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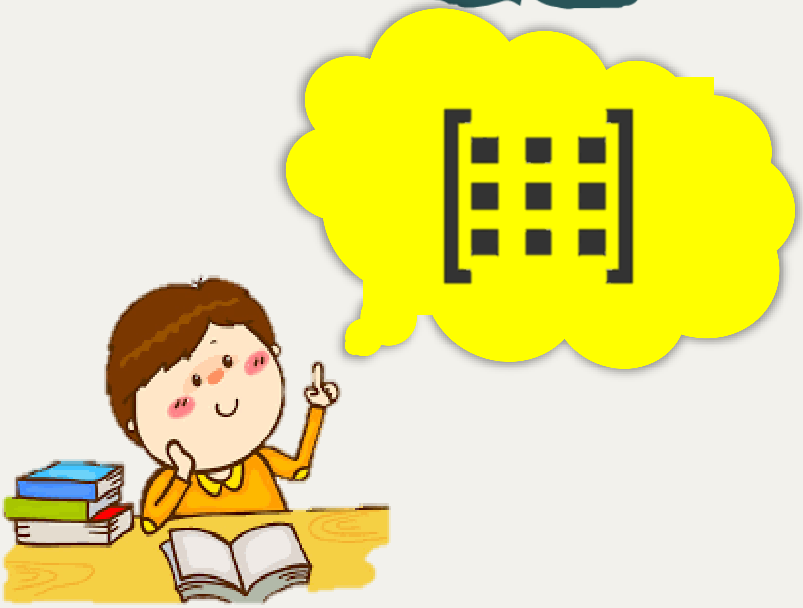
ENGINEERING MATHEMATICS I

MATRICES

VOLUME 2



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VOLUME 2: MATRICES

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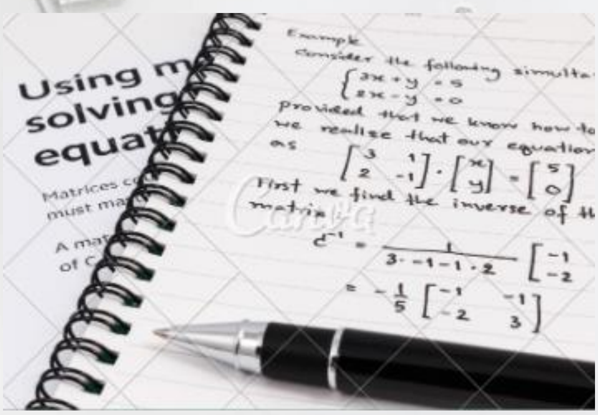

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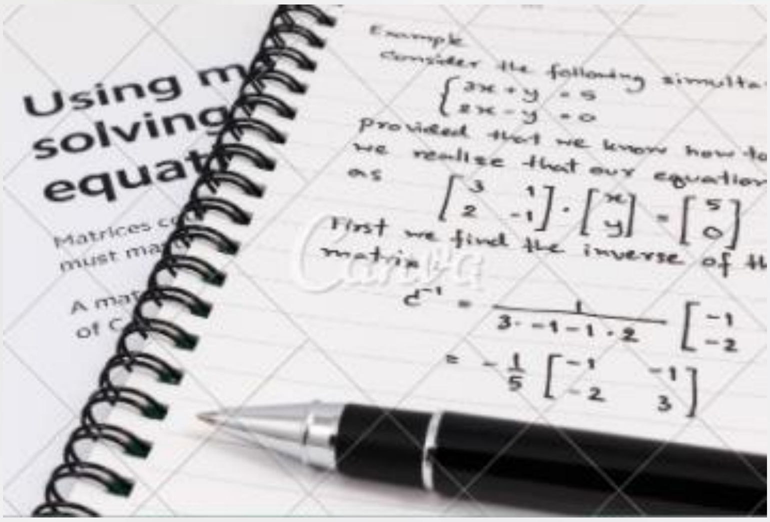
ABSTRACT

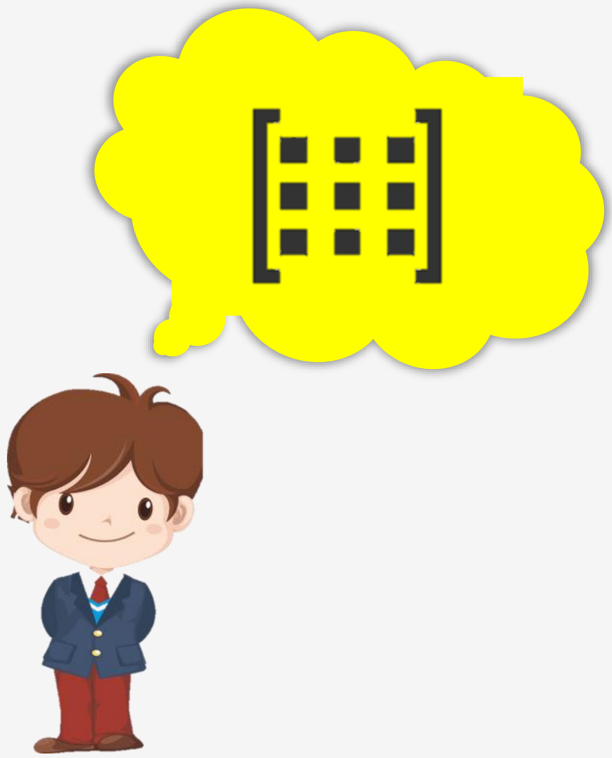
This e-book Engineering Mathematics 1 Volume 2 on Chapter 4 Matrices included the following topics:

- ❖ Understand of Matrices
- ❖ Operation of Matrices
- ❖ Simultaneous Linear Equation using Invers Matrices and Cramer's Rule.

Hopefully with this e-book can help enhance students in the solution of Matrices.



MATRICES



CONTENT

UNDERSTAND MATRIX

- **Matrices** are sets of numbers that are arranged in **rectangular forms**.
- It is a rectangular array of numbers.
- These numbers are arranged inside a round or square bracket.
- Look at the **examples** shown below.

$$\begin{pmatrix} 4 & 2 & -3 & 8 \end{pmatrix} \qquad \begin{pmatrix} 2 \\ 5 \\ -4 \end{pmatrix}$$

$$\begin{pmatrix} 6 & 1 \\ 9 & -6 \\ 1 & 4 \\ 2 & 3 \end{pmatrix} \qquad \begin{pmatrix} -3 & 9 & 7 \\ 5 & -2 & 3 \end{pmatrix}$$

- It is important to study the fundamentals of matrices first and get a good introduction on how to apply simple algebra operations on matrices.

- This can help in solving engineering problems. For example, you can use matrices to solve systems of linear simultaneous equations.

Example of Matrix 3 x 2

$$A = \begin{bmatrix} 2 & 9 \\ 1 & 0 \\ 7 & 3 \end{bmatrix}$$

FUNDAMENTAL OF MATRIX

NOTATION

- A matrix is usually shown by a capital letter (such as A, or B and so on).
- Small letter represent the elements of matrix.
- Each number inside a matrix is called an element of the matrix.
- These number are arranged in rows and columns.

ROWS

- Rows go left – right which that elements of matrix arranged horizontally.
- For example,

$$\begin{pmatrix} 1 & 4 & -4 \\ 0 & -3 & 4 \\ 0 & -2 & 6 \\ 2 & 5 & 2 \end{pmatrix}$$

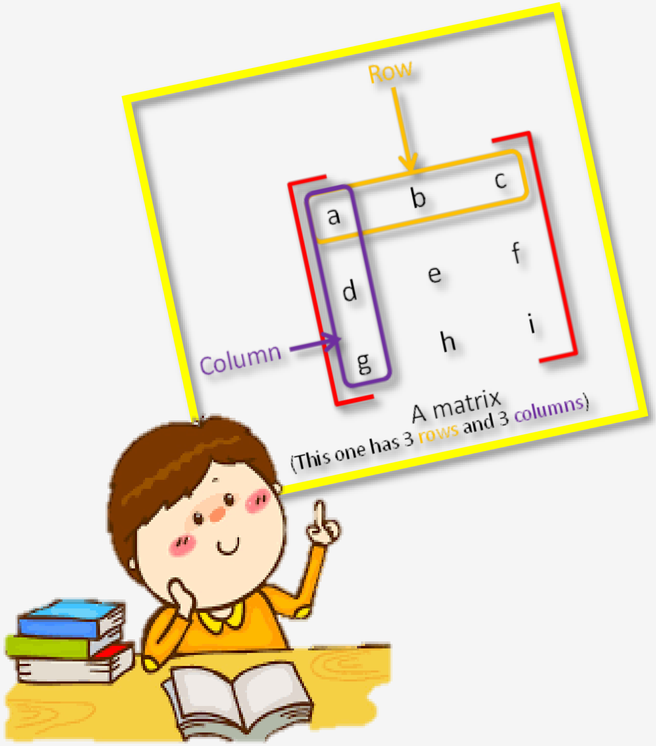
- The shaded region shows the second row of the matrix.

