POLITEKNIK MALAYSIA
UNGKU OMAR

e-Stat
Probability

2022

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e-Stat: Probability

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http://www.puo.edu.my

First publication : 2022

Published by:

Politeknik Ungku Omar
Jalan Raja Musa Mahadi
31400 Ipoh, Perak

e ISBN 978-967-2421-60-3

9 7 8 9 6 7 2 4 2 1 6 0 3

e-Stat: Probability

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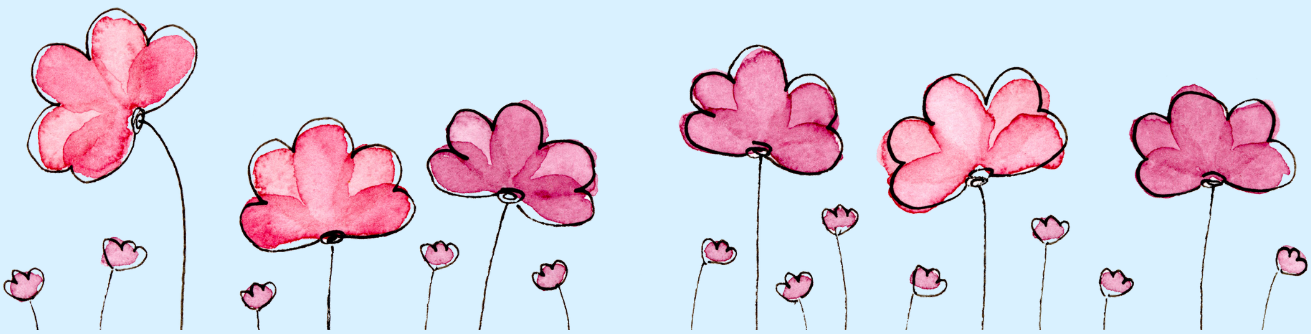
Preface

When relate to actual situations, many students find the probability topic is challenging and complex. Thus, it is a need to create new materials, such as an e-book to explain the core of this topic in easy, clear and simple way as well as giving numerous guided exercises to reinforce students' and readers' understanding.

The contents of this e-book adequately address the main points of the topic of probability. It discuss the theories, ideas and practices with the help of numerous numerical and graphical representations for students to have better understanding. For revision, towards the end of the topic, some practical questions with answers are provided.

We hope that the approach presented in this e-book will benefit the learning process. Hence, we are truly hope that this resource will become the preferred guide for students.

Ziehanie bt Shafiai
Noor Hasnida bt Mohd Najib



Acknowledgement

This e-book would not have been possible without the commitment and support of PUO's E-Learning team. We would like to express our gratitude to our Head of Department, Pn Noor Afida bt Omar and our Programme Coordinator, Cik Noor Faliza bt Omar for offering us the opportunity to improve our teaching and learning materials for the benefit of the students. Finally, we want to show our appreciation to our supportive colleagues and family members for their unconditional support throughout this commitment.

Table of Contents

Definition of Probability

Probability *is a measure of how likely an event is to happen*

PROBABILITY is a numerical value that expresses the ration between the value required and the set of all possible values that could occur.

Concept of Outcomes, Sample Space and Events

EXPERIMENT

- EXPERIMENT is a process that leads to one of several possible outcomes.

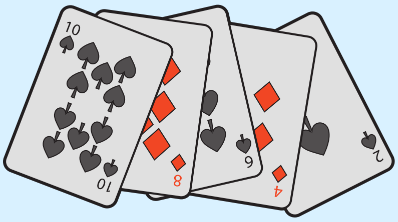
OUTCOMES

- OUTCOMES refer to potential results of an experiment.

SAMPLE SPACE

- SAMPLE SPACE is all the experiments finding.
- Example: Toss a coin it's either head (H) or tail (T) so the samples S = (H, T)

EVENTS



- EVENTS refer to a set of outcomes or the subset of the sample space
- EVENTS consist of one or more results of probability experiment

Probability Using Classical Formula

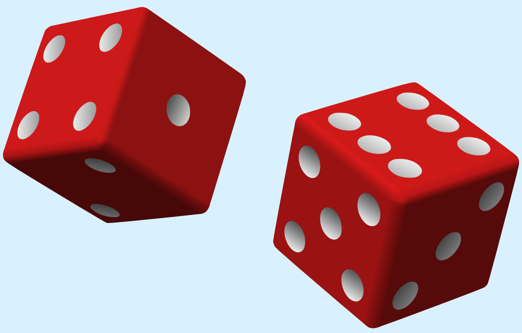
$$P(A) = \frac{\text{number of outcomes in the set}}{\text{number of outcomes in the sample space}}$$

$$P(A) = \frac{n(A)}{n(S)}$$

Example 1

A standard six-sided dice has 6 faces. On each face of the dice is the number 1,2,3,4,5, and 6 dots. If you roll a dice, determine the probability of:

a. getting number 5
b. obtaining an odd number
c. getting an even number



Probability Using Classical Formula

$$P(A) = \frac{\text{number of outcomes in the set}}{\text{number of outcomes in the sample space}}$$

$$P(A) = \frac{n(A)}{n(S)}$$

Solution (a)

Experiment: Roll a dice

Outcomes: 1,2,3,4,5,6

Sample space, S: {1,2,3,4,5,6}

A : Event of getting number 5

A = {5}

n(A) = 1

The probability of getting 5 dots

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A) = \frac{1}{6}$$

Solution (b)

Experiment: Roll a dice

Outcomes: 1,2,3,4,5,6

Sample space, S: {1,2,3,4,5,6}

A : Event of obtaining an odd number

A = {1,3,5}

n(A) = 3

The probability of obtaining an odd number

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A) = \frac{3}{6}$$

$$= \frac{1}{2}$$

Solution (c)

Experiment: Roll a dice

Outcomes: 1,2,3,4,5,6

Sample space, S: {1,2,3,4,5,6}

A : Event of obtaining an even number

A = {2,4,6}

n(A) = 3

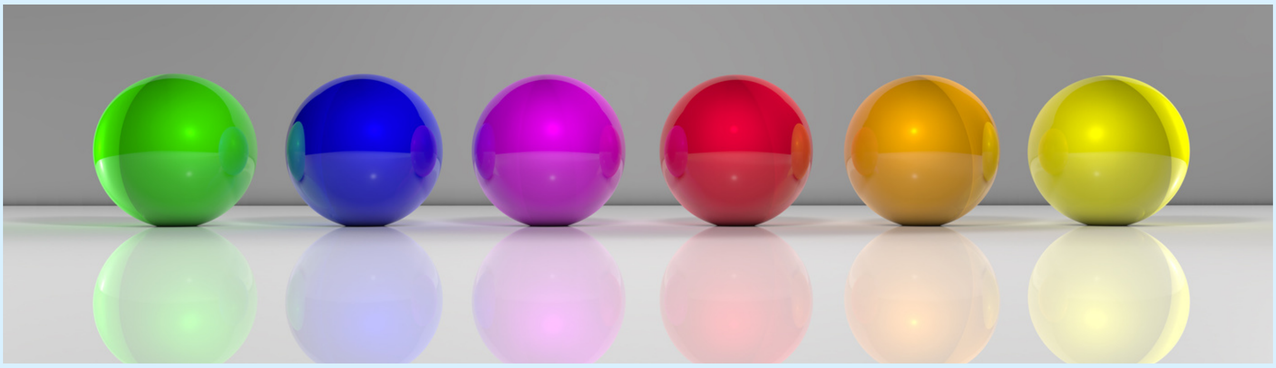
The probability of obtaining an even number

