MALAYSIA POLYTECHNICS SERIES

ELEMENTARY MATHEMATICS

3RD EDITION

LEE KIM THOR

RUHANA BINTI MAT KIA

QA107 .L44 2022 PPD



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Lee Kim Thor
Elementary Mathematics / Lee Kim Thor and Ruhana Binti Mat Kia.

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3rd Edition

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Elementary Mathematics is one of the compulsory courses to be taken in the first semester in order to complete the study of Diploma in Architecture. Students need to fulfil this two credit hours course in no more than eight semester study term.

To fulfil the student's requirement and to boost the student's skill and knowledge, the authors have completed this book. It is hope that this book will ease the student to do their revision.

Above all, the authors would like to express deep sense of thanks and gratitude to the team of publisher and everyone who collaborated in producing this book.

Team of authors and editors,
Ruhana binti Mat Kia
Lee Kim Thor

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CHAPTER ONE

ALGEBRA

1.1 INTRODUCTION TO ALGEBRA

*"**Algebra** is the study of mathematical symbols and the rules for manipulating these symbols"*
(https://en.wikipedia.org/wiki/Algebra).

Therefore, this is the simplest way to describe Algebra. In algebra you will always find the unknowns value, mathematical symbols or shapes and some of it may be empty spaces. Frequently in this syllabus, we will use alphabet as unknowns and also mathematical symbol to represent symbols in algebra. Hence, it is friendlier and much easier.

1.2 FRACTION

The best way to represent a decimal value is using fraction. For example, which is more accurate to represent one third of circle? Is it 0.33333 or $\frac{1}{3}$?

Fraction consist of two parts that is numerator and denominator.

$$\frac{Numerator\ (\boldsymbol{N})}{Denominator\ (\boldsymbol{D})}$$

TYPES OF FRACTION

Proper fraction, N less than **D** ($\boldsymbol{N < D}$)

Example: $\frac{1}{3}$

Improper fraction, N greater than or equal to **D**. ($\boldsymbol{N \geq D}$)

Example: $\frac{4}{3}$

Mixed number, A combination of integer and proper fraction. ONLY Improper Fraction can be converted to mixed number.

Example: $1\frac{1}{3}$

$1\frac{1}{3}$ is a mixed number generated from improper fraction $\frac{4}{3}$.

1.2.1 Numerical Fraction

TIPS: -

1. ***Sum and subtraction Operation***– make sure the value of denominator **(D)** is same. If it is not similar? Multiply fraction by any suitable value in purpose to have same value of Denominator.

   Then add and subtract numerator accordingly.

2. ***Multiplication*** – multiply numerator with numerator; denominator with denominator. Ensure to simplify any value that is possible.

3. ***Division*** – change the division symbol to multiplication.

   How? Twist a fraction which come after division symbol; numerator will become denominator and denominator will become numerator. Solve it like multiplication. Done.

EXERCISE 1.1

Solve of the following without using calculator. Show your step(s).

a) $\frac{1}{2}+\frac{3}{4}$	b) $\frac{2}{3}+\frac{3}{4}$
c) $\frac{2}{3}+\left(\frac{-5}{4}\right)+\frac{3}{6}$	d) $\frac{1}{3}+\frac{3}{7}+\frac{2}{5}$

e) $\frac{2}{3}-\frac{3}{8}$	*f)* $-\frac{2}{3}+\frac{3}{5}$
g) $\frac{2}{5}-\frac{3}{4}-\left(\frac{-6}{7}\right)$	*h)* $\frac{2}{9}-\frac{3}{4}-\frac{2}{3}$
i) $\frac{2}{3}\times\frac{3}{4}$	*j)* $-\frac{2}{3}\times\frac{3}{5}$
k) $\frac{5}{3}\times\left(\frac{3}{-15}\right)\times\frac{-5}{12}$	*l)* $\frac{2}{3}\times\frac{5}{-4}\times\frac{6}{15}$
m) $\frac{2}{3}\div\frac{3}{4}$	*n)* $\frac{-2}{3}\div\frac{4}{9}$

o) $\frac{2}{3} \div \frac{3}{4} \div \frac{36}{27}$	p) $\frac{2}{5} \div \left(\frac{-8}{15}\right) \div \left(\frac{-4}{10}\right)$

1.2.2 Algebraic Fraction

Compare to Numerical Fraction, Algebraic Fraction is a fraction which has unknowns or mathematical symbols within it.
Similar to Numerical Fraction, to simplify Algebraic Fraction, we still apply all the concepts applied for numerical fraction. But one thing for sure, you cannot use calculator to solve or simplify it.

Example 1.1
Simplify the following fraction to the lowest term.

a) $\frac{3}{5x} + \frac{4}{15x}$ b) $\frac{5}{9x} + \frac{3y-4}{27xy}$

Solution: -

a) $\frac{3}{5x} + \frac{4}{15x}$

$= \frac{3 \times 3}{3 \times 5x} + \frac{4}{15x}$
$= \frac{9}{15x} + \frac{4}{15x}$
$= \frac{9+4}{15x}$
$= \frac{13}{15x}$

b) $\frac{5}{9x} + \frac{3y-4}{27xy}$

$= \frac{3y \times 5}{3y \times 9x} + \frac{3y-4}{27xy}$
$= \frac{15y}{27xy} + \frac{3y-4}{27xy}$
$= \frac{15y+3y-4}{27xy}$
$= \frac{18y-4}{27xy}$

Example 1.2

Simplify the following algebraic fractions.

a) $\frac{2u}{3}+\frac{2u-3}{6}$ b) $\frac{2a}{3y+2}-\frac{2a}{3}$

Solution: -

a) $\frac{2u}{3}+\frac{2u-3}{6}$

$=\frac{2\times 2u}{2\times 3}+\frac{2u-3}{6}$

$=\frac{4u}{6}+\frac{2u-3}{6}$

$=\frac{4u+2u-3}{6}$

$=\frac{6u-3}{6}$

$=\frac{3(2u-1)}{6}$

$=\frac{2u-1}{2}$

b) $\frac{2a}{3y+2}-\frac{2a}{3}$

$=\frac{3\times 2a}{3\times(3y+2)}-\frac{2a\times(3y+2)}{3\times(3y+2)}$

$=\frac{6a}{3(3y+2)}-\frac{6ay+6a}{3(3y+2)}$

$=\frac{6a-6ay-6a}{3(3y+2)}$

$=\frac{-6ay}{3(3y+2)}$

$=\frac{-2ay}{3y+2}$

Example 1.3

Simplify the algebraic fractions:

a) $\frac{1}{3p}-\frac{p-6}{9p^2}$ b) $\frac{3(a+4b)}{4p}-\frac{2(3a-8b)}{24q}$

Solution: -

a) $\frac{1}{3p}-\frac{p-6}{9p^2}$

$=\frac{3p\times 1}{3p\times 3p}-\frac{p-6}{9p^2}$

$=\frac{3p}{9p^2}-\frac{p-6}{9p^2}$

$=\frac{3p-p+6}{9p^2}$

$=\frac{2p+6}{9p^2}$

b) $\frac{3(a+4b)}{4p}-\frac{2(3a-8b)}{24q}$

$=\frac{6q\times 3(a+4b)}{6q\times 4p}-\frac{2(3a-8b)\times p}{24q\times p}$

$=\frac{18aq+72bq}{24pq}-\frac{6ap-16bp}{24pq}$

$=\frac{18aq+72bq-6ap+16bp}{24pq}$

$=\frac{2(9aq+36bq-3ap+8bp)}{24pq}$

$=\frac{9aq+36bq-3ap+8bp}{12pq}$

Example 1.4

Simplify the following Algebraic Fraction.

a) $\frac{12p}{3} \times \frac{p-6}{9p^2}$

b) $\frac{3(6a+8b)}{4p} \div \frac{2(3a+4b)}{24p}$

c) $\frac{a+3}{81m} \times \frac{m^2-3m}{(a+3)(a-3)}$

d) $\frac{7abc}{6x^2} \div \frac{36ab^2}{10xy}$

Solution: -

a) $\frac{12p}{3} \times \frac{p-6}{9p^2}$

$= \frac{3 \times 4 \times p}{3} \times \frac{p-6}{3 \times p \times p}$

$= \frac{1 \times 4 \times 1}{1} \times \frac{p-6}{3 \times 1 \times p}$

$= \frac{4(p-6)}{3p}$

b) $\frac{3(6a+8b)}{4p} \div \frac{2(3a+4b)}{24p}$

$= \frac{3 \times 2(3a+4b)}{4p} \times \frac{4p \times 6}{2(3a+4b)}$

$= \frac{3 \times 1}{1} \times \frac{1 \times 6}{1}$

$= 18$

c) $\frac{a+3}{81m} \times \frac{m^2-3m}{(a+3)(a-3)}$

$= \frac{a+3}{81m} \times \frac{m(m-3)}{(a+3)(a-3)}$

$= \frac{1}{81} \times \frac{1(m-3)}{1(a-3)}$

$= \frac{(m-3)}{81(a-3)}$

d) $\frac{7abc}{6x^2} \div \frac{36ab^2}{10xy}$

$= \frac{7abc}{6x^2} \times \frac{10xy}{36ab^2}$

$= \frac{7 \times a \times b \times c}{6 \times x^2} \times \frac{10 \times x \times y}{36 \times a \times b^2}$

$= \frac{7 \times 1 \times 1 \times c}{3 \times x} \times \frac{5 \times 1 \times y}{36 \times 1 \times b}$

$= \frac{7c}{3x} \times \frac{5y}{36b}$

$= \frac{35cy}{108bx}$

EXERCISE 1.2

Simplify the following to the simplest form.

a) $(x-y)^2 - xy$	b) $\frac{3}{4x} - \frac{7}{2xy}$

c) $\frac{h}{3}+\frac{3}{h-4}$	d) $\frac{3-q}{q}+\frac{3q-4}{2q}$
e) $\frac{2h-5}{4x}-\frac{7-2h}{2x}$	f) $\frac{2(x-3y)}{4x}+\frac{4x-3}{8x}$
g) $\frac{26m^5}{3p^5}\times\frac{9p^2}{4m}$	h) $\frac{3(6p-8q)}{4m}\div\frac{6(3p-4q)}{32m^2}$
i) $\frac{3b+9}{27m}\times\frac{m^2+3m}{(b+9)(b+3)}$	j) $\frac{7a+a^2}{6a^2}\div\frac{36b^4-b^2}{5b^4}$
k) $2\left(\frac{4m^2}{3}\div\frac{2m}{9}+m^2\right)\times\frac{1}{4m}$	l) $\frac{h}{3}+\frac{3}{4h}\times\left(\frac{5h}{2}-\frac{7}{3h}\right)$

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1.3 CONVERSION OF FORMULA

Conversion of the formula is one of the mathematical branches which is focus on transposition of the unknowns given by the formula. Commonly, one of the unknowns will be requested to replace the existing Subject of the Formula and become the new subject for the formula.

Conversion of Formula is also known as *Transposition of Formula*.

Example 1.5

Given that $Q = \sqrt{P^2 + (3\pi fL)^2}$, then express P as a subject of the formula.

Solution: -

$$Q = \sqrt{P^2 + (3\pi fL)^2}$$
$$Q^2 = P^2 + (3\pi fL)^2$$
$$Q^2 - (3\pi fL)^2 = P^2$$
$$P = \sqrt{Q^2 - (3\pi fL)^2}$$

Example 1.6

Make $\boldsymbol{u}$ as subject of $t^2 = \sqrt{\frac{u^2}{9}}$.

Solution: -

$$t^2 = \sqrt{\frac{u^2}{9}}$$
$$t^4 = \frac{u^2}{9}$$
$$9t^4 = u^2$$
$$\sqrt{9t^4} = u$$
$$u = 3t^2$$

Example 1.7

Given $E = 3CR^2$, make $\boldsymbol{R}$ as the subject of the formula.

Solution: -

$$E = 3CR^2$$
$$\frac{E}{3C} = R^2$$
$$R = \sqrt{\frac{E}{3C}}$$

c) $\frac{h}{3}+\frac{3}{h-4}$	d) $\frac{3-q}{q}+\frac{3q-4}{2q}$
e) $\frac{2h-5}{4x}-\frac{7-2h}{2x}$	f) $\frac{2(x-3y)}{4x}+\frac{4x-3}{8x}$
g) $\frac{26m^5}{3p^5}\times\frac{9p^2}{4m}$	h) $\frac{3(6p-8q)}{4m}\div\frac{6(3p-4q)}{32m^2}$
i) $\frac{3b+9}{27m}\times\frac{m^2+3m}{(b+9)(b+3)}$	j) $\frac{7a+a^2}{6a^2}\div\frac{36b^4-b^2}{5b^4}$
k) $2\left(\frac{4m^2}{3}\div\frac{2m}{9}+m^2\right)\times\frac{1}{4m}$	l) $\frac{h}{3}+\frac{3}{4h}\times\left(\frac{5h}{2}-\frac{7}{3h}\right)$

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1.3 CONVERSION OF FORMULA

Conversion of the formula is one of the mathematical branches which is focus on transposition of the unknowns given by the formula. Commonly, one of the unknowns will be requested to replace the existing Subject of the Formula and become the new subject for the formula.

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Example 1.5

Given that $Q = \sqrt{P^2 + (3\pi fL)^2}$, then express P as a subject of the formula.

Solution: -

$$Q = \sqrt{P^2 + (3\pi fL)^2}$$
$$Q^2 = P^2 + (3\pi fL)^2$$
$$Q^2 - (3\pi fL)^2 = P^2$$
$$P = \sqrt{Q^2 - (3\pi fL)^2}$$

Example 1.6

Make $\boldsymbol{u}$ as subject of $t^2 = \sqrt{\frac{u^2}{9}}$.

Solution: -

$$t^2 = \sqrt{\frac{u^2}{9}}$$
$$t^4 = \frac{u^2}{9}$$
$$9t^4 = u^2$$
$$\sqrt{9t^4} = u$$
$$u = 3t^2$$

Example 1.7

Given $E = 3CR^2$, make $\boldsymbol{R}$ as the subject of the formula.

Solution: -

$$E = 3CR^2$$
$$\frac{E}{3C} = R^2$$
$$R = \sqrt{\frac{E}{3C}}$$

Example 1.8

Given that total surface area of a solid cylinder is calculated by the following formula, $A = 2\pi r^2 + 2\pi rh$ and volume of a cylinder is calculated by $V = \pi r^2 h$. You are requested to:-

i) Define a formula to calculate height (h) of the cylinder by using total surface area formula of the solid cylinder.

ii) Define a formula to calculate radius (r) of the cylinder by using volume formula of the solid cylinder.

Solution: -

i) Define a formula to calculate height (h) of the cylinder by using total surface area formula of the solid cylinder.

$$A = 2\pi r^2 + 2\pi rh$$
$$A - 2\pi r^2 = 2\pi rh$$
$$h = \frac{A - 2\pi r^2}{2\pi r}$$

ii) Define a formula to calculate radius (r) of the cylinder by using total surface area formula of the solid cylinder.

$$V = \pi r^2 h$$
$$\frac{V}{\pi h} = r^2$$
$$r = \sqrt{\frac{V}{\pi h}}$$

EXERCISE 1.3

a) Given $\frac{4h}{\sqrt{k-1}} = 3$, *express* $\boldsymbol{k}$ in terms of $\boldsymbol{h}$.
b) Given $A = \frac{3}{2}xyz$, express $\boldsymbol{x}$ as subject of the formula.

c) Given $\frac{P_1L_1}{T_1} = \frac{P_2L_2}{T_2}$, make L_2 as the subject of the formula.

d) Given $s = v\left(\frac{t-2}{2+t}\right)$, make t as the subject of the formula.

e) Make $\boldsymbol{p}$ as the subject of the formula for $pr = k(p+q)$.

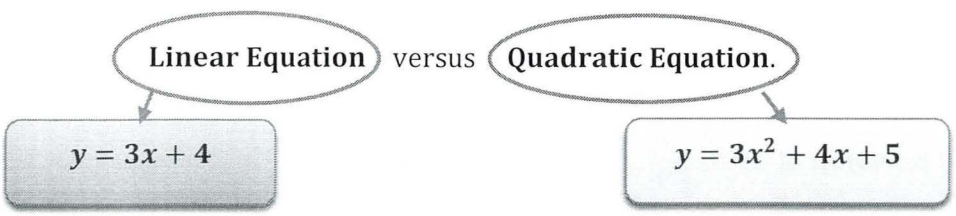
f) Make $\boldsymbol{y}$ as the subject of the formula for $x = \frac{py}{q+y}$.

g) State u as subject of $t^2 = \sqrt{\frac{u^2}{9}}$.

h) Express Z_1 as subject of $\frac{1}{Z_T} = \frac{1}{Z_1} + \frac{1}{Z_2}$.

i) Make $\boldsymbol{c}$ as the subject of the formula for $x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$.

1.4 LINEAR EQUATION



LINEAR EQUATION.

1. The highest power for unknowns in the equations is 1.
2. Number of solutions **ONLY 1**.

QUADRATIC EQUATION.

1. The highest power for unknowns in the equations is 2.
2. Number of solutions **IS 2**, it can be either similar value of solution or different.

Refer to the above illustration in order to have better understanding.

Example 1.9

Find the value of the following equation: -

a) $-2x-73=-89$

b) $\frac{1}{3}(x+3)=9$

Solution: -

a) $-2x-73=-89$

$-2x=-89+73$

$-2x=-16$

$x=\frac{-16}{-2}$

$x=8$

b) $\frac{1}{3}(x+3)=9$

$x+3=9\times 3$

$x+3=27$

$x=24$

Example 1.10

Solve the following equation.

a) $\frac{6}{2x+1}=\frac{16}{x+2}$

b) $\frac{m}{4}+2=\frac{m}{8}$

Solution: -

a) $\frac{6}{2x+1}=\frac{16}{x+2}$

$6(x+2)=16(2x+1)$

$6x+12=32x+16$

$6x-32x=16-12$

$-26x=4$

$x=\frac{4}{-26}$

$x=-\frac{2}{13}$

b) $\frac{m}{4}+2=\frac{m}{8}$

$\frac{m}{4}-\frac{m}{8}=-2$

$\frac{2m}{8}-\frac{m}{8}=-2$

$\frac{2m-m}{8}=-2$

$\frac{m}{8}=-2$

$m=-16$

EXERCISE 1.4

Solve the following equation.

a) $15a - 4 = 3a$	b) $\frac{3b}{4} - 4 = 5b$
c) $\frac{2c - 3}{2} = \frac{4c - 5}{3}$	d) $\frac{7}{3d - 2} = \frac{3}{\frac{1}{2} + 5d}$
e) $\frac{5e}{7} = 8e - 3$	f) $\frac{f - 8}{f + 15} = \frac{4}{3}$
g) $3(g - 4) = \frac{5g - 3}{2}$	h) $\frac{2e - 8}{7} = \frac{e - 3}{2} - 15$

1.5 QUADRATIC EQUATION

There are **THREE** methods used in solving quadratic equation i.e. Factorization Method, Quadratic Formula Method and Completing the Square Method.

However, for Elementary Mathematics, ONLY Factorization and Quadratic Formula Method covered. Completing the Square is not a part of the syllabus.

Quadratic Equation always in format of $y = ax^2 + bx + c$, which x^2 must exist in the equation as highest power. x can be replace with any possible unknown.