COLUMN

- Column go up – down which that elements in matrix arranged vertically.
- For example,

$$\begin{pmatrix} 1 & 4 & -4 \\ 0 & -3 & 4 \\ 0 & -2 & 6 \\ 2 & 5 & 2 \end{pmatrix}$$

- The shaded region shows the second column of the matrix.



SIZE OF MATRIX

1. The size of a matrix is the number of rows and columns that it has. If a matrix has 3 rows and 4 columns, then its size is 3 x 4.

2. Let's look at the following matrix:

$$A = \begin{pmatrix} 1 & 4 & -4 & 7 \\ 8 & 3 & 0 & -1 \\ -5 & 3 & 9 & 6 \end{pmatrix}$$

How many rows and columns do you see?

3. Size of matrix A = 3 x 4, which the **formula of size of matrix is A= row $\times$ column**

$$A = \begin{pmatrix} 1 & 4 & -4 & 7 \\ 8 & 3 & 0 & -1 \\ -5 & 3 & 9 & 6 \end{pmatrix}$$

Row 1

Row 2

Row 3

Column 1 Column 2 Column 3 Column 4

(3 x 4)

4. Elements of matrix A can be represented by the notation of a_{ij} where i = number of row and j = number of column

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$$

(3 x 2)

a_{22} is element of matrix A which located second row and second column

EXAMPLE

State the size of the following matrix.

$$\begin{pmatrix} 4 & 9 & -1 \\ 3 & 6 & 6 \\ -3 & 3 & 9 \\ 7 & -4 & 0 \end{pmatrix}$$

Solution:

- There are 4 rows and 3 columns. Therefore, the size of this matrix is 4 x 3.
- For a matrix A of 4 x 3, you can use the notation A_{4x3} to represent the matrix.

Exercise

1. State the size of each of the following matrices:

   a) $\begin{pmatrix} 2 & 3 \end{pmatrix}$

   b) $\begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix}$

   c) $\begin{pmatrix} 1 & 0 & 2 \\ 3 & 1 & 4 \\ 2 & 7 & 6 \end{pmatrix}$

   d) $\begin{pmatrix} 3 & 2 \\ 1 & 9 \\ 5 & 7 \end{pmatrix}$

2. Referring to matrix $B = \begin{pmatrix} 3 & 5 & -3 \\ 0 & 8 & 4 \\ -1 & 7 & 2 \end{pmatrix}$,

   state the element of:

   a) b_{23}

   b) b_{21}

   c) b_{31}

Scan for answer

http://tiny.cc/c4e1

TYPES OF A MATRIX

1 Square Matrices

- A square matrix is a matrix where the number of rows is equal to the number of columns.

- The following examples are square matrices:

i $\begin{pmatrix} 3 & 4 \\ -3 & -5 \end{pmatrix}$ (2×2)

ii $\begin{pmatrix} 3 & 5 & -3 \\ 0 & 8 & 4 \\ -1 & 7 & 2 \end{pmatrix}$ (3×3)

iii $\begin{pmatrix} 3 & -2 & 16 & 9 \\ -4 & 5 & 12 & -23 \\ 0 & 21 & 17 & 3 \\ 1 & -9 & 4 & 15 \end{pmatrix}$ (4×4)

2 Zero Matrices

- A matrix is known as a zero matrix if all of its elements are zero.

- The following examples are zero matrices:

i $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ (2×2)

ii $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ (3×3)

TYPES OF A MATRIX

3 Identity Matrix

- Identity matrix is a square matrix in which all the elements of the principal diagonal are ones and all other elements are zeros.

- An identity matrix is special because a matrix does not change when multiplying with an identity matrix. For example:

$$\rightarrow \boldsymbol{AI = A}$$
$$\rightarrow \boldsymbol{IA = A}$$

- The following examples are identity matrices:

i $\quad I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} (2 \times 2)$

ii $\quad I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} (3 \times 3)$

4 Diagonal Matrices

- If all the elements of a square matrix consist of zeroes except the diagonal, then this matrix is called a diagonal matrix.

- The following examples are diagonal matrices:

i $\quad \begin{pmatrix} 3 & 0 \\ 0 & -5 \end{pmatrix} (2 \times 2)$

ii $\quad \begin{pmatrix} 3 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 2 \end{pmatrix} (3 \times 3)$

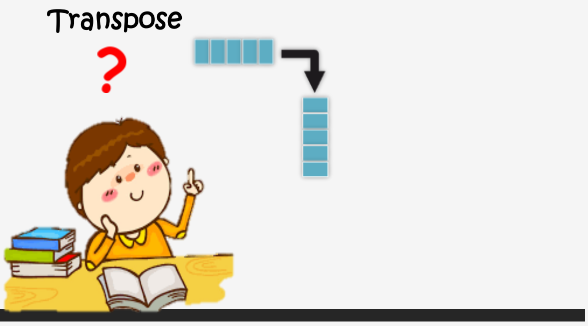
TRANSPOSE OF MATRIX

- When you interchange the rows of a matrix with its columns, you would have converted a matrix $A_{m\times n}$ to another matrix $A_{n\times m}$.
- In other words, a matrix of size $\boldsymbol{m}\times\boldsymbol{n}$ now is size $\boldsymbol{n}\times\boldsymbol{m}$.
- This new matrix is called the transpose of a matrix.
- The symbol for a transpose of a matrix A is A^T.
- **Let's look at the following examples:**

$$A = \begin{bmatrix} 2 & 9 \\ 1 & 0 \\ 7 & 3 \end{bmatrix} \qquad A^T = \begin{bmatrix} 2 & 1 & 7 \\ 9 & 0 & 3 \end{bmatrix}$$

For square matrix,

$$B = \begin{bmatrix} 7 & 6 & -2 \\ 1 & -4 & 0 \\ 8 & 3 & 9 \end{bmatrix} \qquad B^T = \begin{bmatrix} 7 & 1 & 8 \\ 6 & -4 & 3 \\ -2 & 0 & 9 \end{bmatrix}$$



SYMMETRIC MATRIX

- A symmetric matrix is a square matrix that is equal to its transpose.
- Matrix is symmetric if: $A = A^T$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 8 \end{bmatrix}$$

$$=$$

$$A^T = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 8 \end{bmatrix}$$

SYMMETRIC

EXAMPLE

1

Write the transpose of matrix $A=\begin{pmatrix}2 & 0 & 0\\ 2 & 1 & -6\\ 6 & 0 & -1\end{pmatrix}$.

Solution:

Transpose of matrix A, $A^T=\begin{pmatrix}2 & 2 & 6\\ 0 & 1 & 0\\ 0 & -6 & -1\end{pmatrix}$.

2

Determine whether the following matrices are symmetric or not.

a) $\begin{pmatrix}1 & 2 & 3\\ 2 & 3 & 6\end{pmatrix}$

b) $\begin{pmatrix}1 & 8 & 6\\ 8 & 2 & 0\\ 5 & 0 & 3\end{pmatrix}$

c) $\begin{pmatrix}1 & 2 & 3\\ 2 & 0 & 5\\ 3 & 5 & 4\end{pmatrix}$

d) $\begin{pmatrix}1 & 6 & 7\\ 6 & 2 & 3\\ 7 & 3 & 9\end{pmatrix}$

Solution:

a) Not symmetric

b) Not symmetric

c) Symmetric

d) Symmetric

Scan for answer

http://tiny.cc/c4transpose

Exercise

- State the types of the following matrices.

a) $A=\begin{pmatrix}3 & 0\\ 0 & 5\end{pmatrix}$

b) $B=\begin{pmatrix}2 & 4\\ -3 & 2\end{pmatrix}$

c) $C=\begin{pmatrix}0 & 0 & 1\\ 0 & 1 & 0\\ 1 & 0 & 0\end{pmatrix}$

d) $D=\begin{pmatrix}3\\ -5\\ -5\\ 3\end{pmatrix}$

e) $F=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$

- If $A=\begin{pmatrix}1 & 2\\ 4 & -3\\ 3 & 0\end{pmatrix}$, determine .

Scan for answer

http://tiny.cc/c4e2

MATRICES CALCULATOR

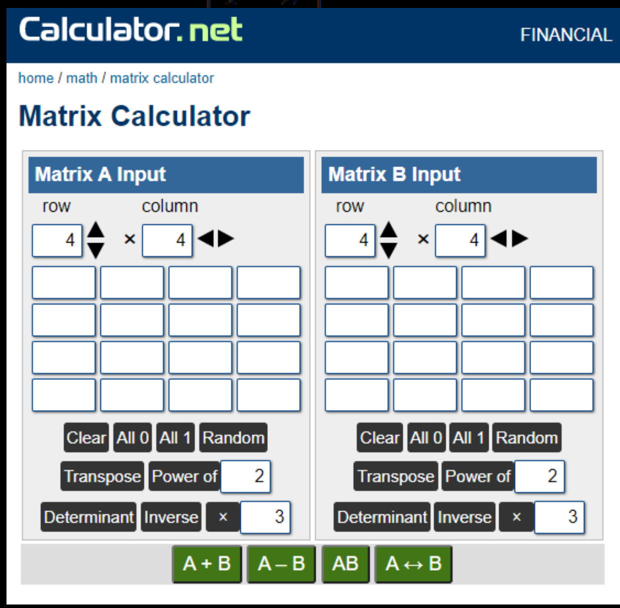
Here !