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A) = \frac{3}{6}$$

$$= \frac{1}{2}$$

Complementary Events



Complementary Events

- Two events are said to be complementary when one event occurs if and only if the other does not.
- The probabilities of two complimentary events add up to 1.

$$P(E') = 1 - P(E)$$
$$P(E) = 1 - P(E')$$
$$P(E) + P(E') = 1$$

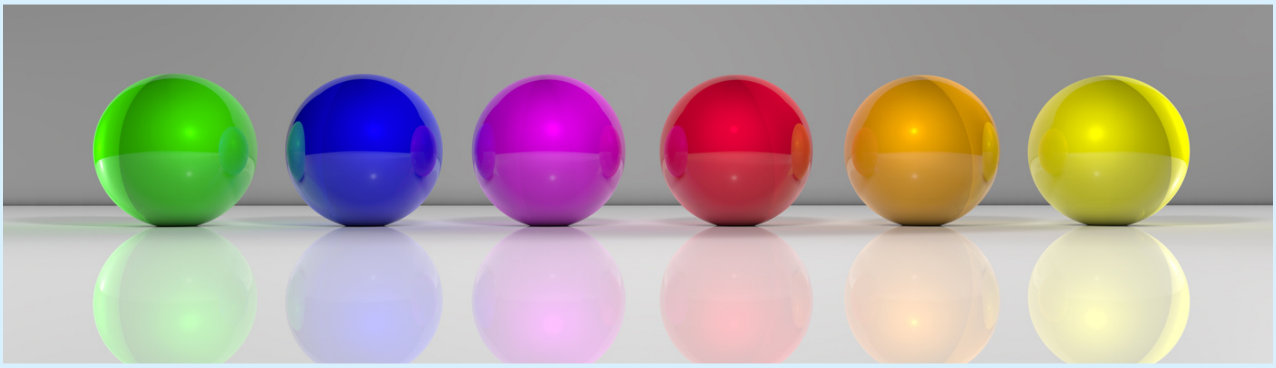
Example 2

The probability of getting a blue ball from a bag of balls is 1/5. Determine the probability of not getting a blue ball.

Solution

P (ball is not blue) = 1 – 1/5
= 4/5

Complementary Events



Example 3

There are 15 similarly shaped blocks in a bag. 5 of them are purple and the remaining are green. The bag is well-shaken and a single block is drawn. Calculate:

a. The probability that the block is purple
b.The probability that the block is not purple

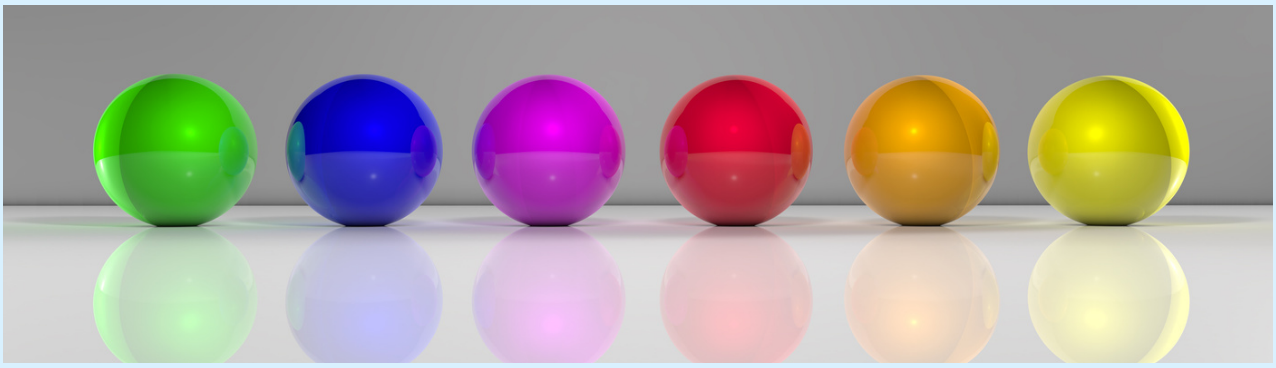
Solution (a)

P (purple) = 5/15
= 1/3

Solution (b)

P (not purple) = 1 – P (purple)
= 1 – 1/3
= 3/3 – 1/3
= 2/3

Complementary Events



Example 4

An airline reports that 95% of its flights arrive on time. Determine the probability that a flight will be late.

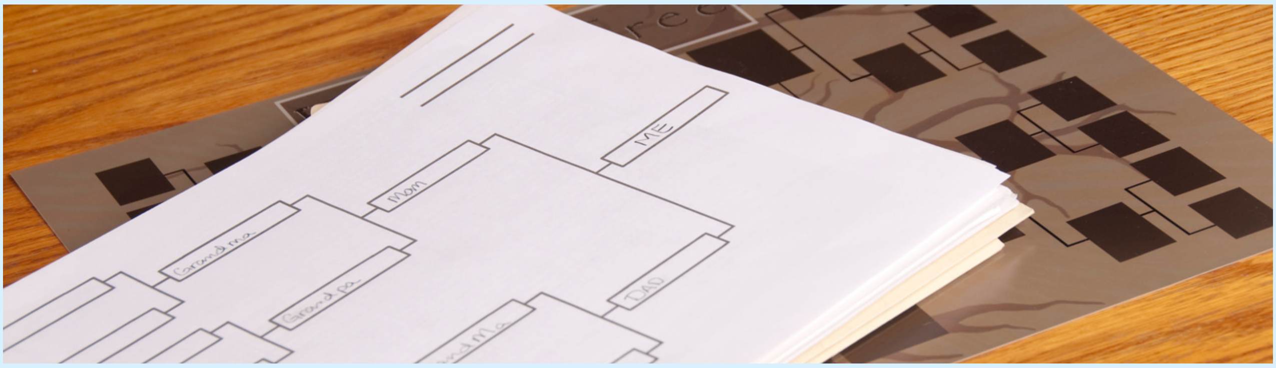
Solution

P (A) = 0.95
P (Not A) = ?

