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ENGINEERING MATHEMATICS

BASIC ALGEBRA

VOLUME 1

AUTHOR
LIM YEONG CHYNG

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ABSTRACT

The book is about Engineering Mathematics Volume 1 on chapter 1 Basic Algebra. The sub topic include The Concept of Basic Algebra, Quadratic Equation and Partial Fraction. The book contains the formulas, examples and exercises regarding the each sub topic.

Hopefully with this book can help students in the solution of Basic Algebra.

TABLE OF CONTENTS

1.1 CONCEPT OF BASIC ALGEBRA

Algebra involves the use of letters in Mathematics. These letters are unknowns and can represent either a single unknown number or a range of unknown numbers. Sometimes algebraic expressions can be simplified; this means that we collect all the similar terms together.

Definition of Algebraic Expression

Algebra is the branch of mathematics that uses letters in place of some unknown numbers. Literal numbers (the letters used in algebra) can either stand for variables (where the value of the letter can be changed, such as the area of a rectangle and the area of a square) or constants (where the value does not change), for example e (which has a constant value of 2.781828...).

Algebraic fraction has the same properties as numerical fraction. The only difference being that the numerator (top) and denominator (bottom) are both algebraic expressions. Fractional algebra is a rational number usually stated in the form of , where p and q are integers.

$$\frac{p}{q} = \frac{\text{Numerator}}{\text{Denominator}}$$

A quantity made up of symbols together with,, or is called an algebraic expression. We can often simplify algebraic expression by 'a collection like terms'. Like terms are multiples of the same variables. For example, , and are all multiples of and so are like terms. Similarly, , and are all multiples of and so are like terms. Like terms can be collected together and added or subtracted in order to simplify them.

The basic laws introduced in arithmetic are generalized in algebra. Let , , , and represent any four numbers. Then

$$p(qr) = (pq)r$$
$$p+q=q+p$$
$$pq = qp$$
$$p(q+r) = pq + pr$$
$$\frac{p+q}{r} = \frac{p}{r} + \frac{q}{r}$$
$$(p+q)(r+s) = pr + ps + qr + qs$$

a. Addition of Fractions

$$\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$$

$$\frac{a}{b} + \frac{c}{d} = \frac{ad}{bd} + \frac{bc}{bd} = \frac{ad+bc}{bd}$$

b. Substraction of Fractions

$$\frac{a}{b} - \frac{c}{b} = \frac{a-c}{b}$$

$$\frac{a}{b} - \frac{c}{d} = \frac{ad}{bd} - \frac{bc}{bd} = \frac{ad-bc}{bd}$$

Example 1

Simplify the following expression $a + 2b - 3a + 6b$

Solution

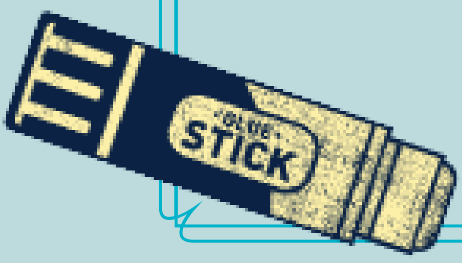
$$\begin{aligned} a + 2b - 3a + 6b &= a - 3a + 2b + 6b \\ &= -2a + 8b \end{aligned}$$

Example 2

Simplify the following expression $\frac{2}{9}x + 7y - 5x - y$

Solution

$$\begin{aligned} \frac{2}{9}x + 7y - 5x - y &= \frac{2}{9}x - 5x + 7y - y \\ &= -\frac{43}{9}x + 6y \end{aligned}$$



Exercise 1

Simplify each of the following algebraic expressions

1) $8x + 12y + 2x + 10$

2) $25p - 3q + 14p + 3q$

3) $6a + b - 2b + \frac{2}{5}a$

4) $27n + \frac{3}{7}m - 13m + \frac{5}{8}n$

Scan for answer Exercise 1

Scan for answer

http://tiny.cc/c1e1

c. Multiplication of Fractions

$$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$$

When we need to multiply or divide negative numbers, care must be taken with the sign of the answer, that is, whether the result is positive or negative. The following rules apply for determining the sign of the answer when multiplying or dividing positive and negative numbers.

$$\begin{aligned}
(\text{positive}) \times (\text{positive}) &= \text{positive} \\
(\text{positive}) \times (\text{negative}) &= \text{negative} \\
(\text{negative}) \times (\text{positive}) &= \text{negative} \\
(\text{negative}) \times (\text{negative}) &= \text{positive}
\end{aligned}$$

In an expression such as it is intended that the multiplies all the bracketed terms:

$$\begin{aligned}
a(b+c) &= ab+ac \\
a(b-c) &= ab-ac
\end{aligned}$$

In the expression it is intended that the term multiplies both and in the second brackets. Therefore

$$(a+b)(c+d) = (a+b)c + (a+b)d$$

Each of these two terms can be expanded further to give

$$(a+b)c = ac + bc$$

and

$$(a+b)d = ad + bd$$

Therefore

$$(a+b)(c+d) = ac + ad + bc + bd$$

d. Division of Fractions

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$$

$$\frac{\text{positive}}{\text{positive}} = \text{positive} \qquad \frac{\text{negative}}{\text{positive}} = \text{negative}$$

$$\frac{\text{positive}}{\text{negative}} = \text{negative} \qquad \frac{\text{negative}}{\text{negative}} = \text{positive}$$

Example 3

Simplify the following expression *6 (5a -2b) +4b*

Solution

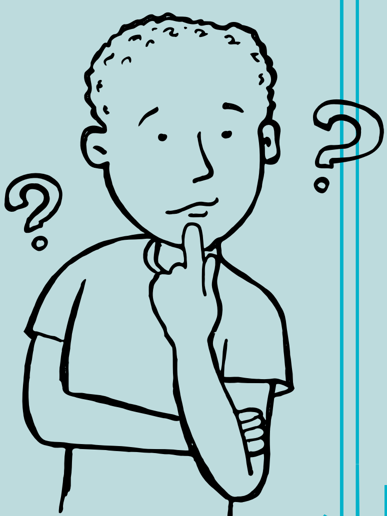
$$\begin{aligned} 6(5a-2b)+4b &= 30a-12b+4b \\ &= 30a-8b \end{aligned}$$

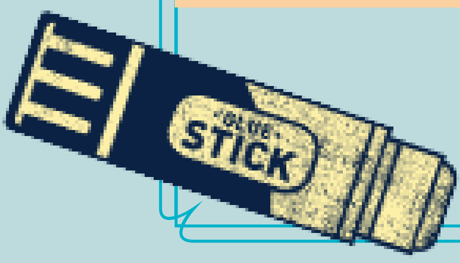
Example 4

Simplify the following expression $\dfrac{10z}{9b} \div \dfrac{yz}{3b}$

Solution

$$\begin{aligned} \frac{10z}{9b} \div \frac{yz}{3b} &= \frac{10z}{9b} \times \frac{3b}{yz} \\ &= \frac{10}{3y} \end{aligned}$$





Exercise 2

Simplify each of the following algebraic expressions

1) $8(p-14)-9(2p+1)$

2) $-5(2q-4r)+6(3q-2r)$

3) $\frac{1}{2}(10x-2y)+3x-y$

4) $-5\left(\frac{3}{5}a+\frac{1}{10}b\right)-6a$

Scan for answer Exercise 2

Scan for answer

http://tiny.cc/c1e2

Example 5

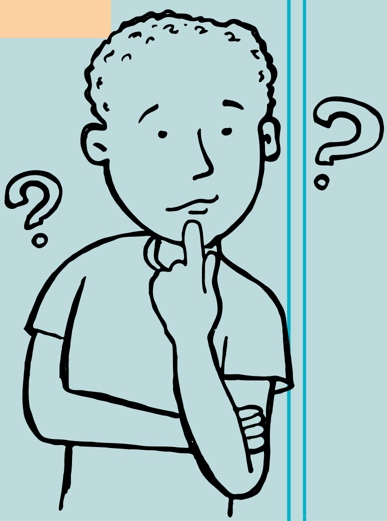
Simplify the following expression $\dfrac{10x}{x+2}+\dfrac{4x^2}{x^2-4}$