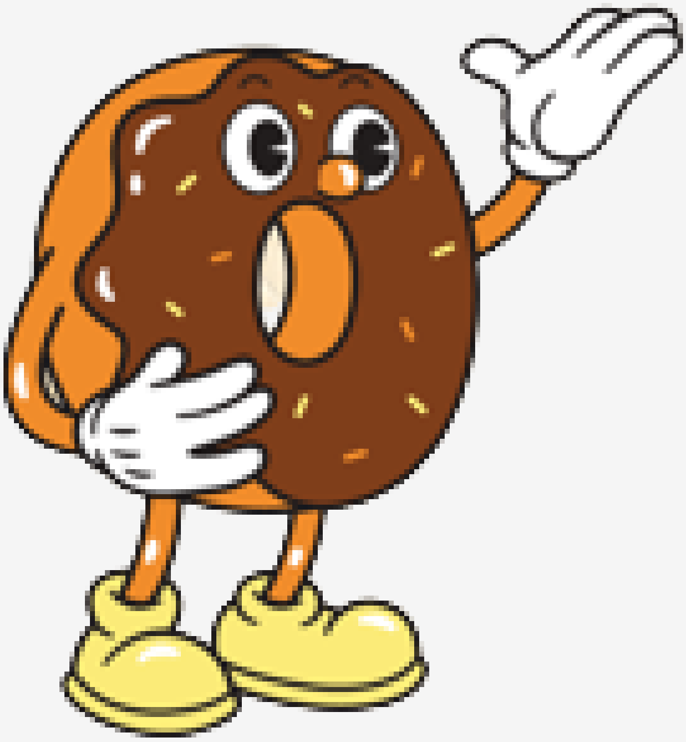
https://www.calculator.net/matrix-calculator.html

WOW!!!!

TRY THIS

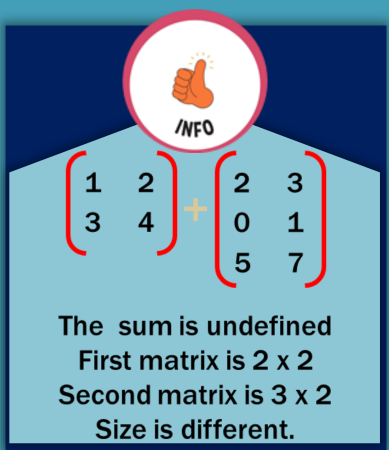
CALCULATOR.NET





OPERATION OF MATRIX

- The first algebra operation we are going to learn is how to add or subtract two matrices.
- Matrix addition and subtraction can only be performed on matrices that have *the same size*.
- The result of a matrix addition or subtraction is a new matrix that is of the same size.
- All we need to do is match the elements that are at the same position in their matrices.



ADDITION

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} a+e & b+f \\ c+g & d+h \end{pmatrix}$$

SUBTRACTION

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} - \begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} a-e & b-f \\ c-g & d-h \end{pmatrix}$$

EXAMPLE

Given that, $A = \begin{pmatrix} 3 & -1 & 4 & 10 \\ 1 & 3 & 3 & -2 \\ -7 & 1 & 5 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} -2 & 5 & -8 & 0 \\ -4 & 1 & -3 & 0 \\ 1 & -3 & 0 & -1 \end{pmatrix}$

Determine:

a) $A + B$

b) $A - B$

Solution:

a)

$$A + B = \begin{pmatrix} 3+(-2) & (-1)+5 & 4+(-8) & 10+0 \\ 1+(-4) & 3+1 & 3+(-3) & (-2)+0 \\ (-7)+1 & 1+(-3) & 5+0 & 3+(-1) \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 4 & -4 & 10 \\ -3 & 4 & 0 & -2 \\ -6 & -2 & 5 & 2 \end{pmatrix}$$

b)

$$A - B = \begin{pmatrix} 3-(-2) & (-1)-5 & 4-(-8) & 10-0 \\ 1-(-4) & 3-1 & 3-(-3) & (-2)-0 \\ (-7)-1 & 1-(-3) & 5-0 & 3-(-1) \end{pmatrix}$$

$$= \begin{pmatrix} 5 & -6 & 12 & 10 \\ 5 & 2 & 6 & -2 \\ -8 & 4 & 5 & 4 \end{pmatrix}$$

OPERATION OF MATRIX (addition & substraction)

More exercises

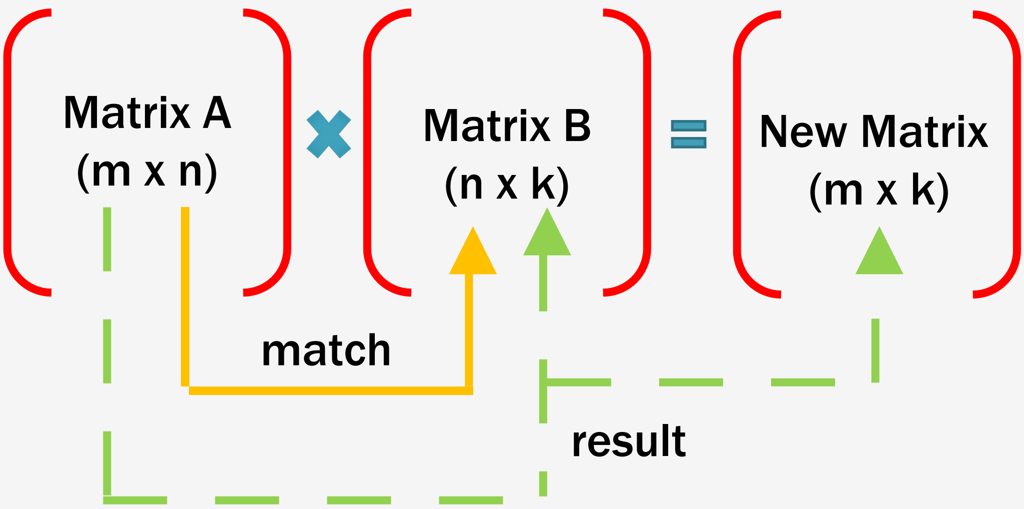
Here !

Link quizziz

https://bit.ly/3jIAQVI

MULTIPLICATION OF MATRIX

1. Matrix multiplication is a little bit more complicated.
2. In order to be able to multiply two matrices ***AB***, we have to ensure that the number of columns in matrix ***A*** is same as the number of rows in matrix ***B*** .
3. That means we can multiply matrix A_{mxn} with matrix B_{nxk} because matrix ***A*** has ***n*** columns and matrix ***B*** has ***n*** rows.
4. The result of the matrix multiplication is a new matrix that has ***m*** rows and ***k*** columns.
5. The multiplication process involves taking a row ***i*** of matrix ***A*** and matching it with a column ***j*** of matrix ***B*** .
6. The result becomes the element ***ij*** of the new matrix.



The multiplication is as follows:

If $A = \begin{pmatrix} a_{11} & a_{12} & \vdots & a_{1n} \\ a_{21} & a_{22} & \vdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & \cdots & \cdots & a_{mn} \end{pmatrix}$ and $B = \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1k} \\ b_{21} & b_{22} & \cdots & b_{2k} \\ \vdots & \vdots & \vdots & \vdots \\ b_{j1} & \cdots & \cdots & b_{jk} \end{pmatrix}$

Matrix $m \times n$ Matrix $j \times k$

Then, $C = \begin{pmatrix} c_{11} & c_{12} & \cdots & c_{1k} \\ c_{21} & c_{22} & \cdots & c_{2k} \\ \vdots & \vdots & \vdots & \vdots \\ c_{m1} & \cdots & \cdots & c_{mk} \end{pmatrix}$

Where

$$c_{11} = (a_{11})(b_{11}) + (a_{12})(b_{21}) + (a_{13})(b_{31}) + \ldots + (a_{1n})(b_{j1})$$

$$c_{12} = (a_{11})(b_{12}) + (a_{12})(b_{22}) + (a_{13})(b_{32}) + \ldots + (a_{1n})(b_{j2})$$

$$c_{21} = (a_{21})(b_{11}) + (a_{22})(b_{21}) + (a_{23})(b_{31}) + \ldots + (a_{2n})(b_{j1})$$

.

.

$$c_{mk} = (a_{m1})(b_{1k}) + (a_{m2})(b_{2k}) + (a_{m3})(b_{3k}) + \ldots + (a_{mn})(b_{jk})$$

Therefore, $C = AB$ with size $m \times k$.

REMEMBER!
Multiplication can only happen if *n* = *j*

EXAMPLE

Find the multiplication of $\mathbf{A}=\begin{pmatrix}5 & 2\\ 0 & 1\end{pmatrix}$ and $\mathbf{B}=\begin{pmatrix}-3 & 7\\ 1 & -1\end{pmatrix}$.