Bear in your mind that ***a, b*** and ***c*** are always numerical value and it is representing the coefficient for x^2 and x, while c is any constant value (c is not coefficient since it does not have any unknown multiplied to. In graph, c is representing the value of y-intercept.)

a always coefficient of x^2

b always coefficient of x

And for **none *x***, it is always the value of ***c***.

$$y = 3x^2 + 4x + 5$$

For this equation, **a = 3, b = 4 and c = 5.**

1.5.1 Factorization Method

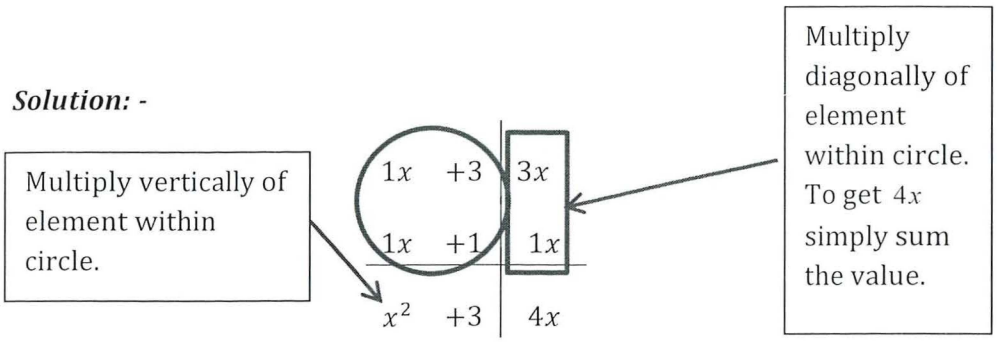
Factorization Method is the best method to solve quadratic equation with the value of unknowns are integers and fractions. While for decimal values, Quadratic Formula can be used to solve the equations.

For any question requested students to use the best method to solve quadratic equation, student must alert to remember that Factorization Method is the best for solution with integers and fraction, and Quadratic Formula Method can be used to solve any quadratic equations that cannot be solved by factorization method.

Example 1.11

Solve the $x^2 + 4x + 3 = 0$ by using Factorization Method.

Solution: -



Okay after you get the value, rewrite as below: -

$(x + 3)(x + 1) = 0$

$x + 3 = 0$ $\qquad$ $x + 1 = 0$

$x = -3$ $\qquad$ $x = -1$

The best way to factorize is **determine** couples of numbers which

1. is multiplied will give value of **c**
2. and sum will give you value of **b**.

a=1; b=4; c=3;

1 x 1 = 1 ←***a***

3 x 1 = 3 ←***c***

3 + 1 = 4 ←***b***

Example 1.12

Find the value of ***x*** for the quadratic equation by using factorization method.

$x^2 - 5x + 6 = 0$

Solution: -

If stated in the question you must use certain method, it is means that you cannot answer or state any method that you love it. You must follow the question or you are considered wrong. Remember, correct final answer is not everything but your understanding is top of all.

$(x - 3)(x - 2) = 0$

$x - 3 = 0$ $\qquad$ $x - 2 = 0$

$x = 3$ $\qquad$ $x = 2$

Example 1.13

Find the value of x for the quadratic equation by using suitable method.

$$2x^2 + x = 6$$

***Solution:* -**

I love this question because for me this question actually gives you option how to solve it. You can use any of the three methods but to solve it by factorization, please check it first.

REMEMBER: -

Factorization **method is only for integer and fraction** not for decimal values.

HINT: -

Check the answer by using your calculator.

So, I want to solve it by factorization.

$2x^2 + x - 6 = 0$ ← convert and rearrange equation to form a quadratic equation.

$(2x - 3)(x + 2) = 0$

$2x - 3 = 0$ $\qquad$ $x + 2 = 0$

$x = \frac{3}{2}$ @ 1.5 $\qquad$ $x = -2$

EXERCISE 1.4

Solve the following quadratic equation by using factorization.

a) $x^2 - 7x + 10 = 0$	b) $7x^2 + 8x - 15 = 0$

c) $21x^2 - 15x - 6 = 0$	d) $(x - 5)(x - 4) = 5x - 13$
e) $x^2 - 5x + 6 = 0$	h) $3x^2 = 25x - 28$
i) $(x + 3)(x - 5) = 7x$	j) $4m(m - 2) = 15 + 3m(2m + 1)$

1.5.2 Formula Method

To solve quadratic equation by using Quadratic Formula, you must remember and make use of this formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

To always rewrite your quadratic equation in the form of $ax^2 + bx + c = 0$, it will help you to replace correct value into formula.

Example 1.12

Find the value of $x$for the quadratic equation by using formula method.

a) $4x^2 + 8x + 3 = 0$

b) $5x^2 - 13x + 7 = 0$

Solution: -

a) $4x^2 + 8x + 3 = 0$

$a = 4\,; b = 8\,; c = 3$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(8) \pm \sqrt{(8)^2 - 4(4)(3)}}{2(4)}$$

$$x = \frac{-8 \pm \sqrt{64 - 48}}{8}$$

$$x = \frac{-8 \pm \sqrt{16}}{8}$$

$$x = \frac{-8 \pm 4}{8}$$

$$x_1 = \frac{-8 + 4}{8} \qquad x_2 = \frac{-8 - 4}{8}$$

$$= \frac{-4}{8} \qquad = \frac{-12}{8}$$

$$= -\frac{1}{2} \qquad = -\frac{3}{2}$$

b) $5x^2 - 13x + 7 = 0$

$a = 5\,; b = -13\,; c = 7$

$$x = \frac{-(-13) \pm \sqrt{(-13)^2 - 4(5)(7)}}{2(5)}$$

$$x = \frac{13 \pm \sqrt{169 - 140}}{10}$$

$$x = \frac{13 \pm \sqrt{29}}{10}$$

$$x = \frac{13 \pm 5.385}{10}$$

$$x_1 = \frac{13 + 5.385}{10} \qquad x_2 = \frac{13 - 5.385}{10}$$

$$= \frac{18.385}{10} \qquad = \frac{7.615}{10}$$

$$= 1.835 \qquad = 0.7615$$

Example 1.13

Solve the following quadratic equation

$$\frac{10x^2 - 5}{2} = \frac{16x + 5}{3}$$

Solution: -

$$\frac{10x^2 - 5}{2} = \frac{16x + 5}{3}$$

$$3(10x^2 - 5) = 2(16x + 5)$$

$$30x^2 - 15 = 32x + 10$$

$$30x^2 - 32x - 25 = 0$$

$$a = 30\,; b = -32\,; c = -25$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-32) \pm \sqrt{(-32)^2 - 4(30)(-25)}}{2(30)}$$

$$x = \frac{32 \pm \sqrt{1024 + 3000}}{60}$$

$$x = \frac{32 \pm \sqrt{4024}}{60}$$

$$x = \frac{32 \pm 63.44}{60}$$

$$x_1 = \frac{32 + 63.44}{60} \qquad x_2 = \frac{32 - 63.44}{60}$$

$$= 1.59 \qquad = -0.52$$

EXERCISE 1.5

Solve the following quadratic equation by using formula method.

a) $x^2 - 13x + 7 = 0$	b) $2x^2 - 7x - 10 = 0$

c) $7x^2 - 8x - 15 = 0$	d) $21x^2 - 15x - 10 = 0$
e) $(x + 5)(x - 4) = 5x$	f) $x^2 - 5x - 6 = 0$
g) $4x^2 + 3x = 2$	h) $20k^2 = 33k - 7$

MASTER YOUR KNOWLEDGE 2

1. Simplify the following algebraic equations.
   a) $(y + x)^2 - x(x + 2y)$
   b) $20jk^2 - 5j^3$
   c) $x(x - 2y) - (x + y)^2$
   d) $(1 - 4u)^2 - 3(u + 5)$

2. Simplify the following Algebraic Fractions into single fraction.
   a) $\frac{3z}{z} - \frac{2z}{4}$
   b) $\frac{8a^3x^2y}{9a^3bc^2}$
   c) $\frac{(4xy^2)(3x^2)}{6y}$
   d) $\frac{3r^2st^2}{8} \div \frac{9rs}{20}$
   e) $\frac{12m^2n+3n}{15m^2n}$
   f) $\frac{2m}{m+k} \div \frac{m^2}{m^2-k^2}$
   g) $\frac{3}{x-1} + \frac{4}{3x}$

3. Express $\frac{2}{m-1} - \frac{5-m}{m^2-1}$ as a single fraction in its simplest form.

4. Given $\frac{1-5r}{r} = \frac{1}{2t}$, express $\boldsymbol{r}$ in terms of $\boldsymbol{t}$

5. Make n as the subject of the formula for the following.
   $$\sqrt{\frac{m}{n-2}} = 3$$

6. Express $\boldsymbol{m}$ in terms of $\boldsymbol{k}$.
   $$\frac{5m - 2}{m + k} = 8$$

7. Make m as the subject of the formula for the following.
   $$n = \frac{m + 2}{3m}$$

8. Solve the following linear equation.
   a) $5x + 2 = 7x$
   b) $2x = 7x + 4$
   c) $3g - 5 = 7(g - 8)$
   d) $m + 4 = 2m + 6$

9. Solve the following quadratic equation by using Factorization Method.

a) $20x^2 + 32x + 11 = 0$

b) $6x^2 - 17x - 3 = 0$

c) $9x^2 - 3x - 42 = 0$

10. Solve the following quadratic equation by using Formula Method.

a) $x^2 - 2x - \frac{5}{2} = 0$

b) $4x^2 + 8x + 3 = 0$

c) $9y^2 - 26y + 11 = 0$

d) $\frac{3}{2}m^2 - 4m - 1 = 0$

e) $20x^2 + 32x + 11 = 0$

f) $6x^2 - 17x - 3 = 0$

g) $9x^2 - 3x - 42 = 0$

CHAPTER TWO

MEASUREMENT

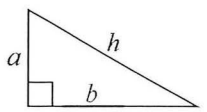
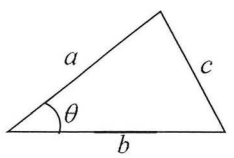
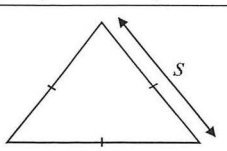
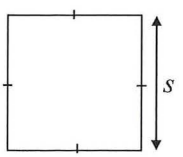
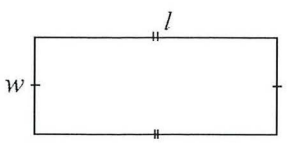
2.1 PERIMETER AND AREA

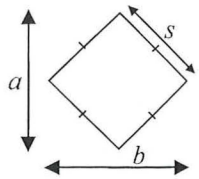
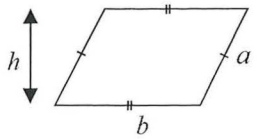
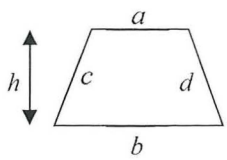
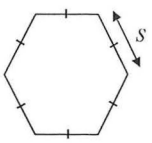
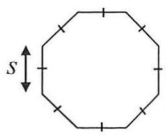
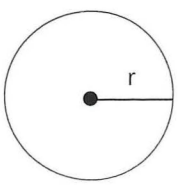
Perimeter is the distance around a two-dimensional shape, or the measurement of the distance around something; the length of the boundary. A **perimeter** is a path that surrounds a two-dimensional shape.

Area is the quantity that expresses the extent of a two-dimensional figure or shape, or planar lamina, in the plane. **Surface area** is its analogue on the two-dimensional surface of a three-dimensional object.

Volume - The amount of 3-dimensional space an object occupies. Capacity.

Table 2.1.1: Formula of Perimeter and Area For 2-Dimensional (2-D) Object.

Shape	Perimeter	Area
 Right triangle	$P = a + b + h$	$A = \frac{1}{2}ab$
 Triangle (isosceles/scalene)	$P = a + b + c$	$A = \frac{1}{2}ab\sin\theta$
 Equilateral triangle	$P = 3 \times s$ @ $P = s + s + s$	$A = \frac{1}{4}\sqrt{3}s^2$
 Square	$P = 4 \times s$ @ $P = s + s + s + s$	$A = s^2$
 Rectangle	$P = (2 \times l) + (2 \times w)$ @ $P = l + l + w + w$	$A = l \times w$

 Rhombus	$P = 4s$	$A = \frac{1}{2}ab$
 Parallelogram	$P = 2a + 2b$	$A = b \times h$
 Trapezoid/Trapezium	$P = a + b + c + d$	$A = \frac{1}{2}(a + b)h$
 Hexagon	$P = 6s$	$A = \frac{3}{2}\sqrt{3}s^2$
 Octagon	$P = 8s$	$A = 2(1 + \sqrt{2})s^2$
 Circle	$P = 2\pi r$ @ $P = \pi d$	$A = \pi r^2$

Example 2.1

Given a rectangle with its width is double its length. The area of rectangle is 98cm^2. Find the width of the rectangle and its perimeter.

Solution: -

Assume $w = 2l$; (because it is stated that width is doubled its length)

Area = 98cm^2

So, according to the formula $A = l \times w$

$$A = l \times 2l$$
$$98 = 2l^2$$
$$l^2 = 49$$
$$l = 7$$

Width = 2 x 7 = 14cm

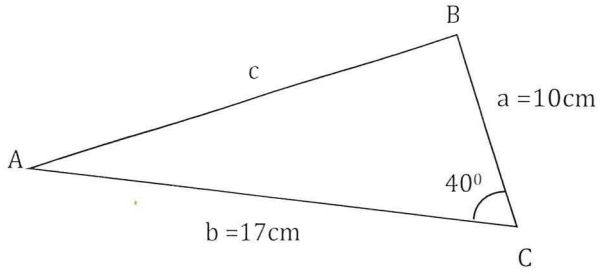
Perimeter = 2w + 2l

= 2 x 14 + 2 x 7

= 42cm

Example 2.2

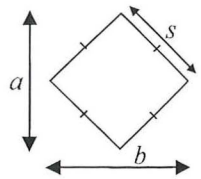
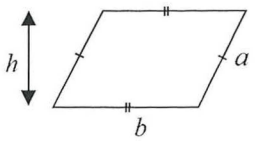
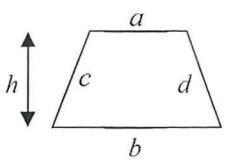
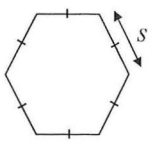
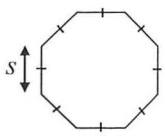
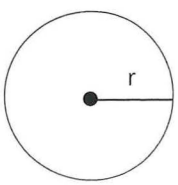
Given the length of a = 10cm, b = 17cm and angle C = 40^0 as shown in figure below. Find the area of triangle.



Solution: -

Area of triangle, $A = \frac{1}{2}ab\ sin\ \theta$

$$A = \frac{1}{2}(10)(17)\ sin\ 40^o$$
$$= 54.64cm^2$$

 Rhombus	$P = 4s$	$A = \frac{1}{2}ab$
 Parallelogram	$P = 2a + 2b$	$A = b \times h$
 Trapezoid/Trapezium	$P = a + b + c + d$	$A = \frac{1}{2}(a + b)h$
 Hexagon	$P = 6s$	$A = \frac{3}{2}\sqrt{3}s^2$
 Octagon	$P = 8s$	$A = 2\left(1 + \sqrt{2}\right)s^2$
 Circle	$P = 2\pi r$ @ $P = \pi d$	$A = \pi r^2$

Example 2.1

Given a rectangle with its width is double its length. The area of rectangle is $98cm^2$. Find the width of the rectangle and its perimeter.

Solution: -

Assume $w = 2l$; (because it is stated that width is doubled its length)

Area = $98cm^2$

So, according to the formula $A = l \times w$

$$A = l \times 2l$$
$$98 = 2l^2$$
$$l^2 = 49$$
$$l = 7$$

Width = 2 x 7 = 14cm

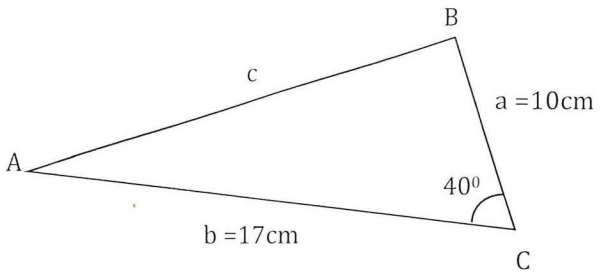
Perimeter = 2w + 2l

= 2 x 14 + 2 x 7

= 42cm

Example 2.2

Given the length of a = 10cm, b = 17cm and angle C = 40^0 as shown in figure below. Find the area of triangle.



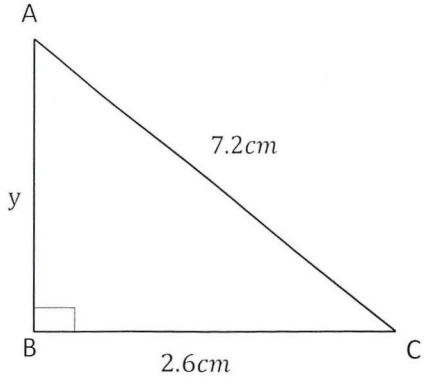
Solution: -

Area of triangle, $A = \frac{1}{2}ab\ sin\ \theta$

$$A = \frac{1}{2}(10)(17)\ sin\ 40^o$$
$$= 54.64cm^2$$

Example 2.3

According to the following right-angled triangle, find its perimeter.



Solution: -

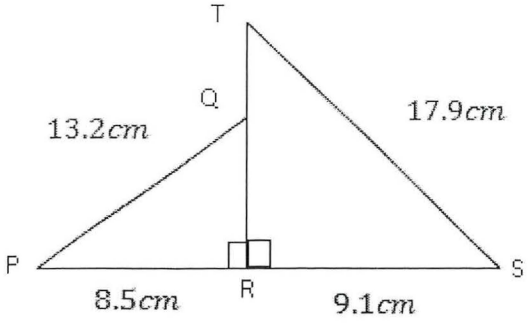
To find out its perimeter, we have to find out the value of ***y***. To get the value of ***y***, we can use Pythagorean Theorem because it is a right-angled triangle.

$$y^2 = 7.2^2 - 2.6^2$$
$$y = \sqrt{7.2^2 - 2.6^2}$$
$$y = \sqrt{45} \text{ @ } 6.714cm$$

So, perimeter of triangle = 7.2cm + 2.6cm + 6.714 cm
= 16.514cm

Example 2.4

According to the following diagram, calculate its perimeter.



Solution: -

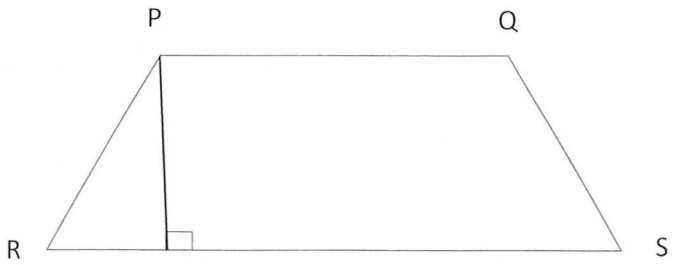
$$QR^2 = 13.2^2 - 8.5^2$$
$$= 174.24 - 72.25$$
$$= 101.9$$
$$QR = \sqrt{101.99} = 10.1cm$$
$$TR^2 = 17.9^2 - 9.1^2$$
$$= 320.41 - 82.81$$
$$= 237.6$$
$$TR = \sqrt{237.6} = 15.41cm$$
$$TQ = TR - QR$$
$$= 15.41cm - 10.1cm = 5.31cm$$

So, perimeter of the diagram is;

$$= 5.31cm + 13.2cm + 8.5cm + 9.1cm + 17.9cm$$
$$= 54.01cm$$

Example 2.5

Diagram below shows a trapezium PQSR. Given that the area of a trapezium is 273 cm^2, $RS = 22\ cm$, $PQ = a\ cm$ and height of trapezium is 13 cm.Find the value of a.



