KEMENTERIAN PENDIDIKAN TINGGI

POLITEKNIK MALAYSIA UNGKU OMAR

ENGINEERING MATHEMATICS

TrigoNometry

VOLUME 3

2023

SYAHIDA BINTI SAID

"Setting goals is the first step in turning the invisible into the visible."

ENGINEERING MATHEMATICS
TRIGONOMETRY
VOLUME 3

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ABSTRACT

This e-book is about Engineering Mathematics Volume 2 on Chapter 2- Trigonometry. This topic explains the fundamental concept of trigonometric functions particularly the six trigonometric basic identities.

This topic also explains about trigonometric identities, sine and cosine rules. Skills using trigonometric identities, sine and cosine rules to solve simple trigonometric equations are discussed.

Hopefully this e-book can help enhance students to understand concept of trigonometric functions and to solve the trigonometry problems.

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2.1 Fundamental of Trigonometric Functions

$$sin\theta = \frac{Opposite}{Hypotenuse}$$

$$cosec\theta = \frac{1}{sin\theta}$$

$$cosec\theta = \frac{Hypotenuse}{Opposite}$$

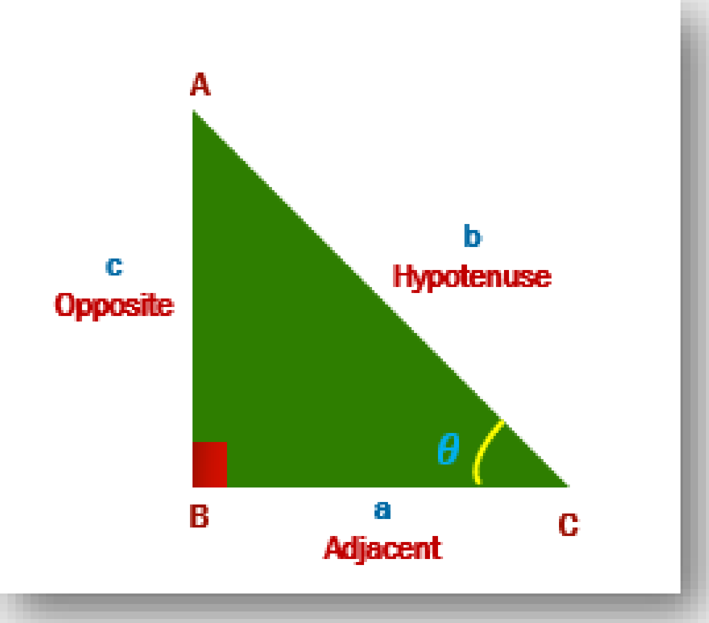


Figure 2.1

$$cos\theta = \frac{Adjacent}{Hypotenuse}$$

$$sec\ \theta = \frac{1}{cos\theta}$$

$$sec\theta = \frac{Hypotenuse}{Adjacent}$$

$$tan\theta = \frac{sin\theta}{cos\theta}$$

$$tan\theta = \frac{Opposite}{Adjacent}$$

$$cot\theta = \frac{1}{tan\theta}$$

$$cot\theta = \frac{Adjacent}{Opposite}$$

- θ - *"theta"* variable for angle
- Capital letter is always used to label the angle.
- The name for the side that is opposite to the angle has the corresponding letter in small case.

EXAMPLE 1

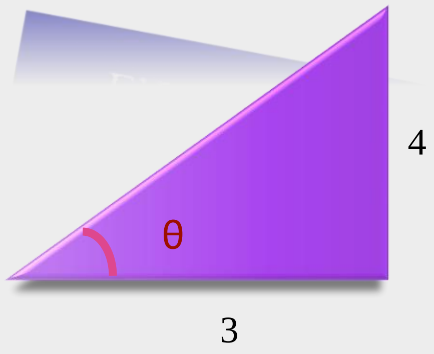


Figure 2.2

Referring to the right angle triangle in Figure 2.2, find the values of *sin θ*, and *cos θ*.