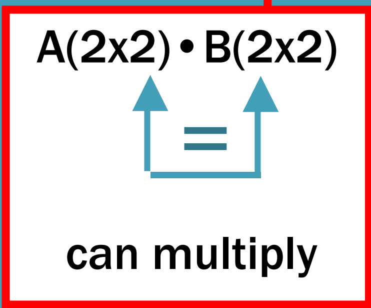
Solution:

$$AB=\begin{pmatrix}5 & 2\\ 0 & 1\end{pmatrix}\begin{pmatrix}-3 & 7\\ 1 & -1\end{pmatrix}$$

$$=\begin{pmatrix}5\cdot(-3)+2\cdot 1 & 5\cdot 7+2\cdot(-1)\\ 0\cdot(-3)+1\cdot 1 & 0\cdot 7+1\cdot(-1)\end{pmatrix}$$

$$=\begin{pmatrix}-15+2 & 35-2\\ 0+1 & 0-1\end{pmatrix}$$

$$=\begin{pmatrix}-13 & 33\\ 1 & -1\end{pmatrix}$$



EXAMPLE

Find the products of matrix A and B, given that,

$$A=\begin{pmatrix}1 & 0 & 1\\ 2 & -7 & 8\\ 0 & 1 & -4\\ 6 & 2 & 1\end{pmatrix} \text{ and } B=\begin{pmatrix}2 & 0\\ -3 & -1\\ 4 & 5\end{pmatrix}.$$

Solution:

The size of matrix A is 4×3 and the size of matrix B is 3×2. Therefore we can multiply them. The new matrix will be of size 4×2.

$$AB=\begin{pmatrix}1(2)+0(-3)+1(4) & 1(0)+0(-1)+1(5)\\ 2(2)+(-7)(-3)+8(4) & 2(0)+(-7)(-1)+8(5)\\ 0(2)+1(-3)+(-4)(4) & 0(0)+1(-1)+(-4)(5)\\ 6(2)+2(-3)+1(4) & 6(0)+2(-1)+1(5)\end{pmatrix}$$

$$=\begin{pmatrix}2+0+4 & 0+0+5\\ 4+21+32 & 0+7+40\\ 0-3-16 & 0-1-20\\ 12-6+4 & 0-2+5\end{pmatrix}$$

$$=\begin{pmatrix}6 & 5\\ 57 & 47\\ -19 & -21\\ 10 & 3\end{pmatrix}$$

Note: We cannot find the product of BA because the number of colum[ns in matrix B is not] the same as the number of rows i

AB ≠ BA

EXERCISE

1. Based on the following matrices,

$A=\begin{pmatrix}3 & 7\\ 9 & 5\end{pmatrix}$, $B=\begin{pmatrix}3 & 4\\ -3 & -5\end{pmatrix}$, $C=\begin{pmatrix}2\\ 6\end{pmatrix}$

and $D=\begin{pmatrix}4 & -2\\ 5 & 7\end{pmatrix}$. Determine:

a) $A+B$

b) $A-C$

c) $D+(B-A)$

d) $B+C$

2. Given that $A=\begin{pmatrix}-2 & -1\\ 3 & 1\end{pmatrix}$ and

$B=\begin{pmatrix}0 & -1\\ 2 & 3\end{pmatrix}$, find:

a) AB

b) BA

3. If $M=\begin{pmatrix}3 & 5 & -3\\ 0 & 2 & 4\\ -1 & 3 & 2\end{pmatrix}$ and $N=\begin{pmatrix}2\\ 1\\ 5\end{pmatrix}$,

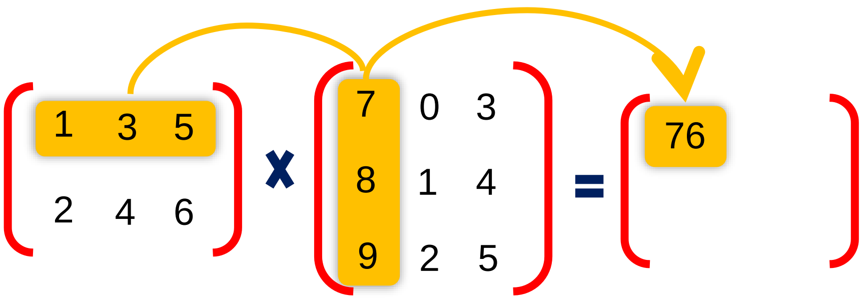
find the product of ***MN*** .

TIPS

Scan for answer

http://tiny.cc/c4e3

How to multiply



$$(1\times 7)+(3\times 8\ )+(5\times 9)$$
$$=7+24+45$$
$$=76$$

TIPS

Calculating multiplication matrices using calculator
Casio
fx-570MS

https://youtu.be/8ilgdQjEG6s

OPERATION OF MATRIX (multiplication)

More exercises

Here !

Link quizziz

https://bit.ly/3fMZH9r

DETERMINANT OF MATRIX

- Determinant is a unique number that can be determined from a square matrix.
- It is used to represent the real-value of the matrix which can be used to solve simple algebra problems later on.
- The symbol for the determinant of matrix *A* is det (*A*) or |*A*| .

Determinant of the matrix 2 x 2

For a matrix of size 2 x 2, the method to find the determinant is:

Let's say, $\boldsymbol{A} = \begin{pmatrix} \boldsymbol{a} & \boldsymbol{b} \\ \boldsymbol{c} & \boldsymbol{d} \end{pmatrix}$

Then, $\det(A) = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$

$= ad - bc$

Determinant of The Matrix 3 x 3

For a matrix of size 3 x 3, the steps to find the determinant are:

- Set polarity
- Choose any column or any row

If $\boldsymbol{A} = \begin{pmatrix} \boldsymbol{a}_{11} & \boldsymbol{a}_{12} & \boldsymbol{a}_{13} \\ \boldsymbol{a}_{21} & \boldsymbol{a}_{22} & \boldsymbol{a}_{23} \\ \boldsymbol{a}_{31} & \boldsymbol{a}_{32} & \boldsymbol{a}_{33} \end{pmatrix}$

Then,

$$|A| = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{21}\begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} + a_{31}\begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}$$

$$= a_{11}(a_{22}a_{33} - a_{23}a_{32}) - a_{21}(a_{12}a_{33} - a_{13}a_{32}) + a_{31}(a_{12}a_{23} - a_{13}a_{22})$$

EXAMPLE

1. Determine determinant, |A| for each of the following:

a) $\boldsymbol{A} = \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}$

b) $\boldsymbol{A} = \begin{pmatrix} 4 & 2 \\ 5 & 9 \end{pmatrix}$

Solution:

a) $|\boldsymbol{A}| = 5(8) - 6(7)$

$= 40 - 42$

$= -2$

b) $|\boldsymbol{A}| = 4(9) - 2(5)$

$= 36 - 10$

$= 26$

EXAMPLE

1. Determine the determinant of matrix

$$A = \begin{pmatrix} 1 & 3 & 2 \\ 4 & 3 & 0 \\ 2 & 1 & 2 \end{pmatrix}$$

Solution:

Choose row number 1

$$\text{Det }(A) = 1\begin{vmatrix} 3 & 0 \\ 1 & 2 \end{vmatrix} - 3\begin{vmatrix} 4 & 0 \\ 2 & 2 \end{vmatrix} + 2\begin{vmatrix} 4 & 3 \\ 2 & 1 \end{vmatrix}$$

$= 1[3(2) - 0(1)] - 3[4(2) - 0(2)] + 2[4(1) - 3(2)]$

$= 6 - 24 + (-4)$

$= -22$

2. If $\boldsymbol{A} = \begin{pmatrix} 1 & 3 & 5 \\ 7 & 9 & 2 \\ 4 & 6 & 8 \end{pmatrix}$, determine $|\boldsymbol{A}|$.

Solution:

Choose row number 1

$$|\boldsymbol{A}| = 1\begin{vmatrix} 9 & 2 \\ 6 & 8 \end{vmatrix} - 3\begin{vmatrix} 7 & 2 \\ 4 & 8 \end{vmatrix} + 5\begin{vmatrix} 7 & 9 \\ 4 & 6 \end{vmatrix}$$

$= 1[9(8) - 2(6)] - 3[7(8) - 2(4)] + 5[7(6) - 9(4)]$

$= 60 - 144 + 30$

$= -54$

To set polarity, it is easier if we choose row or column consists of element "0".

TIPS

EXAMPLE

If $A = \begin{pmatrix} 2 & 8 & 6 \\ 0 & 10 & -5 \\ -3 & -6 & 9 \end{pmatrix}$, determine $|A|$.

Solution:

Choose row number 2 because consist element 0

$$|A| = -0\begin{vmatrix} 6 & 9 \\ -6 & 9 \end{vmatrix} + 10\begin{vmatrix} 2 & 6 \\ -3 & 9 \end{vmatrix} - (-5)\begin{vmatrix} 2 & 8 \\ -3 & -6 \end{vmatrix}$$

$$= 0 + 10\big(18 - (-18)\big) + 5\big(-12 - (-24)\big)$$

$$= 0 + 360 + 60$$

$$= 420$$

- ❑ Try choose other row/column!

Choose column number 1 because consist element 0

$$|A| = 2\begin{vmatrix} 10 & -5 \\ -6 & 9 \end{vmatrix} - 0\begin{vmatrix} 8 & 6 \\ -6 & 9 \end{vmatrix} + (-3)\begin{vmatrix} 8 & 6 \\ 10 & -5 \end{vmatrix}$$

$$= 2(90 - 30) - 0 - 3\big(-40 - (60)\big)$$

$$= 120 - 0 + 300$$

$$= 420$$

If we choose whatever row/column, it gives same answer.