Solution: -

Area of trapezium $= 273$

$$\frac{1}{2}(PQ + RS)(13) = 273$$

$$\frac{1}{2}(a + 22)(13) = 273$$

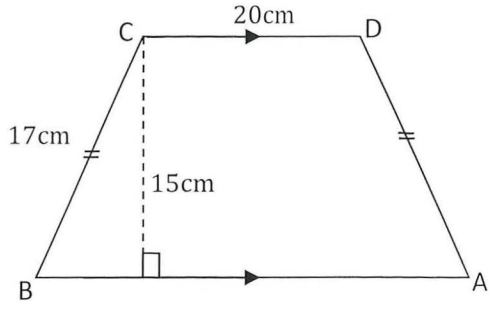
$$a + 22 = 42$$

$$a = 42 - 22$$

$$a = 20\ cm$$

EXERCISE 2.1

1. According to the following diagram, ABCD is a trapezium. Find its perimeter and area.

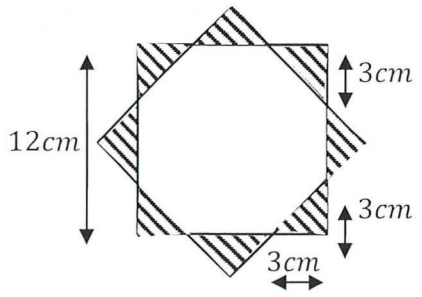


2. Given a circle with its radius is 5cm. Find its circumference and its area.

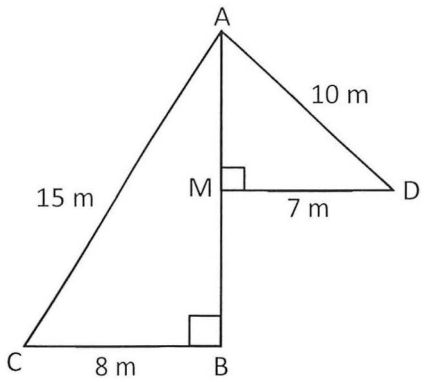
3. A square was formed by 100cm wire. Find its area.

4. A rectangle was formed to have an area of $150cm^2$. If its length is 25cm. Find its width.

5. Two squares are arranged to form a pattern as shown in figure below. Calculate the perimeter and area of the unshaded region.

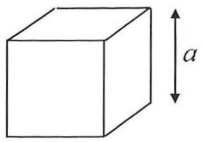
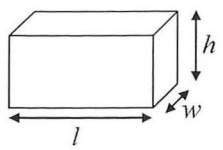
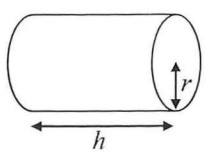
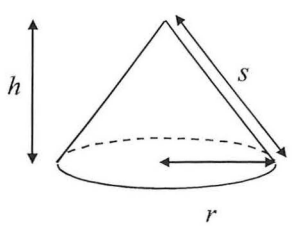


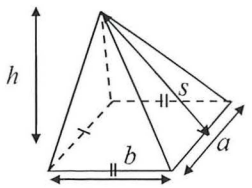
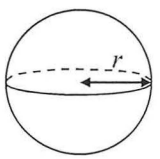
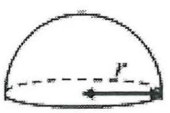
6. In the diagram below, M is the midpoint of AB. Calculate the perimeter of the whole diagram.



2.2 VOLUME FOR SPECIFIC SHAPES

Table 2.2.1: Formula of Total Surface Area and Volume for Solid 3-Dimensional (3-D) Object.

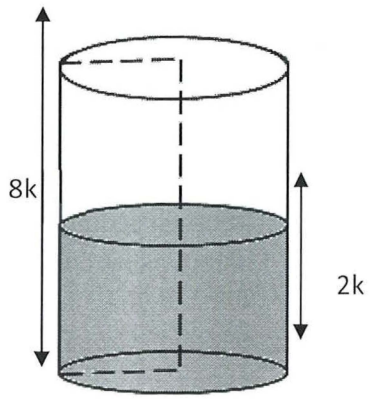
Shape	Surface Area	Volume
 Cube	$A = 6a^2$ - A solid cube has 6 surfaces.	$V = a^3$ - Area of the base (a square) X its height (h)
 Cuboid	$A = 2hw + 2hl + 2lw$ - A solid rectangle always has at least 2 similar size.	$V = h \times l \times w$ - Area of the base (maybe a square or rectangle) X its height (h)
 Cylinder	$A = 2\pi r^2 + 2\pi rh$ - A solid cylinder has 2 circle surfaces and its lateral surface is frequently in rectangle.	$V = \pi r^2 h$ - Area of base (the simplest way, always assume circle as its base); πr^2 x its height (h)
 Circular cone	$A = \pi r^2 + \pi rs$ - A solid circular cone has two surfaces which a circle at its bottom and a slant surface	$V = \frac{1}{3}\pi r^2 h$ - $\frac{1}{3}$ of base area (the simplest way, always assume circle as its base); πr^2x its height (h)

 Pyramid	$A = 2\left(\frac{1}{2}as\right) + 2\left(\frac{1}{2}bs\right) + ab$ $A = as + bs + ab$ - A solid pyramid always has a base on form of square or rectangle; width x length, $A = ab$ - and two similar slant area in form of triangle; $A = 2\left(\frac{1}{2}as\right)$ - more another two similar triangles; $A = 2\left(\frac{1}{2}bs\right)$	$V = \frac{1}{3}abh$ - $\frac{1}{3}$ of base area (the simplest way, always assume rectangle or square as its base. Refer to question detailed.); abx its height (h)
 Sphere	$A = 4\pi r^2$	$V = \frac{4}{3}\pi r^3$
 Hemispshere	$A = 3\pi r^2$	$V = \frac{2}{3}\pi r^3$

Basically, to calculate the volume of a 3-D object, the concept applied is **base area x height**. So, this formula can be used for any object such like cube, cuboid, right-circular cylinder and trapezium.

Example 2.6

Diagram below shows a cylinder tank with 2k water level and diameter = 6k. Prove that the volume of water tank can be added into the tank is 169.646k³ cm³. (Assume $\pi = 3.142$).



Solution: -

From the question, we knew that diameter of cylinder is 6k, therefore the radius is 3k of the diameter since radius is half of cylinder.

r = 3k

$V = \pi r^2 h$

$V = (3.142)(3k)^2(8k) - [(3.142)(3k)^2(2k)]$

$V = (3.142)(9k^2)(8k) - [(3.142)(9k^2)\ \ (2k)]$

$V = 226.195k^3 - 56.549k^3$

$V = 169.646k^3 \text{cm}^3$

Example 2.7

Given a solid right circular cone with total surface area is $90\pi cm^2$, radius is 5cm and slant height is 13cm. Calculate the volume of the cone.

Solution: -

Total Surface area= 90π

$\pi r^2 + \pi rs = 90\pi$

$\pi(5)^2 + \pi(5s) = 90\pi$

$25\pi + (5s)\pi = 90\pi$

$25 + 5s = 90$

$5s = 90 - 25$

$5s = 65$

$s = \frac{65}{5}$

$= 13\ cm$

$h^2 = s^2 - r^2$

$= 13^2 - 5^2$

$= 144$

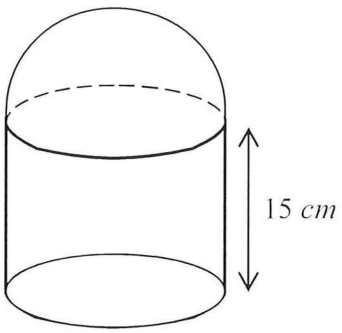
$$h = \sqrt{144}$$
$$= 12cm$$

Volume of cone,

$$V = \frac{1}{3}\pi r^2 h$$
$$= \frac{1}{3}\pi(5)^2(12)$$
$$= 100\pi\ cm^3\ or\ 314.2\ cm^3$$

Example 2.8

The diagram below shows a composite solid consisting of a cylinder and a hemisphere. The cylinder and the hemisphere shared the same radius that is $8cm$. Calculate the volume of the solid.



Solution: -

Volume of cylinder, $V = \pi r^2 h$

$$V = \pi(8)^2 15$$
$$= 3015.92cm^3$$

Volume of half hemisphere, $V = \left(\frac{2}{3}\pi r^3\right) \times \frac{1}{2}$

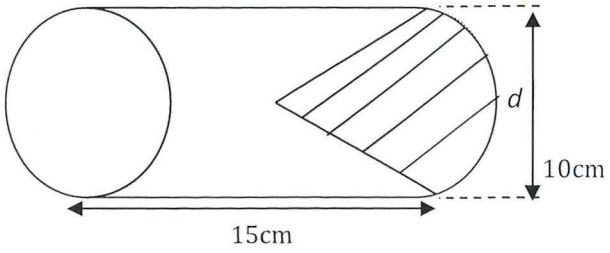
$$V = \left(\frac{2}{3}\pi(8)^3\right) \times \frac{1}{2}$$
$$= \frac{1}{2}\left(\frac{2048\pi}{3}\right)$$
$$= \frac{1024\pi}{3}$$
$$= 1072.33cm^3$$

∴Volume of the solid = volume of cylinder + volume of half cylinder

$$V = 3015.92 - 1072.33$$
$$= 4088.25cm^3$$

Example 2.9

The following diagram shows a cylinder with diameter of 10cm and length of 15 cm. The shaded portion in the form of a cone is hollowed out. Calculate the volume of the remaining solid, in cm^3. (Given height of cone is 6cm)



Solution: -

Volume of cylinder, $V = \pi r^2 h$

$$V = \pi(5)^2(15)$$
$$= 1178.1cm^3$$

Volume of cone, $V = \frac{1}{3}\pi r^2 h$

$$V = \frac{1}{3}\pi(5)^2(6)$$
$$= 157.08cm^3$$

∴Volume of remaining solid = volume of cylinder – volume of cone

$$= 1178.1 - 157.08$$
$$= 1021.02cm^3$$

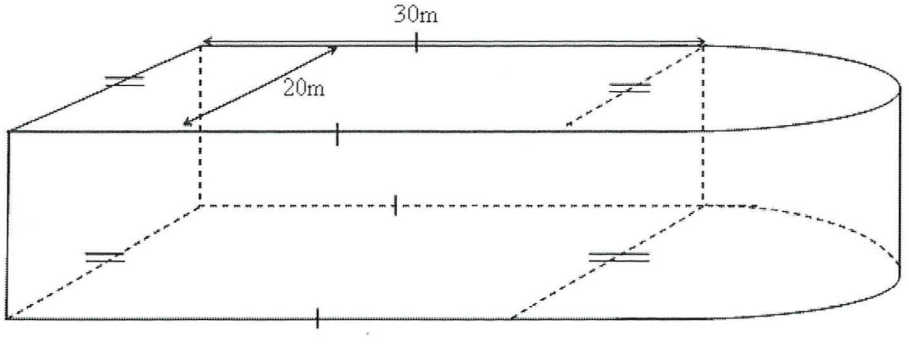
Example 2.10

Ahmad built up a swimming pool in front of his house. He wants to pave the pool with marble. The depth of the pool is 10m. A scale drawing of the pool is shown below (from the surface, pool is design with combined shape of rectangle and semicircle).

According to the diagram: -

a. Find the total cost of the pavement if the cost (labour and materials) for the pavement is RM50 per square metre, and

b. Calculate the volume of water to fill up three quarter of the pool.



Solution: -

Calculate paving cost: -

i) Area of rectangle :-

$$= (20 \times 10) + (30 \times 10) + (30 \times 10) + (30 \times 20)$$
$$= 200 + 300 + 300 + 600$$
$$= 1400m^2$$

ii) Area of half cylinder: -

$$= \frac{1}{2}\pi r^2 + \frac{2\pi rh}{2}$$
$$= \frac{1}{2}\pi 10^2 + \frac{2\pi(10)(10)}{2}$$
$$= 157.08 + 314.16$$
$$= 471.24m^2$$

iii) Total Surface area: -

$$= 471.24m^2 + 1400m^2$$
$$= 1871.24m^2.$$

iv) Pavement cost: -

$$= 1871.24m^2 \times RM50$$
$$= RM93562.00$$

Calculate the volume of water: -

i) Base area of the pool: -

$$= 30 \times 20 + \frac{1}{2}\pi r^2$$
$$= 600 + \frac{1}{2}\pi(10^2)$$
$$= 757.08m^2$$

ii) Volume of water: -

$$= \frac{3}{4} \times 757.08 \times 10$$
$$= 5678.1m^3$$

EXERCISE 2.2

1. A conical container is fully filled up with water. If 500ml water is drained out from the container, calculate the volume in ml, of water left in the container if given diameter of circular top is 16cm and height of container is 17cm. [$100cm^3 = 100ml$]

2. A solid sphere with volume $V = 250m^3$ is built up. Find its total surface area.

3. A based area of a circular cylindrical tank is πr^2 and its radius is less quarter from its height. Given its height is 20meter. If water is poured into the tank is at $100m^3$ per minutes, find out how many minutes its need to fill up until its maximum. [*Answer :- 142minutes*]

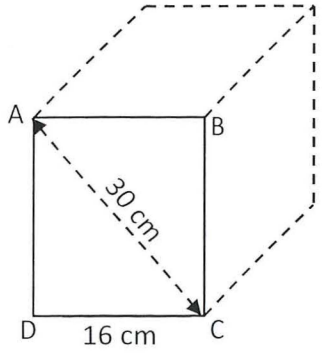
4. A cuboid container has a length which is double its width. The height of the container is also double its length. Find: -

   a. The total surface area of the container if the container loss its cover and its width is 50cm. [Answer = 6500cm^2]

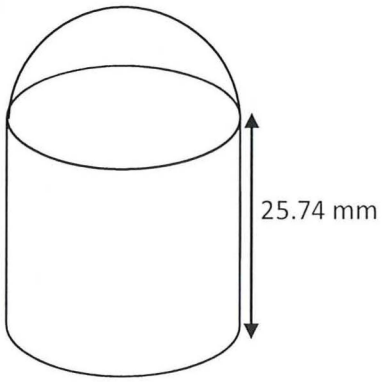
   b. A space needed to place the container if its length is 30cm. [Answer = 450cm^2]

c. A volume of the container if the height is 100cm. [Answer = 125000cm^3]

5. Diagram below shows a rectangle with a given width of 16 cm and AC = 30 cm. Find the volume of the cuboid if the length is twice the width.



6. The shape in figure below are composed of a cylinder and a hemisphere. Find the surface area of the shape, given that the circumference of the circle is 25.74 mm and the radius for both circle and hemisphere are equal.



MASTER YOUR KNOWLEDGE 2

1. The Figure 1a shown, QRS is an equilateral triangle and PQTU is a rhombus. T is the midpoint of QS and PQ = QR. Calculate the perimeter of the entire figure.

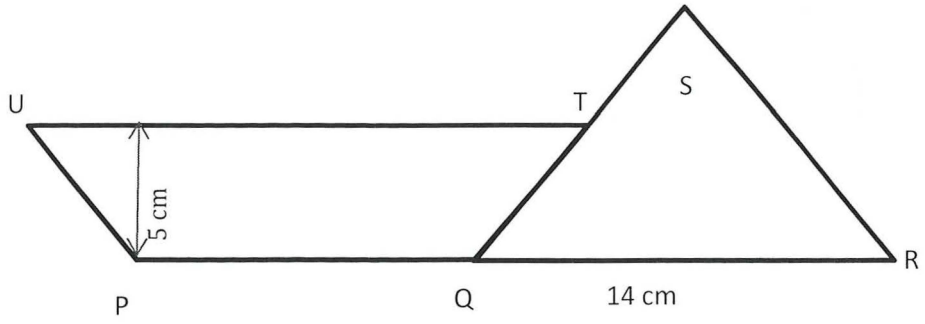


Figure 1a

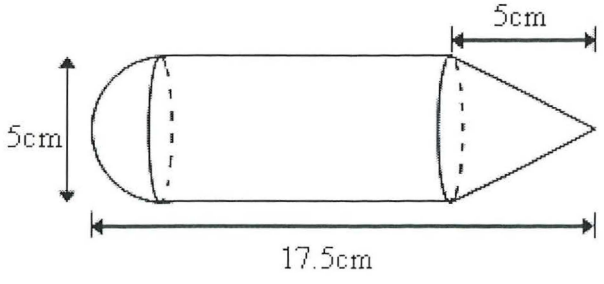
2. A right circular cylinder has base radius 12 cm and height of 22 cm. Determine: -
   a. Its total surface area.
   b. Its volume.

3. A conical container is fully filled up with water. If 200ml water is drained out from the container, calculate the volume in, ml, of water left in the container if given diameter of circular top is 10cm and height of container is 14cm. [$100cm^3 = 100ml$]

4. A circular cylinder tank is filled up with $282.74cm^3$ water. Given the diameter of the cylinder is 6cm. Find the volume of water that should be added to the cylinder until its maximum if the height of the cylinder is 25cm.

5. Given a sphere container with diameter = 16m. Find
   a. its total surface area and
   b. its water capacity if it was filled up by 3 quarter of its maximum volume.

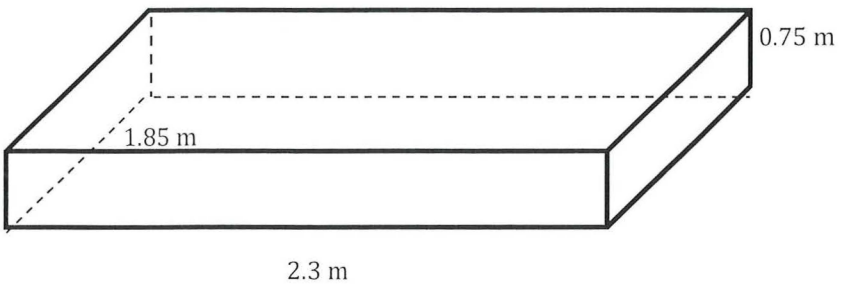
6. The diagram below shows a solid made up of a hemisphere, a right circular cylinder and a right circular cone. Calculate: -
   a. Volume of the solid in cm^3
   b. Total surface area of the solid in cm^2
   (Take $\pi = 3.142$and your answer must correct to 3 decimal places)



7. Hisyamuddin is required to make an aluminium box as the following diagram. The length of the box is 2.3 m, width is 1.85 m and height is 0.75 m. Ignoring of the thickness of the aluminium sheet, calculate;

i. The area of the sheet required for making the box.

ii. The perimeter of the box.

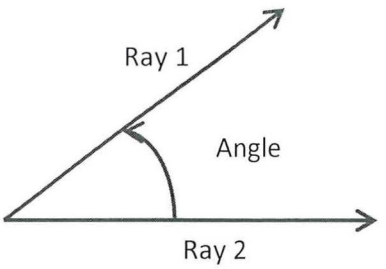


CHAPTER THREE

GEOMETRY

3.1 ANGLES

The magnitude of an **angle** is the amount of rotation that separates the two rays.



The symbol of angle is '$\angle$'.

Angle has no dimension that they are defined as the ratio of lengths. Several units are used to measure angle, for instance degree and radian are the most common.