$$P(A) + P(A') = 1$$
$$P(A') = 1 - P(A)$$
$$= 1 - 0.95$$
$$= 0.05$$

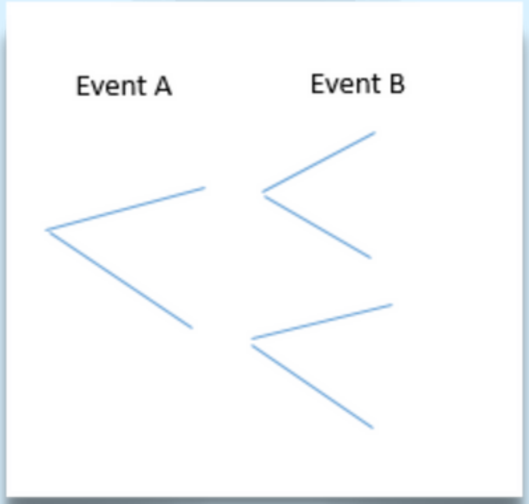
Probability that a flight will be late is 5%.

Tree Diagram

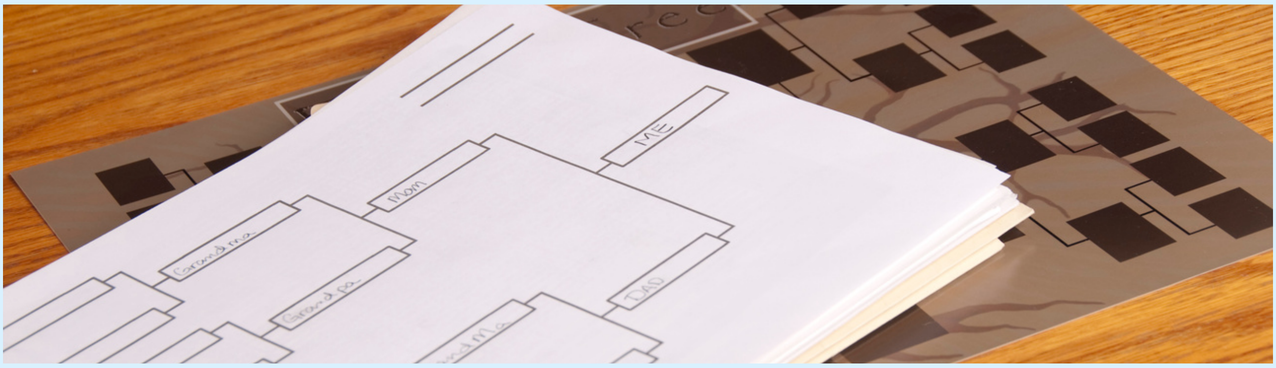


Tree Diagram

- An alternative to the contingency table.
- A possible outcome for each event is represented by each branch in a tree diagram.
- Need to identify each event and its outcome.
- List all possibilities of a sequence of events in a systematic way.



Tree Diagram



Example 5

You toss a coin three times. Find the probability of getting 3 heads by using Tree Diagram

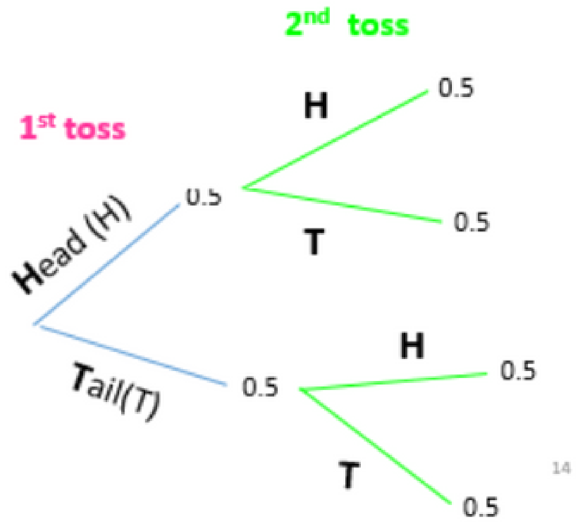
Solution

Step 1

Draw lines to represent the first set of options in the question and label it (Head and Tail). Those probabilities are represented at the ends of each branch.