Scan for answer

http://tiny.cc/c4e4

INFO

To set polarity, we can choose any row/column but it give different formula for determinant.

For odd number row/column such as column number 1
Formula:

$$|A| = a_{11}[\quad] - a_{21}[\quad] + a_{31}[\quad]$$

For even number row/column such as row 2 and column 2.
Formula:

$$|A| = -a_{21}[\quad] + a_{22}[\quad] - a_{23}[\quad]$$

Therefore, the different is positive and negative sign.

EXERCISE

1) Determine the determinant for the following matrices.

a) $\begin{pmatrix} 6 & 13 \\ 4 & 12 \end{pmatrix}$

b) $\begin{pmatrix} 1 & 0 & 3 \\ 1 & 2 & 4 \\ 7 & -2 & 1 \end{pmatrix}$

c) $\begin{pmatrix} -3 & 0 \\ 5 & 3 \end{pmatrix}$

2) If $A = \begin{pmatrix} 2 & 1 & 6 \\ -2 & 3 & 4 \\ 4 & 2 & 7 \end{pmatrix}$, $B = \begin{pmatrix} 4 & 2 \\ -6 & 3 \end{pmatrix}$ and $C = \begin{pmatrix} 1 & -4 & 2 \\ 0 & 4 & 6 \\ 2 & 5 & 0 \end{pmatrix}$,

determine:

a) $|A|$

b) $|B|$

c) $|C|$

INVERSE MATRIX

- The inverse of a square matrix is its complement because if you multiply a matrix by its inverse, the product is an identity matrix.
- In other words, if A is a square matrix and A^{-1} its inverse, then:

$$AA^{-1} = I$$

where I is an identity matrix

Inverse of a square matrix (2x2)

For $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, the inverse can be found using this formula:

$$A^{-1} = \frac{1}{|A|}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$= \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

EXAMPLE

Find inverse matrix for matrix $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}^{-1} = \frac{1}{1(4)-2(3)}\begin{pmatrix} 4 & -2 \\ -3 & 1 \end{pmatrix}$$

$$= \frac{1}{-2}\begin{pmatrix} 4 & -2 \\ -3 & 1 \end{pmatrix}$$

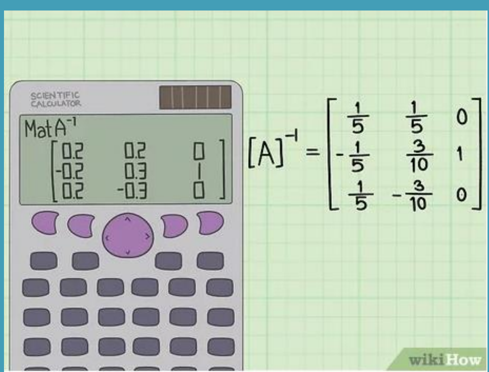
$$= \begin{pmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{pmatrix}$$

Inverse of a square matrix (3x3)

The formula to find A^{-1} for a (3x3) matrix is

$$A^{-1} = \frac{1}{|A|}\text{Adj}(A)$$

Step to find A^{-1}



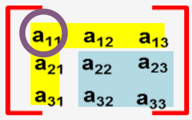
MINOR OF MATRIX

- The minor of a matrix is a new matrix where all the elements are determinants.
- Each determinant is calculated by removing a row and a column from the original matrix.
- For example, in order to determine the element at position ***ij***, you will have to remove row ***i*** and column ***j*** from the matrix.
- Next, calculate the determinant of what is left.

$$If\ A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

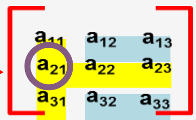
$$then, Minor\ A = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix}$$

$$M_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$$



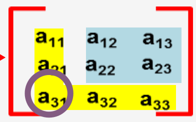
$$M_{11} = ((a_{22} \times a_{33}) - (a_{23} \times a_{32}))$$

$$M_{21} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$$



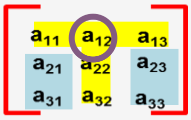
$$M_{21} = ((a_{12} \times a_{33}) - (a_{13} \times a_{32}))$$

$$M_{31} = \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}$$



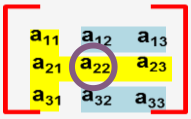
$$M_{31} = ((a_{12} \times a_{23}) - (a_{13} \times a_{22}))$$

$$M_{12} = \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$$



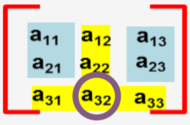
$$M_{12} = ((a_{21} \times a_{33}) - (a_{23} \times a_{31}))$$

$$M_{22} = \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix}$$



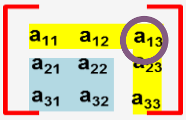
$$M_{22} = ((a_{12} \times a_{33}) - (a_{13} \times a_{32}))$$

$$M_{32} = \begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix}$$



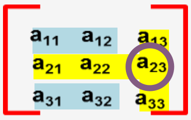
$$M_{32} = ((a_{11} \times a_{23}) - (a_{13} \times a_{21}))$$

$$M_{13} = \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$



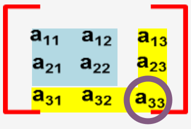
$$M_{13} = ((a_{21} \times a_{32}) - (a_{22} \times a_{31}))$$

$$M_{23} = \begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix}$$



$$M_{23} = ((a_{11} \times a_{32}) - (a_{12} \times a_{31}))$$

$$M_{33} = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$



$$M_{33} = ((a_{11} \times a_{22}) - (a_{12} \times a_{21}))$$

EXAMPLE

If $A = \begin{pmatrix} 1 & 3 & 5 \\ 7 & 9 & 2 \\ 4 & 6 & 8 \end{pmatrix}$, determine minor A.

Solution:

The elements are:

$$M_{11} = \begin{vmatrix} 9 & 2 \\ 6 & 8 \end{vmatrix} = 9(8) - 2(6) = 60$$

$$M_{12} = \begin{vmatrix} 7 & 2 \\ 4 & 8 \end{vmatrix} = 7(8) - 2(4) = 48$$

$$M_{13} = \begin{vmatrix} 7 & 9 \\ 4 & 6 \end{vmatrix} = 7(6) - 9(4) = 6$$

$$M_{21} = \begin{vmatrix} 3 & 5 \\ 6 & 8 \end{vmatrix} = 3(8) - 5(6) = -6$$

$$M_{22} = \begin{vmatrix} 1 & 5 \\ 4 & 8 \end{vmatrix} = 1(8) - 5(4) = -12$$

$$M_{23} = \begin{vmatrix} 1 & 3 \\ 4 & 6 \end{vmatrix} = 1(6) - 3(4) = -6$$

$$M_{31} = \begin{vmatrix} 3 & 5 \\ 9 & 2 \end{vmatrix} = 3(2) - 5(9) = -39$$

$$M_{32} = \begin{vmatrix} 1 & 5 \\ 7 & 2 \end{vmatrix} = 1(2) - 5(7) = -33$$

$$M_{33} = \begin{vmatrix} 1 & 3 \\ 7 & 9 \end{vmatrix} = 1(9) - 3(7) = -12$$

Therefore, $\text{Minor } A = \begin{pmatrix} 60 & 48 & 6 \\ -6 & -12 & -6 \\ -39 & -33 & -12 \end{pmatrix}$

EXAMPLE

If $P = \begin{pmatrix} 1 & 3 & 2 \\ 4 & 3 & 0 \\ 2 & 1 & 2 \end{pmatrix}$, determine minor P .

Solution:

The elements are:

$$M_{11} = \begin{vmatrix} 3 & 0 \\ 1 & 2 \end{vmatrix} = 3(2) - 0(1) = 6$$

$$M_{12} = \begin{vmatrix} 4 & 0 \\ 2 & 2 \end{vmatrix} = 4(2) - 0(2) = 8$$

$$M_{13} = \begin{vmatrix} 4 & 3 \\ 2 & 1 \end{vmatrix} = 4(1) - 3(2) = -2$$

$$M_{21} = \begin{vmatrix} 3 & 2 \\ 1 & 2 \end{vmatrix} = 3(2) - 2(1) = 4$$

$$M_{22} = \begin{vmatrix} 1 & 2 \\ 2 & 2 \end{vmatrix} = 1(2) - 2(2) = -2$$

$$M_{23} = \begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix} = 1(1) - 3(2) = -5$$

$$M_{31} = \begin{vmatrix} 3 & 2 \\ 3 & 0 \end{vmatrix} = 3(0) - 2(3) = -6$$

$$M_{32} = \begin{vmatrix} 1 & 2 \\ 4 & 0 \end{vmatrix} = 1(0) - 2(4) = -8$$

$$M_{33} = \begin{vmatrix} 1 & 3 \\ 4 & 3 \end{vmatrix} = 1(3) - 3(4) = -9$$

Therefore, $\text{Minor } P = \begin{pmatrix} 6 & 8 & -2 \\ 4 & -2 & -5 \\ -6 & -8 & -9 \end{pmatrix}$

EXERCISE

Determine the minor of the following matrices.