- The **degree**, denoted by (∘), minute (,) and second (,,).
  For example; $15^o20'45"$.

- The **radian** is abbreviated *rad*.
  For example; $\frac{\pi}{2} rad$.

One degree is divided into 60 minutes, and one minute into 60 seconds. These units, also called the arc-minute and arc-second, are respectively represented as a single and double prime, or a single and double quotation mark. For example; $52.5042^o = 52^o30'15"$.

The radian is a unit of plane angle. Thus 2π radian is equal to 360 degrees, or one radian is equal to $\frac{180}{\pi}$ degrees.

3.1.1 Convert Radian to Degree

To convert the unit of radian to degree, as we know one radian is equal to $\frac{180^\circ}{\pi}$. Thus, we multiply the given angle by $\frac{180^\circ}{\pi}$.

Example 3.1

Convert $\frac{\pi}{5}$rad to degree.

Solution: -

$$\frac{\pi}{5}\text{rad} = \frac{\pi}{5}\text{rad} \times \frac{180^\circ}{\pi}$$
$$= 36^\circ$$

Example 3.2
Convert 0.17πrad to degree.

Solution: -

$$0.17\pi\text{rad} = 0.17\pi\text{rad} \times \frac{180^\circ}{\pi}$$
$$= 30.6^\circ$$

Example 3.3
Convert 1.588 rad to degree, minute and second.

Solution: -

$$1.588\text{ rad} = 1.588\text{ rad} \times \frac{180^\circ}{\pi}$$
$$= 90.9857^\circ$$
$$= 90^\circ 59' 9''$$

3.1.2 Convert Degree to Radian

Conversely, to convert the angle from degrees to radians, multiply it by $\frac{\pi}{180^\circ}$.

Example 3.4
Convert 60° to radian.

Solution: -

$$60^\circ = 60^\circ \times \frac{\pi}{180^\circ}\text{rad}$$
$$= \frac{\pi}{3}\text{rad}$$

Example 3.5
Convert 22.5° to radian.

Solution: -

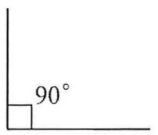
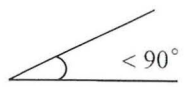
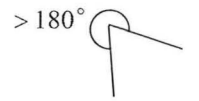
$$22.5^\circ = 22.5^\circ \times \frac{\pi}{180^\circ}\text{rad}$$
$$= \frac{\pi}{8}\text{rad}$$

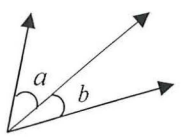
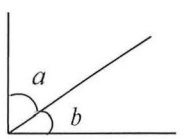
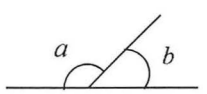
Example 3.6
Convert $40^\circ 20' 30''$ to radian.

Solution: -

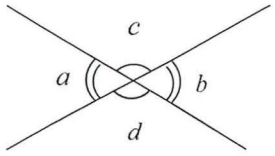
$$40^\circ 20' 30'' = 40^\circ 20' 30'' \times \frac{\pi}{180^\circ}$$
$$= 0.2245\pi\text{rad}$$

3.1.3 Types of Angle

Right angle	
Acute angle	
Obtuse angle	
Reflex angle	

Adjacent angle		Angles that share a common vertex and edge but do not share any interior points are called ***adjacent angle***. (angles a and b are adjacent)
Complementary angle		Two angles that sum to one right angle (90°) are called ***complementary angle***. (a is the complement of b, and b is the complement of a)
Supplementary angle		Two angles that sum to one straight angle (180°) are called ***supplementary angle***. (a and b are supplementary angle)

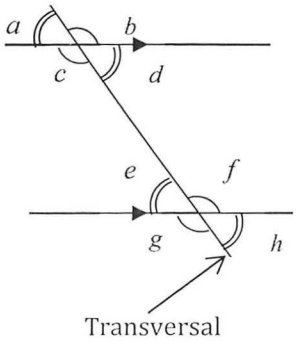
3.1.4 Properties of Angles



Properties of angles between two intersection lines:

$\angle a = \angle b$

$\angle b = \angle d$



Properties of angles between two parallel lines crossed by a straight line

$\angle a = \angle d = \angle e = \angle h$

$\angle b = \angle c = \angle f = \angle g$

- When two lines are crossed by another line, it is called **transversal**.
- The angles in matching corners are called **corresponding angles**, for example the corresponding angles are:
  i. $\angle a$ and $\angle e$,
  ii. $\angle b$ and $\angle f$,
  iii. $\angle c$ and $\angle g$,
  iv. $\angle d$ and $\angle h$.

- The pairs of angles on opposite sides of transversal but **inside** the two lines are called **alternate interior angles** for example
  i. $\angle c$ and $\angle f$,
  ii. $\angle d$ and $\angle e$.

- The pairs of angles on opposite sides of transversal but **outside** the two lines are called **alternate exterior angles** for example
  i. $\angle a$ and $\angle h$,
  ii. $\angle b$ and $\angle g$.

EXERCISE 3.1

1. Convert the angles (degree) to radian:

   a. $60°$

   b. $29°$

   c. $45.54°$

   d. $20°32'45"$

   e. $0.98°$

   f. $244°50"$

2. Convert the angles (radian) to degree, minute and second.

   a. $\frac{2}{3}\pi\text{rad}$

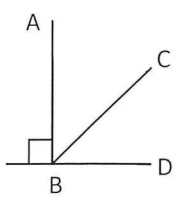
   b. $\frac{\pi}{7}\text{rad}$

   c. $0.2543\pi\text{rad}$

   d. 0.2223rad

3. Classify the type of angles for each below:

   a.

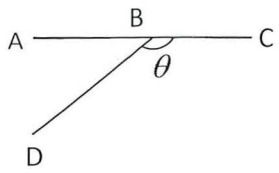


   i. $\angle ABD$

   ii. $\angle ABC$

   iii. $\angle CBD$

b.



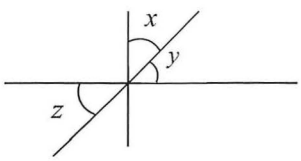
i. $\angle\theta$

ii. $\angle ABD$

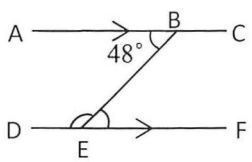
iii. $\angle CBD$

4. Given that $\angle\alpha$ and $\angle\theta$ are supplementary angle. If $\angle\theta = 35°$, find $\angle\alpha$.

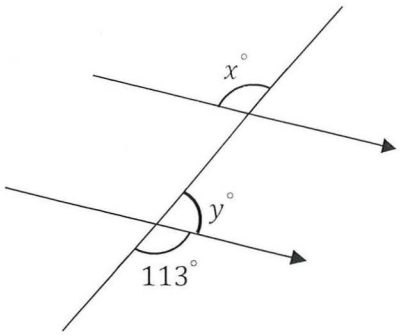
5. Given that $\angle y = 40°$ and $\angle x$ and $\angle y$ are complimentary angle, find the angles of x and z.



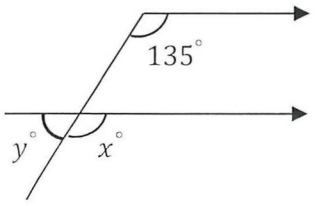
6. In the figure shown below, given that $\angle ABE = 48°$. Calculate the value of $\angle BEF$ and $\angle BED$.



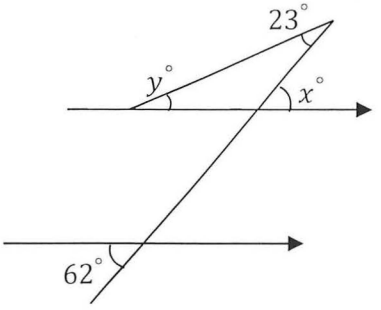
7. Calculate the value of x and y based on the diagram below.



8. Calculate the value of x and y based on the diagram below.

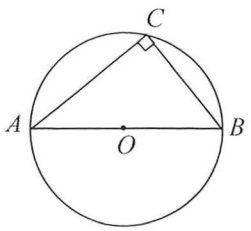


9. Based on figure below, AB and CD are parallel while PQ is a transversal line of AB and CD. Find the value of x and y.



3.2 CIRCLE

3.2.1 Characteristic of Angle and Chord in A Circle

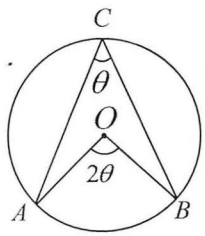


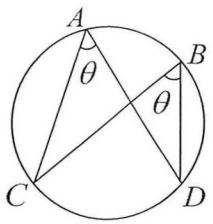
The angle in a semi-circle is 90°.

$(\angle ACB = 90°)$.

The angle subtended by an arc at the centre O is two times the angle subtended by the same arc at the circumference of the circle.

$(\angle AOB = 2\angle ACB)$.





Angles at the circumference of the circle subtended by the same arc are equal.

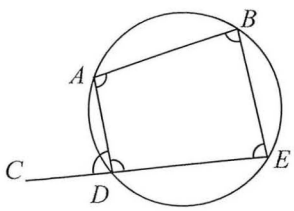
$(\angle CAD = \angle CBD)$.

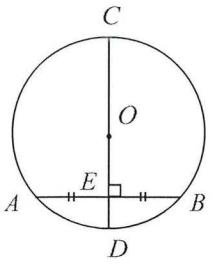
In a cyclic quadrilateral, the sum of the interior opposite angles is 180°. The exterior angle is equal to the opposite angle.

$\angle ADC = \angle ABE$;

$\angle BAD + \angle BED = 180°$;

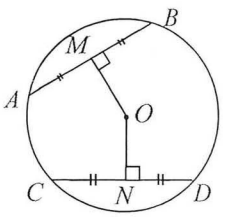
$\angle ADE + \angle ABE = 180°$





If CD is the perpendicular bisector of AB, then $AE = BE$.

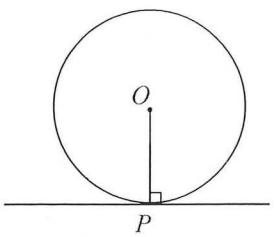
If AB and CD are of the same length, OM is perpendicular to AB and ON is perpendicular to CD, then $OM = ON$.



3.2.2 Characteristic of Tangent Line in a Circle

A tangent to a circle is a straight line which touches the circumference of the circle at one point.

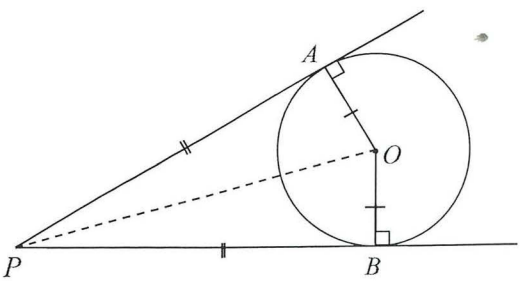
The tangent at point P is perpendicular to the radius OP



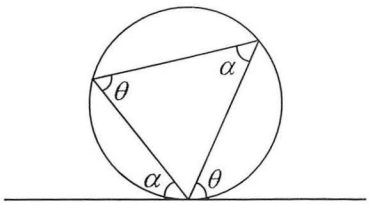
If AP and BP are tangents to a circle, then $AP = BP$;

$\angle AOP = \angle BOP$;

$$\angle APO = \angle BPO$$

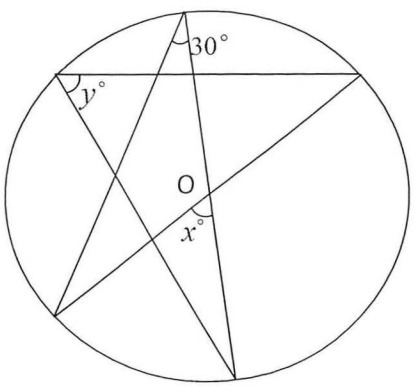


The angle made by tangent and the chord which passes through the point of contact of the tangent is equal to the angle in the alternate segment subtended by the chord.



Example 3.7

Refer to the figure, O is the centre of the circle. Find the angle for x and y.



Solution: -

$$x = 30^o \times 2$$
$$x = 60^o$$

Carefully determine and analyse the diagram. As we may see there, angle x and angle 30^o are subtended by the same arc. Therefore, angle at centre is double the value of angle at circumference.

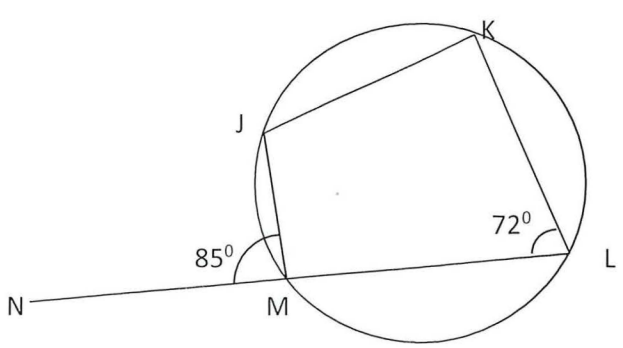
REMEMBER: - Just for same subtended arc.

$$y = \frac{180^o - 60^o}{2}$$
$$y = \frac{120^o}{2}$$
$$y = 60^o$$

While for calculate the value of y.
The remaining value of x, consider that there is straight line, so remaining x is $180^o - 60^o = 120^o$.
Since there are subtended by same arc, angle at circumference (y) is half of centre, therefore y is $\frac{120^o}{2} = 60^o$.

Example 3.8

Given that $\angle JMN = 85^o$and $\angle KLM = 72^o$. Find value of $\angle JML, \angle JKL$ and $\angle KJM$..



Solution: -

$\angle JML = 180^o - 60^o$

$\angle JML = 95^o$

Apply cyclic quadrilateral law.

$\angle JKL = 180^o - 95^o$

$\angle JKL = 85^o$

Alternative:-

$\angle JKL = \angle JMN$

$\angle JKL = 85^o$

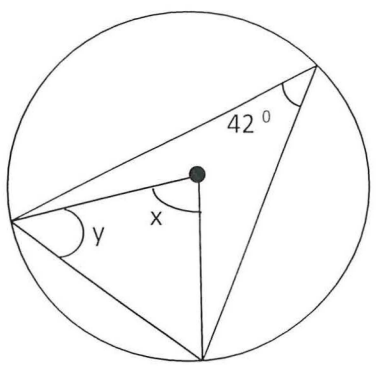
$\angle JKL = 180^o - 72^o$

$\angle JKL = 108^o$

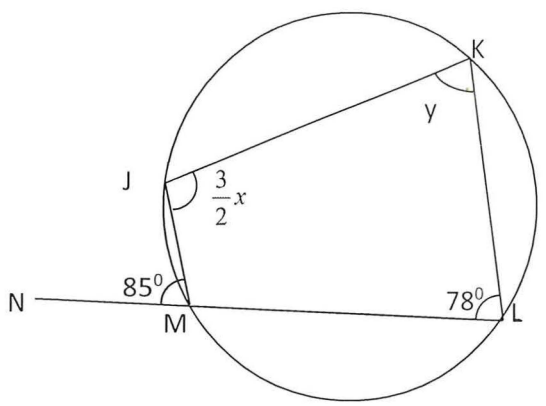
Example 3.9

Find the angle of x and y in each of the diagram below.

a)



b)



Solution: -

a) $x = 48^o \times 2$

$x = 96^o$

$y = \dfrac{180^0 - 96^0}{2}$

$y = 42^o$

b) $\angle JML = 180^0 - 85^0$

$\angle JML = 95^0$

$y + \angle JML = 180^0$

$y + 95^o = 180^o$

$$y = 180^o - 95^o$$
$$y = 85^o$$

Or alternative method, according to cyclic quadrilateral law; $\angle JKL = \angle JMN$

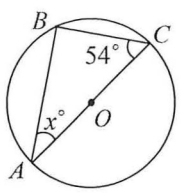
$$y = 85^o$$

To find the value of x.

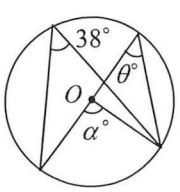
$$\frac{3}{2}x + 78^o = 180^o$$
$$\frac{3}{2}x = 180^o - 78^o$$
$$\frac{3}{2}x = 102^o$$
$$x = \frac{102^o \times 2}{3}$$
$$x = 68^o$$

EXERCISE 3.2

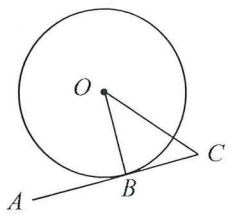
1. In the diagram shown below, O is the centre of the circle. $\angle ACB = 54°$. Find the value of x.



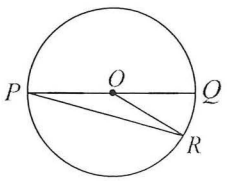
2. In the diagram shown below, find the value of α and θ.



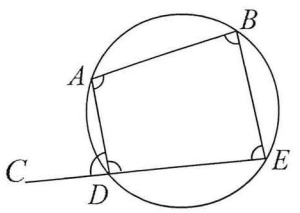
3. In the diagram below, ABC is a tangent to the circle. $\angle BCO = 56°$. Find the value of $\angle BOC$.



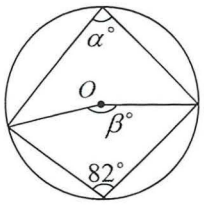
4. In the diagram, $\angle QOR = x°$ and $\angle POR = 3x°$. Find the value of x and $\angle QPR$.



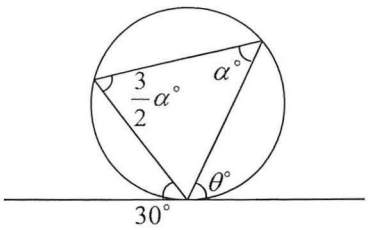
5. Given that $\angle ADC = 82°$ and $\angle BED = 76°$. Find $\angle ADE$, $\angle ABE$ and $\angle BAD$.



6. In the diagram shown below, O is the centre of the circle. Find the value of $\alpha°$ and $\beta°$.

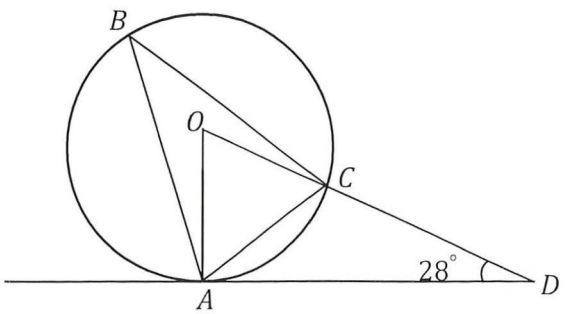


7. In the figure below, find the value of α and β.



8. From the figure below, O is the center of the circle. AD is the tangent and $\measuredangle ADC = 28°$. Find the value of angle:

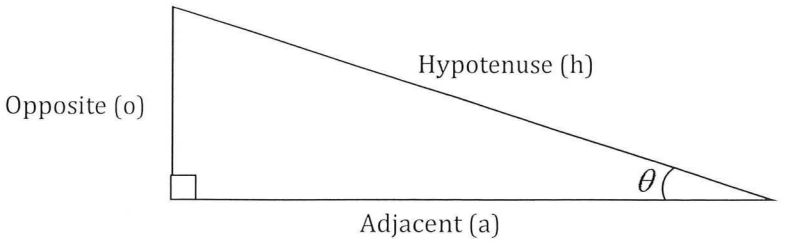
a) $\measuredangle ACD$
b) $\measuredangle ABC$



3.3 PYTHAGOREAN THEOREM

Pythagorean Theorem (American English) or Pythagoras' Theorem (British English) is a relation in Euclidean geometry among the three sides of a right angle (90°).

