Verawaty Ismail
Mohd Hilmi Ariffin
Nurul Khairunnisa Su

FLUID MECHANICS

VOL. 2

Malaysian Polytechnics Version

Verawaty Ismail
Mohd Hilmi Ariffin
Nurul Khairunnisa Su

FLUID MECHANICS
VOL. 2

Malaysian Polytechnics Version

POLITEKNIK MALAYSIA
TUANKU SYED SIRAJUDDIN

Department of
Mechanical Engineering,
PTSS, Perlis.

Terbitan:
Politeknik Tuanku Syed Sirajuddin (PTSS)
Pauh Putra, 02600 Arau, Perlis
e ISBN 978-629-7514-02-4

PREFACE

This e-book is intended to provide Malaysian Polytechnics' students with a clear yet simple explanations of the basic theory and applications of fluid mechanics. The FLUID MECHANICS VOL. 2 e-book covers the last TWO (2) topics in the Polytechnics' Fluid Mechanics syllabus namely; fluid dynamics and energy loss in pipeline.

The explanations in this book are mostly aided by diagrams for better understanding. Also, every topic in this e-book contained solved examples and tutorial questions for the students to enhance their knowledge of the topic. With these features, it is hoped that the book will be useful and tremendous help for the students in the basic of Fluid Mechanics at diploma level.

A few notable reference books on Fluid Mechanics has been referred while writing this e-book, in which are mentioned at the last section of the book. Students are also encouraged to search and refer the books mentioned for further reading.

CONTENTS

Chapter 1:

FLUID DYNAMICS

1.1 INTRODUCTION

Fluid Dynamics is a category of fluid mechanics that basically describes the flow of fluids - liquids and gases. It considers the forces that cause acceleration of a fluid. It has several subdisciplines, most notably aerodynamic (the study of flow of air and other gases) and hydrodynamics (the study of motion of incompressible liquids). Fluid dynamics has a wide range of applications including; calculating forces and moments on aircraft and determining flow rate of petroleum / natural gas through pipelines.

In fluid mechanics, it is more practical to apply the **conservation of energy principle.** By applying the conservation of mass equation, it is easier to balance the incoming and outgoing flowrates in a flow system.

This chapter will first, classify the various types of fluid flow and how to identify flow patterns, followed by, discussion of **Bernoulli equation** applied to solve a variety of fluid flow problems.

1.2 TYPES OF FLOW

In most cases, fluid do not remain static, but rather they flow. There are various ways to classify the flow of a fluid system.

a) Classification of flow based on **space and time**

Uniform flow – properties of fluid (i.e; velocity, temperature, mass, energy, pressure, etc) **do not** change with **location** over a specified region.

For a uniform flow, by its definition, the area of cross section for the flow should remain constant. Thus, the flow of a liquid through a pipeline with constant diameter is a fitting example of a uniform flow (**Figure 1.1**).

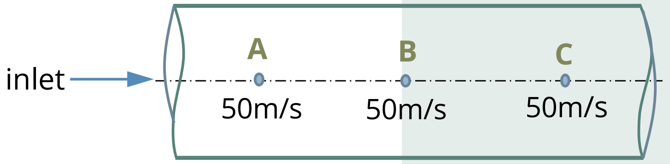


Figure 1.1: At any instant of time, velocity at three different locations (A, B and C) in a cylindrical pipe with constant diameter are the same.

Steady flow – properties of fluid (i.e: velocity, temperature, mass, energy, pressure, etc) at a point **do not** change with **time.** During steady flow, the fluid properties can change from point to point within a device, but at any fixed point they remain constant. **Figure 1.2** shows an example of steady flow through a converging-diverging nozzle.

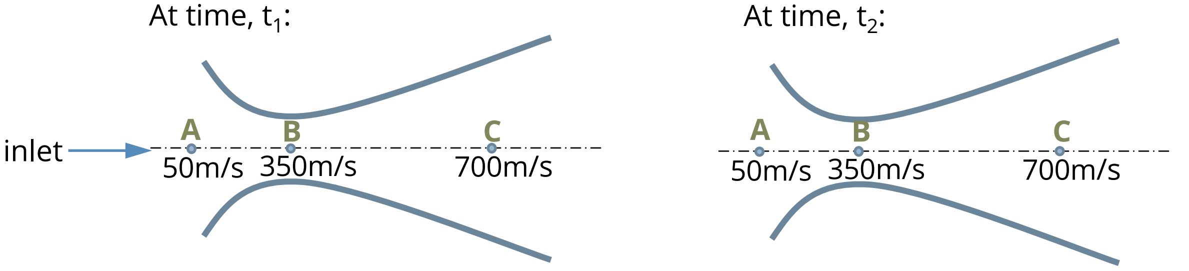


Figure 1.2: At any instant of time, velocity at three different locations, A, B and C in a converging-diverging nozzle are the same.

Steady flow conditions can be approximated by devices that are intended for continuous operation such as pumps, turbines, boilers, nozzles, heat exchangers and condensers.

For a **steady and uniform flow**, the properties of fluid do not change with time and position of the flow. An example (**Figure 1.3**), is the flow of liquid through a pipeline of constant diameter at constant velocity.

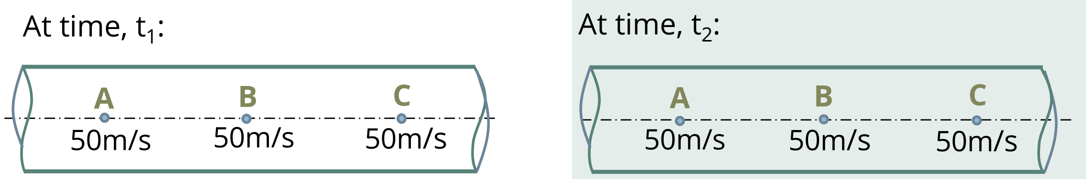


Figure 1.3: In a pipeline with constant diameter, the flow velocity at three different locations is 50 m/s at two different time, t_1 and t_2.

b) Classification of flow related to its **frictional effects**

Laminar – **highly ordered fluid motion** in the form of smooth layers, streamlined at low velocities.

Transition – a region where **both laminar and turbulent flow coexist.**

Turbulence – **highly disordered fluid motion** that typically occurs at high velocities in the form of erratic fluctuations.

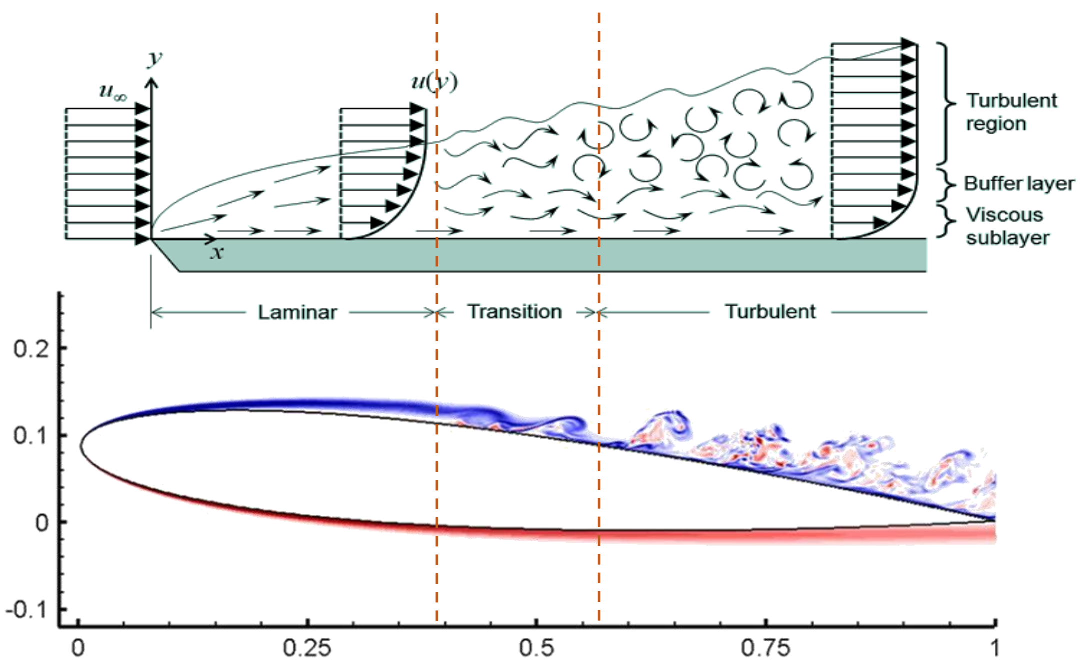
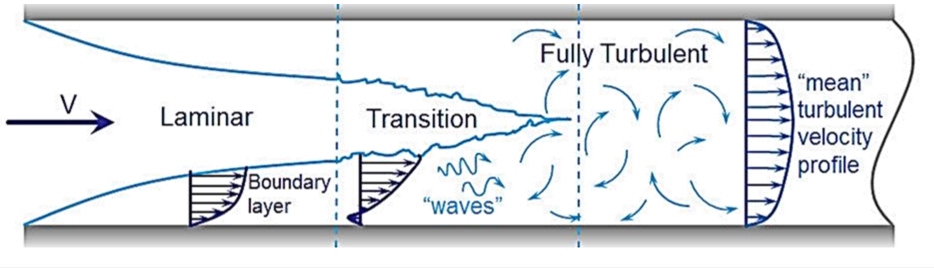


Figure 1.4: Air flow profile on an aerofoil design showing laminar, transition and turbulence flow.

Table 1.1 further explain the characteristics of laminar, transition and turbulent flow of fluid.

Table 1.1 : Laminar, transition and turbulent flow characteristics.

	Laminar	Transition	Turbulence
Motion	Highly ordered fluid motion	Exists between laminar and turbulent	Highly disordered fluid motion.
Characteristics	Smooth layers of fluid	Unpredictable, changeable between laminar and turbulent.	Velocity fluctuates, fluid exhibits complex path.
Discharge	Low	Medium	High
Fluid Molecule	Followed straight path parallel to the boundaries of pipe/tube.	Followed wavy but parallel path that was not parallel to boundaries of the pipe/tube.	Followed erratic paths due to the increase in velocity or decrease in viscosity.



1.3 FLOW RATE

Volume Flow Rate. The rate at which volume of fluid flowing through a cross sectional area, A per unit time, *t* is called the **volume flow rate**. Volume flow rate also referred as *flow* or *discharge*, denoted by Q with the unit of m^3/s.

Consider, a one dimensional flow of viscous fluid through a pipe (**Figure 1.5**).

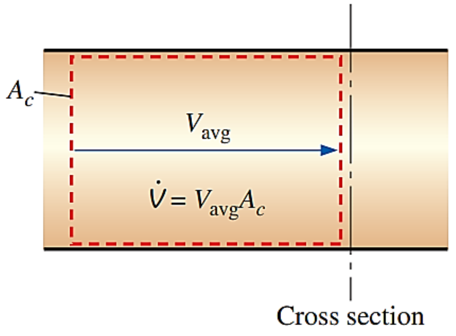


Figure 1.5: The rate of volume of fluid flowing through a cross section per unit time. (source: Y.A Cengel and J.M. Cimbala, 2018)

Volume flow rate (***Q***) is given by,

$$Q = vA \qquad \textbf{Eq. 1-1}$$

Where,

v = the average velocity of fluid (m/s)
A = area of cross section (m^2).

Mass Flow Rate. The amount of mass flowing through a cross sectional area, A per unit time, *t* is called the **mass flow rate**. Mass flow rate also refereed as *mass flow* or *mass discharge*, denoted by $\dot{m}$ with the unit of **kg/s**.

*The dot over the symbol is used to indicate *time rate of change*.

Consider, a one dimensional flow of an incompressible fluid through a pipe (**Figure 1.6**).

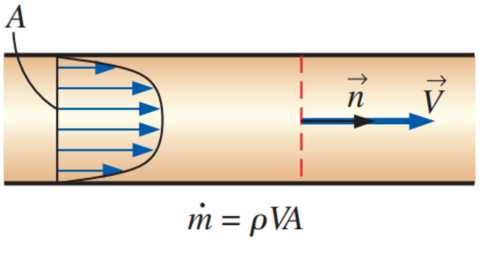


Figure 1.6: The amount of mass of fluid flowing through a cross section per unit time. (source: Y.A Cengel and J.M. Cimbala, 2018)

Volume flow rate (***Q***) is given by,

$$\dot{m} = \rho Q = \rho vA \qquad \textbf{Eq. 1-2}$$

Where,

ρ = density of fluid (kg/m^3)
Q = volume flow rate (m^3/s)
v = average velocity of fluid (m/s)
A = area of cross section (m^2).

Example 1.1:

Water flows in a 10 cm diameter pipe with average velocity of 2 m/s. Calculate the volume flow rate of the water.

Given: v = 2 m/s, d = 10 cm = 0.1 m;

Solution:

Volume flow rate,

$$Q = vA = v(\frac{\pi d^2}{4})$$
$$= (2)\frac{\pi(0.1^2)}{4}$$
$$= 0.0157\ m^3/s(Ans)$$

Cross section area of pipe can be determined by using formula for area of circle,

$$A = \pi r^2 = \frac{\pi d^2}{4}$$

Example 1.2:

A substance with a mass density of 700 kg/m³ is flowing with 2.5 m/s mean speed through a 25 mm diameter nozzle. Determine the mass flow rate of the substance.

Given: v = 2.5 m/s, d = 25 mm = 0.025 m; ρ = 700 kg/m³

Solution:

Volume flow rate,

$$\dot{m} = \rho v A = \rho v \frac{\pi d^2}{4}$$
$$= (700)(2.5)\frac{\pi(0.025^2)}{4}$$
$$= 0.859\ kg/s(Ans)$$

Example 1.3:

Weight of an empty bucket is 0.5 kg. After 10 seconds of collecting water, the weight of the bucket is 5 kg. Calculate the mass flow rate of the fluid.

Given: m_i = 0.5 kg, m_f = 5 kg, duration, t = 10 s

Solution:

Mass flow rate,

$$\dot{m} = \frac{mass\ of\ water\ collected}{time\ taken\ to\ collect\ water}$$
$$= \frac{m_f - m_i}{time} = \frac{5 - 0.5}{10}$$
$$= 0.45\ kg/s(Ans)$$

TUTORIAL 1.1

Q1-1

Oil flows through a pipe at a mean velocity of 1.5 m/s. The diameter of the pipe is 8 cm. Calculate the discharge and mass flow rate of oil, given the specific gravity of oil is 0.85. [Ans: 0.0075 m^3/s, 6.4 kg/s]

Q1-2

Water flows through a pipe at a velocity of 3 m/s. The diameter of the pipe is 115 mm. Calculate the discharge and mass flow rate of water. [Ans: 0.031 m^3/s, 31.16 kg/s]

1.4 THE CONTINUITY EQUATION

The ***conservation of mass principle*** states that, within a region, matter can not be created or destroyed. When a fluid in motion, it must move in such way that the mass is conserved, where mass of all particles within a system remain constant. This principle is used to relate a fluid's velocity to the change in the pipe cross sectional area, assuming;

a) Flow rate / discharge is constant
b) The fluid is in uniform and steady flow.

Consider a steady flow of fluid through a duct in **Figure 1.7** below.

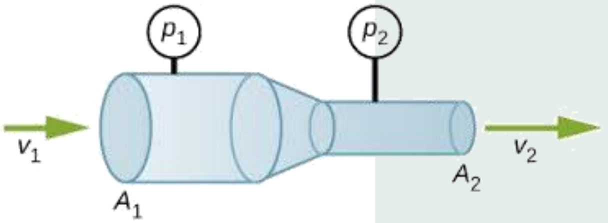


Figure 1.7: Conservation of mass principle a steady flow system.

Applying the principle of mass conservation, total rate of mass entering a control volume system is equal to the total rate of mass leaving the system.

$$\dot{m}_1 = \dot{m}_2 \text{ , or} \qquad \textbf{Eq. 1-3}$$

$$\rho_1 v_1 A_1 = \rho_2 v_2 A_2$$

Assuming there is no flow through the side walls of the duct, the flow entering A_1 will exit at A_2 at the same rate. Hence, the equation of continuity describes that the flow in will equal to flow out,

$$Q_1 = Q_2 \qquad \textbf{Eq. 1-4}$$

Or, this can also expressed as,

$$A_1 v_1 = A_2 v_2$$

In case of a pipe is split unto several junction (**Figure 1.8**),

$$Q_1 = Q_2 + Q_3 \qquad \textbf{Eq. 1-5}$$

$$v_1 A_1 = v_2 A_2 + v_3 A_3$$

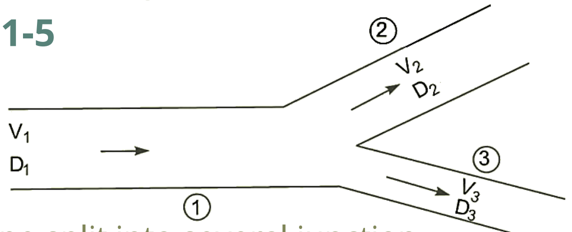
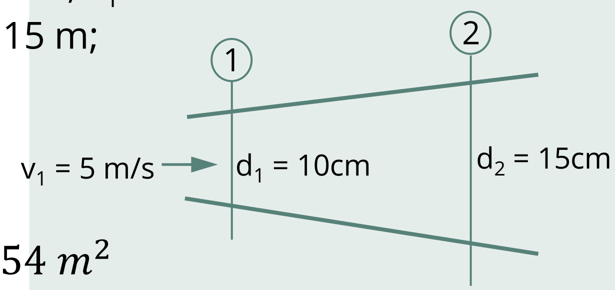


Figure 1.8: Pipe split into several junction.

Example 1.4:

The diameters of a pipe at the sections 1 and 2 are 10 cm and 15 cm respectively. Find the discharge through the pipe if the velocity of water flowing through the pipe at section 1 is 15 m/s. Also, determine also the velocity at the section 2.

Given: at section 1, d_1 = 10 cm = 0.1 m, v_1 = 5 m/s
at section 2, d_2 = 15 cm = 0.15 m;



Solution:

Cross section area at 1 and 2,

$$A_1 = \frac{\pi {d_1}^2}{4} = \frac{\pi 0.1^2}{4} = 0.007854\ m^2$$

$$A_2 = \frac{\pi {d_2}^2}{4} = \frac{\pi 0.15^2}{4} = 0.01767\ m^2$$

Discharge through the pipe,

$$Q = v_1 A_1 = 5 \times 0.007854 = 0.03927\ m^3/s(Ans)$$

Thus, the velocity at section 2,

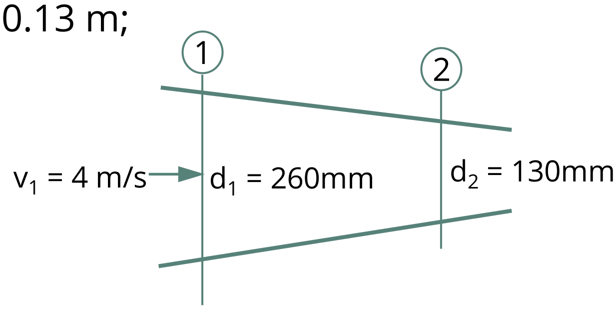
$$v_1 A_1 = v_2 A_2$$

$$v_2 = \frac{v_1 A_1}{A_2} = \frac{0.03927}{0.01767} = 2.22\ m/s(Ans)$$

Example 1.5:

The speed of an incompressible fluid is 4 m/s entering the 260 mm pipe. Calculate the average velocity at the diameter 130 mm pipe?

Given: at section 1, d_1 = 260 mm = 0.26 m, v_1 = 4 m/s
at section 2, d_2 = 130 mm = 0.13 m;



Solution:

Velocity at diameter 130 mm pipe,

$$v_1 A_1 = v_2 A_2$$

$$v_2 = \frac{v_1 \frac{\pi {d_1}^2}{4}}{\frac{\pi {d_2}^2}{4}} = \frac{(4)(0.26^2)}{0.13^2} = 16\ m/s(Ans)$$

Example 1.6:

A 30 cm diameter pipe conveying water, branches into two pipes of diameters 20 cm and 15 cm respectively. If the average velocity in the 30 cm diameter pipe is 2.5 m/s, find the discharge in this pipe. Also, determine the velocity in 15 cm pipe if the average velocity in 20 cm diameter pipe is 2 m/s.

