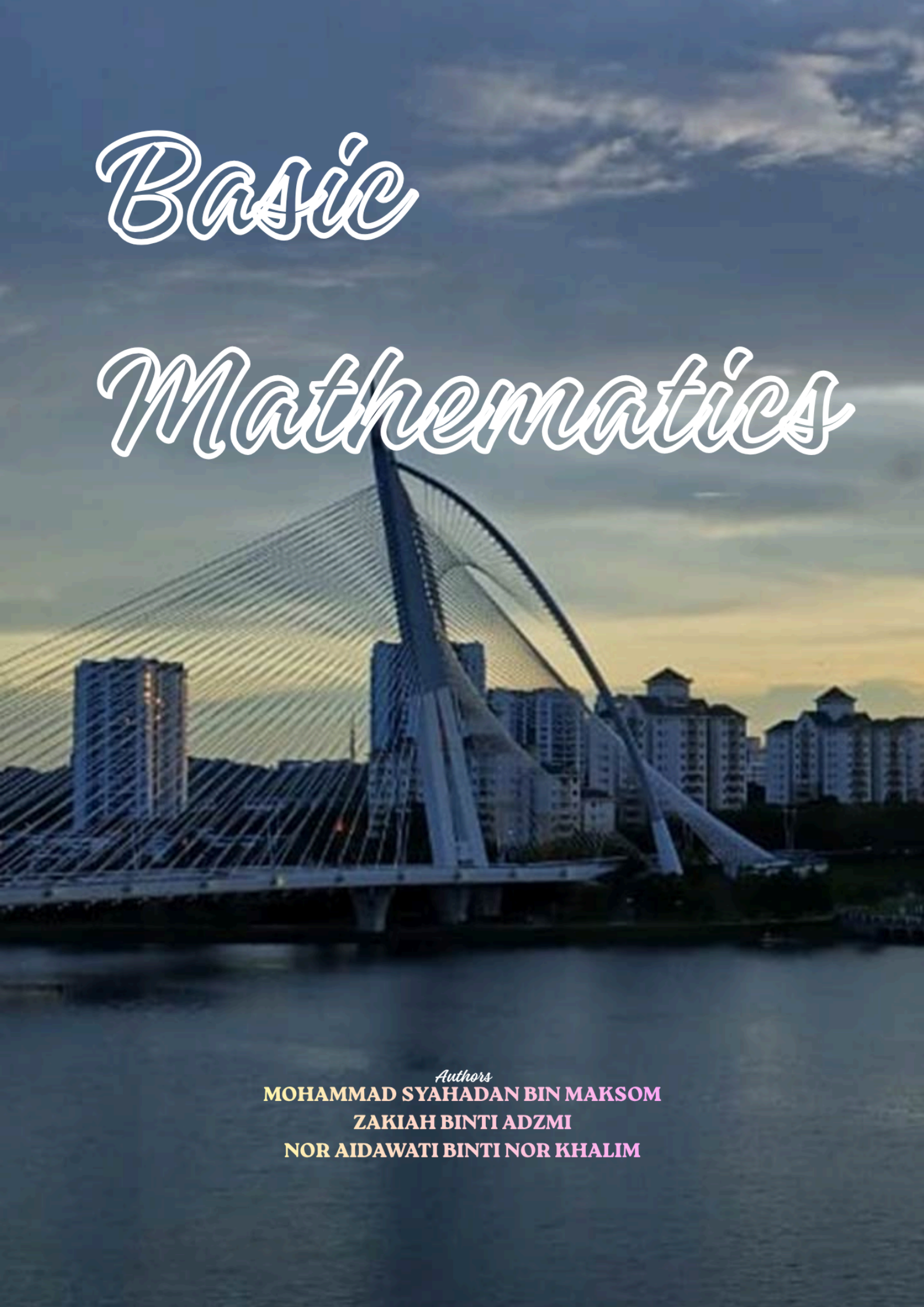


Basic Mathematics

Authors
MOHAMMAD SYAHADAN BIN MAKSOM
ZAKIAH BINTI ADZMI
NOR AIDAWATI BINTI NOR KHALIM

Basic Mathematics

for Polytechnic's Student

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MOHAMMAD SYAHADAN BIN MAKSOM

ZAKIAH BINTI ADZMI

NOR AIDAWATI BINTI NOR KHALIM

Preface

Welcome to **Basic Mathematics**. This textbook has been specifically designed to meet the academic needs of students in **Malaysian Polytechnics**, providing a clear and structured approach to the fundamental mathematical principles required in various engineering and technical diplomas.

Mathematics is often called the language of science and technology. For polytechnic students, mastering this language is not just about passing exams—it is about building the logical foundation necessary to solve real-world technical problems.

What This Book Covers

This book breaks down complex ideas into manageable sections, focusing on four core pillars:

- **Basic Algebra:** The essential "building blocks." We revisit fundamental operations, equations, and inequalities to ensure you have the confidence to manipulate mathematical expressions accurately.
- **Partial Fractions:** A critical tool for calculus and advanced engineering mathematics. You will learn how to decompose complex rational functions into simpler, workable parts.
- **Complex Numbers:** Expanding your horizons beyond real numbers. This section introduces the imaginary unit, polar forms, and their applications in electrical circuits and wave mechanics.
- **Trigonometry:** The study of angles and dimensions. From basic identities to solving trigonometric equations, this chapter is vital for understanding spatial relationships and periodic motions.

Happy Learning!

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CHAPTER 1
ALGEBRA

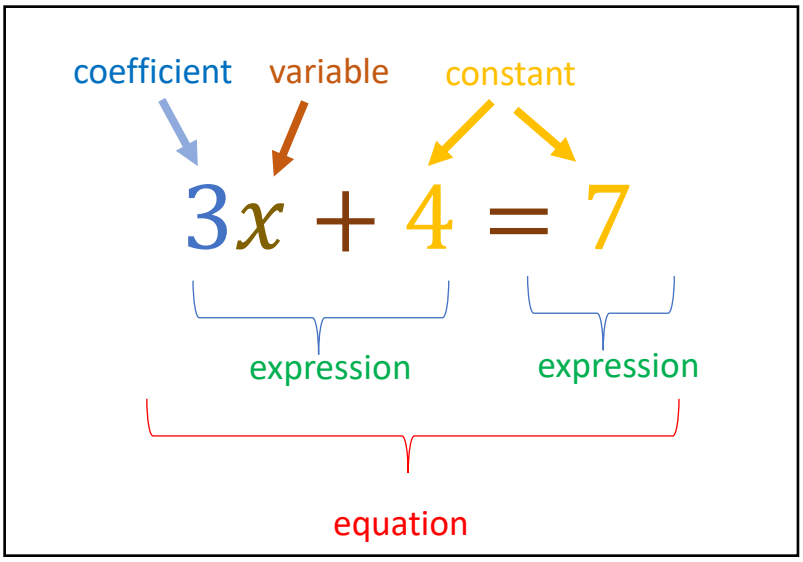


1.0 BASIC ALGEBRA

1.1 SIMPLIFY BASIC ALGEBRA

What is Algebra?

- Algebra is just mathematics and letters.
- Letters (any letters) are called the unknown or the variable. The letter means "we don't know this yet".



- 5s is called an algebraic term.
- Number 5 is known as the coefficient of s.
- Like term (sebutan sama) : 5s, 15s, -2s
- Unlike term (sebutan tak sama) : 5x, 7p, -2m

1.1.1 SIMPLIFY BASIC ALGEBRA

Scan for more info

ALGEBRAIC EXPRESSIONS

REMINDER !!

Rules of Ordering in Mathematics - BODMAS

B	O	D	M	A	S
Brackets (...)	Orders $\sqrt{x}$ x^2	Division ÷	Multiplication ×	Addition +	Subtraction −

Concept of Expanding Brackets and Factorization

$$a\,(x+y) = ax + ay$$

Multiply each term in the bracket by the expression outside

$$(a+b)(x+y) = ax + ay + bx + by$$

Each term in the first bracket has to be multipled by each term in the second bracket.

$$(a+b)^2 = a^2 + 2ab + b^2$$
$$(a-b)^2 = a^2 - 2ab + b^2$$
$$(a+b)(a-b) = a^2 - b^2$$

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Expansion

$$a\,(x+y) = ax + ay$$

Factorization

EXAMPLE

Question	Solution
Simplify : 1. $11x - 5y + 7y - 2x =$	$11x - 5y + 7y - 2x = 11x - 2x - 5y + 7y$ $= 9x + 2y$ *Before adding or subtracting two or more algebraic expressions, combine the like terms.*
2. $4(3p+2) + 7p =$	$4(3p+2) + 7p = 12p + 8 + 7p$ $= 19p + 8$

PRACTICAL EXERCISE

Simplify the following expressions :

1. $5x - (-3x) + 2x$	2. $3n + 2m + n - m$
3. $2 - (x^2 - 2y) - (3x^2 + 2)$	4. $5t - 4s - (-3t) + 2s$
5. $2m + n$	6. $5m^2 + m + 3m^2$
7. $-2s - (3t - s) + 3t^2$	8. $4m - (3n - 4m^2) - 5m$
9. $-2(8 - b^3)$	10. $9x(1 - 2x)$
11. $4(6a^2 - 2a + 5)$	12. $(a^3)^2$
13. $5a^2 \times b$	14. $(a + 1)(6a - 2)$
15. $5(a + 7) + 2(a + 4)$	16. $4(a + 7)^2$

ALGEBRAIC FRACTION

- Algebraic Fractions are fractions with algebraic expressions either on the top (numerator), bottom (denominator) or both.
- If the numerator and denominator have common factors, factorized the expressions completely. Then the fraction can be simplified by cancellation.

Example :

1. $\frac{4st^2}{8s^2t} =$

$= \frac{4\ s\ t\ t}{8\ s\ s\ t}$

$= \frac{4\ \cancel{s}\ \cancel{t}\ t}{8\ \cancel{s}\ s\ \cancel{t}}$ ------- cancelling the common factor

$= \frac{t}{2s}$ ------- simplest form

2. $\frac{x^2-25y^2}{3x-15y}$

$= \frac{(x+5y)(x-5y)}{3(x-5y)}$ ---------factorize the numerator and denominator

$= \frac{(x+5y)(x-\cancel{5y})}{3(x-\cancel{5y})}$ --------- cancelling the common factor

$= \frac{x+5y}{3}$ --------- simplest form

PRACTICAL EXERCISE

Simplify the following expressions :

1. $\frac{7+3x}{5} \times 15$	2. $\frac{2+7x}{3} \times 3$
3. $\frac{x-2}{5} \times 10$	4. $\frac{7-s}{2} \times 4$
5. $\frac{15-3p}{18} \times 3$	6. $\frac{5st-10s}{10t^2-20t}$

7. $\frac{2x+8}{6x+2}$	8. $\frac{16xy^2z}{2xz^3}$
9. $\frac{8x}{16x+24}$	10. $\frac{2x+2}{4x+2}$

1.1.2 SOLVE ALGEBRAIC EXPRESSION

ADDITION AND SUBTRACTION OF ALGEBRAIC FRACTIONS



Scan for more info

SIMPLE RULE IN ADDING FRACTIONS

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$$

SIMPLE RULE IN SUBTRACTING FRACTIONS

$$\frac{a}{b} - \frac{c}{d} = \frac{ad - bc}{bd}$$

(source : https://www.mathsisfun.com/algebra/fractions-algebra.html)

Example :

1. $\frac{2a+1}{5m} + \frac{3a+4}{15m}$

$= \frac{3 \times (2a+1)}{3 \times 5m} + \frac{3a+4}{15m}$ ------- Find a common denominator

$= \frac{6a+3}{15m} + \frac{3a+4}{15m}$

$= \frac{6a+3+3a+4}{15m}$ ------- Write as a single fraction

$= \frac{9a+7}{15m}$ ------- Simplify the numerator

2. $\frac{1}{4v} - \frac{3v-7}{8v^2}$

$= \frac{2v \times 1}{2v \times 4v} - \frac{(3v-7)}{8v^2}$ ------- Be careful of a minus sign before a bracket

$= \frac{2v}{8v^2} - \frac{(3v-7)}{8v^2}$

$= \frac{2v-3v+7}{8v^2}$

$= \frac{-v+7}{8v^2}$

MULTIPLICATION AND DIVISION OF ALGEBRAIC FRACTIONS



MULTIPLYING FRACTIONS

$$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$$

DIVIDING FRACTIONS

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$$

(source : https://www.mathsisfun.com/algebra/fractions-algebra.html)

Example :

1. $\frac{9y}{25x} \times \frac{5}{3y} = \frac{9y \times 5}{25x \times 3y}$ or $\frac{\overset{3}{\cancel{9y}}}{\underset{5}{\cancel{25x}}} \times \frac{\cancel{5}}{\cancel{3y}} = \frac{3}{5x}$

$= \frac{45y}{75xy}$

$= \frac{3}{5x}$

2. $\frac{2ab^2}{a^2-b^2} \times \frac{a+b}{ab} = \frac{2a\cancel{b^2}}{(a-b)\cancel{(a+b)}} \times \frac{\cancel{a+b}}{\cancel{ab}}$