In any right triangle, the area of the square whose side is the hypotenuse is equal to the sum of the areas of the squares whose sides are the two that meet the right angle (opposite side and adjacent side).

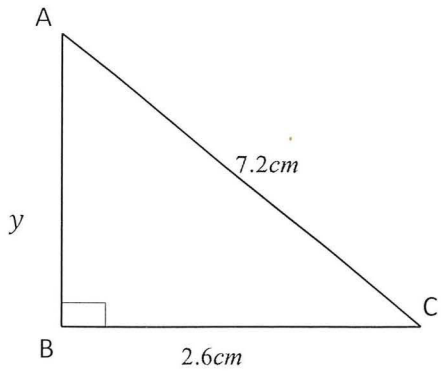


The theorem can be written as an equation:

$$h^2 = a^2 + o^2$$

Example 3.10

In the diagram below $\angle ABC$ is the right angle. Find the value of y.

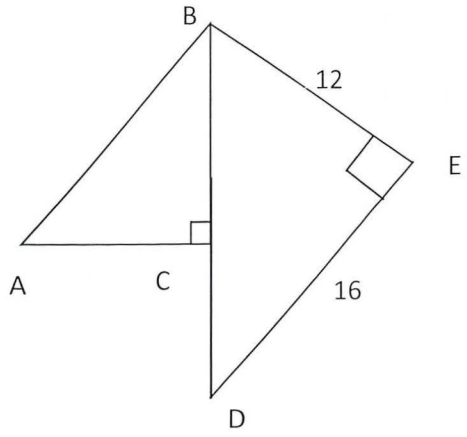


Solution: -

$$y^2 + (2.6)^2 = (7.2)^2$$
$$y^2 + 6.76 = 51.84$$
$$y^2 = 51.84 - 6.76$$
$$y^2 = 45.08$$
$$y = 6.71cm$$

Example 3.11

Given that $BC = CD$ and $AB = BE$.Find the length of AC.



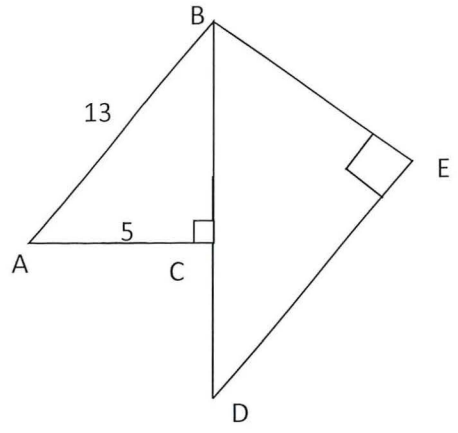
Solution: -

$$BD = \sqrt{12^2 + 16^2}$$
$$BD = \sqrt{400}$$
$$BD = 20cm$$
$$BC = \frac{20}{2}$$
$$BC = 10cm$$

$$DE = \sqrt{12^2 - 10^2}$$
$$DE = \sqrt{44}$$
$$DE = 6.63cm$$

Example 3.12

Given that $BC = CD$ and $AB = BE$.Find the length of DE.



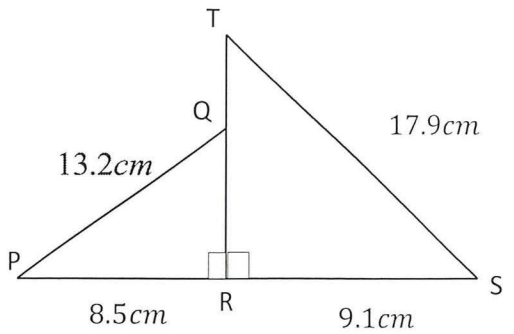
Solution: -

$$BC = \sqrt{13^2 - 5^2}$$
$$BC = \sqrt{144}$$
$$BC = 12cm$$

$$BD = 12 + 12$$
$$= 24cm$$
$$DE = \sqrt{24^2 - 13^2}$$
$$DE = \sqrt{407}$$
$$DE = 20.17cm$$

Example 3.13

Given that PRS is a straight line. $TS = 17.9cm$, $PR = 8.5cm$and $RS = 9.1cm$. Calculate the value of TQ.



Solution: -

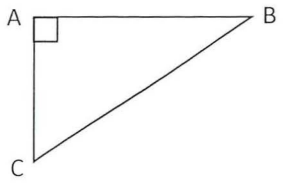
$$QR^2 = 13.2^2 - 8.5^2$$
$$= 174.24 - 72.25$$
$$= 101.9$$
$$QR = \sqrt{101.99}$$
$$= 10.1cm$$

$$TR^2 = 17.9^2 - 9.1^2$$
$$= 320.41 - 82.81$$
$$= 237.6$$
$$TR = \sqrt{237.6}$$
$$= 15.41cm$$

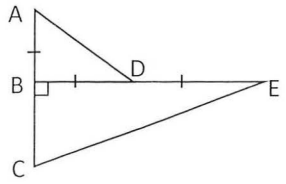
$$TQ = TR - QR$$
$$= 15.41cm - 10.1cm$$
$$= 5.31cm$$
$$y = \sqrt{45.08}$$
$$= 6.714cm$$

EXERCISE 3.3

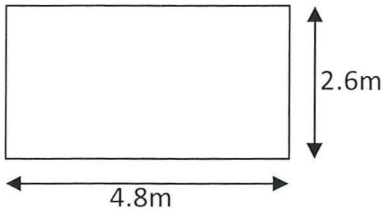
1. By using Pythagoras' Theorem, calculate the distance of BC if given AB=5.4cm and AC=4.8cm.



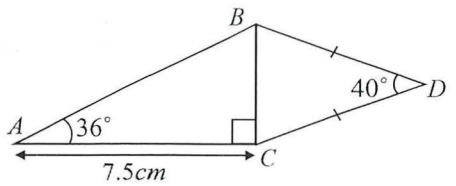
2. In the diagram shown below, $AB = BD = DE = 6cm$. $AC = 14cm$. Find the distance of AD and CE.



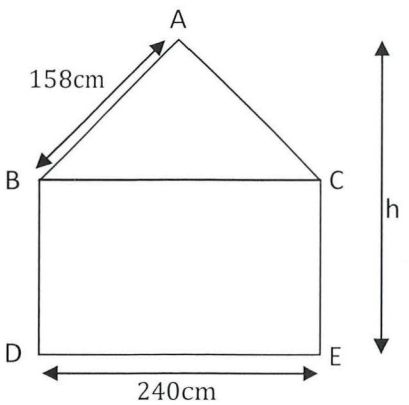
3. Given a rectangular with the length of 4.8m and the width of 2.6m. Find the diagonal distance of the rectangular.



4. In the diagram below, BCD is an isosceles triangle. Find the length of BD.

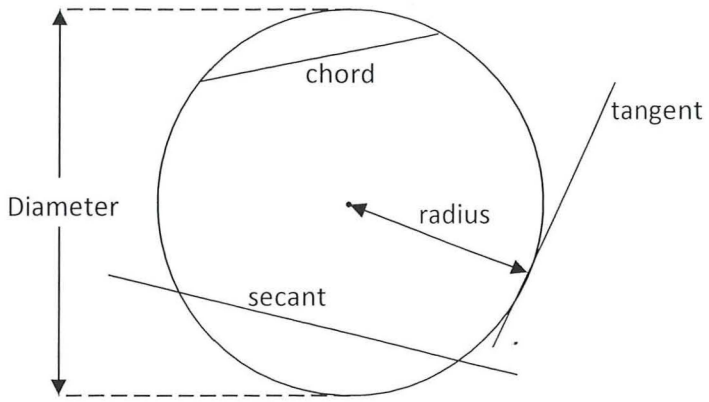


5. In the diagram shown below, $AB = BC$. Find the value of h in cm.



3.4 CIRCLE AND AREA

A circle is a simple shape of Euclidean geometry consisting of those points in a plane which is equidistant from a given point called the centre. The common distance of the points of a circle from its centre is called its radius.



The circumference of a circle is the perimeter of the circle, especially when referring to its length.
Length of circumference, $c = 2\pi r$ @ $c = \pi d$

Whereas, $\pi \Rightarrow$ pi, approximately 3.1416

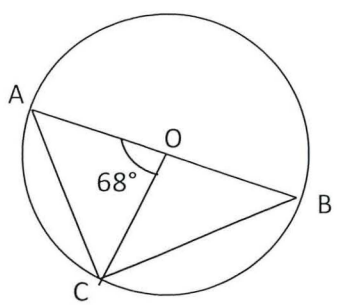
$r \Rightarrow$ radius

$d \Rightarrow$ diameter

The area enclosed by a circle:
Area, $A = \pi r^2$

Whereas, $r \Rightarrow$ radius

Example 3.14
Figure below show right triangle in circle. O is the centre of the circle. $\angle AOC = 68°$ Find $\angle OAC$ and $\angle OBC$.



Solution: -

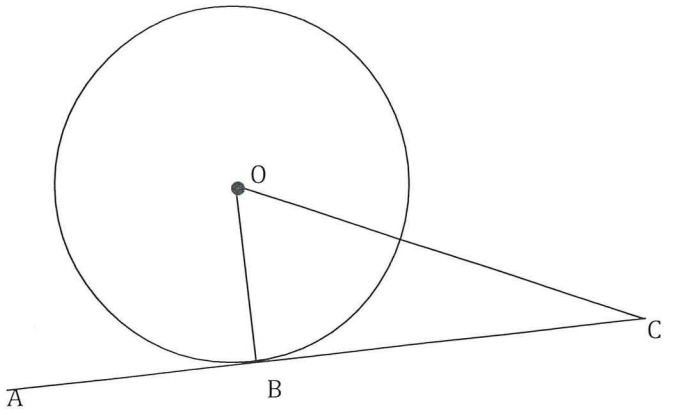
$$\angle OAC = \frac{180^o - 68^o}{2}$$
$$= 56°$$
$$\angle OBC = \angle BCO$$
$$= 90° - 56^o$$
$$= 34^o$$

Example 3.15

In the figure below, O is the centre of the circle. Given that the circumference of the circle is 23cm. Find

i. The radius of the circle

ii. The length of OC if ABC is tangent to the circle and $\angle C = 39^o$.



Solution: -

i. The radius of the circle

circumference $= 2\pi r$

$$23 = 2\pi r$$
$$r = \frac{23}{2\pi}$$
$$r = 3.66cm$$

ii. The length of OC if ABC is tangent to the circle and $\angle C = 39^o$.

$$sin\,39^o = \frac{OB}{OC}$$
$$sin\,39^o = \frac{3.66}{OC}$$
$$OC = \frac{3.66}{sin\,39^o}$$
$$OC = 5.816cm$$

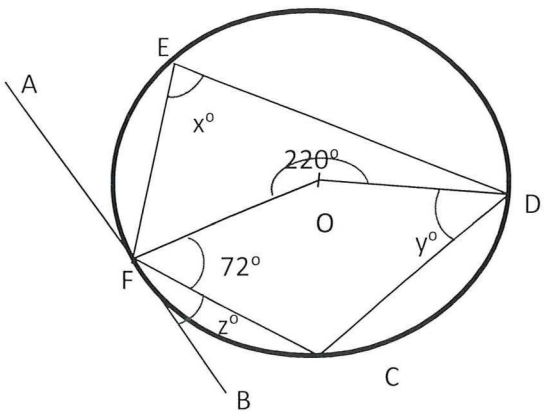
Example 3.16

According to the following diagram, CDEF is a cyclic quadrilateral drawn inside a circle with centre at O. The tangent AB touch the circle at F. Calculate the value of;

i. x

ii. y

iii. z



Solution:-

i. x

$$\angle BOD = 360^o - 220^o$$
$$= 140°$$
$$2x = \angle BOD$$
$$2x = 140^o$$
$$x = 70^o$$

ii. y

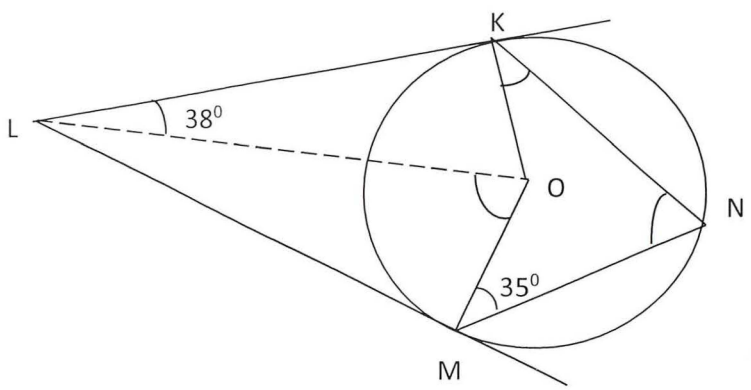
$$y = 180^o - \angle CBO$$
$$y = 180^o - 72^o$$
$$y = 108^o$$

iii. z

$$z = 90^o - 72^o$$
$$z = 18^o$$

Example 3.17

From the Figure 3 shown below, O is the centre of the circle. LK and LM are tangents to the circle. Find the value of $\angle LOM$, $\angle KNM$ and $\angle NKO$.

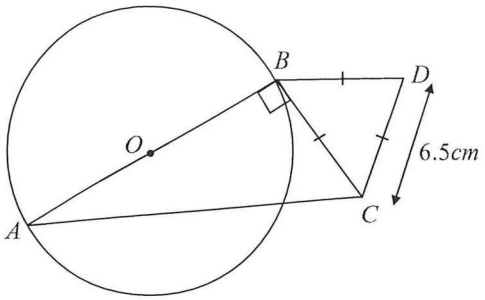


Solution: -

$$\angle LOM = 180^o - 38^o - 90^o$$
$$= 52^o$$
$$\angle KNM = 52^o$$
$$\angle KNO = 52^o - 35^o$$
$$= 17^o$$
$$\angle NKO = \angle KNO$$
$$\angle NKO = 17^o$$

EXERCISE 3.4

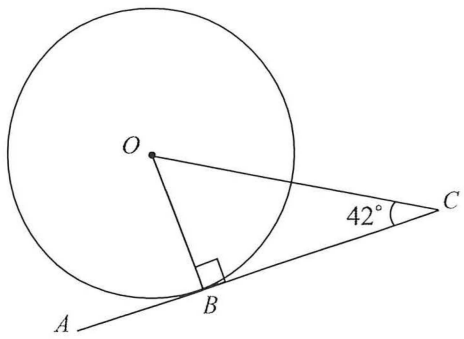
1. In the figure shown below, BCD is an equilateral triangle, ABC is a right triangle, $DC = 6.5cm$ and $AC = 2BC$. Find the circumference and area of the circle:



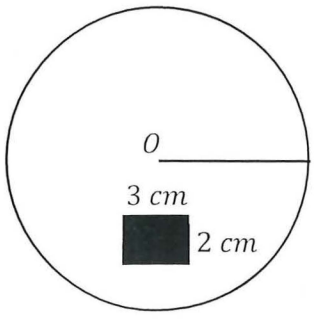
2. In the figure shown below, given that the circumference of the circle is $150mm$, find:

   a. The radius of the circle.

   b. The length of CO if ABC is tangent to the circle and $\angle BCO = 42°$.

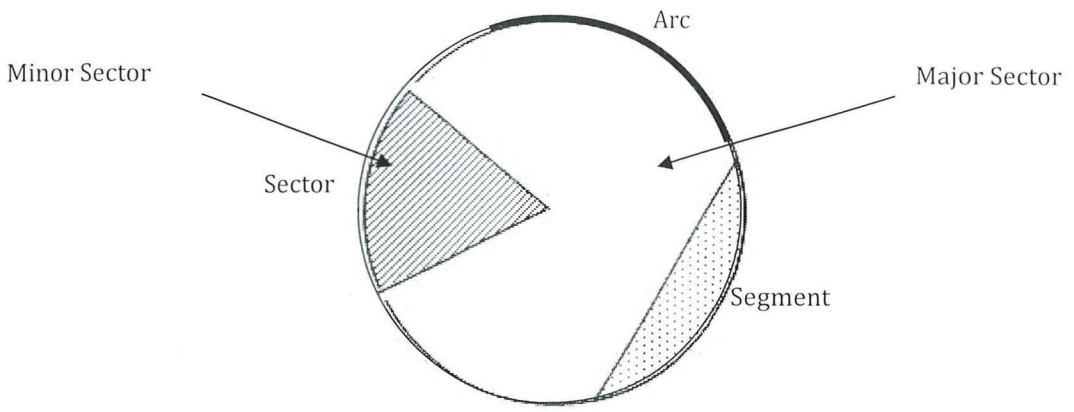


3. A circle with circumference of 52 cm, a rectangular sheet of dimension 3 $cm \times 2\ cm$ is removed as seen in figure below. Find the area of the remaining card.

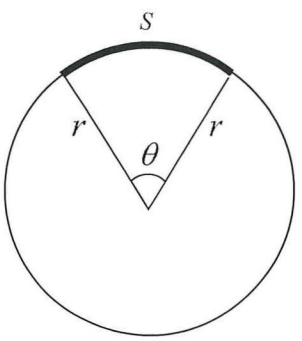


4. Find the circumference of a circle if the area is 37.6 m^2.

3.4.1 Arc length, Area of Sector and Segment



i. An **arc** of a circle is any connected part of the circle's circumference.
ii. A **sector** is a region bounded by two radii and an arc lying between the radii and;
iii. A **segment** is a region bounded by a chord and an arc lying between the chord's endpoints.



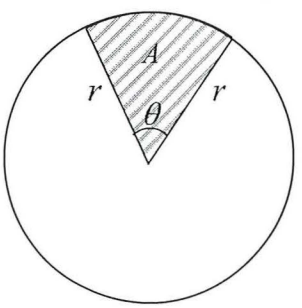
Formula to calculate ARC

1. $s = r\theta$

(if value of angle is given in radian)

2. $s = 2\pi r \times \frac{\theta}{360^\circ}$

(if value of angle is given in degrees)



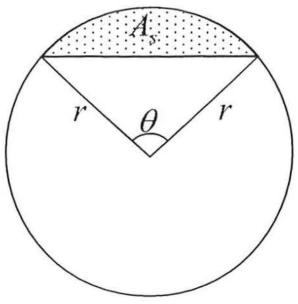
Formula to calculate AREA of SECTOR

1. $A = \frac{1}{2}r^2\theta$

(if value of angle is given in radian)

2. $A = \pi r^2 \times \frac{\theta}{360^\circ}$

(if value of angle is given in degrees)



To get the area of the arc segment, we need to subtract the area of the triangle made up by the circle's centre and the two end points of the arc from the area A.

Area of segment; Angle measured in radians

$$A_s = \left(\frac{1}{2}r^2\theta\right) - \left(\frac{1}{2}r^2 \sin\theta\right)$$

or

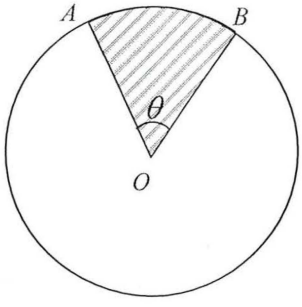
$$A_s = \frac{1}{2}r^2(\theta - \sin\theta)$$

Area of segment: Angle measured in degrees.

$$A_s = \left(\pi r^2 \times \frac{\theta}{360^\circ}\right) - \left(\frac{1}{2}r^2 \sin\theta\right)$$

Example 3.18

In the figure shown on the right, $\theta = \frac{\pi}{5} rad$ and the radius of the circle is $6m$. Find:



a) The length of arc, AB.

b) Area of the sector.