Solution

REMEMBER

$a^2-b^2=(a+b)(a-b)$

$$
\begin{aligned}
\frac{10x}{x+2}+\frac{4x^2}{x^2-4} &= \frac{10x}{x+2}+\frac{4x^2}{(x+2)(x-2)} \\
&= \frac{10x(x-2)}{(x+2)(x-2)}+\frac{4x^2}{(x+2)(x-2)} \\
&= \frac{10x(x-2)+4x^2}{(x+2)(x-2)} \\
&= \frac{10x^2-20x+4x^2}{(x+2)(x-2)} \\
&= \frac{14x^2-20x}{(x+2)(x-2)}
\end{aligned}
$$



Example 6

Simplify the following expression $\frac{5}{y+3}-\frac{10-3y}{y^2-9}$

Solution

REMEMBER

$a^2-b^2=(a+b)(a-b)$

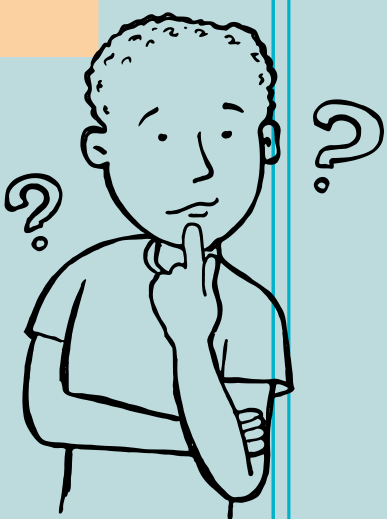
$$\frac{5}{y+3}-\frac{10-3y}{y^2-9}=\frac{5}{y+3}-\frac{10-3y}{(y+3)(y-3)}$$

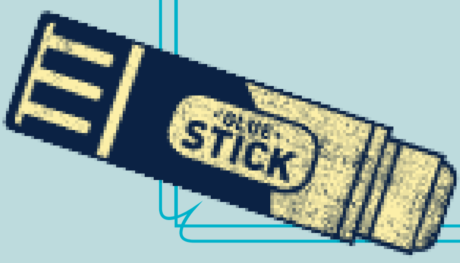
$$=\frac{5(y-3)}{(y+3)(y-3)}-\frac{10-3y}{(y+3)(y-3)}$$

$$=\frac{5y-15-10+3y}{(y+3)(y-3)}$$

$$=\frac{5y+3y-15-10}{(y+3)(y-3)}$$

$$=\frac{8y-25}{(y+3)(y-3)}$$





Exercise 3

Simplify each of the following algebraic expressions

1) $\dfrac{a+3}{a-3} \times \dfrac{3a-5}{a+3}$

2) $\dfrac{m^2}{m-4} \div \dfrac{2m^2}{10}$

3) $\dfrac{x^2-4}{x^2-9} \div \dfrac{x-2}{x-3}$

4) $\dfrac{x-3}{x^2-16} \times \dfrac{x-4}{x^2-9}$

Scan for answer Exercise 3

Scan for answer

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Link Quizizz for practice question

Basic Algebra

https://quizizz.com/admin/quiz/6118c3b3a82bdd001d8b3e39

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1.2 QUADRACTIC EQUATION

A quadratic equation in an unknown (variable) is an equation that has one unknown only and his unknown's supreme power is 2. Quadratic equations are normally express as $ax^2 + bx + c = 0$, where a does not equal zero.

The root of a quadratic equation can be found by using three methods:

(a) Factorization

(b) Quadractic Formula

(c) Completing The Square

Solving Quadratic Equations by using Factorization Method

Many of the simpler quadratic equations with rational roots can be solved by factoring. To solve a quadratic equation by factoring:

Start with the equation in the standard form

Be sure it is set equal to zero.

1. *a* , *b* , *c* are known values. *a* can't be 0.
2. Factor the left hand side (assuming zero is on the right).
3. Set each factor equal zero.
4. Solve to determine the roots (the value of *x*).

Table to identify the value of *a*, *b*, *c*

In disguise	$\longrightarrow$	In Standard Form	a, b and c
$x^2 = 3x - 1$	Move all terms to the left hand side.	$x^2 - 3x + 1 = 0$	$a = 1$ $b = -3$ $c = 1$
$2(w^2 - 2w) = 5$	Expand (undo the bracket), and move 5 to the left.	$2w^2 - 4w - 5 = 0$	$a = 2$ $b = -4$ $c = -5$
$z(z - 1) = 3$	Expand, and move 3 to the left.	$z^2 - z - 3 = 0$	$a = 1$ $b = -1$ $c = -3$

Example 7

Solve the quadratic equation $x^2 - 4x + 3 = 0$
by using factorization

Solution

Roots of denominator:

$$x^2 - 4x + 3 = 0 \quad (\text{Set} = 0)$$

$$(x-1)(x-3) = 0 \quad (\text{Factors})$$

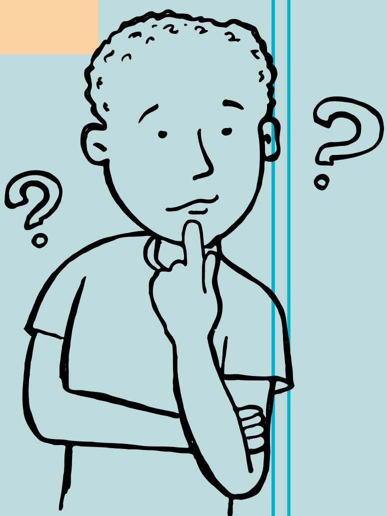
$$x^2 - 4x + 3 = 0$$

$$(x-1)(x-3) = 0$$

$$x - 1 = 0 \quad \text{or} \quad x - 3 = 0$$

$$x = 1 \qquad\qquad x = 3$$

$$\begin{tikzcd}
\boxed{(x-1)(x-3)} \arrow[d, shift right=2ex, "\text{Expand}"'] \\
\boxed{x^2 - 4x + 3} \arrow[u, shift right=2ex, "\text{Factor}"']
\end{tikzcd}$$



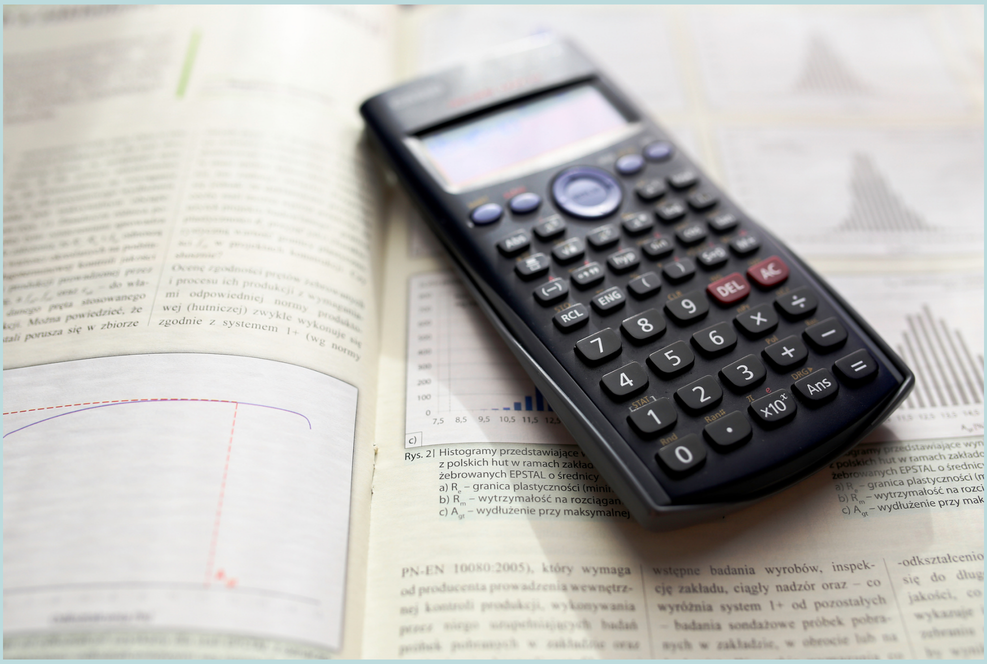
Tips

Steps to Use Calculator for Solving Quadratic Equation, $x^2 - 4x + 3 = 0$

1. Press button ***MODE*** 3 times.
2. Press button "1" for **EQN**.
3. Press ▶ in the **ROUND** button **COPY**, choose button "2" for ***Degree***.
4. Refer to equation form, $ax^2 + bx + c = 0$
5. For *a*, press button "1", and then button "=".
6. For *b*, press buttons "-" and "4", then button "=" .
7. For *c*, press button "3", and then button "=".
8. The screen will display $x_1 = 3$
9. When press ▼ in the **ROUND** button **COPY**, the screen will display $x_2 = 1$.
10. From here, we get the answer for factorization: $(x-3)(x-1)$ or $(x-1)(x-3)$