$= \frac{2b}{a-b}$

3. $\frac{3m^3}{n+1} \div \frac{m}{2} = \frac{3m^3}{n+1} \times \frac{2}{m}$

$= \frac{3m^3 \times 2}{(n+1) \times m}$

$= \frac{(3m^3)(2)}{(n+1)(m)}$

$= \frac{6m^2}{n+1}$

PRACTICAL EXERCISE

Simplify the following expressions :

1. $\frac{2x+1}{y} + \frac{3x+2}{2y}$	2. $\frac{9x}{4} - \frac{x^2-7}{12x}$
3. $\frac{2x}{9} - \frac{5x}{6}$	4. $\frac{x+3}{3} \times \frac{9}{3x+9}$
5. $\frac{x+1}{x-4} + \frac{x-2}{x+4}$	6. $\frac{x+4}{x^2-4} \div \frac{x+4}{x-2}$
7. $\frac{x^2+x}{4y} \div \frac{x^2-1}{8y^2}$	8. $\frac{2m}{m+1} + \frac{3}{(m+1)^2}$

9. $\frac{3}{s+1}+\frac{9}{(s+1)(s+3)}$	10. $\frac{4t+3}{12}-\frac{2t+5}{8}$
11. $\frac{2x+1}{4x^2}+\frac{x+7}{16x}$	12. $\frac{x-2}{x+3}\times\frac{x+1}{x-2}$
13. $\frac{x-3}{15}\div\frac{2x-6}{5}$	14. $\frac{x-2}{4}\times\frac{12}{2x-4}$

1.2 SOLVE QUADRATIC EQUATIONS USING FACTORIZATION

- A quadratic equation is an equation that can be rearranged in standard form, where the highest power of the unknown is 2.
- The standard form of quadratic equation :

$$\boldsymbol{ax^2+bx+c=0}$$

$$a, b, c = known\ numbers, where\ a \neq 0$$

$$x = unknown$$

- There are three methods to determine the roots of a quadratic equation :
  a. Factorization
  b. Using the quadratic formula
  c. Completing the square

- Factorization is the reverse process of expanding brackets.

Solve the quadratic equations by factorization.	
Question	Solution
Example : $x^2 - 2x - 15 = 0$	$x^2 - 2x - 15 = 0$ $(x - 5)(x + 3) = 0$ $x - 5 = 0 \qquad x + 3 = 0$ $x = 5 \qquad x = -3$ The roots of the equation are 5 and -3

x	$-$	5	$-5x$
x	$+$	3	$+3x$
x^2		-15	$-2x$

PRACTICAL EXERCISE

Solve the quadratic equations by factorization.

1. $6m^2 - 17m = -12$	2. $6n^2 - 10 = 11n$
3. $x^2 + 2x - 24 = 0$	4. $x^2 - 25 = 0$
5. $2x^2 - 6x = 0$	6. $x^2 = -5x + 36$

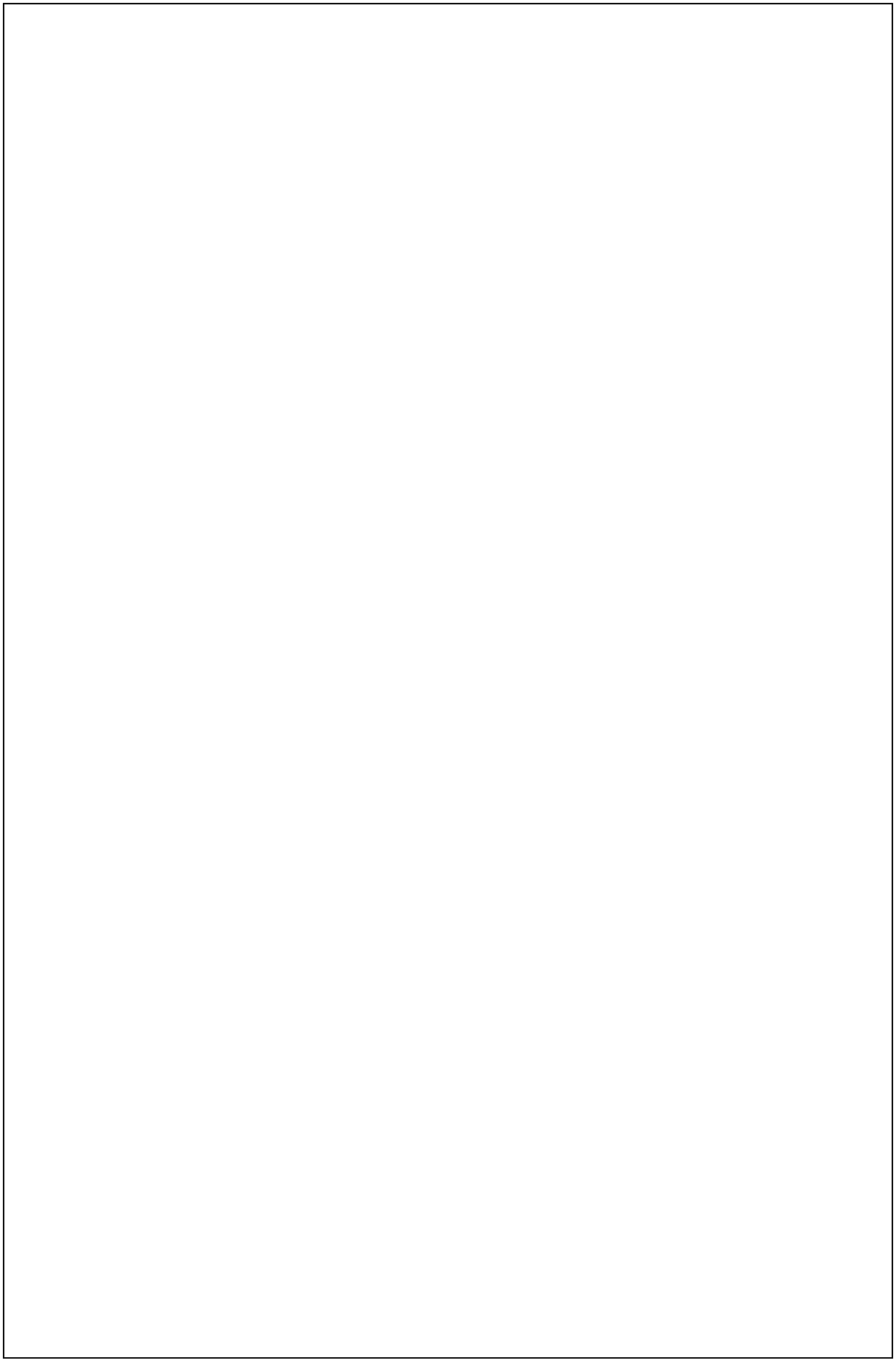
Algebraic Factorisation Exercises 1

Question

a)	x^2-2x+1	**h)**	$9x^2-16(2x-3y)^2$	**o)**	$25k_1^2-625k_2^2$
b)	a^2-1	**i)**	$(x+2y)^2-4(x-y)^2$	**p)**	$-8a^2+16a-8$
c)	y^2+6y+9	**j)**	$36-(a^2-4a)^2$	**q)**	$16c^4-81d^4$
d)	x^2-16	**k)**	$(t-1)^2-(t+1)^2$	**r)**	$1000a^3-1000b^3$
e)	$9x^2-25y^2$	**l)**	u^4-v^4	**s)**	$121f^2+264fg+144g^2$
f)	$36a^2+108ab+81b^2$	**m)**	$27x^3-64y^3$	**t)**	$2a^3+6a^2b+6ab^2+2b^3$
g)	$100-m^2$	**n)**	$(2d)^2-(3c)^2$	**u)**	$8x^3+36x^2y+54xy^2+27y^3$

Answer

a)	$(x-1)^2$	**h)**	$(11x-12y)(12y-5x)$	**o)**	$(5k_1+25k_2)(5k_1-25k_2)$
b)	$(a+1)(a-1)$	**i)**	$3x(4y-x)$	**p)**	$-8(a-1)^2$
c)	$(y+3)^2$	**j)**	$(6-4a+a^2)(6+4a-a^2)$	**q)**	$(4c^2+9d^2)(2c+3d)(2c-3d)$
d)	$(x+4)(x-4)$	**k)**	$-4t$	**r)**	$1,000\,(a-b)\,(a^2+ab+b^2)$
e)	$(3x+5y)(3x-5y)$	**l)**	$(u^2+v^2)(u-v)(u+v)$	**s)**	$(11f+12g)^2$
f)	$(6a+9b)^2$	**m)**	$(3x-4y)(9x^2+12xy+16y^2)$	**t)**	$2(a+b)^3$
g)	$(10+m)(10-m)$	**n)**	$(2d+3c)(2d-3c)$	**u)**	$(2x+3y)^3$