Solution: -

Given that the unit of the angle θ in radian, therefore we can use the formula $s = r\theta$ and $A = \frac{1}{2}r^2\theta$.

a) Length of arc, $AB = r\theta$

$$= 6m \times \frac{\pi}{5} rad$$

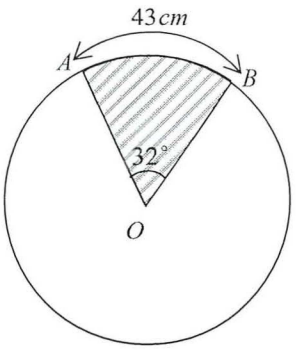
$$= 3.77m$$

b) Area of sector, $A = \frac{1}{2}r^2\theta$

$$= \frac{1}{2} \times (6m)^2 \times \frac{\pi}{5} rad$$

$$= 11.31m^2$$

Example 3.19



Given that the angle $\angle AOB = 32°$ and the length of arc $AB = 43cm$. Find:

a) The diameter of the circle.

b) The area of the shaded region.

Solution: -

Since the unit of the angle is in degree (32°), we may use the formula $s = 2\pi r \times \frac{\theta°}{360°}$ and $A = \pi r^2 \times \frac{\theta°}{360°}$.

a) Given that the length of arc, $AB = 43cm$

Length of arc, $L = 2\pi r \times \frac{\theta°}{360°}$

Therefore,

$$2\pi r \times \frac{\theta°}{360°} = 43cm$$

$$2\pi r \times \frac{32°}{360°} = 43cm$$

$$r = 77cm$$

$\therefore$ Radius, $r = 77cm$

Diameter, $d = 77cm \times 2$

$= 154cm$

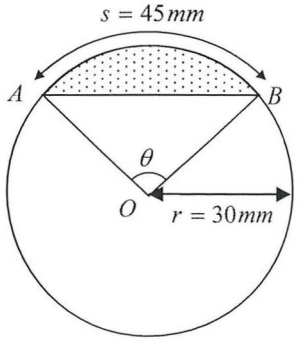
c) Area of shaded region = Area of sector

Area of sector, $A = \pi r^2 \times \frac{\theta°}{360°}$

$= \pi(77cm)^2 \times \frac{32°}{360°}$

$= 1655.7cm^2$

Example 3.20



In the figure shown on the left, θ is an acute angle. Given the arc length, $s = 45mm$, radius of the circle $r = 30mm$ and O is the centre of the circle. Find:

a) Angle, θ.

b) Area of segment.

Solution: -

In this question, we can either use the radians or degrees formula. It's given that the arc length$s = 45mm$. To find the angle θ, we can either use the formula $s = r\theta$ (unit of angle in radian) or $s = 2\pi r \times \frac{\theta^\circ}{360^\circ}$ (unit of angle in degree).

a)

$s = r\theta$

$45mm = 30mm \times \theta$

$\theta = \frac{3}{2} rad$

OR

$s = 2\pi r \times \frac{\theta^\circ}{360^\circ}$

$45mm = 2\pi(30mm) \times \frac{\theta^\circ}{360^\circ}$

$\theta^\circ = 85.9^\circ$

b) If we use the formula, $A_s = \frac{1}{2} r^2(\theta - sin\,\theta)$ to calculate the area of the segment, then we use the answer $\theta = \frac{3}{2} rad$ as the angle's reference.

Area of segment, $A_s = \frac{1}{2} r^2(\theta - sin\,\theta)$

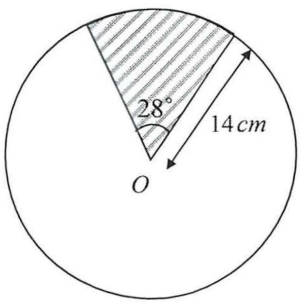
$= \frac{1}{2}(30mm)^2 \left[\frac{3}{2} - sin\frac{3}{2}\right]$

$= \frac{1}{2}(900mm^2)\left[\frac{3}{2} - 0.9975\right]$

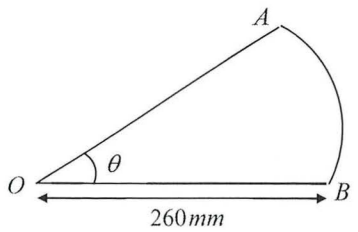
$= 226.13mm^2$

EXERCISE 3.5

1. Calculate the perimeter of the sector and the area of the sector.



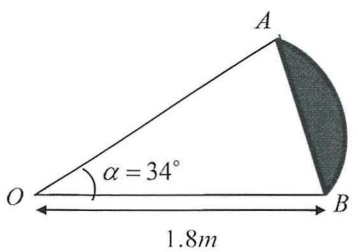
2. Diagram below shows a sector AOB which is centred O. Given that the angle, $\theta = 0.1389\pi$ radians and the radius of the circle is $260mm$. Find:



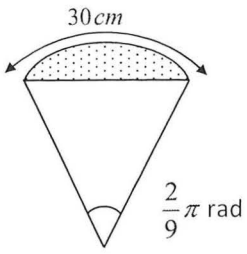
a) The length of arc, AB.

b) Area of the sector.

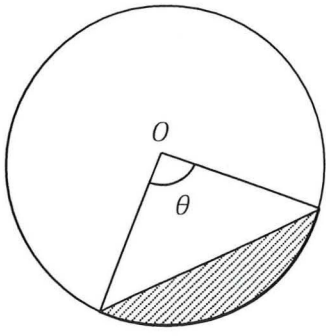
3. AOB is a sector of a circle centred O. Angle, $\alpha = 34°$ and radius, $OB = 1.8m$. Find the perimeter of the shaded region and the area of the shaded region.



4. Figure below shows a sector of a circle. The angle of the minor sector is $\frac{2}{9}\pi$ rad and the length of arc is $30cm$. Calculate the area of the shaded segment.



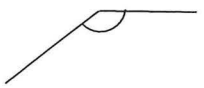
5. The figure shown below has an area of minor sector which is 84 cm^2 and the angle, $\theta = 69^\circ$. Calculate the area of shaded shape.



MASTER YOUR KNOWLEDGE 3

1. Define the following angles either it is right angle, acute angle, obtuse angle, adjacent angle, complementary angle or supplementary angle.

   a. $180° < \theta < 360°$ 

   b. $90° < \theta < 180°$ 

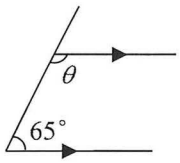
   c. $\angle a + \angle b = 180°$ 

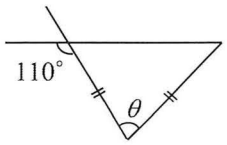
   d. $\theta = 90°$

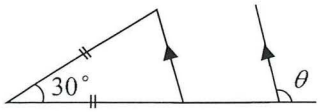
   e. $0° < \theta < 90°$

   f. $x° + y° = 90°$

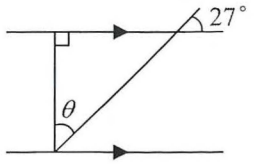
2. Find the value of the angle, θ.

   a. 

   b. 

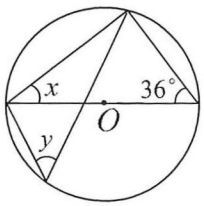
   c. 

d.

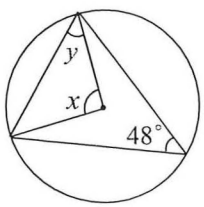


3. Find the angle for x and y in each of the diagram below:

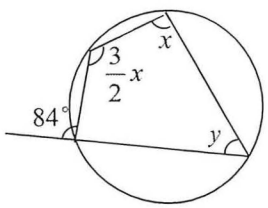
a.



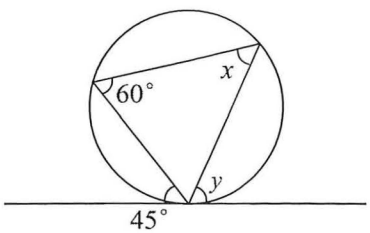
b.



c.

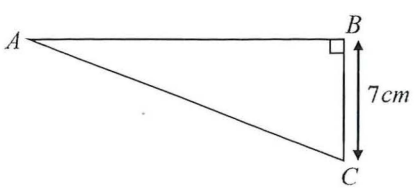


d.

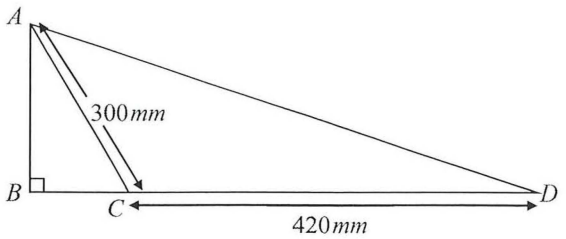


4. Solve the Pythagorean Theorem below:

a. In the diagram below, $BC = 7cm$ and $AC = 2BC$. Find the length of AB.

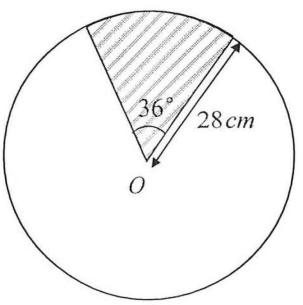


b. In the diagram shown below, given that $AC = 300mm$ and $CD = 420mm$. If $BD = 4BC$, find the length of AB and AD.

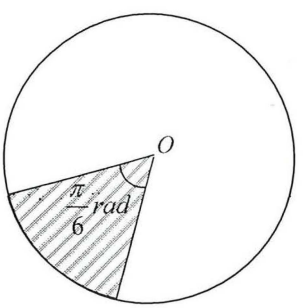


5. Solve the following problems for arc length, and area of a sector and segment.

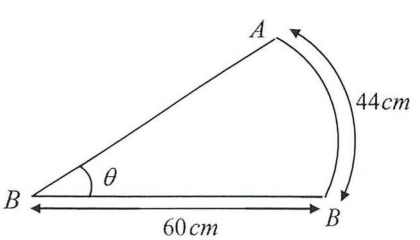
a. In the picture shown below, O is the centre of the circle. The radius of the circle is $28cm$ and the angle of the minor sector is $36°$. Find the arc length of the minor sector and the area of the major sector.



b. Given that O is the centre of the circle and the angle is $\frac{\pi}{6}$ radian. If the area of the shaded region is $4.52m^2$, find the diameter of the circle.



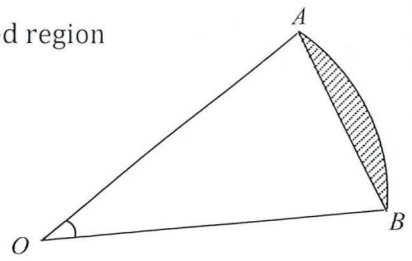
c. In the picture shown below, O is the centre of a circle. Given the arc length is $44cm$ and the radius is $60cm$. Find the value of θ then calculate the area of the sector.



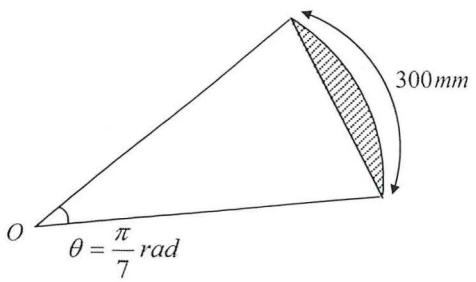
d. In the picture shown below, O is the centre of a circle. Given that the angle of AOB is $36°$ and $AO = BA = 16.8cm$. Find:

i. the perimeter of the shaded region

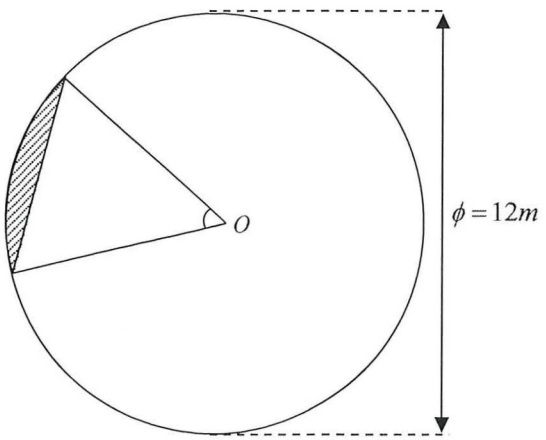
ii. the area of the shaded region



e. In the picture shown below, O is the centre of a circle. The angle of the sector, $\theta = \frac{\pi}{7}$rad and the length of arc is $300mm$. Find the radius of the circle then calculate the area of the segment.



f. In the picture shown below. If the diameter of the circle is $12m$ and the area of the minor sector is $25m^2$, find the area of the shaded region.



CHAPTER FOUR

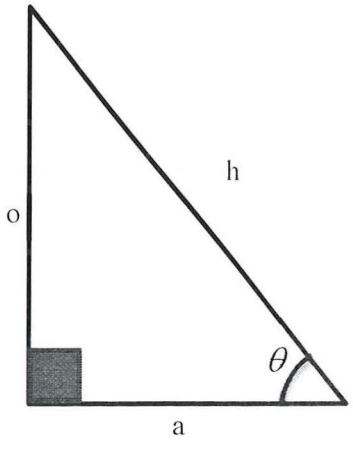
TRIGONOMETRY

4.1 FUNDAMENTAL OF TRIGONOMETRY FUNCTIONS

What is trigonometry?

Trigonometry is a branch of mathematics that studies the relationships between lengths and angles of triangles especially right-angled triangle.
Trigonometric ratio and Pythagorean Theorem will frequently be used in its problem solving.

Right-angled triangle



label	definition
O	Opposite side - This side always opposite to the relevant angle.
A	Adjacent side - This side always closes to the relevant angle. - Bear in your mind that the longest side of right-angles triangle is hypotenuse. - Therefore, adjacent side is always in between hypotenuse and opposite side.
H	Hypotenuse side - This is the longest side of right-angled triangle and it is opposite to the right-angle.

Pythagorean Theorem is only used to find the length of side. This formula can only be used if 2 sides of the triangle are given. Bear in your mind that, Pythagorean Theorem only applied to right-angled triangle only.

i. To find length of hypotenuse

$$h^2 = o^2 + a^2$$
$$h = \sqrt{o^2 + a^2}$$

ii. To find length of opposite side

$$o^2 = h^2 - a^2$$
$$o = \sqrt{h^2 - a^2}$$

iii. To find length of adjacent side

$$a^2 = h^2 - o^2$$
$$a = \sqrt{h^2 - o^2}$$

Example 4.1

Given **ABC** is a right-angle triangle with **B** is a hypotenuse. Find the value of **C** if **A** = 12cm and **B** is 16cm.

Solution: -

You may assume **C** as opposite side or adjacent side. This question just wants you to find the length. So, you may use either second or third formula.

$$C^2 = B^2 - A^2$$
$$C = \sqrt{16^2 - 12^2}$$
$$C = \sqrt{112}$$

So, the value of $C = \sqrt{112}$or 10.583cm.

Since the surd 112 give you a value with decimal point, the best answer for this question is in surd form.

Example 4.2

PQR is a right-angle triangle. Given the adjacent side is 8cm and opposite side is 6cm. Find the value of hypotenuse side.

Solution: -

To find the value of hypotenuse, use the first formula.

$$h^2 = o^2 + a^2$$
$$h = \sqrt{6^2 + 8^2}$$
$$h = \sqrt{100}$$
$$h = 10cm$$

For this question, you cannot state your answer by surd 100, because the value of surd 100 is 10cm. So, your final answer is 10cm.

EXERCISE 4.1

1. Given XYZ is a right-angle triangle with Y is a hypotenuse. Find the value of X is given that Y = 10cm and Z = 7cm.

2. Find the hypotenuse of a right-angle triangle if given that its adjacent side is 17cm and its opposite side is 12cm.

3. A right-angle triangle has a hypotenuse triple times its adjacent. If hypotenuse = 12cm, find its opposite side. Later, calculate the perimeter of the triangle.

4. A right-angle triangle was formed in order to meet the measurement of its adjacent must double its opposite side, and its hypotenuse must double its opposite. Find the value of its opposite if given that its hypotenuse is 57.12cm.

4.2 TRIGONOMETRY FUNCTIONS

Trigonometry ratios cannot exist without its angle. It is totally wrong to write sin θ as sin only.

There are three (3) main trigonometry ratios: -

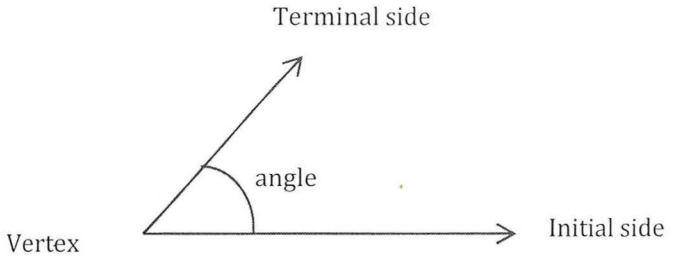
i. Sine
ii. Cosine
iii. Tangent

While there are more three (3) additional trigonometry ratios can be derived from the first three main trigonometry ratios, that are: -

i. **Secant** - derived from $\frac{1}{\text{cosine}}$
ii. **Cosecant** - derived from $\frac{1}{\text{sine}}$
iii. **Cotangent** - derived from $\frac{1}{\text{tangent}}$

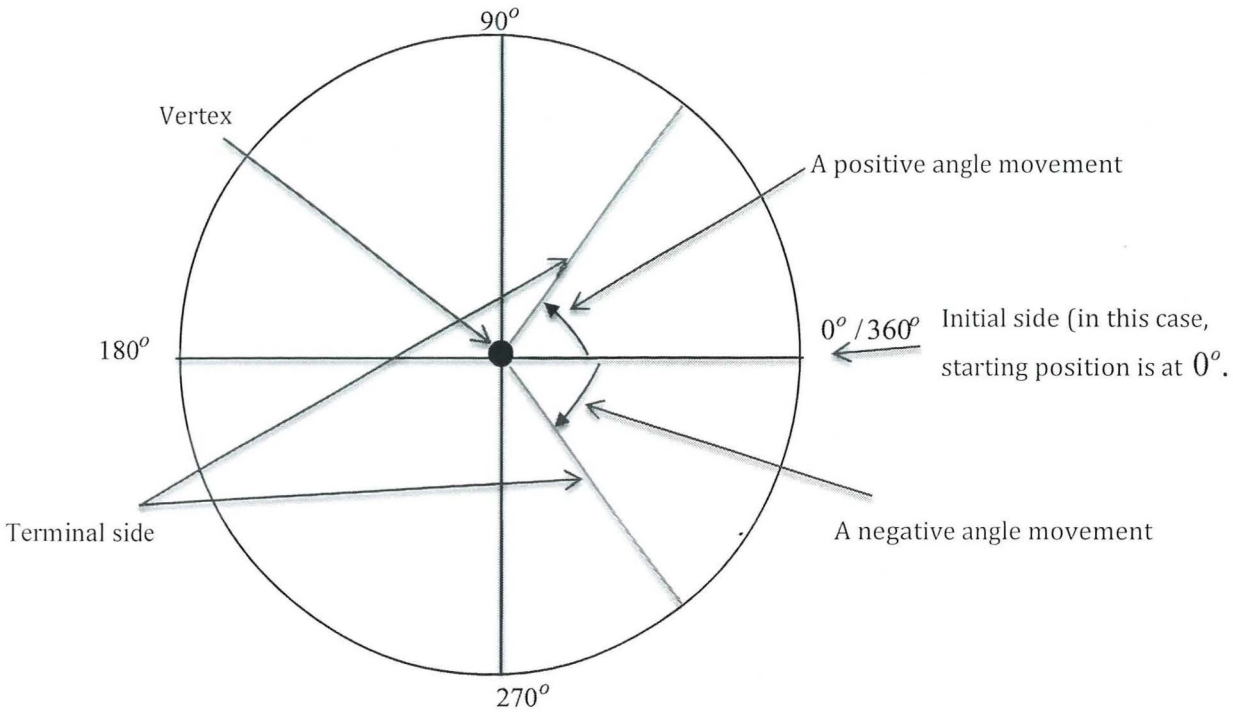
What is angle?