Given: at section 1, d_1 = 30 cm = 0.1 m, v_1 = 2.5 m/s;
at section 2, d_2 = 20 cm = 0.15 m, v_2 = 2 m/s;
at section 3, d_3 = 15 cm = 0.15m;

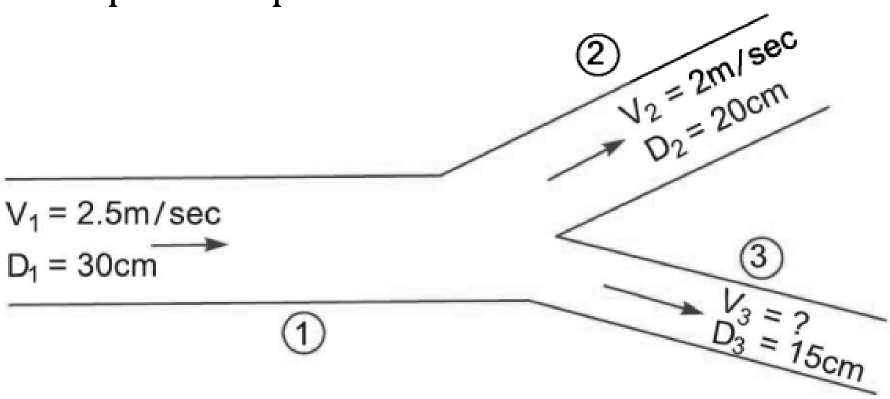
Solution:$_2$

Cross section area at 1, 2 and 3,

$$A_1 = \frac{\pi {d_1}^2}{4} = \frac{\pi 0.3^2}{4} = 0.07069\ m^2$$

$$A_2 = \frac{\pi {d_2}^2}{4} = \frac{\pi 0.2^2}{4} = 0.03142\ m^2$$

$$A_3 = \frac{\pi {d_3}^2}{4} = \frac{\pi 0.15^2}{4} = 0.01767\ m^2$$



Discharge through pipe section 1,

$$Q_1 = v_1 A_1 = 2.5 \times 0.07069 = 0.1767\ m^3/s(Ans)$$

Thus, the velocity at section 3,

$$Q_1 = Q_2 + Q_3$$

$$v_1 A_1 = v_2 A_2 + v_3 A_3$$

$$0.1767 = 2(0.03142) + v_3(0.01767)$$

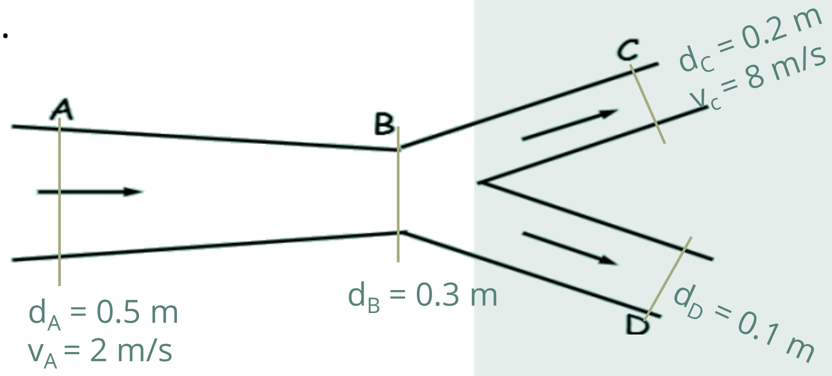
$$v_3 = 6.445\ m/s(Ans)$$

Example 1.7:

A pipe is split into 2 pipes which are BC and BD as shown in the figure. The diameter of pipes at A, B, C and D are 0.5 m, 0.3 m, 0.2 m and 0.1 m respectively. Calculate;

i. The discharge at section A if v_A = 2 m/s

ii. The velocity at section B and section D, if the velocity at section C is 8 m/s.



Given: at section A, d_A = 0.5 m, v_A = 2 m/s;
at section B, d_B = 0.3 m,
at section C, d_C = 0.2 m; v_C = 8 m/s;
at section D, d_D = 0.1 m;

Solution:$_2$

i. Discharge at A,

$$Q_A = v_A A_A = v_A \frac{\pi {d_A}^2}{4} = 2\left(\frac{\pi 0.5^2}{4}\right) = 0.3927\ m^3/s(Ans)$$

ii. Velocity at B,

$$Q_A = Q_B \rightarrow v_A\, A_A = v_B A_B$$

$$0.3927 = v_B \left(\frac{\pi 0.3^2}{4}\right)$$

$$v_B = 5.556\ m/s(Ans)$$

Velocity at D,

$$Q_B = Q_C + Q_D$$

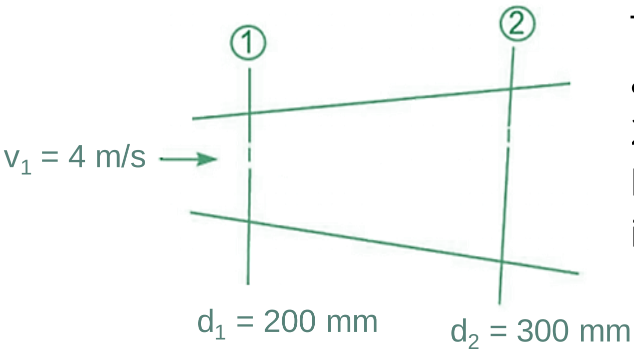
$$v_D A_D = Q_B - v_c A_C$$

$$v_D \left(\frac{\pi 0.1^2}{4}\right) = 0.3927 - 8\left(\frac{\pi 0.2^2}{4}\right)$$

$$v_D = 18\ m/s(Ans)$$

TUTORIAL 1.2

Q1-3



The diameter of a pipe at section 1 and 2 as shown in the figure, are 200 mm and 300 mm respectively. If the velocity of water at section 1 is 4 m/s, determine:

i. Amount of water discharge through the pipe.
ii. Velocity of water at section 2.

[Ans: 0.126 m^3/s, 1.778 m/s]

Q1-4

Raw oil flows through a pipe of 40 mm diameter and exits through a pipe with 25 mm diameter. The volume flowrate is 3.75 litre/s. Calculate the flow velocity for both end of pipes and the density of raw oil if the mass flow rate is 3.23 kg/s.

[Ans: 2.98 m/s, 7.64 m/s, 861.3 kg/m^3]

Q1-5

A 25 cm diameter pipe carries oil with Specific Gravity (SG) of 0.9 at a velocity of 3 m/s. At another section, the diameter is 20 cm. Find the velocity and mass flow rate of oil at this section.

[Ans: 4.69 m/s, 132.5 kg/s]

Q1-6

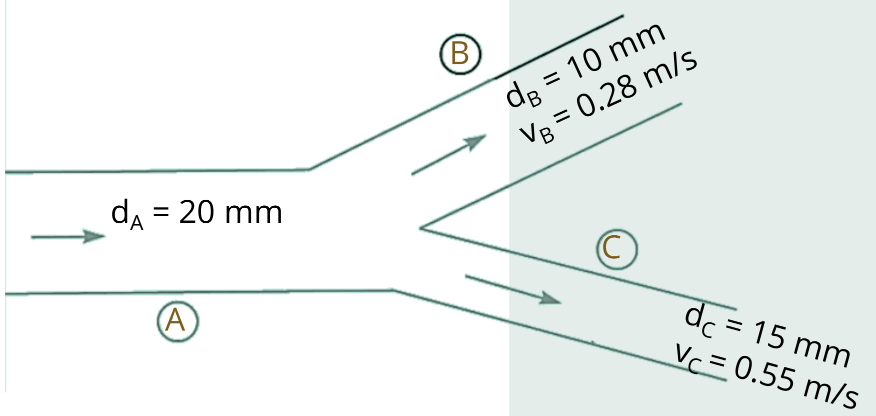
Oil flows in a 20 mm pipe. The pipe is split into two where by the first pipe is 10 mm diameters with a velocity of 0.3 m/s and the other pipe is 15 mm diameter with a velocity of 0.6 m/s. Calculate the flowrate and average velocity of oil at the 20 mm pipe.

[Ans: 0.00013 m^3/s, 0.4137 m/s]

TUTORIAL 1.2

Q1-7

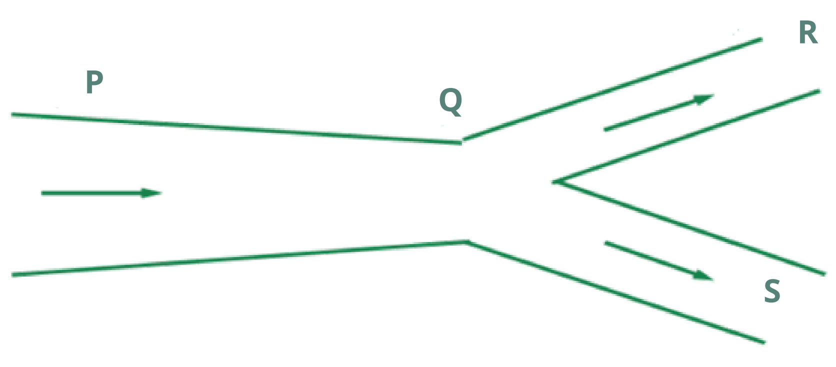
Figure below shows a round pipe A with a diameter of 20 mm. Oil flow splits into two at the end of the pipe. Pipe B with a diameter of 10 mm has a velocity of 0.28 m/s and pipe C with diameter 15 mm has a velocity of 0.55 m/s. Determine the velocity in pipe A, flowrate in pipe B and C. [Ans: 0.38 m/s, 0.00022 m^3/s, 0.000097 m^3/s]



Q1-8

A pipe PQ is split into 2 pipes which are QR and QS as shown below. The diameter of each pipe is as follows;

PQ at P = 0.75 m
PQ at Q = 0.5 m
QR at R = 0.2 m
QS at S = 0.35 m



Calculate;

i. Discharge at section P if v_P = 5 m/s
ii. Velocity at section Q and S if velocity at R = 8 m/s

[Ans: 2.2 m^3/s, 11.25 m/s, 20.35 m/s]

1.5 BERNOULLI'S THEOREM

Bernoulli's theorem, in fluid dynamics, relates the pressure, velocity and elevation in a steady flowing incompressible fluid. It was first derived by Daniel Bernoulli (1738), a Swiss mathematician. The theorem states that *the total energy of a steady flowing fluid, comprising of energy associated with pressure, kinetic energy and potential energy, **remains constant** along a streamline when compressibility and friction are negligible*. Therefore, the various form of mechanical energy of fluid can be converted to each other, but their sum remains constant,

$$\underbrace{\frac{P}{\rho}}_{\text{Flow energy}} + \underbrace{\frac{v^2}{2}}_{\text{Kinetic energy}} + \underbrace{gz}_{\text{Potential energy}} = constant \qquad \text{Eq. 1-6}$$

Bernoulli's theorem is a statement of principle of energy conservation for ideal fluids in steady flow.

$$energy\ before = energy\ after$$

$$\frac{P_1}{\rho} + \frac{{v_1}^2}{2} + gz_1 = \frac{P_2}{\rho} + \frac{{v_2}^2}{2} + gz_2 \qquad \text{Eq. 1-7}$$

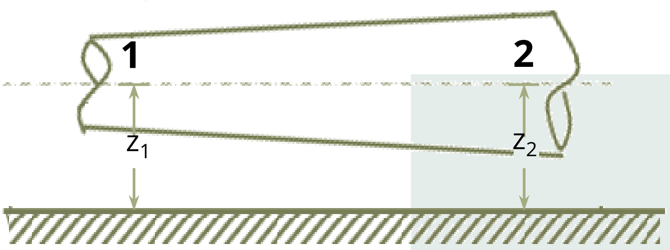
It is sometime convenient to divide Bernoulli by g, and written in the form of,

$$\frac{P_1}{\rho g} + \frac{{v_1}^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{{v_2}^2}{2g} + z_2 \qquad \text{Eq. 1-8}$$

Bernoulli's theorem is the basis for many engineering applications such as the design of an aerofoil, measure fluid flows through pipes, measure aircraft speed, nozzle, etc. In this chapter, we will discuss the application of Bernoulli equation in the following device;

a) Horizontal and inclined pipe
b) Horizontal and inclined venturi meter
c) Orifice meter
d) Pitot tube

Horizontal and Inclined Pipe. In **horizontal pipe**, the elevation is equal, i.e. $z_1 = z_2$.



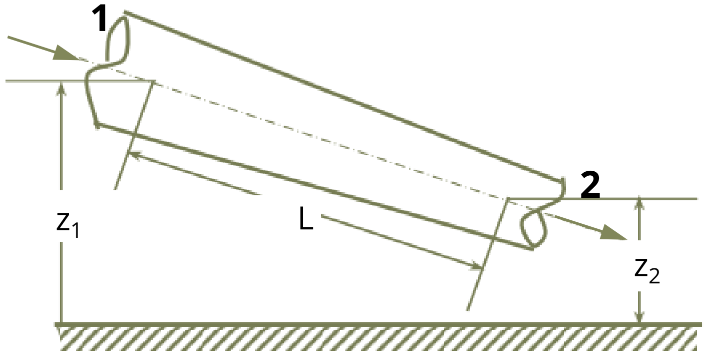
Hence, from **equation 1-8**,

$$\frac{P_1}{\rho g} + \frac{{v_1}^2}{2g} + \cancel{z_1}^{\,0} = \frac{P_2}{\rho g} + \frac{{v_2}^2}{2g} + \cancel{z_2}^{\,0}$$

$$\frac{P_1}{\rho g} - \frac{P_2}{\rho g} = \frac{{v_2}^2}{2g} - \frac{{v_1}^2}{2g}$$

$$P_1 - P_2 = \rho\left[\frac{{v_2}^2}{2} - \frac{{v_1}^2}{2}\right] = \frac{1}{2}\rho {v_2}^2 - \frac{1}{2}\rho {v_1}^2$$

For **inclined pipes**, the elevation is measured from the datum line. Consider, the elevation is given by z_1 and z_2 at section 1 and section 2 respectively,



$$\frac{P_1}{\rho g} + \frac{{v_1}^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{{v_2}^2}{2g} + z_2$$

$$P_1 - P_2 = \rho g\left[\frac{{v_1}^2}{2g} - \frac{{v_2}^2}{2g} + (z_2 - z_1)\right]$$

Example 1.8: (Bernoulli in Horizontal Pipe)

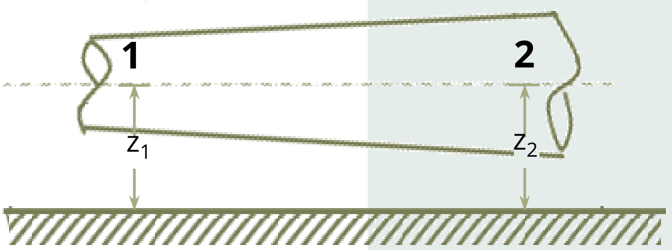
Water flows through section 1 of a horizontal pipe with a diameter of 50 mm at 4 m/s velocity and 2 atm pressure. The diameter of the pipe at section 2 is twice as big as section 1. Determine the gauge pressure at the downstream when the water passes through section 2 of the pipe.

Given: at 1; d_1 = 50 mm = 0.05 m, v_1 = 4 m/s;

P_1 = 2 atm x 101325 Pa = 202650 Pa;

at 2, d_2 = 100 mm = 0.1 m,

Solution:



From continuity equation,

$$v_1 A_1 = v_2 A_2$$

$$v_2 = \frac{A_1}{A_2} v_1 = \left(\frac{d_1}{d_2}\right)^2 v_1 = \left(\frac{0.05}{0.1}\right)^2 4 = 1\ m/s$$

The gauge pressure at section 2, P_2, where $z_1 = z_2$ for horizontal pipe,

$$\frac{P_1}{\rho g} + \frac{{v_1}^2}{2g} + \cancel{z_1}^{0} = \frac{P_2}{\rho g} + \frac{{v_2}^2}{2g} + \cancel{z_2}^{0}$$

$$\frac{P_1}{\rho g} - \frac{P_2}{\rho g} = \frac{{v_2}^2}{2g} - \frac{{v_1}^2}{2g}$$

$$\frac{P_2}{\rho g} = \frac{P_1}{\rho g} + \frac{{v_1}^2 - {v_2}^2}{2g}$$

$$P_2 = 9810\left(\frac{202650}{9810} + \frac{4^2 - 1^2}{2(9.81)}\right)$$

$$= 210150\ Pa = 210.15\ kPa\ (Ans)$$

Example 1.9: (Bernoulli in Inclined Pipe)

Water flows in a circular pipe. At one section the diameter is 0.3 m, the static pressure is 260 kPa gauge, the velocity is 3 m/s and the elevation is 10 m above ground level. The elevation at a section downstream is 0 m, and the pipe diameter is 0.15 m. Find out the gauge pressure at the downstream section.
Neglect the frictional effect, assume density of water to be 999 kg/m^3.

Given:	P_1 = 260 kPa;	v_1 = 3 m/s;	z_1 = 10 m;	d_1 = 0.3 m;
	P_2 = ?;	$v_2 = (A_1/A_2)v_1$;	z_2 = 0 m;	d_2 = 0.15 m;

Solution:

Velocity of v_2,

$$v_2 = \frac{A_1}{A_2}(v_2) = \frac{{d_1}^2}{{d_2}^2}(v_2) = \frac{0.3^2}{0.15^2}(3) = 12\ m/s$$

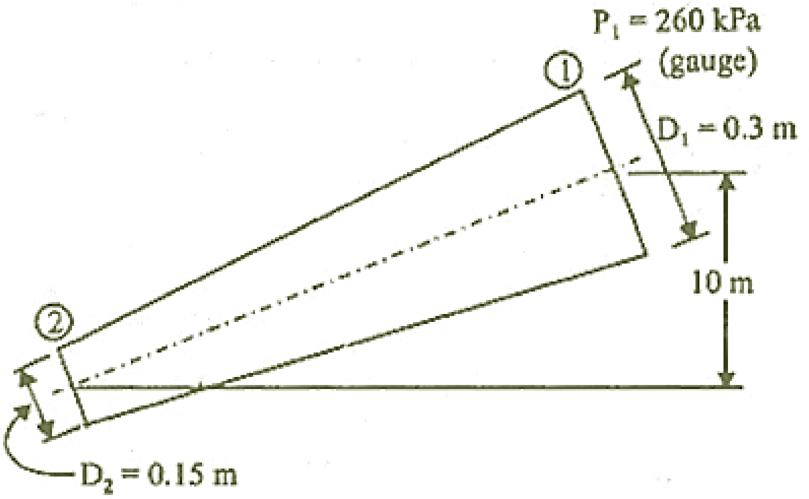
Gauge pressure at downstream, P_2,

$$\frac{P_1}{\rho g} + \frac{{v_1}^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{{v_2}^2}{2g} + z_2$$

$$\frac{260000}{(999)(9.81)} + \frac{3^2}{2(9.81)} + 10 = \frac{P_2}{(999)(9.81)} + \frac{12^2}{2(9.81)} + 0$$

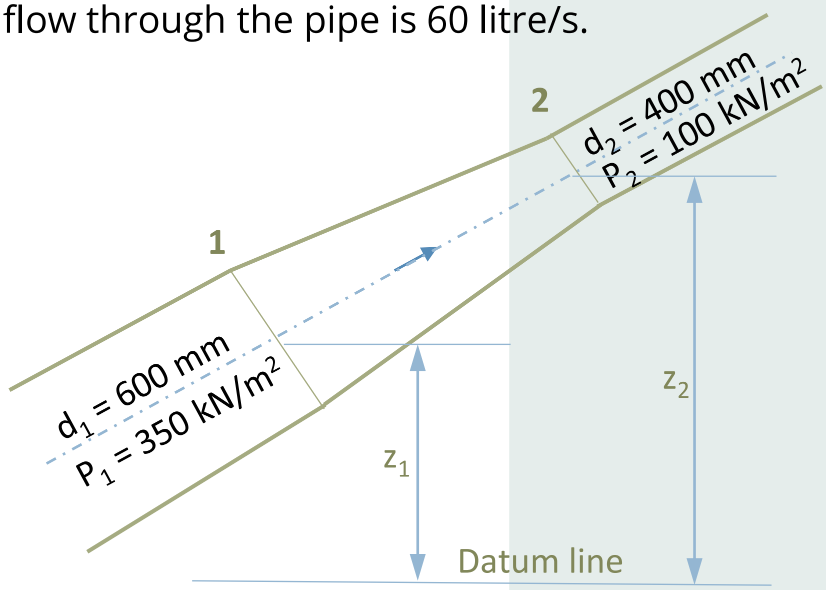
$$26.53 + 0.459 + 10 = \frac{P_2}{(999)(9.81)} + 7.34$$

$$P_2 = 290566\ Pa = 290.56\ kPa\ (Ans)$$



Example 1.10: (Bernoulli in Inclined Pipe)

Water is flowing through a pipe having diameters of 600 mm and 400 mm at the bottom and upper end respectively. The intensity of pressure at the bottom end is 350 kN/m² and the pressure at the upper end is 100 kN/m². Determine the difference in datum head if the rate of flow through the pipe is 60 litre/s.



Given: P_1 = 350 kN/m²; z_1 = 100 m; d_1 = 600 mm = 0.6 m;
P_2 = 100 kN/m²; z_2 = 107 m; d_2 = 400 mm = 0.4 m;
Q = 60 litre/s = 0.06 m³/s; $z_2 - z_1$ = ?;

Solution:

Velocity at 1 and 2,

$$Q = v_1 A_1 \rightarrow v_1 = \frac{Q}{A_1} = \frac{0.06}{\frac{\pi}{4}(0.6^2)} = 0.212\ m/s$$

$$Q = v_2 A_2 \rightarrow v_2 = \frac{Q}{A_2} = \frac{0.06}{\frac{\pi}{4}(0.4^2)} = 0.477\ m/s$$

Pressure at 2,

$$\frac{P_1}{\rho g} + \frac{{v_1}^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{{v_2}^2}{2g} + z_2$$

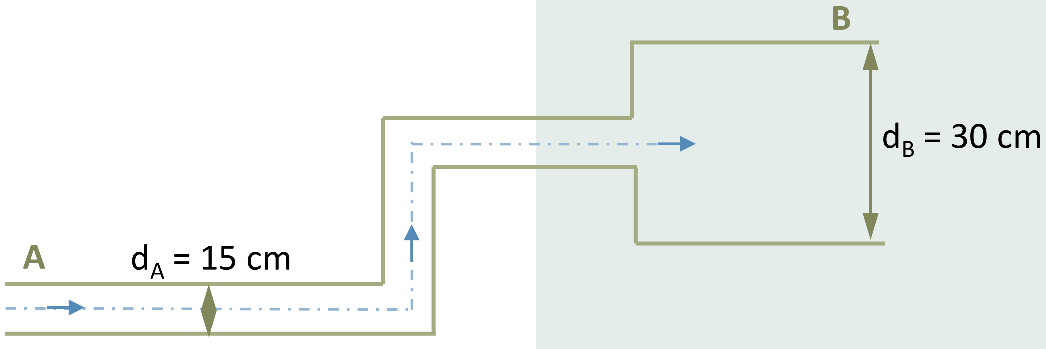
$$\frac{350000}{(1000)(9.81)} + \frac{0.212^2}{2(9.81)} + z_1 = \frac{100000}{(1000)(9.81)} + \frac{0.477^2}{2(9.81)} + z_2$$

$$35.67 + 0.0023 + z_1 = 10.19 + 0.0116 + z_2$$

$$z_2 - z_1 = 25.47\ m(Ans)$$

Example 1.11: (Bernoulli in Different Elevation Pipe)

A pipeline is 15 cm in diameter and it is at an elevation of 100 m at section A. At section B, it is at an elevation of 107 m and has a diameter of 30 cm. When a discharge of 50 litre/s water passes through the pipeline, pressure at A is 35 kPa. Calculate the pressure at B if the flow is from A to B.