Solution:

$$Hypotenuse = \sqrt{5^2 + 3^2} = 5$$

$$\sin\theta = \frac{Opposite}{Hypotenuse}$$

$$\sin\theta = \frac{4}{5}$$

$$\cos\theta = \frac{Adjacent}{Hypotenuse}$$

$$\cos\theta = \frac{3}{5}$$

EXAMPLE 2

Determine the values of θ for the following trigonometric ratios below:

a) $\sin\theta = 0.8660$

b) $\cos\theta = 0.7071$

c) $\sec\theta = 2.0000$

d) $\tan\theta = 3.7321$

Solution:

a) $\sin\theta = 0.8660$

$$\sin\theta = 0.8660$$

$$\theta = \sin^{-1} 0.8660$$

$$= 60°$$

b) $\cos\theta = 0.7071$

$$\cos\theta = 0.7071$$

$$\theta = \cos^{-1} 0.7071$$

$$= 45°$$

c) $\sec\theta = 2.0000$

$$\sec\theta = 2.0000$$

$$\frac{1}{\cos\theta} = 2$$

$$\cos\theta = \frac{1}{2}$$

$$\theta = \cos^{-1}\left(\frac{1}{2}\right)$$

$$= 60°$$

d) $\tan\theta = 3.7321$

$$tan\theta = 3.7321$$

$$\theta = \tan^{-1} 3.7321$$

$$= 75°$$

EXERCISE 1

1.By using scientific calculator, find the values of:

a) Sin25°
b) cos(-145°)
c) sec65°
d) cot120°

2. By using scientific calculator, find the values of θ :

a) $\sin\theta = 0.9784$
b) $\cos\theta = 0.6691$
c) $\tan\theta = 0.4663$
d) $3\operatorname{cosec}\theta = 5$

Answers:
1.a)0.4226
b) -0.8192
c) 2.3662
d) -0.5774

2. a) 78.07°
b) 48.00°
c) 25°
d) 36.87°

Pop Quiz 1

https://www.proprofs.com/quiz-school/ugc/story.php?title=mzy1nzi0ma5at7

"A positive mindset brings positive things."