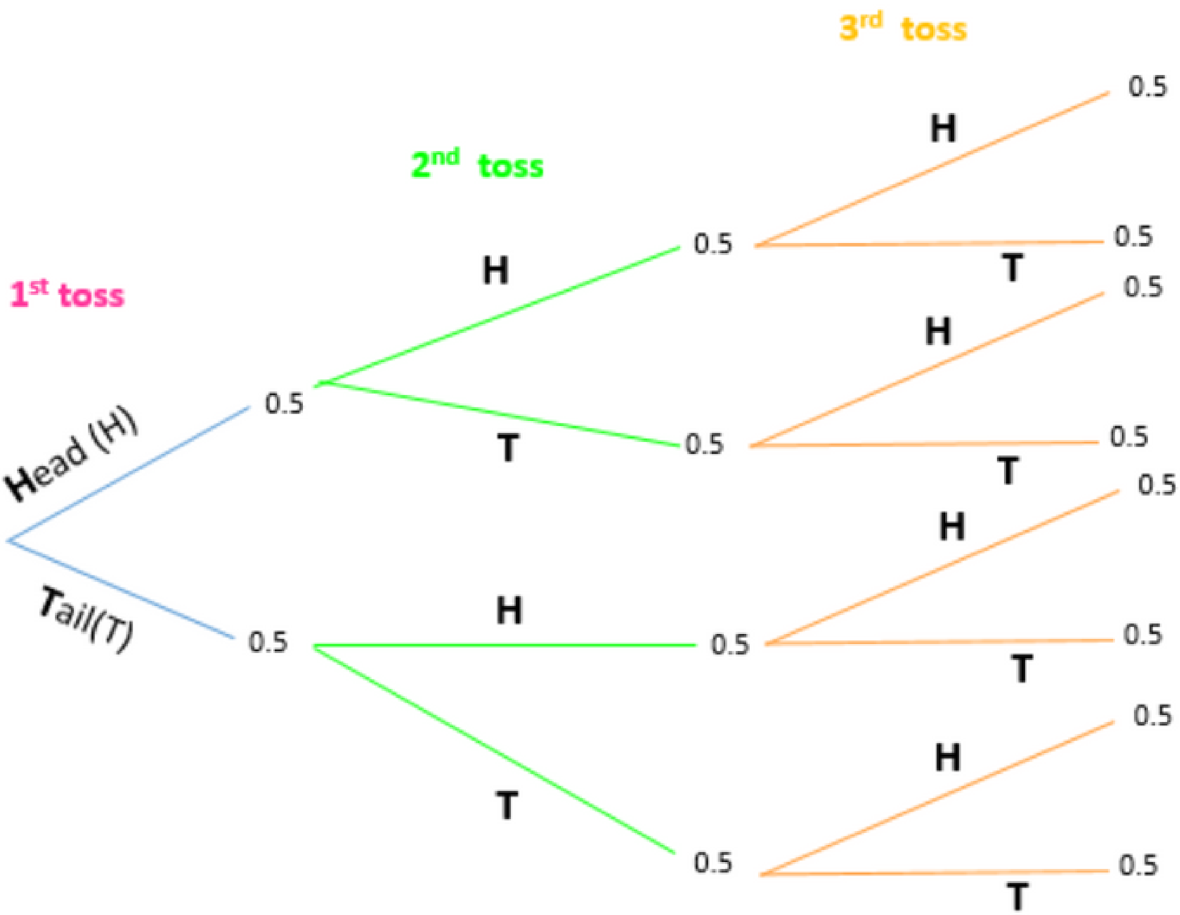
Step 2

Next, add two more branches to each branch to represent the second coin toss.



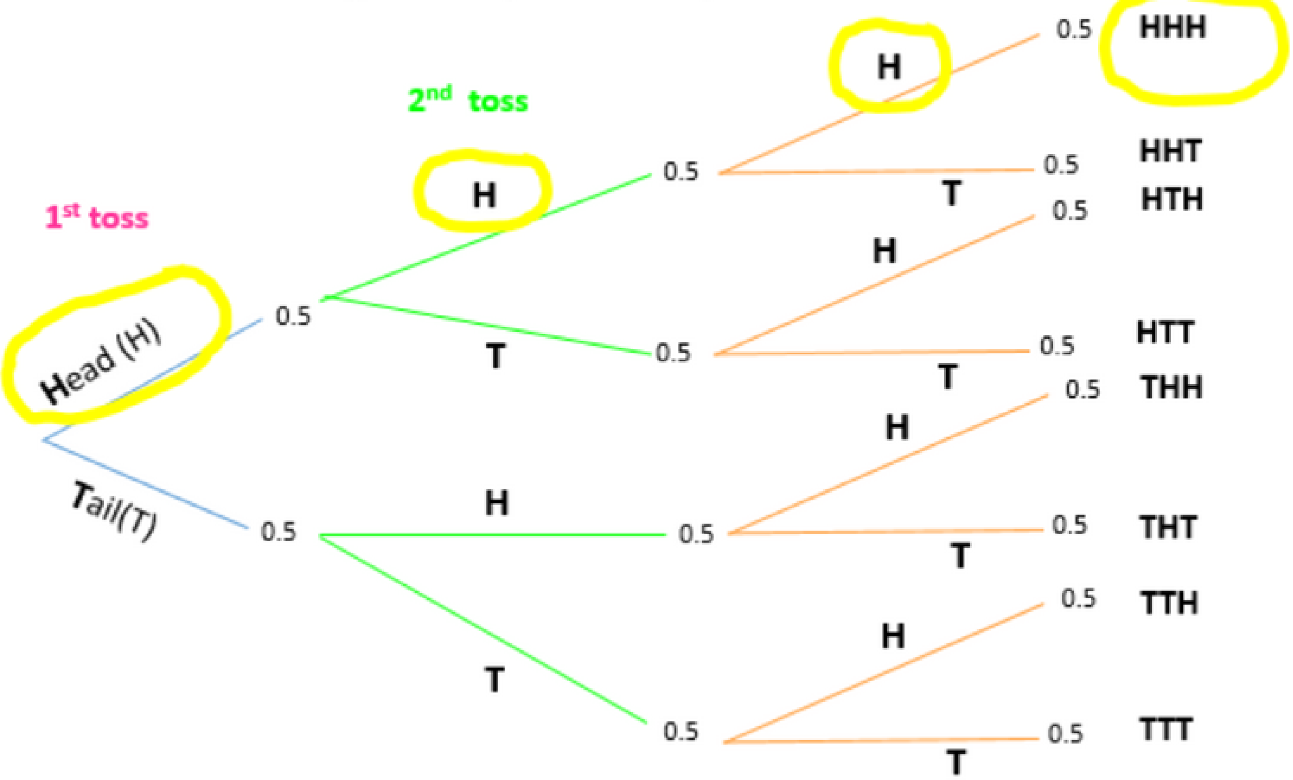
Step 3

Add a third row (to find the probability of throwing 3 heads)



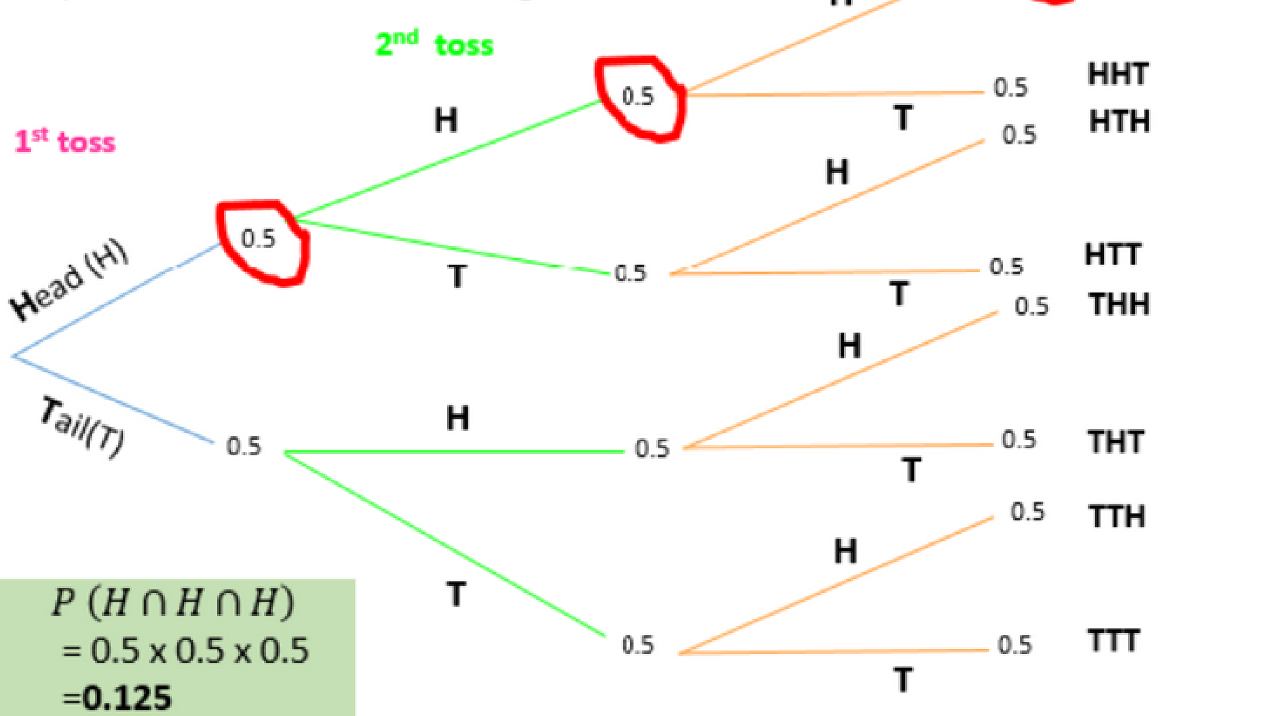
Step 4

Find the outcomes (probability of **3 heads**)