Given: P_A = 35 kPa; z_A = 100 m; d_A = 15 cm = 0.15 m;
P_B = ?; z_B = 107 m; d_B = 30 cm = 0.3 m;
Q = 50 litre/s = 0.05 m^3/s

Solution:

Velocity at A and B,

$$Q = v_A A_A \rightarrow v_A = \frac{Q}{A_A} = \frac{0.05}{\frac{\pi}{4}(0.15^2)} = 2.829\ m/s$$

$$Q = v_B A_B \rightarrow v_B = \frac{Q}{A_B} = \frac{0.05}{\frac{\pi}{4}(0.3^2)} = 0.707\ m/s$$

Pressure at B,

$$\frac{P_A}{\rho g} + \frac{{v_A}^2}{2g} + z_A = \frac{P_B}{\rho g} + \frac{{v_B}^2}{2g} + z_B$$

$$\frac{35000}{(9810)} + \frac{2.289^2}{2(9.81)} + 100 = \frac{P_B}{(9810)} + \frac{0.707^2}{2(9.81)} + 107$$

$$\therefore P_B = -29.9\ kPa\ (Ans)$$

Horizontal and Inclined Venturi Meter. Venturi meter is a flow measuring device, used to **measure the discharge of liquid** through a pipe. It is based on Bernoulli's principle, in which pressure energy converted into kinetic energy in measuring the discharge of liquids through pipelines.

Venturi meter (**Figure 1.8**) consists of a small pipe with two conical portions and short segment of the uniform cross-section in between;

a) **Converging cone:**
   - same base diameter with diverging cone
   - shorter cone with greater cone angle than diverging cone
   - area decreasing, flow speed increase and pressure decrease

b) **Throat:**
   - the smallest area of Venturi meter.
   - area, speed and pressure remains constant.

c) **Diverging cone:**
   - same base diameter with converging cone
   - longer cone with smaller cone angle than diverging cone
   - area increasing, flow speed decreases and pressure increases
   - pressure is restored

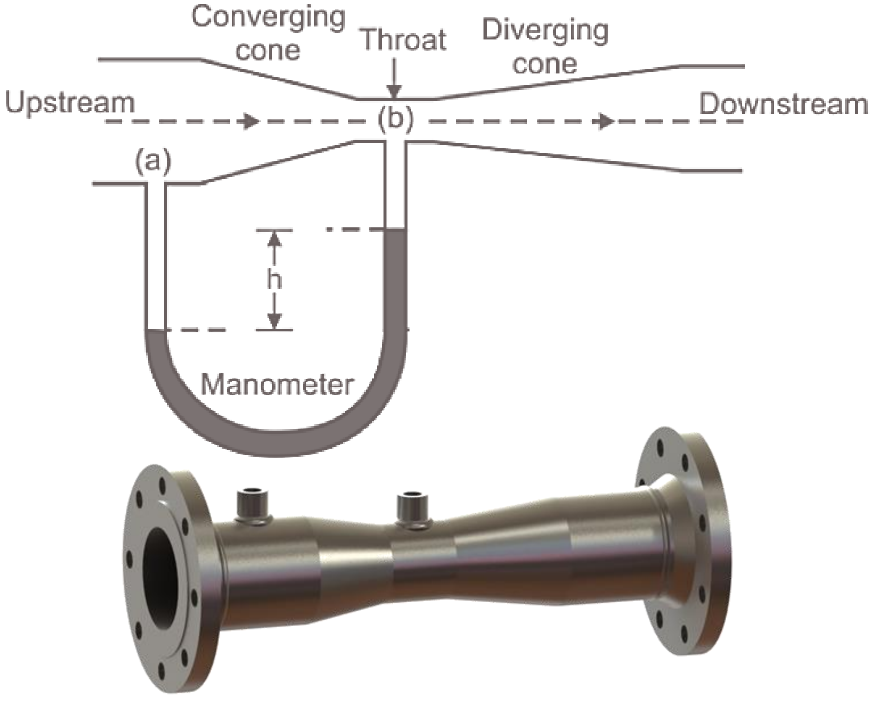
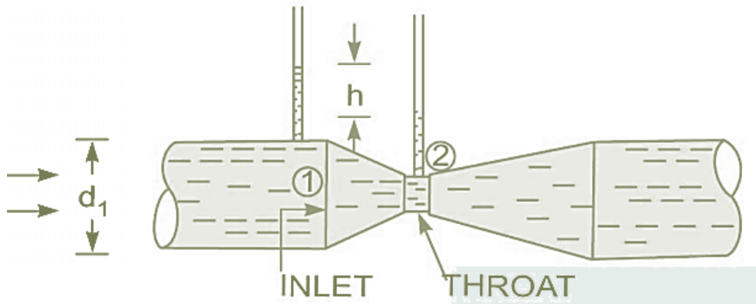


Figure 1.8: Venturi Meter schematic (top) and real one used in practice (bottom).



Applying Bernoulli's equation on horizontal Venturi meter at section 1 and 2, as venturi meter is horizontal, ($z_1 = z_2$) hence,

$$\frac{P_1}{\rho g} + \frac{{v_1}^2}{2g} = \frac{P_2}{\rho g} + \frac{{v_2}^2}{2g} \rightarrow \frac{P_1 - P_2}{\rho g} = \frac{{v_2}^2 - {v_1}^2}{2g}$$

But, the pressure head at section 1 and 2 is, $h = \frac{P_1 - P_2}{\rho g}$

Substituting h into Bernoulli's equation gives,

$$h = \frac{{v_2}^2 - {v_1}^2}{2g}$$

Applying continuity equation, $v_1 = \frac{A_2 v_2}{A_1}$

$$v_2 = \frac{A_1}{\sqrt{{A_1}^2 - {A_2}^2}} \sqrt{2gh}$$

Hence, the discharge of a horizontal Venturi meter,

$$Q = \frac{A_1 A_2}{\sqrt{{A_1}^2 - {A_2}^2}} \sqrt{2gh}$$

If, ratio of area, $m = {}^{A_1}/_{A_2}$, the equation becomes,

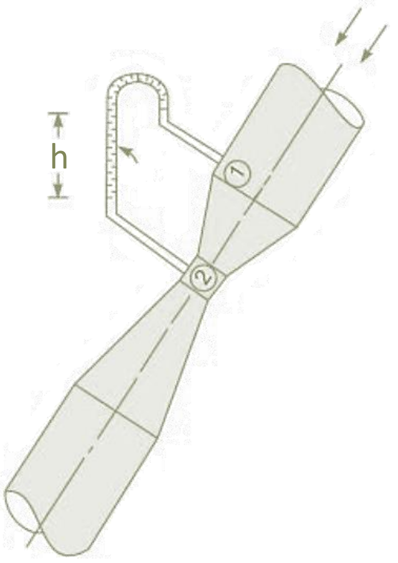
$$Q_{theory} = A_1 \sqrt{\frac{2gH}{(m^2 - 1)}} \qquad \text{Eq. 1-9}$$

Where, pressure head, H is given by,

$$H = \frac{p_1 - p_2}{\gamma} \qquad \text{Eq. 1-10}$$

Or, from manometer,

$$H = x\left(\frac{\gamma_{Hg}}{\gamma_f} - 1\right) = x\left(\frac{SG_{Hg}}{SG_f} - 1\right) \qquad \text{Eq. 1-11}$$



For inclined manometer, using equation 1.9, as the elevation varies, $z_1 \neq z_2$, pressure head of the inclined venturi meter, H is,

$$H = \frac{p_1 - p_2}{\gamma} + (z_1 - z_2)$$

or, value of H given by differential manometer,

$$H = x\left(\frac{\gamma_{Hg}}{\gamma_f} - 1\right) = x(\frac{SG_{Hg}}{SG_f} - 1)$$

Since the Bernoulli's equation **does not** account for any friction losses within the flow, the equation was modified to,

$$Q_{actual} = C_d(A_1)\sqrt{\frac{2gH}{(m^2-1)}} \qquad \textbf{Eq. 1-12}$$

The discharge coefficient, C_d was determined **experimentally** and represents the actual average velocity in the throat to its theoretical velocity.

$$C_d = \frac{(v_2)_{actual}}{(v_2)_{theoretical}} \qquad \textbf{Eq. 1-13}$$

Value of C_d generally provided by the manufacturer as a function of Reynolds number for the pipe.

Example 1.12: (Horizontal Venturi Meter)

A horizontal venturi meter with inlet and throat diameters 30 cm and 15 cm respectively is used to measure the flow of water. The reading of differential manometer connected to the inlet and the throat is 20 cm mercury. Determine the rate of flow. Take C_d = 0.98.

Given: d_1 = 30 cm = 0.3 m; d_2 = 15 cm = 0.15 m; x = 20 cmHg (Mercury); C_d = 0.98;

Solution:

Pressure head,

$$H = x\left(\frac{SG_{Hg}}{SG_f} - 1\right) = 0.2\left(\frac{13.6}{1} - 1\right) = 2.52\ m$$

Ratio of area,

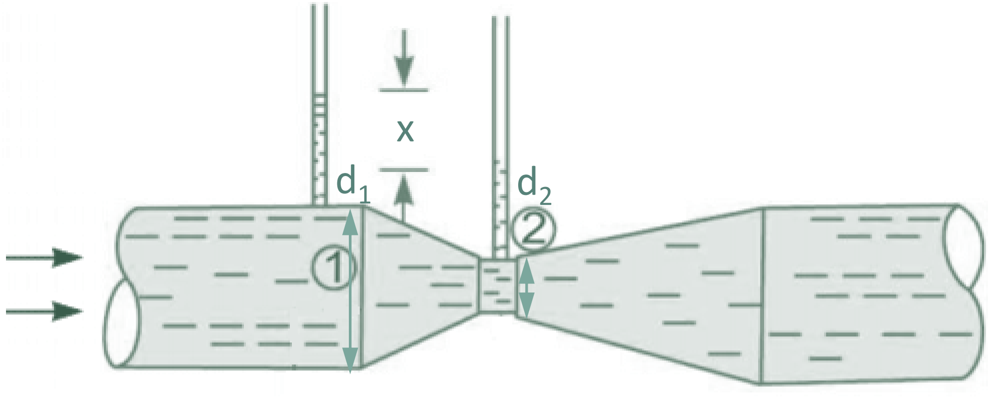
$$m = \left(\frac{d_1}{d_2}\right)^2 = \left(\frac{0.3}{0.15}\right)^2 = 4$$

Thus, the rate of flow, Q,

$$Q_{actual} = C_d(A_1)\sqrt{\frac{2gH}{(m^2-1)}}$$

$$= (0.98)\left(\frac{\pi 0.3^2}{4}\right)\sqrt{\frac{2(9.81)(2.52)}{(4^2-1)}}$$

$$= 0.12577\ m^3/s = 125.577\ litres/s (Ans)$$



Example 1.13: (Horizontal Venturi Meter)

A horizontal venturi meter with inlet diameter 20 cm and throat diameter 10 cm is used to measure the flow of water. The pressure at inlet is 17.658 N/cm^2 and the vacuum pressure at the throat is 30 cm of mercury. Find the discharge of water through the venturi meter. Take C_d = 0.98.

Given: d_1 = 20 cm = 0.2 m; d_2 = 10 cm = 0.1 m; C_d = 0.98;
P_1 = 17.658 N/cm^2 = 176.58 kN/m^2;
P_2 = -30 cmHg = -0.3 mHg;

Solution:

Pressure at 2, P_2,

$$\frac{P_2}{\rho g} = -0.3 \rightarrow P_2 = -0.3(13600)(9.81) = -40024.8\ N/m^2$$

Pressure head,

$$H = \frac{p_1 - p_2}{\gamma} = \frac{176580 - (-40024.8)}{9810} = 22.08\ m$$

Ratio of area,

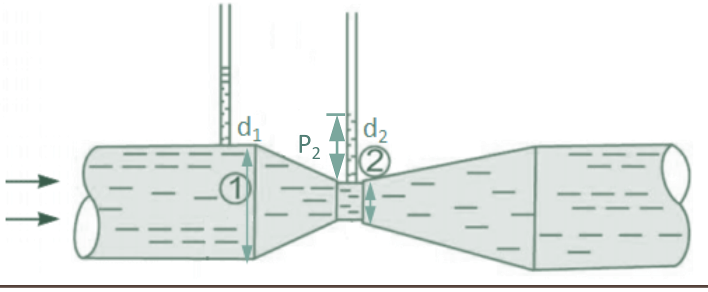
$$m = \left(\frac{d_1}{d_2}\right)^2 = \left(\frac{0.2}{0.1}\right)^2 = 4$$

Discharge, Q is given by,

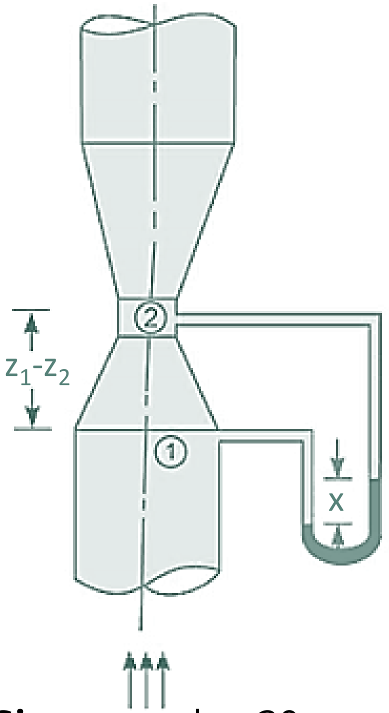
$$Q_{actual} = C_d(A_1)\sqrt{\frac{2gH}{(m^2-1)}}$$

$$= (0.98)\left(\frac{\pi 0.2^2}{4}\right)\sqrt{\frac{2(9.81)(22.08)}{(4^2-1)}}$$

$$= 0.16545 m^3/s = 165.45\ litre/s(Ans)$$



Example 1.14: (Inclined Venturi Meter)



A 30 cm x 15 cm venturi meter is provided in vertical pipe line carrying oil of SG 0.9, the flow being upward direction. The difference in elevation of the throat section and entrance section of the venturi meter is 30 cm. The differential U-tube mercury manometer shows a gauge deflection of 25 cm. Calculate:

i) The discharge of oil, and
ii) The pressure difference between the entrance section and the throat section.
Take the co-efficient of discharge as 0.98 and specific gravity of mercury as 13.6.

Given: d_1 = 30 cm = 0.3 m; d_2 = 15 cm = 0.15 m; $z_1 - z_2$ = -30 cm = -0.3 m; x = 25 cm = 0.25 m; C_d = 0.98; SG_{oil} = 0.9; SG_{Hg} = 13.6;

Solution:

i) Discharge of oil,

using the pressure head readings from manometer,

$$H = x\left(\frac{SG_{Hg}}{SG_f} - 1\right) = 0.25\left(\frac{13.6}{0.9} - 1\right) = 3.5278\ m$$

Ratio of area,

$$m = \left(\frac{d_1}{d_2}\right)^2 = \left(\frac{0.3}{0.15}\right)^2 = 4$$

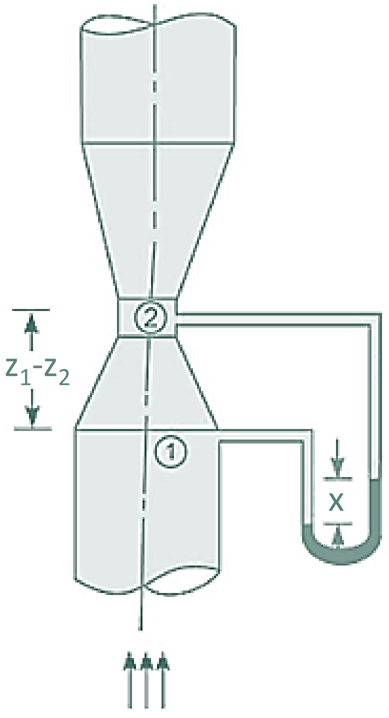
Discharge, Q,

$$Q_{actual} = C_d(A_1)\sqrt{\frac{2gH}{(m^2-1)}}$$

$$= (0.98)\left(\frac{\pi 0.3^2}{4}\right)\sqrt{\frac{2(9.81)(3.5278)}{(4^2-1)}}$$

$$= 0.1488 m^3/s = 148.8\ litres/s (Ans)$$

Example 1.14: (Inclined Venturi Meter) - continued



A 30 cm x 15 cm venturi meter is provided in vertical pipe line carrying oil of SG 0.9, the flow being upward direction. The difference in elevation of the throat section and entrance section of the venturi meter is 30 cm. The differential U-tube mercury manometer shows a gauge deflection of 25 cm. Calculate:

i) The discharge of oil, and
ii) The pressure difference between the entrance section and the throat section.
Take the co-efficient of discharge as 0.98 and specific gravity of mercury as 13.6.

Given: d_1 = 30 cm = 0.3 m; d_2 = 15 cm = 0.15 m;
$z_1 - z_2$ = -30 cm = -0.3 m; x = 25 cm = 0.25 m; C_d = 0.98;
SG_{oil} = 0.9; SG_{Hg} = 13.6;

Solution:

ii) Pressure difference between entrance and throat section,
Pressure head readings from manometer,

$$H = 3.5278\ m$$

Substituting into,

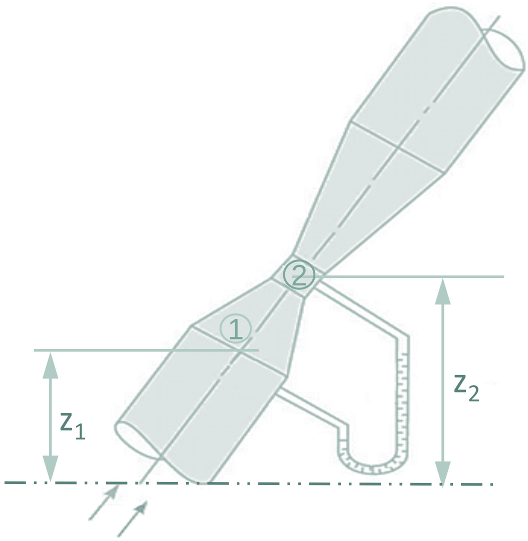
$$H = \frac{p_1 - p_2}{\gamma} + (z_1 - z_2)$$

$$\frac{p_1 - p_2}{\rho g} + (z_1 - z_2) = 3.5278$$

$$p_1 - p_2 = (3.5278 + 0.3)(900)(9.81)$$

$$= 33795.645\ Pa = 33.796\ kPa\ (Ans)$$

Example 1.15: (Inclined Venturi Meter)



An inclined venturi meter measures the flow of oil of SG 0.82 and has an entrance of 125 mm diameter and throat of 50 mm diameter. There are pressure gauges at the entrance and at the throat, which is 300 mm above the entrance. If the coefficient for the venturi meter is 0.97 and pressure difference is 27.5 kN/m^2, calculate the actual discharge in m^3/s.

Given: d_1 = 125 mm = 0.125 m; d_2 = 50 mm = 0.05 m;
$z_1 - z_2$ = -300 mm = -0.3 m; $P_1 - P_2$ = 27.5 kN/m^2;
C_d = 0.97; SG_{oil} = 0.82;

Solution:

Pressure head,

$$H = \frac{p_1 - p_2}{\gamma} + (z_1 - z_2) = \frac{27500}{(0.82)(1000)(9.81)} + (-0.3)$$
$$= 3.1186\ m$$

Ratio of area,

$$m = \left(\frac{d_1}{d_2}\right)^2 = \left(\frac{0.125}{0.05}\right)^2 = 6.25$$

Discharge, Q,

$$Q_{actual} = C_d(A_1)\sqrt{\frac{2gH}{(m^2-1)}}$$

$$= (0.97)\left(\frac{\pi 0.125^2}{4}\right)\sqrt{\frac{2(9.81)(3.1186)}{(6.25^2-1)}}$$
$$= 0.01509 m^3/s (Ans)$$

Orifice Meter. Orifice meter or orifice plate is a device used for **measuring the rate of flow through** a pipe using the same principle as venturi meter. However, it is a cheaper device compared to venturi meter due to its simpler construction and easier to install.

Orifice meter (**Figure 1.9**) consists of a flat plate which has a circular sharp-edged hole called orifice. Pressure is measured upstream at the vena contracta, where streamlines are parallel and the static pressure is constant.

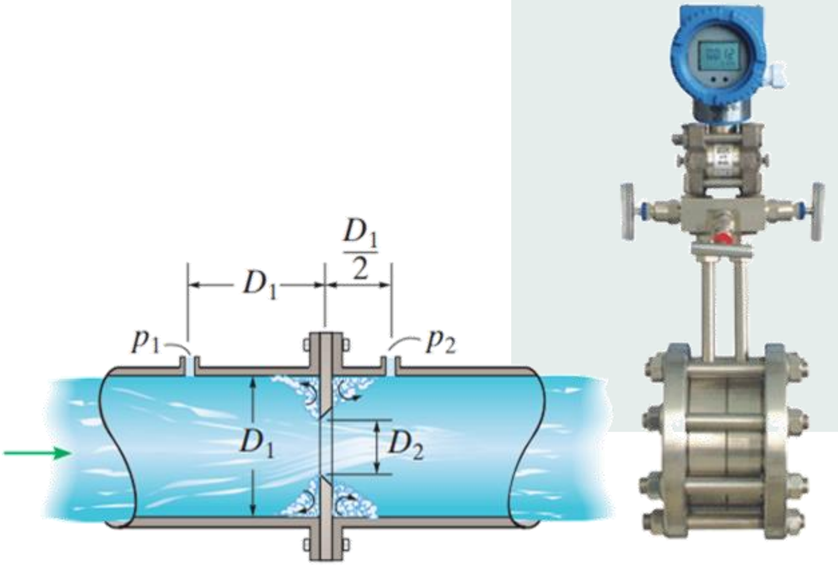


Figure 1.9: Orifice Meter schematic (left) and real one used in practice (right).