2.1.2 Trigonometric Function using Quadrants

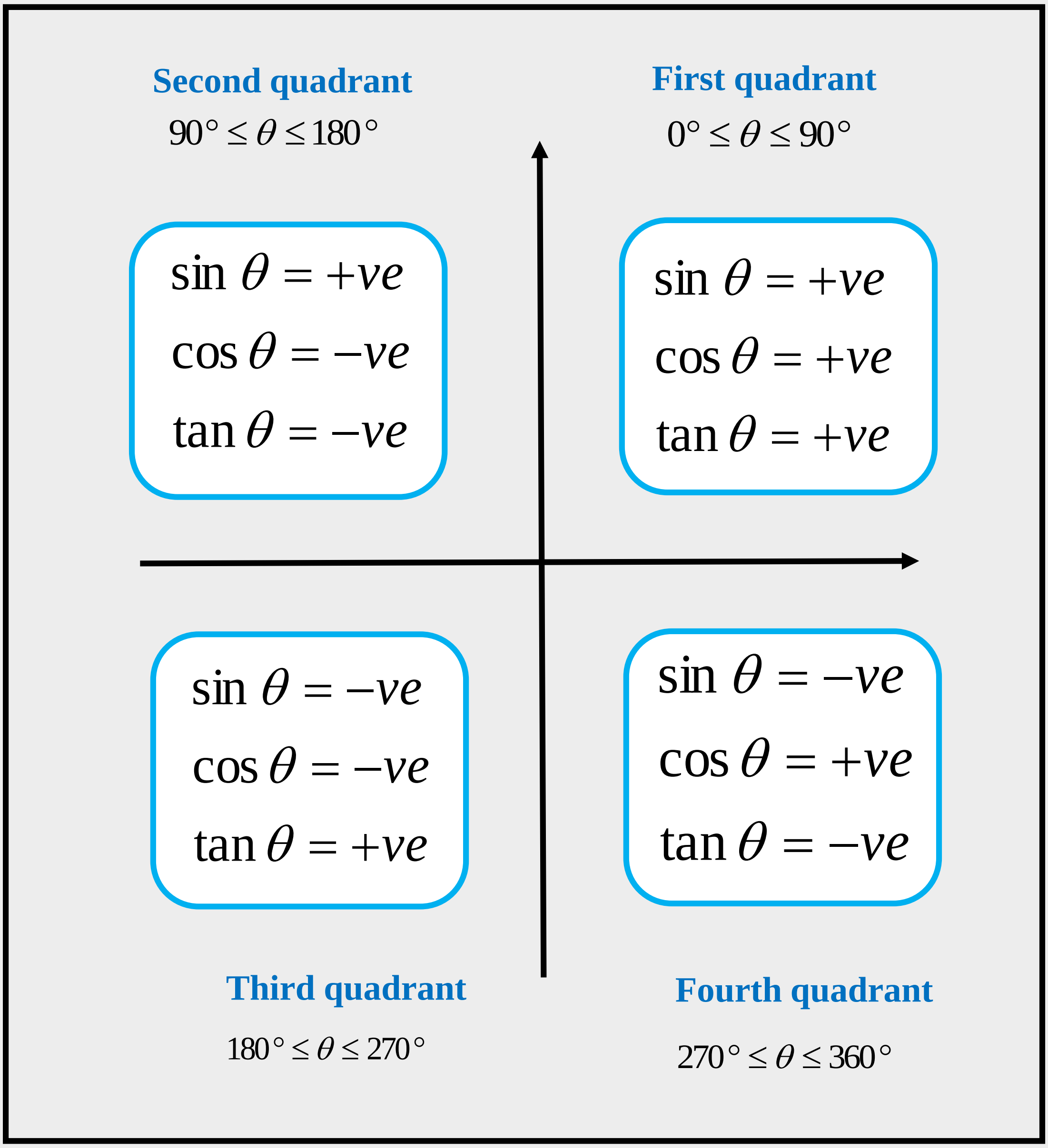
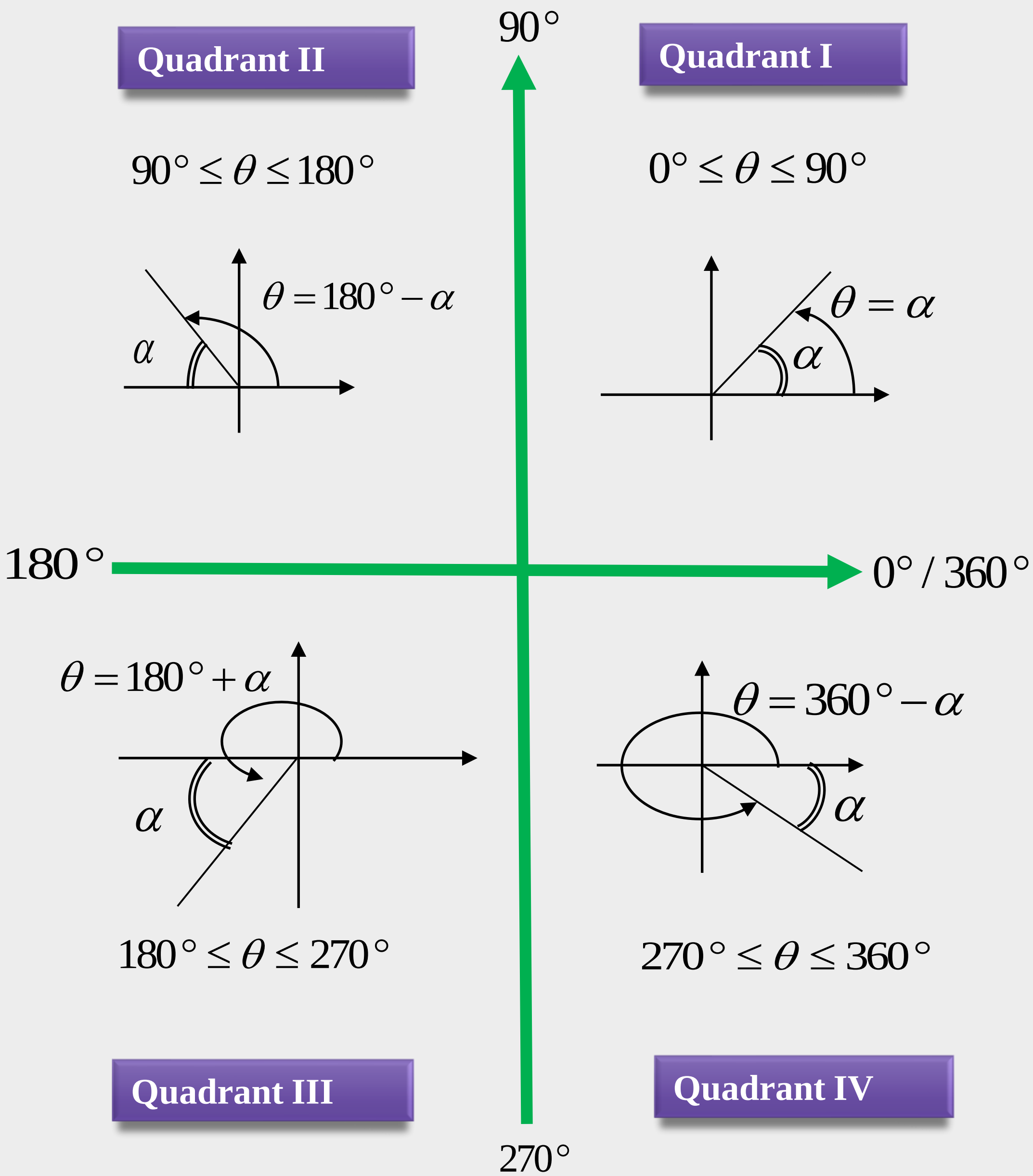


Figure 2.3



Quadrant II
Quadrant I
90°
90° ≤ θ ≤ 180°
0° ≤ θ ≤ 90°
θ = 180° − α
α
θ = α
α
180°
0° / 360°
θ = 180° + α
α
θ = 360° − α
α
180° ≤ θ ≤ 270°
270° ≤ θ ≤ 360°
Quadrant III
Quadrant IV
270°

Figure 2.4

EXAMPLE 3

Find the reference angle for θ=130°.