Step 5

Find the probability of getting **3 heads.** Multiply the probabilities of the first branch that produces the desired result together

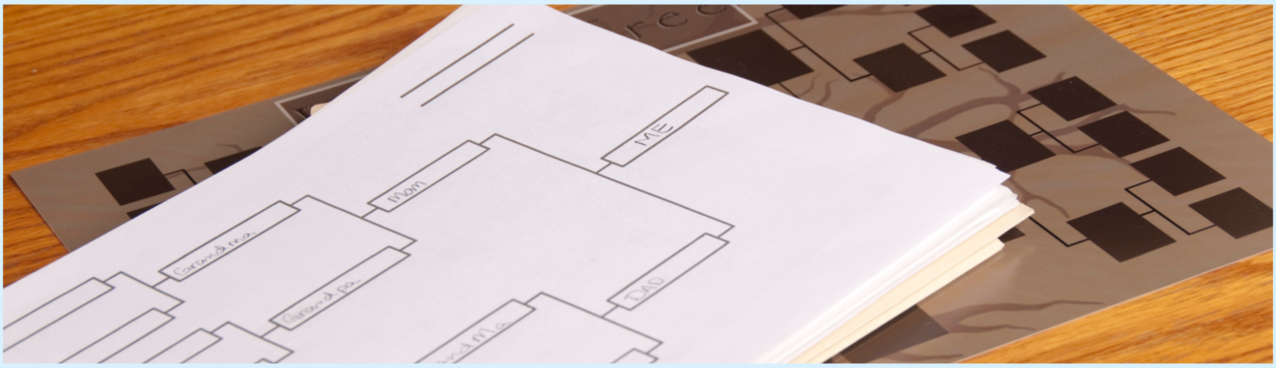


$P\,(H \cap H \cap H)$

$= 0.5 \times 0.5 \times 0.5$

$=$ **0.125**

Tree Diagram



Example 6

A bag contains 4 cards numbered 3, 5, 7, 9. The second bag contains 3 cards numbered 4, 5, 6. One card is drawn at random from each bag.

a. Construct a tree diagram with all of the possible outcomes.

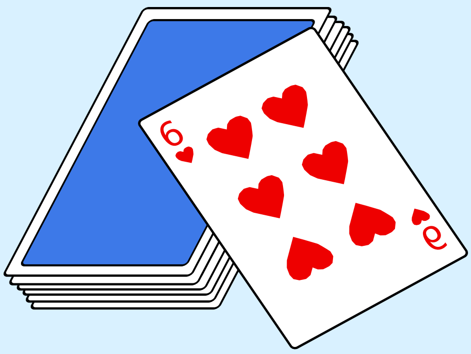
b. Calculate the probability that the two numbers obtained:

i. have different values

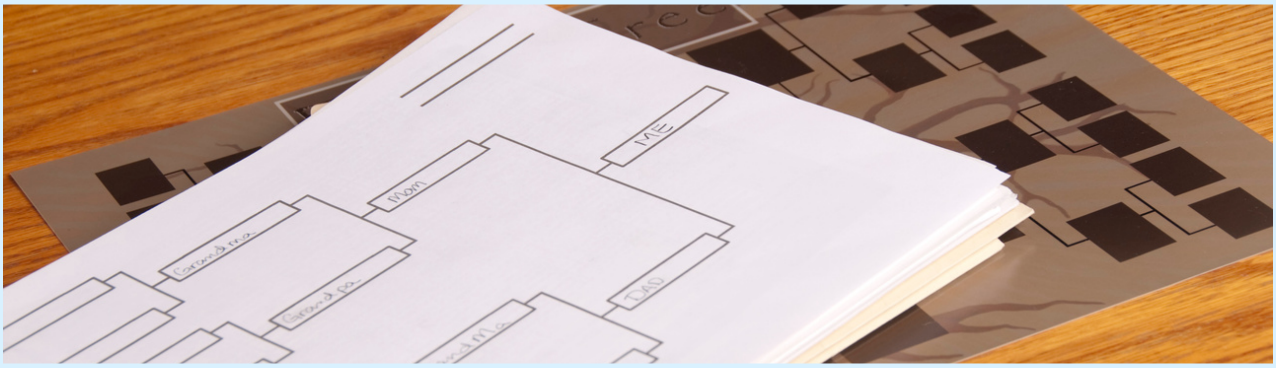
ii. are odd numbers

iii. have a sum greater than 8

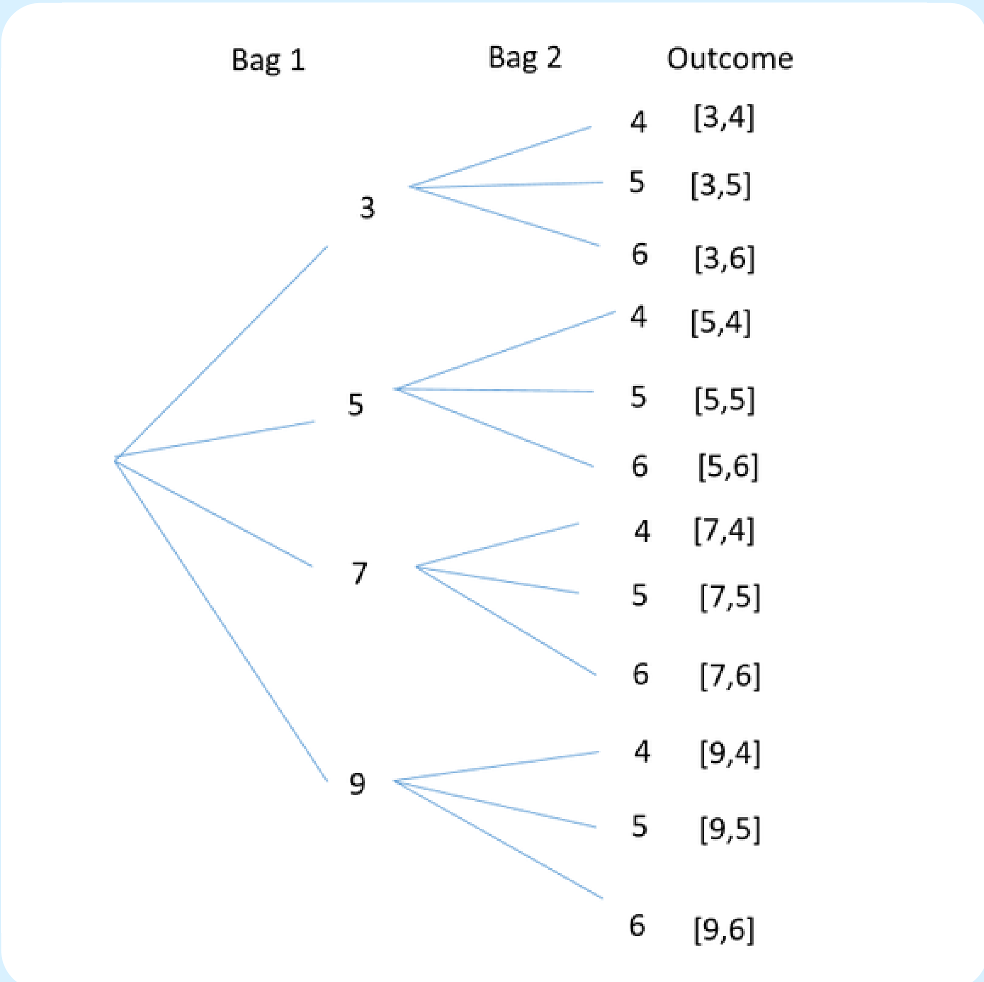
iv. have a sum less than 9



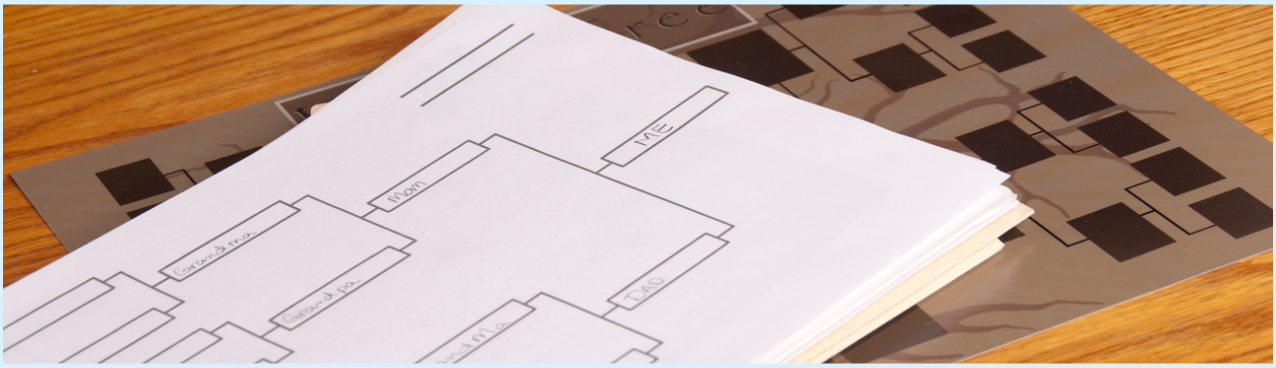
Tree Diagram



Solution (a)



Tree Diagram



Solution (b)(i)

Let S be the sample space and A be the event that the two values are different.

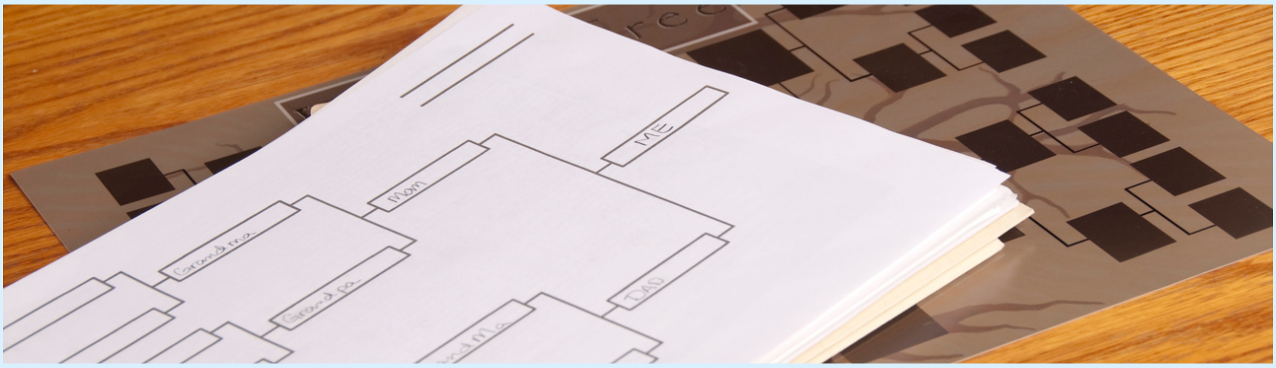
$n(S) = 12$
$n(A) = 11$
$P(A) = \frac{11}{12}$

Solution (b)(ii)

Let B be the event that both values are odd

$n(B) = 4$
$P(B)= \frac{4}{12}$

Tree Diagram



Solution (b)(iii)

Let C be the event the sum of both values is greater than 6

$n(C) = 10$

$P(C) = \frac{10}{12}$

$= \frac{5}{6}$

Solution (b)(iv)

Let D be the event the sum of both values is less than 9

$n(D) = 2$

$P(D) = \frac{2}{12}$

$= \frac{1}{6}$

Two-Way Tables

Two-Way Tables

- Data regarding two different variables can be arranged.
- Organize data based on two categorical variables.
- Also known as contingency tables.

Two-Way Tables

Example 7

Suppose that a company is doing market research on a most effective TV commercial.

The sample of potential customers were chosen randomly. Out of 180 people in the sample, 65 viewed the first version, 30 viewed the second version, and the balance viewed the third.

According to those who viewed the first version, 25 indicated that they were likely to buy the product while, the rest said they were unlikely to buy the product. For those who viewed the second version, 20 said they were likely to buy the product and those who viewed the third version, 54 said the same.