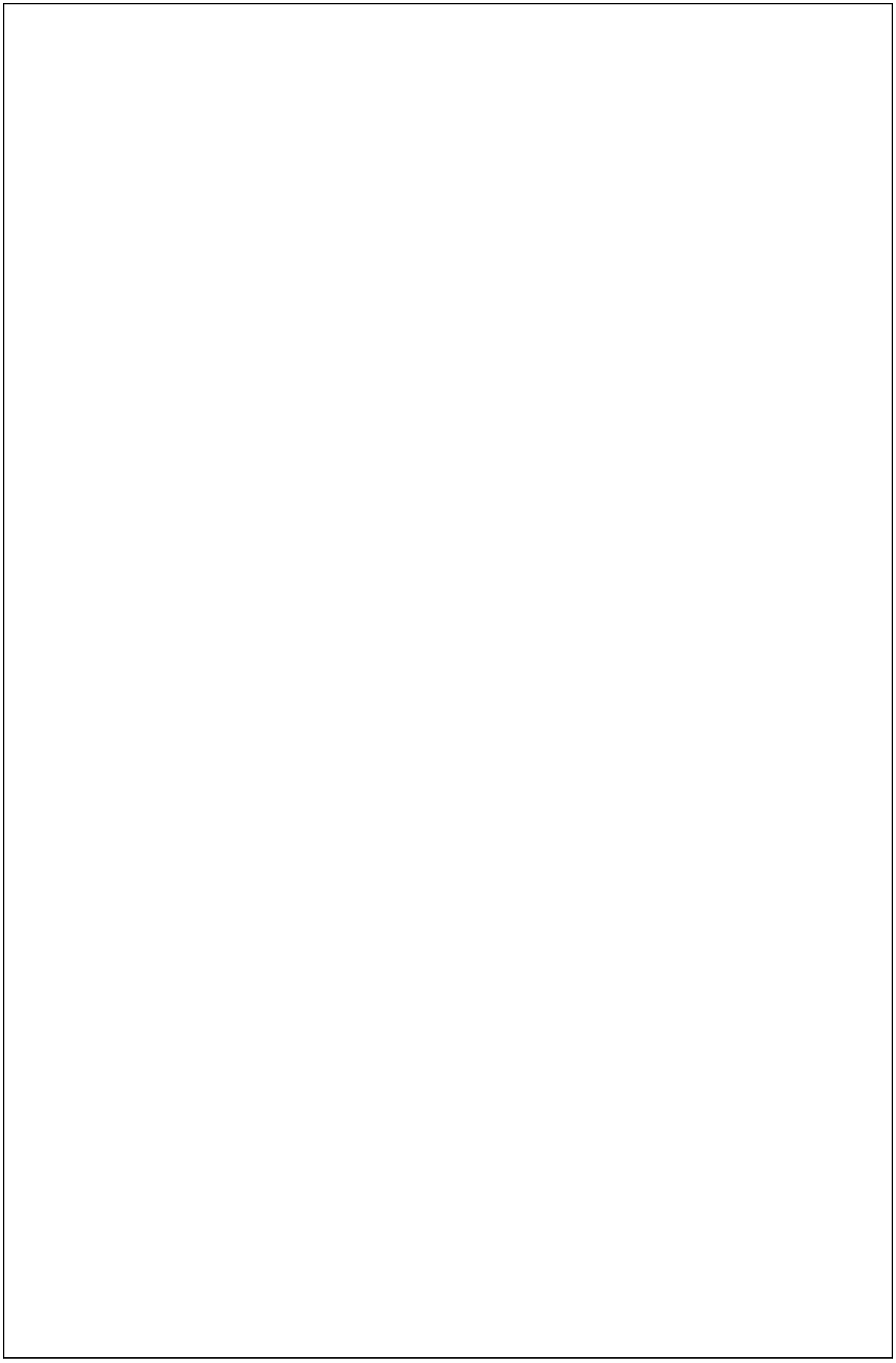
Algebraic Factorisation Exercises 2

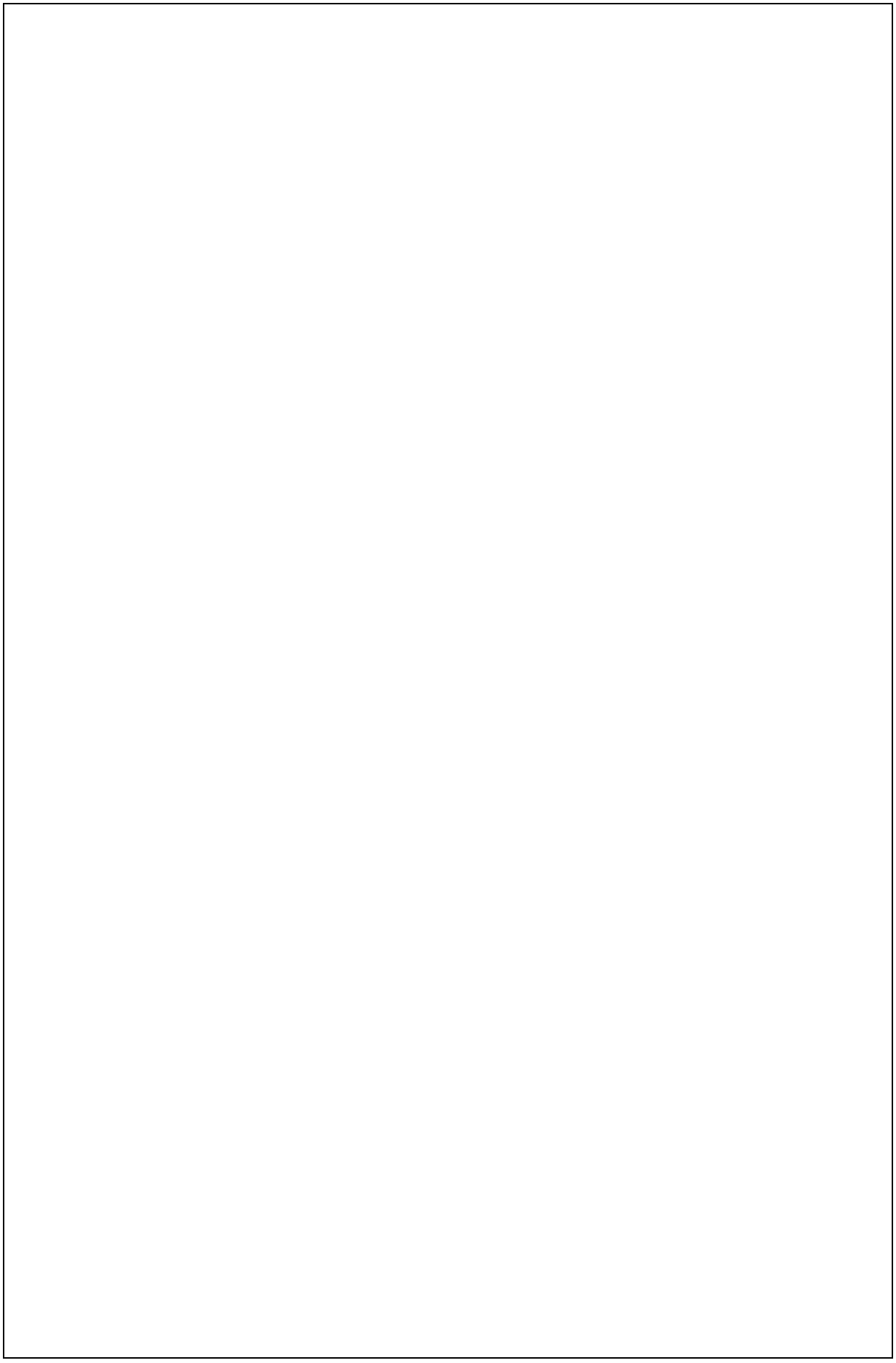
Question

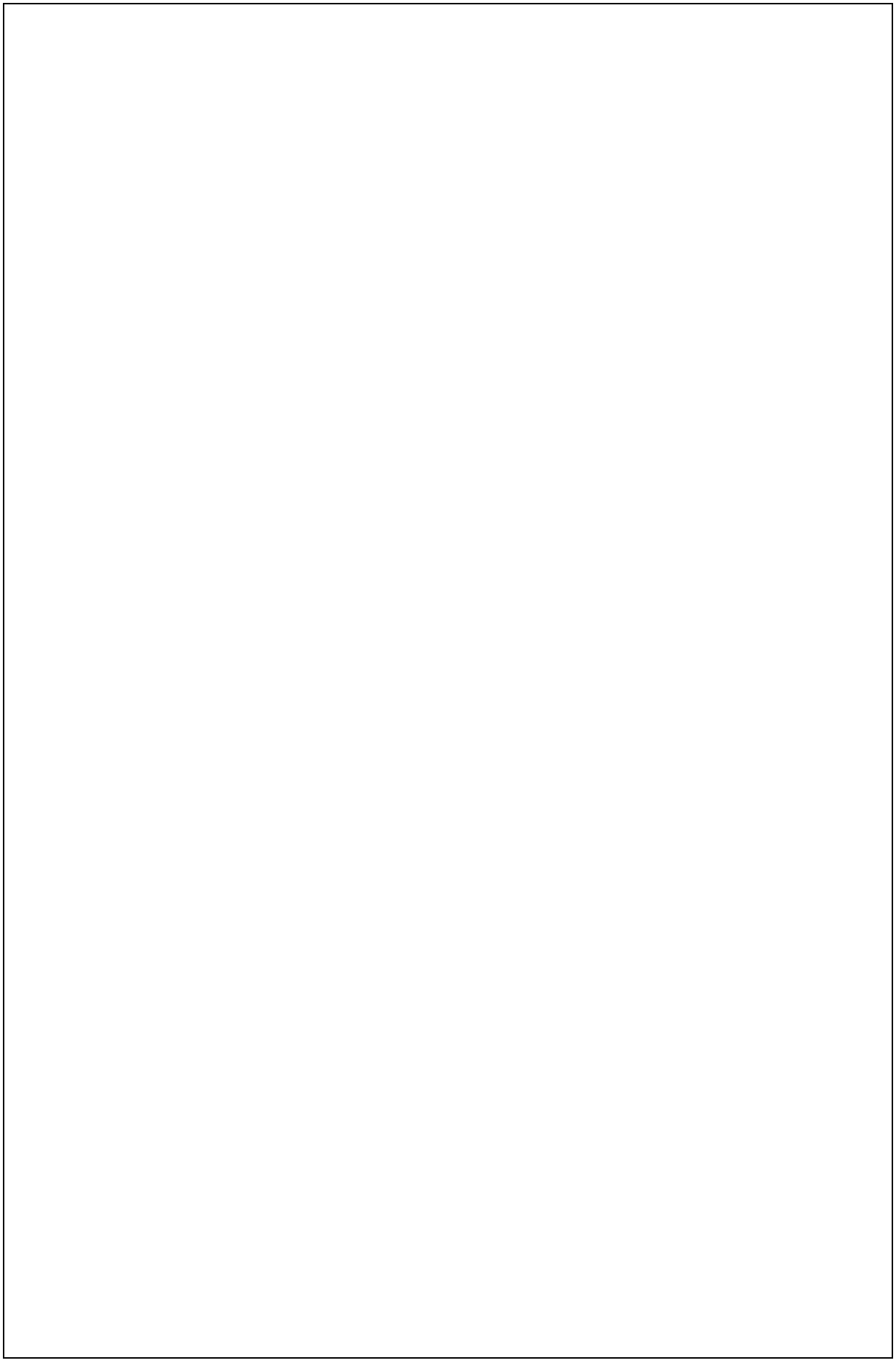
a)	$xy^2 - x^2y$	**j)**	$9a^2b - 27a^2b^2 + 15b^2$	**s)**	$36u^2v^3 + 60uv^2 - 24u^3v^4$
b)	$x^4 + 5x^2 + x^3y$	**k)**	$6x^3y^5 + 4x^4y^3 - 24xy$	**t)**	$8x^5y^3 + 6x^4y^4 - 4x^2y^5$
c)	$a^2b^3 + a^3b^5$	**l)**	$ab - a - b + 1$	**u)**	$2(x-3) - y(x-3)$
d)	$12x^2y^8z^6 - 18xyz^4$	**m)**	$3f + 3 + fg + g$	**v)**	$21a^3b^2 - 14a^4b - 7a^3b^2$
e)	$2p^2r^2s + 8p^5r^7 - 4p^9$	**n)**	$6x^3 + x^2 - 24x - 4$	**w)**	$y(3-z) + x(z-3)$
f)	$15a^4b^2 - 10a^4b + 5a^5b^3$	**o)**	$x(a-1) + a - 1$	**x)**	$6r_1^2r_2^2r_3^2 - 8r_1^3r_2^3 + 4r_1^4r_3^4$
g)	$a^3b^4c^5d^6 - a^6b^5c^4d^3$	**p)**	$k^4 - 4k^3 - k + 4$	**y)**	$5px - py - 5rx + ry$
h)	$s^5t^2u^4v^4 + s^2t^2v^2$	**q)**	$ax - ay - bx + by$	**z)**	$(1+a)x - y(-1-a)$
i)	$9m^2 - 12m + 6$	**r)**	$10a + 15b + 20c$	**Z)**	$x(x^2 - z^2) - z(z^2 - x^2)$

Answer

a)	$xy(y-x)$	**j)**	$3b(3a^2 - 9a^2b + 5b)$	**s)**	$12uv^2(3uv + 5 - 2u^2v^2)$
b)	$x^2(x^2 + 5 + xy)$	**k)**	$2xy(3x^2y^4 + 2x^3y^2 - 12)$	**t)**	$2x^2y^3(4x^3 + 3x^2y - 2y^2)$
c)	$a^2b^3(1 + ab^2)$	**l)**	$(a-1)(b-1)$	**u)**	$(x-3)(2-y)$
d)	$6xyz^4(2xy^7z^2 - 3)$	**m)**	$(f+1)(g+3)$	**v)**	$7a^3b(3b - 2a - b)$
e)	$2p^2(r^2s + 4p^3r^7 - 2p^7)$	**n)**	$(6x+1)(x^2 - 4)$	**w)**	$(3-z)(y-x)$
f)	$5a^4b(3b - 2 + ab^2)$	**o)**	$(a-1)(x+1)$	**x)**	$2r_1^2(3r_2^2r_3^2 - 4r_1r_2^3 + 2r_1^2r_3^4)$
g)	$a^3b^4c^4d^3(cd^3 - a^3b)$	**p)**	$(k-4)(k^3-1)$	**y)**	$(5x-y)(p-r)$
h)	$s^2t^2v^2(s^3u^4v^2 + 1)$	**q)**	$(x-y)(a-b)$	**z)**	$(1+a)(x+y)$
i)	$3(3m^2 - 4m + 2)$	**r)**	$5(2a + 3b + 4c)$	**Z)**	$(x^2 - z^2)(x+z)$