By applying the Bernoulli and continuity equation (**equation 1-12**),

$$Q_{actual} = C_d(A_1)\sqrt{\frac{2gH}{(m^2-1)}}$$

Where,

C_d = discharge coefficient (**equation 1.13**), usually provided by manufacturer.

m = ratio of area, $m = \frac{A_1}{A_2}$ or can be simplified as, $m = \left(\frac{d_1}{d_2}\right)^2$

H = pressure head, $H = \frac{p_1 - p_2}{\gamma}$, or by using differential manometer,

$$H = x\left(\frac{\gamma_{Hg}}{\gamma_f} - 1\right) = x(\frac{SG_{Hg}}{SG_f} - 1)$$

Orifice meter derivation/experiment. There are some assumption to derive orifice meter discharge, experimentally;

a) Fluid must be ideal.
b) Fluid flow is steady and continuous.
c) Inner surface must be frictionless.

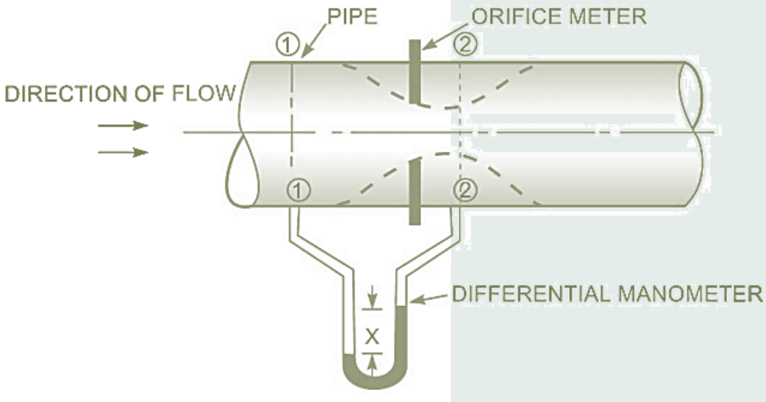


Figure 1.10: Orifice Meter setup

A differential manometer is connected to section 1, at a distance 1.5 to 2 times pipe upstream diameter from orifice (**Figure 1.10**) and section 2, at a distance about half the diameter of orifice on the downstream.

Applying Bernoulli's theorem at section 1 and 2,

$$\frac{P_1}{\rho g} + \frac{{v_1}^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{{v_2}^2}{2g} + z_2$$

$$\left(\frac{P_1}{\rho g} + z_1\right) - \left(\frac{P_2}{\rho g} + z_2\right) = \frac{{v_2}^2}{2g} - \frac{{v_1}^2}{2g}$$

But, $\left(\frac{P_1}{\rho g} + z_1\right) - \left(\frac{P_2}{\rho g} + z_2\right) = h$ is the differential head, hence,

$$h = \frac{{v_2}^2 - {v_1}^2}{2g}$$

$$v_2 = \sqrt{2gh + {v_1}^2}$$

At section 2 is the vena contracta and A_2 represents the area at the vena contracta. If A_o is the area of orifice, then, the coefficient of contraction, C_c,

$$C_c = \frac{A_2}{A_o} \rightarrow A_2 = A_o C_c \qquad \textbf{Eq. 1-14}$$

From continuity equation, $A_1v_1 = A_2v_2$, and substituting A_2 from **Equation 1-14**,

$$v_1 = \frac{A_oC_c}{A_1}v_2$$

$$v_2 = \sqrt{2gh + \frac{{A_o}^2{C_c}^2{v_2}^2}{{A_1}^2}}$$

$$v_2 = \frac{\sqrt{2gh}}{\sqrt{1 - \frac{{A_o}^2}{{A_1}^2}{C_c}^2}} \qquad \textbf{Eq. 1-15}$$

Therefore, the discharge is,

$$Q = A_2v_2 = v_2A_oC_c = \frac{A_oC_c\sqrt{2gh}}{\sqrt{1 - \frac{{A_o}^2}{{A_1}^2}{C_c}^2}} \qquad \textbf{Eq. 1-16}$$

If C_d is the coefficient of discharge for orifice meter, then,

$$C_d = C_c\frac{\sqrt{1 - \frac{{A_o}^2}{{A_1}^2}}}{\sqrt{1 - \frac{{A_o}^2}{{A_1}^2}{C_c}^2}} \rightarrow C_c = C_d\frac{\sqrt{1 - \frac{{A_o}^2}{{A_1}^2}{C_c}^2}}{\sqrt{1 - \frac{{A_o}^2}{{A_1}^2}}}$$

Substituting C_c into the discharge, Q (**Equation 1-16**), the orifice meter equation becomes,

$$Q = C_d\frac{A_oA_1\sqrt{2gh}}{\sqrt{{A_1}^2 - {A_o}^2}} \qquad \textbf{Eq. 1-17}$$

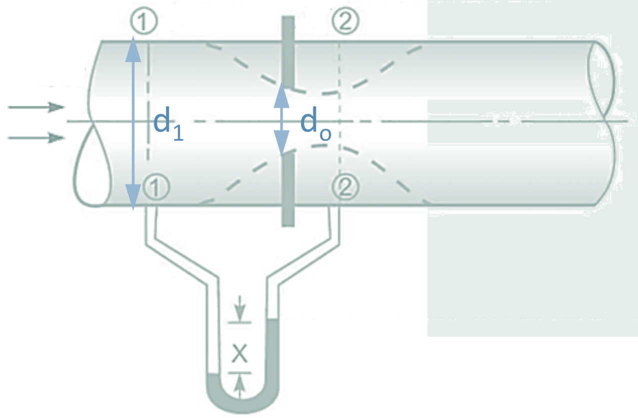
The actual discharge of Orifice Meter can be calculated using **Equation 1-17**. However, the C_d value for orifice meter is usually **lower** than the value of C_d for venturi meter.

Example 1.16: (Orifice Meter)

An orifice meter consists of a 100 mm diameter hole in a 250 mm diameter pipe, and has a coefficient of discharge 0.65. The pipe conveys oil of SG 0.9. The pressure difference between the two sides of the orifice plate is measured by a mercury manometer. If the difference in mercury levels in the gauge is 760 mm, calculate the flow rate of oil in the pipeline.

Given: d_o = 100 mm = 0.1 m; d_1 = 250 mm = 0.25 m; x = 760 mm = 0.76 m; C_d = 0.65;

Solution:



Pressure head,

$$H = x\left(\frac{\gamma_{Hg}}{\gamma_f} - 1\right) = 0.76\left(\frac{13.6}{0.9} - 1\right) = 10.7244\ m$$

Ratio of area,

$$m = \left(\frac{d_1}{d_2}\right)^2 = \left(\frac{0.25}{0.1}\right)^2 = 6.25$$

Hence, discharge, Q,

$$Q_{actual} = C_d(A_1)\sqrt{\frac{2gH}{(m^2-1)}}$$

$$= (0.65)\left(\frac{\pi 0.25^2}{4}\right)\sqrt{\frac{2(9.81)(10.7244)}{(6.25^2-1)}}$$

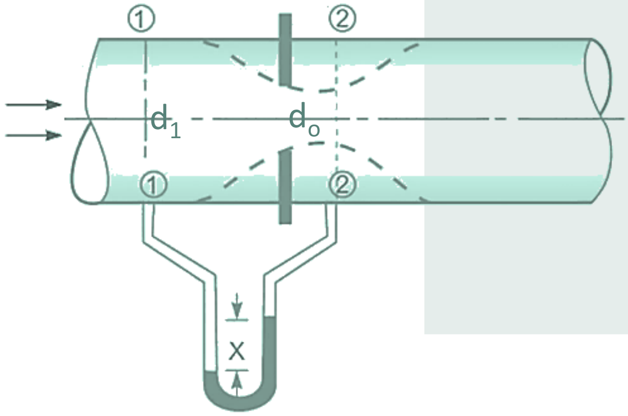
$$= 0.07502\ m^3/s (Ans)$$

Example 1.17: (Orifice Meter)

An orifice meter with orifice diameter of 10 cm is inserted in a pipe of 20 cm diameter. The pressure gauges fitted upstream and downstream of the orifice meter gives readings of 196.2 kN/m^2 and 98.1 kN/m^2 respectively. Co-efficient of discharge for the orifice meter is given as 0.6. Find the discharge through pipe.

Given: d_o = 10 cm = 0.1 m; d_1 = 20 cm = 0.2 m;
P_1 = 196.2 kN/m^2 = 196200 N/m^2;
P_2 = 98.1 kN/m^2 = 98100 N/m^2
C_d = 0.6;

Solution:



Pressure head,

$$H = \frac{p_1 - p_2}{\gamma} = \frac{196200 - 98100}{9810} = 10\ m$$

Ratio of area,

$$m = \left(\frac{d_1}{d_o}\right)^2 = \left(\frac{0.2}{0.1}\right)^2 = 4$$

Hence, discharge, Q,

$$Q_{actual} = C_d(A_1)\sqrt{\frac{2gH}{(m^2-1)}}$$

$$= (0.6)\left(\frac{\pi 0.2^2}{4}\right)\sqrt{\frac{2(9.81)(10)}{(4^2-1)}}$$

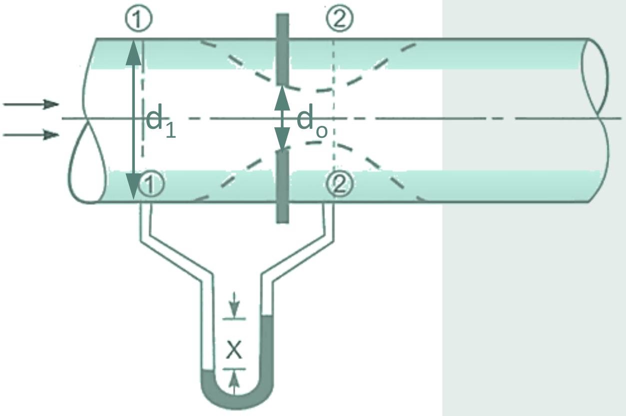
$$= 0.068172\ m^3/s(Ans)$$

Example 1.17: (Orifice Meter)-continued

Or, by using **Equation 1-17**,

Given: d_o = 10 cm = 0.1 m; d_1 = 20 cm = 0.2 m;
P_1 = 196.2 kN/m^2 = 196200 N/m^2;
P_2 = 98.1 kN/m^2 = 98100 N/m^2
C_d = 0.6;

Solution:



Pressure head,

$$H = \frac{p_1 - p_2}{\gamma} = \frac{196200 - 98100}{9810} = 10\ m$$

Area of orifice, Ao,

$$A_o = \frac{\pi {d_o}^2}{4} = \frac{\pi 0.1^2}{4} = 0.0025\pi\ m^2$$

$$A_1 = \frac{\pi {d_1}^2}{4} = \frac{\pi 0.2^2}{4} = 0.01\pi\ m^2$$

Hence, discharge, Q,

$$Q_{actual} = C_d \frac{A_o A_1 \sqrt{2gh}}{\sqrt{{A_1}^2 - {A_o}^2}}$$

$$= (0.6) \frac{(0.0025\pi)(0.01\pi)\sqrt{2(9.81)(10)}}{\sqrt{0.01\pi^2 - 0.0025\pi^2}}$$

$$= 0.068172\ m^3/s (Ans)$$

Pitot Tube. Pitot tube is a device to **measure velocity of flow** at any point in a pipe or channel. It is based on principle that if the velocity of flow at a point become zero, the pressure at that point will be increased. This Is due to the conversion of kinetic energy into pressure energy.

Pitot tube (**Figure 1.10**) consists of a tube, bent at right angle (L-shaped). The lower end is directed to the upstream direction (incoming flow). The liquid rises up in the tube due to the conversion of kinetic energy into pressure energy.

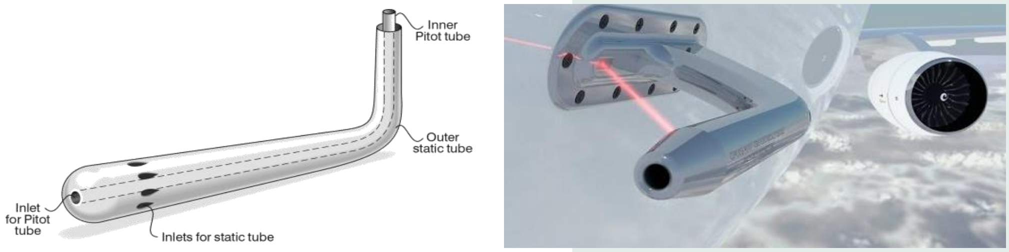


Figure 1.10: Pitot Tube schematic (left) and real one used in practice (right).

In Pitot tube, point 1 and 2 is at the same level ($z_1 = z_2$), where point 2 is the inlet of the Pitot tube (**Figure 1.11**). Assuming $v_2 = 0$ due to all kinetic energy converted to pressure energy, velocity is determined by the rise of liquid in the tube. Applying the Bernoulli equation,

$$\frac{P_1}{\rho g} + \frac{{v_1}^2}{2g} + \cancel{z_1} = \frac{P_2}{\rho g} + \cancel{\frac{{v_2}^2}{2g}} + \cancel{z_2}$$

Pressure head at 1 and 2,

$$at\ 1;\ \frac{P_1}{\rho g} = H \quad and\ at\ 2;\ \frac{P_2}{\rho g} = h + H$$

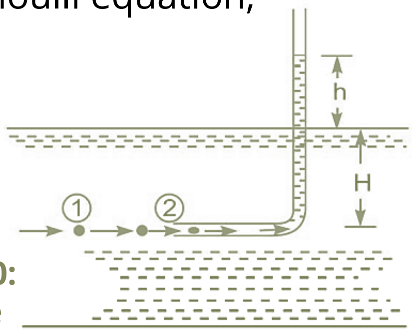


Figure 1.10: Pitot Tube

Substituting pressure head, we get the theoretical value of velocity, v_1,

$$H + \frac{{v_1}^2}{2g} = h + H \rightarrow h = \frac{{v_1}^2}{2g} \rightarrow v_1 = \sqrt{2gh} \qquad \textbf{Eq. 1-18}$$

Actual velocity is given by,

$$v = C_v\sqrt{2gh} \qquad \textbf{Eq. 1-19}$$

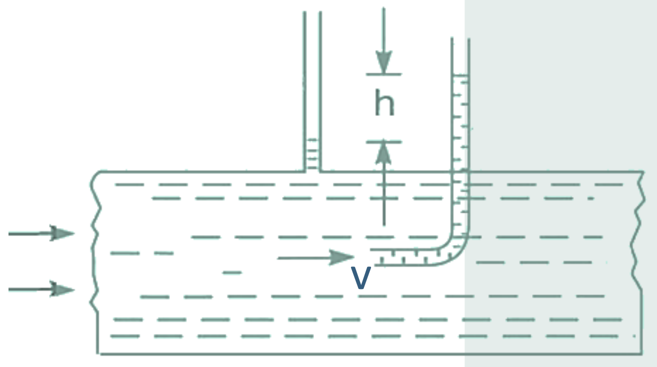
Where, C_v is the coefficient of pitot tube.

Example 1.18: (Pitot Tube)

A pitot-static tube placed in the centre of a 300 mm pipe line has one orifice pointing upstream and other perpendicular to it. The mean velocity in the pipe is 0.80 of the central velocity. Find the discharge through the pipe if the pressure difference between the two tubes is 60 mm of water. Take the coefficient of pitot tube as $C_v = 0.98$.

Given: $d = 300$ mm $= 0.3$ m; $h = 60$ mm water $= 0.06$ m; $v_1 = 0.8 \times$ central velocity; $C_v = 0.98$;

Solution:



Central velocity,

$$v = C_v\sqrt{2gh}$$
$$= 0.98\sqrt{2(9.81)(0.06)}$$
$$= 1.063\ m/s$$

Velocity of flow, v_1,

$$v_1 = 0.8 \times 1.063$$
$$= 0.8504\ m/s$$

Hence, the discharge through pipe,

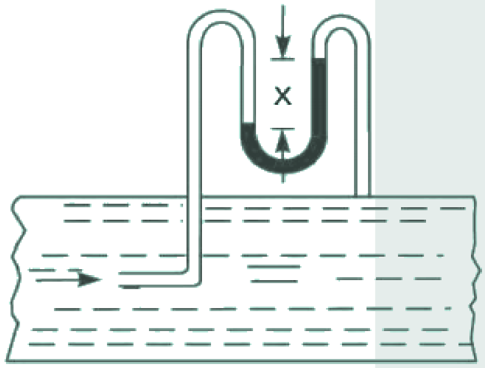
$$Q = v_1A_1$$
$$= 0.8504\left(\frac{\pi 0.3^2}{4}\right)$$
$$= 0.06\ m^3/s(Ans)$$

Example 1.19: (Pitot Tube)

Find the velocity of the flow of an oil through a pipe, when the difference of mercury level in a differential U-tube manometer connected to the two tappings of the pitot tube is 100 mm. Take co-efficient of pitot-tube 0.98 and SH of oil = 0.8.

Given: x = 100 mm = 0.1 m; C_v = 0.98; SG_{oil} = 0.8; SG_{Hg} = 13.6;

Solution:



Difference in pressure head,

$$h = x\left(\frac{\gamma_{Hg}}{\gamma_f} - 1\right) = 0.1\left(\frac{13.6}{0.8} - 1\right) = 1.6\, m$$

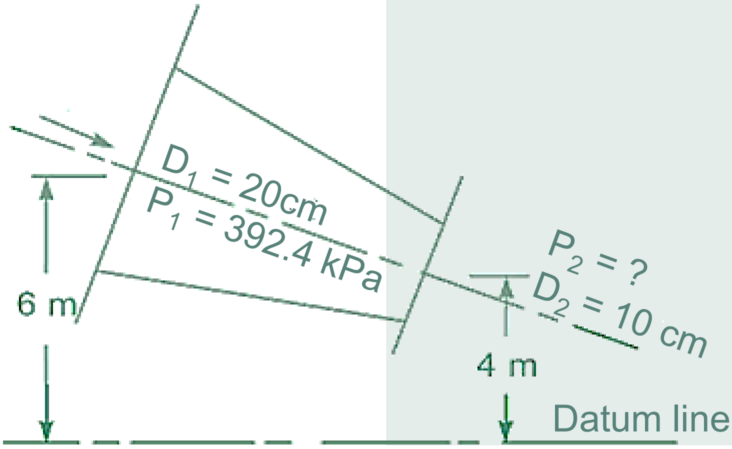
Velocity of flow, v_1,

$$\begin{aligned} v_1 &= C_v\sqrt{2gh} \\ &= 0.98\sqrt{2(9.81)(1.6)} \\ &= 5.4908\, m/s (Ans) \end{aligned}$$

TUTORIAL 1.3

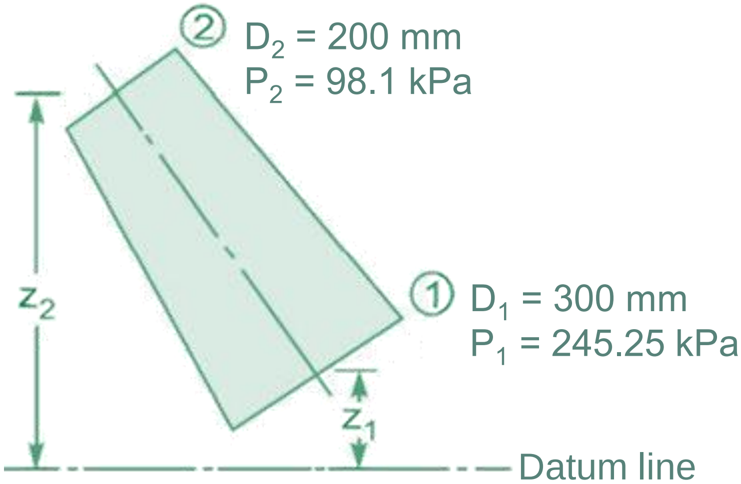
Q1-9

The water is flowing through a pipe having diameters 20 cm and 10 cm at sections 1 and 2 respectively. The rate of flow through pipe is 35 litres/s. The section 1 is 6 m above datum and section 2 is 4 m above datum. If the pressure at section 1 is 392.4 kPa, find the intensity of pressure at section 2. [Ans: 402.7 kPa]



Q1-10

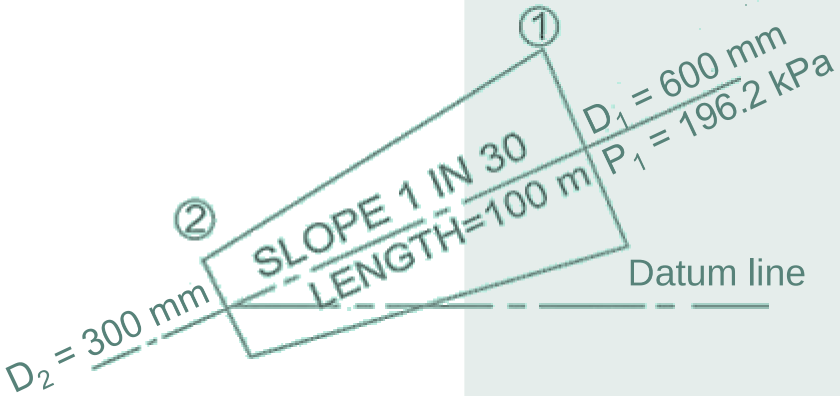
Water is flowing through a pipe having diameter 300 mm and 200 mm at the bottom and upper end respectively. The intensity of pressure at the bottom end is 245.25 kPa and the pressure at the upper end is 98.1 kPa. Determine the difference in datum head if the rate of flow through the pipe is 40 litres/s. [Ans: 14.9 m]



