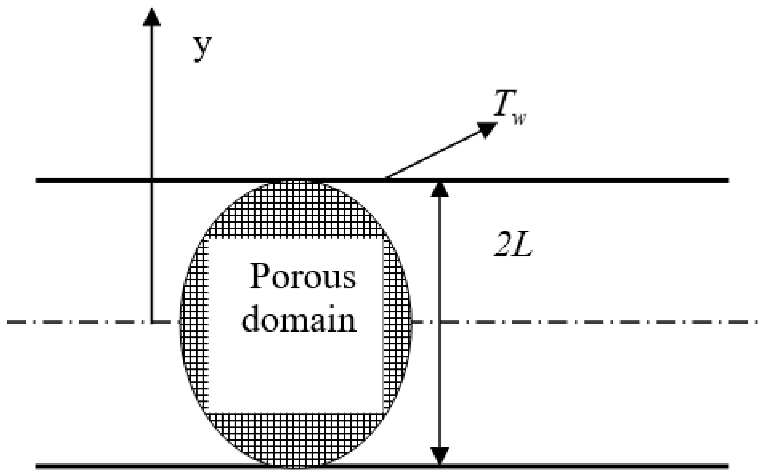

Fig. 1: Domain under consideration

The initial and boundary conditions become:

$$\theta_s(0,Y)=\theta_f(0,Y)=0 \qquad \frac{\partial\theta_s(0,Y)}{\partial\eta}=\frac{\partial\theta_f(0,Y)}{\partial\eta}=0 \tag{3}$$

$$\underline{\theta_s(\eta,1)-1=-Kn\frac{\Omega}{\Pr}Q(\eta,1)} \qquad \underline{\theta_f(\eta,1)-1=-Kn\frac{\Omega}{\Pr}Q(\eta,1)} \qquad \frac{\partial\theta_s(\eta,0)}{\partial Y}=0$$

Equations (1-3) are solved using the Laplace transformation. Now with the observation that $L\{\theta_s(\eta,Y)\}=W_s(S,Y)$ and $L\{\theta_f(\eta,Y)\}=W_f(S,Y)$, Laplace transformation of Eqs. (1-3) gives:

$$\frac{\partial^2 W_f}{\partial Y^2}-\left[(S+Bi)-\frac{(Bi^2)}{(S+Bi)}\right]W_f=0 \tag{4}$$

$$W_s=\frac{(Bi)}{(S+Bi)}W_f \tag{5}$$

Also, the boundary conditions in the Laplace transformation form:

$$\left.\begin{aligned} &W_s(S,1)-\frac{1}{S}=-Kn\frac{\Omega}{\Pr}W_s'(S,1)\\ &W_f(S,1)-\frac{1}{S}=-Kn\frac{\Omega}{\Pr}W_f'(S,1)\\ &\frac{\partial W_s}{\partial Y}(S,0)=0 \end{aligned}\right\} \tag{6}$$

According to the B. C's given in Eq. (6), Eqs. (1-3) are solved to give:

$$W_f=G\left(e^{FY}+e^{-FY}\right) \tag{7}$$

$$W_s = \frac{G(Bi)\left(e^{FY} + e^{-FY}\right)}{(S + Bi)} \tag{8}$$

where $F^2 = \left[(S + Bi) - \frac{(Bi)}{(S + Bi)}\right]$ and $G = \frac{1/S}{\left[\left(e^F + e^{-F}\right) + Kn\frac{\Omega}{\Pr}\left(e^F - e^{-F}\right)\right]}$

By using a computer program, equations (7, 8 and 6) are inverted based on Riemann-sum approximation [23] as:

$$\theta(\eta, Y) \cong \frac{e^{\gamma\eta}}{\eta}\left[\frac{1}{2}W(\gamma, Y) + \operatorname{Re}\sum_{n=1}^{N} W\left(\gamma + \frac{in\pi}{\eta}, Y\right)(-1)^n\right] \tag{9}$$