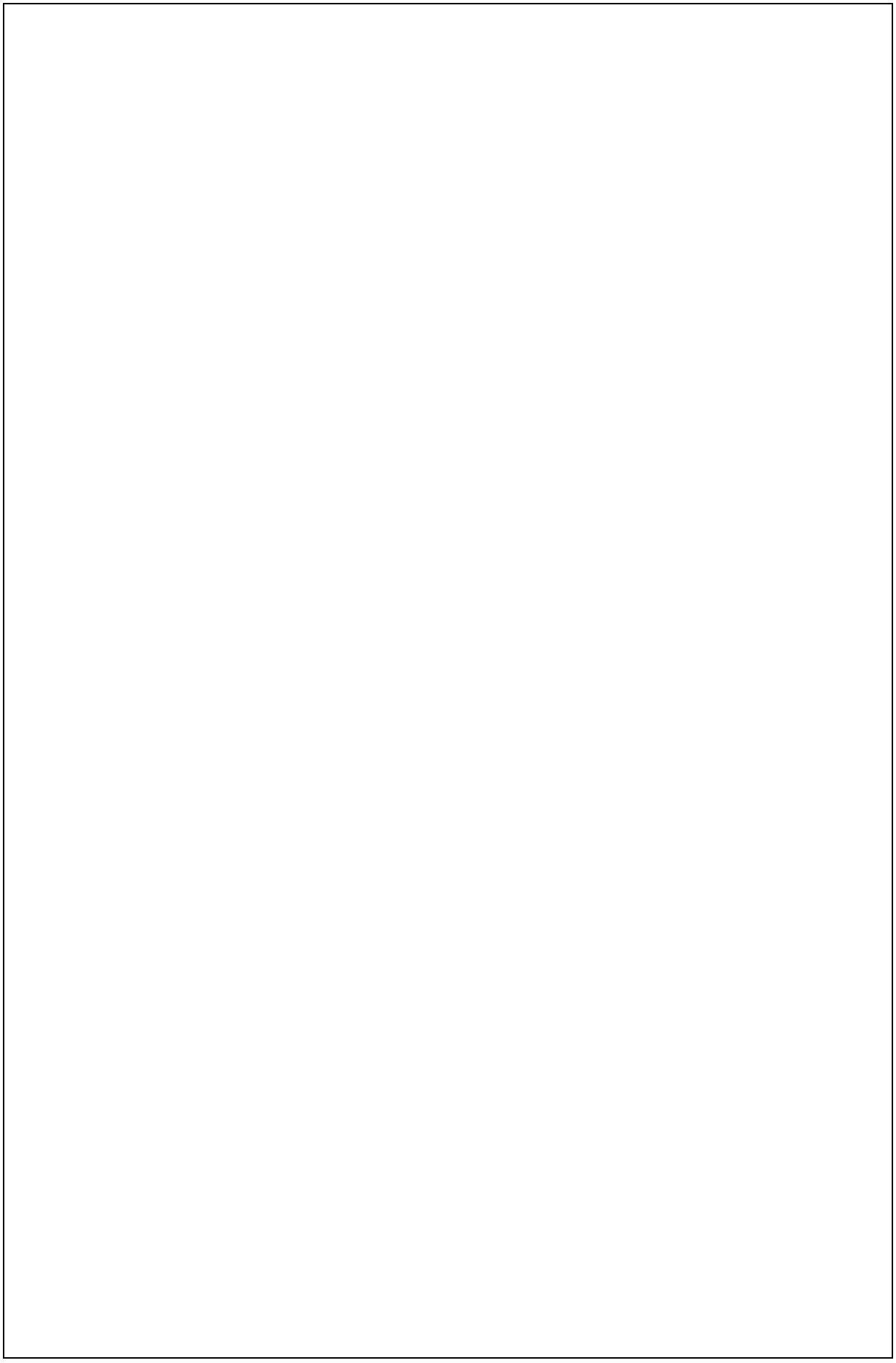
Algebraic Expansion Exercises 1

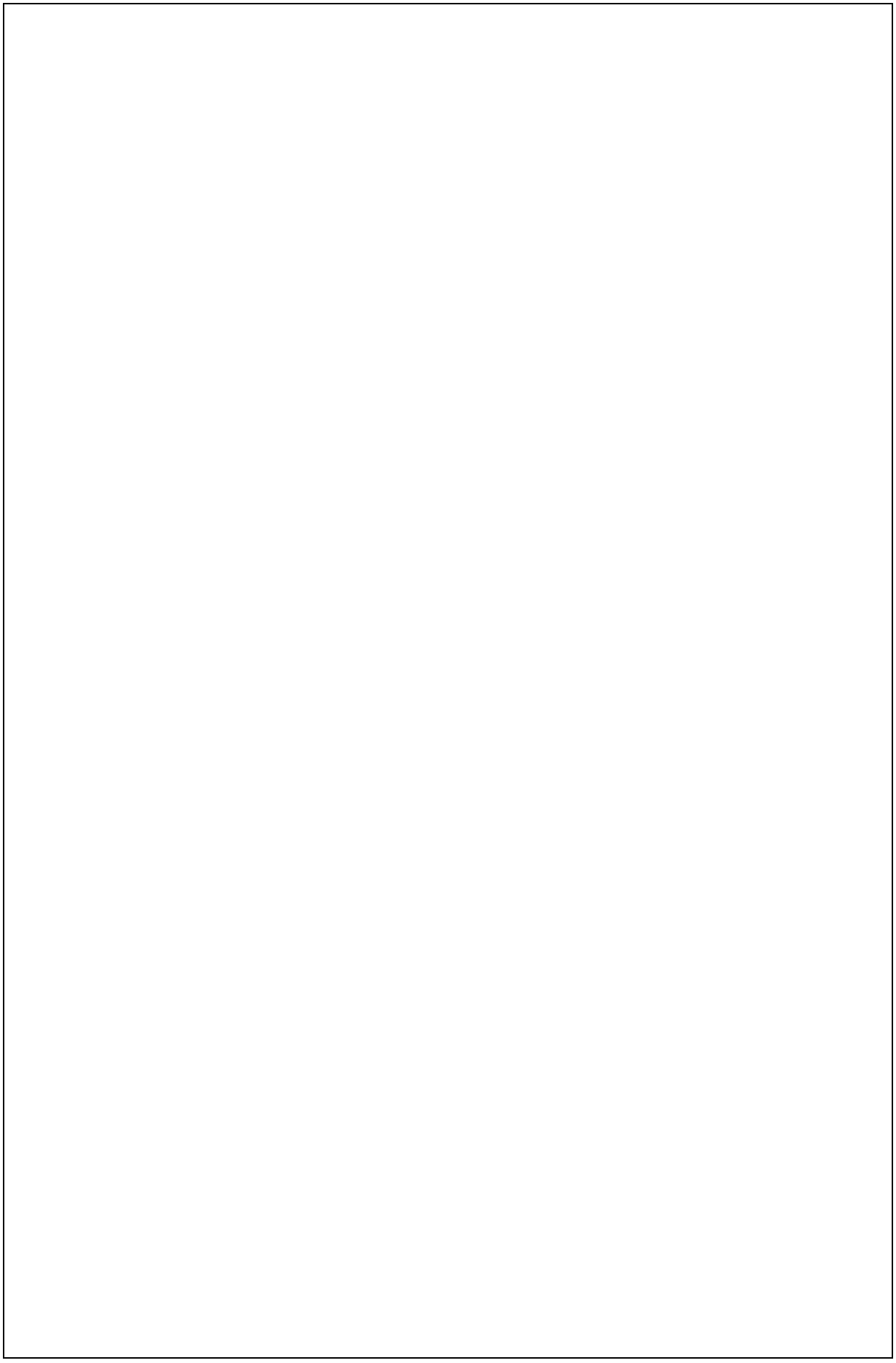
Question

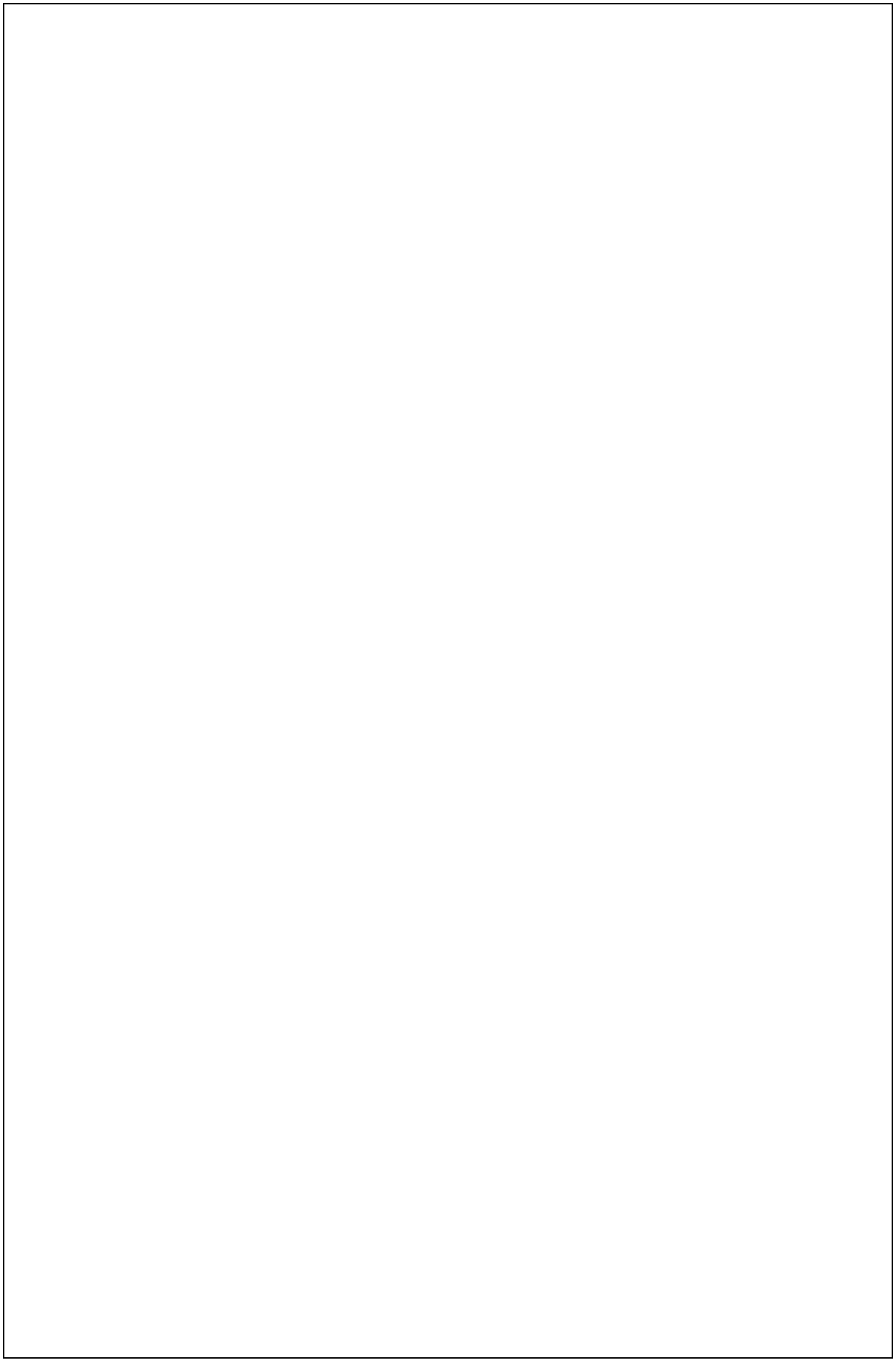
a)	$(2a+b)(3c-2d)$	**j)**	$(4x-3)(3x+2)$	**s)**	$(3x+4y)(3x-4y)$
b)	$(2a+b)(2a-b)$	**k)**	$(xy+1)(z-2)$	**t)**	$e^{-x}(1-x)$
c)	$(2x+1)^2$	**l)**	$(4+a^2)(2+a)(2-a)$	**u)**	$(x-3)(x^2+3x+9)$
d)	$-2(2x-1)(3x+5)$	**m)**	$(3a-2b)^2$	**v)**	$(x+y+1)(x+y-1)$
e)	$(x-2)(3x+2)$	**n)**	$3(x-4)(x+2)$	**w)**	$(3x+4y)(9x^2-12xy+16y^2)$
f)	$(3x-1)(x^2+1)$	**o)**	$3(2x-1)(3x+2)$	**x)**	$2ye^{xy^2}(1+xy^2)$
g)	$3(2x+y)(2x-y)$	**p)**	$2(3x-5)(2x+3)$	**y)**	$(x+y^3)(2y^3-3x)$
h)	$(3ax+1)^2$	**q)**	$2(4a^2-ab-3b^2)$	**z)**	$2(2a-b^2)(4a^2+2ab^2+b^4)$
i)	$(x-5)(x+3)$	**r)**	$(x^2+5)(x^4-5x^2+25)$	**Z)**	$x^{1/3}(3x^{1/3}-2+4x)$

Answer

a)	$6ac+3bc-4ad-2bd$	**j)**	$12x^2-x-6$	**s)**	$9x^2-16y^2$
b)	$4a^2-b^2$	**k)**	$xyz+z-2xy-2$	**t)**	$e^{-x}-xe^{-x}$
c)	$4x^2+4x+1$	**l)**	$16-a^4$	**u)**	x^3-27
d)	$10-14x-12x^2$	**m)**	$9a^2-12ab+4b^2$	**v)**	$(x+y)^2-1$
e)	$3x^2-4x-4$	**n)**	$3x^2-6x-24$	**w)**	$27x^3+64y^3$
f)	$3x^3-x^2+3x-1$	**o)**	$18x^2+3x-6$	**x)**	$2ye^{xy^2}+2xy^3e^{xy^2}$
g)	$12x^2-3y^2$	**p)**	$12x^2-2x-30$	**y)**	$2y^6-xy^3-3x^2$
h)	$9a^2x^2+6ax+1$	**q)**	$8a^2-2ab-6b^2$	**z)**	$16a^3-2b^6$
i)	$x^2-2x-15$	**r)**	x^6+125	**Z)**	$3x^{2/3}-2x^{1/3}+4x^{4/3}$