Watch Youtube Step for Solving Quadratic Equation Using calculator

https://youtu.be/aQlW3S2bVjQ



Example 8

Solve the quadratic equation $4x^2 + 8x + 3 = 0$ by using factorization

Solution

$$4x^2 + 8x + 3 = 0$$
$$(2x+1)(2x+3) = 0$$

$$2x + 1 = 0 \quad \text{or} \quad 2x + 3 = 0$$
$$x = -\frac{1}{2} \qquad\qquad x = -\frac{3}{2}$$

Example 9

Solve the quadratic equation $6x^2 = 47x - 77$ by using factorization

Solution

$$6x^2 = 47x - 77$$

$$6x^2 - 47x + 77 = 0$$

$$(2x - 11)(3x - 7) = 0$$

$$2x - 11 = 0 \quad \text{or} \quad 3x - 7 = 0$$

$$x = \frac{11}{2} \qquad\qquad x = \frac{7}{3}$$

Exercise 4

Solve the following quadratic equations by using factorization

1) $x^2 - 6x + 8 = 0$
2) $x^2 + 5x + 6 = 0$
3) $x^2 - 5x - 24 = 0$
4) $4x^2 + 13x - 35 = 0$
5) $6x^2 - 7x - 20 = 0$

Scan for answer Exercise 4

Scan for answer

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Link Quizizz for practice question

Quadratic Equation

https://quizizz.com/admin/quiz/6118b6ac030b06001d139a05

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Solving Quadratic Equations by using Quadratic Formula Method

The quadratic equation given in form $ax^2 + bx + c = 0$, then the values of x can be find through quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$ax^2 + bx + c = 0$ → Where a, b and c are constants.

Dividing $ax^2 + bx + c = 0$ by a gives: $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$

Rearranging gives: $x^2 + \frac{b}{a}x = -\frac{c}{a}$

Adding to each side of the equation the square of half the coefficient of the term in x to make the *Left-Hand Side* a perfect square gives:

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a}$$

Rearranging gives:

$$\left(x + \frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a}$$

$$= \frac{b^2}{4a^2} - \frac{c}{a}$$

$$= \frac{b^2 - 4ac}{4a^2}$$

Taking the square root of both sides gives:

$$x + \frac{b}{2a} = \pm\sqrt{\left(\frac{b^2 - 4ac}{4a^2}\right)}$$

$$= \pm\frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ ⟸ The Quadratic Formula

Example 10

Solve the quadratic equation $3x^2 - 2x - 7 = 0$ by using quadratic formula.

Solution

$$3x^2 - 2x - 7 = 0$$

$a = 3 \qquad b = -2 \qquad c = -7$

$$
\begin{aligned}
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
&= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(-7)}}{2(3)} \\
&= \frac{2 \pm \sqrt{4 + 84}}{6} \\
&= \frac{2 \pm \sqrt{88}}{6} \\
&= 1.8968 \quad \text{or} \quad -1.2301
\end{aligned}
$$

Watch Youtube Step for Solution Example 10

https://youtu.be/PKD7THD-llg

Example 11

Solve the quadratic equation $x^2 = 5 - 7x$ by using quadratic formula.

Solution

$$x^2 = 5 - 7x$$

$$x^2 + 7x - 5 = 0$$

$a = 1 \qquad b = 7 \qquad c = -5$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-7 \pm \sqrt{7^2 - 4(1)(-5)}}{2(1)}$$

$$= \frac{-7 \pm \sqrt{49 + 20}}{2}$$

$$= \frac{-7 \pm \sqrt{69}}{2}$$

$$= 0.6533 \quad \text{or} \quad -7.6533$$

Exercise 5

Solve the following quadratic equations by using quadratic formula.

1) $3x^2 + 4x + 1 = 0$

2) $2x^2 + 7x - 3 = 0$

3) $x^2 + x - 2 = 0$

4) $4x^2 - 4x - 63 = 0$

5) $15x^2 - 19x + 6 = 0$

6) $7x^2 = 60 + 32x$

7) $18x^2 = 53x + 20$

8) $x^2 = 56 + x$

9) $3x^2 = 2 - 5x$

10) $5x^2 + 52x + 63 = 0$

Scan for answer Exercise 5

Scan for answer

http://tiny.cc/c1e5

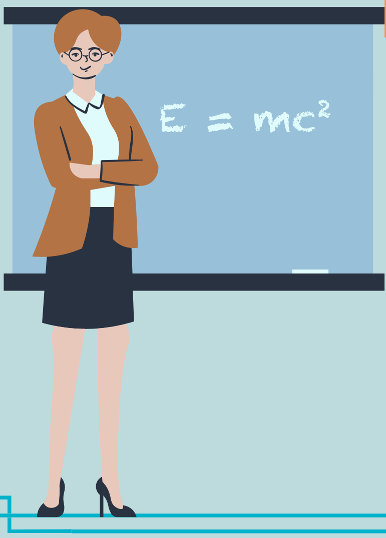
Solving Quadratic Equations by using Completing the Square Method

Completing the Square is a method used to solve a quadratic equation by changing the form of the equation so that the left side is a perfect square trinomial.

Step	Method	Formula
Step 1	Start with	$ax^2 + bx + c = 0$
Step 2	Divide the equation by a	$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$
Step 3	Put $\frac{c}{a}$ on other side	$x^2 + \frac{b}{a}x = -\frac{c}{a}$
Step 4	Add $\left(\frac{b}{2a}\right)^2$ to both sides	$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$
Step 5	Complete the square	$\left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$

Watch Youtube Step for Method Completing the Square

https://youtu.be/tnOkxuRWZWw



If $x^2 = 5$, then $x = \pm\sqrt{5}$

If $(x+2)^2 = 5$, then $x + 2 = \pm\sqrt{5}$ and $x = -2 \pm \sqrt{5}$

Hence if quadratic equation can be rearranged so that one side of equation is a perfect square and the other side of the equation is a number, then the solution of the equation is readily obtained by taking the square roots of each side as in the above examples. The process of rearranging one side of a quadratic equation into a perfect square before solving is called “**completing the square**” .

$$(x+a)^2 = x^2 + 2ax + a^2$$

Thus, to make the quadratic expression $x^2 + 2ax$

into a perfect square it is necessary to add (half the coefficient of x)^2.

That is $\left(\frac{2a}{2}\right)^2$ or a^2

For example $x^2 + 3x$ become a perfect square by adding $\left(\frac{3}{2}\right)^2$.

$$x^2 + 3x + \left(\frac{3}{2}\right)^2 = \left(x + \frac{3}{2}\right)^2$$

Example 12

Solve the quadratic equation $x^2 + 8x - 4 = 0$
by using completing the square.

Solution

Step 1 Rearrange the equation so that all terms are on the same side of the equal sign (and the coefficient of the x^2 term is positive). Hence:

$$x^2 + 8x - 4 = 0$$

Step 2 Make the coefficient of the x^2 term unity. In this case this is the x^2 term unity. Hence:

$$x^2 + 8x - 4 = 0$$

Step 3 Rearrange the equations so that the x^2 and x terms are on one side of the equal sign and the constant is on the other side. Hence:

$$x^2 + 8x = 4$$

Step 4 Add to both sides of the equation (half the coefficient of x)^2.
In this case the coefficient of x is 8. Half the coefficient squared is therefore $\left(\frac{8}{2}\right)^2$. Thus:

$$x^2 + 8x + \left(\frac{8}{2}\right)^2 = 4 + \left(\frac{8}{2}\right)^2$$

$$x^2 + 8x + (4)^2 = 4 + (4)^2$$

The left-hand side is now a perfect square,

$$(x+4)^2 = 4 + 16$$

$$(x+4)^2 = 20$$

$$x + 4 = \pm\sqrt{20}$$

Step 5 Solve:

$$x = -4 \pm \sqrt{20}$$

$$x = 0.4721\ ,\ -8.4721$$

Example 13

Solve the quadratic equation $2x^2 + 5x = 12$ by using completing the square.