In the provided passage, the notation and parameters associated with the Laplace inversion method are explained. Where, *Re* represent the real part of a complex number, *i* represents the imaginary unit, *N* denotes the number of terms used in the Riemann-sum approximation, γ refers to the path in the complex plane used in inverting Laplace transform and η is the specific time at which the lagging phenomenon is being studied. The accuracy of the Riemann-sum approximation in Laplace inversion is influenced by the choice of parameters, particularly the value of γ and the truncation error dictated by *N*. Selecting an appropriate value for γ is crucial to ensure that the Bromwich contour encloses all branch points [23]. For faster convergence, empirical observations suggest that a value satisfying the relation $\gamma\eta \cong 4.7$ often yields satisfactory results [23]. This choice depends on the specific time η at which the lagging phenomenon is under consideration. Additionally, the number N of terms in the Riemann-sum is determined to meet a prescribed threshold for the accumulated partial sum at given values of γ, η and Y.

3. RESULT AND DISCUSSIONS

The investigation into the validity of the TEA in unsteady free convection flow within a porous domain, as described by a parabolic heat-conduction model, is detailed in Figures (2-5). The impact of various parameters on this assumption is elucidated.

In Fig. 2, the transient behavior of solid and fluid temperatures is depicted at different values of a certain parameters, with neglecting the conduction in the fluid domain. The observed trend indicates that as this parameter decreases, the difference between fluid and solid temperature increases. Figure 3 illustrates the influence of the total thermal capacity ratio on the unsteady behavior of solid and fluid temperatures. As the value of this ratio increases, the difference temperature between solid and fluid also increases. The effect of the Biot number on unsteady temperatures difference is presented in Fig.4. It is evident that the temperature difference increases as the Biot number decreases. This obtains that the impact of the Biot number n the temperature difference is negligible at large Biot values.

In Fig.5, the influence of the Knudsen number Kn on temperature difference is demonstrated. The figure clearly show that the temperature difference increases with an increase in Knudsen number.