Solution:

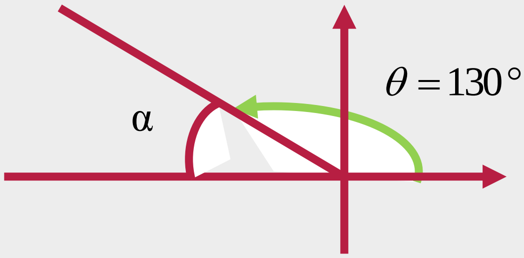


Figure 2.5

θ=130° is in second quadrant.

Reference angle, α for θ is

$$\theta = 180° - \alpha$$

$$130° = 180° - \alpha$$

$$\alpha = 180° - 130°$$

$$= 50°$$

EXERCISE 2

Find the reference angles, α for the following θ :

a) θ =105°

b) θ =228°

c) θ =348°

d) θ =-225°

e) θ = -165°

f) $\theta = \frac{3\pi}{4} radian, \pi$=180°

Answers:
a) 75°
b) 48°
c) 12°
d) 45°
e) 15°
f) 45°

Pop Quiz 2

https://www.proprofs.com/quiz-school/ugc/story.php?title=quiz-2_14888o

"You do not find the happy life. You make it."

2.1.4 Calculate the values of Trigonometric Functions

EXAMPLE 4

Find the value of θ where $0° \le \theta \le 360°$

a) Sin $\theta = 0.5427$ b) Cos $\theta = -0.3407$
c) Sin $(\theta + 15°) = 0.5293$ e) Cos $2\theta = 0.7123$

Solutions:

a) Sin θ= 0.5427
(sin θ is positive in the first and second quadrants)

Reference angle, $\alpha = sin^{-1}(0.5427)$
$\alpha = 32.87°$

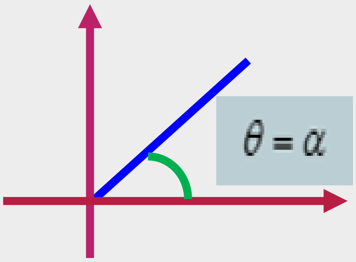


Figure 2.6

First quadrant:
θ= α
=32.87°

Second quadrant:
θ= 180°-α
=180°-32.87°
=147.13°

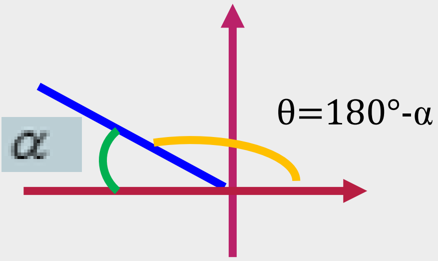


Figure 2.7

b) cos θ= -0.3407

(cos θ is positive in the second and third quadrants)

Reference angle, α=cos^{-1}(0.3407)

α=70.08°

Second quadrant:

θ= 180°-α

=180°-70.08°

=109.92°

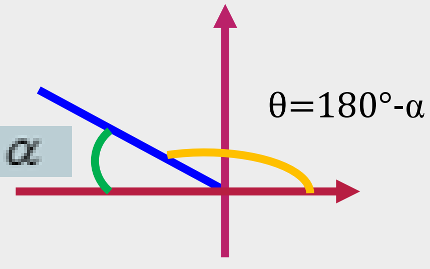


Figure 2.8

Third quadrant:

θ= 180°+α

=180°+70.08°

=250.08°

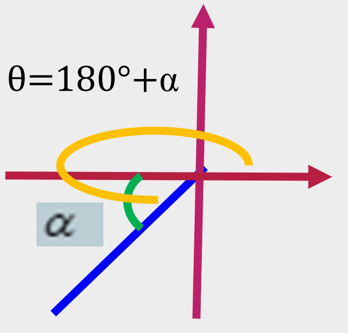


Figure 2.9

c) sin (θ+15°) = 0.5293
(sin (θ+15°) is positive in the first and second quadrants)