Sources: Making two way tables. Retrieved August 13, 2020., from https://www.mathbootcamps.com/making-two-way-tables/

Two-Way Tables

Solution

Step 1: Identify the variables

- Identify the variables
- There are two variables:
  - **commercial viewed**
  - **opinion**

Step 2: Determine the possible values of each variable

- **Commercial viewed:** version 1, version 2, version 3
- **Opinion:** likely to buy the product or unlikely to buy the product

Two-Way Tables

Solution

Step 3: Set up the table

- Pick one variable to be represented by the rows and one to be represented by the columns. It doesn't matter which!
- Then, use the possible values of the variables to represent the rows and columns.
- Finally, be sure to add the total for the columns and the rows

		Version			
Oponion		1	2	3	Total
	Likely to buy				
	Unlikely to buy				
	Total				

Two-Way Tables

Solution

85-54=31

25+20+54=99

		Version			
Oponion		1	2	3	Total
	Likely to buy	25	20	54	99
	Unlikely to buy	40	10	31	81
	Total	65	30	85	180

40+10+31=81

65-25 = 40

30-20=10

180-(65+30)=85

Venn Diagram

Venn Diagram

- Venn diagram visualizes the sample space.
- The overlapping circles or other shapes show the logical connections between two or more set of items.
- It represents the differences and the similarities between the two events.
- It also represents the various events as "union" and "intersections" of circles.

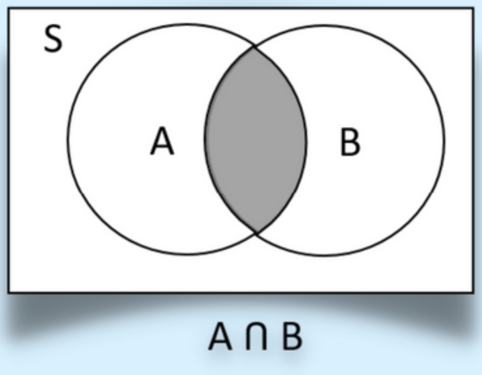
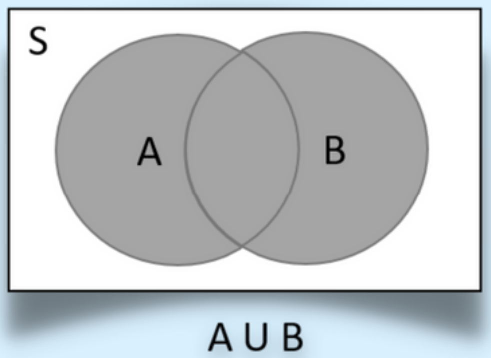


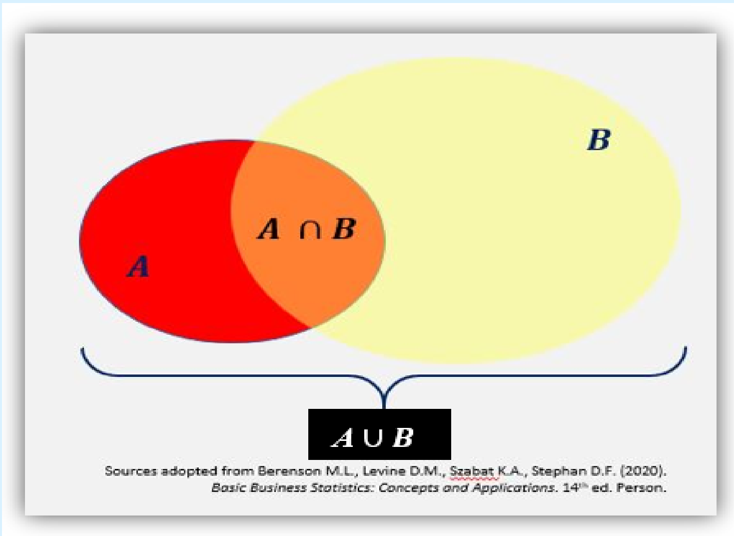
Figure 1: The intersection of two events

Figure 2 : The union of two events



Venn Diagram

Example 8



This figure shows a Venn diagram for a 2 variable situation, with each variable having only two events which is:

- Event A and A’
- Event B and B’

- The red circle represents all events that are part of A.
- The yellow circle represents all events that are part of B.
- They are contained with circle A and circle B which are the intersection of A and B, written as $A \cap B$.
- The total of the two circles is union of A and B, written as $A \cup B$.

Venn Diagram

Example 9

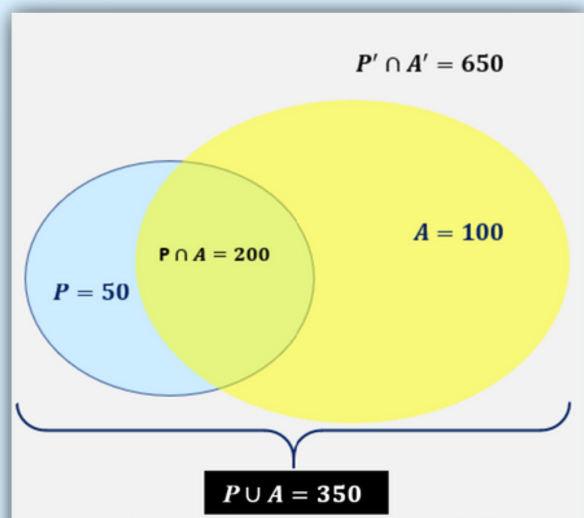
The following data shows the findings of a survey of 1,000 households in term of purchase behavior for Smart TVs.

Planned to purchase	**Actual Purchased**		
	Yes	**No**	**Total**
Yes	200	50	**250**
No	100	650	**750**
Total	**300**	**700**	**1000**

Based on the given data, visualize the Venn Diagram.

Venn Diagram

Solution



Sources adopted from Berenson M.L., Levine D.M., Szabat K.A., Stephan D.F. (2020). *Basic Business Statistics: Concepts and Applications.* 14th ed. Person.

Events:

- Event P = planned to purchase while P'= did not plan to purchase
- Event A= actually purchased while A'= did not actually purchased

The value of the intersection of P and A = 200. It consists of all 200 households who planned to purchase and actually purchased a Smart TV.

$P \cap A$=200

- The balance of event P (planned to purchase) consists of 50 households. It is because:
    - $\sum event\ P$= 300, so balance of event, P =250-200= 50
- The balance of event A (actually purchased) consists of 100 households
    - $\sum event\ A$=300, so balance of event, A=300-200= 100
- The balance 650 households represent those who neither planned to purchase nor actually purchased a Smart TV.

Additional Rules

Additional Rules

- It consists of taking the probability of event A and adding it to the probability of event B and then subtracting the probability of the joint event A and B from the total. It is because the joint probability has already been included in computing, both the probability of event A and the probability of event B.

- Joint probability is the likelihood that two or more occurrence will occur together. The probability that you will get Head on the 1st toss of a coin and Head on the 2nd toss of coin is an example of joint probability.