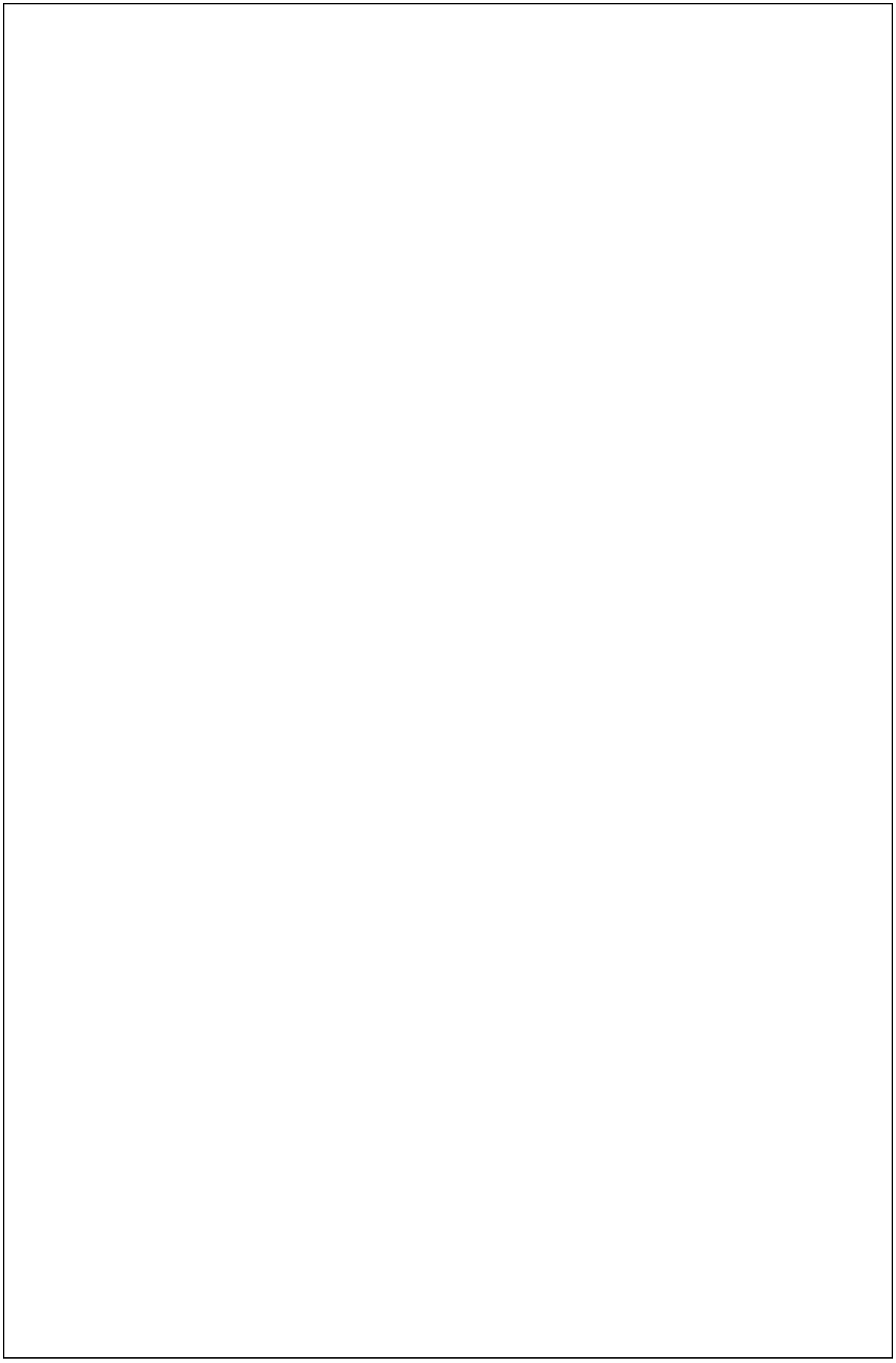
Algebraic Expansion Exercises 1

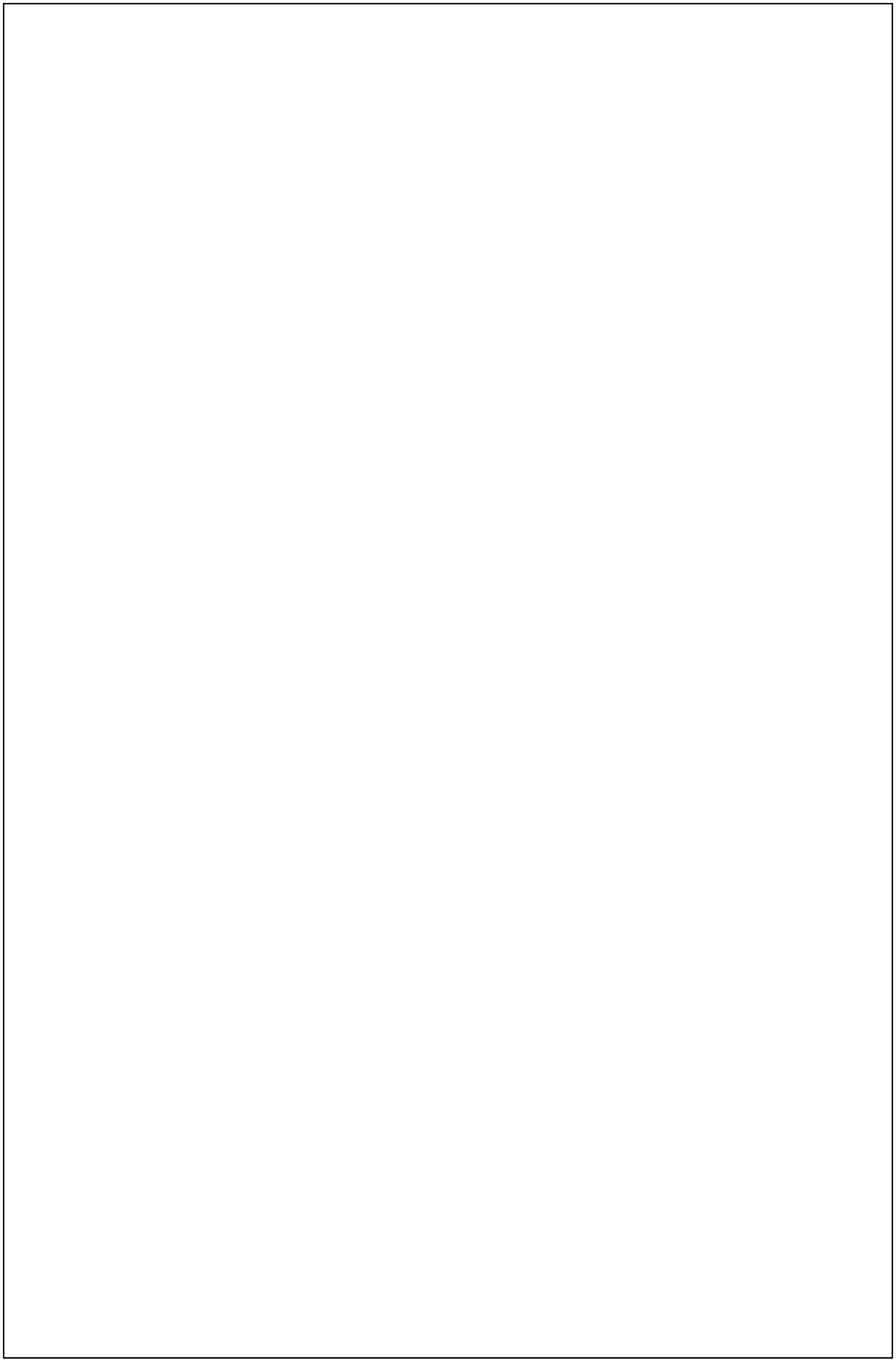
Question

a)	$(a+2b)^2$	**h)**	$(-3x-5y)^2$	**o)**	$(-s+10t)(-s-10t)$
b)	$(2x-3y)^2$	**i)**	$(7u-11v)^2$	**p)**	$(-5-3t)(-5+3t)$
c)	$(10a-9b)^2$	**j)**	$(8c-13d)(8c+13d)$	**q)**	$(12u-14v)^2$
d)	$(5m+2n)(5m-2n)$	**k)**	$(a^4-b^4)^2$	**r)**	$(3x+2y)^3$
e)	$(1-a)(1+a)$	**l)**	$(6xy^2+7x^2y)^2$	**s)**	$(5a-4b)^3$
f)	$(-a+3b)^2$	**m)**	$-(-6u-3v)(6u+3v)$	**t)**	$27a^3-8b^3$
g)	$(-x-4)^2$	**n)**	$(-uv+7a)^2$	**u)**	$64x^6+125y^9$

Answer

a)	$a^2+4ab+4b^2$	**h)**	$9x^2+30xy+25y^2$	**o)**	s^2-100t^2
b)	$4x^2-12xy+9y^2$	**i)**	$49u^2-154uv+121v^2$	**p)**	$25-9t^2$
c)	$100a^2-180ab+81b^2$	**j)**	$64c^2-169d^2$	**q)**	$144u^2-336uv+196v^2$
d)	$25m^2-4n^2$	**k)**	$a^8-2a^4b^4+b^8$	**r)**	$27x^3+54x^2y+36xy^2+8y^3$
e)	$1-a^2$	**l)**	$36x^2y^4+84x^3y^3+49x^4y^2$	**s)**	$125a^3-300a^2b+240ab^2-64b^3$
f)	$a^2-6ab+9b^2$	**m)**	$36u^2+36uv+9v^2$	**t)**	$(3a-2b)(9a^2+6ab+4b^2)$
g)	$x^2+8x+16$	**n)**	$u^2v^2-14auv+49a^2$	**u)**	$(4x^2+5y^3)(16x^4-20x^2y^3+25y^6)$





Solve each equation.

1) $-20 = -4x - 6x$ {2}

2) $6 = 1 - 2n + 5$ {0}

3) $8x - 2 = -9 + 7x$ {−7}

4) $a + 5 = -5a + 5$ {0}

5) $4m - 4 = 4m$ No solution.

6) $p - 1 = 5p + 3p - 8$ {1}

7) $5p - 14 = 8p + 4$ {−6}

8) $p - 4 = -9 + p$ No solution.