1) $A = \begin{pmatrix} 2 & 1 & 6 \\ -2 & 3 & 4 \\ 4 & 2 & 7 \end{pmatrix}$

2) $B = \begin{pmatrix} 1 & 5 & 3 \\ 1 & 0 & 4 \\ 7 & -2 & 1 \end{pmatrix}$

3) $C = \begin{pmatrix} 1 & -4 & 2 \\ 0 & 4 & 6 \\ 2 & 5 & 0 \end{pmatrix}$

Scan for answer

http://tiny.cc/c4e5

Calculating determinant using calculator Casio fx-570MS

https://youtu.be/4QzhnDyaEt0

COFACTOR OF MATRIX

- Once you have found the minor of a matrix, you can easily determine the cofactor of the matrix.
- All the hard work is already done when you determine the minor of a matrix.
- All you need to do now is multiply each element of the matrix minor with a factor, and the cofactor is done.

Let's look at the following example:

If $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$ and minor $A = \begin{pmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{pmatrix}$

Where, $K_{ij} = (-1)^{i+j} M_{ij}$. Then,

cofactor of a matrix $A = \begin{pmatrix} (-1)^2 M_{11} & (-1)^3 M_{12} & (-1)^4 M_{13} \\ (-1)^3 M_{21} & (-1)^4 M_{22} & (-1)^5 M_{23} \\ (-1)^4 M_{31} & (-1)^5 M_{32} & (-1)^6 M_{33} \end{pmatrix}$

EXAMPLE

If $A = \begin{pmatrix} 1 & 3 & 5 \\ 7 & 9 & 2 \\ 4 & 6 & 8 \end{pmatrix}$, determine the cofactor of a matrix A.

Solution:

First, find the minor of a matrix A.

From previous example, Minor $A = \begin{pmatrix} 60 & 48 & 6 \\ -6 & -12 & -6 \\ -39 & -33 & -12 \end{pmatrix}$

Next, multiply each element by its factor $(-1)^{i+j}$.

Therefore, the cofactor of a matrix A:

$$\text{Co-factor } A = \begin{pmatrix} (-1)^2 60 & (-1)^3 48 & (-1)^4 6 \\ (-1)^3 -6 & (-1)^4 -12 & (-1)^5 -6 \\ (-1)^4 -30 & (-1)^5 -33 & (-1)^6 -12 \end{pmatrix}$$

$$= \begin{pmatrix} 60 & -48 & 6 \\ 6 & -12 & 6 \\ -39 & 33 & -12 \end{pmatrix}$$

EXAMPLE

If $P = \begin{pmatrix} 1 & 3 & 2 \\ 4 & 3 & 0 \\ 2 & 1 & 2 \end{pmatrix}$, determine the cofactor of a matrix P.

Solution:

First, find the minor of a matrix P.

From previous example, Minor $P = \begin{pmatrix} 6 & 8 & -2 \\ 4 & -2 & -5 \\ -6 & -8 & -9 \end{pmatrix}$

Next, multiply each element by its factor $(-1)^{i+j}$.

Therefore, the cofactor of a matrix:

$$\text{Co-factor } P = \begin{pmatrix} (-1)^2 6 & (-1)^3 8 & (-1)^4 -2 \\ (-1)^3 4 & (-1)^4 -2 & (-1)^5 -5 \\ (-1)^4 -6 & (-1)^5 -8 & (-1)^6 -9 \end{pmatrix}$$

$$= \begin{pmatrix} 6 & -8 & -2 \\ -4 & -2 & 5 \\ -6 & 8 & -9 \end{pmatrix}$$

Quick way to solve cofactor

$$\text{Minor } A = \begin{bmatrix} 60 & 48 & 6 \\ -6 & -12 & -6 \\ -39 & -33 & -12 \end{bmatrix}$$

$$\downarrow$$

Multiply by positive/negative

$$\begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$$

$$\downarrow$$

$$\text{Cofactor } A = \begin{bmatrix} 60 & -48 & 6 \\ 6 & -12 & 6 \\ -39 & 33 & -12 \end{bmatrix}$$

EXERCISE

Find the cofactor for the following matrices.

1) $A = \begin{pmatrix} 2 & 1 & 6 \\ -2 & 3 & 4 \\ 4 & 2 & 7 \end{pmatrix}$

2) $B = \begin{pmatrix} 1 & 5 & 3 \\ 1 & 0 & 4 \\ 7 & -2 & 1 \end{pmatrix}$

3) $C = \begin{pmatrix} 1 & -4 & 2 \\ 0 & 4 & 6 \\ 2 & 5 & 0 \end{pmatrix}$

Scan for answer

http://tiny.cc/c4e6

MATRIX ADJOINT

- For a square matrix *A* with $n \times n$, you can find the adjoint of matrix *A* when transposing the cofactors of matrix *A*.
- In this case, for matrix *A*, adjoint of matrix A , written as $Adj(A) = K^T$ where *K* is the cofactor for matrix *A* .

If $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$, Minor $A = \begin{pmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{pmatrix}$

and cofactor matrix $A = \begin{pmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{pmatrix}$

Then adjoint matrix *A* ,

$$\text{Adj } (A) = \begin{pmatrix} K_{11} & K_{21} & K_{31} \\ K_{12} & K_{22} & K_{32} \\ K_{13} & K_{23} & K_{33} \end{pmatrix}$$

So formula of adjoint : $\text{Adj}(A) = K^T$

EXAMPLE

If $A=\begin{pmatrix}1 & 3 & 5\\ 7 & 9 & 2\\ 4 & 6 & 8\end{pmatrix}$, determine the adjoint of the matrix A .

Solution:

You will find minor and cofactor for matrix ***A***:

From **previous example**, minor $A=\begin{pmatrix}60 & 48 & 6\\ -6 & -12 & -6\\ -39 & -33 & -12\end{pmatrix}$

From **previous example**, cofactor $A=\begin{pmatrix}60 & -48 & 6\\ 6 & -12 & 6\\ -39 & 33 & -12\end{pmatrix}$

Then, adjoint of matrix A, $\operatorname{Adj}(A)=\begin{pmatrix}60 & 6 & -39\\ -48 & -12 & 33\\ 6 & 6 & -12\end{pmatrix}$

EXAMPLE

If $P=\begin{pmatrix}1 & 3 & 2\\ 4 & 3 & 0\\ 2 & 1 & 2\end{pmatrix}$, determine the adjoint for matrix P .

Solution:

You will find minor and cofactor for matrix ***P***:

From **previous example**, minor $P=\begin{pmatrix}6 & 8 & -2\\ 4 & -2 & -5\\ -6 & -7 & -9\end{pmatrix}$.

From **previous example**, cofactor $P=\begin{pmatrix}6 & -8 & -2\\ -4 & -2 & 5\\ -6 & 7 & -9\end{pmatrix}$.

Then, adjoint of matrix ***P***, $\operatorname{Adj}(P)=\begin{pmatrix}6 & -4 & -6\\ -8 & -2 & 7\\ -2 & 5 & -9\end{pmatrix}$.

EXERCISE

Determine the adjoint for the following matrices

1) $A = \begin{pmatrix} 2 & 1 & 6 \\ -2 & 3 & 4 \\ 4 & 2 & 7 \end{pmatrix}$

2) $B = \begin{pmatrix} 1 & 5 & 3 \\ 1 & 0 & 4 \\ 7 & -2 & 1 \end{pmatrix}$

3) $C = \begin{pmatrix} 1 & -4 & 2 \\ 0 & 4 & 6 \\ 2 & 5 & 0 \end{pmatrix}$

Scan for answer

http://tiny.cc/c4e7

INVERSE OF MATRIX (3 X 3)

The formula to find A^{-1} for a (3x3) matrix is

$$A^{-1} = \frac{1}{|A|}\text{Adj}(A)$$

EXAMPLE

If $A = \begin{pmatrix} 1 & 3 & 5 \\ 7 & 9 & 2 \\ 4 & 6 & 8 \end{pmatrix}$, find A^{-1}

Solution:

Step 1 : Find determinant

The determinant of A,

$$|A| = 1\begin{vmatrix} 9 & 2 \\ 6 & 8 \end{vmatrix} - 3\begin{vmatrix} 7 & 2 \\ 4 & 8 \end{vmatrix} + 5\begin{vmatrix} 7 & 9 \\ 4 & 6 \end{vmatrix}$$

$$= 60 - 144 + 30$$

$$= -54$$

Step 2 : Minor Matrix

From previous example Minor A ,

$$\text{Minor } A = \begin{pmatrix} 60 & 48 & 6 \\ -6 & -12 & -6 \\ -39 & -33 & -12 \end{pmatrix}$$

Step 3 : Cofactor

From previous example Cofactor A ,

$$= \begin{pmatrix} 60 & -48 & 6 \\ 6 & -12 & 6 \\ -39 & 33 & -12 \end{pmatrix}$$

Step 4 : Adjoint

From previous example adjoint A ,

$$\text{Adj}(A) = \begin{pmatrix} 60 & 6 & -39 \\ -48 & -12 & 33 \\ 6 & 6 & -12 \end{pmatrix}$$

Therefore, $A^{-1} = \frac{1}{|A|}\text{Adj}(A)$

$$= \frac{1}{-54}\begin{pmatrix} 60 & 6 & -39 \\ -48 & -12 & 33 \\ 6 & 6 & -12 \end{pmatrix}$$

$$= \begin{pmatrix} -\frac{60}{54} & \frac{6}{54} & -\frac{39}{54} \\ -\frac{48}{54} & -\frac{12}{54} & \frac{33}{54} \\ \frac{6}{54} & \frac{6}{54} & -\frac{12}{54} \end{pmatrix}$$

EXAMPLE

If $P = \begin{pmatrix} 1 & 3 & 2 \\ 4 & 3 & 0 \\ 2 & 1 & 2 \end{pmatrix}$, find P^{-1}.