TUTORIAL 1.3

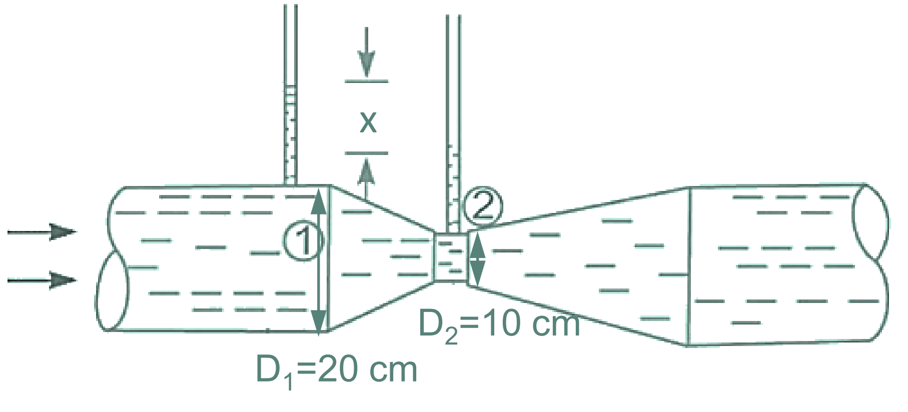
Q1-11

The water is flowing through a taper pipe of length 100 m having diameters 600 mm at the upper end and 300 mm at the lower end, at the rate of 50 litres/s. The pipe has a slope of 1 in 30. Find the pressure at the lower end if the pressure at the higher level is 196.2 kPa. [Ans: 228.7 kPa]



Q1-12

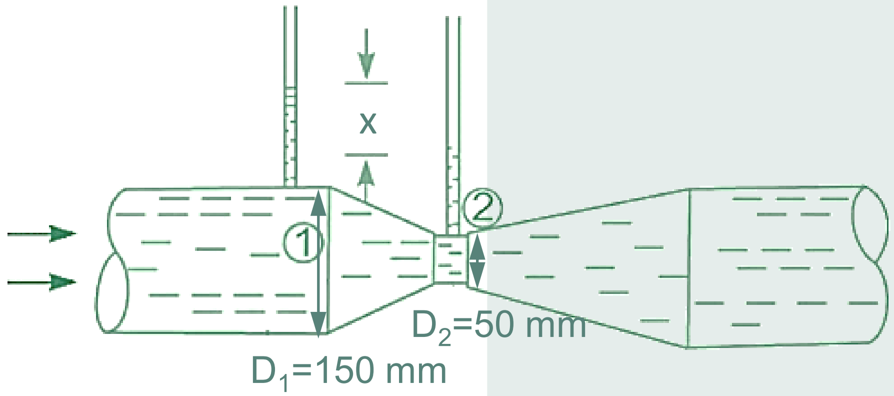
An oil of SG 0.8 is flowing through a venturi meter having inlet diameter 20 cm and throat diameter 10 cm. The oil-mercury differential manometer shows a reading of 25 cm. Calculate the discharge of oil through the horizontal manometer. Take C_d = 0.98. [Ans: 0.07 m^3/s]



TUTORIAL 1.3

Q1-13

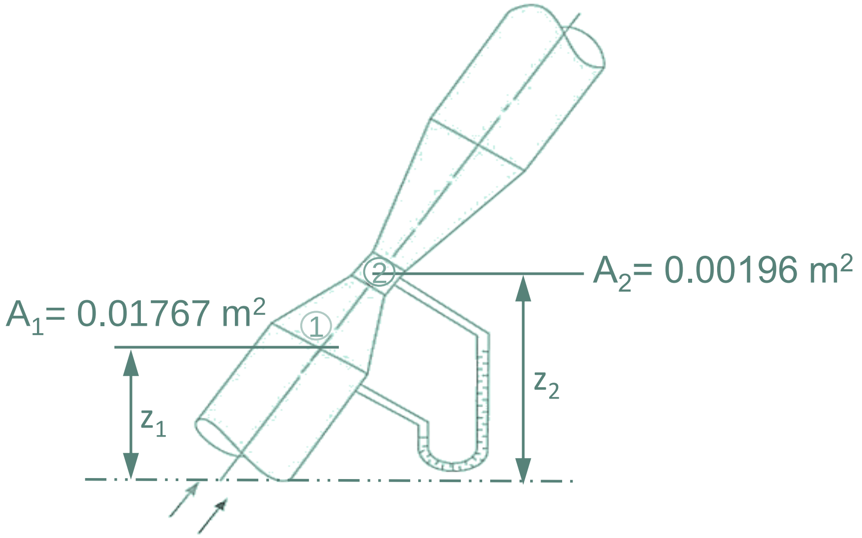
A venturi meter measures flow of oil of specific gravity of 0.86. Its entrance and throat diameter are 150 mm and 50 mm respectively. If the coefficient for the venturi meter is 0.98 and pressure difference is 34.5 kN/m², calculate the actual discharge in m³/s. [Ans: 0.0173 m³/s]



Q1-14

An inclined venturi meter as shown below, measures flow of oil of specific gravity of 0.86. Its entrance and throat area are 0.01767 m² and 0.00196 m² respectively. There is a pressure gauge between the entrance and the throat is at 250 mm above the entrance. If the coefficient of the venturi meter is 0.97 and the pressure difference is 35 kN/m², calculate the actual flow rate of the oil in m³/s.

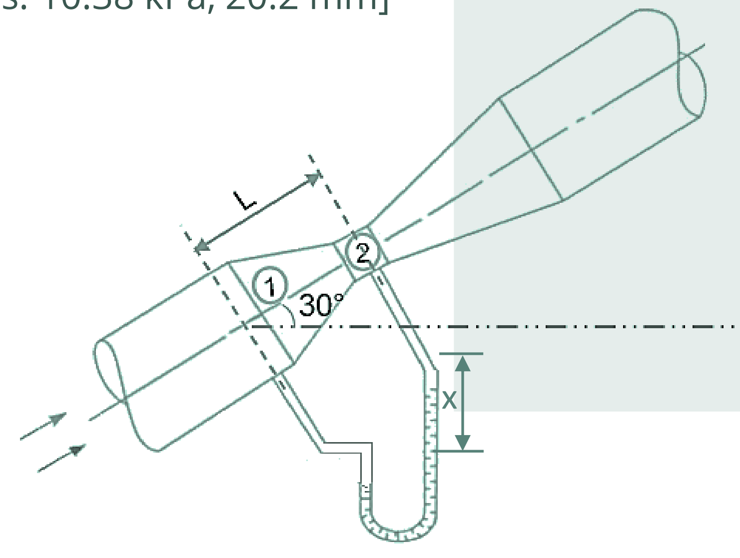
[Ans: 0.0167 m³/s]



TUTORIAL 1.3

Q1-15

An inclined venturi meter at 30° measured the flow of oil with SG 0.8 in a pipe with diameter of d_1 = 200 mm and the diameter of the throat is d_2 = 100 mm. The difference between the throat and the entrance is measured by an oil mercury manometer. Calculate pressure difference and the difference in level of mercury in the manometer if the distance, L is 2 m and the oil flowing in the pipe at 20 litres/s. Take C_d = 0.98. [Ans: 10.38 kPa, 20.2 mm]



Q1-16

An orifice meter with orifice diameter of 15 cm is inserted in a pipe of 30 cm diameter. The pressure difference measured by a mercury oil differential manometer on the two sides of the orifice meter gives a reading of 50 cm of mercury. Find the rate of flow of oil of SG 0.9 when the coefficient of discharge of the orifice meter is 0.64.
[Ans: 0.137 m^3/s]

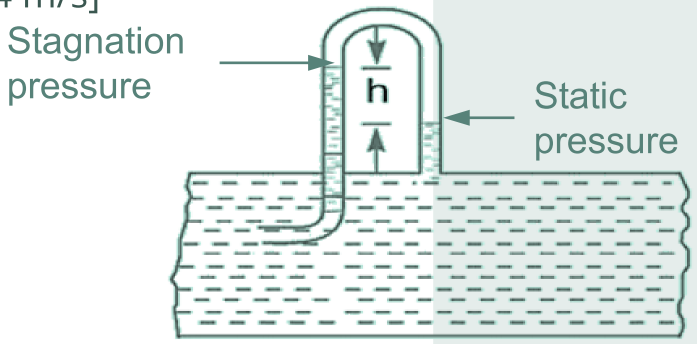
Q1-17

An orifice meter with orifice diameter of 15 cm is inserted in a pipe of 30 cm diameter. The pressure gauges fitted upstream and downstream of the orifice meter give readings of 14.715 N/cm^2 and 9.81 N/cm^2 respectively. Find the rate of flow of water through the pie in litres/s. Take C_d = 0.6. [Ans: 108.5 litres/s]

TUTORIAL 1.3

Q1-18

A Pitot-static tube is used to measure the velocity of water in a pipe. The stagnation pressure head is 6 m and static pressure head is 5 m. Calculate the velocity of flow assuming the coefficient of tube equal to 0.98. [Ans: 4.34 m/s]

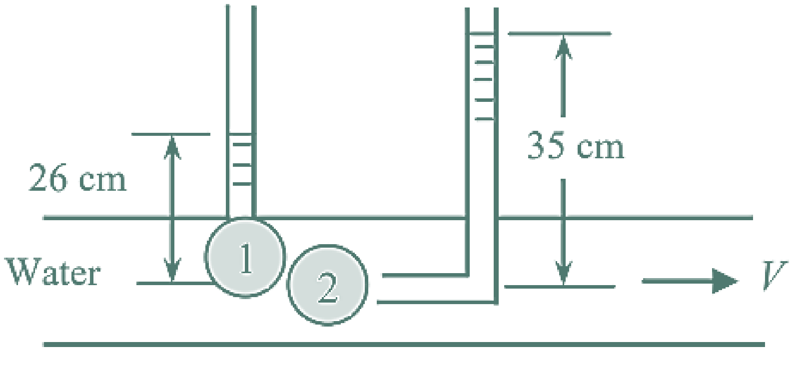


Q1-19

A sub-marine moves horizontally in sea and has its axis 15 m below the surface of water. A pitot-tube properly placed just in front of the sub-marine and along its axis is connected to the two limbs of a U-tube containing mercury. The difference of mercury level is found to be 170 mm. Find the speed of the sub-marine in km/h, knowing that the SG of mercury is 13.6 and that of sea water is 1.026 with respect of fresh water. [Ans: 23 km/h]

Q1-20

A piezometer and Pitot tube are installed into a 4 cm diameter horizontal water pipe, and the height of the water columns are measured to be 26 cm in the piezometer and 35 cm in the Pitot tube (both measured from the top surface of pipe). Determine the velocity at the centre of the pipe. [Ans: 1.33 m/s]



TUTORIAL 1 WORKED SOLUTIONS

Q1-1

Oil flows through a pipe at a mean velocity of 1.5 m/s. The diameter of the pipe is 8 cm. Calculate the discharge and mass flow rate of oil, given the specific gravity of oil is 0.85. [Ans: 0.0075 m^3/s, 6.4 kg/s]

Given: v = 1.5 m/s; d = 8 cm = 0.08 m;
SG_{oil} = 0.85, ρ_{oil} = 850 kg/m^3;

Solution:

The discharge of oil,

$$Q = vA = v(\frac{\pi d^2}{4}) = (1.5)\frac{\pi(0.08^2)}{4} = 0.00754\ m^3/s(Ans)$$

Mass flow rate,

$$\dot{m} = \rho Q = (850)(0.00754) = 6.41\ kg/s(Ans)$$

Q1-5

A 25 cm diameter pipe carries oil with Specific Gravity (SG) of 0.9 at a velocity of 3 m/s. At another section, the diameter is 20 cm. Find the velocity and mass flow rate of oil at this section.
[Ans: 4.69 m/s, 132.5 kg/s]

Given: d_1 = 25 cm = 0.25 m ; d_2 = 20 cm = 0.2 m; v_1 = 3 m/s;
SG_{oil} = 0.9, ρ_{oil} = 900 kg/m^3;

Solution:

Velocity at section 2, v_2,

$$Q_1 = Q_2$$

$$v_1 A_1 = v_2 A_2$$

$$v_2 = \frac{v_1 A_1}{A_2} = \frac{v_1 \frac{\pi {d_1}^2}{4}}{\frac{\pi {d_2}^2}{4}} = \frac{3(\frac{\pi 0.25^2}{4})}{\frac{\pi 0.2^2}{4}} = 4.687\ m/s(Ans)$$

Mass flow rate at section 2,

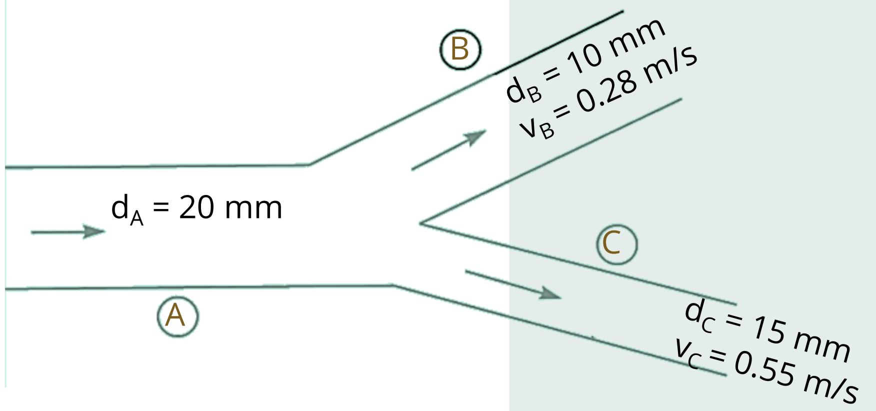
$$\dot{m} = \rho Q_1 = \rho_{oil} v_1 A_1 = 900(3)(0.04909)$$
$$= 132.543\ kg/s(Ans)$$

TUTORIAL 1 WORKED SOLUTIONS

Q1-7

Figure below shows a round pipe A with a diameter of 20 mm. Oil flow splits into two at the end of the pipe. Pipe B with a diameter of 10 mm has a velocity of 0.28 m/s and pipe C with diameter 15 mm has a velocity of 0.55 m/s. Determine the velocity in pipe A, flowrate in pipe B and C. [Ans: 0.38 m/s, 2.2 x 10^{-5} m^3/s, 9.7 x 10^{-5} m^3/s]

Given:



Solution:

Flowrate in pipe B and C,

$$Q_B = v_B A_B$$
$$= 0.28\left(\frac{\pi 0.01^2}{4}\right) = 0.0000219\ m^3/s(Ans)$$
$$Q_c = v_c A_c$$
$$= 0.55\left(\frac{\pi 0.015^2}{4}\right) = 0.00009719\ m^3/s(Ans)$$

Velocity in pipe A,

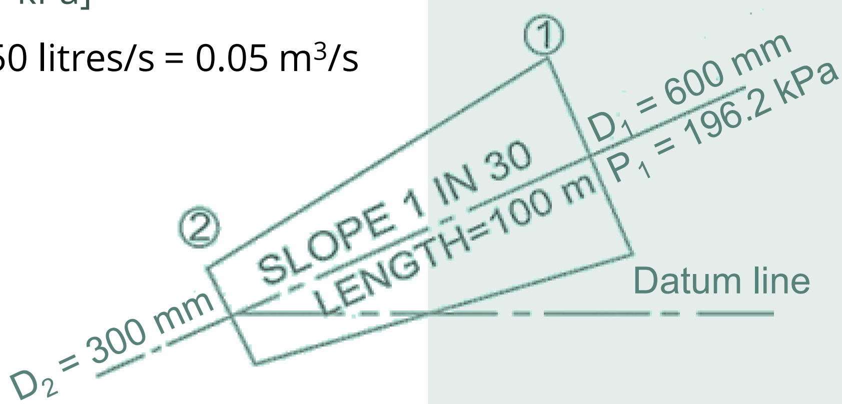
$$Q_A = Q_B + Q_C$$
$$v_A A_A = 0.0000219 + 0.00009719$$
$$v_A = \frac{0.00119}{\left(\frac{\pi 0.02^2}{4}\right)} = 0.379\ m/s(Ans)$$

TUTORIAL 1 WORKED SOLUTIONS

Q1-11

The water is flowing through a taper pipe of length 100 m having diameters 600 mm at the upper end and 300 mm at the lower end, at the rate of 50 litres/s. The pipe has a slope of 1 in 30. Find the pressure at the lower end if the pressure at the higher level is 196.2 kPa. [Ans: 228.7 kPa]

Given: Q = 50 litres/s = 0.05 m^3/s



Solution:

Velocity at section 1 and 2,

$$Q = v_1 A_1 \rightarrow v_1 = \frac{Q}{A_1} = \frac{0.05}{\frac{\pi}{4}(0.6^2)} = 0.1768\, m/s$$

$$Q = v_2 A_2 \rightarrow v_2 = \frac{Q}{A_2} = \frac{0.05}{\frac{\pi}{4}(0.3^2)} = 0.7074\, m/s$$

Slope is 1 in 30 means,

$$z_1 = \frac{1}{30} \times 100 = \frac{10}{3} m$$

Pressure at the lower end,

$$\frac{P_1}{\rho g} + \frac{{v_1}^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{{v_2}^2}{2g} + z_2$$

$$\frac{196200}{(1000)(9.81)} + \frac{0.1768^2}{2(9.81)} + \frac{10}{3} = \frac{P_2}{(1000)(9.81)} + \frac{0.7074^2}{2(9.81)} + 0$$

$$20 + 0.001593 + 3.3334 = \frac{P_2}{9810} + 0.0255$$

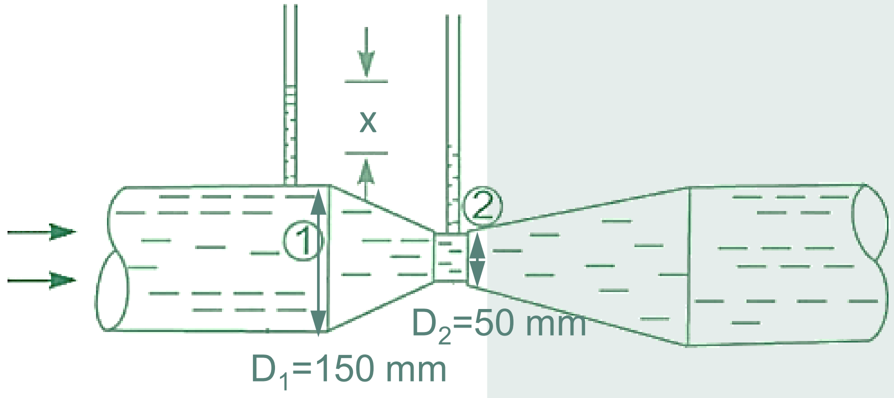
$$P_2 = 228663\, Pa = 228.66\, kPa\ (Ans)$$

TUTORIAL 1 WORKED SOLUTIONS

Q1-13

A venturi meter measures flow of oil of specific gravity of 0.86. Its entrance and throat diameter are 150 mm and 50 mm respectively. If the coefficient for the venturi meter is 0.98 and pressure difference is 34.5 kN/m^2, calculate the actual discharge in m^3/s. [Ans: 0.0173 m^3/s]

Given: $p_1 - p_2 = 34.5$ kN/m^2; $C_v = 0.98$; $SG_{oil} = 0.86$;



Solution:

Pressure head,

$$H = \frac{p_1 - p_2}{\gamma} = \frac{34500}{(0.86)(1000)(9.81)} = 4.0893\ m$$

Ratio of area, m,

$$m = \frac{A_1}{A_2} = \left(\frac{d_1}{d_o}\right)^2 = \frac{0.15^2}{0.05^2} = 9$$

Thus, discharge, Q,

$$Q_{actual} = C_d(A_1)\sqrt{\frac{2gH}{(m^2-1)}}$$

$$= (0.98)\left(\frac{\pi 0.15^2}{4}\right)\sqrt{\frac{2(9.81)(4.0893)}{(9^2-1)}}$$

$$= 0.01734\ m^3/s(Ans)$$

TUTORIAL 1 WORKED SOLUTIONS

Q1-17

An orifice meter with orifice diameter of 15 cm is inserted in a pipe of 30 cm diameter. The pressure gauges fitted upstream and downstream of the orifice meter give readings of 14.715 N/cm^2 and 9.81 N/cm^2 respectively. Find the rate of flow of water through the pie in litres/s. Take C_d = 0.6. [Ans: 108.5 litres/s]

Given: d_o = 15 cm = 0.15 m; d_1 = 30 cm = 0.3 m;
P_1 = 14.715 N/cm^2 = 147150 N/m^2;
P_2 = 9.81 N/cm^2 = 98100 N/m^2; C_d = 0.6;

Solution:

Pressure head,

$$H = \frac{p_1 - p_2}{\gamma} = \frac{147150 - 98100}{9810} = 5\,m$$

Ratio of area,

$$m = \left(\frac{d_1}{d_o}\right)^2 = \left(\frac{0.3}{0.15}\right)^2 = 4$$

Hence, the rate of flow, Q ,

$$Q_{actual} = C_d(A_1)\sqrt{\frac{2gH}{(m^2-1)}}$$

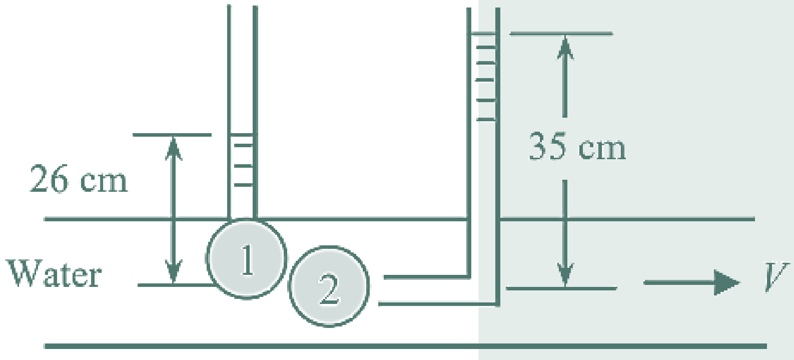
$$= (0.6)\left(\frac{\pi 0.3^2}{4}\right)\sqrt{\frac{2(9.81)(5)}{(4^2-1)}}$$

$$= 0.10846\,m^3/s = 108.466\,litres/s$$

TUTORIAL 1 WORKED SOLUTIONS

Q1-20

A piezometer and Pitot tube are installed into a 4 cm diameter horizontal water pipe, and the height of the water columns are measured to be 26 cm in the piezometer and 35 cm in the Pitot tube (both measured from the top surface of pipe). Determine the velocity at the centre of the pipe. [Ans: 1.33 m/s]



Given: h_{piezo} = 26 cm = 0.26 m; h_{Pitot} = 35 cm = 0.35 m

Solution:

Pressure head difference between piezometer and Pitot tube,

$$h = h_{Pitot} - h_{piezo}$$
$$= 0.35 - 0.26 = 0.09\ m$$

Central velocity (theoretical) of the water pipe,

$$v = \sqrt{2gh}$$
$$= \sqrt{2(9.81)(0.09)}$$
$$= 1.33\ m/s (Ans)$$

Chapter 2:

ENERGY LOSS IN PIPELINE

2.1 INTRODUCTION

Fluid flows through pipelines or ducts is commonly used in heating and cooling applications and fluid distribution networks (**Figure 2.1**). Liquid or gas in such applications is usually forced to flow by mechanical means such as pump or fan through a flow section.