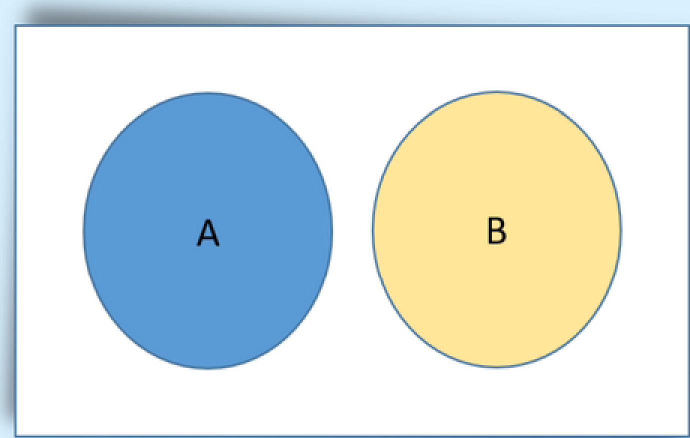
- The key word when using the additional rule is ***or***

- ***The formula:*** $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Additional Rules

Mutually Exclusive

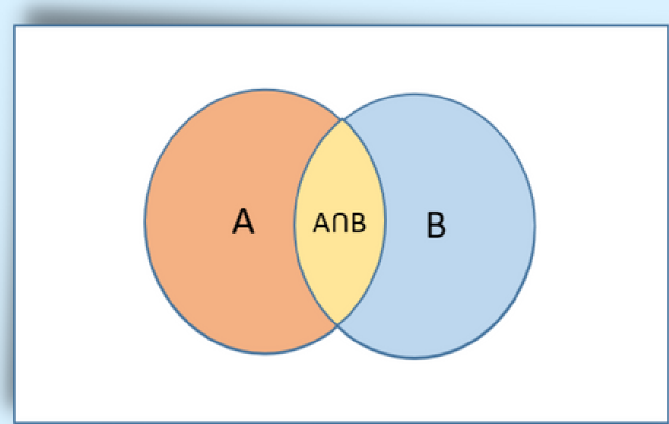
- A set of events are mutually exclusive if they **cannot occur together.**
- For example, when a coin is tossed, the outcome will either Head (H) or Tail (T). It cannot be both as the result.



$P(A \cup B) = P(A) + P(B)$

Non-mutually exclusive

- A set of events are non-mutually exclusive if they **occur at the same time.**
- For example, when a student is either take Principles of Management or Organizational Behavior or both subjects this semester.



$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Additional Rules

Example 10

The following data presents the result of the sample of 1,000 households survey in term of purchase behaviour for Smart TVs.

Planned to purchase	Actual Purchased		
	Yes	**No**	**Total**
Yes	200	50	**250**
No	100	650	**750**
Total	**300**	**700**	**1000**

Determine the probability that a household planned to purchase ***or*** actually purchased a Smart TV.

Additional Rules

Solution

By using this example, determine the probability that a household planned to purchase or actually purchased a Smart TV.

Planned to purchase	Actual Purchased		
	Yes	No	Total
Yes	200	50	250
No	100	650	750
Total	300	700	1000

P(Planned to purchase ∪ actually purchased) = P (Planned to purchase) + P (Actually purchased) – P (Planned to purchase ∩ actually purchased)

$$= \frac{250}{1000} + \frac{300}{1000} - \frac{200}{1000}$$

$$= \frac{350}{1000}$$

Additional Rules

Example 11

The probability that a student passes Statistics is 2/3 and the probability that he passes Accounting is 4/9 . If the probability that he will pass at least one subject is 4/5, what is the **probability that he will pass both subjects**?

Solution

$$n(Statistics) = \frac{2}{3}$$

$$n(Accounting) = \frac{4}{9}$$

$$P(S \cup A) = \frac{4}{5}$$

$$\boldsymbol{P(S \cup A) = P(S) + P(A) - P(S \cap A)}$$

$$\frac{4}{5} = \frac{2}{3} + \frac{4}{9} - P(S \cap A)$$

$$\frac{4}{5} - \frac{2}{3} - \frac{4}{9} = -P(S \cap A)$$

$$\cancel{-}\boldsymbol{\frac{14}{45}} = \cancel{-}\boldsymbol{P(S \cap A)}$$

Multiplication Rules

Multiplication Rules

- The rule of multiplication applies to the situation when we want to know the probability of the intersection of two events; that is, we want to know the probability that two events (Event A and Event B) both occur.

Multiplication Rules

Independent events

- Two events statistically independent when the occurrence of one event is **not affected** by the other event.
- Formula:

$$P(A \cap B) = P(A).P(B)$$

Non-independent events

- A set of events are non-mutually independent when the occurrence of one event **is affected** by the occurrence of another event.
- Formula:

$$P(A \cap B) = P(A).P(B|A)$$

Multiplication Rules

Example 12

Ali have a cowboy hat, a top hat, and a songkok. He also have four shirts which are white, black, green, and pink. If Ali choose one hat and one shirt at random, determine the probability that he choose the songkok and the black shirt.

Solution

$$P(A) = \frac{1}{3}$$

$$P(B) = \frac{1}{4}$$

$$P(A \cap B) = P(A).P(B)$$

$$= \frac{1}{3} \text{x} \frac{1}{4}$$

$$= \frac{1}{12}$$

***The two events are *independent events;* the choice of hat has no effect on the choice of shirt.**

Multiplication Rules

Example 13

A vase contains 6 red marbles and 4 black marbles. Two marbles are drawn **without replacement** from the vase. What is the probability that both of the marbles are black?

Solution

$$P(A) = \frac{4}{10} \qquad P(A \cap B) = P(A).P(B|A)$$

$$P(B|A) = \frac{3}{9} \qquad = \frac{4}{10} \text{x} \frac{3}{9}$$

$$= \frac{2}{15}$$

The two events are **dependent events because the first marble is not replaced.*

Multiplication Rules

Example 14

A vase contains 6 red marbles and 4 black marbles. Two marbles are drawn **with replacement** from the vase. Calculate the probability that both of the marbles are black.

Solution

$P(A) = \frac{4}{10}$

$P(B|A) = \frac{4}{10}$

$$P(A \cap B) = P(A).P(B\,|\,A)$$

$$= \frac{4}{10} \times \frac{4}{10}$$

$$= \frac{4}{25}$$

Summary

- $P(E) = 0$ if the event cannot occur
- $P(E) = 1$ if the event is certain
- Complementary event rule:

  $P(E') = 1 - P(E)$

  $P(E) = 1 - P(E')$

  $P(E) + P(E') = 1$
- The sum of the probabilities of the outcomes in the sample space is 1
- Mutually exclusive events

  $$P(A \cup B) = P(A) + P(B)$$
- Non Mutually exclusive events

  $$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
- Independent events

  $$P(A \cap B) = P(A).P(B)$$
- Dependent events

  $$P(A \cap B) = P(A).P(B|A)$$

Quick Revision

REVIEW EXCERCISE

Answers

ANSWERS

References

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