Reference angle, α=$sin^{-1}(0.5293)$
α=31.96°

First quadrant:
θ+15° = α
θ=α-15°
θ=31.96°-15°
=16.96°

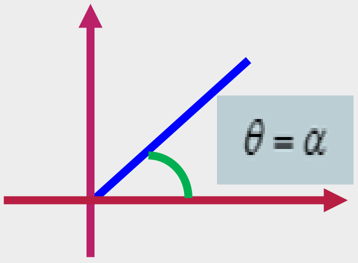


Figure 2.10

Second quadrant:
θ+15° = 180°-α
θ =180°-α- 15°
θ =180°-31.96°- 15°
=133.04°

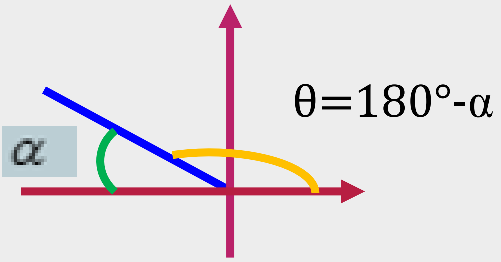


Figure 2.11

c) cos 2θ = 0.7123

(cos 2θ is positive in the first and fourth quadrants)

Reference angle, $\alpha = \cos^{-1}(0.7123)$

$\alpha = 44.58°$

Since 0°≤θ≤360°, so 0°≤2θ≤720°

First rotation: 0°≤θ≤360°,

<u>First quadrant</u>:

2θ= α

2θ=44.58°

θ=31.96°-15°

=16.96°

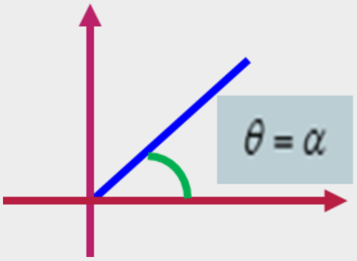


Figure 2.12

<u>Fourth quadrant</u>:

2θ= 360°-α

2θ =360°-44.58°

2θ =315.42°

θ=157.71°

Second rotation: 0°≤2θ≤720°

<u>First quadrant</u>:

2θ= 360°+α

2θ=360°+44.58°

2θ=404.58°

θ=202.29°

<u>Fourth quadrant</u>:

2θ= 360°+360°-α

2θ =720°-44.58°

2θ =675.42°

θ=337.71°

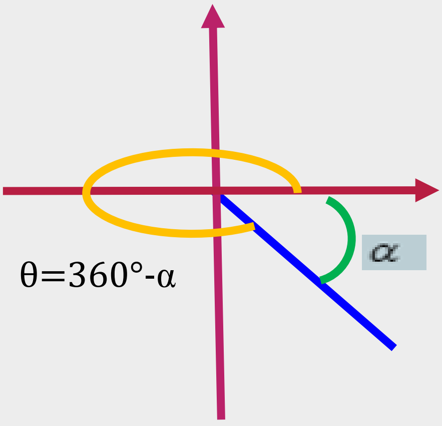


Figure 2.13

EXAMPLE 5

Given that sin $\boldsymbol{\beta}$= - n , such that $cos\theta > 0$, express each of the following in terms of n.

a) cos $\boldsymbol{\beta}$

b) tan $\boldsymbol{\beta}$

Solutions:

Given that sin $\boldsymbol{\beta}$= $\frac{opposite}{hypotenuse} = \frac{-n}{1}$ = - n

Such that $\cos\theta > 0$, means $\cos\theta = +ve$ (cosθ is positive in first and fourth quadrant)
Hence, $\boldsymbol{\beta}$ is in fourth quadrant.