Solution

Step 1 Rearrange the equation so that all terms are on the same side of the equal sign (and the coefficient of the x^2 term is positive). Hence:

$$2x^2 + 5x - 12 = 0$$

Step 2 Make the coefficient of the x^2 term unity. In this case this is achieved by dividing throughout by 2. Hence:

$$\frac{2x^2}{2} + \frac{5x}{2} - \frac{12}{2} = 0$$

$$x^2 + \frac{5}{2}x - 6 = 0$$

Step 3 Rearrange the equations so that the x^2 and x terms are on one side of the equals sign and the constant is on the other side. Hence:

$$x^2 + \frac{5}{2}x = 6$$

Step 4 Add to both sides of the equation (half the coefficient of x)^2.

In this case the coefficient of x is $\frac{5}{2}$.

Half the coefficient squared is therefore $\left(\frac{5}{4}\right)^2$.

Thus: $x^2 + \frac{5}{2}x + \left(\frac{5}{4}\right)^2 = 6 + \left(\frac{5}{4}\right)^2$

The left-hand side is now a perfect square,

$$\left(x + \frac{5}{4}\right)^2 = 6 + \left(\frac{5}{4}\right)^2$$

Evaluate the right-hand side

$$\left(x + \frac{5}{4}\right)^2 = 6 + \frac{25}{16} = \frac{121}{16}$$

$$\left(x + \frac{5}{4}\right) = \pm\sqrt{\frac{121}{16}}$$

$$\left(x + \frac{5}{4}\right) = \pm\frac{11}{4}$$

Step 5 Solve

$$x = -\frac{5}{4} \pm \frac{11}{4}$$

$$x = \frac{3}{2}, -4$$

Exercise 6

Solve the following quadratic equations by using completing the square.

1) $x^2 + x - 30 = 0$

2) $x^2 + 12x + 32 = 0$

3) $2x^2 + 13x - 45 = 0$

4) $6x^2 - 17x + 12 = 0$

5) $8x^2 - 9x = 14$

Scan for answer Exercise 6

Scan for answer

http://tiny.cc/c1e6

Exercise 7

Solve the following quadratic equations by using specific method.

1)	$(x-2)(4x+7)=0$	(Factorization Method)
2)	$10x^2-7x-12=0$	(Quadratic Formula)
3)	$6x^2-9x=20$	(Completing the Square)
4)	$6x^2+x-35=0$	(Factorization Method)
5)	$x^2+2x=48$	(Quadratic Formula)
6)	$14x^2-11x=15$	(Completing the Square)

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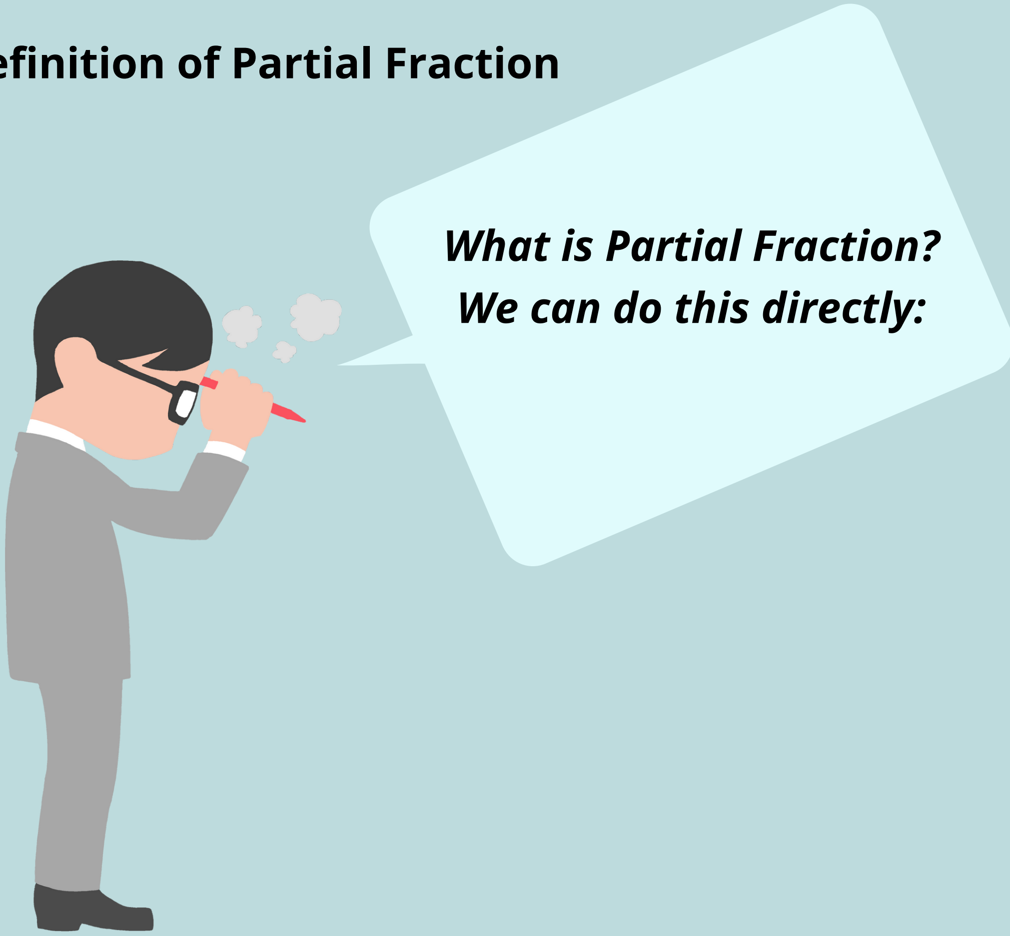
1.3 PARTIAL FRACTION

Type of Fractions

There are 3 different types of fractions:

No	Types of Fraction	Definition	Examples
1	Proper Fractions	**Numerator < Denominator**	$\frac{1}{2}, \frac{2}{5}, \frac{19}{20}, \frac{x}{x^2+2}, \frac{x-1}{x^2+3x-18}$
2	Improper Fraction	**Numerator $\geq$ Denominator**	$\frac{5}{5}, \frac{7}{2}, \frac{x^2-1}{x+1}, \frac{x^2+3x-1}{x^2+2}$
3	Mixed Fraction	**Whole number + a proper fraction**	$2\frac{1}{5}, 123\frac{19}{20}, 2\frac{x+1}{x^2+2x-1}$

Definition of Partial Fraction



Simplify $\dfrac{1}{x-1}+\dfrac{2}{x+2}$

Normally we do:

$$=\frac{1(x+2)+2(x-1)}{(x-1)(x+2)}$$

$$=\frac{x+2+2x-2}{x^2+2x-x-2}$$

$$=\frac{3x}{x^2+x-2}$$

But how do we go in the opposite direction?

$$\frac{1}{x-1}+\frac{2}{x+2} = \frac{3x}{x^2+x-2}$$

Partial Fractions

Single Fraction

The reverse process of moving from $\frac{3x}{x^2+x-2}$ to $\frac{1}{x-1}+\frac{2}{x+2}$

is called resolving into partial fractions. In order to convert an algebraic expression into partial fractions:

(a) The denominator in the above example, x^2+x-2 must factorize as $(x-1)(x+2)$

(b) The numerator must be at least one degree less than the denominator. In the above example *(3x)* is of degree 1 since the highest power of *x* terms is x^1 and (x^2-x-2) is of degree 2.

When the degree of the numerator is equal to or higher than the degree of the denominator, the numerator must be divided by the denominator.

There are basically three types of partial fraction and the form of partial fraction used is summarized in table below.