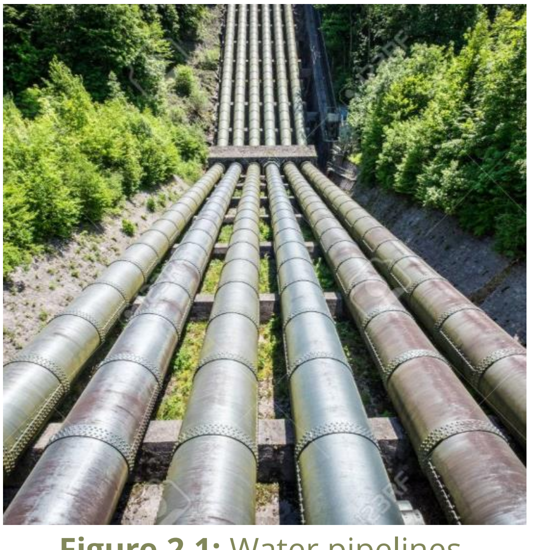


Figure 2.1: Water pipelines.

Most fluids, particularly liquids, are transported in round pipes. This is because pipes with a round/circular cross section can withstand large pressure differences between the inside and outside without undergoing significant distortion.

The start of the chapter will first identify the velocity profile of fluid flow in round pipe system, then introduce the pressure drop correlations and minor losses associated with pipe flow. Finally, we will solve basic problems related to energy loss in pipeline.

2.2 VELOCITY PROFILE IN ROUND PIPE SYSTEM

Fluid velocity in a round pipe **changes from zero at the wall** because of the no-slip condition to **a maximum at the pipe-centre**. In fluid flow, it is convenient to work with an average velocity, v_{avg}, which remains constant for incompressible flow when the cross-sectional area of the pipe is constant.

Consider a fluid entering a round pipe at a uniform velocity (**Figure 2.2**). Because of the no-slip condition, the fluid particles in the layer in contact with the pipe wall come to a complete stop. This layer, in turn, causes the fluid particle in the adjacent layers to slow down gradually as a result of friction. To make up for this velocity reduction, the velocity of the fluid at the middle section of the pipe has to increase to keep the mass flow rate through the pipe constant. As a result, a velocity gradient develops along the pipe, known as velocity profile.

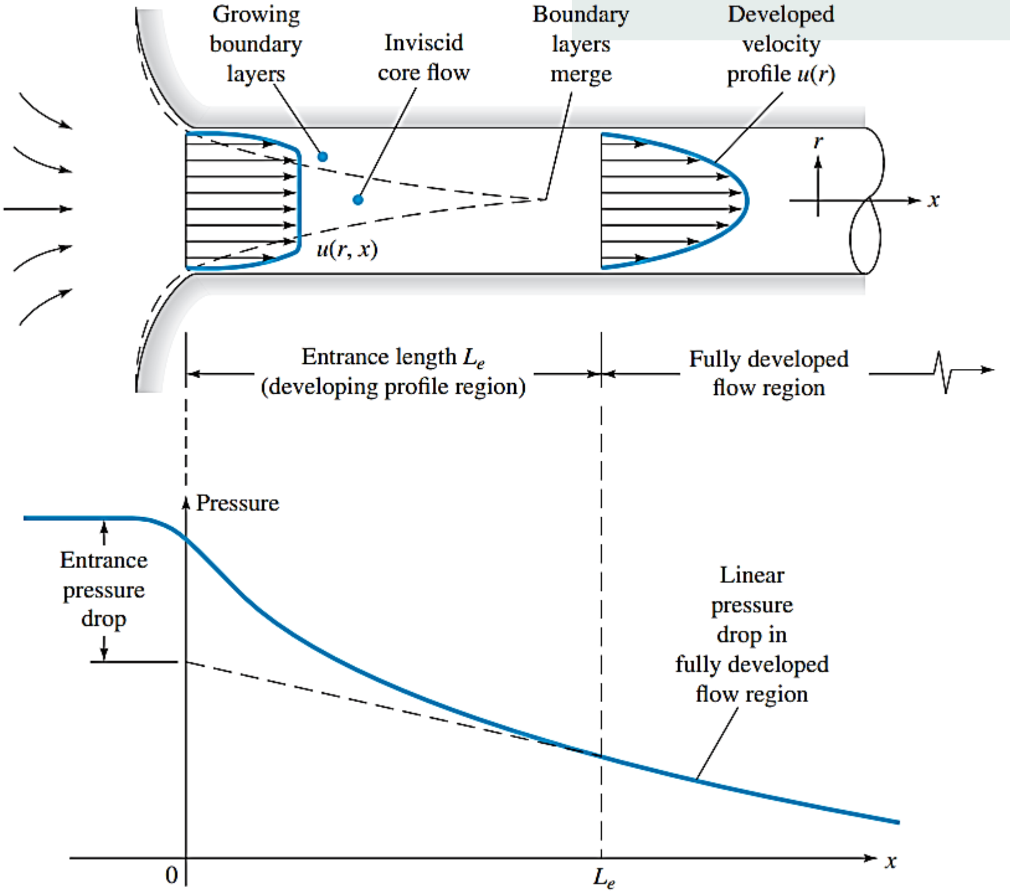


Figure 2.2: Velocity profiles and pressure changes in the entrance of a round-pipe flow. (source: R.C. Hibbeler, 2020)

Laminar vs Turbulent Velocity Profile. In fully developed **laminar flow**, each fluid particle moves at a constant axial velocity along the streamline. The velocity profile remains unchanged in the flow direction and there is no motion in radial direction. Therefore, the velocity component in the normal direction to the pipe axis is zero everywhere.

For laminar flow, velocity distribution at a cross section is **parabolic** in shape (**Figure 2.2**). Maximum velocity is at the centre and it is twice the average velocity in the pipe.

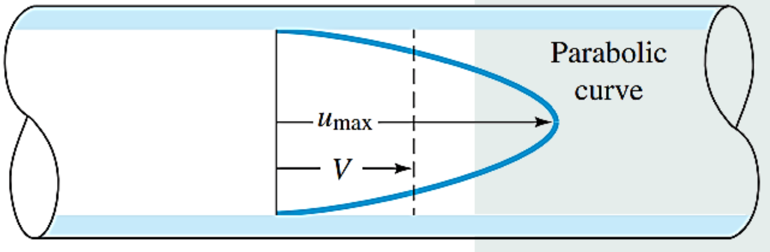


Figure 2.3: Laminar velocity profile. (source: R.C. Hibbeler, 2020)

While laminar flow is ideal, most flows encountered in engineering practice are turbulent, and so, it is important to understand how the turbulence affects wall of the pipes. The velocity profile becomes flatter or 'fuller' in turbulent flow compared to the parabolic in laminar flow. The fullness increases when the flow becomes more turbulent (increase of Reynolds number) and the velocity profile becomes nearly uniform.

Hence, for **turbulent flow**, **flat** (almost) **velocity profile** exists across the section of pipe (**Figure 2.3**). Entire fluid flows at an average velocity is almost equal to maximum velocity.

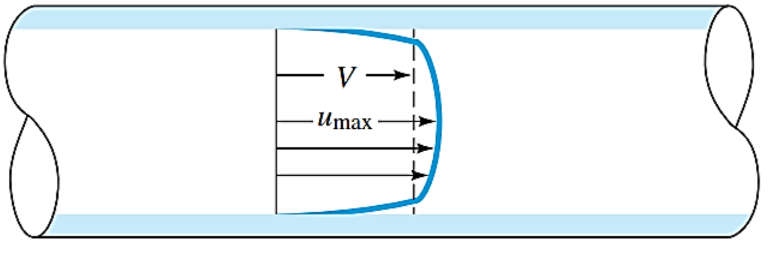


Figure 2.4: Turbulent velocity profile. (source: R.C. Hibbeler, 2020)

2.3 TYPES OF HEAD LOSS IN PIPE SYSTEM

A typical piping system involves pipes of different diameters connected by various fittings or elbows to route the fluid, valves to control the flowrates and pumps to pressurize the fluid. Pressure drop and head loss during flow through pipes and ducts often time occur **due to frictions in the piping system**. The **loss of energy** or also known as **head loss** can be classified as:

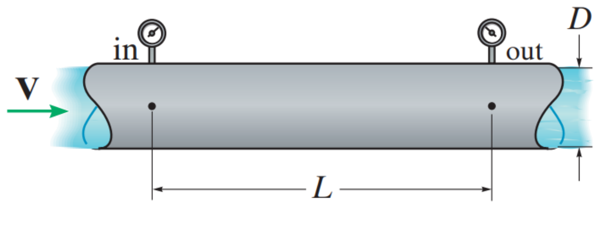
Energy/Head losses

- **Major head losses**
  - **Due to:**
    Friction
    *fluid friction and pipe wall roughness friction.
- **Minor head losses**
  - **Due to:**
    Sudden expansion of pipe
    Sudden contraction of pipe
    Bend in pipe
    Pipe fittings
    Obstruction in pipe

Major Head Loss. In engineering practices, major head loss, h_L refers to energy loss due to **fluid friction** (from fluid viscosity) **and pipe wall roughness**. Thus, it is also known as **friction head loss**. This head loss creates significant pressure drop over the pipe's length, L because the pressure have to work to overcome the frictional resistance that create this loss. For this reason, pressure at the pipe entrance will have to be greater than the exit.

The friction head loss can be calculated using **Darcy-Weisbach equation**, as follows;

$$h_f = \frac{4fL}{d} \cdot \frac{v^2}{2g} \qquad \text{— Eq. 2-1}$$



Where, f = friction factor
L = length of pipe
v = mean/average velocity of flow
d = diameter of pipe

Example 2.1:

Determine the head loss due to friction in a 14 m long and 2 m diameter which carries oil at average velocity of 1.5 m/s. Take f = 0.05.

Given: L = 14 m; d = 2 m; v = 1.5 m/s; f = 0.05;

Solution:

Using Darcy-Weisbach equation,

$$h_f = \frac{4fL}{d} \cdot \frac{v^2}{2g}$$
$$= \frac{4(0.05)(14)}{2} \cdot \frac{1.5^2}{2(9.81)}$$
$$= 0.16055\ m\ (Ans)$$

Example 2.2:

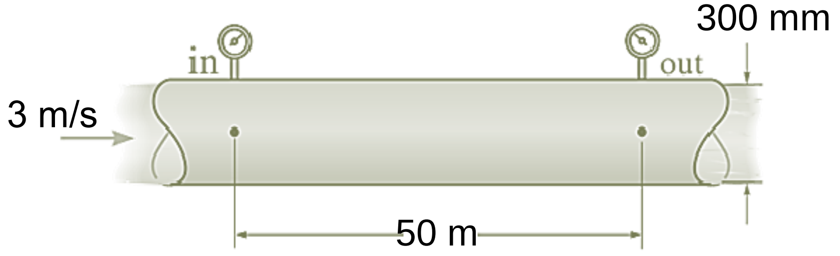
Find the head loss due to friction in a 300 mm diameter pipe and 50 m length, through which water is flowing at average velocity of 3 m/s. Take f = 0.0025.

Given: L = 50 m; d = 300 mm = 0.3 m; v = 3 m/s; f = 0.0025;

Solution:

Using Darcy-Weisbach equation,

$$h_f = \frac{4fL}{d} \cdot \frac{v^2}{2g}$$
$$= \frac{4(0.0025)(50)}{0.3} \cdot \frac{3^2}{2(9.81)}$$
$$= 0.7645\ m\ (Ans)$$



Minor Head Loss. Loss of energy due to changes in velocity in magnitude or direction is called **minor loss of energy** or simply minor head loss. Minor loss of energy includes; head loss due to sudden enlargement, sudden contraction, head loss at the entrance and exit of pipe, obstruction, bends and various pipe fittings.

In case of a long pipe system, these losses are small compared to the head loss due to friction and can even be neglected without serious error. However, these losses are comparable with the major head loss in the case of short pipe system.

Head Loss - Due to Sudden Enlargement. Consider a liquid flow through a pipe which has sudden enlargement (**Figure 2.5**).

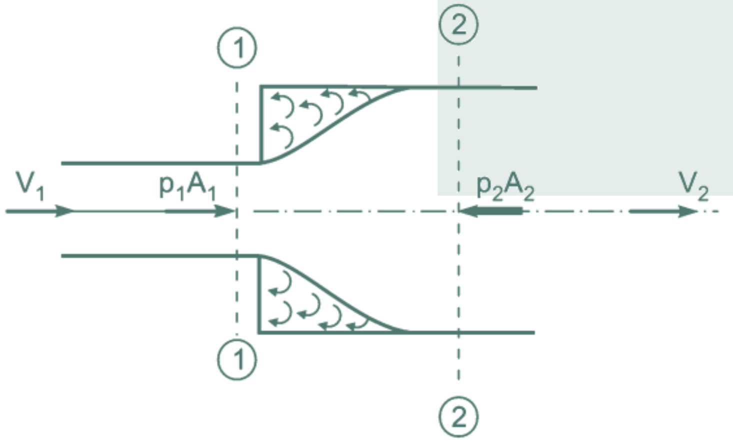


Figure 2.5: Sudden enlargement. (source: R.K Bansal, 2018)

Head loss due to sudden pipe enlargement is given by,

$$h_L = \frac{(v_1 - v_2)^2}{2g} \qquad \textbf{Eq. 2-2}$$

Where, v_1 = velocity at section 1
v_2 = velocity at section 2

Example 2.4: (Head loss due to Sudden Enlargement)

A pipe carrying 0.06 m3/s of water increase suddenly from 75 mm to 180 mm diameter. Find,

i. The head loss due to the sudden enlargement
ii. The difference in pressure between the pipes.

Given: d_1 = 75 mm = 0.075 m, d_2 = 180 mm = 0.18 m, Q = 0.06 m^3/s, P_2-P_1 = ?

Solution:

i. Head loss due to sudden enlargement,

Velocity at 1 & 2,

$$Q = A_1 v_1 \rightarrow v_1 = \frac{Q}{A_1} = \frac{Q}{\pi \frac{{d_1}^2}{4}} = \frac{0.06}{\pi \frac{0.075^2}{4}} = 13.581\, m/s$$

$$Q = A_2 v_2 \rightarrow v_2 = \frac{Q}{A_2} = \frac{Q}{\pi \frac{{d_2}^2}{4}} = \frac{0.06}{\pi \frac{0.18^2}{4}} = 2.358\, m/s$$

Thus, head loss due to sudden enlargement,

$$\therefore h_L = \frac{(v_1 - v_2)^2}{2g} = \frac{(13.581 - 2.358)^2}{2(9.81)} = 6.42\, m\ (Ans)$$

ii. Difference in pressure between pipes,

Using Bernoulli equation for horizontal pipes (z_1 = z_2),

$$\frac{p_1}{\rho g} + \frac{{v_1}^2}{2g} + \cancel{z_1}^{0} = \frac{p_2}{\rho g} + \frac{{v_2}^2}{2g} + \cancel{z_2}^{0} + h_L$$

$$\frac{p_1}{\rho g} + \frac{{v_1}^2}{2g} = \frac{p_2}{\rho g} + \frac{{v_2}^2}{2g} + h_L$$

$$\frac{p_2 - p_1}{(1000)(9.81)} = \frac{13.581^2 - 2.358^2}{2(9.81)} - 6.42$$

$$p_2 - p_1 = 26461.5 N/m^2 = 26.46\, kN/m^2 (Ans)$$

Example 2.5: (Head loss due to Sudden Enlargement)

Find the loss of head when a pipe of diameter 200 mm is suddenly enlarged to a diameter of 400 mm. The rate of flow of water through the pipe is 250 litres/s. The pressure intensity in the smaller pipe is 117.72 kN/m^2. Determine:

i. The head loss due to the sudden enlargement
ii. The pressure intensity in the large pipe.

Given: d_1 = 200 mm = 0.2 m; d_2 = 400 mm = 0.4 m; Q = 250 litres/s = 0.06 m^3/s; P_1 = 117.72 kN/m^2

Solution:

i. Head loss due to sudden enlargement,
Velocity at 1 & 2,

$$Q = A_1 v_1 \rightarrow v_1 = \frac{Q}{A_1} = \frac{Q}{\pi \frac{{d_1}^2}{4}} = \frac{0.25}{\pi \frac{0.2^2}{4}} = 7.958\, m/s$$

$$Q = A_2 v_2 \rightarrow v_2 = \frac{Q}{A_2} = \frac{Q}{\pi \frac{{d_2}^2}{4}} = \frac{0.25}{\pi \frac{0.4^2}{4}} = 1.989\, m/s$$

Thus, head loss due to sudden enlargement,

$$\therefore h_L = \frac{(v_1 - v_2)^2}{2g} = \frac{(7.958 - 1.989)^2}{2(9.81)} = 1.816\, m\ (Ans)$$

ii. Difference in pressure between pipes,
From Bernoulli equation for horizontal pipes ($z_1 = z_2$),

$$\frac{p_1}{\rho g} + \frac{{v_1}^2}{2g} + \cancelto{0}{z_1} = \frac{p_2}{\rho g} + \frac{{v_2}^2}{2g} + \cancelto{0}{z_2} + h_L$$

$$\frac{p_1}{\rho g} + \frac{{v_1}^2}{2g} = \frac{p_2}{\rho g} + \frac{{v_2}^2}{2g} + h_L$$

$$\frac{p_2}{(1000)(9.81)} = \frac{117720}{(1000)(9.81)} + \frac{7.958^2}{2(9.81)} - \frac{1.989^2}{2(9.81)} - 1.816$$

$$p_2 = 129591.86 N/m^2 = 129.59\, kN/m^2 (Ans)$$

Head Loss - Due to Sudden Contraction. Consider a liquid flowing in a pipe which has sudden contraction (**Figure 2.6**).

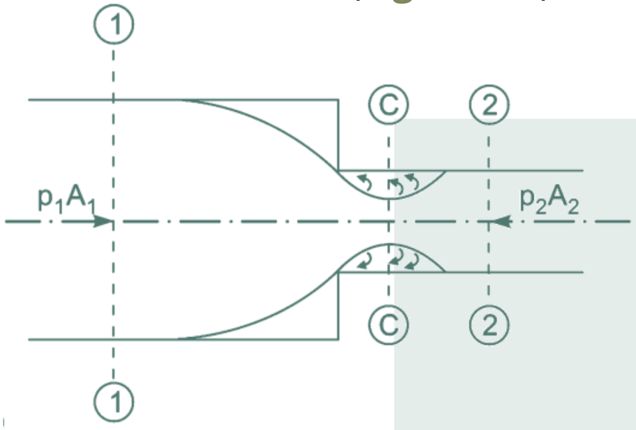


Figure 2.6: Sudden contraction. (source: R.K Bansal, 2018)

Head loss due to sudden pipe contraction (from vena-contracta), is given by,

$$h_c = \frac{{v_2}^2}{2g}\left[\frac{1}{C_c} - 1\right]^2 \qquad \textbf{Eq. 2-3}$$

Where, C_c = coefficient of contraction

v_2 = velocity at section 2

Example 2.6: (Head loss due to Sudden Contraction)

Water flows from an inclined pipeline with a rate of 1800 l/min from a 150 mm diameter pipe to 70 mm diameter pipe. The exit of the pipe is located 5 m higher than the entrance. If the coefficient of contraction is 0.63, calculate:

i. The head loss due to the sudden contraction.
ii. The pressure difference between the .