An angle is the figure formed by two (2) rays, called the sides of the angle, sharing a common endpoint, called vertex of the angle.



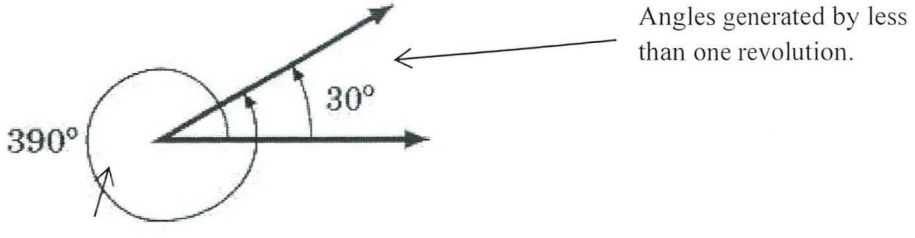
The starting position of the ray is called **Initial Side** of the angle, while the final position is called **Terminal Side** of the angle, and the point at which the initial and terminal sides meet is called the **vertex**.

Angles are considered to be **positive** if generated *anti clockwise* and **negative** if generated *clockwise*.



Angle is measured by degree (α^o), radian (αrad.) and gradian. However, the most frequently used are degree and radian. Therefore, please extra careful when using calculator since it would affect the output

Angles in degree are designated with degree symbol, but angles in radians have no units specified. This is standard practice – when no units are specified for an angle, it is understood that the units are radians.

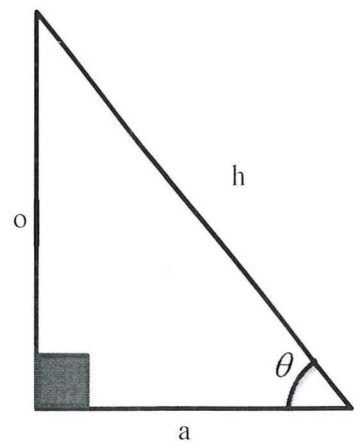


Angles generated by more than one revolution.

A complete rotation or revolution of an angle is 360^o or if in radian is 2π. An angle can has more than one rotation and it has no limit.

Sine, Cosine, Tangent, Secant, Cosecant and Cotangent.

There are six trigonometric functions; *sine, cosine, tangent, secant, cosecant and cotangent.* All these six trigonometric functions for a positive acute angle θ can be defined as ratios of the sides of a right triangle.



$sin\,\theta = \frac{\text{opposite side to } \theta}{\text{hypotenuse, h}}$	$sin\,\theta = \frac{o}{h}$
$cos\,\theta = \frac{\text{adjacent side to } \theta}{\text{hypotenuse, h}}$	$cos\,\theta = \frac{a}{h}$
$tan\,\theta = \frac{\text{opposite side to } \theta}{\text{adjacent side to } \theta}$	$tan\,\theta = \frac{o}{a}$
$csc\,\theta = \frac{1}{\sin\theta}$ $= \frac{1}{o/h}$	$csc\,\theta = \frac{h}{o}$
$sec\,\theta = \frac{1}{cos\,\theta}$ $= \frac{1}{a/h}$	$sec\,\theta = \frac{h}{a}$
$cot\,\theta = \frac{1}{tan\,\theta}$ $= \frac{1}{o/a}$	$cot\,\theta = \frac{a}{o}$

Table 4.1: Trigonometric functions.

Tips to remember: -

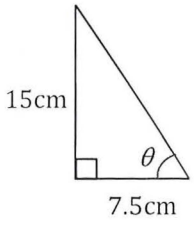
i. The ***hypotenuse side*** is the largest side of the right-angled triangle and it is always opposite to the right angle.

ii. The ***adjacent side*** is the side that is always adjacent to the given acute angle.

iii. The ***opposite angle*** is the side that is always opposite to the given acute angle.

Example 4.3

According to the following right-angled triangle, find the value for each of the trigonometry ratio.



Solution: -

Find the hypotenuse; $h^2 = o^2 + a^2$

$$h = \sqrt{o^2 + a^2}$$
$$= \sqrt{15^2 + 7.5^2}$$
$$= \sqrt{281.25}$$
$$= 16.75cm$$

Trigonometry ratio: -

i. $sin\,\theta$

$$sin\,\theta = \frac{o}{h}$$
$$= \frac{15}{16.75}$$
$$= 0.8955$$

ii. $cos\,\theta$

$$cos\,\theta = \frac{a}{h}$$
$$= \frac{7.5}{16.75}$$
$$= 0.4478$$

iii. $tan\,\theta$

$$tan\,\theta = \frac{o}{a}$$
$$= \frac{15}{7.5}$$
$$= 2$$

iv. $csc\,\theta$

$$csc\,\theta = \frac{1}{sin\,\theta}$$
$$= \frac{1}{0.8955}$$
$$= 1.1167$$

v. $sec\,\theta$

$$sec\,\theta = \frac{1}{cos\,\theta}$$
$$= \frac{1}{0.4478}$$
$$= 2.2331$$

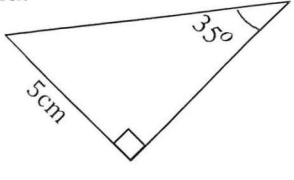
vi. $cot\,\theta$

$$cot\,\theta = \frac{1}{tan\,\theta}$$
$$= \frac{1}{2}$$

Example 4.4

According to the following right-angled triangle, find: -

i. Adjacent side
ii. Hypotenuse side



Solution:

i. Adjacent side

$$\tan\theta = \frac{o}{a}$$

$$\tan 35^o = \frac{5}{a}$$

$$a \times \tan 35^o = 5$$

$$a = \frac{5}{\tan 35^o}$$

$$a = 7.141cm$$

ii. Hypotenuse side

$$h = \sqrt{o^2 + a^2}$$

$$= \sqrt{5^2 + 10.55^2}$$

$$= \sqrt{136.30}$$

$$= 11.67cm$$

Example 4.5

Given that $\sin\theta = \frac{5}{9}$, find the value for $\cot\theta$ and $\sec\theta$. Later, calculate the value for θ.

Solution: -

From $\sin\theta = \frac{5}{9}$, we know that the *opposite side is 5cm* and *hypotenuse side is 9cm*. So, now we need to find the adjacent side by using Pythagorean Theorem.

$$a = \sqrt{h^2 - o^2}$$

$$= \sqrt{9^2 - 5^2}$$

$$= \sqrt{56}$$

$$= 7.48cm$$

Therefore

$$\cot\theta = \frac{1}{\tan\theta}$$

$$= \frac{1}{o/a}$$

$$= \frac{1}{5/7.48}$$

$$= \frac{7.48}{5}$$

$$\approx 1.496$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$= \frac{1}{a/h}$$

$$= \frac{1}{7.48/9}$$

$$= \frac{9}{7.48}$$

$$\approx 1.2032$$

Or alternative method

$$\sin\theta = \frac{5}{9}$$

$$\theta = \sin^{-1}\left(\frac{5}{9}\right)$$

$$= 33.75^o$$

Therefore,

$$\cot\theta = \frac{1}{\tan\theta}$$

$$= \frac{1}{\tan 33.75^o}$$

$$= \frac{1}{7.48/5}$$

$$\approx 1.4966$$

Example 4.6

Given that $cos\ ec\theta = \frac{11}{5}$, find the value for each of the trigonometric ratio.

Solution: -

$$cos\ ec\theta = \frac{11}{5}$$

$$cos\ ec\theta = \frac{1}{sin\ \theta}$$

$$\therefore \frac{1}{sin\ \theta} = \frac{11}{5}$$

Cross multiply;

$$5 = 11 \times sin\ \theta$$

$$sin\ \theta = \frac{5}{11}$$

So, we have *opposite side = 5*, and *hypotenuse side = 11*. Now, find the value of adjacent side.

$$a = \sqrt{h^2 - o^2}$$
$$= \sqrt{11^2 - 5^2}$$
$$= \sqrt{96}$$
$$= 9.80$$

There are **SIX trigonometric ratios**; *sine, cosine, tangent, secant, cosecant and cotangent.* Therefore, your answer should as follow: -

i. $sin\ \theta = \frac{5}{11}$

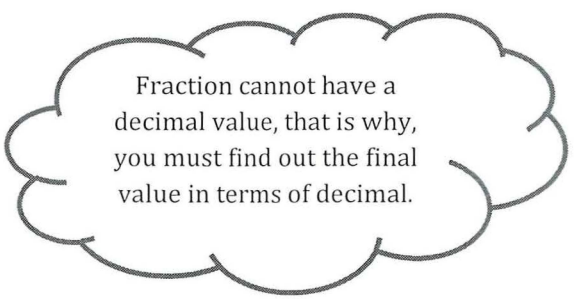
ii. $cos\ \theta = \frac{9.80}{11}$
$= 0.8909$

iii. $tan\ \theta = \frac{5}{9.80}$
$= 0.5102$

iv. $cos\ ec\theta = \frac{11}{5}$

v. $cot\ \theta = \frac{1}{0.5102}$
$= 1.96$

vi. $sec\ \theta = \frac{1}{cos\ \theta}$
$= \frac{1}{0.8909}$
$= 1.1225$



Example 4.7

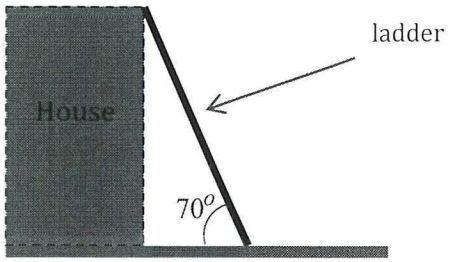
A 15-meter ladder leans against a house and make an angle of 70^o with level ground.

i. Find the distance from the top of the ladder to the ground?

ii. Find the distance of the foot of the ladder to the house?

Express your answer to the nearest meter with 3 decimal places.

Solution: -



From the information given by the question, we can illustrate as above.

So, the ladder becomes the hypotenuse side of the triangle.

i. Find the distance from the top of the ladder to the ground?

$$sin\,\theta = \frac{o}{h}$$

$$sin\,70^o = \frac{o}{15}$$

$$o = sin\,70^o \times 15$$

$$o = 14.095 metre$$

ii. Find the distance of the foot of the ladder to the house?

$$cos\,\theta = \frac{a}{h}$$

$$cos\,70^o = \frac{a}{15}$$

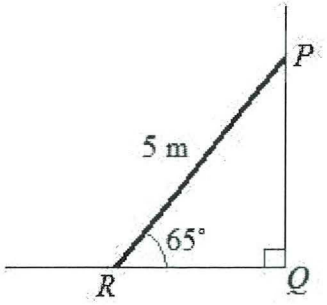
$$a = cos\,70^o \times 15$$

$$a = 5.130 metre$$

Example 4.8

A ladder 5-meter-long, leaning against a vertical wall makes an angle of 65° with the ground.

Find: -



a) The height that the ladder reaches is PQ.

b) The distance of the foot of the ladder from the wall is RQ.

c) The angle that the ladder makes with the wall i.e. angle P.

Solution: -

a) The height that the ladder reach is PQ

$$\sin 65^o = \frac{PQ}{5}$$
$$PQ = \sin 65^o \times 5$$
$$PQ = 4.53m$$

b) The distance of the foot of the ladder from the wall is RQ.

$$\cos 65^o = \frac{RQ}{5}$$
$$RQ = \cos 65^o \times 5$$
$$RQ = 2.11m$$

c) The angle that the ladder makes with the wall i.e. angle P.

$$\angle P = 180^o - 65^o - 90^o$$
$$\angle P = 25^o$$

Example 4.9

Given that $\sin A = \frac{1}{2}$; $90^o < A < 180^o$ and $\cos B = \frac{1}{2}$; $180^o < B < 360^o$. Compute the value of $\tan(B - A)$

Solution: -

$$\sin A = \frac{1}{2}$$
$$A = 30^o$$
$$Quadrant\ \ II: -$$
$$A = 180^o - 30^o$$
$$A = 150^o$$

$$\cos B = \frac{1}{2}$$
$$B = 60^o$$
$$\cos\ \ positive\ \ in\ \ Quadrant\ \ IV$$
$$B = 360^o - 60^o$$
$$B = 300^o$$

$$\tan(B - A^o) = \tan(300^o - 150^o)$$
$$= \tan 150^o$$
$$= -0.5774$$

EXERCISE 4.2

1. Given that a right-angled triangle with adjacent side is 7cm and hypotenuse is 19cm. According to the triangle, find the values for each of trigonometric ratio and its angle.

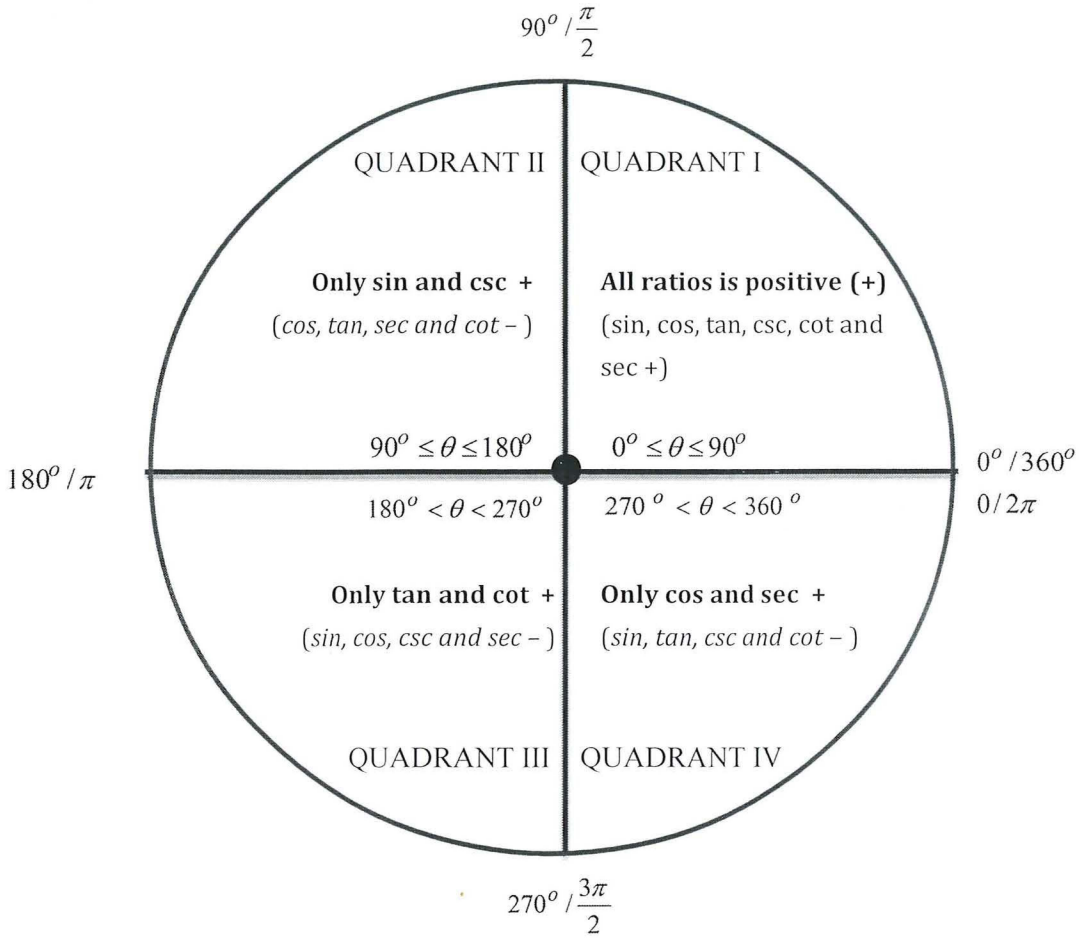
2. Given that a right-angled triangle with an angle of $70^o$and its opposite side is 16cm. Calculate:
   a. The value of adjacent side
   b. The value of hypotenuse side
   c. $csc\ \theta$
   d. $cot\ \theta$
   e. $sec\ \theta$

3. Given that an acute angle of a right-angled triangle is 50^o and its opposite is 8cm, find all the trigonometric ratio for the angle.

4. Ahmad is standing 50 metre from a tower. The elevation angle is 35^o. Ahmad's height is 2metre. Calculate the height of the building. [Answer = 37.01metre].

4.3 IDENTIFYING TRIGONOMETRIC RATIO BY USING QUADRANT

It is very important to know in which quadrant the terminal side of an angle exist, in order to determine either the value of trigonometry ratio for the angle is positive or negative.



Roughly,

i. **First Quadrant** – All trigonometric ratios are positive.
ii. **Second Quadrant** – Only sine is positive and also cosecant because cosecant is generated by using sine function.
iii. **Third Quadrant** – Only tangent and cotangent are positive.
iv. **Fourth Quadrant** - Only cosine and secant are positive.

The trigonometric ratios of negative angles $(-\theta)$ are:-

1. $sin(-\theta) = -sin\,\theta$
2. $cos(-\theta) = cos\,\theta$
3. $tan(-\theta) = -tan\,\theta$

Negative angles mean the movement of the angle is clockwise rotation.

Reference Angle

Reference angle is an acute angle. Acute angle is angle which its value is $0^o \leq \theta \leq 90^o$.