Type	Denominator containing	Expression	Form of partial fraction
1	Linear Factors	$\frac{f(x)}{(x+a)(x-b)(x-c)}$	$\frac{A}{x+a}+\frac{B}{x-b}+\frac{C}{x+c}$
2	Repeated Liner Factors	$\frac{f(x)}{(x+a)^3}$	$\frac{A}{(x+a)}+\frac{B}{(x+a)^2}+\frac{C}{(x+a)^3}$
3	Quadratic Factors	$\frac{f(x)}{(ax^2+bx+c)(x+d)}$	$\frac{Ax+B}{ax^2+bx+c}+\frac{C}{x+d}$

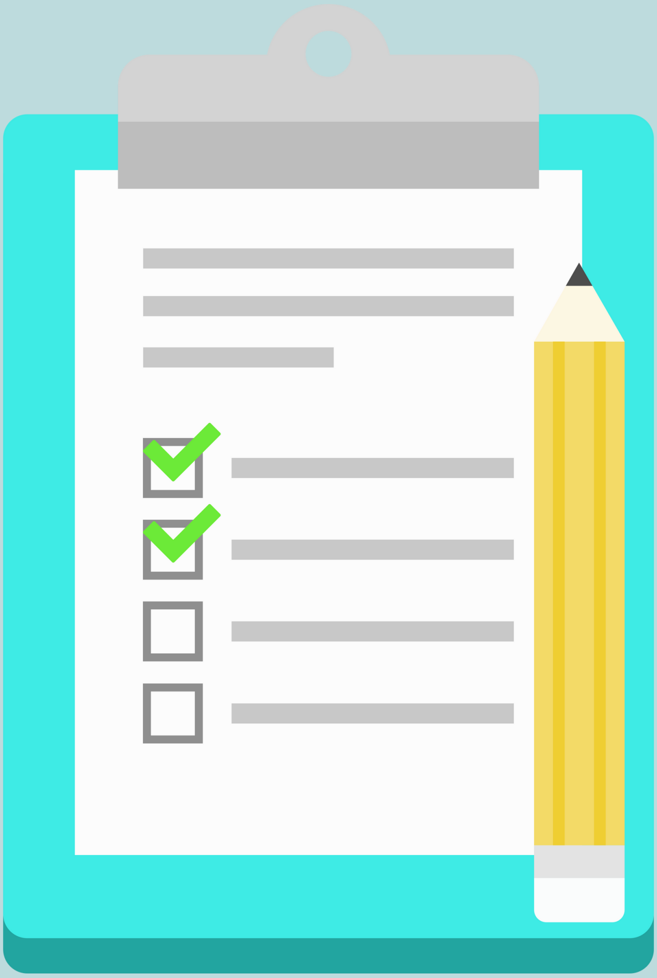
$f(x)$ is assumed to be of less degree than the relevant denominator and A, B and C are constants to be determined.

$ax^2 + bx + c$ is a quadratic expression which cannot factorize without containing surds or imaginary terms.

That is what we discover here:

How to find the “parts” that make the single fraction (the “PARTIAL FRACTION”)?

Resolving an algebraic expression into a partial fraction is used as a preliminary to integrating certain functions.



Construct Partial Fraction using Proper Fraction with:

a. Linear Factor

A linear factor gives a partial fraction in the form $\dfrac{A}{ax+b}$ where A is a constant to be determine. We can calculate the partial fraction of proper fraction whose denominator can be factorized into linear factors.

The following steps are used:

1. Factorized the denominator.
2. Each factor of the denominator produced a partial fraction. A factor produces a partial fraction of the form $\dfrac{A}{ax+b}$ where A is an unknown constant.
3. Evaluate the unknown constants of the partial fractions.
4. This is done by evaluation using a specific value of x or by equating coefficients.

Example 14

Construct $\dfrac{5x-3}{x^2+3x-10}$ into partial fractions.

Solution

Step 1 **Factor the denominator.**

$$\frac{5x-3}{x^2+3x-10}=\frac{5x-3}{(x-2)(x+5)}$$

Roots of Denominator

$x^2+3x-10=0$ *(Set = 0)*

$(x-2)(x+5)=0$ *Factor*

Step 2 **Write one partial fraction for each of those factors.**

$$\frac{5x-3}{(x-2)(x+5)}=\frac{A}{x-2}+\frac{B}{x+5}$$

Step 3 **Multiply through the denominator so that no longer have factors.**

$$5x-3=A(x+5)+B(x-2)$$

Step 4 **Find the constants.**

Substituting the roots ("zero") of the bottom can help:

Roots for $x+5$ is $x=-5$

When $x=-5$

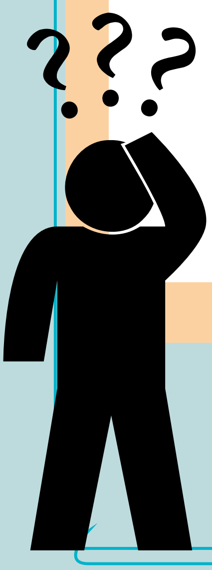
$$\begin{aligned}5(-5)-3&=A(-5+5)+B(-5-2)\\-28&=A(0)+B(-7)\\-28&=0-7B\\B&=4\end{aligned}$$

Roots for $x-2$ is $x=2$

When $x=2$

$$\begin{aligned}5(2)-3&=A(2+5)+B(2-\\7&=A(7)+B(0)\\7&=A(7)+0\\7&=A(7)\\A&=1\end{aligned}$$

$$\therefore\frac{5x-3}{x^2+3x-10}=\frac{1}{x-2}+\frac{4}{x+5}$$



Example 15

Construct $\frac{10x+8}{(x+2)(x-5)}$ into partial fractions.

Solution

$$\frac{10x+8}{(x+2)(x-5)} = \frac{A}{x+2} + \frac{B}{x-5}$$

$$10x+8 = A(x-5) + B(x+2)$$

When $x+2=0$

$$x=-2$$

$$10(-2)+8 = A(-2-5) + B(-2+2)$$

$$-12 = A(-7) + B(0)$$

$$-12 = -7A$$

$$A = \frac{12}{7}$$

When $x-5=0$

$$x=5$$

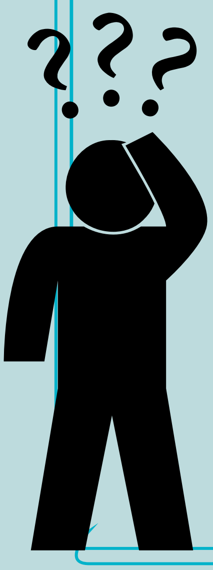
$$10(5)+8 = A(5-5) + B(5+2)$$

$$58 = A(0) + B(7)$$

$$58 = 7B$$

$$B = \frac{58}{7}$$

$$\therefore \frac{10x+8}{(x+2)(x-5)} = \frac{12}{7(x+2)} + \frac{58}{7(x-5)}$$



Example 16

Construct $\dfrac{12}{x^2-9}$ into partial fractions.

Solution

$$\frac{12}{x^2-9}=\frac{12}{(x-3)(x+3)}$$
$$=\frac{A}{x-3}+\frac{B}{x+3}$$
$$12=A(x+3)+B(x-3)$$

Review Algebraic Function:
$a^2-b^2=(a-b)(a+b)$
$x^2-9=(x-3)(x+3)$

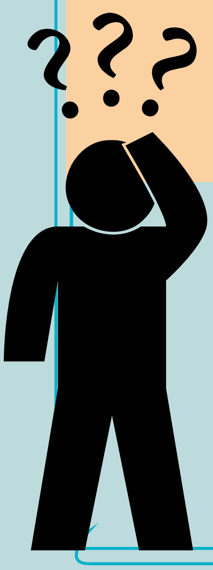
When $x-3=0$
$x=3$

$12=A(3+3)$
$12=A(6)$
$A=2$

When $x+3=0$
$x=-3$

$12=B(-3-3)$
$12=B(-6)$
$B=-2$

$$\therefore \frac{12}{x^2-9}=\frac{2}{x-3}-\frac{2}{x+3}$$



Exercise 8

1) Construct the following fractions into partial fractions.

a) $\dfrac{x}{x^2+3x+2}$

b) $\dfrac{x-5}{x^2+x-6}$

c) $\dfrac{x-1}{(2x-3)(x+4)}$

d) $\dfrac{10}{(x+1)(2x-1)}$

e) $\dfrac{x^2-x+10}{(x-1)(x+4)(x-2)}$

2) Given that partial fraction of $\dfrac{3x}{(x+6)(x-2)}$ are $\dfrac{A}{x+6}+\dfrac{B}{x-2}$. Find the values of A and B when $x=-6$ and $x=2$.