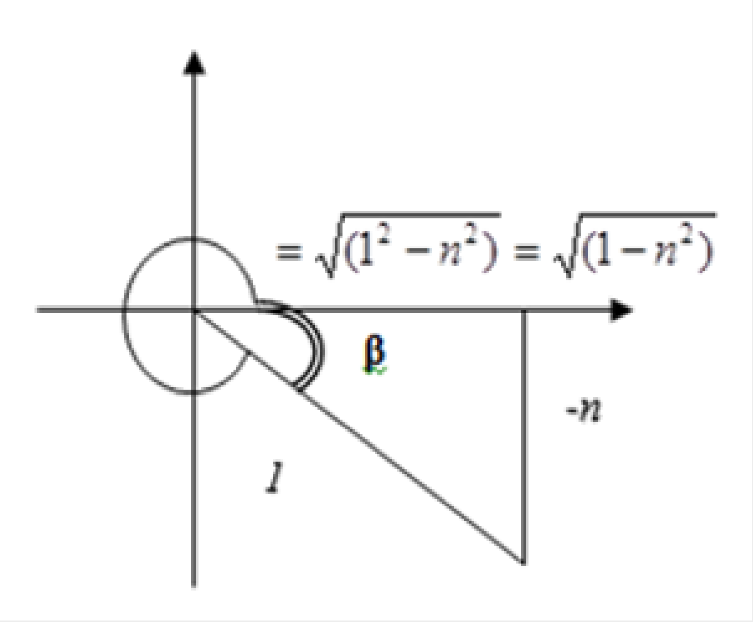


Figure 2.14

a) $\cos\beta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$

$$= \frac{\sqrt{(1-n^2)}}{1}$$

$$= \sqrt{(1-n^2)}$$

b) $tan\beta = \frac{\text{Opposite}}{\text{Adjacent}}$

$$= \frac{-n}{\sqrt{(1-n^2)}}$$

EXAMPLE 6

Find all the angles in the interval $0° \leq \theta \leq 360°$ satisfy the equation below:

$$6\sin^2\theta - \sin\theta - 2 = 0$$

Solutions:

$$6\sin^2\theta - \sin\theta - 2 = 0$$

$$(3\sin\theta - 2)(2\sin\theta + 1) = 0$$

$$\sin\theta = \frac{2}{3} \qquad \text{and} \qquad \sin\theta = -\frac{1}{2}$$

For $\sin\theta = \frac{2}{3}$ → $\sin\theta$ is positive at the first

and second quadrants, Reference angle, $\alpha = 41.81°$

<u>First quadrant</u>

$$\theta = \alpha$$
$$= 41.81°$$

<u>Second quadrant</u>

$$\theta = 180° - \alpha$$
$$= 180° - 41.81°$$
$$= 138.19°$$

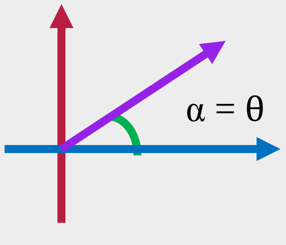


Figure 2.15

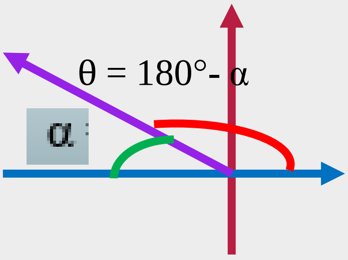


Figure 2.16

For $sin\theta = \frac{1}{2} \rightarrow sin\theta$ is negative at third and fourth quadrants

Reference angle, $\alpha = sin^{-1}\left(\frac{1}{2}\right)$

$$\alpha = 30°$$

<u>Third quadrant</u>

$\theta = 180° + \alpha$
$\theta = 180° + 30°$
$\theta = 210°$

<u>Fourth quadrant</u>

$\theta = 360° - \alpha$
$\theta = 360° - 30°$
$\theta = 330°$

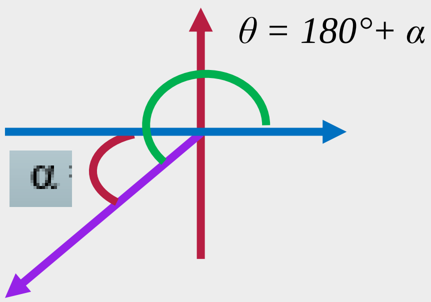


Figure 2.17

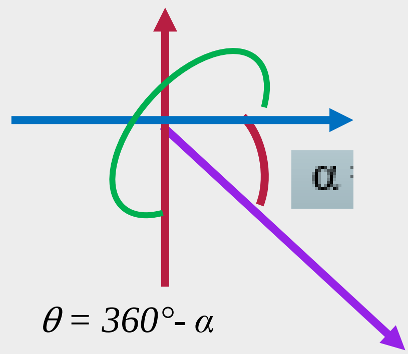


Figure 2.18

EXERCISE 3

Find the value of θ for each of the following where $0° \leq \theta \leq 360°$

a) $\cos \theta = 0.7986$

a) $\tan \theta = -0.9015$

c) $\sin (\theta - 15°) = 0.9675$

c) $\tan 2\theta = 0.7123$

e) $\sec \theta = \text{cosec } 58°$

e) $5\sin\theta = 3\tan \theta$

e) $2\sin^2\theta - \sin \theta = 0$

e) $5\cos^2\theta + 3 \cos \theta = 2$

Answers:
a) $\theta = 37°, 323°$
b) $\theta = 137.97°, 317.97°$
c) $\theta = 90.35°, 119.65°$
d) $\theta = 35.46°, 215.46°, 395.46°, 575.46°$
e) $\theta = 32.01°, 327.99°$
f) $\theta = 53.13°, 306.87°$
g) $\theta = 0°, 90°, 180°$
h) $\theta = 66.4°, 180°, 293.58°$