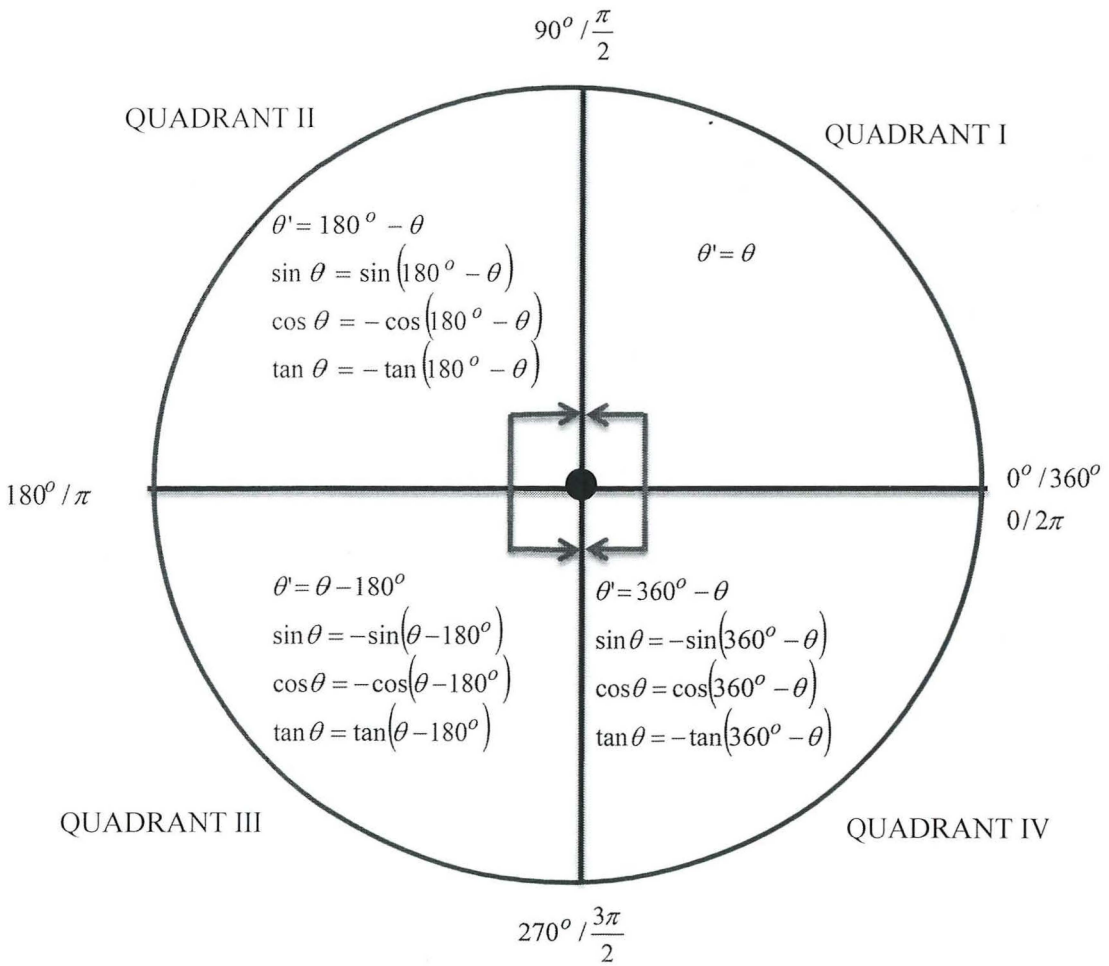
So, what is the value of reference angle?
Yes, value of reference angle is $0^o \leq \theta \leq 90^o$.

Tips: -

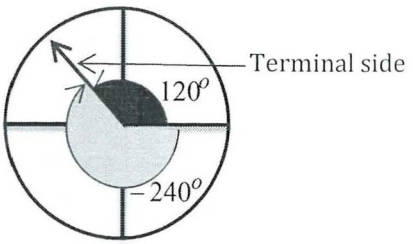
i. The initial side always start with 0^o

ii. The reference angles always exist at x-axis, not at y-axis.

iii. The values of reference angle cannot exceed 90^o and cannot less than 0^o . Reference angle cannot be a negative angle.



Coterminal Angle



If two angles have the same terminal side, so they are called coterminal angle. **BUT**, it must meet the condition that one angle is clockwise rotation and another one is anti-clockwise.

We always used coterminal angle in solving the reference angle when the angle's value is negative.

Example 4.10

For each of the following trigonometric ratio, complete the following: -

a. First, determine in which quadrant the terminal side may exist,
b. Second, state either the trigonometric ratio is positive or negative, and
c. Third, find the value of its reference angle.

i. $sin\,85^o$
ii. $cos\,185^o$
iii. $tan\,325^o$
iv. $cos\,ec285^o$
v. $cot\,185^o$
vi. $sin(-100^o)$
vii. $sec(-220^o)$
viii. $sin\,885^o$

Solution: -

i. $sin\,85^o$

The terminal side is in Quadrant I
All trigonometric ratios are positive in Quadrant I.
So, sine is positive.

Therefore, *Reference angle = given angle,*
Reference angle: -

$$sin\,85^o = +sin\,85^o$$

ii. $cos\,185^o$

The terminal side is in Quadrant III
Cosine is negative in Quadrant III.

Reference angle;

$$cos\,185^o = -cos(185^o - 180^o)$$
$$= -cos5^o$$

iii. $tan\,325^o$

The terminal side is in Quadrant IV
Tangent is negative in Quadrant IV.

Reference angle;

$$tan\,325^o = -tan(360^o - 325^o)$$
$$= -tan\,35^o$$

vi. $sec(-220^o)$

Find its coterminal angle.

$$sec(-220^o) = sec(360^o - 220^o)$$
$$= sec\,140^o$$

The terminal side is in Quadrant II
Cos is negative in Quadrant II.

Reference angle: -

$$sec\,140^o = -sec(180^o - 140^o)$$
$$= -sec\,40^o$$

iv. $cos\ e\ c285^o$

The terminal side is in Quadrant IV
Cosecant is negative in Quadrant IV, according to the sine.

Reference angle: -

$$csc\ 2\,85^o = -csc(360^o - 285^o)$$
$$= -csc\ 7\,5^o$$

v. $cot\ 1\,85^o$

The terminal side is in Quadrant III
Cotangent is positive in Quadrant III, according to the tangent.

Reference angle: -

$$cot\ 1\,85^o = cot(185^o - 180^o)$$
$$= cot 5^o$$

vii. $sin(-100^o)$

For negative angle, first of all, find its coterminal angle.

$$sin(-100^o) = sin(360^o - 100^o)$$
$$= sin\ 260^o$$

The terminal side is in Quadrant III
Sine is negative in Quadrant III.

Reference angle: -

$$sin\ 2\,60^o = -sin(260^o - 180^o)$$
$$= -sin\ 8\,0^o$$

viii. $sin\ 8\,85^o$

Find the angle within range $0^o \leq \theta \leq 360^o$.

$$sin\ 8\,85^o = sin(885^o - 360^o - 360^o)$$
$$= sin\ 1\,65^o$$

The terminal side is in Quadrant II
Sine is positive in Quadrant II.

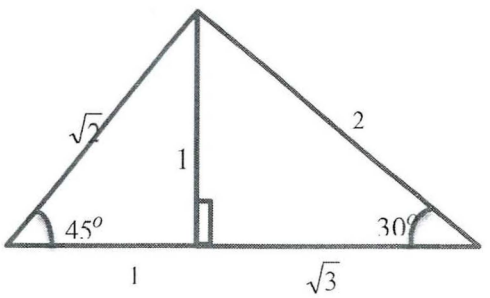
Reference angle: -

$$sin\ 1\,65^o = sin(180^o - 165^o)$$
$$= sin\ 1\,5^o$$

Special Angle

There are three (3) values of angle that being identified as special angle. The values are: - $30^o, 45^o$ and 60^o.

The special angles related problems may be solved by using below triangle. All these values must be memorized by the students. So, the triangle below will help students a lot in memorizing.



$sin\ 45^o = \frac{1}{\sqrt{2}}$ $csc\ 45^o = \frac{\sqrt{2}}{1} = \sqrt{2}$	$cos\ 45^o = \frac{1}{\sqrt{2}}$ $sec\ 45^o = \frac{\sqrt{2}}{1} = \sqrt{2}$	$tan\ 45^o = \frac{1}{1} = 1$ $cot\ 45^o = \frac{1}{1} = 1$
$sin\ 30^o = \frac{1}{2}$ $csc\ 30^o = \frac{2}{1} = 2$	$cos\ 30^o = \frac{\sqrt{3}}{2}$ $sec\ 30^o = \frac{2}{\sqrt{3}}$	$tan\ 30^o = \frac{1}{\sqrt{3}}$ $cot\ 30^o = \frac{\sqrt{3}}{1} = \sqrt{3}$
$sin\ 60^o = \frac{\sqrt{3}}{2}$ $csc\ 60^o = \frac{2}{\sqrt{3}}$	$cos\ 60^o = \frac{1}{2}$ $sec\ 60^o = \frac{2}{1} = 2$	$tan\ 60^o = \frac{\sqrt{3}}{1} = \sqrt{3}$ $cot\ 60^o = \frac{1}{\sqrt{3}}$

Table 4.2: Table of special angle

Tips: -

If you face difficulties in memorizing the value of special angle, you may apply your basic knowledge of trigonometry: -

i. Choose any of trigonometry function that will give you a value of 0.5.Example $sin\ 30^o$ or $cos\ 60^o$.

ii. Convert the 0.5 to fraction; $\frac{1}{2}$.

iii. Draw your right-angled triangle; label your angle and your sides.

iv. For the rest missing sides, use Pythagorean Theorem to calculate and leave your answer in surd form.

v. While for angle 45^o, just memorize $tan\ 45^o = 1$. It is very simple and surely easy.

Example 4.11

Without using calculator, find the value for each of the following: -

i. $sin\,45^o$
ii. $cos\,60^o$
iii. $tan\,30^o$
iv. $csc\,60^o$
v. $cot\,45^o$
vi. $sec\,30^o$

Solution: -

i. $sin\,45^o$

$$= \frac{1}{\sqrt{2}}$$

ii. $cos\,60^o$

$$= \frac{1}{2}$$

iii. $tan\,30^o$

$$= \frac{1}{\sqrt{3}}$$

iv. $csc\,60^o$

$$= \frac{1}{sin\,60^o}$$

$$= \frac{1}{\sqrt{3}/_2}$$

$$= \frac{2}{\sqrt{3}}$$

v. $cot\,45^o$

$$= \frac{1}{tan\,45^o}$$

$$= \frac{1}{1/_1}$$

$$= 1$$

vi. $sec\,30^o$

$$= \frac{1}{cos\,30^o}$$

$$= \frac{1}{\sqrt{3}/_2}$$

$$= \frac{2}{\sqrt{3}}$$

Example 4.12

Without using calculator, find the value for each of the following: -

i. $sin\,120^o$
ii. $cos\,315^o$
iii. $tan\,225^o$
iv. $csc\,390^o$
v. $cot\,120^o$
vi. $sec\,225^o$

Solution: -

i. $sin\,120^o$ --> in Quadrant II

$$= sin(180^o - 120^o)$$

$$= sin\,60^o$$

$$= \frac{\sqrt{3}}{2}$$

ii. $cos\,315^o$ --> in Quadrant IV

$$= cos(360^o - 315^o)$$

$$= cos\,45^o$$

$$= \frac{1}{\sqrt{3}}$$

iii. $tan\,2\,25^o$ – –> in Quadrant III

$= tan(225^o - 180^o)$

$= tan\,4\,5^o$

$= 1$

iv. $csc\,3\,90^o$ – –> in Quadrant I

$= \frac{1}{sin\,3\,90^o}$

$= \frac{1}{sin(390^o - 360^o)}$

$= \frac{1}{sin\,3\,0^o}$

$= \frac{1}{1/2} = 2$

v. $cot\,1\,20^o$ – –> in Quadrant II

$= \frac{1}{-tan(180^o - 120^o)}$

$= \frac{1}{-tan\,6\,0^o}$

$= -\frac{1}{\sqrt{3}/1}$

$= -\frac{1}{\sqrt{3}}$

vi. $sec\,2\,25^o$ – –> in Quadrant III

$= \frac{1}{-cos(225^o - 180^o)}$

$= \frac{1}{-cos\,4\,5^o}$

$= \frac{1}{-1/\sqrt{2}}$

$= -\sqrt{2}$

Example 4.13

Without using calculator, solve the following: -.

i. $sin\,8\,55^o$

ii. $tan(-840^o)$

Solution: -

i. $sin\,8\,55^o = sin(855^o - 720^o)$

$= sin\,1\,35^o$-----> in Quadrant II

$= sin(180^o - 135^o)$

$= sin\,4\,5^o$

$= \frac{1}{\sqrt{2}}$

ii. $tan(-840^o) = tan(1080^o - 840^o)$

$= tan\,2\,40^o$---------> in Quadrant III

$= tan(240^o - 180^o)$

$= tan\,6\,0^o$

$= \frac{\sqrt{3}}{1}$

$= \sqrt{3}$

Example 4.14

Without using calculator, find the value of $tan\ 2\ 25^o - sec\left(-\frac{\pi}{6}\right)$. State your answer in form of square roots.

Solution: -

$$tan\ 2\ 25^o - sec\left(-\frac{\pi}{6}\right)$$

$$= tan(225^o - 180^o) - \frac{1}{cos\left(\frac{\pi}{6}\right)}$$

$$= tan\ 4\ 5^o - \frac{1}{cos\ 3\ 0^o}$$

$$= 1 - \frac{1}{\frac{\sqrt{3}}{2}}$$

$$= 1 - \frac{2}{\sqrt{3}} \quad or = \frac{\sqrt{3} - 2}{\sqrt{3}}$$

EXERCISE 4.3

1. Without using calculate, identify the value for each of the following

i. $sin\ 3\ 0^o$	iv. $-csc\ 6\ 0^o$
ii. $tan\ 4\ 5^o$	v. $-cot\ 3\ 0^o$
iii. $-cos\ 6\ 0^o$	vi. $sec\ 4\ 5^o$

2. Calculate the value for each of the following equation without using calculator: -

i. $sin\,3\,0^o + cos\,1\,50^o$

ii. $tan\,2\,25^o - cos\,3\,30^o$

iii. $1 + sec\,3\,90^o$

iv. $cot\,5\,85^o + cos\,5\,10^o$

4.4 DETERMINE VALUE OF TRIGONOMETRIC FUNCTIONS (INCLUDING RADIAN) BY USING CALCULATOR

Tips: -

i. **Determine** the type of angle, either *degree* or *radian*.

ii. Checked your calculator either **D** or **R** is displayed. *D* is representing degree while *R* is representing radian.

iii. If the given angle and your calculator setting is not match, then set your calculator accordingly by **PUSH [MODE]** button four *(4) times* (for fx-570MS model), and then
   a. Push **[1]** button to choose *Deg* (degree setting).
   b. Push **[2]** button to choose *Rad* (Radian setting).

iv. Please double check the display once again, to prevent the mistake.

Example 4.15

Find the value of

i. $tan\,225^o - sin\,270^o$

ii. $cos(-225^o) + csc\,270^o$

iii. $cot\,125^o - sin\,0.5rad$

iv. $sec\,3\pi + tan\frac{\pi}{3}$

Solution: -

i. $tan\,225^o - sin\,270^o$

Ensure your calculator is in Degree mode by ensuring the D is displayed.

Then, used your calculator as usual;

$= 1 - (-1)$

$= 2$

ii. $cos(-225^o) + csc\,270^o$

Ensure your calculator is in Degree mode by ensuring the D is displayed.

Then, used your calculator as usual;

$= cos(-225^o) + \frac{1}{sin\,270^o}$

$= -0.7071 + (-1)$

$= -1.7071$

iii. $cot\,125^o - sin\,0.5rad$

Ensure your calculator mode is accordingly to type of angles.

$= \frac{1}{tan\,125^o} - sin\,0.5rad$

$= -0.7002 - 0.4794$

$= -1.1796$

iv. $sec\,3\pi + tan\frac{\pi}{3}$

Ensure your calculator is in Radian mode by ensuring the R is displayed.

Then, used your calculator as usual;

$= \frac{1}{cos\,3\pi} + tan\frac{\pi}{3}$

$= -1 + 1.7321$

$= 0.7321$

MASTER YOUR KNOWLEDGE 4

1. Find the value of for θ :-
   a. $tan\,\theta = \frac{6}{8}$
   b. $csc\,\theta = \frac{8}{6}$

2. Without using calculator, solve the following: -
   a. $2\,sin\,30^o + 3\,tan\,45^o$
   b. $cos\,60^o \times tan\,45^o$
   c. $\frac{cos\,45^o \times sin\,30^o}{tan\,60^o}$

3. Given $sin\,\theta = \frac{8}{10}$. Find the value of
   a. $tan\,\theta$
   b. $sec\,\theta$

4. Given a right-angled triangle with acute angle 65^o and adjacent side to the angle is 2cm. Find the value of opposite and hypotenuse side.

5. By using calculator, find the value for the followings: -
   a. $cos\,35°$
   b. $tan(-120°)$
   c. $sec\,77°$
   d. $tan\,5.27\ \ rad$
   e. $sin\,330^o$
   f. $tan-870^o$

6. Without using calculator, calculate each of the following: -
   a. $cos\,\theta = -\frac{1}{\sqrt{2}}; for\ \ 0^o \leq \theta \leq 180^o$
   b. $tan\,\theta = -\frac{1}{\sqrt{3}}; for\ \ 180^o \leq \theta \leq 360^o$
   c. $sin\,\theta = \frac{\sqrt{3}}{2}; for\ \ 0^o \leq \theta \leq 360^o$

7. Given that $sin\,A = \frac{1}{2};\ \ 90^o < A < 180^o$ and $cos\,B = \frac{1}{2};\ \ 180^o < B < 360^o$. Compute the value of $tan(A + B - 45^o)$ without using calculator.

8. A 10-meter ladder leans against a house and makes and angle of 67^o with level ground.
   a. How far is the top of the ladder above the ground? And
   b. How far the foot of the ladder to the house?

   Express your answer to the nearest meter with 3 decimal places.

9. Evaluate and simplify: - $2\,cos^2\,45^o + cos^2\,180^o$

10. If $tan\,\theta = \frac{m}{n}$, find the values of $sin\,\theta$ and $cos\,\theta$ where θ is an acute angle and, m and n are positive.

11. From a point 100 metres away from a tower, the angles of elevation of the top and bottom of a vertical flagstaff on the tower are $38^o22'$ and $34^o20'$. Find the height of flagstaff in the nearest meter.

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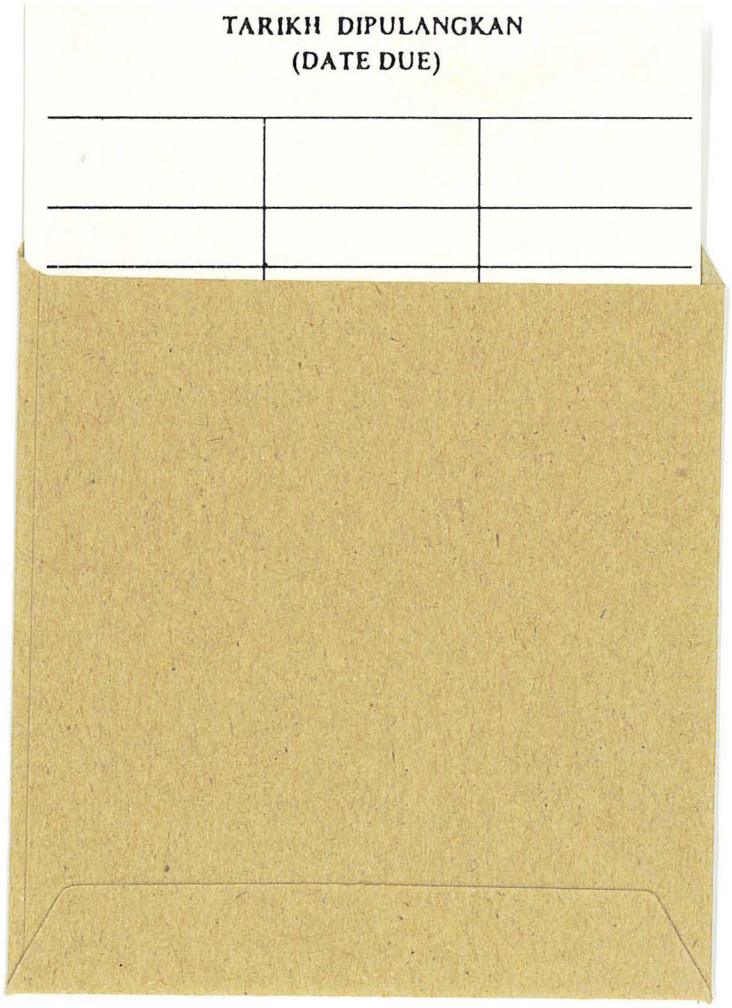
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