3) Given that partial fraction of $\dfrac{3}{(x)(x+1)(2x-1)}$ are $\dfrac{A}{x}+\dfrac{B}{x+1}+\dfrac{C}{2x-1}$. Find the values of A , B and C.

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Partial Fraction

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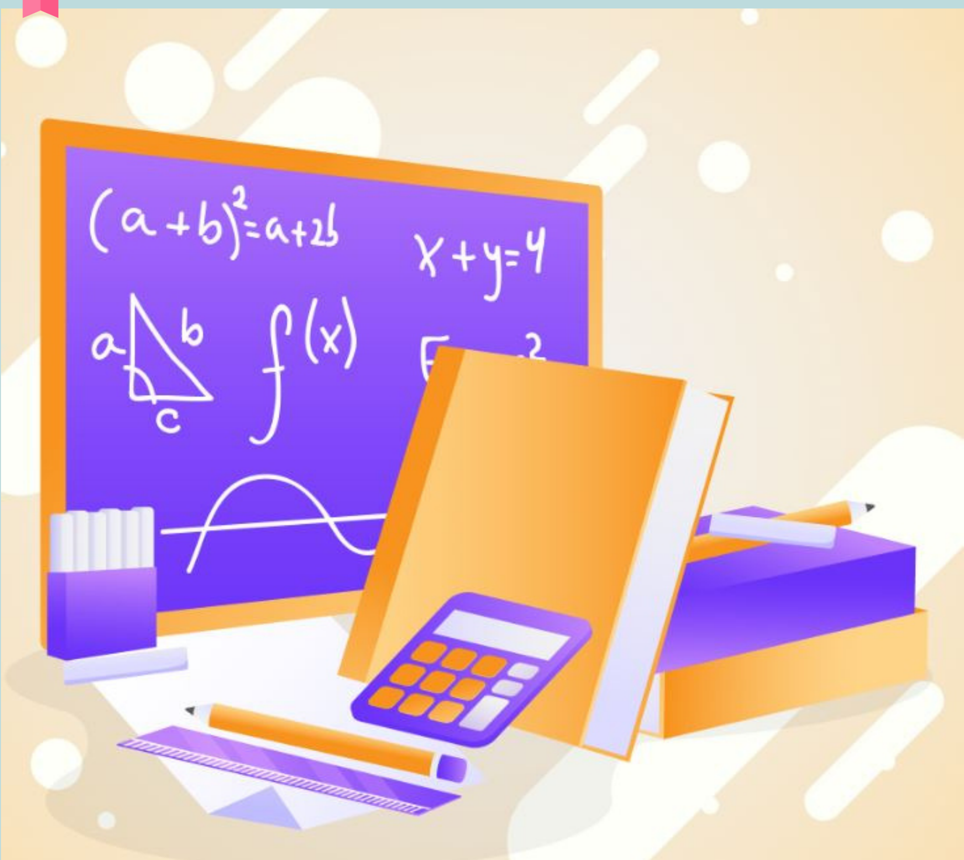
b. Repeated Linear Factors

Repeated Factor	Partial Fraction
$(ax+b)^2$	$\dfrac{A}{ax+b}+\dfrac{B}{(ax+b)^2}$
$(ax+b)^3$	$\dfrac{A}{ax+b}+\dfrac{B}{(ax+b)^2}+\dfrac{C}{(ax+b)^3}$
$(ax+b)^n$	$\dfrac{A}{ax+b}+\dfrac{B}{(ax+b)^2}+\ldots+\dfrac{Z}{(ax+b)^n}$



Example 17

Construct $\dfrac{2x}{(x+5)(x-3)^2}$ into partial fractions.

Solution

Step 1 **Write partial fraction for each of those factors.**

$$\frac{2x}{(x+5)(x-3)^2} = \frac{A}{x+5} + \frac{B}{x-3} + \frac{C}{(x-3)^2}$$

Step 2 **Multiply through the denominator so that no longer factors.**

$$2x = A(x-3)^2 + B(x+5)(x-3) + C(x+5)$$

Step 3 **Find the constants.**

Substituting the roots ("zero") of the denominator can help:

Root for $x+5$ is $x=-5$

When $x=-5$

$$2(-5) = A(-5-3)^2 + B(-5+5)(-5-3) + C(-5+5)$$
$$-10 = A(64) + B(0) + C(0)$$
$$-10 = A(64)$$
$$B = -\frac{5}{32}$$

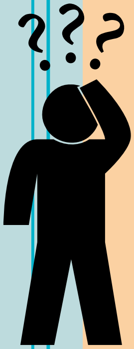
: Find the constants.

Substituting the roots ("zero") of the denominator can help:

Root for $x-3$ is $x=3$

When $x=3$

$$2(3) = A(3-3)^2 + B(3+5)(3-3) + C(3+5)$$
$$6 = A(0) + B(8)(0) + C(8)$$
$$6 = C(8)$$
$$C = \frac{3}{4}$$



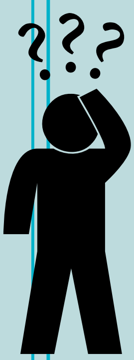
Solution (continue)

Root for x is $x = 0$

When $x = 0$

$$
\begin{aligned}
0 &= A(-3)^2 + B(0+5)(0-3) + C(0+5) \\
0 &= A(9) + B(5)(-3) + C(5) \\
0 &= \left(-\frac{5}{32}\right)(9) + B(-15) + \left(\frac{3}{4}\right)(5) \\
0 &= -\frac{45}{32} - 15B + \frac{15}{4} \\
0 &= \frac{75}{32} - 15B \\
15B &= \frac{75}{32} \\
B &= \frac{5}{32}
\end{aligned}
$$

$$\therefore \frac{2x}{(x+5)(x-3)^2} = -\frac{5}{32(x+5)} + \frac{5}{32(x-3)} + \frac{3}{4(x-3)^2}$$



Example 18

Construct $\dfrac{x-5}{x^2(x+1)}$ into partial fractions.

Solution

$$\frac{x-5}{x^2(x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1}$$

$$x-5 = Ax(x+1) + B(x+1) + Cx^2$$

When $x = 0$

$$0-5 = A(0)(0+1) + B(0+1) + C(0)^2$$
$$-5 = A(0) + B(1) + C(0)$$
$$-5 = B$$

When $x+1=0$

$$x=-1$$

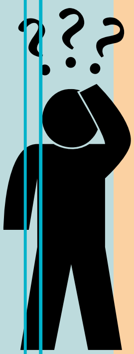
$$-1-5 = C(-1)^2$$
$$-6 = C(1)$$
$$C = -6$$

Expand the equation:

$$x-5 = Ax^2 + Ax + Bx + B + Cx^2$$
$$x-5 = (A+C)x^2 + (A+B)x + B$$

Compare the coefficient: LHS with RHS

LHS		RHS
x^2	0	A+C
x^1	1	A+B
x^0	-5	B



Compare the coefficient:

x^2
$0 = A + C$ $0 = A - 6$ $A = 6$

Notes:

LHS - Left Hand Side

RHS - Right Hand Side

$$\therefore \frac{x-5}{x^2(x+1)} = \frac{6}{x} - \frac{5}{x^2} - \frac{6}{x+1}$$

Exercise 9

1) Construct the following fractions into partial fractions.

a) $\dfrac{2x+1}{(x-1)^2(x+2)}$

b) $\dfrac{x^2}{(x-3)(x+1)^2}$

c) $\dfrac{x^2-3}{(x+5)(x-2)^2}$

d) $\dfrac{8x}{(x-6)^2(x+1)}$

e) $\dfrac{3x-5}{(x-1)(x+4)^2}$

2) Given that partial fraction of $\dfrac{5x^2-6}{(x-1)(x-3)^2}$ are $\dfrac{A}{x-1}+\dfrac{B}{x-3}+\dfrac{C}{(x-3)^2}$. Find the value of A, B and C.