Pop Quiz 3

https://www.proprofs.com/quiz-school/ugc/story.php?title=pop-quiz-3_494

2.2 Solve Trigonometric Equations and Identities

2.2.1 Solve Trigonometric Equations using:

a. Trigonometric Basic Identities

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \cot^2\theta = \mathrm{cosec}^2\theta$$

$$\tan^2\theta + 1 = \sec^2\theta$$

b. Compound Angle

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$

$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$$

c. Double Angle

$$\sin 2A = 2\sin A\cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2\cos^2 A - 1$$

$$= 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

EXAMPLE 7

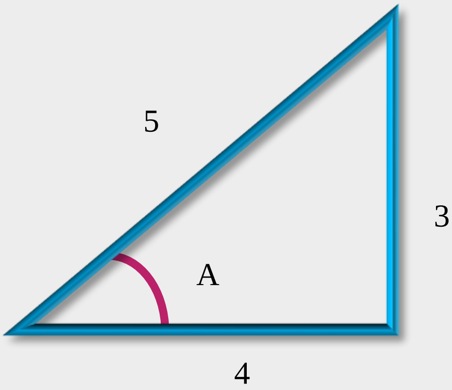


Figure 2.19

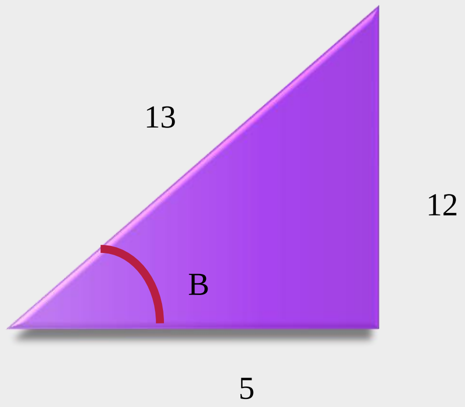


Figure 2.20

Based on the diagrams above, A and B are acute angles, where $\sin A = \frac{3}{5}$ and $\cos B = \frac{5}{13}$, without using a calculator, find the values of:

a) tan *A*
b) sin *B*
c) sin *(A-B)*
d) cos *(A-B)*
e) tan *2B*

Solution:

a) $\tan A = \frac{3}{4}$

b) $\sin B = \frac{12}{13}$

c) $\sin(A - B) = sinAcosB - cosAsinB$

$$= \left(\frac{3}{5}\right)\left(\frac{5}{13}\right) - \left(\frac{4}{5}\right)\left(\frac{12}{13}\right)$$

$$= \frac{-33}{65}$$

d) $\cos(A - B) = cosAcosB + sinAsinB$

$$= \left(\frac{4}{5}\right)\left(\frac{5}{13}\right) + \left(\frac{3}{5}\right)\left(\frac{12}{13}\right)$$

$$= \frac{56}{65}$$

e) $\tan 2B = \frac{2tanB}{1-tan^2B}$

$$= \frac{2\left(\frac{12}{5}\right)}{1-\left(\frac{12}{5}\right)^2} = \frac{-120}{119}$$

EXERCISE 4

1. Find the value of A where $0° \leq A \leq 360°$ if

$$3sin^2A - cos^2A = 0$$

2. Given that $\sin A = \frac{3}{5}$ and $sinB = \frac{5}{13}$, such that A and B are acute angles. Calculate the value of :

i) sin(A+B)
ii) cos(A+B)

Express your answer in fraction.

3. Given that $\cos A = \frac{3}{5}$, such that A are acute angles. Without using tables or calculator, find the value of :

i) sin2A
ii) cos2A

Express your answer in fraction.

Answers:

1. The values of A=30° ,150°, 210°,330°

2. i) sin(A+B)=$\frac{56}{165}$ ii) cos(A+B)=$\frac{33}{165}$

3. i) $\sin 2A = \frac{24}{25}$ ii) $\cos 2A = \frac{-7}{25}$

Pop Quiz 4

https://www.proprofs.com/quiz-school/ugc/story.php?title=pop-quiz-4_2hx

"One small positive thought can change your whole day."