Given: d_1 = 150 mm = 0.15 m; d_2 = 70 mm = 0.07 m; Q = 1800 l/min = 0.03 m^3/s; $z_2 - z_1$ = 5 m; C_c = 0.63;

Solution:

i. Head loss due to sudden enlargement,

Velocity at 1 & 2,

$$Q = A_1 v_1 \rightarrow v_1 = \frac{Q}{A_1} = \frac{Q}{\pi \frac{{d_1}^2}{4}} = \frac{0.03}{\pi \frac{0.15^2}{4}} = 1.6977\, m/s$$

$$Q = A_2 v_2 \rightarrow v_2 = \frac{Q}{A_2} = \frac{Q}{\pi \frac{{d_2}^2}{4}} = \frac{0.03}{\pi \frac{0.07^2}{4}} = 7.7953\, m/s$$

Thus, head loss due to sudden contraction,

$$\therefore h_c = \frac{{v_2}^2}{2g}\left[\frac{1}{C_c} - 1\right]^2 = \frac{7.7953^2}{2(9.81)}\left[\frac{1}{0.63} - 1\right]^2 = 1.0683\, m$$

ii. Difference in pressure between pipes,

From Bernoulli equation,

$$\frac{p_1}{\rho g} + \frac{{v_1}^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{{v_2}^2}{2g} + z_2 + h_c$$

$$\frac{p_1 - p_2}{\rho g} = \frac{{v_2}^2 - {v_1}^2}{2g} + (z_2 - z_1) + h_c$$

$$\frac{p_1 - p_2}{(1000)(9.81)} = \frac{7.7953^2 - 1.6977^2}{2(9.81)} + 5 + 1.0683$$

$$p_1 - p_2 = 88472.28 N/m^2 = 88.472\, kN/m^2 (Ans)$$

Example 2.7: (Head loss due to Sudden Contraction)

A horizontal pipe with a diameter of 500 mm is suddenly contracted to a diameter of 250 mm. The pressure intensities in the large and smaller pipe are 137.34 kN/m^2 and 117.72 kN/m^2 respectively. Find the value of coefficient of contraction, C_c, if volume flow rate of water is 300 litres/s.

Given: d_1 = 500 mm = 0.5 m; d_2 = 250 mm = 0.25 m;
P_1 = 137.34 kN/m^2; P_2 = 117.72 kN/m^2;
Q = 300 litres/s = 0.3 m^3/s;

Solution:

Velocity at 1 & 2,

$$Q = A_1 v_1 \rightarrow v_1 = \frac{Q}{A_1} = \frac{Q}{\pi \frac{{d_1}^2}{4}} = \frac{0.3}{\pi \frac{0.5^2}{4}} = 1.5279\, m/s$$

$$Q = A_2 v_2 \rightarrow v_2 = \frac{Q}{A_2} = \frac{Q}{\pi \frac{{d_2}^2}{4}} = \frac{0.3}{\pi \frac{0.25^2}{4}} = 6.1115\, m/s$$

From Bernoulli equation horizontal pipes ($z_1 = z_2$),

$$\frac{p_1}{\rho g} + \frac{{v_1}^2}{2g} = \frac{p_2}{\rho g} + \frac{{v_2}^2}{2g} + h_c$$

$$\frac{p_1 - p_2}{\rho g} = \frac{{v_2}^2 - {v_1}^2}{2g} + h_c$$

$$\frac{137340 - 117720}{(1000)(9.81)} = \frac{6.1115^2 - 1.5279^2}{2(9.81)} + h_c$$

$$h_c = 0.2153\, m (Ans)$$

Thus, coefficient of contraction,

$$h_c = \frac{{v_2}^2}{2g}\left[\frac{1}{C_c} - 1\right]^2$$

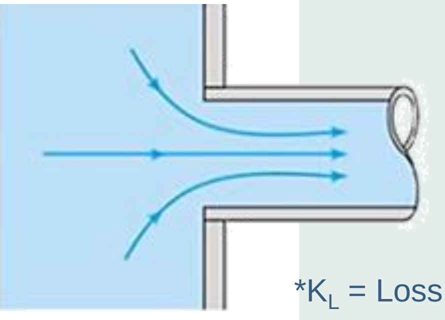
$$0.2153 = \frac{6.1115^2}{2(9.81)}\left[\frac{1}{C_c} - 1\right]^2$$

$$\therefore C_c = 0.7483\ (Ans)$$

Head Loss – At Sharp Inlet/Entrance. Loss of energy often occurs when a liquid enters a pipe which is connected to a large tank or reservoir, similar to the loss of head due to sudden contraction. In practice, the value of head loss at the inlet/entrance of a pipe with sharp cornered entrance is given by,

$$h_i = 0.5\frac{v^2}{2g} \qquad \text{Eq. 2-4}$$

Where, v = velocity of liquid at the inlet of pipe.
0.5 = loss coefficient, K_L for sharp edge.



Example 2.8: (Head loss at Sharp Inlet/Entrance)

Water from a large reservoir flow into a pipeline. If the entrance of the pipe is sharp and the velocity of flow in the pipe is 2 m/s, calculate the head loss.

Given: v = 2 m/s;

Solution:

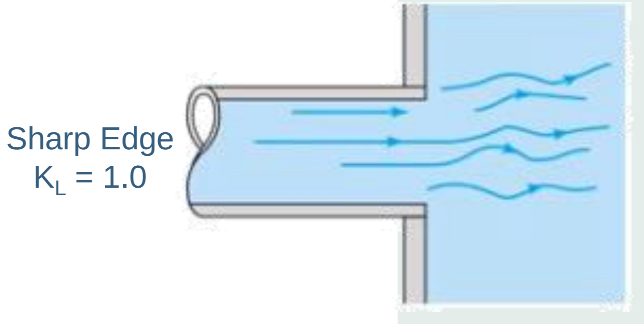
Head loss at the sharp entrance,

$$h_i = 0.5\frac{v^2}{2g}$$
$$= 0.5\frac{2^2}{(2)(9.81)}$$
$$= 0.1019\ m\ (Ans)$$

Head Loss – At Pipe Outlet/Exit. Loss of energy due to the velocity of liquid at the outlet of the pipe which is dissipated either in the form of free jet (if the pipe outlet is free) or it is lost in the tank/reservoir (if the outlet of the pipe connected to the tank/reservoir). For most type of pipe outlet, this loss is equal to,

$$h_o = \frac{v^2}{2g} \qquad \text{Eq. 2-5}$$

Where, v = velocity of liquid at the outlet of pipe.



Example 2.9: (Head loss at Pipe Outlet/Exit)

A pipeline connected to an open tank by sharp exit. Water flows with a velocity of 1.5 m/s into the tank. Determine the head loss.

Given: v = 1.5 m/s;

Solution:

Head loss at the sharp exit,

$$\begin{aligned} h_o &= \frac{v^2}{2g} \\ &= \frac{1.5^2}{(2)(9.81)} \\ &= 0.1147\ m \end{aligned}$$

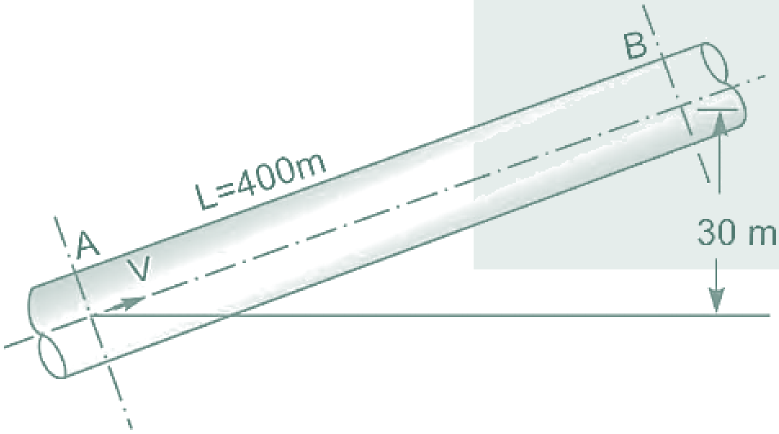
TUTORIAL 2.1

Q2-1

Determine the loss of head due to friction in a 40 m long and 500 mm diameter which carries oil at average velocity of 2.5 m/s. Take f = 0.01.
[Ans: 1.02 m]

Q2-2

A pipeline AB with 300 mm diameter and 400 m length carries water at the rate of 50 litres/s. The flow takes place from A to B where point B is 30 m above A. Find the pressure at A if the pressure at B is 196.2 kN/m^2. Take f = 0.008. [Ans: 501.2 kN/m^2]



Q2-3

The discharge through a pipe is 200 litres/s. Find the loss of head when the pipe is suddenly enlarged from 150 mm to 300 mm.
[Ans: 3.67 m]

Q2-4

The rate of flow of water through a horizontal pipe is 0.3 m^3/s. The diameter of the pipe is suddenly enlarged from 250 mm to 500 mm. The pressure intensity in the smaller pipe is 137,340 N/m^2. Determine:

i. Loss of head due to sudden enlargement
ii. Pressure intensity in the large pipe

[Ans: 1.07 m, 144.3 kN/m^2]

TUTORIAL 2.1

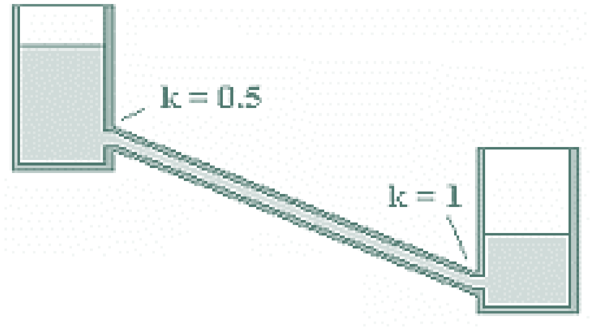
Q2-5

A horizontal pipe of diameter 400 mm is suddenly contracted to a diameter of 200 mm. The pressure intensities in the large and smaller pipe is given as 147.15 kN/m^2 and 127.53 kN/m^2 respectively. Find the loss of head due to contraction if C_c = 0.62. Also, determine the rate of flow of water. [Ans: 0.57 m, 0.172 m^3/s]

Q2-6

A pipe carrying 0.06 m^3/s suddenly contracts from 200 mm to 150 mm diameter. Assuming that the vena contracta is formed in the smaller pipe, calculate the coefficient of contraction if the pressure head at a point upstream of the contraction is 0.655 m greater the downstream of the vena contracta. [Ans: 0.6]

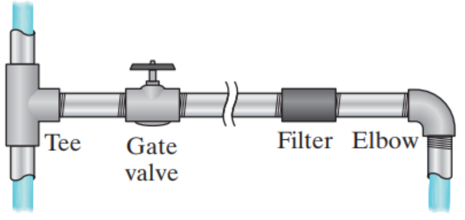
Q2-7



Water from a large reservoir flow into another reservoir through a horizontal pipeline at 5 m/s. If both the entrance and the exit of the pipeline is sharp, calculate the head loss at the pipe entrance and exit. [Ans: 0.64 m, 1.27 m]

2.4 HEAD LOSS IN PIPELINE SYSTEMS

Pipeline systems often used to transport water for industrial and residential usage, hydroelectric power, transport crude oil or lubricants through mechanical equipment, and many other applications. Pipelines that we often encounter, consists of a single diameter pipes with bends, valves, filters and transition.



The flow in a single pipeline must satisfy both continuity equations,

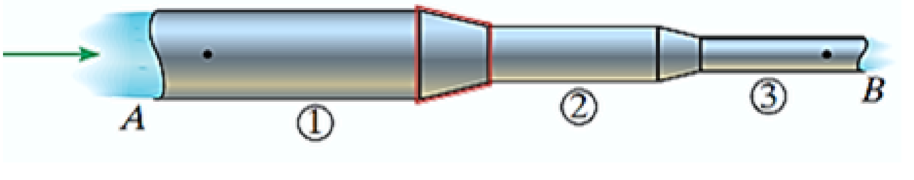
$$Q = v_{in}A_{in} = v_{out}A_{out}$$

and account for all major and minor head losses throughout the system,

$$\frac{p_{in}}{\rho g} + \frac{{v_{in}}^2}{2g} + z_{in} = \frac{p_{out}}{\rho g} + \frac{{v_{out}}^2}{2g} + z_{out} + friction\ loss + minor\ losses$$

In practice, most piping systems can be encountered in the water distributing systems involves numerous **parallel** and **series connections** as well as several supply sources.

Pipes in Series. Typically involve several pipes connected to each other in **succession**.



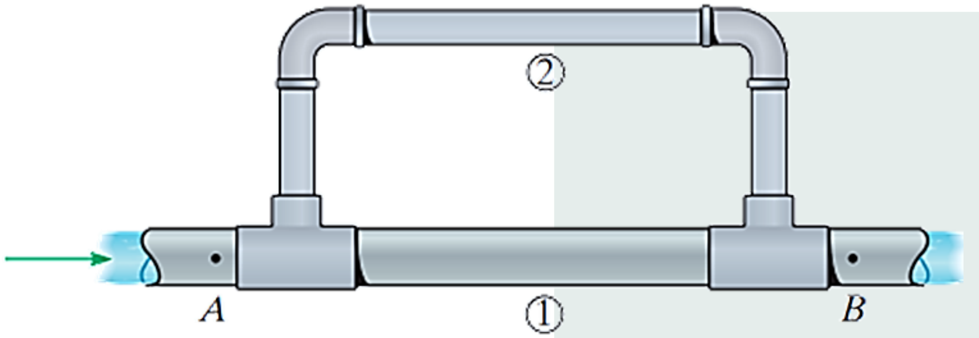
Flow rate remains constant in pipes connected in series regardless of the changes in diameters,

$$Q = Q_1 = Q_2 = Q_3$$

And the **total head loss** is equal to the sum of head losses in individual pipes in the system, including minor losses,

$$h_L = h_{L1} + h_{L2} + h_{L3} + h_{minor}$$

Pipes in Parallel. Pipes which **branches out** into two (or more) and then rejoin at a junction downstream, causing the flow divide into different loops.



Total flow rate is the sum of the flow rates in the individual pipes,

$$Q = Q_A = Q_B = Q_1 + Q_2$$

Head loss in each individual pipe connected in parallel must be the same, since $\Delta P = P_A - P_B$ and the junction pressures P_A and P_B are the same for all individual pipes.

$$h_{L1} = h_{L2}$$

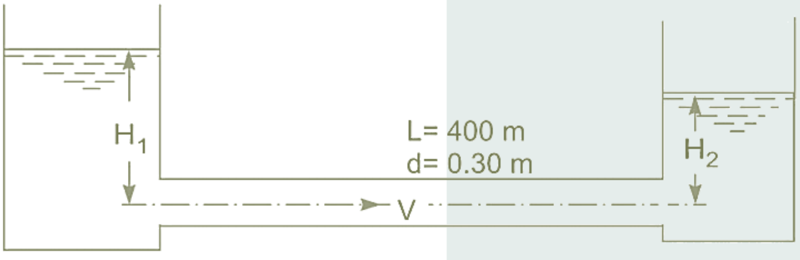
Solving Pipeline Problems. All pipelines systems related problems should be solved by applying Bernoulli's theorem between points for which the total energy is known, including (adding) expressions for any loss of energy due to shock (minor losses) or friction (major losses),

$$\frac{p_A}{\rho g} + \frac{{v_A}^2}{2g} + z_{in} = \frac{p_B}{\rho g} + \frac{{v_B}^2}{2g} + z_{out} + frictional\ loss + shock$$

$$\frac{p_A}{\rho g} + \frac{{v_A}^2}{2g} + z_{in} = \frac{p_B}{\rho g} + \frac{{v_B}^2}{2g} + z_{out} + h_f + \Sigma\, h_{minor} \qquad \textbf{Eq. 2-6}$$

Example 2.10:

Determine the difference in the elevations between the water surfaces In the two tanks which are connected by a horizontal pipe with diameter of 300 mm and 400 m length. The rate of flow of water through the pipes is 300 litres/s. Consider all loses and take the value of $f = 0.008$.



Given: d = 300 mm = 0.3 m; L = 400 m;

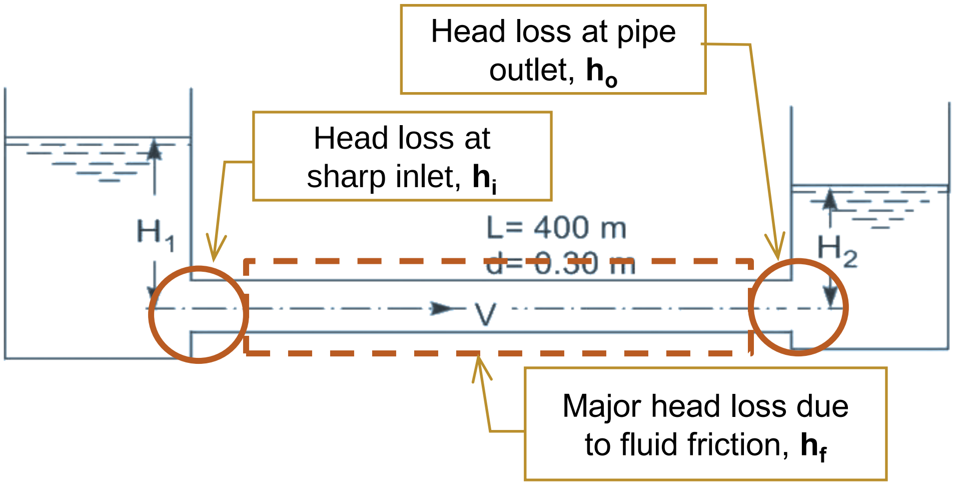
Q = 300 litres/s = 0.3 m³/s; $f = 0.008$;

Solution:

Velocity of fluid,

$$v = \frac{Q}{A} = \frac{0.3}{\frac{\pi(0.3)^2}{4}} = 4.244\ m/s$$

Head losses in the pipeline system:



Major head loss due to friction,

$$h_f = \frac{4fL}{d} \cdot \frac{v^2}{2g} = \frac{4(0.008)(400)}{0.3} \cdot \frac{4.244^2}{2(9.81)} = 39.1688\ m$$

Example 2.10: (continued)

Solution:

Head loss at sharp inlet,

$$h_i = 0.5\frac{v^2}{2g} = 0.5\frac{4.244^2}{(2)(9.81)} = 0.459\, m$$

Head loss at pipe outlet,

$$h_o = \frac{v^2}{2g} = \frac{4.244^2}{(2)(9.81)} = 0.918\, m$$

Since velocity in pipe, $v_1 = v_2$, and the pressure in the surface of reservoirs is equal to atmospheric pressure, $(p_1 = p_2)$, applying the Bernoulli's equation,

$$\cancelto{0}{\frac{p_1}{\rho g}} + \cancelto{0}{\frac{v_1^2}{2g}} + z_1 = \cancelto{0}{\frac{p_2}{\rho g}} + \cancelto{0}{\frac{v_2^2}{2g}} + z_2 + h_f + h_i + h_o$$

$$0 + 0 + z_1 = 0 + 0 + z_2 + h_f + h_i + h_o$$

$$z_1 - z_2 = h_f + h_i + h_o$$

Let the difference in the elevations between water surface = $H_1 - H_2$

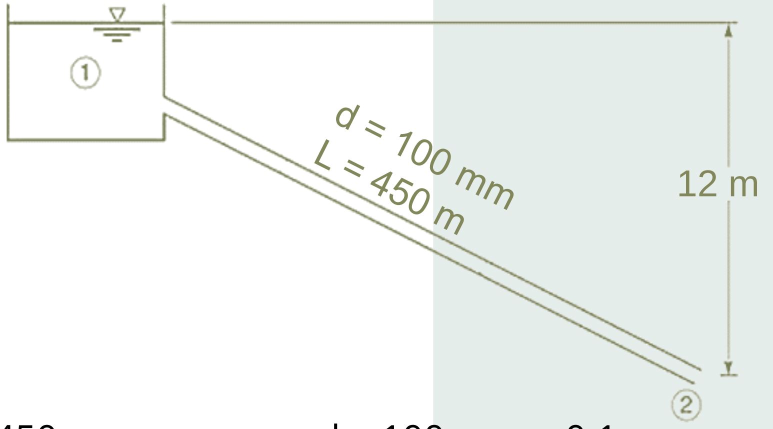
$$H_1 - H_2 = h_f + h_i + h_o$$

Thus, difference in elevation between the water surfaces,

$$H_1 = H_2 + h_f + h_i + h_o$$
$$H_1 - H_2 = h_f + h_i + h_o$$
$$= 39.1688 + 0.459 + 0.918 = 40.546\, m\ (Ans)$$

Example 2.11:

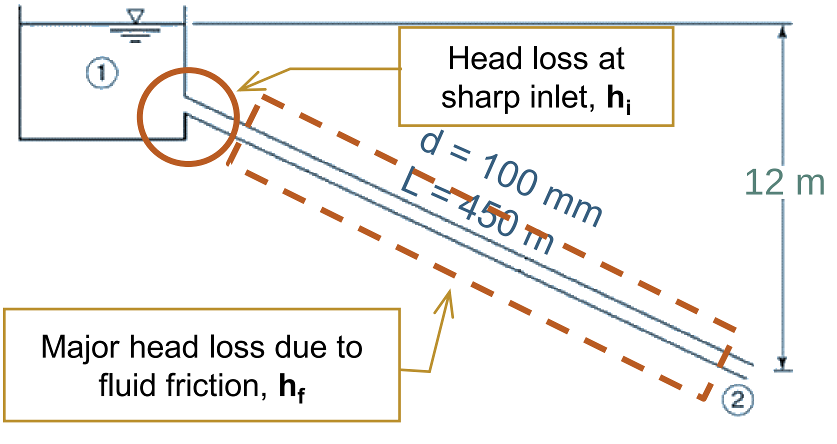
Water from a large reservoir is discharged to atmosphere through 450 m long, 100 mm diameter pipe. The entry from the reservoir to the pipe is sharp and the outlet is 12 m below the surface level in the reservoir in the reservoir. Take f = 0.01, calculate the discharge of the pipe.