3) Given that partial fraction of $\dfrac{7x}{(x+2)(x-8)^2}$ are $\dfrac{A}{x+2}+\dfrac{B}{x-8}+\dfrac{C}{(x-8)^2}$. Find the value of A, B and C.

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Partial Fraction

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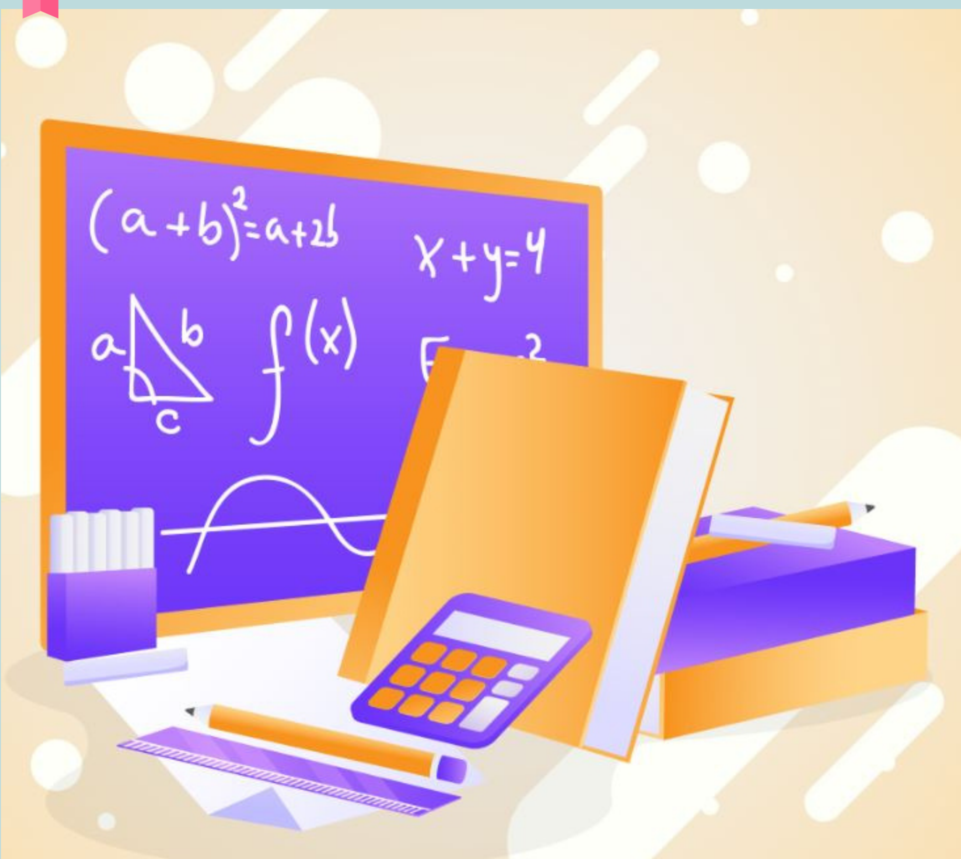
c. Quadratic Factors

An irreducible quadratic, a quadratic factor

$$ax^2 + bx + c = 0$$

that cannot be further factorized into linear factors *(x+a) (x+b)* gives a partial fraction.

$$\frac{Ax + B}{ax^2 + bx + c}$$



Example 19

Construct $\dfrac{9+x}{(x-2)(x^2+1)}$ into partial fractions.

Solution

Step 1 **Write one partial fraction for each of those factors.**

$$\frac{9+x}{(x-2)(x^2+1)}=\frac{A}{x-2}+\frac{Bx+C}{x^2+1}$$

Irreducible quadratic factor
(Can't be factorized)

Linear factor

Step 2 **Multiply through the denominator so that no longer factors.**

$9+x=A(x^2+1)+(Bx+C)(x-2)$

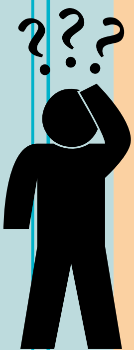
Step 3 **Find the constants**

Substituting the roots ("zero") of the denominator can help:

Root for $x-2$ is $x=2$

When $x=2$

$$9+2=A(2^2+1)+(B(2)+C)(2-2)$$
$$11=A(5)+B(0)$$
$$A=\frac{11}{5}$$



Hence, no have any root for substituting. Then need to expand the equation.

$$9+x=Ax^2+A+Bx^2-2Bx+Cx-2C$$
$$9+x=(A+B)x^2+(-2B+C)x+(A-2C)$$

Solution (continue)

Compare the coefficient: LHS with RHS

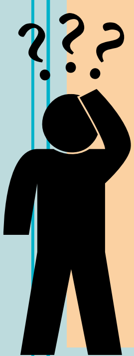
LHS		RHS
x^2	0	A+B
x^1	1	-2B+C
x^0	9	A-2C

Compare the coefficient:

x^2	x^1
$0 = A + B$	$1 = -2B + C$
$0 = \frac{11}{5} + B$	$1 = -2\left(-\frac{11}{5}\right) + C$
$B = -\frac{11}{5}$	$C = -\frac{17}{5}$

$$\therefore \frac{9+x}{(x-2)(x^2+1)} = \frac{11}{5(x-2)} + \frac{-11x-17}{5(x^2+1)}$$

$$= \frac{11}{5(x-2)} - \frac{11x+17}{5(x^2+1)}$$



Example 20

Construct $\dfrac{3x-1}{x^2(x^2+4)}$ into partial fractions.

Solution

$$\frac{3x-1}{x^2(x^2+4)}=\frac{A}{x}+\frac{B}{x^2}+\frac{Cx+D}{x^2+4}$$

$$3x-1=Ax(x^2+4)+B(x^2+4)+(Cx+D)x^2$$

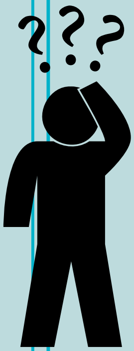
When $x=0$

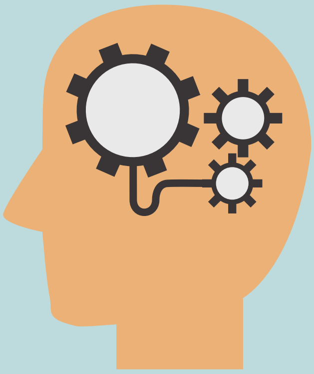
$$-1=A(0)(4)+B(0+4)+(C(0)+D)(0)$$
$$-1=A(0)+B(4)+0$$
$$B=-\frac{1}{4}$$

Expand the equation

$$3x-1=Ax^3+4Ax+Bx^2+4B+Cx^3+Dx^2$$

$$3x-1=(A+C)x^3+(B+D)x^2+4Ax+4B$$





Solution (continue)

Compare the coefficient: LHS with RHS

LHS		RHS
x^3	0	A+C
x^2	0	B+D
x^1	3	4A
x^0	-1	4B

Compare the coefficient:

x^3	x^2	x^1
$0 = A + C$ $0 = \frac{3}{4} + C$ $C = -\frac{3}{4}$	$0 = B + D$ $0 = -\frac{1}{4} + D$ $D = \frac{1}{4}$	$3 = 4A$ $A = \frac{3}{4}$

$$\therefore \frac{3x-1}{x^2(x^2+4)} = \frac{3}{4x} - \frac{1}{4x^2} + \frac{-3x+1}{4(x^2+4)}$$

Exercise 10

1) Construct the following fractions into partial fractions.

a) $\dfrac{x+3}{(x-1)(x^2+4)}$

b) $\dfrac{x-4}{(x^2+1)(x+3)}$

c) $\dfrac{7x^2-5}{(x^2-x+2)(x+4)}$

d) $\dfrac{2x}{(x^2+5)(x-1)}$

e) $\dfrac{5x+2}{(x-3)(x^2+6)}$

2) Given that partial fractions of $\dfrac{5x}{(x+3)(x^2+6x+6)}$ are $\dfrac{A}{x+3}+\dfrac{Bx+C}{x^2+6x+6}$. Find the values of A , B and C

3) Given that partial fractions of $\dfrac{2x+1}{(x-1)(x^2+8)}$ are $\dfrac{A}{x-1}+\dfrac{Bx+C}{x^2+8}$. Find the values of A, B and C.