9) $-8 = -(x + 4)$ {4}

10) $12 = -4(-6x - 3)$ {0}

11) $14 = -(p - 8)$ {−6}

12) $-(7 - 4x) = 9$ {4}

13) $-18 - 6k = 6(1 + 3k)$ {−1}

14) $5n + 34 = -2(1 - 7n)$ {4}

15) $2(4x - 3) - 8 = 4 + 2x$ {3}

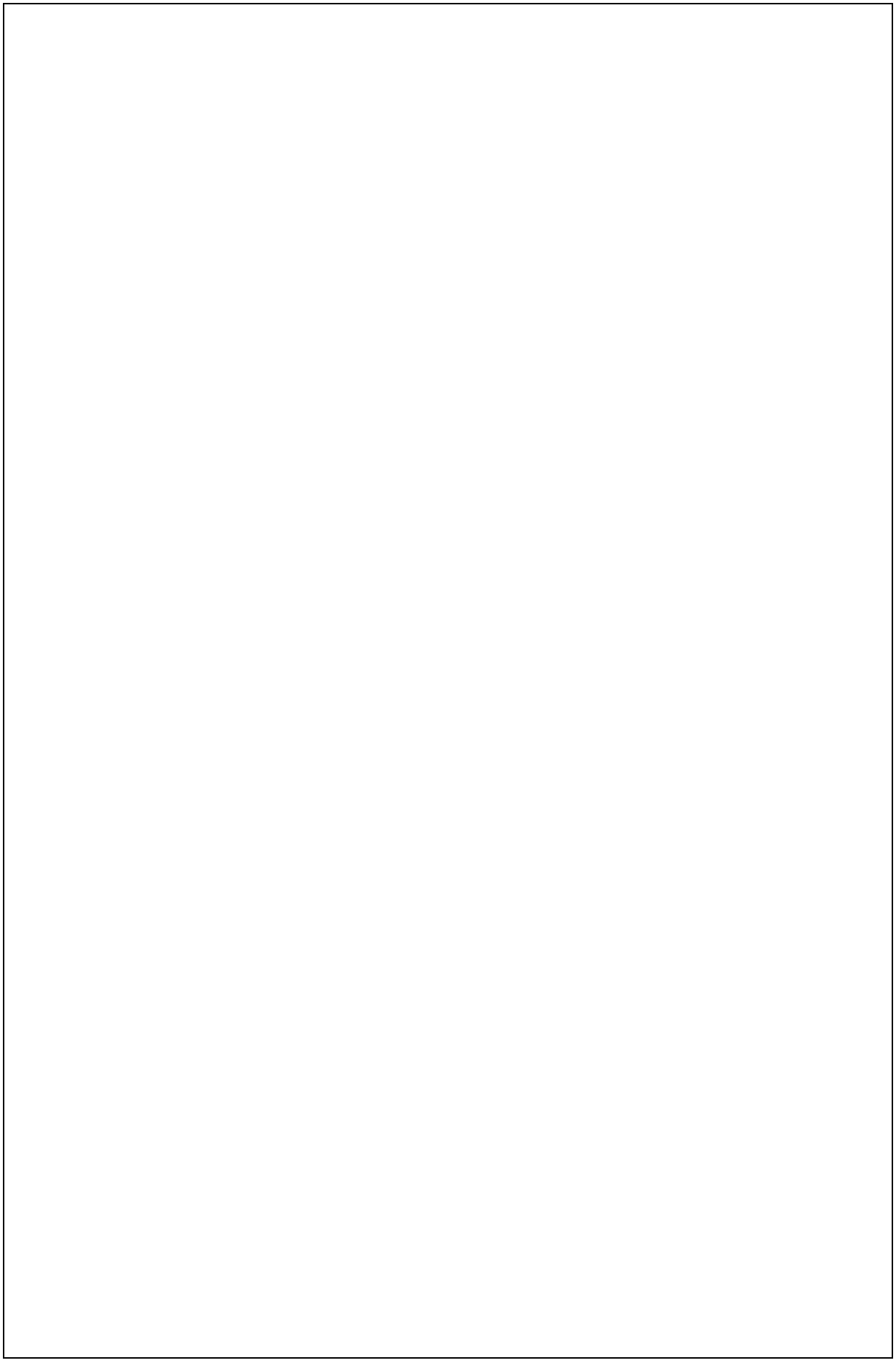
16) $3n - 5 = -8(6 + 5n)$ {−1}

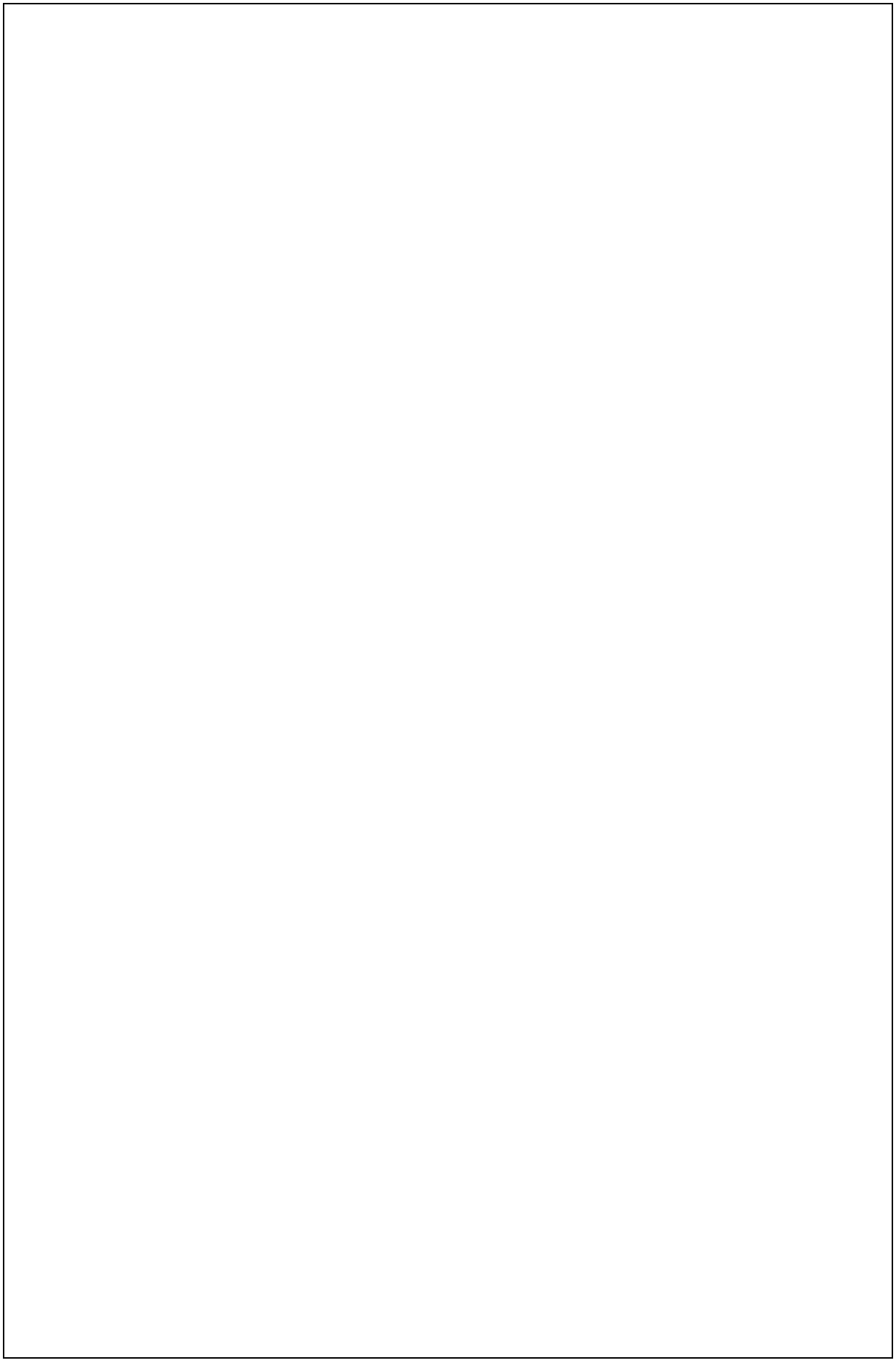
17) $-(1 + 7x) - 6(-7 - x) = 36$ {5}

18) $-3(4x + 3) + 4(6x + 1) = 43$ {4}

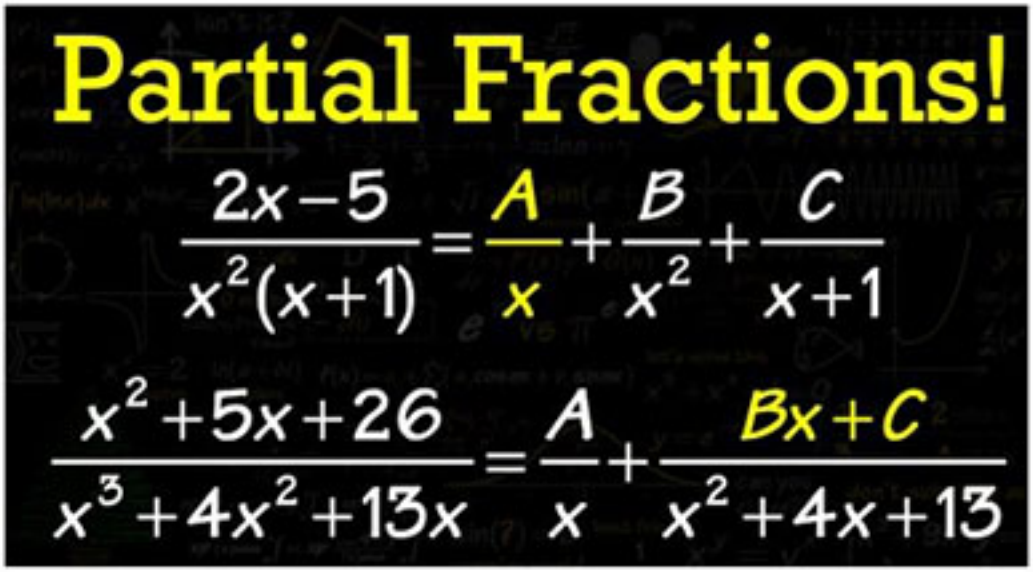
19) $24a - 22 = -4(1 - 6a)$ No solution.

20) $-5(1 - 5x) + 5(-8x - 2) = -4x - 8x$ {−5}





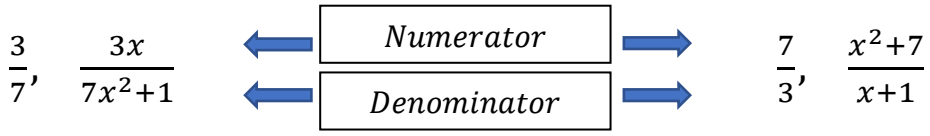
CHAPTER 2
PARTIAL FRACTION



2.0 PARTIAL FRACTION

2.1 DEFINE PROPER AND IMPROPER FRACTION

Proper Fraction	Improper Fraction
Numerator should be less than denominator	Numerator should be greater than or equal to denominator



An algebraic fraction can be broken down (decomposition) into multiple simpler fractions called Partial Fractions. It is the reverse process of combining two fractions into a single fraction. There are three (3) types of Partial Fractions :

1. Proper fraction with linear factors
2. Proper fraction with repeated linear factors
3. Proper fraction with quadratic factors

2.2 CONSTRUCT PARTIAL FRACTION USING PROPER FRACTION

a. Linear Factor

Solve the following partial fractions.	
Question	Solution
$\frac{2x+3}{(x+1)(3x-2)}$	$\frac{2x+3}{(x+1)(3x-2)} = \frac{A}{(x+1)} + \frac{B}{(3x-2)}$ $2x+3 = \left[\frac{A}{(x+1)} + \frac{B}{(3x-2)}\right](x+1)(3x-2)$ $2x+3 = \frac{A(x+1)(3x-2)}{(x+1)} + \frac{B(x+1)(3x-2)}{(3x-2)}$ $2x+3 = A(3x-2) + B(x+1)$ .....Equation 1 When $x+1=0$ $x = -1$ Substitute $x=-1$ into equation 1 $2(-1)+3 = A(3(-1)-2) + B((-1)+1)$

	$1 = -5A$ $A = \frac{-1}{5}$ When $3x - 2 = 0$ $x = \frac{2}{3}$ Substitute $x = \frac{2}{3}$ into equation 1 $2\left(\frac{2}{3}\right) + 3 = A\left(3\left(\frac{2}{3}\right) - 2\right) + B((\frac{2}{3}) + 1)$ $\frac{13}{3} = \frac{5}{3}B$ $\frac{5}{3}B = \frac{13}{3}$ $B = \frac{13}{3} \times \frac{3}{5}$ $B = \frac{13}{5}$ Therefore : $\frac{2x+3}{(x+1)(3x-2)} = \frac{-1}{5(x+1)} + \frac{13}{5(3x-2)}$
$\frac{7x-1}{x(2x-1)(x+3)}$	$\frac{7x-1}{x(2x-1)(x+3)} = \frac{A}{x} + \frac{B}{(2x-1)} + \frac{C}{(x+3)}$ $7x - 1 = \left[\frac{A}{x} + \frac{B}{(2x-1)} + \frac{C}{(x+3)}\right] x(2x-1)(x+3)$ $7x - 1 = \frac{Ax(2x-1)(x+3)}{x} + \frac{Bx(2x-1)(x+3)}{(2x-1)} + \frac{Cx(2x-1)(x+3)}{(x+3)}$ $7x - 1 = A(2x-1)(x+3) + Bx(x+3) + Cx(2x-1)$ …….Equation 1 When $x = 0$ Substitute $x = 0$ into equation 1 $7(0) - 1 = A[(2(0) - 1)(0 + 3)] + B(0)(0 + 3) + C(0)(2(0) - 1)$ $-1 = -3A$ $A = \frac{1}{3}$ When $2x - 1 = 0$ $x = \frac{1}{2}$ Substitute $x = \frac{1}{2}$ into equation 1 $7(\frac{1}{2}) - 1 = A\left[(2\left(\frac{1}{2}\right) - 1)\left(\frac{1}{2} + 3\right)\right] + B(\frac{1}{2})\left(\frac{1}{2} + 3\right) + C(\frac{1}{2})\left(2(\frac{1}{2}) - 1\right)$ $\frac{5}{2} = \frac{7}{4}B$

	$B = \frac{10}{7}$ When $x + 3 = 0$ $x = -3$ Substitute $x = -3$ into equation 1 $7(-3) - 1 = A[(2(-3) - 1)(-3 + 3)] + B(-3)(-3 + 3) + C(-3)(2(-3) - 1$ $-22 = 21C$ $C = -\frac{22}{21}$ Therefore : $\frac{7x - 1}{x(2x - 1)(x + 3)} = \frac{1}{3x} + \frac{10}{7(2x - 1)} - \frac{22}{21(x + 3)}$

PRACTICAL EXERCISE

Solve the following partial fractions.

$\frac{x+5}{(x+1)(x+2)}$	

$\frac{4-3x}{(3-x)(2-3x)}$	
$\frac{7x-1}{(x-2)(x-4)}$	

b. Repeated Linear Factors

Solve the following partial fractions.