2.3 Solve Triangle Problem

2.3.1 Define Sine and Cosine Rules

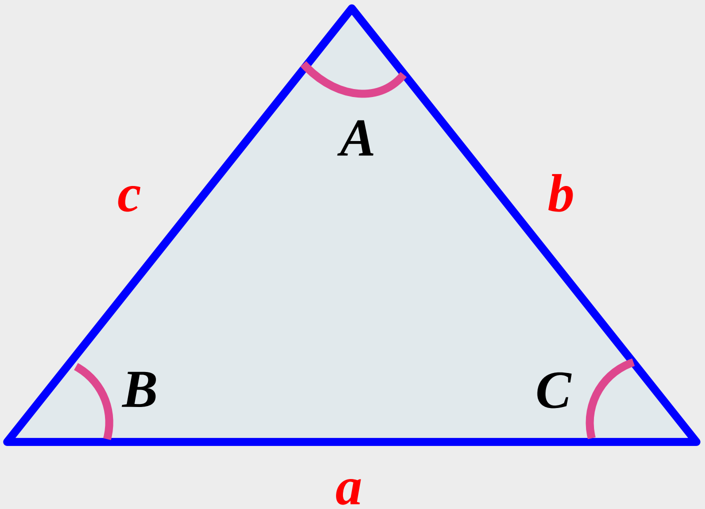


Figure 2.22

a, ***b*** and ***c*** is the lengths of the sides opposite the angles ***A***, ***B*** and ***C*** in a triangle

The Sine Rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

or

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

The Cosine Rule

$$a^2 = b^2 + c^2 - 2bc\ cosA$$

$$b^2 = a^2 + c^2 - 2ac\ cosB$$

$$c^2 = a^2 + b^2 - 2ab\ cosC$$

The Area of a Triangle

Area of Triangle

$$= \frac{1}{2} bcsin\text{A}$$

$$= \frac{1}{2} acsin\text{B}$$

$$= \frac{1}{2} absinC$$

EXAMPLE 8

In the triangle ABC, given that $a = 5cm, b = 3cm$ and $B = 30°$ .Solve the triangle.

Solution:

By using sine rule

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\sin A = \frac{a \sin B}{b}$$

$$A = \sin^{-1}\left(\frac{a \sin B}{b}\right)$$

$$= \sin^{-1}\left(\frac{5 \sin 30°}{3}\right)$$

$$= 56.44°$$

$$C = 180° - 30° - 56.44°$$

$$= 93.56°$$

By using cosine rule

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$c = \sqrt{a^2 + b^2 - 2ab\cos C}$$

$$= \sqrt{5^2 + 3^2 - 2(5)(3)\cos 93.56^\circ}$$

$$= 5.99\ cm$$

$$\text{Area of triangle} = \frac{1}{2}ab\sin C$$

$$= \frac{1}{2}(5)(3)\sin 93.56^\circ$$

$$= 7.49\ \text{cm}^2$$

EXAMPLE 9

In the triangle PQR, given that
PQ is 6cm, angle of P is 110° and angle of R is 30°.Solve the triangle.

Solution:

Length of RQ:

$$\frac{RQ}{\sin P} = \frac{PQ}{\sin R}$$

$$RQ = \frac{PQ \sin P}{\sin R}$$

$$= \frac{6 \sin 110°}{\sin 30°}$$

$$= 11.28\,cm$$

Angle of Q:

$$Q = 180° - 30° - 110°$$

$$= 40°$$

Length of RP:

$$\frac{RP}{\sin Q} = \frac{PQ}{\sin R}$$

$$RP = \frac{PQ \sin Q}{\sin R}$$

$$= \frac{6 \sin 40°}{\sin 30°}$$

$$= 7.71\, cm$$

$$\text{Area of triangle} = \frac{1}{2} pq \sin R$$

$$= \frac{1}{2}(7.71)(11.28) \sin 30°$$

$$= 21.74\, \text{cm}^2$$

EXERCISE 5

1. In the triangle ABC, given that $a = 6cm, b = 8cm$ and angle of $B = 102°$ .Solve the triangle completely.

2. In the triangle ABC, given that $c = 20cm, A = 62.5°$ and $B = 41°$ .Solve the triangle completely.

3. In the triangle ABC, given that $b = 3.5cm, c = 6cm$ and $C = 52°$ .Solve the triangle completely.

4. In the triangle ABC, given that $a = 5cm, b = 3cm$ and $B = 30°$ .Solve the triangle completely.

Answers:

1. $A = 47.19°, C = 78° c = 8cm$

2. $C = 76.5°, a = 18.24cm, b = 13.49cm$

3. $B = 27.37°, A = 100.63°, a = 7.48cm$

4. $A = 56.44°, C = 93.56°, c = 5.99cm$

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ENGINEERING MATHEMATICS
TRIGONOMETRY
VOLUME 3

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