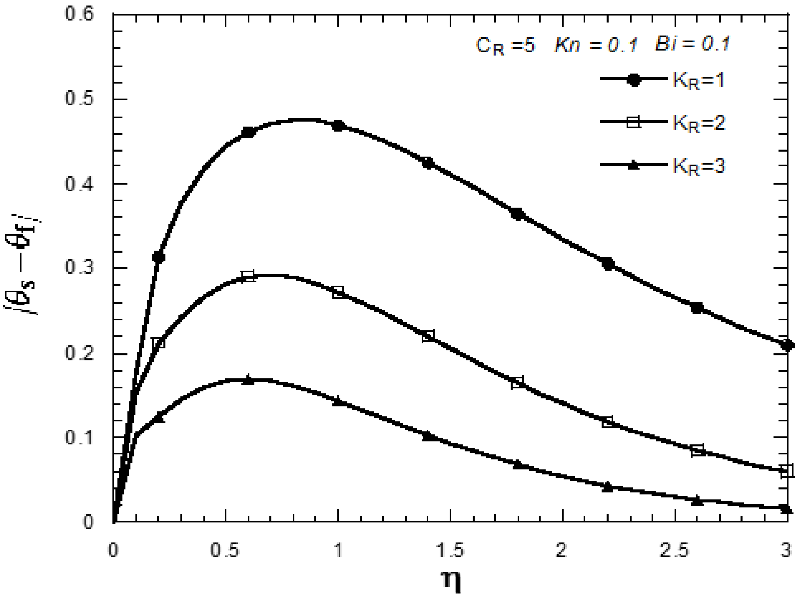

Fig.2: Transient temperature difference at different K_R

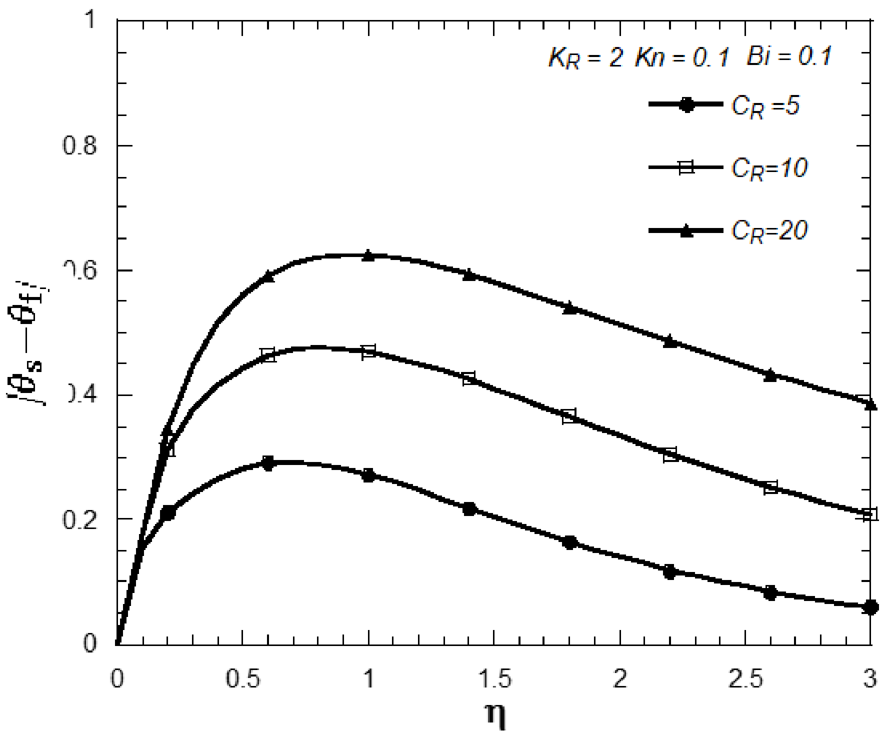

Fig. 3: Transient temperature difference at different C_R

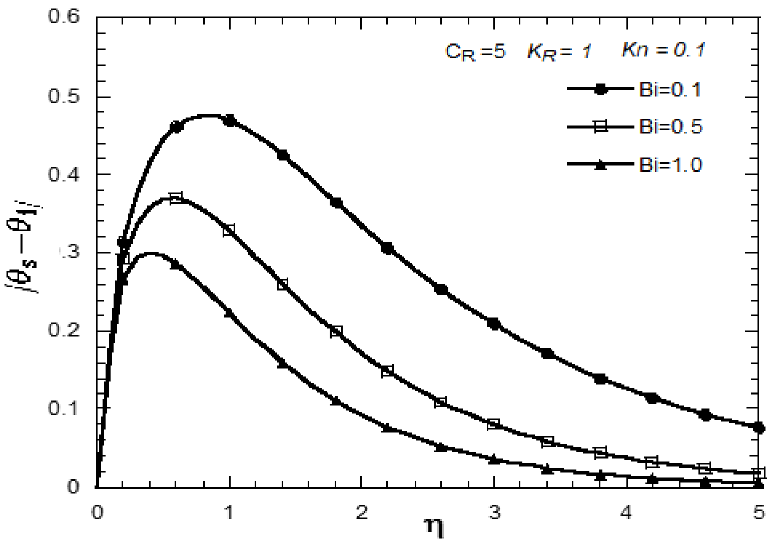

Fig. 4: Transient temperature difference at different Bi

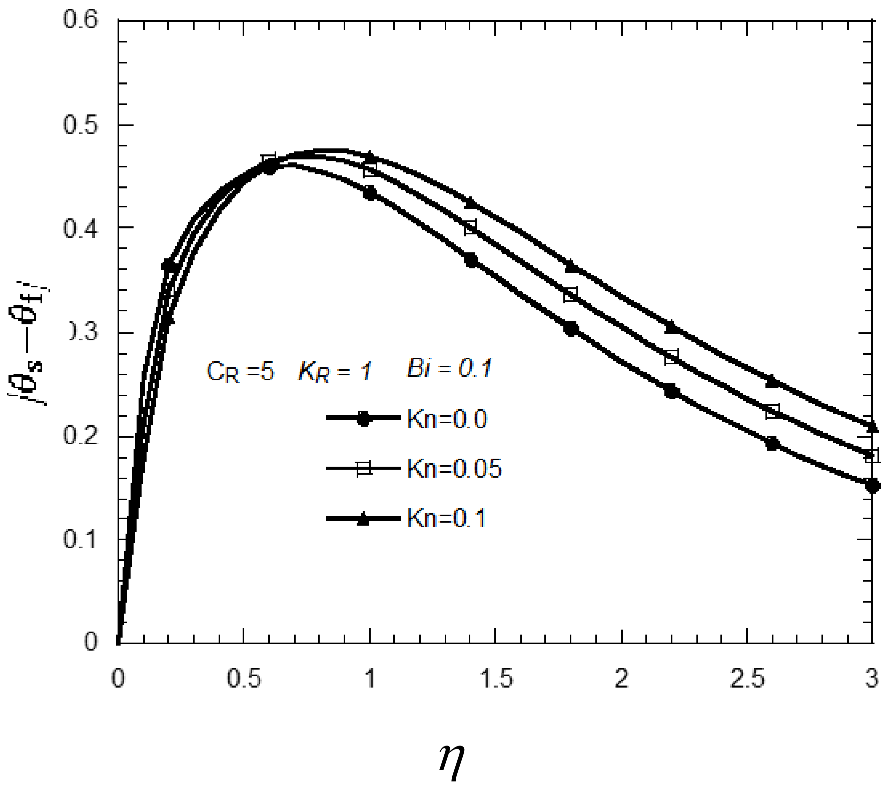

Fig. 5: Transient temperature difference at different Kn

4. CONCLUSIONS

The study investigates the validity of the TEA in unsteady free convection flow within porous channel, as characterized by a parabolic heat conduction model. The analysis reveals that the LTEA is particularly influenced by key parameters, including the volumetric Biot number, thermal conductivity ratio, Knudesn number, and total thermal capacity ratio.

Notably, the most significant impact on the LTEA is observed for the volumetric Biot number, thermal conductivity ratio, Knudsen number, and total capacity ratio. Specifically, large values of the Knudsen number and thermal capacity ratio, along with small values of the Biot number and thermal conductivity ratio, contribute to securing LTEA.

In summary, the study identifies the critical parameters that play a pivotal role in ensuring the LTEA in unsteady free convection flow within a porous channel governed by the wavy heat-conduction model.

NOMENCLATURE

Bi	Biot number, $\frac{h_v L^2}{k_f}$
c	heat capacity, $J/m^3\,K$
C_R	total thermal capacity ratio, $\frac{(1-\varepsilon)\rho_s c_s}{\varepsilon \rho_f c_f}$
K_R	thermal conductivity ratio, $\frac{k_s}{k_f}$
$2L$	slab thickness, m
t	time, s
t_o	reference time, s
T	temperature, K
T_w	wall temperature, K
y	axial coordinate, m
Y	$= \frac{y}{L}$

Greek symbols

η dimensionless time, $\frac{t}{t_o}$

ε porosity

Ω $= \frac{2-\sigma_T}{\sigma_T}\left(\frac{2\gamma}{\gamma+1}\right)$

γ specific heat ratio

σ_T thermal accommodation coefficient

σ_v tangential-momentum- accommodation coefficient.

θ dimensionless temperature, $\frac{T-T_\infty}{T_\infty}$

Subscripts

f fluid domain

s solid domain

w wall

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