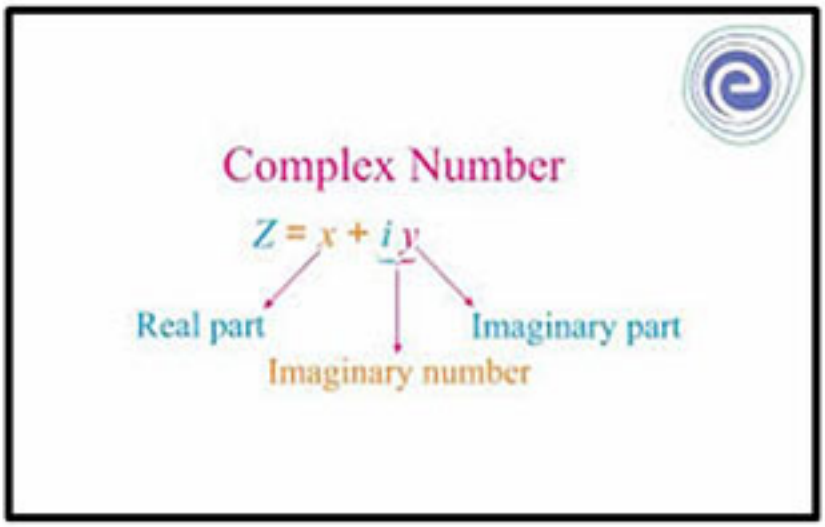
Question	Solution
$\frac{3x^2-5x-5}{(x+2)(x+1)^2}$	$\frac{3x^2-5x-5}{(x+2)(x+1)^2} = \frac{A}{(x+2)} + \frac{B}{(x+1)} + \frac{C}{(x+1)^2}$ $3x^2 - 5x - 5 = \left[\frac{A}{(x+2)} + \frac{B}{(x+1)} + \frac{C}{(x+1)^2}\right](x+2)(x+1)^2$ $3x^2 - 5x - 5 = \left[\frac{A(x+2)(x+1)^2}{(x+2)} + \frac{B(x+2)(x+1)^2}{(x+1)} + \frac{C(x+2)(x+1)^2}{(x+1)^2}\right]$ $3x^2 - 5x - 5 = A(x+1)^2 + B(x+2)(x+1) + C(x+2)$ .....equation 1 When $x + 2 = 0$ $x = -2$ $3(-2)^2 - 5(-2) - 5 = A(-2+1)^2 + B(-2+2)(-2+1) + C(-2+2)$ $17 = A$ When $x + 1 = 0$ $x = -1$ $3(-1)^2 - 5(-1) - 5 = A(-1+1)^2 + B(-1+2)(-1+1) + C(-1+2)$ $3 = C$ To find B, we need to expand equation 1 $3x^2 - 5x - 5 = A(x+1)^2 + B(x+2)(x+1) + C(x+2)$ $3x^2 - 5x - 5 = A(x^2 + 2x + 1) + B(x^2 + 3x + 2) + C(x+2)$ $3x^2 - 5x - 5 = Ax^2 + 2Ax + 1A + Bx^2 + 3Bx + 2B + Cx + 2C$ $= Ax^2 + Bx^2 + 2Ax + 3Bx + Cx + 1A + 2B + 2C$ Constant B (has x^2, x and x^0) Compare coefficient of x^2 $3x^2 = Ax^2 + Bx^2$ $3 = A + B$ $3 = 17 + B$ $B = -14$ Therefore : $\frac{3x^2 - 5x - 5}{(x+2)(x+1)^2} = \frac{17}{(x+2)} - \frac{14}{(x+1)} + \frac{3}{(x+1)^2}$

PRACTICAL EXERCISE

Solve the following partial fractions.

$\frac{x-2}{(4x+3)x^2}$	$\frac{-3x}{(x+3)^2}$

CHAPTER 3
COMPLEX NUMBER



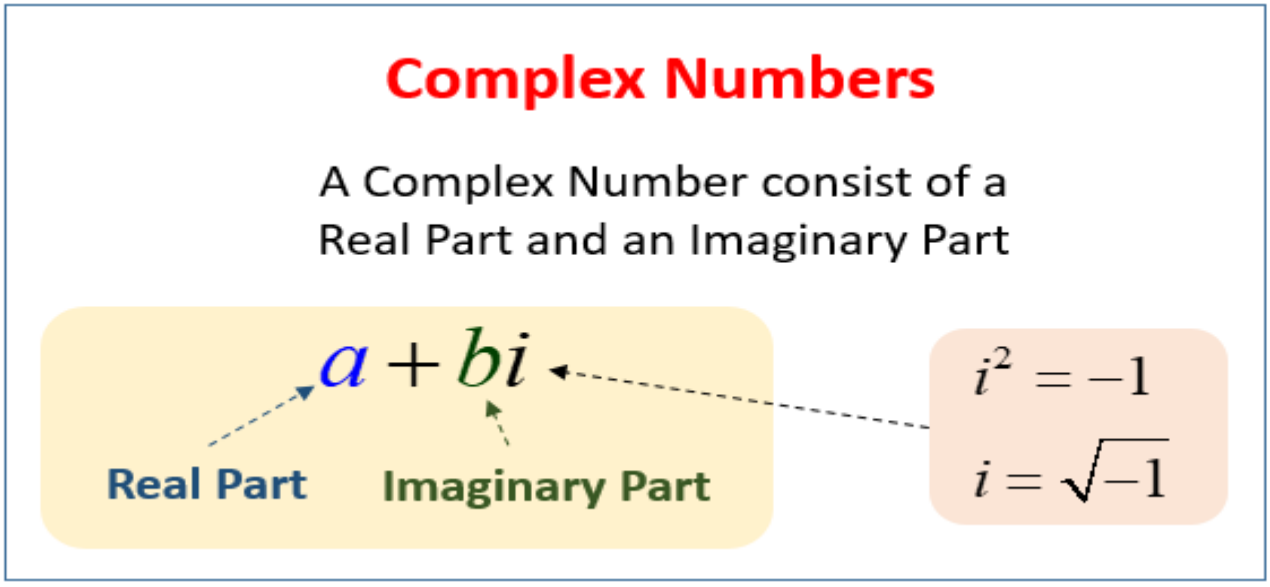
3.0 COMPLEX NUMBER

3.1 EXPLAIN THE CONCEPT OF A COMPLEX NUMBER

Complex numbers are numbers that consist of two parts — a real number and an imaginary number. Complex numbers are the building blocks of more intricate math, such as algebra. They can be applied to many aspects of real life, especiall]y in electronics and electromagnetism.

3.1.1 DESCRIBE BETWEEN REAL PARTS AND IMAGINARY PARTS

A complex Number is any number in the form $a + bi$ where *a* and *b* are any real numbers. *a* is real part and *bi* is imaginary part. The number i is defined by $\sqrt{-1}$.



(Image from https://images.app.goo.gl/ptozdeMfGK7MtX8R6)

3.1.2 RECOGNIZE THAT $i = \sqrt{-1}$

$$i = \sqrt{-1}$$
$$i^2 = -1$$

EXAMPLE

Express in terms of i.	
Question	Solution
a) $\sqrt{-8}$	$\sqrt{-8} = \sqrt{-1(8)}$ $= \sqrt{-1}\sqrt{8} = \sqrt{8}i$
b) $\sqrt{-25}$	$\sqrt{-25} = \sqrt{-1(25)}$ $= \sqrt{-1}\sqrt{25} = 5i$
c) $2 - \sqrt{-31}$	$2 - \sqrt{-31} = 2 - \sqrt{-1(31)}$ $= 2 - \sqrt{-1}\sqrt{31} = 2 - \sqrt{31}i$
d) i^8	i^8 $= (i^2)^4 = (-1)^4$ $= 1$
e) i^{15}	i^{15} $= i(i^{14}) = i(i^2)^7$ $= i(-1)^7$ $= -i$
f) $i^{17} - 32i^4$	$i^{17} - 32i^4$ $= i(i^{16}) - 32(i^4)$ $= i(i^2)^8 - 32(i^2)^2$ $= i(-1)^8 - 32(-1)^2$ $= i(1) - 32(1)$ $= i - 32$

PRACTICAL EXERCISE

Express in terms of i.	
Question	Solution
a) $\sqrt{-36}$	Ans : $6i$
b) $-\sqrt{-5}$	Ans : $-\sqrt{5}i$
c) $-\sqrt{-25}-\sqrt{-16}$	Ans : $-9i$
d) i^{22}	Ans :-1
e) i^{53}	Ans : i
f) $i^{39}-23i$	Ans : $-24i$
g) $-\sqrt{-25}+25i$	Ans : $20i$

3.2 DEMONSTRATE THE OPERATION OF COMPLEX NUMBERS

3.2.1 CALCULATE THE OPERATION OF COMPLEX NUMBERS

Additional and Subtraction of Complex Number

We can add and subtract the complex numbers by collecting like terms.

$$(a + bi) \pm (x + yi) = (a \pm x) + (b \pm y)i$$

EXAMPLE

Solve the problem below	
Question	Solution
a) $(3 + 4i) + (5 + 2i)$	$(3 + 5) + (4 + 2)i = 8 + 6i$ Other solution $\begin{array}{r\|l} 3 & 4i \\ +5 & 2i \\ \hline 8 & 6i \end{array}$
b) $(6 + i) + (5 - 8i)$	$(6 + 5) + [1 + (-8)]i = 11 - 7i$ Other solution $\begin{array}{r\|l} 6 & 1i \\ +5 & -8i \\ \hline 11 & 7i \end{array}$
c) $(5 - 3i) - (-5 - 10i)$	$[5 - (-5)] + [-3 - (-10)]i = 10 + 7i$ Other solution $\begin{array}{r\|l} 5 & -3i \\ -(-5) & -10i \\ \hline 10 & 7i \end{array}$

PRACTICAL EXERCISE

Solve the problem below	
Question	Solution
a) $(3+2i)+(1+5i)$	Ans : $4+7i$
b) $(7-2i)+(3+5i)$	Ans: $10+3i$
c) $(3+7i)+(-2-i)$	Ans: $1+6i$
d) $(-9-4i)-(-6-8i)$	Ans: $-3+4i$
e) $(-3-5i)-(2+3i)$	Ans: $-5-8i$
f) $(2-i)+(-3+2i)—5+3i$	Ans: $-6+4i$

Multiplication of Complex Number

We multiply square roots of negative real numbers by expressing them in terms of i



$$i = \sqrt{-1}$$
$$i^2 = -1$$

EXAMPLE

Multiply the following :	
Question	Solution
a) $-2i(6-5i)$	$= -12i + 10i^2$ $= -12i + 10(-1)$ $= -12i - 10$ $= -10 - 12i$
b) $(1-3i)(-4+2i)$	$= -4 + 2i + 12i - 6i^2$ $= -4 + 2i + 12i - 6(-1)$ $= -4 + 2i + 12i + 6$ $= 2 + 14i$
c) $(5+3i)^2$	$= (5+3i)(5+3i)$ $= 25 + 15i + 15i + 9i^2$ $= 25 + 15i + 15i + 9(-1)$ $= 16 + 30i$

PRACTICAL EXERCISE

Multiply the following :	
Question	Solution
a) $2i(4+3i)$	Ans : $-6+8i$
b) $(3+i)(2+3i)$	Ans: $3+11i$
c) $(-5+3i)(-1-2i)$	Ans: $11+7i$
d) $(3-4i)^2$	Ans : $-7-24i$
e) $(-8+5i)^2$	Ans : $39-80i$

Conjugate and Division of Complex Number

Conjugate

You find the complex conjugate simply **by changing the sign of the imaginary part of the complex number**. The notation for the complex conjugate of z is either $\bar{z}$ or z^*. Example:

Complex Number	Complex Conjugate
$2+i$	$2-i$
$-3+7i$	$-3-7i$
$6-4i$	$6+4i$

(Image from https://images.app.goo.gl/97WjkLq8cQcugweD9)

Division

$$\frac{a+bi}{c+di} = \frac{(a+bi)(c-di)}{(c+di)(c-di)}$$

$$= \frac{(ac+bd)+(bc-ad)i}{c^2+d^2}$$

$$= \frac{ac+bd}{c^2+d^2} + \frac{bc-ad}{c^2+d^2}i.$$

Multiply with conjugate of denominator to get a real number

(Image from https://images.app.goo.gl/Rjau5TercooF5NL9A)

EXAMPLE

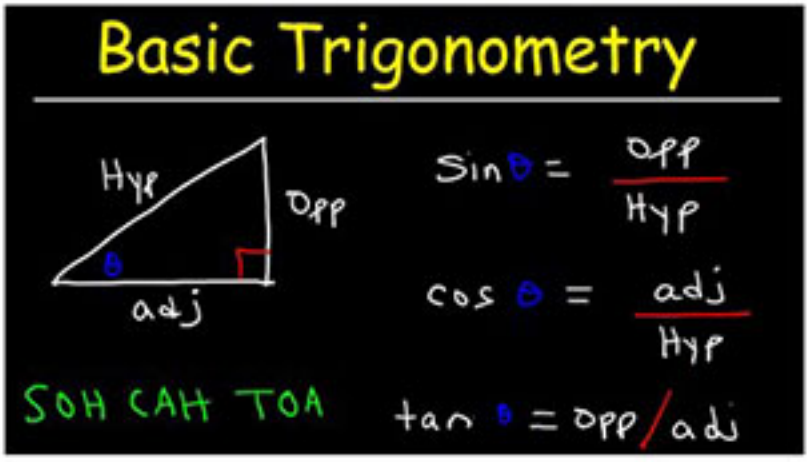
Divide each of the following to the form $a + bi$	
Question	Solution
a) $\frac{1}{3-4i}$	$= \frac{1}{3-4i} \times \frac{3+4i}{3+4i}$ $= \frac{3+4i}{9+12i-12i-16i^2}$ $= \frac{3+4i}{9-16(-1)}$ $= \frac{3+4i}{9+16}$ $= \frac{3+4i}{25}$ $= \frac{3}{25} + \frac{4}{25}i$
b) $\frac{3+5i}{1-2i}$	$= \frac{3+5i}{1-2i} \times \frac{1+2i}{1+2i}$ $= \frac{3+6i+5i+10i^2}{1+2i-2i-4i^2}$ $= \frac{3+11i+10(-1)}{1-4(-1)}$ $= \frac{3+11i-10}{1+4}$ $= \frac{-7+11i}{5}$ $= -\frac{7}{5} + \frac{11}{5}i$