Solution:

The determinant of P,

$$|P| = 1\begin{vmatrix} 3 & 0 \\ 1 & 2 \end{vmatrix} - 3\begin{vmatrix} 4 & 0 \\ 2 & 2 \end{vmatrix} + 2\begin{vmatrix} 4 & 3 \\ 2 & 1 \end{vmatrix}$$

$$= 6 - 24 - 4$$

$$= -22$$

From **previous example**, the adjoint of P:

$$\text{Adj}(P) = \begin{pmatrix} 6 & -4 & -6 \\ -8 & -2 & 7 \\ -2 & 5 & -9 \end{pmatrix}$$

Therefore, $P^{-1} = \frac{1}{|P|}\text{Adj}(P)$

$$= \frac{1}{-22}\begin{pmatrix} 6 & -4 & -6 \\ -8 & -2 & 7 \\ -2 & 5 & -9 \end{pmatrix}$$

$$= \begin{pmatrix} -\frac{6}{22} & \frac{4}{11} & \frac{3}{11} \\ \frac{4}{11} & \frac{1}{11} & -\frac{7}{22} \\ \frac{1}{11} & -\frac{5}{22} & \frac{9}{22} \end{pmatrix}$$

EXERCISE

Find the inverse of the following matrices.

1) $\begin{pmatrix} 4 & 5 \\ -9 & 6 \end{pmatrix}$

2) $\begin{pmatrix} 0 & -2 \\ -1 & -7 \end{pmatrix}$

3) $\begin{pmatrix} 2 & 10 \\ 11 & -8 \end{pmatrix}$

4) $A = \begin{pmatrix} 2 & 1 & 6 \\ -2 & 3 & 4 \\ 4 & 2 & 7 \end{pmatrix}$

5) $B = \begin{pmatrix} 1 & 5 & 3 \\ 1 & 0 & 4 \\ 7 & -2 & 1 \end{pmatrix}$

6) $C = \begin{pmatrix} 1 & -4 & 2 \\ 0 & 4 & 6 \\ 2 & 5 & 0 \end{pmatrix}$

Scan for answer

http://tiny.cc/c4e8

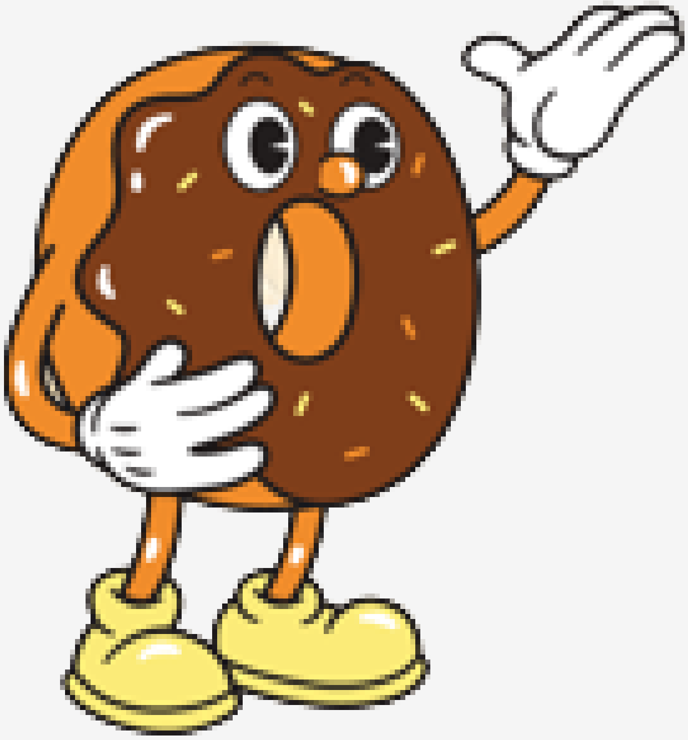
Calculating invers using calculator
Casio
fx-570MS

https://youtu.be/svR_Dp37-LI

MATRICES CALCULATOR Here !

https://matrixcalc.org/en/

WOW!!!!
TRY THIS



Matrix calculator

- Matrix calculator ✓
- Solving systems of linear equations
- Determinant calculator
- Eigenvalues calculator
- Wikipedia:Matrices

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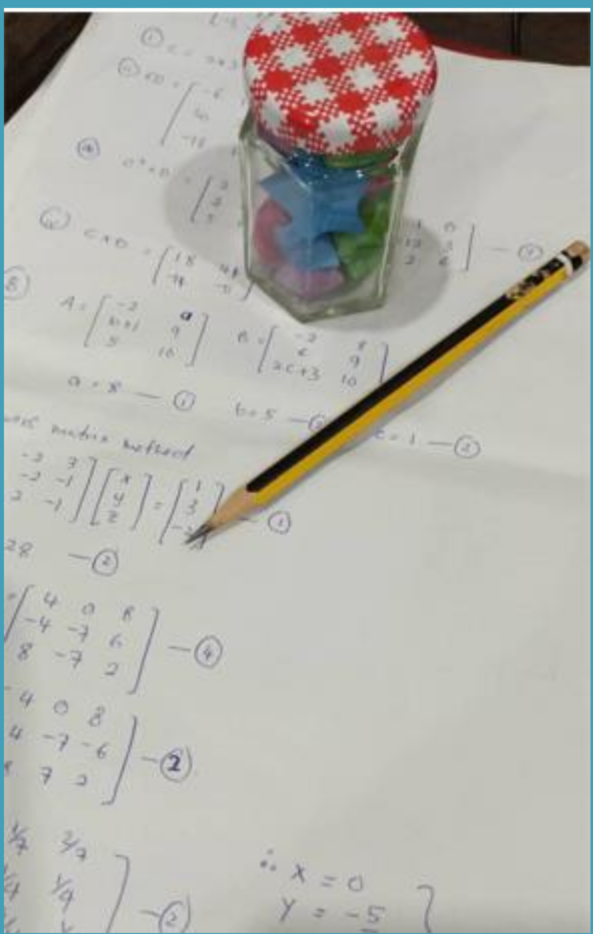
SIMULTANEOUS LINEAR EQUATION – INVERSE MATRIX



Consider one set of three equations:

$$a_{11}x + a_{12}y + a_{13}z = b_1$$
$$a_{21}x + a_{22}y + a_{23}z = b_2$$
$$a_{31}x + a_{32}y + a_{33}z = b_3$$

Then, write the equations in a matrix form:

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

If we take the matrix $\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$ as matrix $\boldsymbol{A}$

matrix $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ as $\boldsymbol{c}$ and matrix $\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ as $\boldsymbol{b}$,

we can write the above matrix form as $\boldsymbol{Ac} = \boldsymbol{b}$

and multiply both sides of the equation with $\boldsymbol{A}^{-1}$.

$$\boldsymbol{A}^{-1}\boldsymbol{Ac} = \boldsymbol{A}^{-1}\boldsymbol{b}$$

$$\boldsymbol{c} = \boldsymbol{A}^{-1}\boldsymbol{b}$$

To obtain $\boldsymbol{c} = \begin{pmatrix} \boldsymbol{x} \\ \boldsymbol{y} \\ \boldsymbol{z} \end{pmatrix}$

we need to multiply inverse of $\boldsymbol{A}$ with $\boldsymbol{b}$.

Therefore, the key to solve this problem

is getting the inverse $\boldsymbol{A}$, which is $\boldsymbol{A}^{-1}$.

EXAMPLE

Determine the solution for the set of linear equations below:

$$x+3y+3z=4$$
$$2x-3y-2z=2$$
$$3x+y+2z=5$$

Solution:

Write in the matrix equation form, $Ac=\boldsymbol{b}$:

$$\begin{pmatrix}1 & 3 & 3\\ 2 & -3 & -2\\ 3 & 1 & 2\end{pmatrix}\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\begin{pmatrix}4\\ 2\\ 5\end{pmatrix}$$

Step ① Matrix $\boldsymbol{A}=\begin{pmatrix}1 & 3 & 3\\ 2 & -3 & -2\\ 3 & 1 & 2\end{pmatrix}$.

Step ② Determine the determinant of matrix ***A*** , $|A|=-1$

Step ③ Determine the minor of matrix $\boldsymbol{A}=\begin{pmatrix}-4 & 10 & 11\\ 3 & -7 & -8\\ 3 & -8 & -9\end{pmatrix}$.