Scan for answer Exercise 10

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Express Improper Fraction in Partial Fraction

The numerator must be a lower degree than the denominator. If not, then it must be divided out.

Example 21

Construct $\dfrac{4x^3+10x+4}{x(2x+1)}$ into partial fractions.

Solution

Step 1

Identify the type of fraction.

The numerator is of degree 3. The denominator is of degree 2. So, this fraction is improper. This means that we are going to divide the numerator by the denominator.

$$\frac{4x^3+10x+4}{x(2x+1)}=\frac{4x^3+10x+4}{2x^2+x}$$

Expand the bottom

$$\begin{array}{r|l} & \quad\quad\quad\quad\quad\quad 2x-1 \\ \cline{2-2} 2x^2+x & 4x^3+0x^2+10x+4 \\ (-) & \underline{4x^3+2x^2} \\ & \quad\quad -2x^2+10x+4 \\ & (-)\ \underline{-2x^2-x} \\ & \quad\quad\quad\quad 11x+4 \end{array}$$

Divisor

Remainder

<u>WHEN TO STOP?</u>

Degree of remainder < Degree of divisor

$$\frac{4x^3+10x+4}{x(2x+1)}=\frac{4x^3+10x+4}{2x^2+x}$$

$$=2x-1+\frac{11x+4}{2x^2+x}$$

Solution (Continue)

Step 2

Take the proper fraction and factor the denominator.

$$\frac{11x+4}{2x^2+x}=\frac{11x+4}{x(2x+1)}$$

Step 3

Write one partial fraction for each of those factors.

$$\frac{11x+4}{2x^2+x}=\frac{11x+4}{x(2x+1)}=\frac{A}{x}+\frac{B}{2x+1}$$

Step 4

Multiply through the denominator so that no longer factors.

$$11x+4=A(2x+1)+B(x)$$

Step 5

Find the constants.

Substituting the roots ("zero") of the bottom can help:

Root for $2x+1$ is $x=-\frac{1}{2}$

When $x=-\frac{1}{2}$

$$11\left(-\frac{1}{2}\right)+4=A(0)+B\left(-\frac{1}{2}\right)$$

$$-\frac{3}{2}=0-\frac{1}{2}B$$

$$B=3$$

Root for x is $x=0$

When $x=0$

$$4=A(1)+B(0)$$

$$A=4$$

Hence,

$$\frac{11x+4}{x(2x+1)}=\frac{4}{x}+\frac{3}{2x+1}$$

Thus,

$$\frac{4x^3+10x+4}{x(2x+1)}=2x-1+\frac{4}{x}+\frac{3}{2x+1}$$

Example 21

Construct $\dfrac{x^2+9x+8}{x^2+x-6}$ into partial fractions.

Solution

The numerator is of degree 2. The denominator is of degree 2. So this fraction is improper. This means that we are going to divide the numerator by the denominator.

$$\begin{array}{r@{\,}r}
 & 1 \\
x^2+x-6 \,\big) & x^2+9x+8 \\
(-) & x^2+x-6 \\
\hline
 & 8x+14
\end{array}$$

Thus,

$$\frac{x^2+9x+8}{x^2+x-6}=1+\frac{8x+14}{x^2+x-6}$$

Take the proper fraction

$$\begin{aligned}
\frac{8x+14}{x^2+x-6} &= \frac{8x+14}{(x-2)(x+3)} \\
&= \frac{A}{x-2}+\frac{B}{x+3}
\end{aligned}$$

$$8x+14 = A(x+3)+B(x-2)$$

When $x-2=0$

$$x=2$$

$$\begin{aligned}
8(2)+14 &= A(2+3)+B(2-2) \\
30 &= A(5)+B(0) \\
30 &= 5A \\
A &= 6
\end{aligned}$$

When $x+3=0$

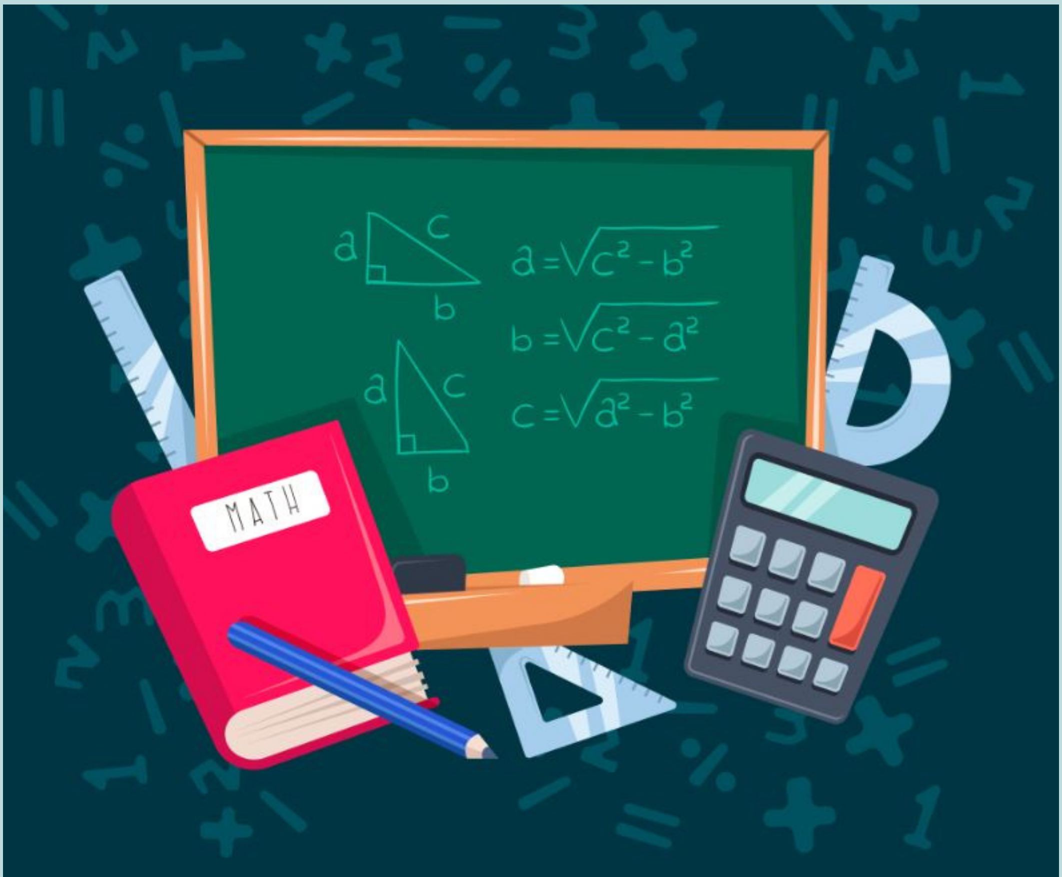
$$x=-3$$

$$\begin{aligned}
8(-3)+14 &= A(-3+3)+B(-3-2) \\
-10 &= A(0)+B(-5) \\
-10 &= -5B \\
B &= 2
\end{aligned}$$

Solution (continue)

Hence, $$\frac{8x+14}{x^2+x-6}=\frac{8x+14}{(x-2)(x+3)}=\frac{6}{x-2}+\frac{2}{x+3}$$

Thus, $$\frac{x^2+9x+8}{x^2+x-6}=1+\frac{6}{x-2}+\frac{2}{x+3}$$



Exercise 11

Construct the following fractions into partial fractions.

1) $$\frac{x^2 - x + 1}{(x+8)(x-2)}$$

2) $$\frac{5x^2 - 2}{(x-1)(x+4)}$$

3) $$\frac{4x^3 + 3x^2 - 2x + 1}{x^2 - x - 6}$$

4) $$\frac{(x+1)(x+2)}{(x+3)(x-1)}$$

5) $$\frac{x^2 - 3x + 4}{x^2 - x - 20}$$

Scan for answer Exercise 11

Scan for answer

http://tiny.cc/c1e11

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