PRACTICAL EXERCISE

Divide and simplify to the form $a + bi$	
Question	Solution
a) $\frac{5+3i}{3+i}$	Ans : $^{9}/_{5} + {}^{2}/_{5}\, i$
b) $\frac{3+4i}{2-i}$	Ans : $^{2}/_{5} + {}^{11}/_{5}\, i$
c) $\frac{6-2i}{3i}$	Ans : $-{}^{2}/_{3} - 2i$

d) $\frac{6i}{3-5i}$	Ans : $-{}^{15}/_{17} + {}^{9}/_{17}\,i$
e) $\frac{8}{3+2i}$	Ans : ${}^{24}/_{13} - {}^{16}/_{13}\,i$

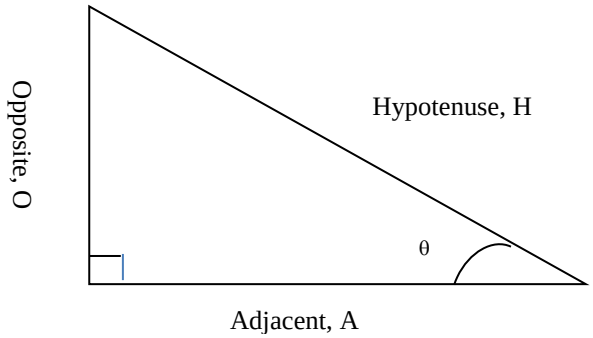
CHAPTER 4
TRIGONOMETRY



4.0 TRIGONOMETRY

- A type of mathematics that deals with the relationship between the angle and sides of triangle used in measuring the height of building, mountains, etc.
- Trigonometry concerns the functions of angles, such as sine, cosine, and tangent

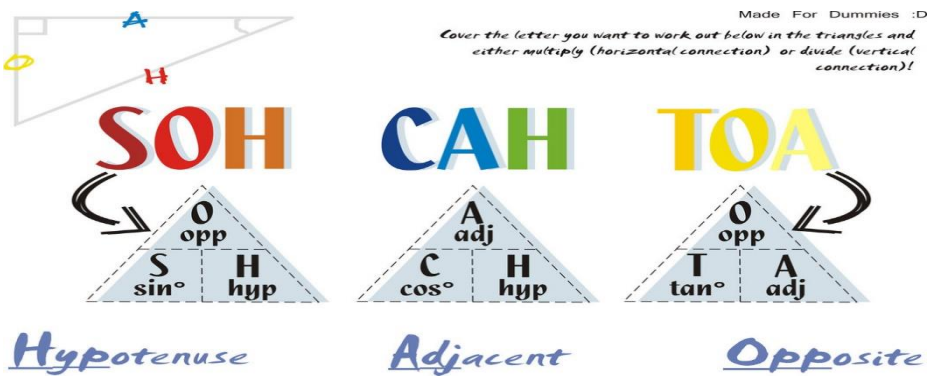
4.1 Define Sine, Cosine, Tangent, Secant, Cosecant and Cotangent



$$sin\ \theta = \frac{opposite}{hypotenuse} \qquad cosecant\ \theta = \frac{1}{sin\theta}$$

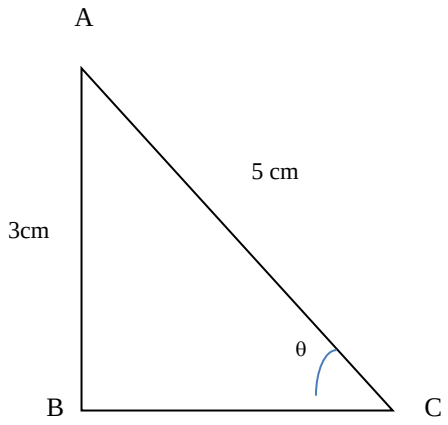
$$\cos\theta = \frac{adjacent}{hypotenuse} \qquad secant\ \theta = \frac{1}{\cos\theta}$$

$$tan\theta = \frac{opposite}{adjacent} = \frac{sin\theta}{cos\theta} \qquad cotangent\ \theta = \frac{1}{tan\theta}$$



(mage from https://fun-practice-test.blogspot.com/2020/10/project-on-trigonometry.html)

EXAMPLE

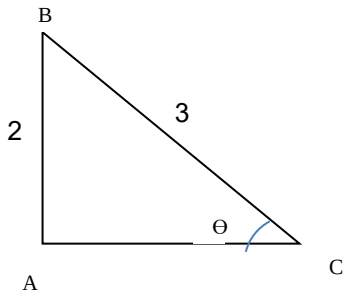
Question	Solution
In the diagram, find $sin\theta, cos\theta, \tan\theta, cotangent\ \theta, secant\ \theta$  	**Step 1: Find the adjacent by using Pythagoras' Theorem.** $adjacent = \sqrt{5^2 - 3^2}$ $adjacent = 4\ cm$ a) $\sin\theta = \frac{opp}{hypo}$ $sin\theta = \frac{3}{5}$ b) $\cos\theta = \frac{adj}{hypo}$ $cos\ \theta = \frac{4}{5}$ c) $tan\theta = \frac{3}{4}$ d) $cot\theta = \frac{1}{tan\theta} = \frac{1}{3/4} = \frac{4}{3}$ e) $sec\theta = \frac{1}{cos\theta} = \frac{1}{4/5} = \frac{5}{4}$ f) $cosec\theta = \frac{1}{sin\theta} = \frac{1}{3/5} = \frac{5}{3}$

PRACTICAL EXERCISE

i. Given $cos\theta = \frac{3}{5}$. Without using the calculator, find the values of the following:

Question	Solution
a) $sin\theta$	Ans : $^4/_5$
b) $\tan\theta$	Ans : $^4/_3$
c) $sec\theta$	Ans : $^5/_3$
d) $cot\theta$	Ans : $^3/_4$

PRACTICAL EXERCISE

Question	Solution
i. Calculate the sine and cosecant of the angle θ 	Ans : $sin\theta = {}^2/_3, cosec\theta = {}^3/_2$

ii. Given that a right-angled triangle ABC has $AC = 12cm$ and $BC = 10cm$. Calculate : a) The angle ABC b) The hypotenuse	Ans : $\measuredangle ABC = 50.19^{\circ}$,hypo = 15.62cm

EXAMPLE

Find the value of each of the following using your calculator. Give the answer correct to 4 decim places.

Question	Solution
a) $sin\ 16°$	0.2756
b) $cosec220°$	$cosec220° = \frac{1}{sin220}$ $\frac{1}{-0.6458} = -1.5557$
c) $\cot 75°21'$	$\cot 75°21' = \frac{1}{\tan 75°21'}$ $\frac{1}{3.8254} = 0.2614$

PRACTICAL EXERCISE

Find the value of each of the following using your calculator. Give the answer correct to 4 decimal places.

Question	Solution
a) $\sin 10°$	
b) $\cos 125°$	
c) $\tan 315°$	
d) $cosec\ 12.8°$	
e) $\sec 9°29'$	
f) $\cot 112°$	

4.1.1 INVERSE TRIGONOMETRIC

EXAMPLE

Find the angle, θ each of the following equation.

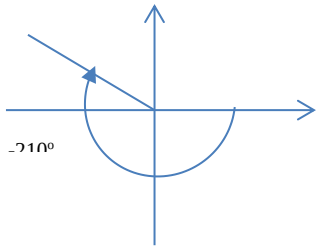
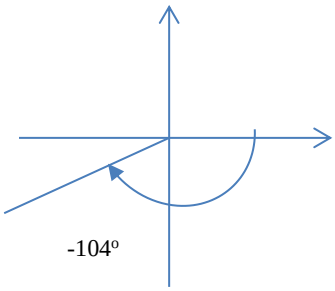
Question	Solution
a) $\sin\theta = 0.7071$	$\sin\theta = 0.7071$ $\theta = sin^{-1}0.7071$ $\theta = 45°$
b) $\cos\theta = 0.5150$	$\cos\theta = 0.5150$ $\theta = cos^{-1}0.5150$ $\theta = 59°$
c) $\cot\theta = 4.2670$	$\cot\theta = 4.2670$ $\frac{1}{\tan\theta} = 4.2670$ $\tan\theta = \frac{1}{4.2670}$ $\theta = tan^{-1}\frac{1}{4.2670}$ $\theta = 13.19°$

PRACTICAL EXERCISE

Find the angle, θ each of the following equation.	
Question	Solution
a) $sin\theta = 0.4226$	Ans :24.999°
b) $tan\theta = 0.8640$	Ans: 40.827°
c) $cosec\theta = 1.0642$	Ans : 69.997°
d) $sec\theta = 5.0856$	Ans :78.66°

4.2 TRIGONOMETRIC FUNCTIONS OF NEGATIVE VALUE

EXAMPLE

Find the value of the following trigonometric functions:	
Question	Solution
a) $sin(-210°)$	 $sin(-210°) = sin(210 - 180)$ $sin(-210°) = \sin 30°$ $sin(-210°) = 0.50$
b) $cot(-104°)$	 $cot(-104°) = \frac{1}{tan(-104°)}$ $cot(-104°) = \frac{1}{tan(180° - 104°)}$ $cot(-104°) = \frac{1}{\tan 76}$ $cot(-104°) = 0.249$

PRACTICAL EXERCISE

Find the value of the following trigonometric functions:	
Question	Solution
a) $cos(-246°)$	
b) $sec(-123°)$	

References

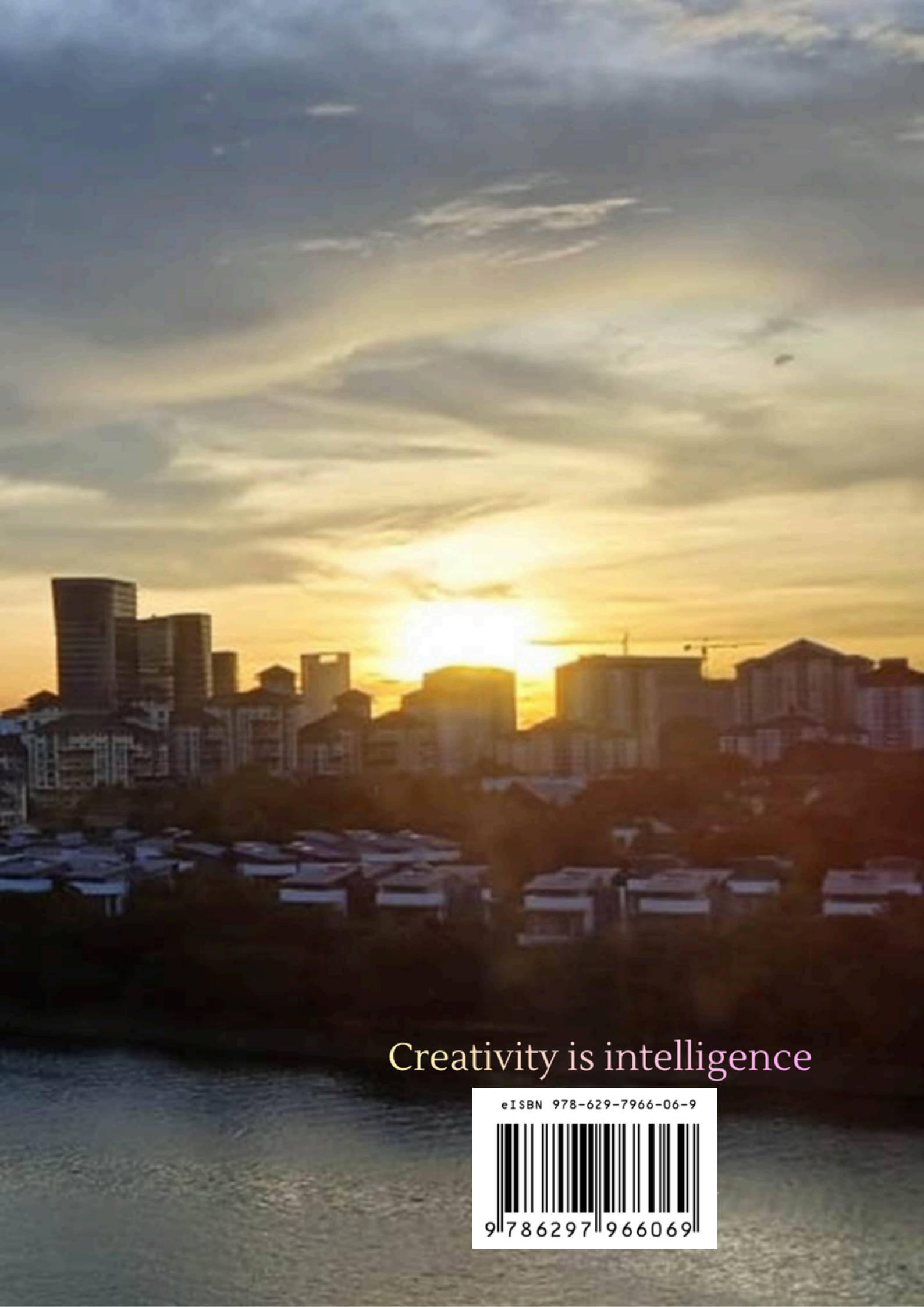
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