Step ④ Determine the cofactor of matrix $\boldsymbol{A}=\begin{pmatrix}-4 & -10 & 11\\ -3 & -7 & 8\\ 3 & 8 & -9\end{pmatrix}$

Step ⑤ Determine the adjoint of matrix $\boldsymbol{A}=\begin{pmatrix}-4 & -3 & 3\\ -10 & -7 & 8\\ 11 & 8 & -9\end{pmatrix}$.

Determine the inverse of matrix $\boldsymbol{A}=\begin{pmatrix}4 & 3 & -3\\ 10 & 7 & -8\\ -11 & -8 & 9\end{pmatrix}$.

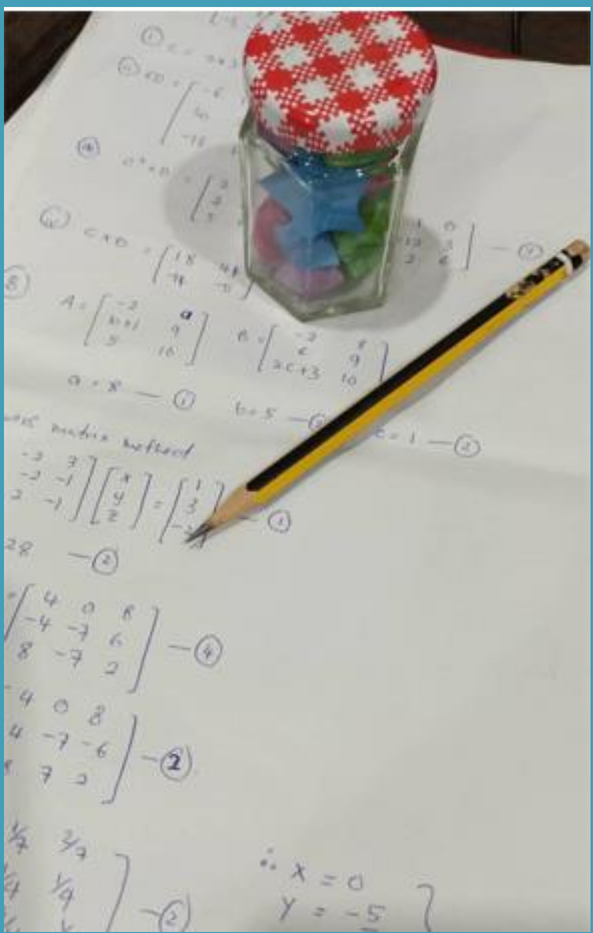
Step ⑦ Determine matrix ***c*** where $c = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

Multiply *A*⁻¹*b* , $c = A^{-1}b$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 & 3 & -3 \\ 10 & 7 & -8 \\ -11 & -8 & 9 \end{pmatrix} \begin{pmatrix} 4 \\ 2 \\ 5 \end{pmatrix} = \begin{pmatrix} 7 \\ 14 \\ -15 \end{pmatrix}$$

Therefore, the answer is $x = 7, y = 14, z = -15$.

SIMULTANEOUS LINEAR EQUATION – CRAMER'S RULE



- Another one method to solve the linear equations using matrix is the Cramer's Rule.
- Cramer's Rule needs skill to obtained determinant in a matrix.

We get the matrix equation as,

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

Let $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$

By replacing the column $\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ in each column of matrix ***A***,

To obtain x_1

$$x_1 = \frac{|A_1|}{|A|}. \text{ Where, } A_1 = \begin{pmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{pmatrix}$$

To obtain x_2 :

$$x_2 = \frac{|A_2|}{|A|}. \text{ Where, } A_2 = \begin{pmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{pmatrix}$$

And to obtain x_3 :

$$x_3 = \frac{|A_3|}{|A|}. \quad \text{Where, } A_3 = \begin{pmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{pmatrix}$$

EXAMPLE

Solve the following linear equations:

$$5x - y + 7z = 4$$
$$6x - 2y + 9z = 5$$
$$2x + 8y - 4z = 8$$

Solution:

Write in matrix equation form, $\boldsymbol{Ax} = \boldsymbol{b}$:

$$\begin{pmatrix} 5 & -1 & 7 \\ 6 & -2 & 9 \\ 2 & 8 & -4 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \\ 8 \end{pmatrix}$$

By take $\boldsymbol{A} = \begin{pmatrix} 5 & -1 & 7 \\ 6 & -2 & 9 \\ 2 & 8 & -4 \end{pmatrix}, \quad |A| = 2$

$$A_1 = \begin{pmatrix} 4 & -1 & 7 \\ 5 & -2 & 9 \\ 8 & 8 & -4 \end{pmatrix} \qquad |A_1| = 44$$

$$A_2 = \begin{pmatrix} 5 & 4 & 7 \\ 6 & 5 & 9 \\ 2 & 8 & -4 \end{pmatrix} \qquad |A_2| = -26$$

$$A_3 = \begin{pmatrix} 5 & -1 & 4 \\ 6 & -2 & 5 \\ 2 & 8 & 8 \end{pmatrix} \qquad |A_3| = -34$$

$$x = \frac{|A_1|}{|A|} = \frac{44}{2} = 22$$

$$y = \frac{|A_2|}{|A|} = \frac{-26}{2} = -13$$

$$z = \frac{|A_3|}{|A|} = \frac{-34}{2} = -17$$

EXERCISE

1) Solve the following linear equations by using **INVERSE MATRIX METHOD**.

a) $x+2y-3z=3$

$2x-y-z=11$

$3x+2y+z=-5$

b) $5x-y+7z=4$

$6x-2y+9z=5$

$2x+8y-4z=8$

2) By using **CRAMER'S RULE**, solve the following linear equations.

a) $2x_1-x_2+3x_3=2$

$x_1+3x_2-x_3=11$

$2x_1-2x_2+5x_3=3$

b) $x-4y-2z=21$

$2x+y+2z=3$

$3x+2y-z=-2$

Scan for answer

http://tiny.cc/c4e9

REVIEW

1) Given matrix $A=\begin{pmatrix}-2 & a\\ b+1 & 8\\ 5 & 10\end{pmatrix}$ and $B=\begin{pmatrix}-2 & 8\\ 6 & 8\\ 2c+3 & 10\end{pmatrix}$. If $A=B$, find the values of a, b and c.

2) Referring to matrix $B=\begin{pmatrix}3 & 5 & -3\\ 0 & 8 & 4\\ -1 & 7 & 2\end{pmatrix}$

   a) Identify the element of:

      i) b_{21}

      ii) b_{12}

      iii) b_{32}

   b) Calculate BI where I is an identity matrix.

   c) Calculate B^T.

   d) Calculate B^T+B.

3) Given $C=\begin{pmatrix}2 & 7 & 5\\ -3 & -1 & 4\end{pmatrix}$ and $D=\begin{pmatrix}-1 & 3\\ 5 & 4\\ -3 & 2\end{pmatrix}$,

   a) express the size of matrix C.

   b) calculate $6D$.

   c) calculate C^T+D.

   d) calculate $C\times D$.

Try this

REVIEW

4) Given the matrices below,

$A=\begin{pmatrix}1 & 2 & 3\\ 2 & 1 & 2\end{pmatrix}$, $B=\begin{pmatrix}3 & 0 & 2\\ 1 & -1 & 4\end{pmatrix}$ and $C=\begin{pmatrix}6 & 2\\ 1 & 3\\ 0 & -2\end{pmatrix}$. Calculate:

a) $A-B$

b) $-C^T$

c) $3B^T+C$

d) AC

5) The determinant of matrix $A=\begin{pmatrix}1 & 0 & 3\\ x & 1 & 2\\ 2 & 1 & 3\end{pmatrix}$ is 10.

a) Calculate the value of x.

b) Convert matrix A into inverse matrix, A^{-1}.Calculate the inverse of matrix

$$T=\begin{pmatrix}2 & 4 & 4\\ 1 & 3 & 2\\ -1 & -2 & -3\end{pmatrix}.$$

Try this

REVIEW

6) Solve the following linear equations by using the inverse matrix method.

$$2x - 4y + 3z = -3$$
$$x + 2y - 5z = 9$$
$$-3x - y + 2z = -9$$

7) Solve the following equations by using Cramer's Rule.

$$-2x + 3y - z = 1$$
$$x + 2y - z = 4$$
$$-2x + 3z = 8$$

8) Solve the following equations by using Cramer's Rule.

$$-2x + 3y - z = 1$$
$$x + 2y - z = 4$$
$$-2x + 3z = 8$$

Try this

REVIEW

9) Solve the following equations by using inverse matrix method.

$$x + 3y + 3z = 4$$

$$2x - 3y - 2z = 2$$

$$3x + y + 2z = 5$$

10) If $|A| = 2$, calculate the values of x, y and z in the following equations by using Cramer's Rule.

$$5x - y + 7z = 4$$

$$6x - 2y + 9z = 5$$

$$2x + 8y - 4z = 8$$

11) Calculate the values of x, y and z in the following equations by using Cramer's Rule.

$$-x - 2y + 4z = 12$$

$$3x - 6y + z = 15$$

$$2x + 5y = -1$$

Try this

Scan for Answer REVIEW

Scan for answer

http://tiny.cc/c4Rev

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