Given: L = 450 m; d = 100 mm = 0.1 m; H_1-H_2 = 12 m; f = 0.01;

Solution:

Head losses in the pipeline system:



Major head loss due to friction,

$$h_f = \frac{4fL}{d} \cdot \frac{v^2}{2g} = \frac{4(0.01)(450)}{0.1} \cdot \frac{v^2}{2g} = 180\frac{v^2}{2g}$$

Head loss at sharp inlet,

$$h_i = 0.5\frac{v^2}{2g}$$

Example 2.11: (continued)

Solution:

Assuming velocity from the reservoir, $v_1 = 0$ (from reservoir), and the pressure in the surface of reservoirs is equal to atmospheric pressure, $(p_1 = p_2)$, applying Bernoulli's equation,

$$\cancelto{0}{\frac{p_1}{\rho g}} + \cancelto{0}{\frac{{v_1}^2}{2g}} + z_1 = \cancelto{0}{\frac{p_2}{\rho g}} + \frac{{v_2}^2}{2g} + z_2 + h_f + h_i$$

$$0 + 0 + z_1 = 0 + \frac{{v_2}^2}{2g} + z_2 + h_f + h_i$$

$$z_1 - z_2 = \frac{{v_2}^2}{2g} + h_f + h_i$$

Let the difference in the elevations between water surface = $H_1 - H_2$

$$H_1 - H_2 = \frac{v^2}{2g} + h_f + h_i$$

$$12 = \frac{v^2}{2g} + 180\frac{v^2}{2g} + 0.5\frac{v^2}{2g}$$

$$12 = \frac{v^2}{2g}(1 + 180 + 0.5)$$

$$v = \sqrt{\frac{12}{181.5}(2)(9.81)} = 1.139\ m/s$$

Thus, the discharge of the pipe,

$$\therefore Q = Av = \frac{\pi d^2}{4}v$$

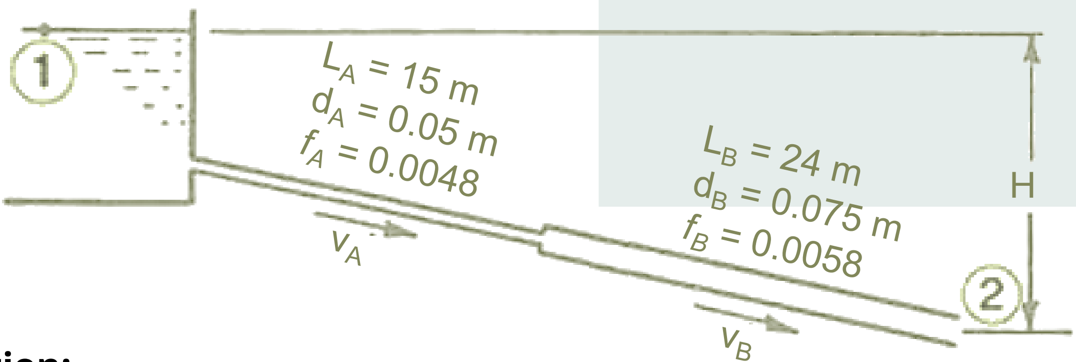
$$= \frac{\pi(0.1)^2}{4}(1.139)$$

$$= 0.00895 m^3/s (Ans)$$

Example 2.12:

Water is discharged from a reservoir into the atmosphere through a 39m long pipe. There is a sharp entrance to the pipe and the diameter is 50 mm for the first 15 m from the entrance. The pipe then enlarges suddenly to 75 mm in diameter for the remainder of its length. Taking into account the loss of head at the entry and at the enlargement, calculate the difference of level between the surface of the reservoir and the pipe exit which will maintain a flow of 0.0028 m^3/s. Take f = 0.0048 for the 50 mm pipe and f = 0.0058 for the 75 mm pipe.

Given: Total L = 39 m; Q = 0.0028 m^3/s;
Q = 300 litres/s =; f = 0.008;



Solution:

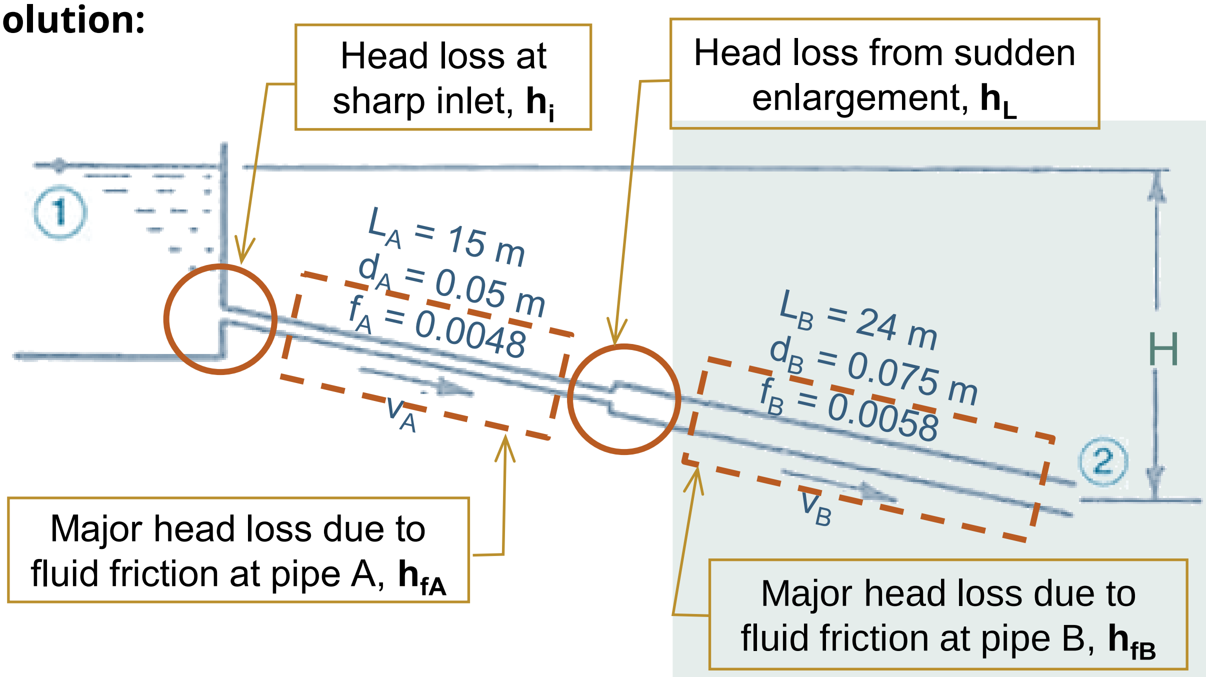
The discharge passing through pipe A and B in series is the same, therefore velocity of fluid,

$$v_A = \frac{Q}{A_A} = \frac{0.0028}{\frac{\pi(0.05)^2}{4}} = 1.426\ m/s$$

$$v_B = \frac{Q}{A_A} = \frac{0.0028}{\frac{\pi(0.075)^2}{4}} = 0.634\ m/s$$

Example 2.12: (continued)

Solution:



Total major head losses at pipe A and B,

$$h_f = h_{fA} + h_{fB}$$

$$= \frac{4f_A L_A}{d_A} \cdot \frac{{v_A}^2}{2g} + \frac{4f_B L_B}{d_B} \cdot \frac{{v_B}^2}{2g}$$

$$= \frac{4(0.0048)(15)}{0.05} \cdot \frac{1.426^2}{2(9.81)} + \frac{4(0.0058)(24)}{0.075} \cdot \frac{0.634^2}{2(9.81)}$$

$$= 0.597 + 0.152 = 0.749\ m$$

Head loss at sharp inlet,

$$h_i = 0.5\frac{{v_A}^2}{2g} = 0.5\frac{1.426^2}{(2)(9.81)} = 0.0518\ m$$

Head loss from sudden enlargement,

$$h_L = \frac{(v_A - v_B)^2}{2g} = \frac{(1.426 - 0.634)^2}{(2)(9.81)} = 0.032\ m$$

Example 2.12: (continued)

Solution:

Since the pressure in the surface of reservoirs and the outlet is equal to atmospheric pressure, $(p_1 = p_2)$, and assuming $v_1 = 0$ (from reservoir) & $v_2 = v_B$, applying the Bernoulli's equation,

$$\cancel{\frac{p_1}{\rho g}}^{0} + \cancel{\frac{v_1^2}{2g}}^{0} + z_1 = \cancel{\frac{p_2}{\rho g}}^{0} + \frac{v_2^{\,2}}{2g} + z_2 + h_f + h_i + h_L$$

$$0 + 0 + z_1 = 0 + \frac{v_2^{\,2}}{2g} + z_2 + h_f + h_i + h_L$$

$$z_1 - z_2 = \frac{v_2^{\,2}}{2g} + h_f + h_i + h_L$$

Let the difference in the elevations between water surface and pipe exit = $H_1 - H_2$,

$$H_1 - H_2 = \frac{v_2^{\,2}}{2g} + h_f + h_i + h_L$$

$$H_1 - H_2 = \frac{0.634^2}{2(9.81)} + 0.749 + 0.0518 + 0.032$$
$$= 0.853\ m\ (Ans)$$

TUTORIAL 2.2

Q2-8

Determine the difference in the elevations between the water surfaces in the two tanks which are connected by a horizontal pipe of diameter 400 mm and length 500 m. The rate of flow of water through the pipe is 200 litres/s. Consider all losses and take the value of $f = 0.009$.
[Ans: 6 m]

Q2-9

A horizontal pipeline 40 m long is connected to a water tank at one end and discharges freely into the atmosphere at the other end. For the first 25 m of its length from the tank, the pipe is 150 mm diameter and its diameter is suddenly enlarged to 300 mm. The height of water level in the tank is 8 m above the centre of the pipe. Considering all losses of head which occur, determine the rate of flow. Take $f = 0.01$ for both sections of the pipe. [Ans: 0.079 m^3/s]

Q2-10

Water from a large reservoir is discharged to atmosphere through a 100 mm diameter and 550 m long pipe. The entry point from the reservoir to the pipe is sharp and the outlet is 15 m below the surface level in the reservoir. Taking $f = 0.01$ in the Darcy-Weisbach equation, calculate the discharge of the water. [Ans: 0.009 m^3/s]

Q2-11

Water is discharged to the atmosphere from a reservoir through a 75m long pipe, which flows at 0.065 m^3/s. The entrance to the pipe is sharp and the diameter is 350 mm for the first 30 m. The pipe is then contracted suddenly to 200 mm in diameter for the remainder of its length. Take $f = 0.004$ for the smaller pipe, $f = 0.003$ for the bigger pipe and $C_c = 0.65$. Calculate:

i. Total head loss in the pipe.
ii. Difference of level between the surface of the reservoir and the pipe exit.

[Ans: 0.88 m, 1.1 m]

TUTORIAL 2 WORKED SOLUTIONS

Q2-1

Determine the loss of head due to friction in a 40 m long and 500 mm diameter which carries oil at average velocity of 2.5 m/s. Take *f* = 0.01. [Ans: 1.02 m]

Given: L = 40 m; d = 500 mm = 0.5 m; v = 2.5 m/s; *f* = 0.01;

Solution:

Using Darcy-Weisbach equation,

$$h_f = \frac{4fL}{d} \cdot \frac{v^2}{2g} = \frac{4(0.01)(40)}{0.5} \cdot \frac{2.5^2}{2(9.81)} = 1.019\ m\ (Ans)$$

Q2-3

The discharge through a pipe is 200 litres/s. Find the loss of head when the pipe is suddenly enlarged from 150 mm to 300 mm.
[Ans: 3.67 m]

Given: Q =200 litres/s = 0.2 m^3/s;
d_1 = 500 mm = 0.5 m; d_2 = 300 mm = 0.3 m;

Solution:

Velocity at 1 and 2,

$$Q = A_1 v_1 \rightarrow v_1 = \frac{Q}{A_1} = \frac{Q}{\pi \frac{{d_1}^2}{4}} = \frac{0.2}{\pi \frac{0.15^2}{4}} = 11.318\ m/s$$

$$Q = A_2 v_2 \rightarrow v_2 = \frac{Q}{A_2} = \frac{Q}{\pi \frac{{d_2}^2}{4}} = \frac{0.2}{\pi \frac{0.3^2}{4}} = 2.829\ m/s$$

Head loss due to sudden enlargement,

$$\therefore h_L = \frac{(v_1 - v_2)^2}{2g}$$

$$= \frac{(11.318 - 2.829)^2}{2(9.81)} = 3.673\ m\ (Ans)$$

TUTORIAL 2 WORKED SOLUTIONS

Q2-6

A pipe carrying 0.06 m³/s suddenly contracts from 200 mm to 150 mm diameter. Assuming that the vena contracta is formed in the smaller pipe, calculate the coefficient of contraction if the pressure head at a point upstream of the contraction is 0.655 m greater the downstream of the vena contracta. [Ans: 0.6]

Given: Q = 0.06 m³/s;

d_1 = 200 mm = 0.2 m; d_2 = 150 mm = 0.15 m;

Pressure head, $\frac{p_1 - p_2}{\rho g} = 0.655$;

Solution:

Velocity at 1 and 2,

$$Q = A_1 v_1 \rightarrow v_1 = \frac{Q}{A_1} = \frac{Q}{\pi \frac{d_1{}^2}{4}} = \frac{0.06}{\pi \frac{0.2^2}{4}} = 1.9099\ m/s$$

$$Q = A_2 v_2 \rightarrow v_2 = \frac{Q}{A_2} = \frac{Q}{\pi \frac{d_2{}^2}{4}} = \frac{0.06}{\pi \frac{0.15^2}{4}} = 3.3953\ m/s$$

Applying Bernoulli's equation for horizontal pipe ($z_1 = z_2$),

$$\frac{p_1}{\rho g} + \frac{v_1{}^2}{2g} + \cancelto{0}{z_1} = \frac{p_2}{\rho g} + \frac{v_2{}^2}{2g} + \cancelto{0}{z_2} + h_c$$

$$\frac{p_1}{\rho g} + \frac{v_1{}^2}{2g} + 0 = \frac{p_2}{\rho g} + \frac{v_2{}^2}{2g} + 0 + \frac{v_2{}^2}{2g}\left[\frac{1}{C_c} - 1\right]^2$$

$$\frac{p_1 - p_2}{\rho g} = \frac{v_2{}^2 - v_1{}^2}{2g} + \frac{v_2{}^2}{2g}\left[\frac{1}{C_c} - 1\right]^2$$

$$0.655 = \frac{3.3953^2 - 1.9099^2}{2(9.81)} + \frac{3.3953^2}{2(9.81)}\left[\frac{1}{C_c} - 1\right]^2$$

$$\frac{1}{C_c} - 1 = \sqrt{\frac{0.655 - 0.4016}{0.5876}} = 0.6566$$

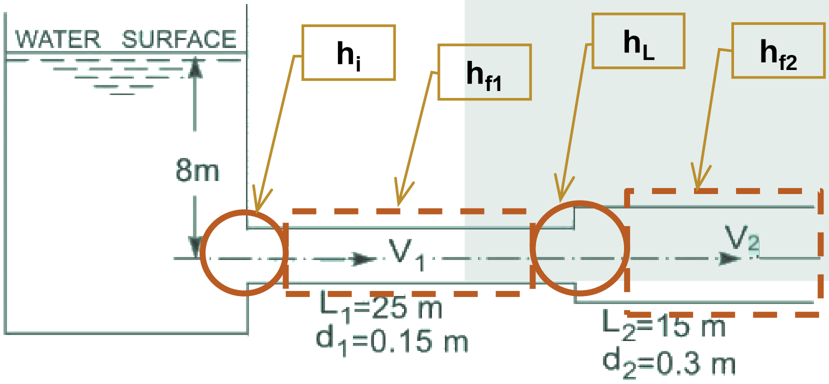
$$C_c = \frac{1}{0.6566 + 1} = 0.604\ (Ans)$$

TUTORIAL 2 WORKED SOLUTIONS

Q2-9

A horizontal pipeline 40 m long is connected to a water tank at one end and discharges freely into the atmosphere at the other end. For the first 25 m of its length from the tank, the pipe is 150 mm diameter and its diameter is suddenly enlarged to 300 mm. The height of water level in the tank is 8 m above the centre of the pipe. Considering all energy loss occurs, determine the rate of flow. Take *f* = 0.01 for both sections of the pipe. [Ans: 0.079 m^3/s]

Given: Total length, L = 40 m;



Solution:

The discharge passing through pipe 1 and 2 in series is the same, therefore,

$$Q_1 = Q_2 = A_1 v_1 = A_2 v_2$$

$$v_1 = \frac{A_2 v_2}{A_1} = \frac{\frac{\pi(0.3)^2}{4}}{\frac{\pi(0.15)^2}{4}} v_2 = 4v_2$$

Major head losses at pipe 1 and 2,

$$h_f = h_{f1} + h_{f2}$$

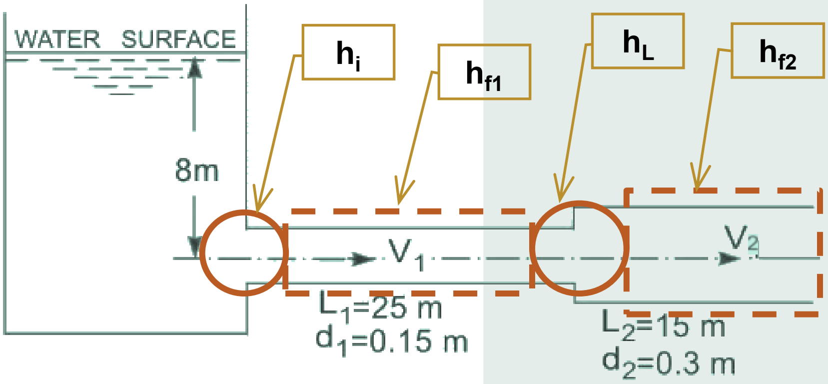
$$= \frac{4f}{2g}\left(\frac{L_1 {v_1}^2}{d_1} + \frac{L_2 {v_2}^2}{d_2}\right)$$

$$= \frac{4(0.01)}{2g}\left(\frac{(25)(4v_B)^2}{0.15} + \frac{(15)(v_B)^2}{0.3}\right) = 108.667 \frac{{v_2}^2}{2g}$$

TUTORIAL 2 WORKED SOLUTIONS

Q2-9 (continued)

Given: Total length, L = 40 m;



Solution:

Head loss at sharp inlet,

$$h_i = 0.5\frac{{v_1}^2}{2g} = 0.5\frac{(4v_2)^2}{2g} = 8\frac{{v_2}^2}{2g}$$

Head loss from sudden enlargement,

$$h_L = \frac{(v_1 - v_2)^2}{2g} = \frac{(4v_2 - v_2)^2}{2g} = 9\frac{{v_2}^2}{2g}$$

Since the pressure in the surface of reservoirs are equal, ($p_1 = p_2$), and assuming $v_1 = 0$ (from reservoir), applying the Bernoulli's equation,

$$\frac{p_1}{\rho g} + \frac{{v_1}^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{{v_2}^2}{2g} + z_2 + h_f + h_i$$

$$0 + 0 + z_1 = 0 + 0 + z_2 + h_f + h_i + h_L$$

$$z_1 - z_2 = \frac{{v_2}^2}{2g} + 108.667\frac{{v_2}^2}{2g} + 8\frac{{v_2}^2}{2g} + 9\frac{{v_2}^2}{2g}$$

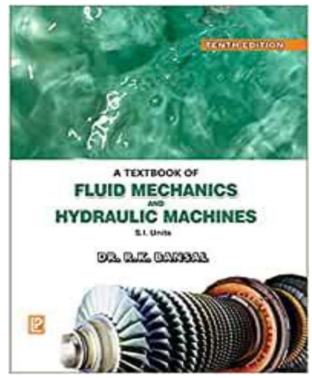
$$8 = 126.667\frac{{v_2}^2}{2g} \rightarrow v_2 = \sqrt{\frac{8}{126.667}(2)(9.81)} = 1.113\ m/s$$

Hence, the discharge,

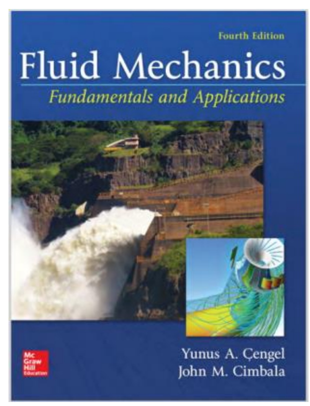
$$Q_1 = Q_2 = A_2 v_2 = \frac{\pi(0.3)^2}{4}(1.113) = 0.07869\ m^3/s(Ans)$$

REFERENCE BOOKS

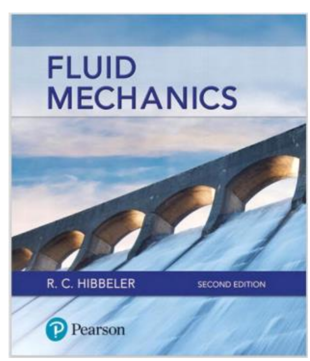
Bansal, R.K., *A textbook of fluid mechanics and hydraulic machines (in S.I. units)*. New Delhi: Laxmi Publications (P) Ltd., 2018.



Çengel, Y. and Cimbala, J., *Fluid Mechanics fundamentals and applications*. New York: McGraw-Hill Education, 2018.



Hibbeler, R. C. and Yap K. B., *Fluid Mechanics*. Harlow, Essex: Pearson Education Limited, 2020.



FLUID MECHANICS VOL. 2
MALAYSIAN POLYTECHNICS VERSION

E ISBN : 978-629-7514-02-4

9 7 8 6 2 9 7 5 1 4 0 2 4

Terbitan:
Politeknik Tuanku Syed Sirajuddin
Pauh Putra, 02600 Arau, Perlis.